Computer algebra independent integration tests

4-Trig-functions/4.1-Sine/4.1.7-d-trig-^m-a+b-c-sin-^n-^p

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [594]. This is test number [79].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 99.33 (590)	% 0.67 (4)
Mathematica	% 98.15 (583)	% 1.85 (11)
Maple	% 87.71 (521)	% 12.29 (73)
Maxima	% 29.29 (174)	% 70.71 (420)
Fricas	% 66.16 (393)	% 33.84 (201)
Sympy	% 10.27 (61)	% 89.73 (533)
Giac	% 48.82 (290)	% 51.18 (304)

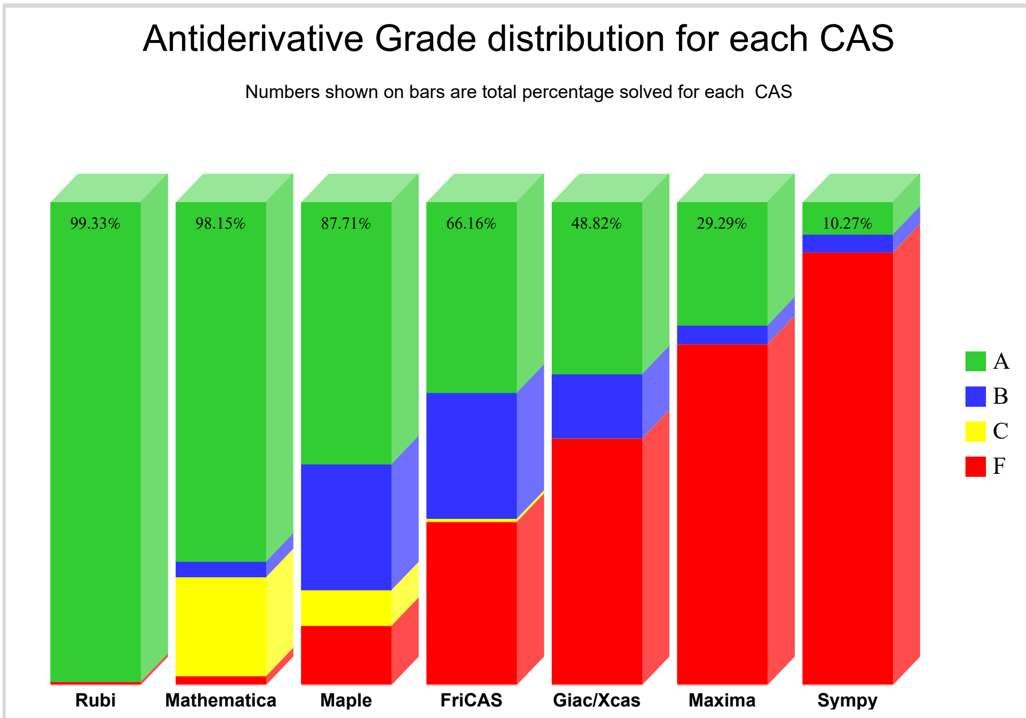
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

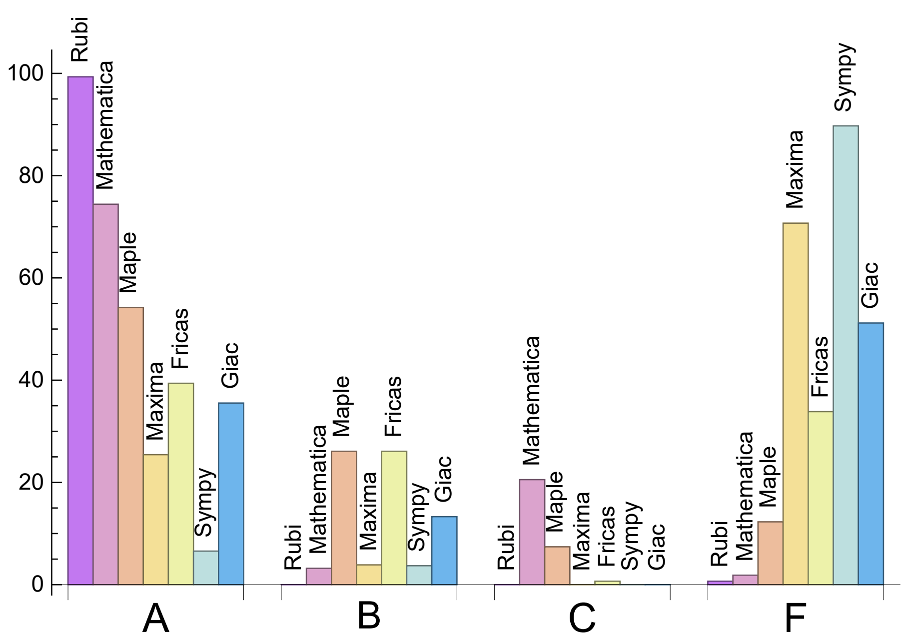
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	99.33	0.	0.	0.67
Mathematica	74.41	3.2	20.54	1.85
Maple	54.21	26.09	7.41	12.29
Maxima	25.42	3.87	0.	70.71
Fricas	39.39	26.09	0.67	33.84
Sympy	6.57	3.7	0.	89.73
Giac	35.52	13.3	0.	51.18

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.16	127.29	0.95	93.5	1.
Mathematica	1.59	1005.86	3.09	98.	0.96
Maple	0.84	8556.94	107.09	120.	1.32
Maxima	1.05	160.5	2.44	50.	1.28
Fricas	3.4	1491.96	9.65	420.	6.71
Sympy	26.11	289.03	7.13	156.	3.17
Giac	1.31	142.89	1.86	89.	1.52

1.4 list of integrals that has no closed form antiderivative

{399, 400, 401, 402, 403, 419, 425, 426, 427, 428, 429, 430, 434, 435, 436, 437, 438, 439, 440, 564, 569, 570, 571, 572, 573, 577, 578, 581, 582, 583, 586, 587, 588, 589, 590}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {399, 400, 401, 402, 403}

Maple {399, 400, 401, 402, 403}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {132, 162, 171, 172, 173, 179, 188, 196, 202, 212, 213, 214, 215, 216, 217, 224, 225, 226, 227, 228, 229, 239, 240, 241, 242, 243, 244, 245, 249, 251, 252, 254, 328, 339, 348, 356, 357, 366, 372, 373, 378, 379, 380, 392, 403, 565, 566, 574, 575, 576, 594}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-s

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

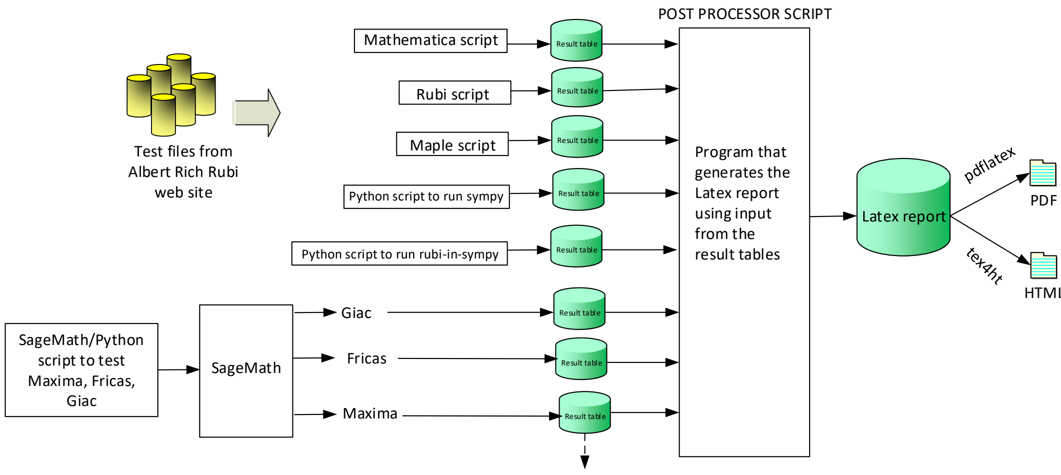
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592 }

B grade: { }

C grade: { }

F grade: { 391, 392, 593, 594}

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 68, 69, 70, 71, 72, 73, 74, 75, 77, 85, 86, 87, 88, 89, 90, 91, 92, 93, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 116, 117, 118, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 174, 178, 203, 204, 205, 206, 207, 208, 209, 210, 211, 218, 219, 220, 221, 222, 223, 230, 231, 232, 233, 234, 235, 236, 256, 260, 261, 262, 263, 264, 266, 267, 268, 269, 271, 272, 273, 274, 275, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 309, 311, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 329, 330, 331, 332, 333, 334, 335, 336, 337, 340, 341, 342, 343, 344, 345, 346, 347, 349, 350, 351, 352, 353, 354, 355, 358, 359, 360, 361, 362, 363, 364, 365, 367, 368, 369, 370, 371, 372, 374, 375, 380, 384, 385, 395, 396, 399, 400, 401, 402, 403, 407, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 527, 528, 529, 530, 531, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 564, 567, 568, 569, 570, 571, 572, 573, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 592 }

B grade: { 40, 54, 66, 67, 119, 120, 179, 265, 270, 276, 277, 308, 310, 312, 378, 379, 565, 566, 593 }

C grade: { 76, 78, 79, 80, 81, 82, 83, 84, 94, 95, 96, 97, 98, 99, 113, 114, 115, 172, 173, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 212, 213, 214, 215, 216, 217, 224, 225, 226, 227, 228, 229, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 257, 258, 259, 294, 328, 338, 339, 348, 356, 357, 366, 373, 383, 386, 387, 388, 389, 390, 391, 392, 393, 394, 397, 398, 404, 405, 406, 408, 409, 410, 484, 521, 522, 523, 524, 525, 526, 532, 533, 534, 535, 536, 537, 561, 562, 563, 574, 575, 576, 591, 594 }

F grade: { 175, 176, 177, 180, 181, 376, 377, 381, 382, 423, 424}

2.1.3 Maple

A grade: { 1, 2, 3, 6, 13, 14, 15, 16, 17, 18, 31, 32, 33, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 107, 114, 116, 117, 118, 121, 127, 128, 129, 130, 131, 139, 140, 141, 142, 143, 148, 149, 151, 152, 154, 157, 158, 159, 160, 161, 162, 163, 164, 168, 170, 195, 196, 197, 198, 199, 200, 201, 202, 215, 217, 236, 238, 246, 256, 261, 262, 263, 264, 265, 267, 268, 271, 272, 273, 274, 275, 276, 279, 280, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 299, 300, 303, 305, 306, 307, 308, 309, 311, 313, 315, 317, 318, 319, 320, 321, 322, 324, 325, 329, 330, 331, 332, 333, 335, 340, 341, 342, 344, 345, 346, 349, 350, 352, 353, 354, 355, 358, 359, 360, 361, 364, 365, 368, 383, 384, 385, 386, 394, 395, 396, 397, 399, 400, 401, 402, 403, 407, 419, 425, 426, 427, 428, 429, 430, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 450, 451, 452, 453, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 491, 492, 493, 494, 495, 496, 497, 498, 502, 503, 504, 505, 507, 508, 509, 513, 514, 515, 516, 519, 520, 524, 525, 526, 527, 528, 529, 530, 531, 541, 542, 552, 553, 554, 557, 564, 569, 570, 571, 572, 573, 577, 579, 580, 581, 582, 583, 586, 587, 588, 589, 590 }

B grade: { 4, 5, 34, 84, 85, 106, 108, 109, 110, 111, 112, 113, 115, 120, 122, 123, 124, 125, 126, 132, 133, 134, 135, 136, 137, 138, 144, 145, 146, 147, 153, 155, 156, 165, 166, 167, 169, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 216, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 237, 247, 259, 260, 266, 270, 277, 278, 298, 301, 302, 304, 310, 312, 314, 316, 323, 326, 327, 328, 334, 336, 337, 338, 339, 343, 347, 348, 356, 357, 362, 363, 366, 367, 369, 370, 371, 387, 393, 398, 404, 405, 406, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 446, 447, 448, 449, 454, 455, 456, 488, 489, 490, 499, 500, 501, 506, 510, 511, 512, 517, 521, 522, 523, 532, 533, 534, 535, 536, 537, 538, 539, 540, 562, 591, 593 }

C grade: { 7, 8, 9, 10, 11, 12, 119, 150, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 239, 241, 242, 243, 249, 250, 251, 252, 253, 254, 255, 257, 258, 269, 281, 351, 388, 389, 390, 391, 392, 518, 594 }

F grade: { 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 240, 244, 245, 248, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 420, 421, 422, 423, 424, 431, 432, 433, 543, 544, 545, 546, 547, 548, 549, 550, 551, 555, 556, 558, 559, 560, 561, 563, 565, 566, 567, 568, 574, 575, 576, 578, 584, 585, 592 }

2.1.4 Maxima

A grade: { 3, 4, 13, 14, 15, 16, 17, 18, 31, 32, 33, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 113, 114, 115, 116, 117, 118, 154, 163, 164, 236, 256, 261, 262, 263, 264, 266, 267, 268, 269, 271, 272, 273, 274, 275, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 355, 364, 365, 419, 425, 426, 427, 428, 429, 430, 434, 435, 436, 437, 438, 439, 440, 442, 443, 444, 445, 446, 447, 448, 457, 458, 464, 465, 466, 467, 468, 469, 478, 479, 564, 569, 570, 571, 572, 573, 577, 578, 581, 582, 583, 586, 587, 588, 589, 590

B grade: { 5, 6, 34, 119, 120, 121, 265, 270, 276, 277, 441, 462, 463, 473, 474, 475, 476, 477, 483, 484, 485, 486, 487 }

C grade: { }

F grade: { 1, 2, 7, 8, 9, 10, 11, 12, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 257, 258, 259, 260, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 356, 357, 358, 359, 360, 361, 362, 363, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 420, 421, 422, 423, 424, 431, 432, 433, 449, 450, 451, 452, 453, 454, 455, 456, 459, 460, 461, 470, 471, 472, 480, 481, 482, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 565, 566, 567, 568, 574, 575, 576, 579, 580, 584, 585, 591, 592, 593, 594 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 5, 6, 13, 14, 15, 17, 18, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 88, 97, 100, 116, 117, 118, 120, 121, 122, 125, 126, 132, 133, 136, 137, 147, 154, 163, 256, 261, 262, 263, 264, 266, 267, 268, 269, 271, 272, 273, 274, 275, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 310, 312, 317, 319, 324, 328, 334, 338, 339, 355, 364, 365, 385, 419, 425, 426, 427, 428, 429, 430, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 499, 500, 501, 502, 503, 504, 510, 511, 512, 513, 514, 515, 521, 555, 558, 559, 560, 564, 569, 570, 571, 572, 573, 577, 578, 579, 580, 581, 582, 583, 586, 587, 588, 589, 590 }

B grade: { 4, 16, 67, 84, 89, 90, 91, 92, 93, 94, 95, 96, 98, 99, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 119, 123, 124, 134, 135, 144, 145, 146, 153, 155, 156, 162, 164, 165, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 265, 270, 276, 277, 309, 311, 313, 314, 315, 316, 318, 320, 321, 322, 323, 325, 326, 327, 335, 336, 337, 345, 346, 347, 348, 354, 356, 357, 363, 366, 395, 396, 404, 405, 406, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 522, 523, 524, 525, 526, 532, 533, 534, 535, 536, 537, 556, 557 }

C grade: { 383, 386, 387, 397 }

F grade: { 7, 8, 9, 10, 11, 12, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 127, 128, 129, 130, 131, 138, 139, 140, 141, 142, 143, 148, 149, 150, 151, 152, 157, 158, 159, 160, 161, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 257, 258, 259, 260, 329, 330, 331, 332, 333, 340, 341, 342, 343, 344, 349, 350, 351, 352, 353, 358, 359, 360, 361, 362, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 384, 388, 389, 390, 391, 392, 393, 394, 398, 399, 400, 401, 402, 403, 407, 420, 421, 422, 423, 424, 431, 432, 433, 494, 495, 496, 497, 498, 505, 506, 507, 508, 509, 516, 517, 518, 519, 520, 527, 528, 529, 530, 531, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 561, 562, 563, 565, 566, 567, 568, 574, 575, 576, 584, 585, 591, 592, 593, 594 }

2.1.6 Sympy

A grade: { 3, 29, 31, 35, 36, 37, 38, 42, 43, 44, 45, 50, 51, 52, 55, 56, 57, 58, 64, 65, 68, 69, 70, 74, 89, 269, 281, 285, 286, 287, 288, 293, 294, 295, 307, 319, 385, 407, 577 }

B grade: { 32, 33, 34, 61, 62, 75, 76, 77, 261, 262, 263, 264, 265, 267, 268, 274, 275, 276, 277, 279, 280, 282 }

C grade: { }

F grade: { 1, 2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 39, 40, 41, 46, 47, 48, 49, 53, 54, 59, 60, 63, 66, 67, 71, 72, 73, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 266, 270, 271, 272, 273, 278, 283, 284, 289, 290, 291, 292, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 13, 14, 15, 31, 32, 33, 34, 37, 38, 42, 43, 44, 45, 46, 47, 48, 49, 51, 52, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 68, 69, 70, 72, 73, 74, 75, 76, 77, 80, 81, 85, 86, 87, 90, 91, 92, 93, 96, 97, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 113, 114, 115, 120, 122, 123, 132, 133, 144, 145, 153, 154, 261, 262, 263, 264, 266, 267, 268, 271, 272, 273, 274, 275, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 290, 291, 292, 293, 294, 295, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 314, 315, 316, 317, 318, 319, 320, 321, 324, 325, 334, 335, 345, 346, 354, 355, 364, 365, 383, 384, 385, 386, 387, 393, 394, 395, 396, 397, 398, 400, 401, 402, 403, 419, 425, 426, 427, 428, 429, 430, 434, 435, 438, 439, 440, 445, 446, 452, 453, 454, 458, 460, 465, 466, 467, 468, 469, 471, 476, 477, 479, 481, 484, 487, 491, 502, 513, 524, 535, 552, 553, 554, 558, 564, 569, 570, 571, 572, 573, 577, 578, 581, 582, 583, 586, 587, 588, 589, 590 }

B grade: { 29, 35, 36, 39, 40, 41, 50, 53, 54, 66, 67, 71, 78, 79, 82, 83, 84, 88, 89, 94, 95, 98, 99, 112, 116, 117, 118, 121, 163, 164, 236, 256, 265, 269, 270, 276, 277, 289, 296, 310, 311, 312, 313, 322, 323, 404, 405, 406, 407, 408, 409, 410, 441, 442, 443, 444, 447, 448, 449, 450, 451, 455, 456, 457, 459, 461, 462, 463, 464, 470, 472, 473, 474, 475, 480, 482, 485, 486, 512 }

C grade: { }

F grade: { 7, 8, 9, 10, 11, 12, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 119, 124, 125, 126, 127, 128, 129, 130, 131, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 146, 147, 148, 149, 150, 151, 152, 155, 156, 157, 158, 159, 160, 161, 162, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 257, 258, 259, 260, 326, 327, 328, 329, 330, 331, 332, 333, 336, 337, 338, 339, 340, 341, 342, 343, 344, 347, 348, 349, 350, 351, 352, 353, 356, 357, 358, 359, 360, 361, 362, 363, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 388, 389, 390, 391, 392, 399, 411, 412, 413, 414, 415, 416, 417, 418, 420, 421, 422, 423, 424, 431, 432, 433, 436, 437, 478, 483, 488, 489, 490, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 503, 504, 505, 506, 507, 508, 509, 510, 511, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 555, 556, 557, 559, 560, 561, 562, 563, 565, 566, 567, 568, 574, 575, 576, 579, 580, 584, 585, 591, 592, 593, 594 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	36	32	0	117	0	61
normalized size	1	1.	0.68	0.6	0.	2.21	0.	1.15
time (sec)	N/A	0.029	0.033	0.632	0.	1.625	0.	1.164

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	26	24	0	81	0	32
normalized size	1	1.	0.76	0.71	0.	2.38	0.	0.94
time (sec)	N/A	0.019	0.038	0.628	0.	1.558	0.	1.223

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	16	18	51	20	23
normalized size	1	1.	1.	1.14	1.29	3.64	1.43	1.64
time (sec)	N/A	0.01	0.004	0.294	1.424	1.578	0.72	1.242

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	30	49	35	193	0	20
normalized size	1	1.	1.76	2.88	2.06	11.35	0.	1.18
time (sec)	N/A	0.011	0.015	0.599	1.599	1.716	0.	1.23

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	55	70	424	158	0	82
normalized size	1	1.	1.31	1.67	10.1	3.76	0.	1.95
time (sec)	N/A	0.024	0.058	1.116	1.644	1.665	0.	1.277

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	77	89	1257	221	0	108
normalized size	1	1.	1.26	1.46	20.61	3.62	0.	1.77
time (sec)	N/A	0.03	0.195	1.156	2.559	1.701	0.	1.3

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	65	149	0	0	0	0
normalized size	1	1.	0.53	1.21	0.	0.	0.	0.
time (sec)	N/A	0.041	0.189	0.224	0.	0.	0.	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	54	337	0	0	0	0
normalized size	1	1.	0.74	4.62	0.	0.	0.	0.
time (sec)	N/A	0.025	0.1	0.213	0.	0.	0.	0.

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	41	118	0	0	0	0
normalized size	1	1.	0.82	2.36	0.	0.	0.	0.
time (sec)	N/A	0.018	0.031	0.271	0.	0.	0.	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	37	330	0	0	0	0
normalized size	1	1.	0.77	6.88	0.	0.	0.	0.
time (sec)	N/A	0.017	0.027	0.257	0.	0.	0.	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	48	360	0	0	0	0
normalized size	1	1.	0.62	4.68	0.	0.	0.	0.
time (sec)	N/A	0.025	0.068	0.283	0.	0.	0.	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	60	1301	0	0	0	0
normalized size	1	1.	0.49	10.58	0.	0.	0.	0.
time (sec)	N/A	0.042	0.208	0.248	0.	0.	0.	0.

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	53	57	115	238	0	77
normalized size	1	1.	0.4	0.43	0.87	1.8	0.	0.58
time (sec)	N/A	0.044	0.165	0.292	1.448	1.801	0.	1.108

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	38	41	74	163	0	36
normalized size	1	1.	0.49	0.53	0.95	2.09	0.	0.46
time (sec)	N/A	0.028	0.095	0.144	1.437	1.77	0.	1.102

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	25	27	30	103	0	20
normalized size	1	1.	0.69	0.75	0.83	2.86	0.	0.56
time (sec)	N/A	0.013	0.016	0.158	1.441	1.67	0.	1.1

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	12	95	0	0
normalized size	1	1.	1.	0.94	0.75	5.94	0.	0.
time (sec)	N/A	0.014	0.007	0.159	1.451	1.548	0.	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	34	29	31	196	0	0
normalized size	1	1.	0.5	0.43	0.46	2.88	0.	0.
time (sec)	N/A	0.02	0.033	0.119	1.448	1.621	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	47	41	47	294	0	0
normalized size	1	1.	0.4	0.35	0.4	2.49	0.	0.
time (sec)	N/A	0.028	0.05	0.16	1.449	1.629	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	74	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.179	0.468	0.	0.	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	72	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	0.118	0.19	0.	0.	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	68	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	0.072	0.213	0.	0.	0.	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	72	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.042	0.075	0.192	0.	0.	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	71	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	0.109	0.181	0.	0.	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	73	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	0.134	0.184	0.	0.	0.	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	71	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.035	0.064	0.312	0.	0.	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	61	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.033	0.078	0.796	0.	0.	0.	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	67	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.035	0.084	0.829	0.	0.	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	65	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.033	0.089	0.917	0.	0.	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	0	0	34	61	518
normalized size	1	1.	1.	0.	0.	1.36	2.44	20.72
time (sec)	N/A	0.019	0.034	0.292	0.	1.643	6.304	8.92

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	73	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	0.057	0.333	0.	0.	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	18	23	42	15	23
normalized size	1	1.	1.	1.12	1.44	2.62	0.94	1.44
time (sec)	N/A	0.009	0.003	0.019	0.978	1.649	0.156	1.142

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	26	43	54	76	110	34
normalized size	1	1.	0.79	1.3	1.64	2.3	3.33	1.03
time (sec)	N/A	0.026	0.003	0.021	0.949	1.646	1.066	1.125

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	34	72	93	104	233	46
normalized size	1	1.	0.74	1.57	2.02	2.26	5.07	1.
time (sec)	N/A	0.032	0.003	0.02	0.95	1.621	4.281	1.121

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	42	105	140	135	376	58
normalized size	1	1.	0.71	1.78	2.37	2.29	6.37	0.98
time (sec)	N/A	0.042	0.003	0.02	0.958	1.746	13.81	1.138

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	58	45	68	113	314	201
normalized size	1	1.	0.94	0.73	1.1	1.82	5.06	3.24
time (sec)	N/A	0.087	0.064	0.042	0.967	1.604	125.581	1.164

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	43	35	54	88	143	142
normalized size	1	1.	0.93	0.76	1.17	1.91	3.11	3.09
time (sec)	N/A	0.08	0.044	0.04	0.941	1.861	42.933	1.182

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	25	23	36	55	36	39
normalized size	1	1.	0.93	0.85	1.33	2.04	1.33	1.44
time (sec)	N/A	0.065	0.033	0.04	0.949	1.911	15.002	1.103

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	16	20	30	34	20
normalized size	1	1.	1.	1.23	1.54	2.31	2.62	1.54
time (sec)	N/A	0.034	0.013	0.03	0.949	1.654	3.187	1.117

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	46	51	62	157	0	84
normalized size	1	1.	1.59	1.76	2.14	5.41	0.	2.9
time (sec)	N/A	0.058	0.043	0.06	0.949	1.627	0.	1.167

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	146	87	95	266	0	201
normalized size	1	1.	2.52	1.5	1.64	4.59	0.	3.47
time (sec)	N/A	0.096	0.265	0.077	0.968	1.84	0.	1.193

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	132	123	122	382	0	244
normalized size	1	1.	1.61	1.5	1.49	4.66	0.	2.98
time (sec)	N/A	0.095	4.472	0.08	0.964	1.799	0.	1.21

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	44	84	97	140	1161	85
normalized size	1	1.	0.6	1.15	1.33	1.92	15.9	1.16
time (sec)	N/A	0.09	0.189	0.046	1.454	1.641	107.4	1.149

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	34	56	66	111	502	68
normalized size	1	1.	0.69	1.14	1.35	2.27	10.24	1.39
time (sec)	N/A	0.082	0.127	0.041	1.433	1.595	39.793	1.165

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	27	30	35	74	100	35
normalized size	1	1.	1.35	1.5	1.75	3.7	5.	1.75
time (sec)	N/A	0.063	0.014	0.038	1.428	1.618	7.736	1.175

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	18	45	41	18
normalized size	1	1.	1.	1.08	1.38	3.46	3.15	1.38
time (sec)	N/A	0.022	0.006	0.039	0.948	1.6	2.676	1.121

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	16	25	38	77	0	26
normalized size	1	1.	0.57	0.89	1.36	2.75	0.	0.93
time (sec)	N/A	0.075	0.027	0.06	0.957	1.581	0.	1.153

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	49	35	57	140	0	57
normalized size	1	1.	1.07	0.76	1.24	3.04	0.	1.24
time (sec)	N/A	0.08	0.043	0.075	0.955	1.622	0.	1.182

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	70	45	70	200	0	70
normalized size	1	1.	1.13	0.73	1.13	3.23	0.	1.13
time (sec)	N/A	0.084	0.037	0.078	0.962	1.573	0.	1.228

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	59	47	70	117	0	77
normalized size	1	1.	0.91	0.72	1.08	1.8	0.	1.18
time (sec)	N/A	0.084	0.05	0.043	0.945	1.629	0.	1.161

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	42	37	55	96	156	143
normalized size	1	1.	0.89	0.79	1.17	2.04	3.32	3.04
time (sec)	N/A	0.069	0.037	0.042	0.95	1.628	93.172	1.173

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	31	29	38	70	309	38
normalized size	1	1.	0.94	0.88	1.15	2.12	9.36	1.15
time (sec)	N/A	0.062	0.03	0.04	0.973	1.533	25.345	1.157

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	22	38	156	22
normalized size	1	1.	1.	0.94	1.22	2.11	8.67	1.22
time (sec)	N/A	0.042	0.011	0.032	0.952	1.596	19.495	1.158

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	61	67	80	198	0	144
normalized size	1	1.	1.3	1.43	1.7	4.21	0.	3.06
time (sec)	N/A	0.062	0.041	0.065	0.951	1.691	0.	1.171

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	208	104	116	312	0	236
normalized size	1	1.	2.67	1.33	1.49	4.	0.	3.03
time (sec)	N/A	0.085	0.492	0.086	0.957	1.761	0.	1.219

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	46	73	86	149	1275	92
normalized size	1	1.	0.67	1.06	1.25	2.16	18.48	1.33
time (sec)	N/A	0.082	0.21	0.046	1.467	1.701	144.461	1.167

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	42	46	50	120	551	59
normalized size	1	1.	1.11	1.21	1.32	3.16	14.5	1.55
time (sec)	N/A	0.056	0.014	0.036	1.439	1.588	61.364	1.157

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	22	85	94	22
normalized size	1	1.	1.	0.94	1.22	4.72	5.22	1.22
time (sec)	N/A	0.067	0.018	0.033	1.138	1.535	31.774	1.121

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	26	25	34	86	238	34
normalized size	1	1.	0.81	0.78	1.06	2.69	7.44	1.06
time (sec)	N/A	0.025	0.038	0.04	0.956	1.578	13.451	1.129

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	50	37	54	113	0	65
normalized size	1	1.	1.06	0.79	1.15	2.4	0.	1.38
time (sec)	N/A	0.078	0.039	0.066	0.96	1.551	0.	1.115

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	46	47	70	176	0	46
normalized size	1	1.	0.71	0.72	1.08	2.71	0.	0.71
time (sec)	N/A	0.081	0.025	0.076	0.956	1.64	0.	1.161

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	31	20	30	78	362	30
normalized size	1	1.	1.07	0.69	1.03	2.69	12.48	1.03
time (sec)	N/A	0.021	0.005	0.03	0.951	1.565	24.013	1.11

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	41	24	38	97	675	38
normalized size	1	1.	1.11	0.65	1.03	2.62	18.24	1.03
time (sec)	N/A	0.022	0.005	0.032	0.941	1.894	108.262	1.111

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	32	46	123	0	46
normalized size	1	1.	1.	0.63	0.9	2.41	0.	0.9
time (sec)	N/A	0.026	0.006	0.031	0.958	1.866	0.	1.157

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	77	54	58	115	107	90
normalized size	1	1.	1.51	1.06	1.14	2.25	2.1	1.76
time (sec)	N/A	0.045	0.029	0.029	0.963	1.927	2.338	1.103

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	54	34	46	69	58	54
normalized size	1	1.	1.74	1.1	1.48	2.23	1.87	1.74
time (sec)	N/A	0.022	0.019	0.028	0.937	1.708	0.901	1.13

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	63	35	51	124	0	78
normalized size	1	1.	2.42	1.35	1.96	4.77	0.	3.
time (sec)	N/A	0.025	0.032	0.046	0.949	1.732	0.	1.152

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	118	63	78	246	0	163
normalized size	1	1.	2.95	1.58	1.95	6.15	0.	4.08
time (sec)	N/A	0.031	0.037	0.055	0.936	1.702	0.	1.183

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	70	86	140	171	258	92
normalized size	1	1.	0.79	0.97	1.57	1.92	2.9	1.03
time (sec)	N/A	0.053	0.111	0.029	1.438	1.669	10.697	1.122

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	45	65	100	119	158	58
normalized size	1	1.	0.74	1.07	1.64	1.95	2.59	0.95
time (sec)	N/A	0.04	0.094	0.027	1.435	1.649	2.126	1.12

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	33	32	39	72	51	34
normalized size	1	1.	1.1	1.07	1.3	2.4	1.7	1.13
time (sec)	N/A	0.015	0.028	0.022	0.944	1.575	0.429	1.126

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	22	31	76	0	53
normalized size	1	1.	1.	1.38	1.94	4.75	0.	3.31
time (sec)	N/A	0.023	0.018	0.046	1.427	1.584	0.	1.158

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	49	35	38	132	0	50
normalized size	1	1.	1.14	0.81	0.88	3.07	0.	1.16
time (sec)	N/A	0.036	0.026	0.052	0.95	1.584	0.	1.153

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	95	56	61	208	0	82
normalized size	1	1.	1.46	0.86	0.94	3.2	0.	1.26
time (sec)	N/A	0.042	0.028	0.053	0.943	1.611	0.	1.138

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	17	23	54	15	23
normalized size	1	1.	1.	0.89	1.21	2.84	0.79	1.21
time (sec)	N/A	0.009	0.003	0.017	0.943	1.586	0.1	1.133

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	43	42	53	116	110	57
normalized size	1	1.	0.86	0.84	1.06	2.32	2.2	1.14
time (sec)	N/A	0.016	0.058	0.025	0.938	1.627	1.281	1.139

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	80	73	96	207	246	103
normalized size	1	1.	0.92	0.84	1.1	2.38	2.83	1.18
time (sec)	N/A	0.083	0.102	0.024	0.938	1.661	5.622	1.11

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	113	110	146	323	410	159
normalized size	1	1.	0.81	0.79	1.04	2.31	2.93	1.14
time (sec)	N/A	0.167	0.15	0.026	0.956	1.748	18.413	1.104

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	180	110	0	626	0	448
normalized size	1	1.	1.7	1.04	0.	5.91	0.	4.23
time (sec)	N/A	0.113	1.395	0.085	0.	1.856	0.	1.16

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	150	70	0	491	0	234
normalized size	1	1.	1.95	0.91	0.	6.38	0.	3.04
time (sec)	N/A	0.092	0.507	0.075	0.	1.873	0.	1.171

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	125	45	0	381	0	77
normalized size	1	1.	2.4	0.87	0.	7.33	0.	1.48
time (sec)	N/A	0.07	0.258	0.069	0.	1.794	0.	1.161

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	97	29	0	273	0	50
normalized size	1	1.	2.62	0.78	0.	7.38	0.	1.35
time (sec)	N/A	0.04	0.157	0.063	0.	1.779	0.	1.126

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	143	67	0	440	0	135
normalized size	1	1.	2.6	1.22	0.	8.	0.	2.45
time (sec)	N/A	0.064	0.309	0.102	0.	1.91	0.	1.132

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	224	142	0	824	0	265
normalized size	1	1.	2.64	1.67	0.	9.69	0.	3.12
time (sec)	N/A	0.116	2.31	0.122	0.	1.942	0.	1.21

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	657	255	0	1482	0	451
normalized size	1	1.	5.26	2.04	0.	11.86	0.	3.61
time (sec)	N/A	0.188	6.291	0.122	0.	2.114	0.	1.261

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	133	361	0	1077	0	315
normalized size	1	1.	0.82	2.21	0.	6.61	0.	1.93
time (sec)	N/A	0.365	1.195	0.102	0.	1.932	0.	1.178

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	95	196	0	884	0	212
normalized size	1	1.	0.81	1.68	0.	7.56	0.	1.81
time (sec)	N/A	0.224	0.445	0.092	0.	1.926	0.	1.207

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	69	94	0	738	0	154
normalized size	1	1.	0.9	1.22	0.	9.58	0.	2.
time (sec)	N/A	0.114	0.31	0.079	0.	1.876	0.	1.198

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	50	0	617	0	109
normalized size	1	1.	1.	1.09	0.	13.41	0.	2.37
time (sec)	N/A	0.075	0.145	0.075	0.	1.859	0.	1.196

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	30	0	568	3907	86
normalized size	1	1.	1.	0.83	0.	15.78	108.53	2.39
time (sec)	N/A	0.024	0.078	0.075	0.	1.795	49.856	1.172

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	53	52	0	748	0	112
normalized size	1	1.	1.	0.98	0.	14.11	0.	2.11
time (sec)	N/A	0.073	0.296	0.109	0.	1.784	0.	1.184

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	119	85	0	1046	0	150
normalized size	1	1.	1.55	1.1	0.	13.58	0.	1.95
time (sec)	N/A	0.106	0.655	0.116	0.	1.86	0.	1.151

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	147	138	0	1400	0	209
normalized size	1	1.	1.35	1.27	0.	12.84	0.	1.92
time (sec)	N/A	0.125	1.542	0.123	0.	1.882	0.	1.169

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	137	207	0	1839	0	290
normalized size	1	1.	0.98	1.48	0.	13.14	0.	2.07
time (sec)	N/A	0.156	1.729	0.158	0.	2.018	0.	1.194

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	194	118	0	1162	0	435
normalized size	1	1.	1.52	0.92	0.	9.08	0.	3.4
time (sec)	N/A	0.186	1.574	0.085	0.	2.073	0.	1.182

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	172	94	0	941	0	462
normalized size	1	1.	1.69	0.92	0.	9.23	0.	4.53
time (sec)	N/A	0.146	0.904	0.083	0.	1.972	0.	1.164

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	160	80	0	721	0	126
normalized size	1	1.	1.93	0.96	0.	8.69	0.	1.52
time (sec)	N/A	0.089	0.496	0.081	0.	1.879	0.	1.138

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	149	68	0	635	0	107
normalized size	1	1.	2.01	0.92	0.	8.58	0.	1.45
time (sec)	N/A	0.054	0.274	0.072	0.	1.826	0.	1.132

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	194	150	0	1081	0	332
normalized size	1	1.	1.88	1.46	0.	10.5	0.	3.22
time (sec)	N/A	0.129	0.768	0.116	0.	2.312	0.	1.169

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	390	226	0	1928	0	691
normalized size	1	1.	2.55	1.48	0.	12.6	0.	4.52
time (sec)	N/A	0.243	1.636	0.14	0.	2.565	0.	1.231

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	106	187	0	1413	0	301
normalized size	1	1.	0.72	1.26	0.	9.55	0.	2.03
time (sec)	N/A	0.292	1.476	0.099	0.	2.035	0.	1.167

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	93	140	0	1165	0	189
normalized size	1	1.	1.	1.51	0.	12.53	0.	2.03
time (sec)	N/A	0.138	0.869	0.086	0.	1.952	0.	1.168

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	74	77	0	965	0	147
normalized size	1	1.	0.95	0.99	0.	12.37	0.	1.88
time (sec)	N/A	0.088	0.517	0.088	0.	1.85	0.	1.128

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	84	119	0	1060	0	153
normalized size	1	1.	0.97	1.37	0.	12.18	0.	1.76
time (sec)	N/A	0.063	0.417	0.089	0.	1.883	0.	1.108

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	130	155	144	0	1330	0	242
normalized size	1	1.02	1.22	1.13	0.	10.47	0.	1.91
time (sec)	N/A	0.146	1.157	0.118	0.	1.96	0.	1.209

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	202	179	0	1912	0	235
normalized size	1	1.	1.25	1.1	0.	11.8	0.	1.45
time (sec)	N/A	0.203	1.245	0.147	0.	2.07	0.	1.194

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	134	363	0	2175	0	302
normalized size	1	1.	0.91	2.45	0.	14.7	0.	2.04
time (sec)	N/A	0.279	2.584	0.096	0.	2.22	0.	1.204

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	97	136	0	1567	0	205
normalized size	1	1.	0.88	1.24	0.	14.25	0.	1.86
time (sec)	N/A	0.096	1.249	0.089	0.	1.973	0.	1.176

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	112	278	0	1719	0	258
normalized size	1	1.	0.85	2.12	0.	13.12	0.	1.97
time (sec)	N/A	0.149	1.372	0.088	0.	2.186	0.	1.2

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	125	334	0	1883	0	285
normalized size	1	1.	0.87	2.32	0.	13.08	0.	1.98
time (sec)	N/A	0.146	1.242	0.093	0.	2.318	0.	1.163

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	214	367	0	2275	0	313
normalized size	1	1.	1.09	1.87	0.	11.61	0.	1.6
time (sec)	N/A	0.255	1.705	0.13	0.	2.244	0.	1.188

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	201	705	0	3070	0	464
normalized size	1	1.	0.98	3.42	0.	14.9	0.	2.25
time (sec)	N/A	0.296	1.398	0.099	0.	2.421	0.	1.125

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	279	312	1249	0	4771	0	707
normalized size	1	1.	1.12	4.48	0.	17.1	0.	2.53
time (sec)	N/A	0.531	1.892	0.098	0.	2.985	0.	1.162

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	29	33	14	173	0	14
normalized size	1	1.	2.64	3.	1.27	15.73	0.	1.27
time (sec)	N/A	0.026	0.064	0.463	1.414	1.634	0.	1.136

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	53	51	34	219	0	34
normalized size	1	1.	1.77	1.7	1.13	7.3	0.	1.13
time (sec)	N/A	0.03	0.048	0.938	1.41	1.794	0.	1.197

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	39	57	15	251	0	15
normalized size	1	1.	2.6	3.8	1.	16.73	0.	1.
time (sec)	N/A	0.03	0.093	0.797	1.419	1.818	0.	1.305

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	36	32	42	107	0	113
normalized size	1	1.	0.68	0.6	0.79	2.02	0.	2.13
time (sec)	N/A	0.054	0.022	0.677	1.617	1.675	0.	1.262

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	26	24	23	74	0	77
normalized size	1	1.	0.76	0.71	0.68	2.18	0.	2.26
time (sec)	N/A	0.036	0.009	0.541	1.61	1.668	0.	1.217

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	15	8	43	0	36
normalized size	1	1.	1.	1.15	0.62	3.31	0.	2.77
time (sec)	N/A	0.026	0.004	0.392	1.592	1.626	0.	1.145

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	46	20	51	182	0	0
normalized size	1	1.	2.88	1.25	3.19	11.38	0.	0.
time (sec)	N/A	0.031	0.019	0.11	1.58	1.653	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	91	70	410	124	0	59
normalized size	1	1.	2.17	1.67	9.76	2.95	0.	1.4
time (sec)	N/A	0.037	0.065	0.994	1.628	1.759	0.	1.232

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	72	89	1260	151	0	174
normalized size	1	1.	1.18	1.46	20.66	2.48	0.	2.85
time (sec)	N/A	0.05	0.152	1.084	2.562	1.763	0.	1.286

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	119	311	0	1214	0	177
normalized size	1	1.	0.95	2.49	0.	9.71	0.	1.42
time (sec)	N/A	0.128	0.42	1.677	0.	4.632	0.	1.742

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	93	182	0	1069	0	112
normalized size	1	1.	1.19	2.33	0.	13.71	0.	1.44
time (sec)	N/A	0.061	0.253	1.282	0.	2.895	0.	1.736

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	99	174	0	2862	0	0
normalized size	1	1.	1.19	2.1	0.	34.48	0.	0.
time (sec)	N/A	0.097	0.116	1.883	0.	3.549	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	100	227	0	849	0	0
normalized size	1	1.	1.19	2.7	0.	10.11	0.	0.
time (sec)	N/A	0.1	0.277	1.444	0.	2.258	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	127	379	0	1239	0	0
normalized size	1	1.	0.89	2.65	0.	8.66	0.	0.
time (sec)	N/A	0.132	0.518	1.583	0.	3.553	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	259	199	413	0	0	0	0
normalized size	1	1.	0.77	1.59	0.	0.	0.	0.
time (sec)	N/A	0.316	1.453	1.037	0.	0.	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	159	266	0	0	0	0
normalized size	1	1.	1.	1.67	0.	0.	0.	0.
time (sec)	N/A	0.191	0.813	1.19	0.	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	61	71	0	0	0	0
normalized size	1	1.	1.2	1.39	0.	0.	0.	0.
time (sec)	N/A	0.035	0.087	0.609	0.	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	137	156	0	0	0	0
normalized size	1	1.	0.79	0.9	0.	0.	0.	0.
time (sec)	N/A	0.163	0.599	1.209	0.	0.	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	188	342	0	0	0	0
normalized size	1	1.	0.8	1.46	0.	0.	0.	0.
time (sec)	N/A	0.26	3.231	1.098	0.	0.	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	169	169	152	446	0	1400	0	266
normalized size	1	1.	0.9	2.64	0.	8.28	0.	1.57
time (sec)	N/A	0.148	0.754	1.565	0.	12.448	0.	1.736

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	113	309	0	1208	0	173
normalized size	1	1.	0.99	2.71	0.	10.6	0.	1.52
time (sec)	N/A	0.076	0.401	1.507	0.	4.648	0.	1.71

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	141	255	0	3204	0	0
normalized size	1	1.	1.16	2.09	0.	26.26	0.	0.
time (sec)	N/A	0.144	0.899	1.888	0.	5.047	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	147	287	0	3572	0	0
normalized size	1	1.	1.15	2.24	0.	27.91	0.	0.
time (sec)	N/A	0.151	1.214	1.911	0.	5.788	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	114	376	0	1216	0	0
normalized size	1	1.	0.89	2.94	0.	9.5	0.	0.
time (sec)	N/A	0.125	0.704	1.58	0.	4.333	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	161	565	0	1773	0	0
normalized size	1	1.	0.82	2.87	0.	9.	0.	0.
time (sec)	N/A	0.175	1.26	2.063	0.	10.587	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	325	249	602	0	0	0	0
normalized size	1	1.	0.77	1.85	0.	0.	0.	0.
time (sec)	N/A	0.482	2.74	1.324	0.	0.	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	201	429	0	0	0	0
normalized size	1	1.	0.92	1.97	0.	0.	0.	0.
time (sec)	N/A	0.304	1.38	1.236	0.	0.	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	156	266	0	0	0	0
normalized size	1	1.	1.01	1.73	0.	0.	0.	0.
time (sec)	N/A	0.164	0.78	1.211	0.	0.	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	141	174	0	0	0	0
normalized size	1	1.	0.78	0.96	0.	0.	0.	0.
time (sec)	N/A	0.192	1.325	1.124	0.	0.	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	201	408	0	0	0	0
normalized size	1	1.	0.85	1.73	0.	0.	0.	0.
time (sec)	N/A	0.293	4.505	1.169	0.	0.	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	194	437	0	0	0	0
normalized size	1	1.	0.92	2.08	0.	0.	0.	0.
time (sec)	N/A	0.282	1.44	1.256	0.	0.	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	105	186	0	1076	0	126
normalized size	1	1.	1.27	2.24	0.	12.96	0.	1.52
time (sec)	N/A	0.096	0.279	1.256	0.	2.325	0.	1.838

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	53	99	0	900	0	68
normalized size	1	1.	1.29	2.41	0.	21.95	0.	1.66
time (sec)	N/A	0.047	0.113	0.861	0.	1.948	0.	1.769

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	48	112	0	558	0	0
normalized size	1	1.	1.17	2.73	0.	13.61	0.	0.
time (sec)	N/A	0.076	0.15	0.875	0.	1.778	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	102	231	0	860	0	0
normalized size	1	1.	1.15	2.6	0.	9.66	0.	0.
time (sec)	N/A	0.104	0.31	1.43	0.	2.065	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	163	268	0	0	0	0
normalized size	1	1.	0.79	1.3	0.	0.	0.	0.
time (sec)	N/A	0.199	0.977	1.089	0.	0.	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	78	93	0	0	0	0
normalized size	1	1.	0.7	0.84	0.	0.	0.	0.
time (sec)	N/A	0.126	0.221	0.835	0.	0.	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	60	60	0	0	0	0
normalized size	1	1.	1.18	1.18	0.	0.	0.	0.
time (sec)	N/A	0.033	0.085	0.266	0.	0.	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	138	140	0	0	0	0
normalized size	1	1.	0.78	0.79	0.	0.	0.	0.
time (sec)	N/A	0.179	0.501	1.098	0.	0.	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	195	354	0	0	0	0
normalized size	1	1.	0.8	1.45	0.	0.	0.	0.
time (sec)	N/A	0.267	3.929	1.183	0.	0.	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	96	156	0	1339	0	154
normalized size	1	1.	1.22	1.97	0.	16.95	0.	1.95
time (sec)	N/A	0.099	0.415	1.646	0.	2.628	0.	1.929

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	41	31	43	136	0	72
normalized size	1	1.	1.21	0.91	1.26	4.	0.	2.12
time (sec)	N/A	0.045	0.114	0.839	0.945	1.56	0.	1.358

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	93	165	0	1008	0	0
normalized size	1	1.	1.18	2.09	0.	12.76	0.	0.
time (sec)	N/A	0.094	0.339	2.155	0.	2.287	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	134	274	0	1474	0	0
normalized size	1	1.	1.	2.04	0.	11.	0.	0.
time (sec)	N/A	0.164	0.659	2.581	0.	3.835	0.	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	197	405	0	0	0	0
normalized size	1	1.	0.72	1.48	0.	0.	0.	0.
time (sec)	N/A	0.314	1.262	1.309	0.	0.	0.	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	136	241	0	0	0	0
normalized size	1	1.	0.67	1.19	0.	0.	0.	0.
time (sec)	N/A	0.2	0.673	1.25	0.	0.	0.	0.

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	138	191	0	0	0	0
normalized size	1	1.	0.9	1.25	0.	0.	0.	0.
time (sec)	N/A	0.192	0.443	1.262	0.	0.	0.	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	90	103	0	0	0	0
normalized size	1	1.	0.89	1.02	0.	0.	0.	0.
time (sec)	N/A	0.057	0.148	1.217	0.	0.	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	170	199	0	0	0	0
normalized size	1	1.	0.72	0.85	0.	0.	0.	0.
time (sec)	N/A	0.268	1.198	1.267	0.	0.	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	137	137	133	243	0	2045	0	0
normalized size	1	1.	0.97	1.77	0.	14.93	0.	0.
time (sec)	N/A	0.137	0.754	2.746	0.	6.052	0.	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	64	56	163	323	0	201
normalized size	1	1.	0.79	0.69	2.01	3.99	0.	2.48
time (sec)	N/A	0.095	0.31	1.161	0.994	2.673	0.	1.354

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	60	55	85	315	0	185
normalized size	1	1.	0.82	0.75	1.16	4.32	0.	2.53
time (sec)	N/A	0.058	0.172	1.041	0.959	2.536	0.	1.346

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	127	249	0	1723	0	0
normalized size	1	1.	0.98	1.93	0.	13.36	0.	0.
time (sec)	N/A	0.153	0.59	2.726	0.	4.166	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	192	698	0	0	0	0
normalized size	1	1.	0.67	2.45	0.	0.	0.	0.
time (sec)	N/A	0.332	2.014	1.5	0.	0.	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	182	623	0	0	0	0
normalized size	1	1.	0.68	2.32	0.	0.	0.	0.
time (sec)	N/A	0.276	1.637	1.527	0.	0.	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	174	483	0	0	0	0
normalized size	1	1.	0.79	2.19	0.	0.	0.	0.
time (sec)	N/A	0.29	1.461	1.424	0.	0.	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	172	547	0	0	0	0
normalized size	1	1.	0.77	2.45	0.	0.	0.	0.
time (sec)	N/A	0.256	1.322	1.27	0.	0.	0.	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	214	527	0	0	0	0
normalized size	1	1.	0.66	1.64	0.	0.	0.	0.
time (sec)	N/A	0.411	2.287	1.819	0.	0.	0.	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	122	122	113	0	0	0	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.116	0.442	1.32	0.	0.	0.	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	220	220	98	0	0	0	0	0
normalized size	1	1.	0.45	0.	0.	0.	0.	0.
time (sec)	N/A	0.225	0.547	1.431	0.	0.	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	131	131	98	0	0	0	0	0
normalized size	1	1.	0.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.11	0.373	2.264	0.	0.	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	74	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.046	0.174	1.348	0.	0.	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	83	83	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.078	4.745	0.892	0.	0.	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	83	83	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.084	6.185	0.895	0.	0.	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	83	83	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.085	8.754	0.563	0.	0.	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	102	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.103	0.539	1.232	0.	0.	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	99	99	240	0	0	0	0	0
normalized size	1	1.	2.42	0.	0.	0.	0.	0.
time (sec)	N/A	0.166	0.721	1.733	0.	0.	0.	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	97	97	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.104	4.7	0.791	0.	0.	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.1	6.416	0.605	0.	0.	0.	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	335	335	219	366	0	0	0	0
normalized size	1	1.	0.65	1.09	0.	0.	0.	0.
time (sec)	N/A	0.699	0.488	0.178	0.	0.	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	273	273	255	163	0	0	0	0
normalized size	1	1.	0.93	0.6	0.	0.	0.	0.
time (sec)	N/A	0.567	0.278	0.161	0.	0.	0.	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	259	140	106	0	0	0	0
normalized size	1	1.	0.54	0.41	0.	0.	0.	0.
time (sec)	N/A	0.458	0.18	0.153	0.	0.	0.	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	172	78	0	0	0	0
normalized size	1	1.	0.64	0.29	0.	0.	0.	0.
time (sec)	N/A	0.262	0.184	0.156	0.	0.	0.	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	264	98	0	0	0	0
normalized size	1	1.	1.	0.37	0.	0.	0.	0.
time (sec)	N/A	0.365	0.264	0.188	0.	0.	0.	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	181	144	0	0	0	0
normalized size	1	1.	0.63	0.5	0.	0.	0.	0.
time (sec)	N/A	0.404	0.396	0.213	0.	0.	0.	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	344	344	290	217	0	0	0	0
normalized size	1	1.	0.84	0.63	0.	0.	0.	0.
time (sec)	N/A	0.478	1.994	0.21	0.	0.	0.	0.

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	164	166	0	0	0	0
normalized size	1	1.	0.56	0.57	0.	0.	0.	0.
time (sec)	N/A	0.394	0.273	0.161	0.	0.	0.	0.

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	186	106	0	0	0	0
normalized size	1	1.	0.66	0.38	0.	0.	0.	0.
time (sec)	N/A	0.422	0.257	0.155	0.	0.	0.	0.

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	231	76	0	0	0	0
normalized size	1	1.	0.96	0.32	0.	0.	0.	0.
time (sec)	N/A	0.272	0.184	0.141	0.	0.	0.	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	126	83	0	0	0	0
normalized size	1	1.	0.51	0.34	0.	0.	0.	0.
time (sec)	N/A	0.261	0.125	0.151	0.	0.	0.	0.

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	196	119	0	0	0	0
normalized size	1	1.	0.7	0.42	0.	0.	0.	0.
time (sec)	N/A	0.429	0.31	0.193	0.	0.	0.	0.

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	296	333	176	0	0	0	0
normalized size	1	1.	1.12	0.59	0.	0.	0.	0.
time (sec)	N/A	0.393	2.05	0.207	0.	0.	0.	0.

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	228	159	0	1758	0	0
normalized size	1	1.	1.29	0.9	0.	9.93	0.	0.
time (sec)	N/A	0.252	0.47	0.116	0.	3.01	0.	0.

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	148	148	310	115	0	1662	0	0
normalized size	1	1.	2.09	0.78	0.	11.23	0.	0.
time (sec)	N/A	0.176	0.284	0.09	0.	2.762	0.	0.

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	198	103	0	1592	0	0
normalized size	1	1.	1.43	0.75	0.	11.54	0.	0.
time (sec)	N/A	0.179	0.25	0.085	0.	2.916	0.	0.

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	285	78	0	1435	0	0
normalized size	1	1.	2.48	0.68	0.	12.48	0.	0.
time (sec)	N/A	0.117	0.171	0.085	0.	2.416	0.	0.

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	183	90	0	1480	0	0
normalized size	1	1.	1.46	0.72	0.	11.84	0.	0.
time (sec)	N/A	0.102	0.162	0.095	0.	2.561	0.	0.

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	318	120	0	1601	0	0
normalized size	1	1.	2.34	0.88	0.	11.77	0.	0.
time (sec)	N/A	0.178	0.25	0.121	0.	3.18	0.	0.

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	242	170	0	1905	0	0
normalized size	1	1.	1.32	0.92	0.	10.35	0.	0.
time (sec)	N/A	0.21	0.317	0.132	0.	3.945	0.	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	229	229	409	232	0	2292	0	0
normalized size	1	1.	1.79	1.01	0.	10.01	0.	0.
time (sec)	N/A	0.247	1.126	0.142	0.	4.91	0.	0.

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	172	605	0	2712	0	0
normalized size	1	1.	0.93	3.29	0.	14.74	0.	0.
time (sec)	N/A	0.293	0.892	0.119	0.	3.424	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	157	551	0	2641	0	0
normalized size	1	1.	1.01	3.55	0.	17.04	0.	0.
time (sec)	N/A	0.204	0.806	0.097	0.	3.637	0.	0.

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	143	517	0	2385	0	0
normalized size	1	1.	1.13	4.07	0.	18.78	0.	0.
time (sec)	N/A	0.185	0.412	0.095	0.	3.088	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	137	492	0	2390	0	0
normalized size	1	1.	1.1	3.94	0.	19.12	0.	0.
time (sec)	N/A	0.112	0.349	0.096	0.	3.274	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	128	492	0	2388	0	0
normalized size	1	1.	1.11	4.28	0.	20.77	0.	0.
time (sec)	N/A	0.09	0.256	0.094	0.	3.062	0.	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	143	518	0	2635	0	0
normalized size	1	1.	1.03	3.73	0.	18.96	0.	0.
time (sec)	N/A	0.178	1.043	0.122	0.	3.461	0.	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	165	542	0	2907	0	0
normalized size	1	1.	1.11	3.64	0.	19.51	0.	0.
time (sec)	N/A	0.193	1.523	0.138	0.	3.33	0.	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	174	585	0	3224	0	0
normalized size	1	1.	0.98	3.29	0.	18.11	0.	0.
time (sec)	N/A	0.203	4.566	0.137	0.	3.644	0.	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	277	624	0	3502	0	0
normalized size	1	1.	1.41	3.17	0.	17.78	0.	0.
time (sec)	N/A	0.235	6.32	0.141	0.	3.474	0.	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	236	236	486	482	0	5901	0	0
normalized size	1	1.	2.06	2.04	0.	25.	0.	0.
time (sec)	N/A	0.48	1.114	0.114	0.	5.969	0.	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	210	210	565	394	0	5509	0	0
normalized size	1	1.	2.69	1.88	0.	26.23	0.	0.
time (sec)	N/A	0.335	0.569	0.111	0.	5.102	0.	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	217	217	469	440	0	5237	0	0
normalized size	1	1.	2.16	2.03	0.	24.13	0.	0.
time (sec)	N/A	0.264	0.63	0.105	0.	5.035	0.	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	B	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	186	186	345	213	0	4251	0	0
normalized size	1	1.	1.85	1.15	0.	22.85	0.	0.
time (sec)	N/A	0.185	0.327	0.109	0.	3.839	0.	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	221	221	469	488	0	4799	0	0
normalized size	1	1.	2.12	2.21	0.	21.71	0.	0.
time (sec)	N/A	0.265	0.445	0.146	0.	5.244	0.	0.

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	325	325	600	450	0	6010	0	0
normalized size	1	1.	1.85	1.38	0.	18.49	0.	0.
time (sec)	N/A	0.336	0.831	0.147	0.	9.265	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	262	644	0	8038	0	0
normalized size	1	1.	0.82	2.01	0.	25.12	0.	0.
time (sec)	N/A	0.446	4.956	0.125	0.	9.068	0.	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	238	674	0	6782	0	0
normalized size	1	1.	1.02	2.89	0.	29.11	0.	0.
time (sec)	N/A	0.351	2.586	0.119	0.	8.757	0.	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	225	478	0	5956	0	0
normalized size	1	1.	1.15	2.45	0.	30.54	0.	0.
time (sec)	N/A	0.229	4.235	0.108	0.	5.513	0.	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	255	534	0	7386	0	0
normalized size	1	1.	1.16	2.44	0.	33.73	0.	0.
time (sec)	N/A	0.297	2.089	0.127	0.	8.092	0.	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	230	618	0	7862	0	0
normalized size	1	1.	1.1	2.94	0.	37.44	0.	0.
time (sec)	N/A	0.26	2.824	0.129	0.	8.202	0.	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	274	708	0	8411	0	0
normalized size	1	1.	1.16	3.	0.	35.64	0.	0.
time (sec)	N/A	0.534	2.16	0.154	0.	10.543	0.	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	B	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	315	315	785	1164	0	10985	0	0
normalized size	1	1.	2.49	3.7	0.	34.87	0.	0.
time (sec)	N/A	0.575	1.492	0.145	0.	10.578	0.	0.

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	B	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	290	290	630	814	0	9296	0	0
normalized size	1	1.	2.17	2.81	0.	32.06	0.	0.
time (sec)	N/A	0.435	1.139	0.121	0.	7.899	0.	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	B	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	313	313	786	1167	0	10311	0	0
normalized size	1	1.	2.51	3.73	0.	32.94	0.	0.
time (sec)	N/A	0.472	1.376	0.125	0.	10.201	0.	0.

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	B	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	288	288	631	1153	0	9106	0	0
normalized size	1	1.	2.19	4.	0.	31.62	0.	0.
time (sec)	N/A	0.498	1.12	0.181	0.	8.154	0.	0.

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	B	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	313	313	784	1281	0	9480	0	0
normalized size	1	1.	2.5	4.09	0.	30.29	0.	0.
time (sec)	N/A	0.459	1.243	0.188	0.	10.569	0.	0.

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	B	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	617	617	920	1139	0	12519	0	0
normalized size	1	1.	1.49	1.85	0.	20.29	0.	0.
time (sec)	N/A	0.837	4.28	0.182	0.	25.205	0.	0.

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	319	319	331	1634	0	12051	0	0
normalized size	1	1.	1.04	5.12	0.	37.78	0.	0.
time (sec)	N/A	0.53	4.012	0.135	0.	14.083	0.	0.

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	350	1909	0	13869	0	0
normalized size	1	1.	1.02	5.57	0.	40.43	0.	0.
time (sec)	N/A	0.765	3.856	0.138	0.	21.107	0.	0.

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	316	1624	0	12400	0	0
normalized size	1	1.	1.01	5.19	0.	39.62	0.	0.
time (sec)	N/A	0.695	5.047	0.131	0.	13.617	0.	0.

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	347	347	457	1906	0	15023	0	0
normalized size	1	1.	1.32	5.49	0.	43.29	0.	0.
time (sec)	N/A	0.724	6.43	0.145	0.	24.384	0.	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	319	319	333	1803	0	15823	0	0
normalized size	1	1.	1.04	5.65	0.	49.6	0.	0.
time (sec)	N/A	0.652	3.124	0.148	0.	24.443	0.	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	357	357	357	1959	0	16077	0	0
normalized size	1	1.	1.	5.49	0.	45.03	0.	0.
time (sec)	N/A	1.294	5.277	0.206	0.	26.895	0.	0.

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	45	24	18	23	138	0	69
normalized size	1	1.8	0.96	0.72	0.92	5.52	0.	2.76
time (sec)	N/A	0.019	0.053	0.04	1.415	1.996	0.	1.1

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	487	487	148	1677	0	1893	0	0
normalized size	1	1.	0.3	3.44	0.	3.89	0.	0.
time (sec)	N/A	1.137	0.3	0.169	0.	3.245	0.	0.

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	309	309	45	239	0	0	0	0
normalized size	1	1.	0.15	0.77	0.	0.	0.	0.
time (sec)	N/A	0.202	0.073	0.151	0.	0.	0.	0.

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	477	477	47242	439	0	0	0	0
normalized size	1	1.	99.04	0.92	0.	0.	0.	0.
time (sec)	N/A	0.388	31.593	0.686	0.	0.	0.	0.

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	521	521	118912	0	0	0	0	0
normalized size	1	1.	228.24	0.	0.	0.	0.	0.
time (sec)	N/A	0.678	31.962	0.81	0.	0.	0.	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	484	484	47246	837	0	0	0	0
normalized size	1	1.	97.62	1.73	0.	0.	0.	0.
time (sec)	N/A	0.426	31.72	0.468	0.	0.	0.	0.

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	431	431	89374	398	0	0	0	0
normalized size	1	1.	207.36	0.92	0.	0.	0.	0.
time (sec)	N/A	0.302	31.874	0.525	0.	0.	0.	0.

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	171	171	13300	163	0	0	0	0
normalized size	1	1.	77.78	0.95	0.	0.	0.	0.
time (sec)	N/A	0.105	25.209	0.349	0.	0.	0.	0.

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	469	469	63281	0	0	0	0	0
normalized size	1	1.	134.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.434	31.479	0.75	0.	0.	0.	0.

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	776	776	119171	0	0	0	0	0
normalized size	1	1.	153.57	0.	0.	0.	0.	0.
time (sec)	N/A	1.043	32.729	0.727	0.	0.	0.	0.

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	499	499	287	881	0	0	0	0
normalized size	1	1.	0.58	1.77	0.	0.	0.	0.
time (sec)	N/A	0.665	2.778	4.555	0.	0.	0.	0.

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	304	396	0	0	0	0
normalized size	1	1.	1.88	2.44	0.	0.	0.	0.
time (sec)	N/A	0.079	8.404	2.356	0.	0.	0.	0.

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	493	493	498	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.419	16.219	0.707	0.	0.	0.	0.

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	384	384	149	109	0	0	0	0
normalized size	1	1.	0.39	0.28	0.	0.	0.	0.
time (sec)	N/A	0.714	0.212	0.123	0.	0.	0.	0.

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	148	68	0	0	0	0
normalized size	1	1.	0.87	0.4	0.	0.	0.	0.
time (sec)	N/A	0.257	0.221	0.376	0.	0.	0.	0.

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	245	245	174	85	0	0	0	0
normalized size	1	1.	0.71	0.35	0.	0.	0.	0.
time (sec)	N/A	0.528	0.266	0.118	0.	0.	0.	0.

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	379	379	149	109	0	0	0	0
normalized size	1	1.	0.39	0.29	0.	0.	0.	0.
time (sec)	N/A	0.477	0.191	0.125	0.	0.	0.	0.

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	148	71	0	0	0	0
normalized size	1	1.	0.85	0.41	0.	0.	0.	0.
time (sec)	N/A	0.26	0.173	0.358	0.	0.	0.	0.

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	213	213	174	88	0	0	0	0
normalized size	1	1.	0.82	0.41	0.	0.	0.	0.
time (sec)	N/A	0.218	0.214	0.114	0.	0.	0.	0.

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	411	133	0	0	0	0
normalized size	1	1.	2.11	0.68	0.	0.	0.	0.
time (sec)	N/A	0.383	0.159	0.079	0.	0.	0.	0.

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	79	72	96	466	0	250
normalized size	1	1.	0.77	0.7	0.93	4.52	0.	2.43
time (sec)	N/A	0.104	0.165	0.044	1.468	1.886	0.	1.143

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	129	141	71	0	0	0	0
normalized size	1	0.59	0.65	0.33	0.	0.	0.	0.
time (sec)	N/A	0.2	0.148	0.066	0.	0.	0.	0.

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	413	133	0	0	0	0
normalized size	1	1.	2.21	0.71	0.	0.	0.	0.
time (sec)	N/A	0.269	0.134	0.082	0.	0.	0.	0.

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	117	255	0	0	0	0
normalized size	1	1.	1.65	3.59	0.	0.	0.	0.
time (sec)	N/A	0.135	0.276	0.139	0.	0.	0.	0.

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	64	255	0	0	0	0
normalized size	1	1.	0.72	2.87	0.	0.	0.	0.
time (sec)	N/A	0.077	0.166	0.078	0.	0.	0.	0.

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	35	26	38	80	580	38
normalized size	1	1.	0.92	0.68	1.	2.11	15.26	1.
time (sec)	N/A	0.054	0.004	0.034	0.991	2.459	83.045	1.103

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	27	20	30	61	311	30
normalized size	1	1.	0.93	0.69	1.03	2.1	10.72	1.03
time (sec)	N/A	0.052	0.003	0.035	1.043	2.536	35.157	1.141

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	19	14	19	39	124	19
normalized size	1	1.	1.06	0.78	1.06	2.17	6.89	1.06
time (sec)	N/A	0.05	0.003	0.033	1.014	2.347	16.336	1.136

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	8	14	15	8
normalized size	1	1.	1.	1.17	1.33	2.33	2.5	1.33
time (sec)	N/A	0.043	0.002	0.031	0.983	2.154	6.936	1.139

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	37	8	28	59	19	31
normalized size	1	1.	5.29	1.14	4.	8.43	2.71	4.43
time (sec)	N/A	0.026	0.004	0.028	1.018	1.93	0.431	1.135

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	61	66	69	143	0	63
normalized size	1	1.	1.74	1.89	1.97	4.09	0.	1.8
time (sec)	N/A	0.058	0.121	0.052	0.967	1.944	0.	1.1

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	26	40	50	62	473	49
normalized size	1	1.	0.79	1.21	1.52	1.88	14.33	1.48
time (sec)	N/A	0.055	0.003	0.04	1.505	1.928	30.213	1.096

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	18	25	28	36	153	32
normalized size	1	1.	0.9	1.25	1.4	1.8	7.65	1.6
time (sec)	N/A	0.048	0.003	0.037	1.481	1.829	8.197	1.125

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	8	7	7	2	19
normalized size	1	1.	1.	1.6	1.4	1.4	0.4	3.8
time (sec)	N/A	0.041	0.001	0.033	1.473	1.584	2.113	1.109

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	45	44	50	113	0	51
normalized size	1	1.	2.05	2.	2.27	5.14	0.	2.32
time (sec)	N/A	0.043	0.041	0.046	0.995	1.975	0.	1.12

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	21	14	19	57	0	19
normalized size	1	1.	1.17	0.78	1.06	3.17	0.	1.06
time (sec)	N/A	0.049	0.004	0.042	0.968	1.809	0.	1.168

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	31	20	30	76	0	30
normalized size	1	1.	1.07	0.69	1.03	2.62	0.	1.03
time (sec)	N/A	0.052	0.004	0.046	0.979	1.832	0.	1.165

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	27	20	30	63	0	30
normalized size	1	1.	0.93	0.69	1.03	2.17	0.	1.03
time (sec)	N/A	0.046	0.004	0.035	0.962	1.833	0.	1.112

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	19	14	19	42	144	19
normalized size	1	1.	1.06	0.78	1.06	2.33	8.	1.06
time (sec)	N/A	0.044	0.003	0.033	0.986	1.822	74.165	1.099

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	8	16	19	8
normalized size	1	1.	1.	1.17	1.33	2.67	3.17	1.33
time (sec)	N/A	0.039	0.002	0.03	0.996	1.784	30.179	1.097

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	37	8	28	62	22	31
normalized size	1	1.	5.29	1.14	4.	8.86	3.14	4.43
time (sec)	N/A	0.039	0.004	0.033	0.975	1.922	11.15	1.142

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	45	44	55	116	117	51
normalized size	1	1.	2.05	2.	2.5	5.27	5.32	2.32
time (sec)	N/A	0.032	0.006	0.036	1.002	1.791	1.258	1.11

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	61	66	77	146	0	63
normalized size	1	1.	1.74	1.89	2.2	4.17	0.	1.8
time (sec)	N/A	0.048	0.007	0.049	0.985	1.922	0.	1.117

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	26	40	58	65	549	42
normalized size	1	1.	0.79	1.21	1.76	1.97	16.64	1.27
time (sec)	N/A	0.05	0.003	0.039	1.512	1.943	123.222	1.101

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	18	25	34	39	178	30
normalized size	1	1.	0.9	1.25	1.7	1.95	8.9	1.5
time (sec)	N/A	0.043	0.003	0.037	1.481	1.825	47.708	1.162

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	8	7	9	3	7
normalized size	1	1.	1.	1.6	1.4	1.8	0.6	1.4
time (sec)	N/A	0.037	0.	0.035	1.449	1.671	19.101	1.097

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	8	28	20	8
normalized size	1	1.	1.	1.17	1.33	4.67	3.33	1.33
time (sec)	N/A	0.043	0.002	0.033	0.99	1.852	7.592	1.096

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	31	20	30	78	0	30
normalized size	1	1.	1.07	0.69	1.03	2.69	0.	1.03
time (sec)	N/A	0.047	0.003	0.043	0.978	1.782	0.	1.114

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	41	24	38	97	0	38
normalized size	1	1.	1.11	0.65	1.03	2.62	0.	1.03
time (sec)	N/A	0.048	0.004	0.047	0.981	1.872	0.	1.129

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	87	112	165	205	354	117
normalized size	1	1.	0.8	1.03	1.51	1.88	3.25	1.07
time (sec)	N/A	0.065	0.311	0.046	1.455	1.946	13.746	1.137

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	64	92	131	159	250	90
normalized size	1	1.	0.77	1.11	1.58	1.92	3.01	1.08
time (sec)	N/A	0.051	0.149	0.045	1.468	1.938	4.957	1.138

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	46	70	93	113	150	55
normalized size	1	1.	0.81	1.23	1.63	1.98	2.63	0.96
time (sec)	N/A	0.044	0.08	0.046	1.494	1.892	1.835	1.126

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	33	32	39	72	51	35
normalized size	1	1.	1.1	1.07	1.3	2.4	1.7	1.17
time (sec)	N/A	0.015	0.029	0.02	0.981	1.848	0.363	1.083

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	36	30	41	85	0	66
normalized size	1	1.	2.	1.67	2.28	4.72	0.	3.67
time (sec)	N/A	0.033	0.015	0.05	1.481	1.755	0.	1.123

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	41	46	36	97	0	51
normalized size	1	1.	1.37	1.53	1.2	3.23	0.	1.7
time (sec)	N/A	0.032	0.074	0.056	0.979	1.873	0.	1.168

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	64	76	58	143	0	86
normalized size	1	1.	1.28	1.52	1.16	2.86	0.	1.72
time (sec)	N/A	0.044	0.178	0.06	0.993	1.945	0.	1.136

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	86	104	81	189	0	119
normalized size	1	1.	1.19	1.44	1.12	2.62	0.	1.65
time (sec)	N/A	0.055	0.309	0.063	0.985	1.891	0.	1.168

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	96	167	228	278	481	146
normalized size	1	1.	0.62	1.07	1.46	1.78	3.08	0.94
time (sec)	N/A	0.15	0.318	0.055	1.472	2.304	18.224	1.15

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	79	134	171	204	314	113
normalized size	1	1.	0.68	1.16	1.47	1.76	2.71	0.97
time (sec)	N/A	0.146	0.269	0.05	1.479	1.939	5.989	1.153

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	58	78	92	143	168	81
normalized size	1	1.	0.81	1.08	1.28	1.99	2.33	1.12
time (sec)	N/A	0.02	0.116	0.03	0.973	1.956	1.694	1.108

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	48	87	100	159	0	134
normalized size	1	1.	0.94	1.71	1.96	3.12	0.	2.63
time (sec)	N/A	0.089	0.319	0.06	1.482	1.852	0.	1.132

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	57	76	72	169	0	108
normalized size	1	1.	1.27	1.69	1.6	3.76	0.	2.4
time (sec)	N/A	0.062	0.325	0.072	1.472	1.972	0.	1.145

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	67	101	74	194	0	116
normalized size	1	1.	1.26	1.91	1.4	3.66	0.	2.19
time (sec)	N/A	0.057	0.354	0.072	1.005	1.812	0.	1.163

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	92	149	109	261	0	171
normalized size	1	1.	1.15	1.86	1.36	3.26	0.	2.14
time (sec)	N/A	0.076	0.479	0.075	0.977	1.953	0.	1.152

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	107	195	139	316	0	227
normalized size	1	1.	1.01	1.84	1.31	2.98	0.	2.14
time (sec)	N/A	0.093	0.506	0.075	0.988	1.904	0.	1.17

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	109	136	0	566	0	132
normalized size	1	1.	1.4	1.74	0.	7.26	0.	1.69
time (sec)	N/A	0.09	0.275	0.043	0.	2.445	0.	1.117

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	79	202	0	775	0	177
normalized size	1	1.	0.91	2.32	0.	8.91	0.	2.03
time (sec)	N/A	0.187	0.189	0.085	0.	2.513	0.	1.116

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	84	85	0	393	0	78
normalized size	1	1.	1.56	1.57	0.	7.28	0.	1.44
time (sec)	N/A	0.073	0.174	0.043	0.	2.303	0.	1.131

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	55	111	0	617	0	124
normalized size	1	1.	0.93	1.88	0.	10.46	0.	2.1
time (sec)	N/A	0.108	0.152	0.079	0.	2.055	0.	1.125

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	45	0	262	0	41
normalized size	1	1.	1.	1.25	0.	7.28	0.	1.14
time (sec)	N/A	0.056	0.023	0.045	0.	2.005	0.	1.112

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	58	0	508	0	84
normalized size	1	1.	1.	1.49	0.	13.03	0.	2.15
time (sec)	N/A	0.06	0.056	0.077	0.	2.172	0.	1.12

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	17	0	190	87	22
normalized size	1	1.	1.	0.68	0.	7.6	3.48	0.88
time (sec)	N/A	0.029	0.007	0.027	0.	2.032	1.637	1.121

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	96	55	0	308	0	66
normalized size	1	1.	2.4	1.38	0.	7.7	0.	1.65
time (sec)	N/A	0.051	0.124	0.055	0.	2.284	0.	1.1

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	38	0	641	0	61
normalized size	1	1.	1.	0.97	0.	16.44	0.	1.56
time (sec)	N/A	0.061	0.08	0.08	0.	2.385	0.	1.126

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	147	112	0	556	0	138
normalized size	1	1.	2.41	1.84	0.	9.11	0.	2.26
time (sec)	N/A	0.087	0.315	0.062	0.	2.81	0.	1.147

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	59	75	0	834	0	181
normalized size	1	1.	1.	1.27	0.	14.14	0.	3.07
time (sec)	N/A	0.081	0.21	0.099	0.	2.158	0.	1.115

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	214	204	0	825	0	239
normalized size	1	1.	2.3	2.19	0.	8.87	0.	2.57
time (sec)	N/A	0.145	1.23	0.068	0.	2.969	0.	1.104

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	90	147	0	1098	0	343
normalized size	1	1.	1.03	1.69	0.	12.62	0.	3.94
time (sec)	N/A	0.113	0.371	0.089	0.	2.329	0.	1.114

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	90	211	0	1127	0	236
normalized size	1	1.	0.8	1.87	0.	9.97	0.	2.09
time (sec)	N/A	0.223	0.294	0.091	0.	2.41	0.	1.122

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	118	120	0	641	0	111
normalized size	1	1.	1.64	1.67	0.	8.9	0.	1.54
time (sec)	N/A	0.119	0.328	0.05	0.	2.121	0.	1.126

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	79	132	0	882	0	147
normalized size	1	1.	1.05	1.76	0.	11.76	0.	1.96
time (sec)	N/A	0.106	0.312	0.089	0.	2.105	0.	1.114

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	59	65	0	455	0	76
normalized size	1	1.	1.	1.1	0.	7.71	0.	1.29
time (sec)	N/A	0.058	0.065	0.05	0.	2.115	0.	1.112

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	59	51	0	749	0	104
normalized size	1	1.	1.09	0.94	0.	13.87	0.	1.93
time (sec)	N/A	0.056	0.143	0.077	0.	2.198	0.	1.116

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	39	0	381	340	51
normalized size	1	1.	1.	0.81	0.	7.94	7.08	1.06
time (sec)	N/A	0.035	0.058	0.035	0.	2.026	31.522	1.155

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	130	122	0	826	0	147
normalized size	1	1.	1.78	1.67	0.	11.32	0.	2.01
time (sec)	N/A	0.09	0.561	0.067	0.	2.556	0.	1.106

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	76	112	0	1152	0	153
normalized size	1	1.	1.	1.47	0.	15.16	0.	2.01
time (sec)	N/A	0.124	0.519	0.091	0.	2.319	0.	1.148

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	183	180	0	1274	0	262
normalized size	1	1.	1.68	1.65	0.	11.69	0.	2.4
time (sec)	N/A	0.163	1.012	0.079	0.	3.144	0.	1.141

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	97	193	0	1481	0	365
normalized size	1	1.	1.01	2.01	0.	15.43	0.	3.8
time (sec)	N/A	0.148	0.971	0.102	0.	2.451	0.	1.147

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	125	155	0	1239	0	130
normalized size	1	1.	1.07	1.32	0.	10.59	0.	1.11
time (sec)	N/A	0.118	0.464	1.184	0.	5.15	0.	1.186

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	96	62	0	1106	0	88
normalized size	1	1.	1.33	0.86	0.	15.36	0.	1.22
time (sec)	N/A	0.055	0.271	0.088	0.	2.823	0.	1.189

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	129	155	0	3027	0	0
normalized size	1	1.	1.57	1.89	0.	36.91	0.	0.
time (sec)	N/A	0.092	0.278	3.635	0.	3.976	0.	0.

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	164	290	0	852	0	0
normalized size	1	1.	2.	3.54	0.	10.39	0.	0.
time (sec)	N/A	0.094	2.329	3.897	0.	2.744	0.	0.

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	143	143	669	570	0	1079	0	0
normalized size	1	1.	4.68	3.99	0.	7.55	0.	0.
time (sec)	N/A	0.139	14.378	4.52	0.	5.383	0.	0.

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	260	199	432	0	0	0	0
normalized size	1	1.18	0.9	1.96	0.	0.	0.	0.
time (sec)	N/A	0.275	1.443	1.106	0.	0.	0.	0.

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	199	158	265	0	0	0	0
normalized size	1	1.25	0.99	1.67	0.	0.	0.	0.
time (sec)	N/A	0.184	0.883	1.167	0.	0.	0.	0.

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	61	71	0	0	0	0
normalized size	1	1.	1.2	1.39	0.	0.	0.	0.
time (sec)	N/A	0.035	0.086	0.635	0.	0.	0.	0.

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	171	134	294	0	0	0	0
normalized size	1	1.31	1.02	2.24	0.	0.	0.	0.
time (sec)	N/A	0.165	0.488	1.91	0.	0.	0.	0.

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	236	187	368	0	0	0	0
normalized size	1	1.2	0.95	1.88	0.	0.	0.	0.
time (sec)	N/A	0.223	1.889	2.336	0.	0.	0.	0.

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	149	277	0	1393	0	181
normalized size	1	1.	0.95	1.76	0.	8.87	0.	1.15
time (sec)	N/A	0.138	0.848	1.391	0.	15.831	0.	1.239

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	93	90	0	1218	0	113
normalized size	1	1.	0.89	0.87	0.	11.71	0.	1.09
time (sec)	N/A	0.069	0.512	0.089	0.	5.08	0.	1.196

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	233	451	0	3399	0	0
normalized size	1	1.	1.93	3.73	0.	28.09	0.	0.
time (sec)	N/A	0.144	0.661	3.106	0.	6.545	0.	0.

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	210	402	0	3646	0	0
normalized size	1	1.	1.65	3.17	0.	28.71	0.	0.
time (sec)	N/A	0.146	0.97	3.408	0.	7.192	0.	0.

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	63	406	0	1014	0	0
normalized size	1	1.	0.52	3.33	0.	8.31	0.	0.
time (sec)	N/A	0.121	0.13	2.654	0.	6.284	0.	0.

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	195	195	938	693	0	1310	0	0
normalized size	1	1.	4.81	3.55	0.	6.72	0.	0.
time (sec)	N/A	0.176	15.936	3.241	0.	25.533	0.	0.

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	321	321	247	590	0	0	0	0
normalized size	1	1.	0.77	1.84	0.	0.	0.	0.
time (sec)	N/A	0.389	2.6	1.266	0.	0.	0.	0.

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	259	200	429	0	0	0	0
normalized size	1	1.	0.77	1.66	0.	0.	0.	0.
time (sec)	N/A	0.285	1.321	1.421	0.	0.	0.	0.

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	156	266	0	0	0	0
normalized size	1	1.	1.01	1.73	0.	0.	0.	0.
time (sec)	N/A	0.165	0.731	1.099	0.	0.	0.	0.

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	144	466	0	0	0	0
normalized size	1	1.	0.79	2.56	0.	0.	0.	0.
time (sec)	N/A	0.18	0.84	2.187	0.	0.	0.	0.

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	190	375	0	0	0	0
normalized size	1	1.	0.81	1.59	0.	0.	0.	0.
time (sec)	N/A	0.256	2.037	2.043	0.	0.	0.	0.

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	79	98	0	1133	0	96
normalized size	1	1.	1.	1.24	0.	14.34	0.	1.22
time (sec)	N/A	0.093	0.114	1.06	0.	3.	0.	1.329

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	34	0	953	0	51
normalized size	1	1.	1.	0.89	0.	25.08	0.	1.34
time (sec)	N/A	0.046	0.016	0.088	0.	2.495	0.	1.181

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	113	0	595	0	0
normalized size	1	1.	1.	2.69	0.	14.17	0.	0.
time (sec)	N/A	0.072	0.032	2.483	0.	2.749	0.	0.

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	91	91	436	360	0	906	0	0
normalized size	1	1.	4.79	3.96	0.	9.96	0.	0.
time (sec)	N/A	0.111	9.847	2.79	0.	3.271	0.	0.

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	208	170	316	0	0	0	0
normalized size	1	1.24	1.01	1.88	0.	0.	0.	0.
time (sec)	N/A	0.194	0.854	1.154	0.	0.	0.	0.

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	153	83	113	0	0	0	0
normalized size	1	1.34	0.73	0.99	0.	0.	0.	0.
time (sec)	N/A	0.141	0.2	1.05	0.	0.	0.	0.

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	60	60	0	0	0	0
normalized size	1	1.	1.18	1.18	0.	0.	0.	0.
time (sec)	N/A	0.035	0.075	0.236	0.	0.	0.	0.

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	180	141	278	0	0	0	0
normalized size	1	1.29	1.01	1.99	0.	0.	0.	0.
time (sec)	N/A	0.169	0.609	2.016	0.	0.	0.	0.

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	252	205	405	0	0	0	0
normalized size	1	1.19	0.97	1.91	0.	0.	0.	0.
time (sec)	N/A	0.248	2.176	2.41	0.	0.	0.	0.

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	88	90	0	1324	0	96
normalized size	1	1.	1.17	1.2	0.	17.65	0.	1.28
time (sec)	N/A	0.1	0.179	0.946	0.	4.05	0.	1.376

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	28	36	116	0	39
normalized size	1	1.	1.	0.97	1.24	4.	0.	1.34
time (sec)	N/A	0.044	0.03	0.085	0.97	2.435	0.	1.367

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	78	78	480	398	0	1068	0	0
normalized size	1	1.	6.15	5.1	0.	13.69	0.	0.
time (sec)	N/A	0.101	7.44	3.955	0.	3.166	0.	0.

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	134	134	224	3219	0	1438	0	0
normalized size	1	1.	1.67	24.02	0.	10.73	0.	0.
time (sec)	N/A	0.176	7.47	9.885	0.	6.27	0.	0.

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	184	415	0	0	0	0
normalized size	1	1.	0.67	1.51	0.	0.	0.	0.
time (sec)	N/A	0.29	1.109	1.301	0.	0.	0.	0.

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	139	274	0	0	0	0
normalized size	1	1.	0.69	1.36	0.	0.	0.	0.
time (sec)	N/A	0.206	0.557	1.103	0.	0.	0.	0.

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	133	145	0	0	0	0
normalized size	1	1.	0.71	0.77	0.	0.	0.	0.
time (sec)	N/A	0.184	0.302	1.058	0.	0.	0.	0.

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	90	103	0	0	0	0
normalized size	1	1.	0.89	1.02	0.	0.	0.	0.
time (sec)	N/A	0.06	0.146	1.351	0.	0.	0.	0.

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	167	468	0	0	0	0
normalized size	1	1.	0.7	1.95	0.	0.	0.	0.
time (sec)	N/A	0.239	1.195	2.345	0.	0.	0.	0.

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	128	383	0	1828	0	0
normalized size	1	1.	0.98	2.95	0.	14.06	0.	0.
time (sec)	N/A	0.135	0.81	3.361	0.	12.556	0.	0.

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	51	120	144	250	0	78
normalized size	1	1.	0.7	1.64	1.97	3.42	0.	1.07
time (sec)	N/A	0.092	0.107	3.362	0.975	4.283	0.	1.39

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	47	56	74	243	0	65
normalized size	1	1.	0.72	0.86	1.14	3.74	0.	1.
time (sec)	N/A	0.055	0.047	0.087	0.978	3.646	0.	1.352

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	126	126	1291	899	0	1752	0	0
normalized size	1	1.	10.25	7.13	0.	13.9	0.	0.
time (sec)	N/A	0.159	9.315	4.599	0.	6.394	0.	0.

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	283	194	712	0	0	0	0
normalized size	1	1.16	0.8	2.93	0.	0.	0.	0.
time (sec)	N/A	0.31	2.086	1.57	0.	0.	0.	0.

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	263	171	485	0	0	0	0
normalized size	1	1.18	0.77	2.17	0.	0.	0.	0.
time (sec)	N/A	0.283	1.388	1.336	0.	0.	0.	0.

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	257	175	549	0	0	0	0
normalized size	1	1.18	0.81	2.53	0.	0.	0.	0.
time (sec)	N/A	0.242	1.421	1.753	0.	0.	0.	0.

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	172	547	0	0	0	0
normalized size	1	1.	0.77	2.45	0.	0.	0.	0.
time (sec)	N/A	0.266	1.225	1.664	0.	0.	0.	0.

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	328	245	1082	0	0	0	0
normalized size	1	1.14	0.85	3.76	0.	0.	0.	0.
time (sec)	N/A	0.357	3.3	2.833	0.	0.	0.	0.

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	115	115	228	0	0	0	0	0
normalized size	1	1.	1.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.09	0.833	1.342	0.	0.	0.	0.

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	214	214	191	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.209	0.403	1.368	0.	0.	0.	0.

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	119	120	0	0	0	0	0
normalized size	1	0.96	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.104	0.194	2.313	0.	0.	0.	0.

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	67	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	0.027	1.371	0.	0.	0.	0.

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.074	4.417	0.878	0.	0.	0.	0.

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.082	6.208	0.901	0.	0.	0.	0.

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	90	90	199	0	0	0	0	0
normalized size	1	1.	2.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.082	0.616	1.292	0.	0.	0.	0.

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	90	90	195	0	0	0	0	0
normalized size	1	1.	2.17	0.	0.	0.	0.	0.
time (sec)	N/A	0.082	0.507	1.817	0.	0.	0.	0.

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	90	90	145	0	0	0	0	0
normalized size	1	1.	1.61	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	0.522	0.546	0.	0.	0.	0.

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	90	90	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.08	4.372	0.786	0.	0.	0.	0.

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	90	90	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.081	5.663	0.639	0.	0.	0.	0.

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	203	278	0	7471	0	298
normalized size	1	1.	0.93	1.27	0.	34.11	0.	1.36
time (sec)	N/A	0.29	0.243	0.101	0.	12.522	0.	1.197

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	139	141	0	0	0	211
normalized size	1	1.	0.83	0.84	0.	0.	0.	1.26
time (sec)	N/A	0.148	0.136	0.099	0.	0.	0.	1.141

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	116	120	0	1018	260	185
normalized size	1	1.	0.81	0.83	0.	7.07	1.81	1.28
time (sec)	N/A	0.1	0.056	0.047	0.	2.841	15.964	1.131

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	290	268	374	0	9420	0	417
normalized size	1	1.	0.92	1.29	0.	32.48	0.	1.44
time (sec)	N/A	0.329	0.213	0.128	0.	14.505	0.	1.18

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	385	385	333	668	0	20282	0	689
normalized size	1	1.	0.86	1.74	0.	52.68	0.	1.79
time (sec)	N/A	0.504	2.323	0.146	0.	25.194	0.	1.23

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	764	764	300	123	0	0	0	0
normalized size	1	1.	0.39	0.16	0.	0.	0.	0.
time (sec)	N/A	1.53	0.142	0.214	0.	0.	0.	0.

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	484	484	231	83	0	0	0	0
normalized size	1	1.	0.48	0.17	0.	0.	0.	0.
time (sec)	N/A	0.634	0.101	0.191	0.	0.	0.	0.

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	126	83	0	0	0	0
normalized size	1	1.	0.51	0.34	0.	0.	0.	0.
time (sec)	N/A	0.251	0.158	0.161	0.	0.	0.	0.

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	299	0	432	164	0	0	0	0
normalized size	1	0.	1.44	0.55	0.	0.	0.	0.
time (sec)	N/A	0.045	0.253	0.213	0.	0.	0.	0.

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	1093	0	679	346	0	0	0	0
normalized size	1	0.	0.62	0.32	0.	0.	0.	0.
time (sec)	N/A	0.046	1.691	0.241	0.	0.	0.	0.

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	402	490	0	0	0	374
normalized size	1	1.	1.4	1.7	0.	0.	0.	1.3
time (sec)	N/A	0.335	3.598	0.145	0.	0.	0.	1.236

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	258	327	0	0	0	308
normalized size	1	1.	1.08	1.37	0.	0.	0.	1.29
time (sec)	N/A	0.225	1.037	0.145	0.	0.	0.	1.195

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	184	179	0	1609	0	228
normalized size	1	1.	1.01	0.98	0.	8.79	0.	1.25
time (sec)	N/A	0.162	0.842	0.141	0.	2.707	0.	1.189

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	152	157	0	1588	0	219
normalized size	1	1.	0.86	0.89	0.	9.02	0.	1.24
time (sec)	N/A	0.115	0.475	0.064	0.	2.737	0.	1.229

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	587	587	503	934	0	22565	0	760
normalized size	1	1.	0.86	1.59	0.	38.44	0.	1.29
time (sec)	N/A	0.688	4.368	0.182	0.	22.688	0.	1.265

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	747	747	657	1309	0	0	0	1064
normalized size	1	1.	0.88	1.75	0.	0.	0.	1.42
time (sec)	N/A	1.022	6.378	0.208	0.	0.	0.	1.279

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	25	0	394	550	0	0	0	0
normalized size	1	0.	15.76	22.	0.	0.	0.	0.
time (sec)	N/A	0.044	0.352	0.244	0.	0.	0.	0.

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	F(-1)	F(-1)	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	25	0	273	236	0	0	0	0
normalized size	1	0.	10.92	9.44	0.	0.	0.	0.
time (sec)	N/A	0.043	0.245	0.247	0.	0.	0.	0.

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	F(-1)	F(-1)	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	16	0	502	658	0	0	0	0
normalized size	1	0.	31.38	41.12	0.	0.	0.	0.
time (sec)	N/A	0.012	0.46	0.198	0.	0.	0.	0.

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	F(-1)	F(-1)	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	25	0	845	1276	0	0	0	0
normalized size	1	0.	33.8	51.04	0.	0.	0.	0.
time (sec)	N/A	0.043	1.591	0.269	0.	0.	0.	0.

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	F(-1)	F(-1)	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	25	0	1158	1549	0	0	0	0
normalized size	1	0.	46.32	61.96	0.	0.	0.	0.
time (sec)	N/A	0.043	1.703	0.323	0.	0.	0.	0.

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	207	350	0	3235	0	486
normalized size	1	1.	1.58	2.67	0.	24.69	0.	3.71
time (sec)	N/A	0.19	0.268	0.084	0.	5.499	0.	6.47

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	189	252	0	2337	0	420
normalized size	1	1.	1.67	2.23	0.	20.68	0.	3.72
time (sec)	N/A	0.162	0.211	0.069	0.	3.874	0.	6.395

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	160	160	0	1411	0	378
normalized size	1	1.	1.68	1.68	0.	14.85	0.	3.98
time (sec)	N/A	0.102	0.082	0.071	0.	2.876	0.	5.905

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	54	81	0	0	165	302
normalized size	1	1.	0.76	1.14	0.	0.	2.32	4.25
time (sec)	N/A	0.067	0.024	0.033	0.	0.	16.052	6.585

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	184	229	0	2836	0	500
normalized size	1	1.	1.57	1.96	0.	24.24	0.	4.27
time (sec)	N/A	0.153	0.182	0.095	0.	4.221	0.	6.212

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	255	415	0	5469	0	635
normalized size	1	1.	1.46	2.37	0.	31.25	0.	3.63
time (sec)	N/A	0.214	1.01	0.108	0.	9.822	0.	5.737

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	317	660	0	8404	0	851
normalized size	1	1.	1.27	2.65	0.	33.75	0.	3.42
time (sec)	N/A	0.297	5.65	0.119	0.	17.973	0.	6.379

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	233	880	0	6926	0	0
normalized size	1	1.	0.92	3.49	0.	27.48	0.	0.
time (sec)	N/A	0.44	0.919	0.132	0.	13.998	0.	0.

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	200	750	0	5565	0	0
normalized size	1	1.	1.08	4.03	0.	29.92	0.	0.
time (sec)	N/A	0.326	0.682	0.127	0.	8.867	0.	0.

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	194	483	0	3996	0	0
normalized size	1	1.	1.25	3.12	0.	25.78	0.	0.
time (sec)	N/A	0.286	0.489	0.125	0.	5.907	0.	0.

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	171	449	0	2688	0	0
normalized size	1	1.	1.35	3.54	0.	21.17	0.	0.
time (sec)	N/A	0.235	0.252	0.132	0.	3.918	0.	0.

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	158	226	0	1277	0	0
normalized size	1	1.	1.26	1.81	0.	10.22	0.	0.
time (sec)	N/A	0.109	0.282	0.112	0.	2.765	0.	0.

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	175	393	0	5536	0	0
normalized size	1	1.	1.23	2.77	0.	38.99	0.	0.
time (sec)	N/A	0.232	0.498	0.128	0.	5.377	0.	0.

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	205	581	0	9225	0	0
normalized size	1	1.	1.27	3.61	0.	57.3	0.	0.
time (sec)	N/A	0.348	1.011	0.157	0.	7.969	0.	0.

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	253	839	0	13045	0	0
normalized size	1	1.	1.24	4.11	0.	63.95	0.	0.
time (sec)	N/A	0.378	1.347	0.148	0.	14.173	0.	0.

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	8.211	1.721	0.	0.	0.	0.

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	191	141	0	0	0	0	0
normalized size	1	0.97	0.72	0.	0.	0.	0.	0.
time (sec)	N/A	0.222	0.147	2.021	0.	0.	0.	0.

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	106	0	0	0	0	0
normalized size	1	1.	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.098	0.049	3.415	0.	0.	0.	0.

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	67	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.042	0.016	1.985	0.	0.	0.	0.

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	158	158	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.15	5.95	1.202	0.	0.	0.	0.

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	239	239	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.217	8.727	1.212	0.	0.	0.	0.

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	5.38	1.881	0.	0.	0.	0.

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	17.34	2.797	0.	0.	0.	0.

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.011	1.101	0.727	0.	0.	0.	0.

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	5.737	1.075	0.	0.	0.	0.

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	7.616	0.832	0.	0.	0.	0.

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	4.369	1.533	0.	0.	0.	0.

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	155	0	0	0	0	0
normalized size	1	1.	0.69	0.	0.	0.	0.	0.
time (sec)	N/A	0.174	0.218	0.886	0.	0.	0.	0.

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	114	0	0	0	0	0
normalized size	1	1.	0.77	0.	0.	0.	0.	0.
time (sec)	N/A	0.118	0.086	0.757	0.	0.	0.	0.

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	69	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	0.022	0.7	0.	0.	0.	0.

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	2.844	0.52	0.	0.	0.	0.

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	5.288	0.54	0.	0.	0.	0.

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	19.306	0.807	0.	0.	0.	0.

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.048	11.833	0.701	0.	0.	0.	0.

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	1.586	0.463	0.	0.	0.	0.

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.048	2.631	0.539	0.	0.	0.	0.

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	3.728	0.557	0.	0.	0.	0.

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	113	170	369	420	0	814
normalized size	1	1.	0.88	1.33	2.88	3.28	0.	6.36
time (sec)	N/A	0.132	0.286	0.086	1.013	4.124	0.	6.97

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	78	109	215	288	0	531
normalized size	1	1.	0.83	1.16	2.29	3.06	0.	5.65
time (sec)	N/A	0.102	0.276	0.085	1.036	2.97	0.	3.145

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	52	66	111	193	0	316
normalized size	1	1.	0.81	1.03	1.73	3.02	0.	4.94
time (sec)	N/A	0.076	0.093	0.078	1.008	2.259	0.	1.601

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	37	47	58	99	0	149
normalized size	1	1.	0.86	1.09	1.35	2.3	0.	3.47
time (sec)	N/A	0.039	0.032	0.065	0.984	1.874	0.	1.14

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	37	50	96	0	51
normalized size	1	1.	1.	0.97	1.32	2.53	0.	1.34
time (sec)	N/A	0.043	0.022	0.044	1.019	1.828	0.	1.107

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	50	161	76	223	0	146
normalized size	1	1.	0.79	2.56	1.21	3.54	0.	2.32
time (sec)	N/A	0.075	0.141	0.094	0.986	2.022	0.	1.185

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	72	302	124	491	0	277
normalized size	1	1.	0.81	3.39	1.39	5.52	0.	3.11
time (sec)	N/A	0.089	0.564	0.099	0.998	2.288	0.	1.223

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	100	489	185	861	0	477
normalized size	1	1.	0.83	4.04	1.53	7.12	0.	3.94
time (sec)	N/A	0.112	0.268	0.108	1.011	2.84	0.	1.254

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	147	252	0	1453	0	637
normalized size	1	1.	1.22	2.1	0.	12.11	0.	5.31
time (sec)	N/A	0.125	2.401	0.118	0.	2.286	0.	9.942

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	111	161	0	1122	0	400
normalized size	1	1.	1.14	1.66	0.	11.57	0.	4.12
time (sec)	N/A	0.107	0.785	0.121	0.	2.073	0.	4.582

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	75	94	0	879	0	221
normalized size	1	1.	1.01	1.27	0.	11.88	0.	2.99
time (sec)	N/A	0.098	0.31	0.109	0.	1.911	0.	2.189

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	53	53	0	728	0	116
normalized size	1	1.	1.	1.	0.	13.74	0.	2.19
time (sec)	N/A	0.076	0.129	0.104	0.	1.878	0.	1.354

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	52	82	0	698	0	115
normalized size	1	1.	1.	1.58	0.	13.42	0.	2.21
time (sec)	N/A	0.068	0.183	0.11	0.	1.77	0.	1.133

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	72	147	0	973	0	162
normalized size	1	1.	1.01	2.07	0.	13.7	0.	2.28
time (sec)	N/A	0.079	0.292	0.121	0.	1.83	0.	1.17

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	101	239	0	1413	0	231
normalized size	1	1.	1.05	2.49	0.	14.72	0.	2.41
time (sec)	N/A	0.096	0.897	0.126	0.	1.904	0.	1.19

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	135	342	0	2006	0	321
normalized size	1	1.	1.15	2.92	0.	17.15	0.	2.74
time (sec)	N/A	0.112	1.086	0.132	0.	2.043	0.	1.2

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	51	48	93	122	0	184
normalized size	1	1.	0.8	0.75	1.45	1.91	0.	2.88
time (sec)	N/A	0.11	0.094	2.165	1.015	1.652	0.	2.749

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	29	35	62	86	0	53
normalized size	1	1.	0.76	0.92	1.63	2.26	0.	1.39
time (sec)	N/A	0.105	0.081	2.115	1.003	1.596	0.	1.503

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	21	0	36	0	53
normalized size	1	1.	1.	1.11	0.	1.89	0.	2.79
time (sec)	N/A	0.063	0.04	0.131	0.	1.605	0.	1.134

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	55	55	0	146	0	69
normalized size	1	1.	1.1	1.1	0.	2.92	0.	1.38
time (sec)	N/A	0.079	0.069	1.521	0.	1.652	0.	1.098

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	88	83	0	230	0	231
normalized size	1	1.	1.01	0.95	0.	2.64	0.	2.66
time (sec)	N/A	0.118	0.432	1.508	0.	1.696	0.	1.243

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	75	120	2639	232	0	339
normalized size	1	1.	0.62	1.	21.99	1.93	0.	2.82
time (sec)	N/A	0.129	0.333	1.229	6.591	1.732	0.	3.916

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	55	84	1116	204	0	285
normalized size	1	1.	0.6	0.92	12.26	2.24	0.	3.13
time (sec)	N/A	0.124	0.195	1.273	1.918	1.685	0.	1.983

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	40	54	99	147	0	184
normalized size	1	1.	0.7	0.95	1.74	2.58	0.	3.23
time (sec)	N/A	0.103	0.046	1.321	1.699	1.699	0.	1.378

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	35	43	57	101	0	122
normalized size	1	1.	0.61	0.75	1.	1.77	0.	2.14
time (sec)	N/A	0.112	0.075	0.707	1.487	1.578	0.	1.194

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	47	55	77	166	0	178
normalized size	1	1.	0.52	0.6	0.85	1.82	0.	1.96
time (sec)	N/A	0.115	0.076	0.624	1.56	1.659	0.	1.24

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	67	65	92	221	0	235
normalized size	1	1.	0.54	0.52	0.74	1.78	0.	1.9
time (sec)	N/A	0.122	0.196	0.803	1.502	1.649	0.	1.24

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	43	51	93	127	0	96
normalized size	1	1.	0.66	0.78	1.43	1.95	0.	1.48
time (sec)	N/A	0.115	0.081	1.846	1.032	1.611	0.	2.915

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	31	41	62	99	0	77
normalized size	1	1.	0.74	0.98	1.48	2.36	0.	1.83
time (sec)	N/A	0.107	0.064	1.779	1.029	1.612	0.	1.652

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	20	0	61	0	55
normalized size	1	1.	1.	1.11	0.	3.39	0.	3.06
time (sec)	N/A	0.065	0.027	0.148	0.	1.571	0.	1.218

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	49	40	0	208	0	43
normalized size	1	1.	1.58	1.29	0.	6.71	0.	1.39
time (sec)	N/A	0.076	0.043	0.797	0.	1.732	0.	1.133

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	80	69	0	207	0	158
normalized size	1	1.	1.21	1.05	0.	3.14	0.	2.39
time (sec)	N/A	0.114	0.176	1.388	0.	1.666	0.	1.343

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	66	103	2049	208	0	271
normalized size	1	1.	0.73	1.13	22.52	2.29	0.	2.98
time (sec)	N/A	0.139	0.13	1.333	2.683	1.758	0.	2.141

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	43	65	711	173	0	228
normalized size	1	1.	0.69	1.05	11.47	2.79	0.	3.68
time (sec)	N/A	0.119	0.045	1.423	1.733	1.685	0.	1.458

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	32	122	77	0	90
normalized size	1	1.	1.	1.28	4.88	3.08	0.	3.6
time (sec)	N/A	0.102	0.029	0.246	1.72	1.544	0.	1.279

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	37	44	709	143	0	134
normalized size	1	1.	0.62	0.73	11.82	2.38	0.	2.23
time (sec)	N/A	0.117	0.06	0.717	1.709	1.581	0.	1.345

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	49	54	1669	205	0	173
normalized size	1	1.	0.51	0.56	17.39	2.14	0.	1.8
time (sec)	N/A	0.121	0.075	0.714	1.777	1.616	0.	1.405

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	51	51	93	132	0	0
normalized size	1	1.	0.75	0.75	1.37	1.94	0.	0.
time (sec)	N/A	0.127	0.107	1.991	1.035	1.675	0.	0.

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	34	41	65	103	0	78
normalized size	1	1.	0.77	0.93	1.48	2.34	0.	1.77
time (sec)	N/A	0.119	0.111	1.908	1.034	1.591	0.	1.306

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	21	0	69	0	49
normalized size	1	1.	1.	1.	0.	3.29	0.	2.33
time (sec)	N/A	0.074	0.03	0.119	0.	1.562	0.	1.25

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	55	75	0	155	0	80
normalized size	1	1.	1.04	1.42	0.	2.92	0.	1.51
time (sec)	N/A	0.092	0.067	2.359	0.	1.686	0.	1.127

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	82	69	0	212	0	174
normalized size	1	1.	1.24	1.05	0.	3.21	0.	2.64
time (sec)	N/A	0.127	0.172	1.306	0.	1.609	0.	1.358

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	59	104	2068	205	0	0
normalized size	1	1.	0.56	0.98	19.51	1.93	0.	0.
time (sec)	N/A	0.157	0.095	1.285	2.657	1.716	0.	0.

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	44	65	297	170	0	95
normalized size	1	1.	0.7	1.03	4.71	2.7	0.	1.51
time (sec)	N/A	0.132	0.076	1.17	1.72	1.743	0.	1.309

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	29	35	516	117	0	153
normalized size	1	1.	0.76	0.92	13.58	3.08	0.	4.03
time (sec)	N/A	0.126	0.034	0.528	1.668	1.656	0.	1.379

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	41	67	1435	185	0	204
normalized size	1	1.	0.53	0.87	18.64	2.4	0.	2.65
time (sec)	N/A	0.144	0.093	0.855	1.75	1.686	0.	1.424

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	51	57	2735	247	0	248
normalized size	1	1.	0.44	0.5	23.78	2.15	0.	2.16
time (sec)	N/A	0.152	0.136	0.731	1.823	1.677	0.	1.499

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	143	721	0	871	0	0
normalized size	1	1.	0.81	4.07	0.	4.92	0.	0.
time (sec)	N/A	0.211	0.6	4.911	0.	8.29	0.	0.

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	84	403	0	609	0	0
normalized size	1	1.	0.71	3.42	0.	5.16	0.	0.
time (sec)	N/A	0.111	0.399	4.355	0.	4.748	0.	0.

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	60	134	0	370	0	0
normalized size	1	1.	1.03	2.31	0.	6.38	0.	0.
time (sec)	N/A	0.056	0.058	3.81	0.	3.364	0.	0.

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	53	61	0	347	0	66
normalized size	1	1.	0.98	1.13	0.	6.43	0.	1.22
time (sec)	N/A	0.065	0.047	1.001	0.	6.169	0.	1.099

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	77	130	0	595	0	0
normalized size	1	1.	0.7	1.18	0.	5.41	0.	0.
time (sec)	N/A	0.103	0.199	1.22	0.	8.425	0.	0.

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	103	230	0	965	0	0
normalized size	1	1.	0.62	1.39	0.	5.85	0.	0.
time (sec)	N/A	0.155	0.589	1.502	0.	11.546	0.	0.

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	198	380	0	0	0	0
normalized size	1	1.	0.85	1.62	0.	0.	0.	0.
time (sec)	N/A	0.277	2.041	2.718	0.	0.	0.	0.

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	140	294	0	0	0	0
normalized size	1	1.	0.82	1.72	0.	0.	0.	0.
time (sec)	N/A	0.159	0.484	1.994	0.	0.	0.	0.

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	61	71	0	0	0	0
normalized size	1	1.	1.2	1.39	0.	0.	0.	0.
time (sec)	N/A	0.033	0.085	0.66	0.	0.	0.	0.

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	143	156	0	0	0	0
normalized size	1	1.	0.82	0.9	0.	0.	0.	0.
time (sec)	N/A	0.172	0.568	1.21	0.	0.	0.	0.

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	197	351	0	0	0	0
normalized size	1	1.	0.85	1.51	0.	0.	0.	0.
time (sec)	N/A	0.268	3.295	1.322	0.	0.	0.	0.

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	160	711	0	953	0	0
normalized size	1	1.	0.73	3.23	0.	4.33	0.	0.
time (sec)	N/A	0.258	2.184	3.613	0.	7.071	0.	0.

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	116	567	0	672	0	0
normalized size	1	1.	0.78	3.83	0.	4.54	0.	0.
time (sec)	N/A	0.136	0.535	3.349	0.	4.529	0.	0.

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	79	423	0	478	0	0
normalized size	1	1.	0.94	5.04	0.	5.69	0.	0.
time (sec)	N/A	0.078	0.159	2.803	0.	4.042	0.	0.

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	69	91	0	439	0	96
normalized size	1	1.	0.88	1.17	0.	5.63	0.	1.23
time (sec)	N/A	0.079	0.128	1.359	0.	6.375	0.	1.115

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	90	179	0	699	0	0
normalized size	1	1.	0.64	1.28	0.	4.99	0.	0.
time (sec)	N/A	0.127	0.481	1.432	0.	9.405	0.	0.

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	123	280	0	1089	0	0
normalized size	1	1.	0.59	1.35	0.	5.24	0.	0.
time (sec)	N/A	0.192	0.842	1.688	0.	12.947	0.	0.

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	211	419	0	0	0	0
normalized size	1	1.	0.77	1.52	0.	0.	0.	0.
time (sec)	N/A	0.369	2.874	2.241	0.	0.	0.	0.

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	174	515	0	0	0	0
normalized size	1	1.	0.78	2.32	0.	0.	0.	0.
time (sec)	N/A	0.231	2.939	2.208	0.	0.	0.	0.

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	156	266	0	0	0	0
normalized size	1	1.	1.01	1.73	0.	0.	0.	0.
time (sec)	N/A	0.167	0.767	1.317	0.	0.	0.	0.

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	173	204	0	0	0	0
normalized size	1	1.	0.78	0.91	0.	0.	0.	0.
time (sec)	N/A	0.257	2.266	1.258	0.	0.	0.	0.

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	218	419	0	0	0	0
normalized size	1	1.	0.79	1.52	0.	0.	0.	0.
time (sec)	N/A	0.351	5.054	1.303	0.	0.	0.	0.

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	108	644	0	792	0	0
normalized size	1	1.	0.81	4.81	0.	5.91	0.	0.
time (sec)	N/A	0.174	0.493	3.356	0.	3.039	0.	0.

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	77	353	0	563	0	0
normalized size	1	1.	0.95	4.36	0.	6.95	0.	0.
time (sec)	N/A	0.098	0.226	2.783	0.	2.533	0.	0.

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	38	113	0	286	0	132
normalized size	1	1.	1.06	3.14	0.	7.94	0.	3.67
time (sec)	N/A	0.051	0.04	2.641	0.	2.085	0.	1.263

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	42	0	255	0	42
normalized size	1	1.	1.	1.27	0.	7.73	0.	1.27
time (sec)	N/A	0.064	0.036	0.762	0.	2.524	0.	1.094

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	71	114	0	537	0	0
normalized size	1	1.	0.95	1.52	0.	7.16	0.	0.
time (sec)	N/A	0.09	0.169	1.213	0.	2.733	0.	0.

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	101	219	0	923	0	0
normalized size	1	1.	0.8	1.74	0.	7.33	0.	0.
time (sec)	N/A	0.129	0.367	1.662	0.	3.086	0.	0.

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	188	377	0	0	0	0
normalized size	1	1.	0.76	1.53	0.	0.	0.	0.
time (sec)	N/A	0.238	2.183	2.59	0.	0.	0.	0.

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	100	222	0	0	0	0
normalized size	1	1.	0.92	2.04	0.	0.	0.	0.
time (sec)	N/A	0.118	0.393	2.065	0.	0.	0.	0.

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	60	60	0	0	0	0
normalized size	1	1.	1.18	1.18	0.	0.	0.	0.
time (sec)	N/A	0.034	0.077	0.207	0.	0.	0.	0.

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	101	120	0	0	0	0
normalized size	1	1.	0.95	1.13	0.	0.	0.	0.
time (sec)	N/A	0.115	0.372	1.09	0.	0.	0.	0.

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	186	351	0	0	0	0
normalized size	1	1.	0.78	1.46	0.	0.	0.	0.
time (sec)	N/A	0.268	4.107	1.243	0.	0.	0.	0.

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	107	3763	0	1345	0	0
normalized size	1	1.	0.6	21.26	0.	7.6	0.	0.
time (sec)	N/A	0.229	0.486	12.073	0.	4.167	0.	0.

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	75	2199	0	1026	0	0
normalized size	1	1.	0.64	18.64	0.	8.69	0.	0.
time (sec)	N/A	0.122	0.117	9.714	0.	3.273	0.	0.

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	54	1317	0	672	0	0
normalized size	1	1.	0.86	20.9	0.	10.67	0.	0.
time (sec)	N/A	0.068	0.071	8.316	0.	2.397	0.	0.

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	46	64	0	536	0	77
normalized size	1	1.	0.81	1.12	0.	9.4	0.	1.35
time (sec)	N/A	0.076	0.057	1.235	0.	2.543	0.	1.11

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	70	159	0	956	0	0
normalized size	1	1.	0.64	1.45	0.	8.69	0.	0.
time (sec)	N/A	0.123	0.102	1.547	0.	2.653	0.	0.

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	94	288	0	1539	0	0
normalized size	1	1.	0.56	1.72	0.	9.22	0.	0.
time (sec)	N/A	0.163	0.34	1.669	0.	2.915	0.	0.

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	292	197	368	0	0	0	0
normalized size	1	1.	0.67	1.26	0.	0.	0.	0.
time (sec)	N/A	0.315	2.326	2.556	0.	0.	0.	0.

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	145	278	0	0	0	0
normalized size	1	1.	0.65	1.24	0.	0.	0.	0.
time (sec)	N/A	0.226	0.886	2.174	0.	0.	0.	0.

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	90	103	0	0	0	0
normalized size	1	1.	0.89	1.02	0.	0.	0.	0.
time (sec)	N/A	0.062	0.149	1.346	0.	0.	0.	0.

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	142	141	0	0	0	0
normalized size	1	1.	0.68	0.67	0.	0.	0.	0.
time (sec)	N/A	0.242	0.764	1.23	0.	0.	0.	0.

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	199	353	0	0	0	0
normalized size	1	1.	0.67	1.19	0.	0.	0.	0.
time (sec)	N/A	0.374	4.018	1.395	0.	0.	0.	0.

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	107	2139	0	2287	0	0
normalized size	1	1.	0.49	9.81	0.	10.49	0.	0.
time (sec)	N/A	0.277	0.485	7.612	0.	4.639	0.	0.

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	76	1256	0	1746	0	0
normalized size	1	1.	0.5	8.21	0.	11.41	0.	0.
time (sec)	N/A	0.154	0.118	6.123	0.	3.423	0.	0.

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	56	895	0	1226	0	0
normalized size	1	1.	0.62	9.84	0.	13.47	0.	0.
time (sec)	N/A	0.085	0.082	5.415	0.	2.623	0.	0.

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	49	271	0	911	0	100
normalized size	1	1.	0.59	3.27	0.	10.98	0.	1.2
time (sec)	N/A	0.088	0.064	2.888	0.	2.578	0.	1.109

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	69	1038	0	1540	0	0
normalized size	1	1.	0.48	7.26	0.	10.77	0.	0.
time (sec)	N/A	0.137	0.262	4.118	0.	2.777	0.	0.

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	117	901	0	2334	0	0
normalized size	1	1.	0.56	4.33	0.	11.22	0.	0.
time (sec)	N/A	0.196	0.868	4.849	0.	3.168	0.	0.

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	348	348	235	666	0	0	0	0
normalized size	1	1.	0.68	1.91	0.	0.	0.	0.
time (sec)	N/A	0.426	3.446	3.062	0.	0.	0.	0.

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	292	199	851	0	0	0	0
normalized size	1	1.	0.68	2.91	0.	0.	0.	0.
time (sec)	N/A	0.309	2.695	2.592	0.	0.	0.	0.

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	172	547	0	0	0	0
normalized size	1	1.	0.77	2.45	0.	0.	0.	0.
time (sec)	N/A	0.259	1.385	1.721	0.	0.	0.	0.

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	209	411	0	0	0	0
normalized size	1	1.	0.73	1.43	0.	0.	0.	0.
time (sec)	N/A	0.366	2.704	1.698	0.	0.	0.	0.

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	348	348	226	633	0	0	0	0
normalized size	1	1.	0.65	1.82	0.	0.	0.	0.
time (sec)	N/A	0.523	3.116	1.55	0.	0.	0.	0.

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	121	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.122	0.502	1.605	0.	0.	0.	0.

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	83	0	0	0	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.094	0.236	0.889	0.	0.	0.	0.

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	61	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	0.062	1.234	0.	0.	0.	0.

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	0.059	1.125	0.	0.	0.	0.

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	73	0	0	0	0	0
normalized size	1	1.	0.77	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	0.445	0.809	0.	0.	0.	0.

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	102	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.127	0.539	0.517	0.	0.	0.	0.

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	102	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.105	0.289	0.664	0.	0.	0.	0.

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	98	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.1	0.198	0.686	0.	0.	0.	0.

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	102	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.102	0.237	0.506	0.	0.	0.	0.

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	143	126	0	0	0	194
normalized size	1	1.	0.93	0.82	0.	0.	0.	1.27
time (sec)	N/A	0.191	0.297	0.078	0.	0.	0.	1.112

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	34	0	0	0	51
normalized size	1	1.	1.	0.76	0.	0.	0.	1.13
time (sec)	N/A	0.074	0.028	0.48	0.	0.	0.	1.095

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	21	0	0	0	32
normalized size	1	1.	1.	0.75	0.	0.	0.	1.14
time (sec)	N/A	0.072	0.018	0.115	0.	0.	0.	1.084

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	55	0	0	514	0	0
normalized size	1	1.	0.93	0.	0.	8.71	0.	0.
time (sec)	N/A	0.089	0.054	0.754	0.	2.922	0.	0.

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	85	0	0	895	0	0
normalized size	1	1.	0.96	0.	0.	10.06	0.	0.
time (sec)	N/A	0.117	0.186	0.72	0.	3.047	0.	0.

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	65	72	0	589	0	0
normalized size	1	1.	1.27	1.41	0.	11.55	0.	0.
time (sec)	N/A	0.056	0.091	0.203	0.	2.668	0.	0.

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	0	0	369	0	42
normalized size	1	1.	1.	0.	0.	10.54	0.	1.2
time (sec)	N/A	0.071	0.025	0.717	0.	2.474	0.	1.145

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	66	0	0	640	0	0
normalized size	1	1.	0.94	0.	0.	9.14	0.	0.
time (sec)	N/A	0.099	0.082	0.71	0.	2.543	0.	0.

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	141	0	0	913	0	0
normalized size	1	1.	1.31	0.	0.	8.45	0.	0.
time (sec)	N/A	0.176	2.993	0.736	0.	2.779	0.	0.

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	411	411	291	0	0	0	0	0
normalized size	1	1.	0.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.38	6.074	0.71	0.	0.	0.	0.

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	304	396	0	0	0	0
normalized size	1	1.	1.88	2.44	0.	0.	0.	0.
time (sec)	N/A	0.083	2.787	2.372	0.	0.	0.	0.

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	477	477	378	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.366	11.231	0.71	0.	0.	0.	0.

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.042	5.412	1.996	0.	0.	0.	0.

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	279	279	2007	0	0	0	0	0
normalized size	1	1.	7.19	0.	0.	0.	0.	0.
time (sec)	N/A	0.294	17.269	1.094	0.	0.	0.	0.

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	141	141	466	0	0	0	0	0
normalized size	1	1.	3.3	0.	0.	0.	0.	0.
time (sec)	N/A	0.119	8.081	1.684	0.	0.	0.	0.

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.065	0.091	1.563	0.	0.	0.	0.

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	119	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.102	0.65	1.055	0.	0.	0.	0.

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	3.397	0.656	0.	0.	0.	0.

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.042	1.591	0.886	0.	0.	0.	0.

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.011	2.182	0.784	0.	0.	0.	0.

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	1.416	0.904	0.	0.	0.	0.

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	2.228	0.632	0.	0.	0.	0.

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	306	306	3544	0	0	0	0	0
normalized size	1	1.	11.58	0.	0.	0.	0.	0.
time (sec)	N/A	0.429	19.974	180.	0.	0.	0.	0.

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	215	215	2368	0	0	0	0	0
normalized size	1	1.	11.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.295	15.447	180.	0.	0.	0.	0.

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	124	124	1395	0	0	0	0	0
normalized size	1	1.	11.25	0.	0.	0.	0.	0.
time (sec)	N/A	0.161	13.521	180.	0.	0.	0.	0.

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	2.379	0.75	0.	0.	0.	0.

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	F	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	23.488	180.	0.	0.	0.	0.

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	45	38	0	252	0	0
normalized size	1	1.	0.96	0.81	0.	5.36	0.	0.
time (sec)	N/A	0.081	0.028	0.124	0.	1.949	0.	0.

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	0	190	0	0
normalized size	1	1.	1.	0.83	0.	6.55	0.	0.
time (sec)	N/A	0.079	0.017	0.04	0.	1.932	0.	0.

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	3.887	2.234	0.	0.	0.	0.

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	2.764	0.533	0.	0.	0.	0.

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	2.489	0.634	0.	0.	0.	0.

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	0.058	0.624	0.	0.	0.	0.

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	129	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.152	1.012	0.56	0.	0.	0.	0.

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	2.771	0.537	0.	0.	0.	0.

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	2.07	0.519	0.	0.	0.	0.

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	1.226	0.461	0.	0.	0.	0.

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	2.006	0.53	0.	0.	0.	0.

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	2.728	0.569	0.	0.	0.	0.

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	114	102	327	0	0	0	0
normalized size	1	1.07	0.95	3.06	0.	0.	0.	0.
time (sec)	N/A	0.181	0.185	0.415	0.	0.	0.	0.

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	135	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.211	0.78	3.26	0.	0.	0.	0.

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	B	F	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	79	0	325	4061599	0	0	0	0
normalized size	1	0.	4.11	51412.7	0.	0.	0.	0.
time (sec)	N/A	0.699	1.639	1.51	0.	0.	0.	0.

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	C	C	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	79	0	529	257865	0	0	0	0
normalized size	1	0.	6.7	3264.11	0.	0.	0.	0.
time (sec)	N/A	0.697	1.603	0.871	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [238] had the largest ratio of [0.75]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.	10	0.3
2	A	3	3	1.	10	0.3
3	A	2	2	1.	10	0.2
4	A	2	2	1.	10	0.2
5	A	3	3	1.	10	0.3
6	A	4	3	1.	10	0.3
7	A	6	3	1.	10	0.3

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
8	A	4	3	1.	10	0.3
9	A	3	3	1.	10	0.3
10	A	3	3	1.	10	0.3
11	A	4	3	1.	10	0.3
12	A	6	3	1.	10	0.3
13	A	7	3	1.	10	0.3
14	A	5	3	1.	10	0.3
15	A	3	3	1.	10	0.3
16	A	3	3	1.	10	0.3
17	A	3	2	1.	10	0.2
18	A	3	2	1.	10	0.2
19	A	2	2	1.	14	0.143
20	A	2	2	1.	14	0.143
21	A	2	2	1.	14	0.143
22	A	2	2	1.	14	0.143
23	A	2	2	1.	14	0.143
24	A	2	2	1.	14	0.143
25	A	2	2	1.	12	0.167
26	A	2	2	1.	12	0.167
27	A	2	2	1.	12	0.167
28	A	2	2	1.	12	0.167
29	A	2	2	1.	14	0.143
30	A	2	2	1.	14	0.143
31	A	3	2	1.	9	0.222
32	A	4	3	1.	11	0.273
33	A	5	3	1.	11	0.273
34	A	6	3	1.	11	0.273
35	A	4	3	1.	24	0.125
36	A	4	3	1.	24	0.125
37	A	4	3	1.	24	0.125
38	A	3	3	1.	22	0.136
39	A	4	4	1.	22	0.182
40	A	5	5	1.	24	0.208
41	A	6	5	1.	24	0.208
42	A	6	5	1.	24	0.208
43	A	5	5	1.	24	0.208
44	A	4	4	1.	24	0.167
45	A	3	3	1.	15	0.2
46	A	4	3	1.	24	0.125
47	A	4	3	1.	24	0.125
48	A	4	3	1.	24	0.125
49	A	4	3	1.	24	0.125
50	A	4	3	1.	24	0.125
51	A	3	2	1.	24	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
52	A	3	3	1.	22	0.136
53	A	5	4	1.	22	0.182
54	A	6	5	1.	24	0.208
55	A	6	5	1.	24	0.208
56	A	4	3	1.	24	0.125
57	A	3	3	1.	24	0.125
58	A	3	2	1.	15	0.133
59	A	4	3	1.	24	0.125
60	A	4	3	1.	24	0.125
61	A	3	2	1.	11	0.182
62	A	3	2	1.	11	0.182
63	A	3	2	1.	11	0.182
64	A	3	2	1.	21	0.095
65	A	2	1	1.	19	0.053
66	A	2	2	1.	19	0.105
67	A	2	2	1.	21	0.095
68	A	4	3	1.	21	0.143
69	A	3	3	1.	21	0.143
70	A	3	2	1.	12	0.167
71	A	2	2	1.	21	0.095
72	A	3	3	1.	21	0.143
73	A	3	2	1.	21	0.095
74	A	3	2	1.	8	0.25
75	A	1	1	1.	10	0.1
76	A	2	2	1.	10	0.2
77	A	3	3	1.	10	0.3
78	A	4	3	1.	23	0.13
79	A	4	3	1.	23	0.13
80	A	3	3	1.	23	0.13
81	A	2	2	1.	21	0.095
82	A	4	4	1.	21	0.19
83	A	5	5	1.	23	0.217
84	A	6	6	1.	23	0.261
85	A	7	6	1.	23	0.261
86	A	6	6	1.	23	0.261
87	A	5	5	1.	23	0.217
88	A	3	3	1.	23	0.13
89	A	2	2	1.	14	0.143
90	A	3	3	1.	23	0.13
91	A	4	3	1.	23	0.13
92	A	4	3	1.	23	0.13
93	A	4	3	1.	23	0.13
94	A	5	4	1.	23	0.174
95	A	5	4	1.	23	0.174

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	3	3	1.	23	0.13
97	A	3	3	1.	21	0.143
98	A	5	5	1.	21	0.238
99	A	6	6	1.	23	0.261
100	A	6	6	1.	23	0.261
101	A	5	5	1.	23	0.217
102	A	4	4	1.	23	0.174
103	A	4	4	1.	14	0.286
104	A	4	4	1.02	23	0.174
105	A	5	4	1.	23	0.174
106	A	6	6	1.	23	0.261
107	A	4	3	1.	23	0.13
108	A	5	4	1.	23	0.174
109	A	5	5	1.	14	0.357
110	A	5	5	1.	23	0.217
111	A	6	5	1.	14	0.357
112	A	7	5	1.	14	0.357
113	A	2	2	1.	13	0.154
114	A	3	3	1.	13	0.231
115	A	2	2	1.	21	0.095
116	A	5	4	1.	13	0.308
117	A	4	4	1.	13	0.308
118	A	3	3	1.	13	0.231
119	A	3	3	1.	13	0.231
120	A	4	4	1.	13	0.308
121	A	5	4	1.	13	0.308
122	A	5	5	1.	25	0.2
123	A	4	4	1.	23	0.174
124	A	6	6	1.	23	0.261
125	A	4	4	1.	25	0.16
126	A	5	5	1.	25	0.2
127	A	8	8	1.	25	0.32
128	A	6	6	1.	25	0.24
129	A	2	2	1.	16	0.125
130	A	8	8	1.	25	0.32
131	A	8	8	1.	25	0.32
132	A	6	5	1.	25	0.2
133	A	5	4	1.	23	0.174
134	A	7	7	1.	23	0.304
135	A	7	7	1.	25	0.28
136	A	5	4	1.	25	0.16
137	A	6	5	1.	25	0.2
138	A	9	8	1.	25	0.32
139	A	7	6	1.	25	0.24

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
140	A	6	6	1.	16	0.375
141	A	7	7	1.	25	0.28
142	A	8	8	1.	25	0.32
143	A	7	7	1.	16	0.438
144	A	4	4	1.	25	0.16
145	A	3	3	1.	23	0.13
146	A	3	3	1.	23	0.13
147	A	4	4	1.	25	0.16
148	A	7	7	1.	25	0.28
149	A	5	5	1.	25	0.2
150	A	2	2	1.	16	0.125
151	A	8	8	1.	25	0.32
152	A	8	8	1.	25	0.32
153	A	4	4	1.	25	0.16
154	A	2	2	1.	23	0.087
155	A	4	4	1.	23	0.174
156	A	6	6	1.	25	0.24
157	A	8	8	1.	25	0.32
158	A	7	7	1.	25	0.28
159	A	6	6	1.	25	0.24
160	A	4	4	1.	16	0.25
161	A	8	8	1.	25	0.32
162	A	5	5	1.	25	0.2
163	A	3	3	1.	25	0.12
164	A	3	3	1.	23	0.13
165	A	6	6	1.	23	0.261
166	A	8	8	1.	25	0.32
167	A	8	8	1.	25	0.32
168	A	7	6	1.	25	0.24
169	A	7	7	1.	16	0.438
170	A	9	9	1.	25	0.36
171	A	3	3	1.	25	0.12
172	A	5	5	1.	23	0.217
173	A	4	4	1.	23	0.174
174	A	3	3	1.	21	0.143
175	A	3	3	1.	21	0.143
176	A	3	3	1.	23	0.13
177	A	3	3	1.	23	0.13
178	A	3	3	1.	23	0.13
179	A	3	3	1.	23	0.13
180	A	3	3	1.	23	0.13
181	A	3	3	1.	23	0.13
182	A	17	7	1.	23	0.304
183	A	15	7	1.	23	0.304

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
184	A	13	5	1.	23	0.217
185	A	11	4	1.	21	0.19
186	A	14	6	1.	21	0.286
187	A	15	7	1.	23	0.304
188	A	18	8	1.	23	0.348
189	A	15	6	1.	23	0.261
190	A	14	5	1.	23	0.217
191	A	11	5	1.	23	0.217
192	A	11	4	1.	14	0.286
193	A	15	6	1.	23	0.261
194	A	16	7	1.	23	0.304
195	A	6	5	1.	24	0.208
196	A	6	5	1.	24	0.208
197	A	6	5	1.	24	0.208
198	A	4	4	1.	24	0.167
199	A	4	4	1.	22	0.182
200	A	7	6	1.	22	0.273
201	A	7	6	1.	24	0.25
202	A	7	6	1.	24	0.25
203	A	12	6	1.	24	0.25
204	A	9	6	1.	24	0.25
205	A	7	5	1.	24	0.208
206	A	4	3	1.	24	0.125
207	A	4	3	1.	15	0.2
208	A	6	4	1.	24	0.167
209	A	6	4	1.	24	0.167
210	A	6	4	1.	24	0.167
211	A	6	4	1.	24	0.167
212	A	7	6	1.	24	0.25
213	A	5	5	1.	24	0.208
214	A	5	5	1.	24	0.208
215	A	5	5	1.	24	0.208
216	A	5	5	1.	22	0.227
217	A	11	7	1.	22	0.318
218	A	14	9	1.	24	0.375
219	A	6	5	1.	24	0.208
220	A	7	6	1.	24	0.25
221	A	5	4	1.	24	0.167
222	A	5	4	1.	15	0.267
223	A	7	5	1.	24	0.208
224	A	6	6	1.	24	0.25
225	A	6	6	1.	24	0.25
226	A	6	6	1.	24	0.25
227	A	6	5	1.	24	0.208

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
228	A	6	6	1.	22	0.273
229	A	16	7	1.	22	0.318
230	A	9	7	1.	24	0.292
231	A	6	5	1.	24	0.208
232	A	6	5	1.	24	0.208
233	A	6	5	1.	24	0.208
234	A	6	5	1.	15	0.333
235	A	8	6	1.	24	0.25
236	A	3	3	1.8	10	0.3
237	A	10	6	1.	10	0.6
238	A	10	6	1.	8	0.75
239	A	5	5	1.	23	0.217
240	A	8	7	1.	23	0.304
241	A	5	5	1.	25	0.2
242	A	4	4	1.	25	0.16
243	A	2	2	1.	23	0.087
244	A	4	4	1.	23	0.174
245	A	7	7	1.	25	0.28
246	A	4	4	1.	25	0.16
247	A	2	2	1.	16	0.125
248	A	5	5	1.	25	0.2
249	A	17	4	1.	10	0.4
250	A	7	3	1.	10	0.3
251	A	9	3	1.	10	0.3
252	A	17	4	1.	11	0.364
253	A	7	3	1.	11	0.273
254	A	9	3	1.	11	0.273
255	A	15	5	1.	8	0.625
256	A	7	3	1.	8	0.375
257	A	9	3	0.59	8	0.375
258	A	15	5	1.	10	0.5
259	A	8	6	1.	10	0.6
260	A	10	6	1.	10	0.6
261	A	3	2	1.	16	0.125
262	A	3	2	1.	16	0.125
263	A	3	2	1.	16	0.125
264	A	2	2	1.	16	0.125
265	A	2	2	1.	14	0.143
266	A	4	3	1.	16	0.188
267	A	4	3	1.	16	0.188
268	A	3	3	1.	16	0.188
269	A	2	2	1.	16	0.125
270	A	3	3	1.	14	0.214
271	A	3	2	1.	16	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
272	A	3	2	1.	16	0.125
273	A	3	2	1.	16	0.125
274	A	3	2	1.	16	0.125
275	A	2	2	1.	16	0.125
276	A	2	2	1.	16	0.125
277	A	3	3	1.	14	0.214
278	A	4	3	1.	14	0.214
279	A	4	3	1.	16	0.188
280	A	3	3	1.	16	0.188
281	A	2	2	1.	16	0.125
282	A	3	3	1.	16	0.188
283	A	3	2	1.	16	0.125
284	A	3	2	1.	16	0.125
285	A	6	4	1.	21	0.19
286	A	5	4	1.	21	0.19
287	A	4	4	1.	21	0.19
288	A	3	2	1.	12	0.167
289	A	3	3	1.	21	0.143
290	A	2	1	1.	21	0.048
291	A	3	2	1.	21	0.095
292	A	3	2	1.	21	0.095
293	A	6	5	1.	23	0.217
294	A	5	5	1.	23	0.217
295	A	1	1	1.	14	0.071
296	A	5	4	1.	23	0.174
297	A	4	3	1.	23	0.13
298	A	3	2	1.	23	0.087
299	A	3	2	1.	23	0.087
300	A	3	2	1.	23	0.087
301	A	4	3	1.	15	0.2
302	A	6	6	1.	15	0.4
303	A	4	3	1.	15	0.2
304	A	5	5	1.	15	0.333
305	A	3	3	1.	15	0.2
306	A	4	4	1.	15	0.267
307	A	2	2	1.	13	0.154
308	A	4	4	1.	13	0.308
309	A	3	3	1.	15	0.2
310	A	5	5	1.	15	0.333
311	A	4	3	1.	15	0.2
312	A	6	6	1.	15	0.4
313	A	4	3	1.	15	0.2
314	A	6	6	1.	15	0.4
315	A	5	4	1.	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
316	A	5	5	1.	15	0.333
317	A	3	3	1.	15	0.2
318	A	3	3	1.	15	0.2
319	A	3	3	1.	13	0.231
320	A	5	5	1.	13	0.385
321	A	5	4	1.	15	0.267
322	A	6	6	1.	15	0.4
323	A	5	4	1.	15	0.267
324	A	5	5	1.	25	0.2
325	A	4	4	1.	23	0.174
326	A	6	5	1.	23	0.217
327	A	4	4	1.	25	0.16
328	A	5	5	1.	25	0.2
329	A	8	8	1.18	25	0.32
330	A	7	7	1.25	25	0.28
331	A	2	2	1.	16	0.125
332	A	8	8	1.31	25	0.32
333	A	8	8	1.2	25	0.32
334	A	6	5	1.	25	0.2
335	A	5	4	1.	23	0.174
336	A	7	6	1.	23	0.261
337	A	7	6	1.	25	0.24
338	A	5	4	1.	25	0.16
339	A	6	5	1.	25	0.2
340	A	9	8	1.	25	0.32
341	A	8	8	1.	25	0.32
342	A	6	6	1.	16	0.375
343	A	7	7	1.	25	0.28
344	A	8	8	1.	25	0.32
345	A	4	4	1.	25	0.16
346	A	3	3	1.	23	0.13
347	A	3	3	1.	23	0.13
348	A	4	4	1.	25	0.16
349	A	7	7	1.24	25	0.28
350	A	6	6	1.34	25	0.24
351	A	2	2	1.	16	0.125
352	A	8	8	1.29	25	0.32
353	A	8	8	1.19	25	0.32
354	A	4	4	1.	25	0.16
355	A	2	2	1.	23	0.087
356	A	4	4	1.	23	0.174
357	A	6	6	1.	25	0.24
358	A	8	8	1.	25	0.32
359	A	7	7	1.	25	0.28

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
360	A	7	7	1.	25	0.28
361	A	4	4	1.	16	0.25
362	A	8	8	1.	25	0.32
363	A	5	5	1.	25	0.2
364	A	3	3	1.	25	0.12
365	A	3	3	1.	23	0.13
366	A	6	6	1.	23	0.261
367	A	8	8	1.16	25	0.32
368	A	8	8	1.18	25	0.32
369	A	8	8	1.18	25	0.32
370	A	7	7	1.	16	0.438
371	A	9	8	1.14	25	0.32
372	A	3	3	1.	25	0.12
373	A	5	5	1.	23	0.217
374	A	4	4	0.96	23	0.174
375	A	3	3	1.	21	0.143
376	A	3	3	1.	21	0.143
377	A	3	3	1.	23	0.13
378	A	3	3	1.	23	0.13
379	A	3	3	1.	23	0.13
380	A	3	3	1.	14	0.214
381	A	3	3	1.	23	0.13
382	A	3	3	1.	23	0.13
383	A	11	10	1.	23	0.435
384	A	9	9	1.	23	0.391
385	A	7	7	1.	21	0.333
386	A	11	10	1.	21	0.476
387	A	11	10	1.	23	0.435
388	A	38	8	1.	23	0.348
389	A	24	7	1.	23	0.304
390	A	11	4	1.	14	0.286
391	F	0	0	N/A	0	N/A
392	F	0	0	N/A	0	N/A
393	A	10	9	1.	23	0.391
394	A	8	8	1.	23	0.348
395	A	9	9	1.	23	0.391
396	A	8	8	1.	21	0.381
397	A	18	11	1.	21	0.524
398	A	18	11	1.	23	0.478
399	A	0	0	0.	0	0.
400	A	0	0	0.	0	0.
401	A	0	0	0.	0	0.
402	A	0	0	0.	0	0.
403	A	0	0	0.	0	0.

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
404	A	6	5	1.	24	0.208
405	A	6	5	1.	24	0.208
406	A	4	4	1.	24	0.167
407	A	4	4	1.	22	0.182
408	A	7	6	1.	22	0.273
409	A	7	6	1.	24	0.25
410	A	7	6	1.	24	0.25
411	A	16	6	1.	24	0.25
412	A	12	6	1.	24	0.25
413	A	9	6	1.	24	0.25
414	A	7	5	1.	24	0.208
415	A	4	3	1.	24	0.125
416	A	6	4	1.	24	0.167
417	A	6	4	1.	24	0.167
418	A	6	4	1.	24	0.167
419	A	0	0	0.	0	0.
420	A	8	7	0.97	23	0.304
421	A	7	6	1.	23	0.261
422	A	3	3	1.	21	0.143
423	A	7	6	1.	21	0.286
424	A	9	6	1.	23	0.261
425	A	0	0	0.	0	0.
426	A	0	0	0.	0	0.
427	A	0	0	0.	0	0.
428	A	0	0	0.	0	0.
429	A	0	0	0.	0	0.
430	A	0	0	0.	0	0.
431	A	9	6	1.	23	0.261
432	A	7	6	1.	23	0.261
433	A	3	3	1.	21	0.143
434	A	0	0	0.	0	0.
435	A	0	0	0.	0	0.
436	A	0	0	0.	0	0.
437	A	0	0	0.	0	0.
438	A	0	0	0.	0	0.
439	A	0	0	0.	0	0.
440	A	0	0	0.	0	0.
441	A	3	2	1.	23	0.087
442	A	3	2	1.	23	0.087
443	A	3	2	1.	23	0.087
444	A	4	3	1.	21	0.143
445	A	4	4	1.	21	0.19
446	A	3	2	1.	23	0.087
447	A	3	2	1.	23	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
448	A	3	2	1.	23	0.087
449	A	4	3	1.	23	0.13
450	A	4	3	1.	23	0.13
451	A	4	3	1.	23	0.13
452	A	3	3	1.	23	0.13
453	A	3	3	1.	23	0.13
454	A	4	3	1.	23	0.13
455	A	5	3	1.	23	0.13
456	A	6	3	1.	23	0.13
457	A	5	4	1.	26	0.154
458	A	5	4	1.	26	0.154
459	A	4	4	1.	24	0.167
460	A	5	5	1.	24	0.208
461	A	7	7	1.	26	0.269
462	A	7	6	1.	26	0.231
463	A	6	6	1.	26	0.231
464	A	5	5	1.	26	0.192
465	A	5	4	1.	26	0.154
466	A	5	4	1.	26	0.154
467	A	5	4	1.	26	0.154
468	A	5	4	1.	26	0.154
469	A	5	4	1.	26	0.154
470	A	4	4	1.	24	0.167
471	A	4	4	1.	24	0.167
472	A	6	6	1.	26	0.231
473	A	5	4	1.	26	0.154
474	A	4	4	1.	26	0.154
475	A	4	4	1.	26	0.154
476	A	4	3	1.	26	0.115
477	A	5	4	1.	26	0.154
478	A	5	4	1.	26	0.154
479	A	5	4	1.	26	0.154
480	A	4	4	1.	24	0.167
481	A	5	5	1.	24	0.208
482	A	6	6	1.	26	0.231
483	A	5	5	1.	26	0.192
484	A	5	5	1.	26	0.192
485	A	4	4	1.	26	0.154
486	A	5	4	1.	26	0.154
487	A	5	4	1.	26	0.154
488	A	6	6	1.	25	0.24
489	A	5	5	1.	25	0.2
490	A	4	4	1.	23	0.174
491	A	4	4	1.	23	0.174

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
492	A	5	5	1.	25	0.2
493	A	6	6	1.	25	0.24
494	A	8	8	1.	25	0.32
495	A	7	7	1.	25	0.28
496	A	2	2	1.	16	0.125
497	A	7	7	1.	25	0.28
498	A	8	8	1.	25	0.32
499	A	7	6	1.	25	0.24
500	A	6	5	1.	25	0.2
501	A	5	4	1.	23	0.174
502	A	5	4	1.	23	0.174
503	A	6	5	1.	25	0.2
504	A	7	6	1.	25	0.24
505	A	9	9	1.	25	0.36
506	A	8	8	1.	25	0.32
507	A	6	6	1.	16	0.375
508	A	8	8	1.	25	0.32
509	A	9	9	1.	25	0.36
510	A	5	5	1.	25	0.2
511	A	4	4	1.	25	0.16
512	A	3	3	1.	23	0.13
513	A	3	3	1.	23	0.13
514	A	4	4	1.	25	0.16
515	A	5	5	1.	25	0.2
516	A	8	8	1.	25	0.32
517	A	4	4	1.	25	0.16
518	A	2	2	1.	16	0.125
519	A	5	5	1.	25	0.2
520	A	8	8	1.	25	0.32
521	A	6	6	1.	25	0.24
522	A	5	5	1.	25	0.2
523	A	4	4	1.	23	0.174
524	A	4	4	1.	23	0.174
525	A	5	5	1.	25	0.2
526	A	6	6	1.	25	0.24
527	A	9	8	1.	25	0.32
528	A	8	8	1.	25	0.32
529	A	4	4	1.	16	0.25
530	A	8	8	1.	25	0.32
531	A	9	8	1.	25	0.32
532	A	7	6	1.	25	0.24
533	A	6	5	1.	25	0.2
534	A	5	4	1.	23	0.174
535	A	5	4	1.	23	0.174

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
536	A	6	5	1.	25	0.2
537	A	7	6	1.	25	0.24
538	A	10	8	1.	25	0.32
539	A	9	8	1.	25	0.32
540	A	7	7	1.	16	0.438
541	A	9	9	1.	25	0.36
542	A	10	9	1.	25	0.36
543	A	3	3	1.	25	0.12
544	A	3	3	1.	23	0.13
545	A	2	2	1.	21	0.095
546	A	2	2	1.	21	0.095
547	A	3	3	1.	23	0.13
548	A	3	3	1.	23	0.13
549	A	3	3	1.	23	0.13
550	A	3	3	1.	23	0.13
551	A	3	3	1.	23	0.13
552	A	11	10	1.	15	0.667
553	A	5	5	1.	15	0.333
554	A	4	4	1.	15	0.267
555	A	5	5	1.	23	0.217
556	A	4	4	1.	25	0.16
557	A	3	3	1.	23	0.13
558	A	4	4	1.	23	0.174
559	A	5	5	1.	25	0.2
560	A	6	6	1.	25	0.24
561	A	4	4	1.	25	0.16
562	A	2	2	1.	16	0.125
563	A	6	6	1.	25	0.24
564	A	0	0	0.	0	0.
565	A	11	10	1.	23	0.435
566	A	7	6	1.	21	0.286
567	A	3	3	1.	21	0.143
568	A	6	6	1.	23	0.261
569	A	0	0	0.	0	0.
570	A	0	0	0.	0	0.
571	A	0	0	0.	0	0.
572	A	0	0	0.	0	0.
573	A	0	0	0.	0	0.
574	A	10	5	1.	23	0.217
575	A	8	5	1.	23	0.217
576	A	6	5	1.	21	0.238
577	A	0	0	0.	0	0.
578	A	0	0	0.	0	0.
579	A	5	5	1.	15	0.333

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
580	A	4	4	1.	15	0.267
581	A	0	0	0.	0	0.
582	A	0	0	0.	0	0.
583	A	0	0	0.	0	0.
584	A	3	3	1.	21	0.143
585	A	7	6	1.	23	0.261
586	A	0	0	0.	0	0.
587	A	0	0	0.	0	0.
588	A	0	0	0.	0	0.
589	A	0	0	0.	0	0.
590	A	0	0	0.	0	0.
591	A	7	7	1.07	37	0.189
592	A	3	3	1.	35	0.086
593	F	0	0	N/A	0	N/A
594	F	0	0	N/A	0	N/A

Chapter 3

Listing of integrals

3.1 $\int \left(a\sin^2(x)\right)^{5/2}\,dx$

Optimal. Leaf size=53

$$-\frac{8}{15}a^2\cot(x)\sqrt{a\sin^2(x)}-\frac{1}{5}\cot(x)\left(a\sin^2(x)\right)^{5/2}-\frac{4}{15}a\cot(x)\left(a\sin^2(x)\right)^{3/2}$$

```
[Out] (-8*a^2*Cot[x]*Sqrt[a*Sin[x]^2])/15 - (4*a*Cot[x]*(a*Sin[x]^2)^(3/2))/15 -
(Cot[x]*(a*Sin[x]^2)^(5/2))/5
```

Rubi [A] time = 0.0290957, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3203, 3207, 2638}

$$-\frac{8}{15}a^2\cot(x)\sqrt{a\sin^2(x)}-\frac{1}{5}\cot(x)\left(a\sin^2(x)\right)^{5/2}-\frac{4}{15}a\cot(x)\left(a\sin^2(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[x]^2)^(5/2),x]
```

```
[Out] (-8*a^2*Cot[x]*Sqrt[a*Sin[x]^2])/15 - (4*a*Cot[x]*(a*Sin[x]^2)^(3/2))/15 -
(Cot[x]*(a*Sin[x]^2)^(5/2))/5
```

Rule 3203

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(Cot[e + f*x
]*(b*Sin[e + f*x]^2)^p)/(2*f*p), x] + Dist[(b*(2*p - 1))/(2*p), Int[(b*Sin[
e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] && !IntegerQ[p] && GtQ[
p, 1]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \left(a\sin^2(x)\right)^{5/2}\,dx &= -\frac{1}{5}\cot(x)\left(a\sin^2(x)\right)^{5/2}+\frac{1}{5}(4a)\int \left(a\sin^2(x)\right)^{3/2}\,dx\\ &= -\frac{4}{15}a\cot(x)\left(a\sin^2(x)\right)^{3/2}-\frac{1}{5}\cot(x)\left(a\sin^2(x)\right)^{5/2}+\frac{1}{15}\left(8a^2\right)\int \sqrt{a\sin^2(x)}\,dx\\ &= -\frac{4}{15}a\cot(x)\left(a\sin^2(x)\right)^{3/2}-\frac{1}{5}\cot(x)\left(a\sin^2(x)\right)^{5/2}+\frac{1}{15}\left(8a^2\csc(x)\sqrt{a\sin^2(x)}\right)\int \sin(x)\,dx\\ &= -\frac{8}{15}a^2\cot(x)\sqrt{a\sin^2(x)}-\frac{4}{15}a\cot(x)\left(a\sin^2(x)\right)^{3/2}-\frac{1}{5}\cot(x)\left(a\sin^2(x)\right)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.0326258, size = 36, normalized size = 0.68

$$-\frac{1}{240}a^2(150\cos(x)-25\cos(3x)+3\cos(5x))\csc(x)\sqrt{a\sin^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[x]^2)^(5/2),x]
```

```
[Out] -(a^2*(150*Cos[x] - 25*Cos[3*x] + 3*Cos[5*x])*Csc[x]*Sqrt[a*Sin[x]^2])/240
```

Maple [A] time = 0.632, size = 32, normalized size = 0.6

$$-\frac{a^3\cos(x)\sin(x)\left(3\,(\sin(x))^4+4\,(\sin(x))^2+8\right)}{15}\frac{1}{\sqrt{a\,(\sin(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(x)^2)^(5/2),x)
```

```
[Out] -1/15*a^3*sin(x)*cos(x)*(3*sin(x)^4+4*sin(x)^2+8)/(a*sin(x)^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin(x)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(x)^2)^(5/2), x)
```

Fricas [A] time = 1.6246, size = 117, normalized size = 2.21

$$-\frac{\left(3\,a^2\cos\left(x\right)^5-10\,a^2\cos\left(x\right)^3+15\,a^2\cos\left(x\right)\right)\sqrt{-a\cos\left(x\right)^2+a}}{15\,\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/15*(3*a^2*cos(x)^5 - 10*a^2*cos(x)^3 + 15*a^2*cos(x))*sqrt(-a*cos(x)^2 +
 a)/sin(x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16377, size = 61, normalized size = 1.15

$$\frac{1}{15}\left(8\,a^2\mathrm{sgn}\left(\sin\left(x\right)\right)-\left(3\,a^2\cos\left(x\right)^5-10\,a^2\cos\left(x\right)^3+15\,a^2\cos\left(x\right)\right)\mathrm{sgn}\left(\sin\left(x\right)\right)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/15*(8*a^2*sgn(sin(x)) - (3*a^2*cos(x)^5 - 10*a^2*cos(x)^3 + 15*a^2*cos(x)
)*sgn(sin(x)))*sqrt(a)
```

3.2 $\int \left(a\sin^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=34

$$-\frac{1}{3}\cot(x)\left(a\sin^2(x)\right)^{3/2}-\frac{2}{3}a\cot(x)\sqrt{a\sin^2(x)}$$

```
[Out] (-2*a*Cot[x]*Sqrt[a*Sin[x]^2])/3 - (Cot[x]*(a*Sin[x]^2)^(3/2))/3
```

Rubi [A] time = 0.0185186, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3203, 3207, 2638}

$$-\frac{1}{3}\cot(x)\left(a\sin^2(x)\right)^{3/2}-\frac{2}{3}a\cot(x)\sqrt{a\sin^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[x]^2)^(3/2),x]
```

```
[Out] (-2*a*Cot[x]*Sqrt[a*Sin[x]^2])/3 - (Cot[x]*(a*Sin[x]^2)^(3/2))/3
```

Rule 3203

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(Cot[e + f*x
]*(b*Sin[e + f*x]^2)^p)/(2*f*p), x] + Dist[(b*(2*p - 1))/(2*p), Int[(b*Sin[
e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && GtQ[
p, 1]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \left(a\sin^2(x)\right)^{3/2}\,dx &= -\frac{1}{3}\cot(x)\left(a\sin^2(x)\right)^{3/2}+\frac{1}{3}(2a)\int\sqrt{a\sin^2(x)}\,dx\\
&= -\frac{1}{3}\cot(x)\left(a\sin^2(x)\right)^{3/2}+\frac{1}{3}\left(2a\csc(x)\sqrt{a\sin^2(x)}\right)\int\sin(x)\,dx\\
&= -\frac{2}{3}a\cot(x)\sqrt{a\sin^2(x)}-\frac{1}{3}\cot(x)\left(a\sin^2(x)\right)^{3/2}
\end{aligned}$$

Mathematica [A] time = 0.0378638, size = 26, normalized size = 0.76

$$\frac{1}{12}a(\cos(3x)-9\cos(x))\csc(x)\sqrt{a\sin^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[x]^2)^(3/2),x]
```

```
[Out] (a*(-9*Cos[x] + Cos[3*x])*Csc[x]*Sqrt[a*Sin[x]^2])/12
```

Maple [A] time = 0.628, size = 24, normalized size = 0.7

$$-\frac{a^2\cos(x)\sin(x)\left(2+(\sin(x))^2\right)}{3}\frac{1}{\sqrt{a(\sin(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(x)^2)^(3/2),x)
```

```
[Out] -1/3*a^2*sin(x)*cos(x)*(2+sin(x)^2)/(a*sin(x)^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin(x)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(x)^2)^(3/2), x)
```

Fricas [A] time = 1.55797, size = 81, normalized size = 2.38

$$\frac{\left(a\cos(x)^3-3\,a\cos(x)\right)\sqrt{-a\cos(x)^2+a}}{3\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(a*cos(x)^3 - 3*a*cos(x))*sqrt(-a*cos(x)^2 + a)/sin(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin^2(x)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)**2)**(3/2),x)
```

```
[Out] Integral((a*sin(x)**2)**(3/2), x)
```

Giac [A] time = 1.22319, size = 32, normalized size = 0.94

$$\frac{1}{3}\left(\left(\cos\left(x\right)^{3}-3\,\cos\left(x\right)\right)\mathrm{sgn}\left(\sin\left(x\right)\right)+2\,\mathrm{sgn}\left(\sin\left(x\right)\right)\right)a^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*((cos(x)^3 - 3*cos(x))*sgn(sin(x)) + 2*sgn(sin(x)))*a^(3/2)
```

3.3 $\int \sqrt{a \sin^2(x)}\, dx$

Optimal. Leaf size=14

$$-\cot(x)\sqrt{a\sin^2(x)}$$

```
[Out] -(Cot[x]*Sqrt[a*Sin[x]^2])
```

Rubi [A] time = 0.0098667, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3207, 2638}

$$-\cot(x)\sqrt{a\sin^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Sin[x]^2],x]

[Out] -(Cot[x]*Sqrt[a*Sin[x]^2])
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a\sin^2(x)}\, dx &= \left(\csc(x)\sqrt{a\sin^2(x)}\right)\int \sin(x)\, dx\\ &= -\cot(x)\sqrt{a\sin^2(x)}\end{aligned}$$

Mathematica [A] time = 0.0043304, size = 14, normalized size = 1.

$$-\cot(x)\sqrt{a\sin^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Sin[x]^2],x]

[Out] -(Cot[x]*Sqrt[a*Sin[x]^2])
```

Maple [A] time = 0.294, size = 16, normalized size = 1.1

$$-a\cos(x)\sin(x)\frac{1}{\sqrt{a(\sin(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(x)^2)^(1/2),x)
```

```
[Out] -1/(a*sin(x)^2)^(1/2)*a*cos(x)*sin(x)
```

Maxima [A] time = 1.42444, size = 18, normalized size = 1.29

$$-\frac{\sqrt{a}}{\sqrt{\tan(x)^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(a)/sqrt(tan(x)^2 + 1)
```

Fricas [A] time = 1.5783, size = 51, normalized size = 3.64

$$-\frac{\sqrt{-a\cos(x)^2+a}\cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-a*cos(x)^2 + a)*cos(x)/sin(x)
```

Sympy [A] time = 0.719972, size = 20, normalized size = 1.43

$$-\frac{\sqrt{a}\sqrt{\sin^2(x)}\cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)**2)**(1/2),x)
```

```
[Out] -sqrt(a)*sqrt(sin(x)**2)*cos(x)/sin(x)
```

Giac [A] time = 1.24183, size = 23, normalized size = 1.64

$$-(\cos(x)\operatorname{sgn}(\sin(x)) - \operatorname{sgn}(\sin(x)))\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(cos(x)*sgn(sin(x)) - sgn(sin(x)))*sqrt(a)
```

3.4 $\int \frac{1}{\sqrt{a \sin^2(x)}}\,dx$

Optimal. Leaf size=17

$$-\frac{\sin(x)\tanh^{-1}(\cos(x))}{\sqrt{a\sin^2(x)}}$$

```
[Out] -((ArcTanh[Cos[x]]*Sin[x])/Sqrt[a*Sin[x]^2])
```

Rubi [A] time = 0.0113021, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3207, 3770}

$$-\frac{\sin(x)\tanh^{-1}(\cos(x))}{\sqrt{a\sin^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Sin[x]^2],x]
```

```
[Out] -((ArcTanh[Cos[x]]*Sin[x])/Sqrt[a*Sin[x]^2])
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a\sin^2(x)}}\,dx &= \frac{\sin(x)\int \csc(x)\,dx}{\sqrt{a\sin^2(x)}}\\ &= -\frac{\tanh^{-1}(\cos(x))\sin(x)}{\sqrt{a\sin^2(x)}}\end{aligned}$$

Mathematica [A] time = 0.0148339, size = 30, normalized size = 1.76

$$\frac{\sin(x)\left(\log\left(\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)\right)\right)}{\sqrt{a\sin^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Sin[x]^2],x]
```

```
[Out] ((-Log[Cos[x/2]] + Log[Sin[x/2]])*Sin[x])/Sqrt[a*Sin[x]^2]
```

Maple [B] time = 0.599, size = 49, normalized size = 2.9

$$-\frac{\sin(x)}{\cos(x)}\sqrt{a(\cos(x))^2}\ln\left(2\,\frac{\sqrt{a}\sqrt{a(\cos(x))^2}+a}{\sin(x)}\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{a(\sin(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(x)^2)^(1/2),x)
```

```
[Out] -sin(x)*(a*cos(x)^2)^(1/2)/a^(1/2)*ln(2*(a^(1/2)*(a*cos(x)^2)^(1/2)+a)/sin(
x))/cos(x)/(a*sin(x)^2)^(1/2)
```

Maxima [A] time = 1.59911, size = 35, normalized size = 2.06

$$\frac{\sqrt{-a}(\arctan(\sin(x),\cos(x)+1)-\arctan(\sin(x),\cos(x)-1))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(-a)*(arctan2(sin(x), cos(x) + 1) - arctan2(sin(x), cos(x) - 1))/a
```

Fricas [B] time = 1.71577, size = 193, normalized size = 11.35

$$\left[\frac{\sqrt{-a\cos(x)^2+a}\log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)}{2\,a\sin(x)},\ \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{-a\cos(x)^2+a}\sqrt{-a}\cos(x)}{a\sin(x)}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(-a*cos(x)^2 + a)*log(-(cos(x) - 1)/(cos(x) + 1))/(a*sin(x)), sqrt
(-a)*arctan(sqrt(-a*cos(x)^2 + a)*sqrt(-a)*cos(x)/(a*sin(x)))/a]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{a\sin^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*sin(x)**2), x)
```

Giac [A] time = 1.23031, size = 20, normalized size = 1.18

$$\frac{\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)}{\sqrt{a}\mathrm{sgn}\left(\sin\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] log(abs(tan(1/2*x)))/(sqrt(a)*sgn(sin(x)))
```

3.5 $\int \frac{1}{\left(a\sin^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=42

$$-\frac{\cot(x)}{2a\sqrt{a\sin^2(x)}}-\frac{\sin(x)\tanh^{-1}(\cos(x))}{2a\sqrt{a\sin^2(x)}}$$

```
[Out] -Cot[x]/(2*a*Sqrt[a*Sin[x]^2]) - (ArcTanh[Cos[x]]*Sin[x])/(2*a*Sqrt[a*Sin[x
]^2])
```

Rubi [A] time = 0.0242362, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3204, 3207, 3770}

$$-\frac{\cot(x)}{2a\sqrt{a\sin^2(x)}}-\frac{\sin(x)\tanh^{-1}(\cos(x))}{2a\sqrt{a\sin^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[x]^2)^(-3/2),x]
```

```
[Out] -Cot[x]/(2*a*Sqrt[a*Sin[x]^2]) - (ArcTanh[Cos[x]]*Sin[x])/(2*a*Sqrt[a*Sin[x
]^2])
```

Rule 3204

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(Cot[e + f*x]
*(b*Sin[e + f*x]^2)^(p + 1))/(b*f*(2*p + 1)), x] + Dist[(2*(p + 1))/(b*(2*p
 + 1)), Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&  !
IntegerQ[p] && LtQ[p, -1]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a\sin^2(x)\right)^{3/2}}\,dx = -\frac{\cot(x)}{2a\sqrt{a\sin^2(x)}} + \frac{\int \frac{1}{\sqrt{a\sin^2(x)}}\,dx}{2a}$$
$$= -\frac{\cot(x)}{2a\sqrt{a\sin^2(x)}} + \frac{\sin(x)\int \csc(x)\,dx}{2a\sqrt{a\sin^2(x)}}$$
$$= -\frac{\cot(x)}{2a\sqrt{a\sin^2(x)}} - \frac{\tanh^{-1}(\cos(x))\sin(x)}{2a\sqrt{a\sin^2(x)}}$$

Mathematica [A] time = 0.058478, size = 55, normalized size = 1.31

$$-\frac{\sin^3(x)\left(\csc^2\left(\frac{x}{2}\right) - \sec^2\left(\frac{x}{2}\right) - 4\log\left(\sin\left(\frac{x}{2}\right)\right) + 4\log\left(\cos\left(\frac{x}{2}\right)\right)\right)}{8\left(a\sin^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[x]^2)^(-3/2),x]
```

```
[Out] -((Csc[x/2]^2 + 4*Log[Cos[x/2]] - 4*Log[Sin[x/2]] - Sec[x/2]^2)*Sin[x]^3)/(
8*(a*Sin[x]^2)^(3/2))
```

Maple [B] time = 1.116, size = 70, normalized size = 1.7

$$-\frac{1}{2\sin(x)\cos(x)}\sqrt{a(\cos(x))^2}\left(\ln\left(2\frac{\sqrt{a}\sqrt{a(\cos(x))^2}+a}{\sin(x)}\right)(\sin(x))^2 a + \sqrt{a}\sqrt{a(\cos(x))^2}\right)a^{-\frac{5}{2}}\frac{1}{\sqrt{a(\sin(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(x)^2)^(3/2),x)
```

```
[Out] -1/2/a^(5/2)/sin(x)*(a*cos(x)^2)^(1/2)*(ln(2*(a^(1/2)*(a*cos(x)^2)^(1/2)+a)
/sin(x))*sin(x)^2*a+a^(1/2)*(a*cos(x)^2)^(1/2))/cos(x)/(a*sin(x)^2)^(1/2)
```

Maxima [B] time = 1.64417, size = 424, normalized size = 10.1

$$-\frac{\left(\left(2\left(2\cos(2x)-1\right)\cos(4x) - \cos(4x)^2 - 4\cos(2x)^2 - \sin(4x)^2 + 4\sin(4x)\sin(2x) - 4\sin(2x)^2 + 4\cos(2x) - 1\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*((2*(2*cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - 4*cos(2*x)^2 - sin(4*x)^2
 + 4*sin(4*x)*sin(2*x) - 4*sin(2*x)^2 + 4*cos(2*x) - 1)*arctan2(sin(x), cos
(x) + 1) - (2*(2*cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - 4*cos(2*x)^2 - sin(4
*x)^2 + 4*sin(4*x)*sin(2*x) - 4*sin(2*x)^2 + 4*cos(2*x) - 1)*arctan2(sin(x)
, cos(x) - 1) + 2*(sin(3*x) + sin(x))*cos(4*x) - 2*(cos(3*x) + cos(x))*sin(
```

```
4*x) - 2*(2*cos(2*x) - 1)*sin(3*x) + 4*cos(3*x)*sin(2*x) + 4*cos(x)*sin(2*x
) - 4*cos(2*x)*sin(x) + 2*sin(x))*sqrt(-a)/(a^2*cos(4*x)^2 + 4*a^2*cos(2*x)
^2 + a^2*sin(4*x)^2 - 4*a^2*sin(4*x)*sin(2*x) + 4*a^2*sin(2*x)^2 - 4*a^2*co
s(2*x) + a^2 - 2*(2*a^2*cos(2*x) - a^2)*cos(4*x))
```

Fricas [A] time = 1.6649, size = 158, normalized size = 3.76

$$\frac{\sqrt{-a\cos(x)^2+a}\left(\left(\cos(x)^2-1\right)\log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)+2\cos(x)\right)}{4\left(a^2\cos(x)^2-a^2\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(-a*cos(x)^2 + a)*((cos(x)^2 - 1)*log(-(cos(x) - 1)/(cos(x) + 1)) +
 2*cos(x))/((a^2*cos(x)^2 - a^2)*sin(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin^2(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)**2)**(3/2),x)
```

```
[Out] Integral((a*sin(x)**2)**(-3/2), x)
```

Giac [A] time = 1.27683, size = 82, normalized size = 1.95

$$\frac{\frac{\tan\left(\frac{1}{2}x\right)^2}{\operatorname{sgn}\left(\tan\left(\frac{1}{2}x\right)\right)}+\frac{2\log\left(\tan\left(\frac{1}{2}x\right)^2\right)}{\operatorname{sgn}\left(\tan\left(\frac{1}{2}x\right)\right)}-\frac{2\tan\left(\frac{1}{2}x\right)^2+1}{\operatorname{sgn}\left(\tan\left(\frac{1}{2}x\right)\right)\tan\left(\frac{1}{2}x\right)^2}}{8a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(tan(1/2*x)^2/sgn(tan(1/2*x)) + 2*log(tan(1/2*x)^2)/sgn(tan(1/2*x)) - (
2*tan(1/2*x)^2 + 1)/(sgn(tan(1/2*x))*tan(1/2*x)^2))/a^(3/2)
```

3.6 $\int \frac{1}{\left(a\sin^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=61

$$-\frac{3\cot(x)}{8a^2\sqrt{a\sin^2(x)}}-\frac{3\sin(x)\tanh^{-1}(\cos(x))}{8a^2\sqrt{a\sin^2(x)}}-\frac{\cot(x)}{4a\left(a\sin^2(x)\right)^{3/2}}$$

```
[Out] -Cot[x]/(4*a*(a*Sin[x]^2)^(3/2)) - (3*Cot[x])/(8*a^2*Sqrt[a*Sin[x]^2]) - (3
*ArcTanh[Cos[x]]*Sin[x])/(8*a^2*Sqrt[a*Sin[x]^2])
```

Rubi [A] time = 0.0302122, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3204, 3207, 3770}

$$-\frac{3\cot(x)}{8a^2\sqrt{a\sin^2(x)}}-\frac{3\sin(x)\tanh^{-1}(\cos(x))}{8a^2\sqrt{a\sin^2(x)}}-\frac{\cot(x)}{4a\left(a\sin^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[x]^2)^(-5/2),x]
```

```
[Out] -Cot[x]/(4*a*(a*Sin[x]^2)^(3/2)) - (3*Cot[x])/(8*a^2*Sqrt[a*Sin[x]^2]) - (3
*ArcTanh[Cos[x]]*Sin[x])/(8*a^2*Sqrt[a*Sin[x]^2])
```

Rule 3204

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(Cot[e + f*x]
*(b*Sin[e + f*x]^2)^(p + 1))/(b*f*(2*p + 1)), x] + Dist[(2*(p + 1))/(b*(2*p
 + 1)), Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&  !
IntegerQ[p] && LtQ[p, -1]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a\sin^2(x)\right)^{5/2}}\,dx = -\frac{\cot(x)}{4a\left(a\sin^2(x)\right)^{3/2}} + \frac{3\int \frac{1}{\left(a\sin^2(x)\right)^{3/2}}\,dx}{4a}$$
$$= -\frac{\cot(x)}{4a\left(a\sin^2(x)\right)^{3/2}} - \frac{3\cot(x)}{8a^2\sqrt{a\sin^2(x)}} + \frac{3\int \frac{1}{\sqrt{a\sin^2(x)}}\,dx}{8a^2}$$
$$= -\frac{\cot(x)}{4a\left(a\sin^2(x)\right)^{3/2}} - \frac{3\cot(x)}{8a^2\sqrt{a\sin^2(x)}} + \frac{(3\sin(x))\int \csc(x)\,dx}{8a^2\sqrt{a\sin^2(x)}}$$
$$= -\frac{\cot(x)}{4a\left(a\sin^2(x)\right)^{3/2}} - \frac{3\cot(x)}{8a^2\sqrt{a\sin^2(x)}} - \frac{3\tanh^{-1}(\cos(x))\sin(x)}{8a^2\sqrt{a\sin^2(x)}}$$

Mathematica [A] time = 0.194872, size = 77, normalized size = 1.26

$$-\frac{\csc(x)\sqrt{a\sin^2(x)}\left(\csc^4\left(\frac{x}{2}\right)+6\csc^2\left(\frac{x}{2}\right)-\sec^4\left(\frac{x}{2}\right)-6\sec^2\left(\frac{x}{2}\right)+24\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)\right)}{64a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[x]^2)^(-5/2),x]
```

```
[Out] -(Csc[x]*(6*Csc[x/2]^2 + Csc[x/2]^4 + 24*(Log[Cos[x/2]] - Log[Sin[x/2]]) -
6*Sec[x/2]^2 - Sec[x/2]^4)*Sqrt[a*Sin[x]^2])/(64*a^3)
```

Maple [A] time = 1.156, size = 89, normalized size = 1.5

$$-\frac{1}{8\,(\sin(x))^3\cos(x)}\sqrt{a(\cos(x))^2}\left(3\,\ln\left(2\,\frac{\sqrt{a}\sqrt{a(\cos(x))^2}+a}{\sin(x)}\right)(\sin(x))^4\,a+3\,\sqrt{a(\cos(x))^2}\,(\sin(x))^2\,\sqrt{a}+2\,\sqrt{a}\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(x)^2)^(5/2),x)
```

```
[Out] -1/8/a^(7/2)/sin(x)^3*(a*cos(x)^2)^(1/2)*(3*ln(2*(a^(1/2)*(a*cos(x)^2)^(1/2
)+a)/sin(x))*sin(x)^4*a+3*(a*cos(x)^2)^(1/2)*sin(x)^2*a^(1/2)+2*a^(1/2)*(a*
cos(x)^2)^(1/2))/cos(x)/(a*sin(x)^2)^(1/2)
```

Maxima [B] time = 2.55918, size = 1257, normalized size = 20.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/8*(3*(2*(4*cos(6*x) - 6*cos(4*x) + 4*cos(2*x) - 1)*cos(8*x) - cos(8*x)^2
 + 8*(6*cos(4*x) - 4*cos(2*x) + 1)*cos(6*x) - 16*cos(6*x)^2 + 12*(4*cos(2*x
```

```
) - 1)*cos(4*x) - 36*cos(4*x)^2 - 16*cos(2*x)^2 + 4*(2*sin(6*x) - 3*sin(4*x
) + 2*sin(2*x))*sin(8*x) - sin(8*x)^2 + 16*(3*sin(4*x) - 2*sin(2*x))*sin(6*
x) - 16*sin(6*x)^2 - 36*sin(4*x)^2 + 48*sin(4*x)*sin(2*x) - 16*sin(2*x)^2 +
 8*cos(2*x) - 1)*arctan2(sin(x), cos(x) + 1) - 3*(2*(4*cos(6*x) - 6*cos(4*x
) + 4*cos(2*x) - 1)*cos(8*x) - cos(8*x)^2 + 8*(6*cos(4*x) - 4*cos(2*x) + 1)
*cos(6*x) - 16*cos(6*x)^2 + 12*(4*cos(2*x) - 1)*cos(4*x) - 36*cos(4*x)^2 -
16*cos(2*x)^2 + 4*(2*sin(6*x) - 3*sin(4*x) + 2*sin(2*x))*sin(8*x) - sin(8*x
)^2 + 16*(3*sin(4*x) - 2*sin(2*x))*sin(6*x) - 16*sin(6*x)^2 - 36*sin(4*x)^2
 + 48*sin(4*x)*sin(2*x) - 16*sin(2*x)^2 + 8*cos(2*x) - 1)*arctan2(sin(x), c
os(x) - 1) + 2*(3*sin(7*x) - 11*sin(5*x) - 11*sin(3*x) + 3*sin(x))*cos(8*x)
 + 12*(2*sin(6*x) - 3*sin(4*x) + 2*sin(2*x))*cos(7*x) + 8*(11*sin(5*x) + 11
*sin(3*x) - 3*sin(x))*cos(6*x) + 44*(3*sin(4*x) - 2*sin(2*x))*cos(5*x) - 12
*(11*sin(3*x) - 3*sin(x))*cos(4*x) - 2*(3*cos(7*x) - 11*cos(5*x) - 11*cos(3
*x) + 3*cos(x))*sin(8*x) - 6*(4*cos(6*x) - 6*cos(4*x) + 4*cos(2*x) - 1)*sin
(7*x) - 8*(11*cos(5*x) + 11*cos(3*x) - 3*cos(x))*sin(6*x) - 22*(6*cos(4*x)
- 4*cos(2*x) + 1)*sin(5*x) + 12*(11*cos(3*x) - 3*cos(x))*sin(4*x) + 22*(4*c
os(2*x) - 1)*sin(3*x) - 88*cos(3*x)*sin(2*x) + 24*cos(x)*sin(2*x) - 24*cos(
2*x)*sin(x) + 6*sin(x))*sqrt(-a)/(a^3*cos(8*x)^2 + 16*a^3*cos(6*x)^2 + 36*a
^3*cos(4*x)^2 + 16*a^3*cos(2*x)^2 + a^3*sin(8*x)^2 + 16*a^3*sin(6*x)^2 + 36
*a^3*sin(4*x)^2 - 48*a^3*sin(4*x)*sin(2*x) + 16*a^3*sin(2*x)^2 - 8*a^3*cos(
2*x) + a^3 - 2*(4*a^3*cos(6*x) - 6*a^3*cos(4*x) + 4*a^3*cos(2*x) - a^3)*cos
(8*x) - 8*(6*a^3*cos(4*x) - 4*a^3*cos(2*x) + a^3)*cos(6*x) - 12*(4*a^3*cos(
2*x) - a^3)*cos(4*x) - 4*(2*a^3*sin(6*x) - 3*a^3*sin(4*x) + 2*a^3*sin(2*x))
*sin(8*x) - 16*(3*a^3*sin(4*x) - 2*a^3*sin(2*x))*sin(6*x))
```

Fricas [A] time = 1.70054, size = 221, normalized size = 3.62

$$\frac{\sqrt{-a\cos(x)^2+a}\left(6\cos(x)^3+3\left(\cos(x)^4-2\cos(x)^2+1\right)\log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)-10\cos(x)\right)}{16\left(a^3\cos(x)^4-2a^3\cos(x)^2+a^3\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/16*sqrt(-a*cos(x)^2 + a)*(6*cos(x)^3 + 3*(cos(x)^4 - 2*cos(x)^2 + 1)*log(
-(cos(x) - 1)/(cos(x) + 1)) - 10*cos(x))/((a^3*cos(x)^4 - 2*a^3*cos(x)^2 +
a^3)*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29953, size = 108, normalized size = 1.77

$$\frac{\operatorname{sgn}\left(\tan\left(\frac{1}{2}x\right)\right)\tan\left(\frac{1}{2}x\right)^{4}+8\operatorname{sgn}\left(\tan\left(\frac{1}{2}x\right)\right)\tan\left(\frac{1}{2}x\right)^{2}+\frac{12\log\left(\tan\left(\frac{1}{2}x\right)^{2}\right)}{\operatorname{sgn}\left(\tan\left(\frac{1}{2}x\right)\right)}-\frac{18\tan\left(\frac{1}{2}x\right)^{4}+8\tan\left(\frac{1}{2}x\right)^{2}+1}{\operatorname{sgn}\left(\tan\left(\frac{1}{2}x\right)\right)\tan\left(\frac{1}{2}x\right)^{4}}}{64a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/64*(sgn(tan(1/2*x))*tan(1/2*x)^4 + 8*sgn(tan(1/2*x))*tan(1/2*x)^2 + 12*lo
g(tan(1/2*x)^2)/sgn(tan(1/2*x)) - (18*tan(1/2*x)^4 + 8*tan(1/2*x)^2 + 1)/(s
gn(tan(1/2*x))*tan(1/2*x)^4))/a^(5/2)
```

3.7 $\int \left(a \sin^3(x)\right)^{5/2} dx$

Optimal. Leaf size=123

$$-\frac{2}{15}a^2 \sin^5(x)\cos(x)\sqrt{a\sin^3(x)}-\frac{26}{165}a^2 \sin^3(x)\cos(x)\sqrt{a\sin^3(x)}-\frac{78}{385}a^2 \sin(x)\cos(x)\sqrt{a\sin^3(x)}-\frac{26}{77}a^2 \cot(x)\sqrt{a\sin$$

```
[Out] (-26*a^2*Cot[x]*Sqrt[a*Sin[x]^3])/77 - (26*a^2*EllipticF[Pi/4 - x/2, 2]*Sqr
t[a*Sin[x]^3])/(77*Sin[x]^(3/2)) - (78*a^2*Cos[x]*Sin[x]*Sqrt[a*Sin[x]^3])/
385 - (26*a^2*Cos[x]*Sin[x]^3*Sqrt[a*Sin[x]^3])/165 - (2*a^2*Cos[x]*Sin[x]^
5*Sqrt[a*Sin[x]^3])/15
```

Rubi [A] time = 0.0407798, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3207, 2635, 2641}

$$-\frac{2}{15}a^2 \sin^5(x)\cos(x)\sqrt{a\sin^3(x)}-\frac{26}{165}a^2 \sin^3(x)\cos(x)\sqrt{a\sin^3(x)}-\frac{78}{385}a^2 \sin(x)\cos(x)\sqrt{a\sin^3(x)}-\frac{26}{77}a^2 \cot(x)\sqrt{a\sin$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[x]^3)^(5/2),x]
```

```
[Out] (-26*a^2*Cot[x]*Sqrt[a*Sin[x]^3])/77 - (26*a^2*EllipticF[Pi/4 - x/2, 2]*Sqr
t[a*Sin[x]^3])/(77*Sin[x]^(3/2)) - (78*a^2*Cos[x]*Sin[x]*Sqrt[a*Sin[x]^3])/
385 - (26*a^2*Cos[x]*Sin[x]^3*Sqrt[a*Sin[x]^3])/165 - (2*a^2*Cos[x]*Sin[x]^
5*Sqrt[a*Sin[x]^3])/15
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \left(a\sin^3(x)\right)^{5/2} dx = \frac{\left(a^2\sqrt{a\sin^3(x)}\right)\int \sin^{\frac{15}{2}}(x)\,dx}{\sin^{\frac{3}{2}}(x)}$$

$$= -\frac{2}{15}a^2\cos(x)\sin^5(x)\sqrt{a\sin^3(x)} + \frac{\left(13a^2\sqrt{a\sin^3(x)}\right)\int \sin^{\frac{11}{2}}(x)\,dx}{15\sin^{\frac{3}{2}}(x)}$$

$$= -\frac{26}{165}a^2\cos(x)\sin^3(x)\sqrt{a\sin^3(x)} - \frac{2}{15}a^2\cos(x)\sin^5(x)\sqrt{a\sin^3(x)} + \frac{\left(39a^2\sqrt{a\sin^3(x)}\right)\int \sin^{\frac{7}{2}}(x)$$

$$55\sin^{\frac{3}{2}}(x)$$

$$= -\frac{78}{385}a^2\cos(x)\sin(x)\sqrt{a\sin^3(x)} - \frac{26}{165}a^2\cos(x)\sin^3(x)\sqrt{a\sin^3(x)} - \frac{2}{15}a^2\cos(x)\sin^5(x)\sqrt{a\sin$$

$$= -\frac{26}{77}a^2\cot(x)\sqrt{a\sin^3(x)} - \frac{78}{385}a^2\cos(x)\sin(x)\sqrt{a\sin^3(x)} - \frac{26}{165}a^2\cos(x)\sin^3(x)\sqrt{a\sin^3(x)} -$$

$$= -\frac{26}{77}a^2\cot(x)\sqrt{a\sin^3(x)} - \frac{26a^2F\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)\sqrt{a\sin^3(x)}}{77\sin^{\frac{3}{2}}(x)} - \frac{78}{385}a^2\cos(x)\sin(x)\sqrt{a\sin^3(x)} - \frac{26}{165}$$

Mathematica [A] time = 0.188814, size = 65, normalized size = 0.53

$$\frac{a\left(a\sin^3(x)\right)^{3/2}\left(\sqrt{\sin(x)}(-15465\cos(x)+3657\cos(3x)-749\cos(5x)+77\cos(7x))-12480F\left(\frac{1}{4}(\pi-2x)\middle|2\right)\right)}{36960\sin^{\frac{9}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[x]^3)^(5/2),x]
```

```
[Out] (a*(-12480*EllipticF[(Pi - 2*x)/4, 2] + (-15465*Cos[x] + 3657*Cos[3*x] - 74
9*Cos[5*x] + 77*Cos[7*x])*Sqrt[Sin[x]])*(a*Sin[x]^3)^(3/2))/(36960*Sin[x]^(
9/2))
```

Maple [C] time = 0.224, size = 149, normalized size = 1.2

$$-\frac{1}{1155\,(\sin(x))^7(-1+\cos(x))}\left(-154\,(\cos(x))^8+195\,i\sqrt{2}\sin(x)\sqrt{\frac{-i(-1+\cos(x))}{\sin(x)}}\sqrt{\frac{i\cos(x)+\sin(x)-i}{\sin(x)}}\sqrt{-\frac{i\cos}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(x)^3)^(5/2),x)
```

```
[Out] -1/1155*(-154*cos(x)^8+195*I*2^(1/2)*sin(x)*(-I*(-1+cos(x))/sin(x))^(1/2)*(
(I*cos(x)+sin(x)-I)/sin(x))^(1/2)*(-(I*cos(x)-sin(x)-I)/sin(x))^(1/2)*Ellip
ticF(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))+154*cos(x)^7+644*cos(x
)^6-644*cos(x)^5-1060*cos(x)^4+1060*cos(x)^3+960*cos(x)^2-960*cos(x))*(a*si
n(x)^3)^(5/2)/sin(x)^7/(-1+cos(x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(x\right)^{3}\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^3)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(x)^3)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(a^{2}\cos\left(x\right)^{6} - 3\,a^{2}\cos\left(x\right)^{4} + 3\,a^{2}\cos\left(x\right)^{2} - a^{2}\right)\sqrt{-\left(a\cos\left(x\right)^{2} - a\right)\sin\left(x\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*cos(x)^6 - 3*a^2*cos(x)^4 + 3*a^2*cos(x)^2 - a^2)*sqrt(-(a*c
os(x)^2 - a)*sin(x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)**3)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(x\right)^{3}\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^3)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(x)^3)^(5/2), x)
```

3.8 $\int \left(a\sin^3(x)\right)^{3/2}\,dx$

Optimal. Leaf size=73

$$-\frac{2}{9}a\sin^2(x)\cos(x)\sqrt{a\sin^3(x)}-\frac{14}{45}a\cos(x)\sqrt{a\sin^3(x)}-\frac{14aE\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)\sqrt{a\sin^3(x)}}{15\sin^{\frac{3}{2}}(x)}$$

```
[Out] (-14*a*Cos[x]*Sqrt[a*Sin[x]^3])/45 - (14*a*EllipticE[Pi/4 - x/2, 2]*Sqrt[a*
Sin[x]^3])/(15*Sin[x]^(3/2)) - (2*a*Cos[x]*Sin[x]^2*Sqrt[a*Sin[x]^3])/9
```

Rubi [A] time = 0.0247842, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3207, 2635, 2639}

$$-\frac{2}{9}a\sin^2(x)\cos(x)\sqrt{a\sin^3(x)}-\frac{14}{45}a\cos(x)\sqrt{a\sin^3(x)}-\frac{14aE\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)\sqrt{a\sin^3(x)}}{15\sin^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[x]^3)^(3/2),x]
```

```
[Out] (-14*a*Cos[x]*Sqrt[a*Sin[x]^3])/45 - (14*a*EllipticE[Pi/4 - x/2, 2]*Sqrt[a*
Sin[x]^3])/(15*Sin[x]^(3/2)) - (2*a*Cos[x]*Sin[x]^2*Sqrt[a*Sin[x]^3])/9
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \left(a\sin^3(x)\right)^{3/2}\,dx &= \frac{\left(a\sqrt{a\sin^3(x)}\right)\int \sin^{\frac{9}{2}}(x)\,dx}{\sin^{\frac{3}{2}}(x)}\\
&= -\frac{2}{9}a\cos(x)\sin^2(x)\sqrt{a\sin^3(x)}+\frac{\left(7a\sqrt{a\sin^3(x)}\right)\int \sin^{\frac{5}{2}}(x)\,dx}{9\sin^{\frac{3}{2}}(x)}\\
&= -\frac{14}{45}a\cos(x)\sqrt{a\sin^3(x)}-\frac{2}{9}a\cos(x)\sin^2(x)\sqrt{a\sin^3(x)}+\frac{\left(7a\sqrt{a\sin^3(x)}\right)\int \sqrt{\sin(x)}\,dx}{15\sin^{\frac{3}{2}}(x)}\\
&= -\frac{14}{45}a\cos(x)\sqrt{a\sin^3(x)}-\frac{14aE\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)\sqrt{a\sin^3(x)}}{15\sin^{\frac{3}{2}}(x)}-\frac{2}{9}a\cos(x)\sin^2(x)\sqrt{a\sin^3(x)}
\end{aligned}
$$

Mathematica [A] time = 0.100251, size = 54, normalized size = 0.74

$$
\frac{\left(a\sin^3(x)\right)^{3/2}\left(\sqrt{\sin(x)}(5\sin(4x)-38\sin(2x))-168E\left(\frac{1}{4}(\pi-2x)\middle|2\right)\right)}{180\sin^{\frac{9}{2}}(x)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[x]^3)^(3/2),x]
```

```
[Out] ((a*Sin[x]^3)^(3/2)*(-168*EllipticE[(Pi - 2*x)/4, 2] + Sqrt[Sin[x]]*(-38*Si
n[2*x] + 5*Sin[4*x])))/(180*Sin[x]^(9/2))
```

Maple [C] time = 0.213, size = 337, normalized size = 4.6

$$
-\frac{1}{45\left(\sin(x)\right)^5}\left(42\sqrt{2}\cos(x)\sqrt{\frac{-i\cos(x)+\sin(x)+i}{\sin(x)}}\mathrm{EllipticE}\left(\sqrt{\frac{i\cos(x)+\sin(x)-i}{\sin(x)}},1/2\sqrt{2}\right)\sqrt{\frac{i\cos(x)+\sin(x)-}{\sin(x)}}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(x)^3)^(3/2),x)
```

```
[Out] -1/45*(42*2^(1/2)*cos(x)*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*EllipticE(((I*
cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))*((I*cos(x)+sin(x)-I)/sin(x))^(1
/2)*(-I*(-1+cos(x))/sin(x))^(1/2)-21*2^(1/2)*cos(x)*((-I*cos(x)+sin(x)+I)/s
in(x))^(1/2)*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*(-I*(-1+cos(x))/sin(x))^(1/
2)*EllipticF(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))+42*2^(1/2)*((-
I*cos(x)+sin(x)+I)/sin(x))^(1/2)*EllipticE(((I*cos(x)+sin(x)-I)/sin(x))^(1/
2),1/2*2^(1/2))*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*(-I*(-1+cos(x))/sin(x))^
(1/2)-21*2^(1/2)*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*((I*cos(x)+sin(x)-I)/s
in(x))^(1/2)*(-I*(-1+cos(x))/sin(x))^(1/2)*EllipticF(((I*cos(x)+sin(x)-I)/s
in(x))^(1/2),1/2*2^(1/2))+10*cos(x)^5-34*cos(x)^3+66*cos(x)-42)*(a*sin(x)^3
)^(3/2)/sin(x)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \left(a\sin(x)^3\right)^{\frac{3}{2}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(x)^3)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(a\cos\left(x\right)^{2}-a\right)\sqrt{-\left(a\cos\left(x\right)^{2}-a\right)\sin\left(x\right)}\sin\left(x\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(a*cos(x)^2 - a)*sqrt(-(a*cos(x)^2 - a)*sin(x))*sin(x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)**3)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(x\right)^{3}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(x)^3)^(3/2), x)
```

3.9 $\int \sqrt{a \sin^3(x)}\, dx$

Optimal. Leaf size=50

$$-\frac{2}{3} \cot(x)\sqrt{a \sin^3(x)} - \frac{2F\left(\frac{\pi}{4} - \frac{x}{2}\middle|2\right)\sqrt{a \sin^3(x)}}{3 \sin^{\frac{3}{2}}(x)}$$

```
[Out] (-2*Cot[x]*Sqrt[a*Sin[x]^3])/3 - (2*EllipticF[Pi/4 - x/2, 2]*Sqrt[a*Sin[x]^
3])/(3*Sin[x]^(3/2))
```

Rubi [A] time = 0.0176481, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3207, 2635, 2641}

$$-\frac{2}{3} \cot(x)\sqrt{a \sin^3(x)} - \frac{2F\left(\frac{\pi}{4} - \frac{x}{2}\middle|2\right)\sqrt{a \sin^3(x)}}{3 \sin^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Sin[x]^3],x]
```

```
[Out] (-2*Cot[x]*Sqrt[a*Sin[x]^3])/3 - (2*EllipticF[Pi/4 - x/2, 2]*Sqrt[a*Sin[x]^
3])/(3*Sin[x]^(3/2))
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{a\sin^3(x)}\,dx = \frac{\sqrt{a\sin^3(x)}\int \sin^{\frac{3}{2}}(x)\,dx}{\sin^{\frac{3}{2}}(x)}$$
$$= -\frac{2}{3}\cot(x)\sqrt{a\sin^3(x)} + \frac{\sqrt{a\sin^3(x)}\int \frac{1}{\sqrt{\sin(x)}}\,dx}{3\sin^{\frac{3}{2}}(x)}$$
$$= -\frac{2}{3}\cot(x)\sqrt{a\sin^3(x)} - \frac{2F\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)\sqrt{a\sin^3(x)}}{3\sin^{\frac{3}{2}}(x)}$$

Mathematica [A] time = 0.0305087, size = 41, normalized size = 0.82

$$-\frac{2\sqrt{a\sin^3(x)}\left(F\left(\frac{1}{4}(\pi-2x)\middle|2\right)+\sqrt{\sin(x)}\cos(x)\right)}{3\sin^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Sin[x]^3],x]
```

```
[Out] (-2*(EllipticF[(Pi - 2*x)/4, 2] + Cos[x]*Sqrt[Sin[x]])*Sqrt[a*Sin[x]^3])/(3
*Sin[x]^(3/2))
```

Maple [C] time = 0.271, size = 118, normalized size = 2.4

$$-\frac{\sqrt{8}}{6\sin(x)(-1+\cos(x))}\left(i\sqrt{\frac{-i(-1+\cos(x))}{\sin(x)}}\sin(x)\sqrt{-\frac{i\cos(x)-\sin(x)-i}{\sin(x)}}\mathrm{EllipticF}\left(\sqrt{\frac{i\cos(x)+\sin(x)-i}{\sin(x)}},\frac{\sqrt{2}}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(x)^3)^(1/2),x)
```

```
[Out] -1/6*8^(1/2)*(I*(-I*(-1+cos(x))/sin(x))^(1/2)*sin(x)*(-(I*cos(x)-sin(x)-I)/
sin(x))^(1/2)*EllipticF(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))*((I
*cos(x)+sin(x)-I)/sin(x))^(1/2)+cos(x)^2*2^(1/2)-cos(x)*2^(1/2))*(a*sin(x)^
3)^(1/2)/sin(x)/(-1+cos(x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin(x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(x)^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-\left(a\cos\left(x\right)^{2}-a\right)\sin\left(x\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-(a*cos(x)^2 - a)*sin(x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\sin^{3}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)**3)**(1/2),x)
```

```
[Out] Integral(sqrt(a*sin(x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\sin\left(x\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sin(x)^3), x)
```

3.10 $\int \frac{1}{\sqrt{a\sin^3(x)}}\,dx$

Optimal. Leaf size=48

$$\frac{2\sin^{\frac{3}{2}}(x)E\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)}{\sqrt{a\sin^3(x)}}-\frac{2\sin(x)\cos(x)}{\sqrt{a\sin^3(x)}}$$

```
[Out] (-2*Cos[x]*Sin[x])/Sqrt[a*Sin[x]^3] + (2*EllipticE[Pi/4 - x/2, 2]*Sin[x]^(3
/2))/Sqrt[a*Sin[x]^3]
```

Rubi [A] time = 0.0173165, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3207, 2636, 2639}

$$\frac{2\sin^{\frac{3}{2}}(x)E\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)}{\sqrt{a\sin^3(x)}}-\frac{2\sin(x)\cos(x)}{\sqrt{a\sin^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Sin[x]^3],x]
```

```
[Out] (-2*Cos[x]*Sin[x])/Sqrt[a*Sin[x]^3] + (2*EllipticE[Pi/4 - x/2, 2]*Sin[x]^(3
/2))/Sqrt[a*Sin[x]^3]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a\sin^3(x)}}\,dx = \frac{\sin^{\frac{3}{2}}(x)\int \frac{1}{\sin^{\frac{3}{2}}(x)}\,dx}{\sqrt{a\sin^3(x)}}$$

$$= -\frac{2\cos(x)\sin(x)}{\sqrt{a\sin^3(x)}} - \frac{\sin^{\frac{3}{2}}(x)\int \sqrt{\sin(x)}\,dx}{\sqrt{a\sin^3(x)}}$$

$$= -\frac{2\cos(x)\sin(x)}{\sqrt{a\sin^3(x)}} + \frac{2E\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)\sin^{\frac{3}{2}}(x)}{\sqrt{a\sin^3(x)}}$$

Mathematica [A] time = 0.0265732, size = 37, normalized size = 0.77

$$\frac{2\sin^{\frac{3}{2}}(x)E\left(\frac{1}{4}(\pi-2x)\middle|2\right)-\sin(2x)}{\sqrt{a\sin^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Sin[x]^3],x]
```

```
[Out] (2*EllipticE[(Pi - 2*x)/4, 2]*Sin[x]^(3/2) - Sin[2*x])/Sqrt[a*Sin[x]^3]
```

Maple [C] time = 0.257, size = 330, normalized size = 6.9

$$\sin(x)\left(2\sqrt{2}\cos(x)\sqrt{\frac{-i(-1+\cos(x))}{\sin(x)}}\sqrt{\frac{i\cos(x)+\sin(x)-i}{\sin(x)}}\sqrt{-\frac{i\cos(x)-\sin(x)-i}{\sin(x)}}\mathrm{EllipticE}\left(\sqrt{\frac{i\cos(x)+\sin(x)-}{\sin(x)}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(x)^3)^(1/2),x)
```

```
[Out] (2*2^(1/2)*cos(x)*(-I*(-1+cos(x))/sin(x))^(1/2)*((I*cos(x)+sin(x)-I)/sin(x)
)^(1/2)*(-(I*cos(x)-sin(x)-I)/sin(x))^(1/2)*EllipticE(((I*cos(x)+sin(x)-I)/
sin(x))^(1/2),1/2*2^(1/2))-2^(1/2)*cos(x)*(-I*(-1+cos(x))/sin(x))^(1/2)*((I
*cos(x)+sin(x)-I)/sin(x))^(1/2)*(-(I*cos(x)-sin(x)-I)/sin(x))^(1/2)*Ellipti
cF(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))+2*2^(1/2)*(-I*(-1+cos(x)
)/sin(x))^(1/2)*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*(-(I*cos(x)-sin(x)-I)/si
n(x))^(1/2)*EllipticE(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))-2^(1/
2)*(-I*(-1+cos(x))/sin(x))^(1/2)*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*(-(I*co
s(x)-sin(x)-I)/sin(x))^(1/2)*EllipticF(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1
/2*2^(1/2))-2)*sin(x)/(a*sin(x)^3)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\sin(x)^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a*sin(x)^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-\left(a\cos\left(x\right)^{2}-a\right)\sin\left(x\right)}}{\left(a\cos\left(x\right)^{2}-a\right)\sin\left(x\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-(a*cos(x)^2 - a)*sin(x))/((a*cos(x)^2 - a)*sin(x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\sin^{3}\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)**3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*sin(x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\sin\left(x\right)^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(a*sin(x)^3), x)
```

3.11 $\int \frac{1}{\left(a\sin^3(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=77

$$-\frac{10\cos(x)}{21a\sqrt{a\sin^3(x)}}-\frac{10\sin^{\frac{3}{2}}(x)F\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)}{21a\sqrt{a\sin^3(x)}}-\frac{2\cot(x)\csc(x)}{7a\sqrt{a\sin^3(x)}}$$

```
[Out] (-10*Cos[x])/(21*a*Sqrt[a*Sin[x]^3]) - (2*Cot[x]*Csc[x])/(7*a*Sqrt[a*Sin[x]
^3]) - (10*EllipticF[Pi/4 - x/2, 2]*Sin[x]^(3/2))/(21*a*Sqrt[a*Sin[x]^3])
```

Rubi [A] time = 0.0254569, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3207, 2636, 2641}

$$-\frac{10\cos(x)}{21a\sqrt{a\sin^3(x)}}-\frac{10\sin^{\frac{3}{2}}(x)F\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)}{21a\sqrt{a\sin^3(x)}}-\frac{2\cot(x)\csc(x)}{7a\sqrt{a\sin^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[x]^3)^(-3/2),x]
```

```
[Out] (-10*Cos[x])/(21*a*Sqrt[a*Sin[x]^3]) - (2*Cot[x]*Csc[x])/(7*a*Sqrt[a*Sin[x]
^3]) - (10*EllipticF[Pi/4 - x/2, 2]*Sin[x]^(3/2))/(21*a*Sqrt[a*Sin[x]^3])
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a\sin^3(x)\right)^{3/2}}\,dx = \frac{\sin^{\frac{3}{2}}(x)\int \frac{1}{\sin^{\frac{9}{2}}(x)}\,dx}{a\sqrt{a\sin^3(x)}}$$

$$= -\frac{2\cot(x)\csc(x)}{7a\sqrt{a\sin^3(x)}} + \frac{\left(5\sin^{\frac{3}{2}}(x)\right)\int \frac{1}{\sin^{\frac{5}{2}}(x)}\,dx}{7a\sqrt{a\sin^3(x)}}$$

$$= -\frac{10\cos(x)}{21a\sqrt{a\sin^3(x)}} - \frac{2\cot(x)\csc(x)}{7a\sqrt{a\sin^3(x)}} + \frac{\left(5\sin^{\frac{3}{2}}(x)\right)\int \frac{1}{\sqrt{\sin(x)}}\,dx}{21a\sqrt{a\sin^3(x)}}$$

$$= -\frac{10\cos(x)}{21a\sqrt{a\sin^3(x)}} - \frac{2\cot(x)\csc(x)}{7a\sqrt{a\sin^3(x)}} - \frac{10F\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)\sin^{\frac{3}{2}}(x)}{21a\sqrt{a\sin^3(x)}}$$

Mathematica [A] time = 0.0678059, size = 48, normalized size = 0.62

$$-\frac{2\sin^2(x)\left(3\cot(x)+5\sin(x)\cos(x)+5\sin^{\frac{5}{2}}(x)F\left(\frac{1}{4}(\pi-2x)\middle|2\right)\right)}{21\left(a\sin^3(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[x]^3)^(-3/2),x]
```

```
[Out] (-2*Sin[x]^2*(3*Cot[x] + 5*Cos[x]*Sin[x] + 5*EllipticF[(Pi - 2*x)/4, 2]*Sin
[x]^(5/2)))/(21*(a*Sin[x]^3)^(3/2))
```

Maple [C] time = 0.283, size = 360, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(x)^3)^(3/2),x)
```

```
[Out] -1/21*(cos(x)+1)^2*(-1+cos(x))^2*(5*I*2^(1/2)*sin(x)*cos(x)^3*((I*cos(x)+si
n(x)-I)/sin(x))^(1/2)*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*(-I*(-1+cos(x))/s
in(x))^(1/2)*EllipticF(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))+5*I*
2^(1/2)*sin(x)*cos(x)^2*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*((-I*cos(x)+sin(
x)+I)/sin(x))^(1/2)*(-I*(-1+cos(x))/sin(x))^(1/2)*EllipticF(((I*cos(x)+sin(
x)-I)/sin(x))^(1/2),1/2*2^(1/2))-5*I*2^(1/2)*sin(x)*cos(x)*((I*cos(x)+sin(x
)-I)/sin(x))^(1/2)*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*(-I*(-1+cos(x))/sin(
x))^(1/2)*EllipticF(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))-5*I*((I
*cos(x)+sin(x)-I)/sin(x))^(1/2)*2^(1/2)*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)
*(-I*(-1+cos(x))/sin(x))^(1/2)*EllipticF(((I*cos(x)+sin(x)-I)/sin(x))^(1/2)
,1/2*2^(1/2))*sin(x)-10*cos(x)^3+16*cos(x))/(a*sin(x)^3)^(3/2)/sin(x)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin(x)^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(x)^3)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-\left(a\cos\left(x\right)^{2}-a\right)\sin\left(x\right)}}{a^{2}\cos\left(x\right)^{6}-3\,a^{2}\cos\left(x\right)^{4}+3\,a^{2}\cos\left(x\right)^{2}-a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-(a*cos(x)^2 - a)*sin(x))/(a^2*cos(x)^6 - 3*a^2*cos(x)^4 + 3
*a^2*cos(x)^2 - a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(a\sin^{3}\left(x\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)**3)**(3/2),x)
```

```
[Out] Integral((a*sin(x)**3)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(a\sin\left(x\right)^{3}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(x)^3)^(-3/2), x)
```

3.12 $\int \frac{1}{\left(a\sin^3(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=123

$$-\frac{154\sin(x)\cos(x)}{195a^2\sqrt{a\sin^3(x)}}-\frac{154\cot(x)}{585a^2\sqrt{a\sin^3(x)}}+\frac{154\sin^{\frac{3}{2}}(x)E\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)}{195a^2\sqrt{a\sin^3(x)}}-\frac{2\cot(x)\csc^4(x)}{13a^2\sqrt{a\sin^3(x)}}-\frac{22\cot(x)\csc^2(x)}{117a^2\sqrt{a\sin^3(x)}}$$

```
[Out] (-154*Cot[x])/(585*a^2*Sqrt[a*Sin[x]^3]) - (22*Cot[x]*Csc[x]^2)/(117*a^2*Sq
rt[a*Sin[x]^3]) - (2*Cot[x]*Csc[x]^4)/(13*a^2*Sqrt[a*Sin[x]^3]) - (154*Cos[
x]*Sin[x])/(195*a^2*Sqrt[a*Sin[x]^3]) + (154*EllipticE[Pi/4 - x/2, 2]*Sin[x
]^(3/2))/(195*a^2*Sqrt[a*Sin[x]^3])
```

Rubi [A] time = 0.0420212, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3207, 2636, 2639}

$$-\frac{154\sin(x)\cos(x)}{195a^2\sqrt{a\sin^3(x)}}-\frac{154\cot(x)}{585a^2\sqrt{a\sin^3(x)}}+\frac{154\sin^{\frac{3}{2}}(x)E\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)}{195a^2\sqrt{a\sin^3(x)}}-\frac{2\cot(x)\csc^4(x)}{13a^2\sqrt{a\sin^3(x)}}-\frac{22\cot(x)\csc^2(x)}{117a^2\sqrt{a\sin^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[x]^3)^(-5/2),x]
```

```
[Out] (-154*Cot[x])/(585*a^2*Sqrt[a*Sin[x]^3]) - (22*Cot[x]*Csc[x]^2)/(117*a^2*Sq
rt[a*Sin[x]^3]) - (2*Cot[x]*Csc[x]^4)/(13*a^2*Sqrt[a*Sin[x]^3]) - (154*Cos[
x]*Sin[x])/(195*a^2*Sqrt[a*Sin[x]^3]) + (154*EllipticE[Pi/4 - x/2, 2]*Sin[x
]^(3/2))/(195*a^2*Sqrt[a*Sin[x]^3])
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a\sin^3(x)\right)^{5/2}}\,dx = \frac{\sin^{\frac{3}{2}}(x)\int \frac{1}{\sin^{\frac{15}{2}}(x)}\,dx}{a^2\sqrt{a\sin^3(x)}}$$

$$= -\frac{2\cot(x)\csc^4(x)}{13a^2\sqrt{a\sin^3(x)}} + \frac{\left(11\sin^{\frac{3}{2}}(x)\right)\int \frac{1}{\sin^{\frac{11}{2}}(x)}\,dx}{13a^2\sqrt{a\sin^3(x)}}$$

$$= -\frac{22\cot(x)\csc^2(x)}{117a^2\sqrt{a\sin^3(x)}} - \frac{2\cot(x)\csc^4(x)}{13a^2\sqrt{a\sin^3(x)}} + \frac{\left(77\sin^{\frac{3}{2}}(x)\right)\int \frac{1}{\sin^{\frac{7}{2}}(x)}\,dx}{117a^2\sqrt{a\sin^3(x)}}$$

$$= -\frac{154\cot(x)}{585a^2\sqrt{a\sin^3(x)}} - \frac{22\cot(x)\csc^2(x)}{117a^2\sqrt{a\sin^3(x)}} - \frac{2\cot(x)\csc^4(x)}{13a^2\sqrt{a\sin^3(x)}} + \frac{\left(77\sin^{\frac{3}{2}}(x)\right)\int \frac{1}{\sin^{\frac{3}{2}}(x)}\,dx}{195a^2\sqrt{a\sin^3(x)}}$$

$$= -\frac{154\cot(x)}{585a^2\sqrt{a\sin^3(x)}} - \frac{22\cot(x)\csc^2(x)}{117a^2\sqrt{a\sin^3(x)}} - \frac{2\cot(x)\csc^4(x)}{13a^2\sqrt{a\sin^3(x)}} - \frac{154\cos(x)\sin(x)}{195a^2\sqrt{a\sin^3(x)}} - \frac{\left(77\sin^{\frac{3}{2}}(x)\right)\int \sqrt{s}}{195a^2\sqrt{a\sin}}$$

$$= -\frac{154\cot(x)}{585a^2\sqrt{a\sin^3(x)}} - \frac{22\cot(x)\csc^2(x)}{117a^2\sqrt{a\sin^3(x)}} - \frac{2\cot(x)\csc^4(x)}{13a^2\sqrt{a\sin^3(x)}} - \frac{154\cos(x)\sin(x)}{195a^2\sqrt{a\sin^3(x)}} + \frac{154E\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)s}{195a^2\sqrt{a\sin^3}}$$

Mathematica [A] time = 0.208115, size = 60, normalized size = 0.49

$$-\frac{2\left(231\sin(x)\cos(x)+\cot(x)\left(45\csc^4(x)+55\csc^2(x)+77\right)-231\sin^{\frac{3}{2}}(x)E\left(\frac{1}{4}(\pi-2x)\middle|2\right)\right)}{585a^2\sqrt{a\sin^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[x]^3)^(-5/2),x]
```

```
[Out] (-2*(Cot[x]*(77 + 55*Csc[x]^2 + 45*Csc[x]^4) + 231*Cos[x]*Sin[x] - 231*Elli
pticE[(Pi - 2*x)/4, 2]*Sin[x]^(3/2)))/(585*a^2*Sqrt[a*Sin[x]^3])
```

Maple [C] time = 0.248, size = 1301, normalized size = 10.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(x)^3)^(5/2),x)
```

```
[Out] -1/585*(462*2^(1/2)*cos(x)^7*(-I*(-1+cos(x))/sin(x))^(1/2)*((I*cos(x)+sin(x
)-I)/sin(x))^(1/2)*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*EllipticE(((I*cos(x)
+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))-231*2^(1/2)*cos(x)^7*(-I*(-1+cos(x))/
sin(x))^(1/2)*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*((-I*cos(x)+sin(x)+I)/sin(
x))^(1/2)*EllipticF(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))+462*2^(
1/2)*cos(x)^6*(-I*(-1+cos(x))/sin(x))^(1/2)*((I*cos(x)+sin(x)-I)/sin(x))^(1
/2)*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*EllipticE(((I*cos(x)+sin(x)-I)/sin(
x))^(1/2),1/2*2^(1/2))-231*2^(1/2)*cos(x)^6*(-I*(-1+cos(x))/sin(x))^(1/2)*(
```

```
(I*cos(x)+sin(x)-I)/sin(x))^(1/2)*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*Ellip
ticF(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))-1386*2^(1/2)*cos(x)^5*
(-I*(-1+cos(x))/sin(x))^(1/2)*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*((-I*cos(x
)+sin(x)+I)/sin(x))^(1/2)*EllipticE(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*
2^(1/2))+693*2^(1/2)*cos(x)^5*(-I*(-1+cos(x))/sin(x))^(1/2)*((I*cos(x)+sin(
x)-I)/sin(x))^(1/2)*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*EllipticF(((I*cos(x
)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))-1386*2^(1/2)*cos(x)^4*(-I*(-1+cos(x)
)/sin(x))^(1/2)*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*((-I*cos(x)+sin(x)+I)/si
n(x))^(1/2)*EllipticE(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))+693*2
^(1/2)*cos(x)^4*(-I*(-1+cos(x))/sin(x))^(1/2)*((I*cos(x)+sin(x)-I)/sin(x))^
(1/2)*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*EllipticF(((I*cos(x)+sin(x)-I)/si
n(x))^(1/2),1/2*2^(1/2))+1386*2^(1/2)*cos(x)^3*(-I*(-1+cos(x))/sin(x))^(1/2
)*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*El
lipticE(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))-693*2^(1/2)*cos(x)^
3*(-I*(-1+cos(x))/sin(x))^(1/2)*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*((-I*cos
(x)+sin(x)+I)/sin(x))^(1/2)*EllipticF(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/
2*2^(1/2))+1386*2^(1/2)*cos(x)^2*(-I*(-1+cos(x))/sin(x))^(1/2)*((I*cos(x)+s
in(x)-I)/sin(x))^(1/2)*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*EllipticE(((I*co
s(x)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))-693*2^(1/2)*cos(x)^2*(-I*(-1+cos(
x))/sin(x))^(1/2)*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*((-I*cos(x)+sin(x)+I)/
sin(x))^(1/2)*EllipticF(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))-462
*2^(1/2)*cos(x)*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*EllipticE(((I*cos(x)+si
n(x)-I)/sin(x))^(1/2),1/2*2^(1/2))*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*(-I*(
-1+cos(x))/sin(x))^(1/2)+231*2^(1/2)*cos(x)*((-I*cos(x)+sin(x)+I)/sin(x))^(
1/2)*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*(-I*(-1+cos(x))/sin(x))^(1/2)*Ellip
ticF(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))-462*cos(x)^6-462*2^(1/
2)*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*EllipticE(((I*cos(x)+sin(x)-I)/sin(x
))^(1/2),1/2*2^(1/2))*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*(-I*(-1+cos(x))/si
n(x))^(1/2)+231*2^(1/2)*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*((I*cos(x)+sin(
x)-I)/sin(x))^(1/2)*(-I*(-1+cos(x))/sin(x))^(1/2)*EllipticF(((I*cos(x)+sin(
x)-I)/sin(x))^(1/2),1/2*2^(1/2))+154*cos(x)^5+1386*cos(x)^4-418*cos(x)^3-13
86*cos(x)^2+354*cos(x)+462)*sin(x)/(a*sin(x)^3)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin\left(x\right)^{3}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^3)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(x)^3)^(-5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-\left(a\cos\left(x\right)^{2}-a\right)\sin\left(x\right)}}{\left(a^{3}\cos\left(x\right)^{8}-4\,a^{3}\cos\left(x\right)^{6}+6\,a^{3}\cos\left(x\right)^{4}-4\,a^{3}\cos\left(x\right)^{2}+a^{3}\right)\sin\left(x\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-(a*cos(x)^2 - a)*sin(x))/((a^3*cos(x)^8 - 4*a^3*cos(x)^6 + 6
*a^3*cos(x)^4 - 4*a^3*cos(x)^2 + a^3)*sin(x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)**3)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin\left(x\right)^3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^3)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(x)^3)^(-5/2), x)
```

3.13 $\int \left(a\sin^4(x)\right)^{5/2}\,dx$

Optimal. Leaf size=132

$$-\frac{1}{10}a^2\sin^7(x)\cos(x)\sqrt{a\sin^4(x)}-\frac{9}{80}a^2\sin^5(x)\cos(x)\sqrt{a\sin^4(x)}-\frac{21}{160}a^2\sin^3(x)\cos(x)\sqrt{a\sin^4(x)}-\frac{21}{128}a^2\sin(x)\mathrm{co}$$

```
[Out] (-63*a^2*Cot[x]*Sqrt[a*Sin[x]^4])/256 + (63*a^2*x*Csc[x]^2*Sqrt[a*Sin[x]^4]
)/256 - (21*a^2*Cos[x]*Sin[x]*Sqrt[a*Sin[x]^4])/128 - (21*a^2*Cos[x]*Sin[x]
^3*Sqrt[a*Sin[x]^4])/160 - (9*a^2*Cos[x]*Sin[x]^5*Sqrt[a*Sin[x]^4])/80 - (a
^2*Cos[x]*Sin[x]^7*Sqrt[a*Sin[x]^4])/10
```

Rubi [A] time = 0.0438645, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3207, 2635, 8}

$$-\frac{1}{10}a^2\sin^7(x)\cos(x)\sqrt{a\sin^4(x)}-\frac{9}{80}a^2\sin^5(x)\cos(x)\sqrt{a\sin^4(x)}-\frac{21}{160}a^2\sin^3(x)\cos(x)\sqrt{a\sin^4(x)}-\frac{21}{128}a^2\sin(x)\mathrm{co}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[x]^4)^(5/2),x]
```


```
[Out] (-63*a^2*Cot[x]*Sqrt[a*Sin[x]^4])/256 + (63*a^2*x*Csc[x]^2*Sqrt[a*Sin[x]^4]
)/256 - (21*a^2*Cos[x]*Sin[x]*Sqrt[a*Sin[x]^4])/128 - (21*a^2*Cos[x]*Sin[x]
^3*Sqrt[a*Sin[x]^4])/160 - (9*a^2*Cos[x]*Sin[x]^5*Sqrt[a*Sin[x]^4])/80 - (a
^2*Cos[x]*Sin[x]^7*Sqrt[a*Sin[x]^4])/10
```


Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \left(a\sin^4(x)\right)^{5/2} dx = \left(a^2\csc^2(x)\sqrt{a\sin^4(x)}\right)\int \sin^{10}(x)\,dx$$
$$= -\frac{1}{10}a^2\cos(x)\sin^7(x)\sqrt{a\sin^4(x)} + \frac{1}{10}\left(9a^2\csc^2(x)\sqrt{a\sin^4(x)}\right)\int \sin^8(x)\,dx$$
$$= -\frac{9}{80}a^2\cos(x)\sin^5(x)\sqrt{a\sin^4(x)} - \frac{1}{10}a^2\cos(x)\sin^7(x)\sqrt{a\sin^4(x)} + \frac{1}{80}\left(63a^2\csc^2(x)\sqrt{a\sin^4(x)}\right)\int$$
$$= -\frac{21}{160}a^2\cos(x)\sin^3(x)\sqrt{a\sin^4(x)} - \frac{9}{80}a^2\cos(x)\sin^5(x)\sqrt{a\sin^4(x)} - \frac{1}{10}a^2\cos(x)\sin^7(x)\sqrt{a\sin^4(x}$$
$$= -\frac{21}{128}a^2\cos(x)\sin(x)\sqrt{a\sin^4(x)} - \frac{21}{160}a^2\cos(x)\sin^3(x)\sqrt{a\sin^4(x)} - \frac{9}{80}a^2\cos(x)\sin^5(x)\sqrt{a\sin^4(x}$$
$$= -\frac{63}{256}a^2\cot(x)\sqrt{a\sin^4(x)} - \frac{21}{128}a^2\cos(x)\sin(x)\sqrt{a\sin^4(x)} - \frac{21}{160}a^2\cos(x)\sin^3(x)\sqrt{a\sin^4(x)} - \frac{9}{80}$$
$$= -\frac{63}{256}a^2\cot(x)\sqrt{a\sin^4(x)} + \frac{63}{256}a^2x\csc^2(x)\sqrt{a\sin^4(x)} - \frac{21}{128}a^2\cos(x)\sin(x)\sqrt{a\sin^4(x)} - \frac{21}{160}a^2\,\text{c}$$

Mathematica [A] time = 0.164697, size = 53, normalized size = 0.4

$$\frac{a(2520x - 2100\sin(2x) + 600\sin(4x) - 150\sin(6x) + 25\sin(8x) - 2\sin(10x))\csc^6(x)\left(a\sin^4(x)\right)^{3/2}}{10240}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[x]^4)^(5/2),x]
```

```
[Out] (a*Csc[x]^6*(a*Sin[x]^4)^(3/2)*(2520*x - 2100*Sin[2*x] + 600*Sin[4*x] - 150
*Sin[6*x] + 25*Sin[8*x] - 2*Sin[10*x]))/10240
```

Maple [A] time = 0.292, size = 57, normalized size = 0.4

$$-\frac{128\,(\cos(x))^9\sin(x) - 656\,(\cos(x))^7\sin(x) + 1368\,(\cos(x))^5\sin(x) - 1490\,\sin(x)(\cos(x))^3 + 965\,\sin(x)\cos(x) - 3}{1280\,(\sin(x))^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(x)^4)^(5/2),x)
```

```
[Out] -1/1280*(a*sin(x)^4)^(5/2)*(128*cos(x)^9*sin(x)-656*cos(x)^7*sin(x)+1368*co
s(x)^5*sin(x)-1490*sin(x)*cos(x)^3+965*sin(x)*cos(x)-315*x)/sin(x)^10
```

Maxima [A] time = 1.44827, size = 115, normalized size = 0.87

$$\frac{63}{256}a^{\frac{5}{2}}x - \frac{965\,a^{\frac{5}{2}}\tan(x)^9 + 2370\,a^{\frac{5}{2}}\tan(x)^7 + 2688\,a^{\frac{5}{2}}\tan(x)^5 + 1470\,a^{\frac{5}{2}}\tan(x)^3 + 315\,a^{\frac{5}{2}}\tan(x)}{1280\left(\tan(x)^{10} + 5\tan(x)^8 + 10\tan(x)^6 + 10\tan(x)^4 + 5\tan(x)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] 63/256*a^(5/2)*x - 1/1280*(965*a^(5/2)*tan(x)^9 + 2370*a^(5/2)*tan(x)^7 + 2
688*a^(5/2)*tan(x)^5 + 1470*a^(5/2)*tan(x)^3 + 315*a^(5/2)*tan(x))/(tan(x)^
10 + 5*tan(x)^8 + 10*tan(x)^6 + 10*tan(x)^4 + 5*tan(x)^2 + 1)
```

Fricas [A] time = 1.80149, size = 238, normalized size = 1.8

$$-\frac{\sqrt{a\cos(x)^4 - 2\,a\cos(x)^2 + a}\left(315\,a^2x - \left(128\,a^2\cos(x)^9 - 656\,a^2\cos(x)^7 + 1368\,a^2\cos(x)^5 - 1490\,a^2\cos(x)^3 + 965\right.\right.}{1280\left(\cos(x)^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/1280*sqrt(a*cos(x)^4 - 2*a*cos(x)^2 + a)*(315*a^2*x - (128*a^2*cos(x)^9
- 656*a^2*cos(x)^7 + 1368*a^2*cos(x)^5 - 1490*a^2*cos(x)^3 + 965*a^2*cos(x)
)*sin(x))/(cos(x)^2 - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)**4)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10804, size = 77, normalized size = 0.58

$$\frac{1}{10240}\left(2520\,a^2x - 2\,a^2\sin(10\,x) + 25\,a^2\sin(8\,x) - 150\,a^2\sin(6\,x) + 600\,a^2\sin(4\,x) - 2100\,a^2\sin(2\,x)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] 1/10240*(2520*a^2*x - 2*a^2*sin(10*x) + 25*a^2*sin(8*x) - 150*a^2*sin(6*x)
+ 600*a^2*sin(4*x) - 2100*a^2*sin(2*x))*sqrt(a)
```

3.14 $\int \left(a\sin^4(x)\right)^{3/2}\,dx$

Optimal. Leaf size=78

$$-\frac{1}{6}a\sin^3(x)\cos(x)\sqrt{a\sin^4(x)}-\frac{5}{24}a\sin(x)\cos(x)\sqrt{a\sin^4(x)}-\frac{5}{16}a\cot(x)\sqrt{a\sin^4(x)}+\frac{5}{16}ax\csc^2(x)\sqrt{a\sin^4(x)}$$

```
[Out] (-5*a*Cot[x]*Sqrt[a*Sin[x]^4])/16 + (5*a*x*Csc[x]^2*Sqrt[a*Sin[x]^4])/16 -
(5*a*Cos[x]*Sin[x]*Sqrt[a*Sin[x]^4])/24 - (a*Cos[x]*Sin[x]^3*Sqrt[a*Sin[x]^
4])/6
```

Rubi [A] time = 0.0277571, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3207, 2635, 8}

$$-\frac{1}{6}a\sin^3(x)\cos(x)\sqrt{a\sin^4(x)}-\frac{5}{24}a\sin(x)\cos(x)\sqrt{a\sin^4(x)}-\frac{5}{16}a\cot(x)\sqrt{a\sin^4(x)}+\frac{5}{16}ax\csc^2(x)\sqrt{a\sin^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[x]^4)^(3/2),x]
```

```
[Out] (-5*a*Cot[x]*Sqrt[a*Sin[x]^4])/16 + (5*a*x*Csc[x]^2*Sqrt[a*Sin[x]^4])/16 -
(5*a*Cos[x]*Sin[x]*Sqrt[a*Sin[x]^4])/24 - (a*Cos[x]*Sin[x]^3*Sqrt[a*Sin[x]^
4])/6
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \left(a\sin^4(x)\right)^{3/2} dx = \left(a\csc^2(x)\sqrt{a\sin^4(x)}\right)\int \sin^6(x)\,dx$$
$$= -\frac{1}{6}a\cos(x)\sin^3(x)\sqrt{a\sin^4(x)} + \frac{1}{6}\left(5a\csc^2(x)\sqrt{a\sin^4(x)}\right)\int \sin^4(x)\,dx$$
$$= -\frac{5}{24}a\cos(x)\sin(x)\sqrt{a\sin^4(x)} - \frac{1}{6}a\cos(x)\sin^3(x)\sqrt{a\sin^4(x)} + \frac{1}{8}\left(5a\csc^2(x)\sqrt{a\sin^4(x)}\right)\int \sin$$
$$= -\frac{5}{16}a\cot(x)\sqrt{a\sin^4(x)} - \frac{5}{24}a\cos(x)\sin(x)\sqrt{a\sin^4(x)} - \frac{1}{6}a\cos(x)\sin^3(x)\sqrt{a\sin^4(x)} + \frac{1}{16}\left(5a\right.$$
$$= -\frac{5}{16}a\cot(x)\sqrt{a\sin^4(x)} + \frac{5}{16}ax\csc^2(x)\sqrt{a\sin^4(x)} - \frac{5}{24}a\cos(x)\sin(x)\sqrt{a\sin^4(x)} - \frac{1}{6}a\cos(x)\,\mathrm{s}$$

Mathematica [A] time = 0.095272, size = 38, normalized size = 0.49

$$-\frac{1}{192}(-60x + 45\sin(2x) - 9\sin(4x) + \sin(6x))\csc^6(x)\left(a\sin^4(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[x]^4)^(3/2),x]
```

```
[Out] -(Csc[x]^6*(a*Sin[x]^4)^(3/2)*(-60*x + 45*Sin[2*x] - 9*Sin[4*x] + Sin[6*x])
)/192
```

Maple [A] time = 0.144, size = 41, normalized size = 0.5

$$-\frac{8\,(\cos(x))^5\sin(x) - 26\,\sin(x)(\cos(x))^3 + 33\,\sin(x)\cos(x) - 15\,x}{48\,(\sin(x))^6}\left(a(\sin(x))^4\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(x)^4)^(3/2),x)
```

```
[Out] -1/48*(a*sin(x)^4)^(3/2)*(8*cos(x)^5*sin(x)-26*sin(x)*cos(x)^3+33*sin(x)*co
s(x)-15*x)/sin(x)^6
```

Maxima [A] time = 1.43658, size = 74, normalized size = 0.95

$$\frac{5}{16}a^{\frac{3}{2}}x - \frac{33\,a^{\frac{3}{2}}\tan(x)^5 + 40\,a^{\frac{3}{2}}\tan(x)^3 + 15\,a^{\frac{3}{2}}\tan(x)}{48\left(\tan(x)^6 + 3\,\tan(x)^4 + 3\,\tan(x)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] 5/16*a^(3/2)*x - 1/48*(33*a^(3/2)*tan(x)^5 + 40*a^(3/2)*tan(x)^3 + 15*a^(3/
2)*tan(x))/(tan(x)^6 + 3*tan(x)^4 + 3*tan(x)^2 + 1)
```

Fricas [A] time = 1.77046, size = 163, normalized size = 2.09

$$-\frac{\sqrt{a\cos(x)^4 - 2\,a\cos(x)^2 + a}\left(15\,ax - \left(8\,a\cos(x)^5 - 26\,a\cos(x)^3 + 33\,a\cos(x)\right)\sin(x)\right)}{48\left(\cos(x)^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/48*sqrt(a*cos(x)^4 - 2*a*cos(x)^2 + a)*(15*a*x - (8*a*cos(x)^5 - 26*a*co
s(x)^3 + 33*a*cos(x))*sin(x))/(cos(x)^2 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin^4(x)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)**4)**(3/2),x)
```

```
[Out] Integral((a*sin(x)**4)**(3/2), x)
```

Giac [A] time = 1.10186, size = 36, normalized size = 0.46

$$\frac{1}{192}\, a^{\frac{3}{2}}(60\,x - \sin(6\,x) + 9\,\sin(4\,x) - 45\,\sin(2\,x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] 1/192*a^(3/2)*(60*x - sin(6*x) + 9*sin(4*x) - 45*sin(2*x))
```

3.15 $\int \sqrt{a \sin^4(x)} \, dx$

Optimal. Leaf size=36

$$\frac{1}{2} x \csc^2(x) \sqrt{a \sin^4(x)} - \frac{1}{2} \cot(x) \sqrt{a \sin^4(x)}$$

```
[Out] -(Cot[x]*Sqrt[a*Sin[x]^4])/2 + (x*Csc[x]^2*Sqrt[a*Sin[x]^4])/2
```

Rubi [A] time = 0.0131223, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3207, 2635, 8}

$$\frac{1}{2} x \csc^2(x) \sqrt{a \sin^4(x)} - \frac{1}{2} \cot(x) \sqrt{a \sin^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Sin[x]^4],x]
```

```
[Out] -(Cot[x]*Sqrt[a*Sin[x]^4])/2 + (x*Csc[x]^2*Sqrt[a*Sin[x]^4])/2
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a \sin^4(x)} \, dx &= \left(\csc^2(x) \sqrt{a \sin^4(x)}\right) \int \sin^2(x) \, dx \\
&= -\frac{1}{2} \cot(x) \sqrt{a \sin^4(x)} + \frac{1}{2} \left(\csc^2(x) \sqrt{a \sin^4(x)}\right) \int 1 \, dx \\
&= -\frac{1}{2} \cot(x) \sqrt{a \sin^4(x)} + \frac{1}{2} x \csc^2(x) \sqrt{a \sin^4(x)}
\end{aligned}$$

Mathematica [A] time = 0.0164401, size = 25, normalized size = 0.69

$$\frac{1}{2} \csc(x) \sqrt{a \sin^4(x)} (x \csc(x) - \cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Sin[x]^4],x]
```

```
[Out] (Csc[x]*(-Cos[x] + x*Csc[x])*Sqrt[a*Sin[x]^4])/2
```

Maple [A] time = 0.158, size = 27, normalized size = 0.8

$$-\frac{\sqrt{16}\left(\sin\left(x\right)\cos\left(x\right)-x\right)}{8\left(\sin\left(x\right)\right)^{2}}\sqrt{a\left(\sin\left(x\right)\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(x)^4)^(1/2),x)
```

```
[Out] -1/8*16^(1/2)*(a*sin(x)^4)^(1/2)*(sin(x)*cos(x)-x)/sin(x)^2
```

Maxima [A] time = 1.44084, size = 30, normalized size = 0.83

$$\frac{1}{2}\sqrt{a}x-\frac{\sqrt{a}\tan\left(x\right)}{2\left(\tan\left(x\right)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(a)*x - 1/2*sqrt(a)*tan(x)/(tan(x)^2 + 1)
```

Fricas [A] time = 1.6705, size = 103, normalized size = 2.86

$$\frac{\sqrt{a\cos\left(x\right)^{4}-2\,a\cos\left(x\right)^{2}+a}\left(\cos\left(x\right)\sin\left(x\right)-x\right)}{2\left(\cos\left(x\right)^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(a*cos(x)^4 - 2*a*cos(x)^2 + a)*(cos(x)*sin(x) - x)/(cos(x)^2 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\sin^{4}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)**4)**(1/2),x)
```

```
[Out] Integral(sqrt(a*sin(x)**4), x)
```

Giac [A] time = 1.09986, size = 20, normalized size = 0.56

$$\frac{1}{4}\sqrt{a}(2x - \sin(2x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(a)*(2*x - sin(2*x))
```

3.16 $\int \frac{1}{\sqrt{a\sin^4(x)}}\,dx$

Optimal. Leaf size=16

$$-\frac{\sin(x)\cos(x)}{\sqrt{a\sin^4(x)}}$$

```
[Out] -((Cos[x]*Sin[x])/Sqrt[a*Sin[x]^4])
```

Rubi [A] time = 0.0136734, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3207, 3767, 8}

$$-\frac{\sin(x)\cos(x)}{\sqrt{a\sin^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Sin[x]^4],x]
```

```
[Out] -((Cos[x]*Sin[x])/Sqrt[a*Sin[x]^4])
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a\sin^4(x)}}\,dx &= \frac{\sin^2(x)\int \csc^2(x)\,dx}{\sqrt{a\sin^4(x)}}\\ &= -\frac{\sin^2(x)\,\text{Subst}\left(\int 1\,dx, x, \cot(x)\right)}{\sqrt{a\sin^4(x)}}\\ &= -\frac{\cos(x)\sin(x)}{\sqrt{a\sin^4(x)}}\end{aligned}$$

Mathematica [A] time = 0.0071261, size = 16, normalized size = 1.

$$-\frac{\sin(x)\cos(x)}{\sqrt{a\sin^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Sin[x]^4],x]
```

```
[Out] -((Cos[x]*Sin[x])/Sqrt[a*Sin[x]^4])
```

Maple [A] time = 0.159, size = 15, normalized size = 0.9

$$-\sin(x)\cos(x)\frac{1}{\sqrt{a(\sin(x))^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(x)^4)^(1/2),x)
```

```
[Out] -cos(x)*sin(x)/(a*sin(x)^4)^(1/2)
```

Maxima [A] time = 1.4513, size = 12, normalized size = 0.75

$$-\frac{1}{\sqrt{a}\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/(sqrt(a)*tan(x))
```

Fricas [B] time = 1.54785, size = 95, normalized size = 5.94

$$\frac{\sqrt{a\cos(x)^4-2\,a\cos(x)^2+a}\cos(x)}{\left(a\cos(x)^2-a\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(a*cos(x)^4 - 2*a*cos(x)^2 + a)*cos(x)/((a*cos(x)^2 - a)*sin(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{a\sin^4(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)**4)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*sin(x)**4), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.17 $\int \frac{1}{\left(a \sin^4(x)\right)^{3/2}} \, dx$

Optimal. Leaf size=68

$$-\frac{\sin(x)\cos(x)}{a\sqrt{a\sin^4(x)}}-\frac{\cos^2(x)\cot^3(x)}{5a\sqrt{a\sin^4(x)}}-\frac{2\cos^2(x)\cot(x)}{3a\sqrt{a\sin^4(x)}}$$

```
[Out] (-2*Cos[x]^2*Cot[x])/(3*a*Sqrt[a*Sin[x]^4]) - (Cos[x]^2*Cot[x]^3)/(5*a*Sqrt
[a*Sin[x]^4]) - (Cos[x]*Sin[x])/(a*Sqrt[a*Sin[x]^4])
```

Rubi [A] time = 0.0195453, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3207, 3767}

$$-\frac{\sin(x)\cos(x)}{a\sqrt{a\sin^4(x)}}-\frac{\cos^2(x)\cot^3(x)}{5a\sqrt{a\sin^4(x)}}-\frac{2\cos^2(x)\cot(x)}{3a\sqrt{a\sin^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[x]^4)^(-3/2),x]
```

```
[Out] (-2*Cos[x]^2*Cot[x])/(3*a*Sqrt[a*Sin[x]^4]) - (Cos[x]^2*Cot[x]^3)/(5*a*Sqrt
[a*Sin[x]^4]) - (Cos[x]*Sin[x])/(a*Sqrt[a*Sin[x]^4])
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a\sin^4(x)\right)^{3/2}}\,dx &= \frac{\sin^2(x)\int \csc^6(x)\,dx}{a\sqrt{a\sin^4(x)}}\\ &= -\frac{\sin^2(x)\,\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx,x,\cot(x)\right)}{a\sqrt{a\sin^4(x)}}\\ &= -\frac{2\cos^2(x)\cot(x)}{3a\sqrt{a\sin^4(x)}}-\frac{\cos^2(x)\cot^3(x)}{5a\sqrt{a\sin^4(x)}}-\frac{\cos(x)\sin(x)}{a\sqrt{a\sin^4(x)}}\end{aligned}$$

Mathematica [A] time = 0.0334892, size = 34, normalized size = 0.5

$$-\frac{\sin^5(x)\cos(x)\left(3\csc^4(x)+4\csc^2(x)+8\right)}{15\left(a\sin^4(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[x]^4)^(-3/2),x]
```

```
[Out] -(Cos[x]*(8 + 4*Csc[x]^2 + 3*Csc[x]^4)*Sin[x]^5)/(15*(a*Sin[x]^4)^(3/2))
```

Maple [A] time = 0.119, size = 29, normalized size = 0.4

$$-\frac{\left(8\,(\cos(x))^4-20\,(\cos(x))^2+15\right)\sin(x)\cos(x)}{15}\left(a\,(\sin(x))^4\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(x)^4)^(3/2),x)
```

```
[Out] -1/15*(8*cos(x)^4-20*cos(x)^2+15)*sin(x)*cos(x)/(a*sin(x)^4)^(3/2)
```

Maxima [A] time = 1.44756, size = 31, normalized size = 0.46

$$-\frac{15\,\tan(x)^4+10\,\tan(x)^2+3}{15\,a^{\frac{3}{2}}\tan(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*tan(x)^4 + 10*tan(x)^2 + 3)/(a^(3/2)*tan(x)^5)
```

Fricas [A] time = 1.62095, size = 196, normalized size = 2.88

$$\frac{\sqrt{a\cos(x)^4-2\,a\cos(x)^2+a}\left(8\,\cos(x)^5-20\,\cos(x)^3+15\,\cos(x)\right)}{15\left(a^2\cos(x)^6-3\,a^2\cos(x)^4+3\,a^2\cos(x)^2-a^2\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*sqrt(a*cos(x)^4 - 2*a*cos(x)^2 + a)*(8*cos(x)^5 - 20*cos(x)^3 + 15*cos
(x))/((a^2*cos(x)^6 - 3*a^2*cos(x)^4 + 3*a^2*cos(x)^2 - a^2)*sin(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sin^4(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)**4)**(3/2),x)
```

```
[Out] Integral((a*sin(x)**4)**(-3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.18 $\int \frac{1}{\left(a\sin^4(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=118

$$-\frac{\sin(x)\cos(x)}{a^2\sqrt{a\sin^4(x)}}-\frac{\cos^2(x)\cot^7(x)}{9a^2\sqrt{a\sin^4(x)}}-\frac{4\cos^2(x)\cot^5(x)}{7a^2\sqrt{a\sin^4(x)}}-\frac{6\cos^2(x)\cot^3(x)}{5a^2\sqrt{a\sin^4(x)}}-\frac{4\cos^2(x)\cot(x)}{3a^2\sqrt{a\sin^4(x)}}$$

```
[Out] (-4*Cos[x]^2*Cot[x])/(3*a^2*Sqrt[a*Sin[x]^4]) - (6*Cos[x]^2*Cot[x]^3)/(5*a^
2*Sqrt[a*Sin[x]^4]) - (4*Cos[x]^2*Cot[x]^5)/(7*a^2*Sqrt[a*Sin[x]^4]) - (Cos
[x]^2*Cot[x]^7)/(9*a^2*Sqrt[a*Sin[x]^4]) - (Cos[x]*Sin[x])/(a^2*Sqrt[a*Sin[
x]^4])
```

Rubi [A] time = 0.0282477, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3207, 3767}

$$-\frac{\sin(x)\cos(x)}{a^2\sqrt{a\sin^4(x)}}-\frac{\cos^2(x)\cot^7(x)}{9a^2\sqrt{a\sin^4(x)}}-\frac{4\cos^2(x)\cot^5(x)}{7a^2\sqrt{a\sin^4(x)}}-\frac{6\cos^2(x)\cot^3(x)}{5a^2\sqrt{a\sin^4(x)}}-\frac{4\cos^2(x)\cot(x)}{3a^2\sqrt{a\sin^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[x]^4)^(-5/2),x]
```

```
[Out] (-4*Cos[x]^2*Cot[x])/(3*a^2*Sqrt[a*Sin[x]^4]) - (6*Cos[x]^2*Cot[x]^3)/(5*a^
2*Sqrt[a*Sin[x]^4]) - (4*Cos[x]^2*Cot[x]^5)/(7*a^2*Sqrt[a*Sin[x]^4]) - (Cos
[x]^2*Cot[x]^7)/(9*a^2*Sqrt[a*Sin[x]^4]) - (Cos[x]*Sin[x])/(a^2*Sqrt[a*Sin[
x]^4])
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{\left(a\sin^4(x)\right)^{5/2}}\,dx = \frac{\sin^2(x)\int \csc^{10}(x)\,dx}{a^2\sqrt{a\sin^4(x)}}$$
$$= -\frac{\sin^2(x)\,\text{Subst}\left(\int \left(1+4x^2+6x^4+4x^6+x^8\right)dx, x, \cot(x)\right)}{a^2\sqrt{a\sin^4(x)}}$$
$$= -\frac{4\cos^2(x)\cot(x)}{3a^2\sqrt{a\sin^4(x)}} - \frac{6\cos^2(x)\cot^3(x)}{5a^2\sqrt{a\sin^4(x)}} - \frac{4\cos^2(x)\cot^5(x)}{7a^2\sqrt{a\sin^4(x)}} - \frac{\cos^2(x)\cot^7(x)}{9a^2\sqrt{a\sin^4(x)}} - \frac{\cos(x)\sin(x)}{a^2\sqrt{a\sin^4(x)}}$$

Mathematica [A] time = 0.049783, size = 47, normalized size = 0.4

$$-\frac{\sin(x)\cos(x)\left(35\csc^8(x)+40\csc^6(x)+48\csc^4(x)+64\csc^2(x)+128\right)}{315a^2\sqrt{a\sin^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[x]^4)^(-5/2),x]
```

```
[Out] -(Cos[x]*(128 + 64*Csc[x]^2 + 48*Csc[x]^4 + 40*Csc[x]^6 + 35*Csc[x]^8)*Sin[
x])/(315*a^2*Sqrt[a*Sin[x]^4])
```

Maple [A] time = 0.16, size = 41, normalized size = 0.4

$$-\frac{\left(128(\cos(x))^8-576(\cos(x))^6+1008(\cos(x))^4-840(\cos(x))^2+315\right)\sin(x)\cos(x)}{315}\left(a(\sin(x))^4\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(x)^4)^(5/2),x)
```

```
[Out] -1/315*(128*cos(x)^8-576*cos(x)^6+1008*cos(x)^4-840*cos(x)^2+315)*sin(x)*co
s(x)/(a*sin(x)^4)^(5/2)
```

Maxima [A] time = 1.44887, size = 47, normalized size = 0.4

$$-\frac{315\tan(x)^8+420\tan(x)^6+378\tan(x)^4+180\tan(x)^2+35}{315\,a^{\frac{5}{2}}\tan(x)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/315*(315*tan(x)^8 + 420*tan(x)^6 + 378*tan(x)^4 + 180*tan(x)^2 + 35)/(a^
(5/2)*tan(x)^9)
```

Fricas [A] time = 1.62853, size = 294, normalized size = 2.49

$$\frac{\left(128\,\cos\left(x\right)^{9}-576\,\cos\left(x\right)^{7}+1008\,\cos\left(x\right)^{5}-840\,\cos\left(x\right)^{3}+315\,\cos\left(x\right)\right)\sqrt{a\cos\left(x\right)^{4}-2\,a\cos\left(x\right)^{2}+a}}{315\,\left(a^{3}\cos\left(x\right)^{10}-5\,a^{3}\cos\left(x\right)^{8}+10\,a^{3}\cos\left(x\right)^{6}-10\,a^{3}\cos\left(x\right)^{4}+5\,a^{3}\cos\left(x\right)^{2}-a^{3}\right)\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/315*(128*cos(x)^9 - 576*cos(x)^7 + 1008*cos(x)^5 - 840*cos(x)^3 + 315*cos
(x))*sqrt(a*cos(x)^4 - 2*a*cos(x)^2 + a)/((a^3*cos(x)^10 - 5*a^3*cos(x)^8 +
 10*a^3*cos(x)^6 - 10*a^3*cos(x)^4 + 5*a^3*cos(x)^2 - a^3)*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)**4)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.19 $\int \left(c\sin^m(a+bx)\right)^{5/2}\,dx$

Optimal. Leaf size=89

$$\frac{2c^2\cos(a+bx)\sin^{2m+1}(a+bx)\sqrt{c\sin^m(a+bx)}\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(5m+2);\frac{1}{4}(5m+6);\sin^2(a+bx)\right)}{b(5m+2)\sqrt{\cos^2(a+bx)}}$$

```
[Out] (2*c^2*Cos[a + b*x]*Hypergeometric2F1[1/2, (2 + 5*m)/4, (6 + 5*m)/4, Sin[a
+ b*x]^2]*Sin[a + b*x]^(1 + 2*m)*Sqrt[c*Sin[a + b*x]^m])/(b*(2 + 5*m)*Sqrt[
Cos[a + b*x]^2])
```

Rubi [A] time = 0.0390368, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3208, 2643}

$$\frac{2c^2\cos(a+bx)\sin^{2m+1}(a+bx)\sqrt{c\sin^m(a+bx)}\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(5m+2);\frac{1}{4}(5m+6);\sin^2(a+bx)\right)}{b(5m+2)\sqrt{\cos^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x]^m)^(5/2),x]
```

```
[Out] (2*c^2*Cos[a + b*x]*Hypergeometric2F1[1/2, (2 + 5*m)/4, (6 + 5*m)/4, Sin[a
+ b*x]^2]*Sin[a + b*x]^(1 + 2*m)*Sqrt[c*Sin[a + b*x]^m])/(b*(2 + 5*m)*Sqrt[
Cos[a + b*x]^2])
```

Rule 3208

```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Sin[e + f*x])^n)^FracPart[p])/(c*Sin[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \left(c\sin^m(a+bx)\right)^{5/2}\,dx = \left(c^2\sin^{-\frac{m}{2}}(a+bx)\sqrt{c\sin^m(a+bx)}\right)\int \sin^{\frac{5m}{2}}(a+bx)\,dx$$
$$= \frac{2c^2\cos(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2+5m);\frac{1}{4}(6+5m);\sin^2(a+bx)\right)\sin^{1+2m}(a+bx)\sqrt{c\sin^m(a+b}}{b(2+5m)\sqrt{\cos^2(a+bx)}}$$

Mathematica [A] time = 0.178953, size = 74, normalized size = 0.83

$$\frac{2\sqrt{\cos^2(a+bx)}\tan(a+bx)\left(c\sin^m(a+bx)\right)^{5/2}\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(5m+2);\frac{1}{4}(5m+6);\sin^2(a+bx)\right)}{b(5m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x]^m)^(5/2),x]
```

```
[Out] (2*Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, (2 + 5*m)/4, (6 + 5*m)/4, Si
n[a + b*x]^2]*(c*Sin[a + b*x]^m)^(5/2)*Tan[a + b*x])/(b*(2 + 5*m))
```

Maple [F] time = 0.468, size = 0, normalized size = 0.

$$\int \left(c\left(\sin \left(bx + a\right)\right)^{m}\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a)^m)^(5/2),x)
```

```
[Out] int((c*sin(b*x+a)^m)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c \sin \left(bx + a\right)^{m}\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^m)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a)^m)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^m)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)**m)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c \sin(bx + a)^m\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^m)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^m)^(5/2), x)
```

3.20 $\int \left(c \sin^m(a+bx)\right)^{3/2} dx$

Optimal. Leaf size=83

$$\frac{2c\cos(a+bx)\sin^{m+1}(a+bx)\sqrt{c\sin^m(a+bx)}\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(3m+2);\frac{3(m+2)}{4};\sin^2(a+bx)\right)}{b(3m+2)\sqrt{\cos^2(a+bx)}}$$

```
[Out] (2*c*Cos[a + b*x]*Hypergeometric2F1[1/2, (2 + 3*m)/4, (3*(2 + m))/4, Sin[a
+ b*x]^2]*Sin[a + b*x]^(1 + m)*Sqrt[c*Sin[a + b*x]^m])/(b*(2 + 3*m)*Sqrt[Co
s[a + b*x]^2])
```

Rubi [A] time = 0.0379316, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3208, 2643}

$$\frac{2c\cos(a+bx)\sin^{m+1}(a+bx)\sqrt{c\sin^m(a+bx)}\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(3m+2);\frac{3(m+2)}{4};\sin^2(a+bx)\right)}{b(3m+2)\sqrt{\cos^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x]^m)^(3/2),x]
```

```
[Out] (2*c*Cos[a + b*x]*Hypergeometric2F1[1/2, (2 + 3*m)/4, (3*(2 + m))/4, Sin[a
+ b*x]^2]*Sin[a + b*x]^(1 + m)*Sqrt[c*Sin[a + b*x]^m])/(b*(2 + 3*m)*Sqrt[Co
s[a + b*x]^2])
```

Rule 3208

```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Sin[e + f*x])^n)^FracPart[p])/(c*Sin[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \left(c\sin^m(a+bx)\right)^{3/2} dx &= \left(c\sin^{-\frac{m}{2}}(a+bx)\sqrt{c\sin^m(a+bx)}\right)\int \sin^{\frac{3m}{2}}(a+bx)\,dx\\ &= \frac{2c\cos(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2+3m);\frac{3(2+m)}{4};\sin^2(a+bx)\right)\sin^{1+m}(a+bx)\sqrt{c\sin^m(a+bx)}}{b(2+3m)\sqrt{\cos^2(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.118059, size = 72, normalized size = 0.87

$$\frac{2\sqrt{\cos^2(a+bx)}\tan(a+bx)\left(c\sin^m(a+bx)\right)^{3/2}\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(3m+2);\frac{3(m+2)}{4};\sin^2(a+bx)\right)}{b(3m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x]^m)^(3/2),x]
```

```
[Out] (2*Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, (2 + 3*m)/4, (3*(2 + m))/4,
Sin[a + b*x]^2]*(c*Sin[a + b*x]^m)^(3/2)*Tan[a + b*x])/(b*(2 + 3*m))
```

Maple [F] time = 0.19, size = 0, normalized size = 0.

$$\int \left(c (\sin (bx + a))^m\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a)^m)^(3/2),x)
```

```
[Out] int((c*sin(b*x+a)^m)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c \sin (bx + a)^m\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^m)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a)^m)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^m)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)**m)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c\sin\left(bx+a\right)^m\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^m)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^m)^(3/2), x)
```

3.21 $\int \sqrt{c\sin^m(a+bx)}\,dx$

Optimal. Leaf size=74

$$\frac{2\sin(a+bx)\cos(a+bx)\sqrt{c\sin^m(a+bx)}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{4};\frac{m+6}{4};\sin^2(a+bx)\right)}{b(m+2)\sqrt{\cos^2(a+bx)}}$$

```
[Out] (2*Cos[a + b*x]*Hypergeometric2F1[1/2, (2 + m)/4, (6 + m)/4, Sin[a + b*x]^2
]*Sin[a + b*x]*Sqrt[c*Sin[a + b*x]^m])/(b*(2 + m)*Sqrt[Cos[a + b*x]^2])
```

Rubi [A] time = 0.0360372, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3208, 2643}

$$\frac{2\sin(a+bx)\cos(a+bx)\sqrt{c\sin^m(a+bx)}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{4};\frac{m+6}{4};\sin^2(a+bx)\right)}{b(m+2)\sqrt{\cos^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*Sin[a + b*x]^m],x]
```

```
[Out] (2*Cos[a + b*x]*Hypergeometric2F1[1/2, (2 + m)/4, (6 + m)/4, Sin[a + b*x]^2
]*Sin[a + b*x]*Sqrt[c*Sin[a + b*x]^m])/(b*(2 + m)*Sqrt[Cos[a + b*x]^2])
```

Rule 3208

```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Sin[e + f*x])^n)^FracPart[p])/(c*Sin[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \sqrt{c\sin^m(a+bx)}\,dx &= \left(\sin^{-\frac{m}{2}}(a+bx)\sqrt{c\sin^m(a+bx)}\right)\int \sin^{\frac{m}{2}}(a+bx)\,dx\\ &= \frac{2\cos(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{4};\frac{6+m}{4};\sin^2(a+bx)\right)\sin(a+bx)\sqrt{c\sin^m(a+bx)}}{b(2+m)\sqrt{\cos^2(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.0716404, size = 68, normalized size = 0.92

$$\frac{2\sqrt{\cos^2(a+bx)}\tan(a+bx)\sqrt{c\sin^m(a+bx)}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{4};\frac{m+6}{4};\sin^2(a+bx)\right)}{b(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*Sin[a + b*x]^m],x]
```

```
[Out] (2*Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, (2 + m)/4, (6 + m)/4, Sin[a
+ b*x]^2]*Sqrt[c*Sin[a + b*x]^m]*Tan[a + b*x])/(b*(2 + m))
```

Maple [F] time = 0.213, size = 0, normalized size = 0.

$$\int \sqrt{c\,(\sin(bx+a))^{m}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a)^m)^(1/2),x)
```

```
[Out] int((c*sin(b*x+a)^m)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c\sin(bx+a)^{m}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^m)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*sin(b*x + a)^m), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^m)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c\sin^{m}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)**m)**(1/2),x)
```

```
[Out] Integral(sqrt(c*sin(a + b*x)**m), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c \sin(bx+a)^m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^m)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*sin(b*x + a)^m), x)
```

3.22 $\int \frac{1}{\sqrt{c \sin^m(a+bx)}} dx$

Optimal. Leaf size=80

$$\frac{2 \sin(a+bx) \cos(a+bx) \,_2F_1\left(\frac{1}{2},\frac{2-m}{4};\frac{6-m}{4};\sin^2(a+bx)\right)}{b(2-m)\sqrt{\cos^2(a+bx)}\sqrt{c \sin^m(a+bx)}}$$

```
[Out] (2*Cos[a + b*x]*Hypergeometric2F1[1/2, (2 - m)/4, (6 - m)/4, Sin[a + b*x]^2
]*Sin[a + b*x])/(b*(2 - m)*Sqrt[Cos[a + b*x]^2]*Sqrt[c*Sin[a + b*x]^m])
```

Rubi [A] time = 0.041786, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3208, 2643}

$$\frac{2 \sin(a+bx) \cos(a+bx) \,_2F_1\left(\frac{1}{2},\frac{2-m}{4};\frac{6-m}{4};\sin^2(a+bx)\right)}{b(2-m)\sqrt{\cos^2(a+bx)}\sqrt{c \sin^m(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[c*Sin[a + b*x]^m],x]
```

```
[Out] (2*Cos[a + b*x]*Hypergeometric2F1[1/2, (2 - m)/4, (6 - m)/4, Sin[a + b*x]^2
]*Sin[a + b*x])/(b*(2 - m)*Sqrt[Cos[a + b*x]^2]*Sqrt[c*Sin[a + b*x]^m])
```

Rule 3208

```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Sin[e + f*x])^n)^FracPart[p])/(c*Sin[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{c \sin^m(a+bx)}} \, dx &= \frac{\sin^{\frac{m}{2}}(a+bx) \int \sin^{-\frac{m}{2}}(a+bx) \, dx}{\sqrt{c \sin^m(a+bx)}}\\ &= \frac{2 \cos(a+bx) \,_2F_1\left(\frac{1}{2},\frac{2-m}{4};\frac{6-m}{4};\sin^2(a+bx)\right) \sin(a+bx)}{b(2-m)\sqrt{\cos^2(a+bx)}\sqrt{c \sin^m(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.0749101, size = 72, normalized size = 0.9

$$-\frac{2\sqrt{\cos^2(a+bx)} \tan(a+bx) \,_2F_1\left(\frac{1}{2},\frac{2-m}{4};\frac{6-m}{4};\sin^2(a+bx)\right)}{b(m-2)\sqrt{c \sin^m(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[c*Sin[a + b*x]^m],x]
```

```
[Out] (-2*Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, (2 - m)/4, (6 - m)/4, Sin[a
 + b*x]^2]*Tan[a + b*x])/(b*(-2 + m)*Sqrt[c*Sin[a + b*x]^m])
```

Maple [F] time = 0.192, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c\left(\sin\left(bx+a\right)\right)^{m}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*sin(b*x+a)^m)^(1/2),x)
```

```
[Out] int(1/(c*sin(b*x+a)^m)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c\sin\left(bx+a\right)^{m}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a)^m)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(c*sin(b*x + a)^m), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a)^m)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c\sin^{m}\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a)**m)**(1/2),x)
```

```
[Out] Integral(1/sqrt(c*sin(a + b*x)**m), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c \sin(bx+a)^m}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a)^m)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(c*sin(b*x + a)^m), x)
```

3.23 $\int \frac{1}{\left(c\sin^m(a+bx)\right)^{3/2}}\,dx$

Optimal. Leaf size=89

$$\frac{2\cos(a+bx)\sin^{1-m}(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2-3m);\frac{3(2-m)}{4};\sin^2(a+bx)\right)}{bc(2-3m)\sqrt{\cos^2(a+bx)}\sqrt{c\sin^m(a+bx)}}$$

```
[Out] (2*Cos[a + b*x]*Hypergeometric2F1[1/2, (2 - 3*m)/4, (3*(2 - m))/4, Sin[a +
b*x]^2]*Sin[a + b*x]^(1 - m))/(b*c*(2 - 3*m)*Sqrt[Cos[a + b*x]^2]*Sqrt[c*Si
n[a + b*x]^m])
```

Rubi [A] time = 0.0405227, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3208, 2643}

$$\frac{2\cos(a+bx)\sin^{1-m}(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2-3m);\frac{3(2-m)}{4};\sin^2(a+bx)\right)}{bc(2-3m)\sqrt{\cos^2(a+bx)}\sqrt{c\sin^m(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x]^m)^(-3/2),x]
```

```
[Out] (2*Cos[a + b*x]*Hypergeometric2F1[1/2, (2 - 3*m)/4, (3*(2 - m))/4, Sin[a +
b*x]^2]*Sin[a + b*x]^(1 - m))/(b*c*(2 - 3*m)*Sqrt[Cos[a + b*x]^2]*Sqrt[c*Si
n[a + b*x]^m])
```

Rule 3208

```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Sin[e + f*x])^n)^FracPart[p])/(c*Sin[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(c\sin^m(a+bx)\right)^{3/2}}\,dx &= \frac{\sin^{\frac{m}{2}}(a+bx)\int \sin^{-\frac{3m}{2}}(a+bx)\,dx}{c\sqrt{c\sin^m(a+bx)}}\\ &= \frac{2\cos(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2-3m);\frac{3(2-m)}{4};\sin^2(a+bx)\right)\sin^{1-m}(a+bx)}{bc(2-3m)\sqrt{\cos^2(a+bx)}\sqrt{c\sin^m(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.108938, size = 71, normalized size = 0.8

$$\frac{\sqrt{\cos^2(a+bx)}\tan(a+bx)\,_2F_1\left(\frac{1}{2},\frac{1}{4}(2-3m);-\frac{3}{4}(m-2);\sin^2(a+bx)\right)}{\left(b-\frac{3bm}{2}\right)\left(c\sin^m(a+bx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x]^m)^(-3/2),x]
```

```
[Out] (Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, (2 - 3*m)/4, (-3*(-2 + m))/4,
Sin[a + b*x]^2]*Tan[a + b*x])/((b - (3*b*m)/2)*(c*Sin[a + b*x]^m)^(3/2))
```

Maple [F] time = 0.181, size = 0, normalized size = 0.

$$\int \left(c\left(\sin\left(bx+a\right)\right)^m\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*sin(b*x+a)^m)^(3/2),x)
```

```
[Out] int(1/(c*sin(b*x+a)^m)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c\sin\left(bx+a\right)^m\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a)^m)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a)^m)^(-3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a)^m)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c\sin^m\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a)**m)**(3/2),x)
```

```
[Out] Integral((c*sin(a + b*x)**m)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c\sin\left(bx+a\right)^{m}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a)^m)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^m)^(-3/2), x)
```

3.24 $\int \frac{1}{\left(c\sin^m(a+bx)\right)^{5/2}}\,dx$

Optimal. Leaf size=89

$$\frac{2\cos(a+bx)\sin^{1-2m}(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2-5m);\frac{1}{4}(6-5m);\sin^2(a+bx)\right)}{bc^2(2-5m)\sqrt{\cos^2(a+bx)}\sqrt{c\sin^m(a+bx)}}$$

```
[Out] (2*Cos[a + b*x]*Hypergeometric2F1[1/2, (2 - 5*m)/4, (6 - 5*m)/4, Sin[a + b*
x]^2]*Sin[a + b*x]^(1 - 2*m))/(b*c^2*(2 - 5*m)*Sqrt[Cos[a + b*x]^2]*Sqrt[c*
Sin[a + b*x]^m])
```

Rubi [A] time = 0.0408314, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3208, 2643}

$$\frac{2\cos(a+bx)\sin^{1-2m}(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2-5m);\frac{1}{4}(6-5m);\sin^2(a+bx)\right)}{bc^2(2-5m)\sqrt{\cos^2(a+bx)}\sqrt{c\sin^m(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x]^m)^(-5/2),x]
```

```
[Out] (2*Cos[a + b*x]*Hypergeometric2F1[1/2, (2 - 5*m)/4, (6 - 5*m)/4, Sin[a + b*
x]^2]*Sin[a + b*x]^(1 - 2*m))/(b*c^2*(2 - 5*m)*Sqrt[Cos[a + b*x]^2]*Sqrt[c*
Sin[a + b*x]^m])
```

Rule 3208

```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Sin[e + f*x])^n)^FracPart[p])/(c*Sin[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(c\sin^m(a+bx)\right)^{5/2}}\,dx &= \frac{\sin^{\frac{m}{2}}(a+bx)\int \sin^{-\frac{5m}{2}}(a+bx)\,dx}{c^2\sqrt{c\sin^m(a+bx)}}\\ &= \frac{2\cos(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2-5m);\frac{1}{4}(6-5m);\sin^2(a+bx)\right)\sin^{1-2m}(a+bx)}{bc^2(2-5m)\sqrt{\cos^2(a+bx)}\sqrt{c\sin^m(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.134078, size = 73, normalized size = 0.82

$$\frac{\sqrt{\cos^2(a+bx)}\tan(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2-5m);\frac{1}{4}(6-5m);\sin^2(a+bx)\right)}{\left(b-\frac{5bm}{2}\right)\left(c\sin^m(a+bx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x]^m)^(-5/2),x]
```

```
[Out] (Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, (2 - 5*m)/4, (6 - 5*m)/4, Sin[
a + b*x]^2]*Tan[a + b*x])/((b - (5*b*m)/2)*(c*Sin[a + b*x]^m)^(5/2))
```

Maple [F] time = 0.184, size = 0, normalized size = 0.

$$\int \left(c\left(\sin\left(bx+a\right)\right)^{m}\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*sin(b*x+a)^m)^(5/2),x)
```

```
[Out] int(1/(c*sin(b*x+a)^m)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c\sin\left(bx+a\right)^{m}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a)^m)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a)^m)^(-5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a)^m)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a)**m)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c\sin\left(bx+a\right)^{m}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a)^m)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^m)^(-5/2), x)
```

3.25 $\int \left(b\sin^n(c+dx)\right)^p\,dx$

Optimal. Leaf size=77

$$\frac{\sin(c+dx)\cos(c+dx)\left(b\sin^n(c+dx)\right)^p\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+1);\frac{1}{2}(np+3);\sin^2(c+dx)\right)}{d(np+1)\sqrt{\cos^2(c+dx)}}$$

```
[Out] (Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sin[c + d*x]
^2]*Sin[c + d*x]*(b*Sin[c + d*x]^n)^p)/(d*(1 + n*p)*Sqrt[Cos[c + d*x]^2])
```

Rubi [A] time = 0.0354642, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3208, 2643}

$$\frac{\sin(c+dx)\cos(c+dx)\left(b\sin^n(c+dx)\right)^p\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+1);\frac{1}{2}(np+3);\sin^2(c+dx)\right)}{d(np+1)\sqrt{\cos^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sin[c + d*x]^n)^p,x]
```

```
[Out] (Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sin[c + d*x]
^2]*Sin[c + d*x]*(b*Sin[c + d*x]^n)^p)/(d*(1 + n*p)*Sqrt[Cos[c + d*x]^2])
```

Rule 3208

```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Sin[e + f*x])^n)^FracPart[p])/(c*Sin[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \left(b\sin^n(c+dx)\right)^p\,dx &= \left(\sin^{-np}(c+dx)\left(b\sin^n(c+dx)\right)^p\right)\int \sin^{np}(c+dx)\,dx\\ &= \frac{\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+np);\frac{1}{2}(3+np);\sin^2(c+dx)\right)\sin(c+dx)\left(b\sin^n(c+dx)\right)^p}{d(1+np)\sqrt{\cos^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0641188, size = 71, normalized size = 0.92

$$\frac{\sqrt{\cos^2(c+dx)}\tan(c+dx)\left(b\sin^n(c+dx)\right)^p\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+1);\frac{1}{2}(np+3);\sin^2(c+dx)\right)}{d(np+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sin[c + d*x]^n)^p,x]
```

```
[Out] (Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sin[
c + d*x]^2]*(b*Sin[c + d*x]^n)^p*Tan[c + d*x])/(d*(1 + n*p))
```

Maple [F] time = 0.312, size = 0, normalized size = 0.

$$\int \left(b\left(\sin\left(dx+c\right)\right)^{n}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(d*x+c)^n)^p,x)
```

```
[Out] int((b*sin(d*x+c)^n)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(dx+c\right)^{n}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(d*x+c)^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^n)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sin\left(dx+c\right)^{n}\right)^{p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(d*x+c)^n)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sin(d*x + c)^n)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin^{n}\left(c+dx\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(d*x+c)**n)**p,x)
```

```
[Out] Integral((b*sin(c + d*x)**n)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sin (dx + c)^n)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(d*x+c)^n)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^n)^p, x)
```

3.26 $\int \left(c\sin^2(a+bx)\right)^p\,dx$

Optimal. Leaf size=77

$$\frac{\sin(a+bx)\cos(a+bx)\left(c\sin^2(a+bx)\right)^p{}_2F_1\left(\frac{1}{2},\frac{1}{2}(2p+1);\frac{1}{2}(2p+3);\sin^2(a+bx)\right)}{b(2p+1)\sqrt{\cos^2(a+bx)}}$$

```
[Out] (Cos[a + b*x]*Hypergeometric2F1[1/2, (1 + 2*p)/2, (3 + 2*p)/2, Sin[a + b*x]
^2]*Sin[a + b*x]*(c*Sin[a + b*x]^2)^p)/(b*(1 + 2*p)*Sqrt[Cos[a + b*x]^2])
```

Rubi [A] time = 0.0334552, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3207, 2643}

$$\frac{\sin(a+bx)\cos(a+bx)\left(c\sin^2(a+bx)\right)^p{}_2F_1\left(\frac{1}{2},\frac{1}{2}(2p+1);\frac{1}{2}(2p+3);\sin^2(a+bx)\right)}{b(2p+1)\sqrt{\cos^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x]^2)^p,x]
```

```
[Out] (Cos[a + b*x]*Hypergeometric2F1[1/2, (1 + 2*p)/2, (3 + 2*p)/2, Sin[a + b*x]
^2]*Sin[a + b*x]*(c*Sin[a + b*x]^2)^p)/(b*(1 + 2*p)*Sqrt[Cos[a + b*x]^2])
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \left(c\sin^2(a+bx)\right)^p\,dx &= \left(\sin^{-2p}(a+bx)\left(c\sin^2(a+bx)\right)^p\right)\int \sin^{2p}(a+bx)\,dx\\ &= \frac{\cos(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+2p);\frac{1}{2}(3+2p);\sin^2(a+bx)\right)\sin(a+bx)\left(c\sin^2(a+bx)\right)^p}{b(1+2p)\sqrt{\cos^2(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.0777694, size = 61, normalized size = 0.79

$$\frac{\sqrt{\cos^2(a+bx)}\tan(a+bx)\left(c\sin^2(a+bx)\right)^p{}_2F_1\left(\frac{1}{2},p+\frac{1}{2};p+\frac{3}{2};\sin^2(a+bx)\right)}{2bp+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x]^2)^p,x]
```

```
[Out] (Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, 1/2 + p, 3/2 + p, Sin[a + b*x]
^2]*(c*Sin[a + b*x]^2)^p*Tan[a + b*x])/(b + 2*b*p)
```

Maple [F] time = 0.796, size = 0, normalized size = 0.

$$\int \left(c (\sin (bx + a))^2\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a)^2)^p,x)
```

```
[Out] int((c*sin(b*x+a)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c \sin (bx + a)^2\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a)^2)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(-c \cos (bx + a)^2 + c\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-c*cos(b*x + a)^2 + c)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c \sin^2 (a + bx)\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)**2)**p,x)
```

```
[Out] Integral((c*sin(a + b*x)**2)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c \sin \left(bx+a\right)^{2}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^2)^p, x)
```

3.27 $\int \left(c\sin^3(a+bx)\right)^p\,dx$

Optimal. Leaf size=75

$$\frac{\sin(a+bx)\cos(a+bx)\left(c\sin^3(a+bx)\right)^p\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(3p+1);\frac{3(p+1)}{2};\sin^2(a+bx)\right)}{b(3p+1)\sqrt{\cos^2(a+bx)}}$$

```
[Out] (Cos[a + b*x]*Hypergeometric2F1[1/2, (1 + 3*p)/2, (3*(1 + p))/2, Sin[a + b*
x]^2]*Sin[a + b*x]*(c*Sin[a + b*x]^3)^p)/(b*(1 + 3*p)*Sqrt[Cos[a + b*x]^2])
```

Rubi [A] time = 0.0346214, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3207, 2643}

$$\frac{\sin(a+bx)\cos(a+bx)\left(c\sin^3(a+bx)\right)^p\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(3p+1);\frac{3(p+1)}{2};\sin^2(a+bx)\right)}{b(3p+1)\sqrt{\cos^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x]^3)^p,x]
```

```
[Out] (Cos[a + b*x]*Hypergeometric2F1[1/2, (1 + 3*p)/2, (3*(1 + p))/2, Sin[a + b*
x]^2]*Sin[a + b*x]*(c*Sin[a + b*x]^3)^p)/(b*(1 + 3*p)*Sqrt[Cos[a + b*x]^2])
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \left(c\sin^3(a+bx)\right)^p\,dx &= \left(\sin^{-3p}(a+bx)\left(c\sin^3(a+bx)\right)^p\right)\int \sin^{3p}(a+bx)\,dx\\ &= \frac{\cos(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+3p);\frac{3(1+p)}{2};\sin^2(a+bx)\right)\sin(a+bx)\left(c\sin^3(a+bx)\right)^p}{b(1+3p)\sqrt{\cos^2(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.0840974, size = 67, normalized size = 0.89

$$\frac{\sqrt{\cos^2(a+bx)}\tan(a+bx)\left(c\sin^3(a+bx)\right)^p\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(3p+1);\frac{3(p+1)}{2};\sin^2(a+bx)\right)}{3bp+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x]^3)^p,x]
```

```
[Out] (Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, (1 + 3*p)/2, (3*(1 + p))/2, Si
n[a + b*x]^2]*(c*Sin[a + b*x]^3)^p*Tan[a + b*x])/(b + 3*b*p)
```

Maple [F] time = 0.829, size = 0, normalized size = 0.

$$\int \left(c\left(\sin\left(bx+a\right)\right)^{3}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a)^3)^p,x)
```

```
[Out] int((c*sin(b*x+a)^3)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c\sin\left(bx+a\right)^{3}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a)^3)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(-\left(c\cos\left(bx+a\right)^{2}-c\right)\sin\left(bx+a\right)\right)^{p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^p,x, algorithm="fricas")
```

```
[Out] integral((-(c*cos(b*x + a)^2 - c)*sin(b*x + a))^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c\sin^{3}\left(a+bx\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)**3)**p,x)
```

```
[Out] Integral((c*sin(a + b*x)**3)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c\sin\left(bx+a\right)^3\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^p,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^3)^p, x)
```

3.28 $\int \left(c\sin^4(a+bx)\right)^p\,dx$

Optimal. Leaf size=77

$$\frac{\sin(a+bx)\cos(a+bx)\left(c\sin^4(a+bx)\right)^p{}_2F_1\left(\frac{1}{2},\frac{1}{2}(4p+1);\frac{1}{2}(4p+3);\sin^2(a+bx)\right)}{b(4p+1)\sqrt{\cos^2(a+bx)}}$$

```
[Out] (Cos[a + b*x]*Hypergeometric2F1[1/2, (1 + 4*p)/2, (3 + 4*p)/2, Sin[a + b*x]
^2]*Sin[a + b*x]*(c*Sin[a + b*x]^4)^p)/(b*(1 + 4*p)*Sqrt[Cos[a + b*x]^2])
```

Rubi [A] time = 0.0332862, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3207, 2643}

$$\frac{\sin(a+bx)\cos(a+bx)\left(c\sin^4(a+bx)\right)^p{}_2F_1\left(\frac{1}{2},\frac{1}{2}(4p+1);\frac{1}{2}(4p+3);\sin^2(a+bx)\right)}{b(4p+1)\sqrt{\cos^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x]^4)^p,x]
```

```
[Out] (Cos[a + b*x]*Hypergeometric2F1[1/2, (1 + 4*p)/2, (3 + 4*p)/2, Sin[a + b*x]
^2]*Sin[a + b*x]*(c*Sin[a + b*x]^4)^p)/(b*(1 + 4*p)*Sqrt[Cos[a + b*x]^2])
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \left(c\sin^4(a+bx)\right)^p\,dx &= \left(\sin^{-4p}(a+bx)\left(c\sin^4(a+bx)\right)^p\right)\int \sin^{4p}(a+bx)\,dx\\ &= \frac{\cos(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+4p);\frac{1}{2}(3+4p);\sin^2(a+bx)\right)\sin(a+bx)\left(c\sin^4(a+bx)\right)^p}{b(1+4p)\sqrt{\cos^2(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.089348, size = 65, normalized size = 0.84

$$\frac{\sqrt{\cos^2(a+bx)}\tan(a+bx)\left(c\sin^4(a+bx)\right)^p{}_2F_1\left(\frac{1}{2},2p+\frac{1}{2};2p+\frac{3}{2};\sin^2(a+bx)\right)}{4bp+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x]^4)^p,x]
```

```
[Out] (Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, 1/2 + 2*p, 3/2 + 2*p, Sin[a +
b*x]^2]*(c*Sin[a + b*x]^4)^p*Tan[a + b*x])/(b + 4*b*p)
```

Maple [F] time = 0.917, size = 0, normalized size = 0.

$$\int \left(c (\sin (bx + a))^4\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a)^4)^p,x)
```

```
[Out] int((c*sin(b*x+a)^4)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c \sin (bx + a)^4\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^4)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a)^4)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(c \cos (bx + a)^4 - 2 c \cos (bx + a)^2 + c\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^4)^p,x, algorithm="fricas")
```

```
[Out] integral((c*cos(b*x + a)^4 - 2*c*cos(b*x + a)^2 + c)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c \sin^4 (a + bx)\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)**4)**p,x)
```

```
[Out] Integral((c*sin(a + b*x)**4)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c \sin \left(bx + a\right)^4\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^4)^p,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^4)^p, x)
```

3.29 $\int \left(c \sin^n(a+bx)\right)^{\frac{1}{n}} \, dx$

Optimal. Leaf size=25

$$-\frac{\cot(a+bx)\left(c\sin^n(a+bx)\right)^{\frac{1}{n}}}{b}$$

```
[Out] -((Cot[a + b*x]*(c*Sin[a + b*x]^n)^n^(-1))/b)
```

Rubi [A] time = 0.0189031, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3208, 2638}

$$-\frac{\cot(a+bx)\left(c\sin^n(a+bx)\right)^{\frac{1}{n}}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x]^n)^n^(-1),x]
```

```
[Out] -((Cot[a + b*x]*(c*Sin[a + b*x]^n)^n^(-1))/b)
```

Rule 3208

```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Sin[e + f*x])^n)^FracPart[p])/(c*Sin[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \left(c\sin^n(a+bx)\right)^{\frac{1}{n}}\,dx &= \left(\csc(a+bx)\left(c\sin^n(a+bx)\right)^{\frac{1}{n}}\right)\int \sin(a+bx)\,dx\\
&= -\frac{\cot(a+bx)\left(c\sin^n(a+bx)\right)^{\frac{1}{n}}}{b}
\end{aligned}$$

Mathematica [A] time = 0.0337482, size = 25, normalized size = 1.

$$-\frac{\cot(a+bx)\left(c\sin^n(a+bx)\right)^{\frac{1}{n}}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x]^n)^n^(-1),x]
```

```
[Out] -((Cot[a + b*x]*(c*Sin[a + b*x]^n)^n^(-1))/b)
```

Maple [F] time = 0.292, size = 0, normalized size = 0.

$$\int \sqrt[n]{c\left(\sin\left(bx+a\right)\right)^{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a)^n)^(1/n),x)
```

```
[Out] int((c*sin(b*x+a)^n)^(1/n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c\sin\left(bx+a\right)^{n}\right)^{\left(\frac{1}{n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^n)^(1/n),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a)^n)^(1/n), x)
```

Fricas [A] time = 1.643, size = 34, normalized size = 1.36

$$-\frac{c^{\left(\frac{1}{n}\right)}\cos\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^n)^(1/n),x, algorithm="fricas")
```

```
[Out] -c^(1/n)*cos(b*x + a)/b
```

Sympy [A] time = 6.30427, size = 61, normalized size = 2.44

$$\begin{cases} x\left(c\sin^{n}\left(a\right)\right)^{\frac{1}{n}} & \text{for}\ b=0 \\ x\left(0^{n}c\right)^{\frac{1}{n}} & \text{for}\ a=-bx\vee a=-bx+\pi \\ -\frac{c^{\frac{1}{n}}\left(\sin^{n}\left(a+bx\right)\right)^{\frac{1}{n}}\cos\left(a+bx\right)}{b\sin\left(a+bx\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)**n)**(1/n),x)
```

```
[Out] Piecewise((x*(c*sin(a)**n)**(1/n), Eq(b, 0)), (x*(0**n*c)**(1/n), Eq(a, -b*
x) | Eq(a, -b*x + pi)), (-c**(1/n)*(sin(a + b*x)**n)**(1/n)*cos(a + b*x)/(b
*sin(a + b*x)), True))
```

Giac [B] time = 8.92025, size = 518, normalized size = 20.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^n)^(1/n),x, algorithm="giac")
```

```
[Out] (abs(c)^(1/n)*tan(1/2*b*x + 1/2*a + 1/4*pi*sgn(c)/n - 1/4*pi/n)^2*tan(1/2*b
*x + 1/2*a)^4 - 2*abs(c)^(1/n)*tan(1/2*b*x + 1/2*a + 1/4*pi*sgn(c)/n - 1/4*
pi/n)^2*tan(1/2*b*x + 1/2*a)^2 + 4*abs(c)^(1/n)*tan(1/2*b*x + 1/2*a + 1/4*p
i*sgn(c)/n - 1/4*pi/n)*tan(1/2*b*x + 1/2*a)^3 - abs(c)^(1/n)*tan(1/2*b*x +
1/2*a)^4 + abs(c)^(1/n)*tan(1/2*b*x + 1/2*a + 1/4*pi*sgn(c)/n - 1/4*pi/n)^2
 - 4*abs(c)^(1/n)*tan(1/2*b*x + 1/2*a + 1/4*pi*sgn(c)/n - 1/4*pi/n)*tan(1/2
*b*x + 1/2*a) + 2*abs(c)^(1/n)*tan(1/2*b*x + 1/2*a)^2 - abs(c)^(1/n))/(b*ta
n(1/2*b*x + 1/2*a + 1/4*pi*sgn(c)/n - 1/4*pi/n)^2*tan(1/2*b*x + 1/2*a)^4 +
2*b*tan(1/2*b*x + 1/2*a + 1/4*pi*sgn(c)/n - 1/4*pi/n)^2*tan(1/2*b*x + 1/2*a
)^2 + b*tan(1/2*b*x + 1/2*a)^4 + b*tan(1/2*b*x + 1/2*a + 1/4*pi*sgn(c)/n -
1/4*pi/n)^2 + 2*b*tan(1/2*b*x + 1/2*a)^2 + b)
```

3.30 $\int (a(b\sin(c+dx))^p)^n\,dx$

Optimal. Leaf size=79

$$\frac{\sin(c+dx)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+1);\frac{1}{2}(np+3);\sin^2(c+dx)\right)(a(b\sin(c+dx))^p)^n}{d(np+1)\sqrt{\cos^2(c+dx)}}$$

```
[Out] (Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sin[c + d*x]
^2]*Sin[c + d*x]*(a*(b*Sin[c + d*x])^p)^n)/(d*(1 + n*p)*Sqrt[Cos[c + d*x]^2
])
```

Rubi [A] time = 0.0370264, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3208, 2643}

$$\frac{\sin(c+dx)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+1);\frac{1}{2}(np+3);\sin^2(c+dx)\right)(a(b\sin(c+dx))^p)^n}{d(np+1)\sqrt{\cos^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*(b*Sin[c + d*x])^p)^n,x]
```

```
[Out] (Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sin[c + d*x]
^2]*Sin[c + d*x]*(a*(b*Sin[c + d*x])^p)^n)/(d*(1 + n*p)*Sqrt[Cos[c + d*x]^2
])
```

Rule 3208

```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Sin[e + f*x])^n)^FracPart[p])/(c*Sin[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int (a(b\sin(c+dx))^p)^n\,dx &= \left((b\sin(c+dx))^{-np}(a(b\sin(c+dx))^p)^n\right)\int (b\sin(c+dx))^{np}\,dx\\ &= \frac{\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+np);\frac{1}{2}(3+np);\sin^2(c+dx)\right)\sin(c+dx)(a(b\sin(c+dx))^p)^n}{d(1+np)\sqrt{\cos^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0574693, size = 73, normalized size = 0.92

$$\frac{\sqrt{\cos^2(c+dx)}\tan(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+1);\frac{1}{2}(np+3);\sin^2(c+dx)\right)(a(b\sin(c+dx))^p)^n}{d(np+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*(b*Sin[c + d*x])^p)^n,x]
```

```
[Out] (Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sin[
c + d*x]^2]*(a*(b*Sin[c + d*x])^p)^n*Tan[c + d*x])/(d*(1 + n*p))
```

Maple [F] time = 0.333, size = 0, normalized size = 0.

$$\int \left(a \left(b \sin \left(dx + c\right)\right)^{p}\right)^{n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*(b*sin(d*x+c))^p)^n,x)
```

```
[Out] int((a*(b*sin(d*x+c))^p)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\left(b \sin \left(dx + c\right)\right)^{p} a\right)^{n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*sin(d*x+c))^p)^n,x, algorithm="maxima")
```

```
[Out] integrate(((b*sin(d*x + c))^p*a)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(\left(b \sin \left(dx + c\right)\right)^{p} a\right)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*sin(d*x+c))^p)^n,x, algorithm="fricas")
```

```
[Out] integral(((b*sin(d*x + c))^p*a)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \left(b \sin \left(c + dx\right)\right)^{p}\right)^{n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*sin(d*x+c))**p)**n,x)
```

```
[Out] Integral((a*(b*sin(c + d*x))**p)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left((b \sin (dx + c))^p a\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*sin(d*x+c))^p)^n,x, algorithm="giac")
```

```
[Out] integrate(((b*sin(d*x + c))^p*a)^n, x)
```

3.31 $\int \left(a - a\sin^2(x)\right) dx$

Optimal. Leaf size=16

$$\frac{ax}{2} + \frac{1}{2}a\sin(x)\cos(x)$$

```
[Out] (a*x)/2 + (a*Cos[x]*Sin[x])/2
```

Rubi [A] time = 0.0085589, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2635, 8}

$$\frac{ax}{2} + \frac{1}{2}a\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[a - a*Sin[x]^2,x]
```

```
[Out] (a*x)/2 + (a*Cos[x]*Sin[x])/2
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned} \int \left(a - a\sin^2(x)\right) dx &= ax - a\int \sin^2(x)\,dx \\ &= ax + \frac{1}{2}a\cos(x)\sin(x) - \frac{1}{2}a\int 1\,dx \\ &= \frac{ax}{2} + \frac{1}{2}a\cos(x)\sin(x) \end{aligned}$$

Mathematica [A] time = 0.0028456, size = 16, normalized size = 1.

$$a\left(\frac{x}{2} + \frac{1}{4}\sin(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a - a*Sin[x]^2,x]
```

```
[Out] a*(x/2 + Sin[2*x]/4)
```

Maple [A] time = 0.019, size = 18, normalized size = 1.1

$$ax - a\left(-\frac{\sin(x)\cos(x)}{2} + \frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a-a*sin(x)^2,x)
```

```
[Out] a*x-a*(-1/2*sin(x)*cos(x)+1/2*x)
```

Maxima [A] time = 0.97762, size = 23, normalized size = 1.44

$$-\frac{1}{4}a(2x - \sin(2x)) + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a-a*sin(x)^2,x, algorithm="maxima")
```

```
[Out] -1/4*a*(2*x - sin(2*x)) + a*x
```

Fricas [A] time = 1.64897, size = 42, normalized size = 2.62

$$\frac{1}{2}a\cos(x)\sin(x) + \frac{1}{2}ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a-a*sin(x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*a*cos(x)*sin(x) + 1/2*a*x
```

Sympy [A] time = 0.155767, size = 15, normalized size = 0.94

$$ax - a\left(\frac{x}{2} - \frac{\sin(x)\cos(x)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a-a*sin(x)**2,x)
```

```
[Out] a*x - a*(x/2 - sin(x)*cos(x)/2)
```

Giac [A] time = 1.1417, size = 23, normalized size = 1.44

$$-\frac{1}{4}a(2x - \sin(2x)) + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a-a*sin(x)^2,x, algorithm="giac")
```

```
[Out] -1/4*a*(2*x - sin(2*x)) + a*x
```

3.32 $\int \left(a-a \sin^2(x)\right)^2 \, dx$

Optimal. Leaf size=33

$$\frac{3a^2x}{8}+\frac{1}{4}a^2\sin(x)\cos^3(x)+\frac{3}{8}a^2\sin(x)\cos(x)$$

```
[Out] (3*a^2*x)/8 + (3*a^2*Cos[x]*Sin[x])/8 + (a^2*Cos[x]^3*Sin[x])/4
```

Rubi [A] time = 0.0256267, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3175, 2635, 8}

$$\frac{3a^2x}{8}+\frac{1}{4}a^2\sin(x)\cos^3(x)+\frac{3}{8}a^2\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[x]^2)^2,x]
```

```
[Out] (3*a^2*x)/8 + (3*a^2*Cos[x]*Sin[x])/8 + (a^2*Cos[x]^3*Sin[x])/4
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \left(a-a\sin^2(x)\right)^2 \, dx &= a^2 \int \cos^4(x) \, dx\\
&= \frac{1}{4}a^2\cos^3(x)\sin(x)+\frac{1}{4}\left(3a^2\right)\int \cos^2(x) \, dx\\
&= \frac{3}{8}a^2\cos(x)\sin(x)+\frac{1}{4}a^2\cos^3(x)\sin(x)+\frac{1}{8}\left(3a^2\right)\int 1 \, dx\\
&= \frac{3a^2x}{8}+\frac{3}{8}a^2\cos(x)\sin(x)+\frac{1}{4}a^2\cos^3(x)\sin(x)
\end{aligned}$$

Mathematica [A] time = 0.0027889, size = 26, normalized size = 0.79

$$a^2\left(\frac{3x}{8}+\frac{1}{4}\sin(2x)+\frac{1}{32}\sin(4x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sin[x]^2)^2,x]
```

```
[Out] a^2*((3*x)/8 + Sin[2*x]/4 + Sin[4*x]/32)
```

Maple [A] time = 0.021, size = 43, normalized size = 1.3

$$a^2\left(-\frac{\cos(x)}{4}\left((\sin(x))^3+\frac{3\sin(x)}{2}\right)+\frac{3x}{8}\right)-2a^2(-1/2\sin(x)\cos(x)+x/2)+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(x)^2)^2,x)
```

```
[Out] a^2*(-1/4*(sin(x)^3+3/2*sin(x))*cos(x)+3/8*x)-2*a^2*(-1/2*sin(x)*cos(x)+1/2
*x)+a^2*x
```

Maxima [A] time = 0.948838, size = 54, normalized size = 1.64

$$\frac{1}{32}a^2(12x+\sin(4x)-8\sin(2x))-\frac{1}{2}a^2(2x-\sin(2x))+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/32*a^2*(12*x + sin(4*x) - 8*sin(2*x)) - 1/2*a^2*(2*x - sin(2*x)) + a^2*x
```

Fricas [A] time = 1.64587, size = 76, normalized size = 2.3

$$\frac{3}{8}a^2x+\frac{1}{8}\left(2a^2\cos(x)^3+3a^2\cos(x)\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] 3/8*a^2*x + 1/8*(2*a^2*cos(x)^3 + 3*a^2*cos(x))*sin(x)
```

Sympy [B] time = 1.06638, size = 110, normalized size = 3.33

$$\frac{3a^2x\sin^4(x)}{8}+\frac{3a^2x\sin^2(x)\cos^2(x)}{4}-a^2x\sin^2(x)+\frac{3a^2x\cos^4(x)}{8}-a^2x\cos^2(x)+a^2x-\frac{5a^2\sin^3(x)\cos(x)}{8}-\frac{3a^2\text{s}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)**2)**2,x)
```

```
[Out] 3*a**2*x*sin(x)**4/8 + 3*a**2*x*sin(x)**2*cos(x)**2/4 - a**2*x*sin(x)**2 +
3*a**2*x*cos(x)**4/8 - a**2*x*cos(x)**2 + a**2*x - 5*a**2*sin(x)**3*cos(x)/
8 - 3*a**2*sin(x)*cos(x)**3/8 + a**2*sin(x)*cos(x)
```

Giac [A] time = 1.12513, size = 34, normalized size = 1.03

$$\frac{3}{8}a^2x + \frac{1}{32}a^2\sin(4x) + \frac{1}{4}a^2\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] 3/8*a^2*x + 1/32*a^2*sin(4*x) + 1/4*a^2*sin(2*x)
```

3.33 $\int \left(a-a\sin^2(x)\right)^3 \, dx$

Optimal. Leaf size=46

$$\frac{5a^3x}{16}+\frac{1}{6}a^3\sin(x)\cos^5(x)+\frac{5}{24}a^3\sin(x)\cos^3(x)+\frac{5}{16}a^3\sin(x)\cos(x)$$

```
[Out] (5*a^3*x)/16 + (5*a^3*Cos[x]*Sin[x])/16 + (5*a^3*Cos[x]^3*Sin[x])/24 + (a^3
*Cos[x]^5*Sin[x])/6
```

Rubi [A] time = 0.0324165, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3175, 2635, 8}

$$\frac{5a^3x}{16}+\frac{1}{6}a^3\sin(x)\cos^5(x)+\frac{5}{24}a^3\sin(x)\cos^3(x)+\frac{5}{16}a^3\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[x]^2)^3,x]
```

```
[Out] (5*a^3*x)/16 + (5*a^3*Cos[x]*Sin[x])/16 + (5*a^3*Cos[x]^3*Sin[x])/24 + (a^3
*Cos[x]^5*Sin[x])/6
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \left(a-a\sin^2(x)\right)^3 \, dx &= a^3\int \cos^6(x)\,dx\\
&=\frac{1}{6}a^3\cos^5(x)\sin(x)+\frac{1}{6}\left(5a^3\right)\int \cos^4(x)\,dx\\
&=\frac{5}{24}a^3\cos^3(x)\sin(x)+\frac{1}{6}a^3\cos^5(x)\sin(x)+\frac{1}{8}\left(5a^3\right)\int \cos^2(x)\,dx\\
&=\frac{5}{16}a^3\cos(x)\sin(x)+\frac{5}{24}a^3\cos^3(x)\sin(x)+\frac{1}{6}a^3\cos^5(x)\sin(x)+\frac{1}{16}\left(5a^3\right)\int 1\,dx\\
&=\frac{5a^3x}{16}+\frac{5}{16}a^3\cos(x)\sin(x)+\frac{5}{24}a^3\cos^3(x)\sin(x)+\frac{1}{6}a^3\cos^5(x)\sin(x)
\end{aligned}$$

Mathematica [A] time = 0.0027158, size = 34, normalized size = 0.74

$$a^3\left(\frac{5x}{16}+\frac{15}{64}\sin(2x)+\frac{3}{64}\sin(4x)+\frac{1}{192}\sin(6x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sin[x]^2)^3,x]
```

```
[Out] a^3*((5*x)/16 + (15*Sin[2*x])/64 + (3*Sin[4*x])/64 + Sin[6*x]/192)
```

Maple [A] time = 0.02, size = 72, normalized size = 1.6

$$-a^3\left(-\frac{\cos(x)}{6}\left((\sin(x))^5+\frac{5(\sin(x))^3}{4}+\frac{15\sin(x)}{8}\right)+\frac{5x}{16}\right)+3a^3\left(-1/4\left((\sin(x))^3+3/2\sin(x)\right)\cos(x)+3/8x\right)-3a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(x)^2)^3,x)
```

```
[Out] -a^3*(-1/6*(sin(x)^5+5/4*sin(x)^3+15/8*sin(x))*cos(x)+5/16*x)+3*a^3*(-1/4*(
sin(x)^3+3/2*sin(x))*cos(x)+3/8*x)-3*a^3*(-1/2*sin(x)*cos(x)+1/2*x)+a^3*x
```

Maxima [A] time = 0.949992, size = 93, normalized size = 2.02

$$-\frac{1}{192}\left(4\sin(2x)^3+60x+9\sin(4x)-48\sin(2x)\right)a^3+\frac{3}{32}a^3(12x+\sin(4x)-8\sin(2x))-\frac{3}{4}a^3(2x-\sin(2x))+a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/192*(4*sin(2*x)^3 + 60*x + 9*sin(4*x) - 48*sin(2*x))*a^3 + 3/32*a^3*(12*
x + sin(4*x) - 8*sin(2*x)) - 3/4*a^3*(2*x - sin(2*x)) + a^3*x
```

Fricas [A] time = 1.62099, size = 104, normalized size = 2.26

$$\frac{5}{16}a^3x+\frac{1}{48}\left(8a^3\cos(x)^5+10a^3\cos(x)^3+15a^3\cos(x)\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^3,x, algorithm="fricas")
```

```
[Out] 5/16*a^3*x + 1/48*(8*a^3*cos(x)^5 + 10*a^3*cos(x)^3 + 15*a^3*cos(x))*sin(x)
```

Sympy [B] time = 4.28107, size = 233, normalized size = 5.07

$$-\frac{5a^3x\sin^6(x)}{16}-\frac{15a^3x\sin^4(x)\cos^2(x)}{16}+\frac{9a^3x\sin^4(x)}{8}-\frac{15a^3x\sin^2(x)\cos^4(x)}{16}+\frac{9a^3x\sin^2(x)\cos^2(x)}{4}-\frac{3a^3x\sin^2(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)**2)**3,x)
```

```
[Out] -5*a**3*x*sin(x)**6/16 - 15*a**3*x*sin(x)**4*cos(x)**2/16 + 9*a**3*x*sin(x)
**4/8 - 15*a**3*x*sin(x)**2*cos(x)**4/16 + 9*a**3*x*sin(x)**2*cos(x)**2/4 -
 3*a**3*x*sin(x)**2/2 - 5*a**3*x*cos(x)**6/16 + 9*a**3*x*cos(x)**4/8 - 3*a*
*3*x*cos(x)**2/2 + a**3*x + 11*a**3*sin(x)**5*cos(x)/16 + 5*a**3*sin(x)**3*
cos(x)**3/6 - 15*a**3*sin(x)**3*cos(x)/8 + 5*a**3*sin(x)*cos(x)**5/16 - 9*a
**3*sin(x)*cos(x)**3/8 + 3*a**3*sin(x)*cos(x)/2
```

Giac [A] time = 1.12124, size = 46, normalized size = 1.

$$\frac{5}{16}a^3x + \frac{1}{192}a^3\sin(6x) + \frac{3}{64}a^3\sin(4x) + \frac{15}{64}a^3\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^3,x, algorithm="giac")
```

```
[Out] 5/16*a^3*x + 1/192*a^3*sin(6*x) + 3/64*a^3*sin(4*x) + 15/64*a^3*sin(2*x)
```

3.34 $\int \left(a-a\sin^2(x)\right)^4\,dx$

Optimal. Leaf size=59

$$\frac{35a^4x}{128}+\frac{1}{8}a^4\sin(x)\cos^7(x)+\frac{7}{48}a^4\sin(x)\cos^5(x)+\frac{35}{192}a^4\sin(x)\cos^3(x)+\frac{35}{128}a^4\sin(x)\cos(x)$$

```
[Out] (35*a^4*x)/128 + (35*a^4*Cos[x]*Sin[x])/128 + (35*a^4*Cos[x]^3*Sin[x])/192
+ (7*a^4*Cos[x]^5*Sin[x])/48 + (a^4*Cos[x]^7*Sin[x])/8
```

Rubi [A] time = 0.0415468, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3175, 2635, 8}

$$\frac{35a^4x}{128}+\frac{1}{8}a^4\sin(x)\cos^7(x)+\frac{7}{48}a^4\sin(x)\cos^5(x)+\frac{35}{192}a^4\sin(x)\cos^3(x)+\frac{35}{128}a^4\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[x]^2)^4,x]
```

```
[Out] (35*a^4*x)/128 + (35*a^4*Cos[x]*Sin[x])/128 + (35*a^4*Cos[x]^3*Sin[x])/192
+ (7*a^4*Cos[x]^5*Sin[x])/48 + (a^4*Cos[x]^7*Sin[x])/8
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \left(a-a\sin^2(x)\right)^4\,dx &= a^4\int \cos^8(x)\,dx\\
&=\frac{1}{8}a^4\cos^7(x)\sin(x)+\frac{1}{8}\left(7a^4\right)\int \cos^6(x)\,dx\\
&=\frac{7}{48}a^4\cos^5(x)\sin(x)+\frac{1}{8}a^4\cos^7(x)\sin(x)+\frac{1}{48}\left(35a^4\right)\int \cos^4(x)\,dx\\
&=\frac{35}{192}a^4\cos^3(x)\sin(x)+\frac{7}{48}a^4\cos^5(x)\sin(x)+\frac{1}{8}a^4\cos^7(x)\sin(x)+\frac{1}{64}\left(35a^4\right)\int \cos^2(x)\,dx\\
&=\frac{35}{128}a^4\cos(x)\sin(x)+\frac{35}{192}a^4\cos^3(x)\sin(x)+\frac{7}{48}a^4\cos^5(x)\sin(x)+\frac{1}{8}a^4\cos^7(x)\sin(x)+\frac{1}{128}(3\\
&=\frac{35a^4x}{128}+\frac{35}{128}a^4\cos(x)\sin(x)+\frac{35}{192}a^4\cos^3(x)\sin(x)+\frac{7}{48}a^4\cos^5(x)\sin(x)+\frac{1}{8}a^4\cos^7(x)\sin(x)
\end{aligned}$$

Mathematica [A] time = 0.0030452, size = 42, normalized size = 0.71

$$a^4\left(\frac{35x}{128}+\frac{7}{32}\sin(2x)+\frac{7}{128}\sin(4x)+\frac{1}{96}\sin(6x)+\frac{\sin(8x)}{1024}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sin[x]^2)^4,x]
```

```
[Out] a^4*((35*x)/128 + (7*Sin[2*x])/32 + (7*Sin[4*x])/128 + Sin[6*x]/96 + Sin[8*
x]/1024)
```

Maple [B] time = 0.02, size = 105, normalized size = 1.8

$$a^4\left(-\frac{\cos(x)}{8}\left((\sin(x))^7+\frac{7(\sin(x))^5}{6}+\frac{35(\sin(x))^3}{24}+\frac{35\sin(x)}{16}\right)+\frac{35x}{128}\right)-4a^4\left(-1/6\left((\sin(x))^5+5/4(\sin(x))^3+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(x)^2)^4,x)
```

```
[Out] a^4*(-1/8*(sin(x)^7+7/6*sin(x)^5+35/24*sin(x)^3+35/16*sin(x))*cos(x)+35/128
*x)-4*a^4*(-1/6*(sin(x)^5+5/4*sin(x)^3+15/8*sin(x))*cos(x)+5/16*x)+6*a^4*(-
1/4*(sin(x)^3+3/2*sin(x))*cos(x)+3/8*x)-4*a^4*(-1/2*sin(x)*cos(x)+1/2*x)+a^
4*x
```

Maxima [B] time = 0.958191, size = 140, normalized size = 2.37

$$\frac{1}{3072}\left(128\sin(2x)^3+840x+3\sin(8x)+168\sin(4x)-768\sin(2x)\right)a^4-\frac{1}{48}\left(4\sin(2x)^3+60x+9\sin(4x)-4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^4,x, algorithm="maxima")
```

```
[Out] 1/3072*(128*sin(2*x)^3 + 840*x + 3*sin(8*x) + 168*sin(4*x) - 768*sin(2*x))*
a^4 - 1/48*(4*sin(2*x)^3 + 60*x + 9*sin(4*x) - 48*sin(2*x))*a^4 + 3/16*a^4*
(12*x + sin(4*x) - 8*sin(2*x)) - a^4*(2*x - sin(2*x)) + a^4*x
```

Fricas [A] time = 1.74637, size = 135, normalized size = 2.29

$$\frac{35}{128}a^4x+\frac{1}{384}\left(48a^4\cos(x)^7+56a^4\cos(x)^5+70a^4\cos(x)^3+105a^4\cos(x)\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^4,x, algorithm="fricas")
```

```
[Out] 35/128*a^4*x + 1/384*(48*a^4*cos(x)^7 + 56*a^4*cos(x)^5 + 70*a^4*cos(x)^3 +
 105*a^4*cos(x))*sin(x)
```

Sympy [B] time = 13.8102, size = 376, normalized size = 6.37

$$\frac{35a^4x\sin^8(x)}{128} + \frac{35a^4x\sin^6(x)\cos^2(x)}{32} - \frac{5a^4x\sin^6(x)}{4} + \frac{105a^4x\sin^4(x)\cos^4(x)}{64} - \frac{15a^4x\sin^4(x)\cos^2(x)}{4} + \frac{9a^4x\sin^4(}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)**2)**4,x)
```

```
[Out] 35*a**4*x*sin(x)**8/128 + 35*a**4*x*sin(x)**6*cos(x)**2/32 - 5*a**4*x*sin(x
)**6/4 + 105*a**4*x*sin(x)**4*cos(x)**4/64 - 15*a**4*x*sin(x)**4*cos(x)**2/
4 + 9*a**4*x*sin(x)**4/4 + 35*a**4*x*sin(x)**2*cos(x)**6/32 - 15*a**4*x*sin
(x)**2*cos(x)**4/4 + 9*a**4*x*sin(x)**2*cos(x)**2/2 - 2*a**4*x*sin(x)**2 +
35*a**4*x*cos(x)**8/128 - 5*a**4*x*cos(x)**6/4 + 9*a**4*x*cos(x)**4/4 - 2*a
**4*x*cos(x)**2 + a**4*x - 93*a**4*sin(x)**7*cos(x)/128 - 511*a**4*sin(x)**
5*cos(x)**3/384 + 11*a**4*sin(x)**5*cos(x)/4 - 385*a**4*sin(x)**3*cos(x)**5
/384 + 10*a**4*sin(x)**3*cos(x)**3/3 - 15*a**4*sin(x)**3*cos(x)/4 - 35*a**4
*sin(x)*cos(x)**7/128 + 5*a**4*sin(x)*cos(x)**5/4 - 9*a**4*sin(x)*cos(x)**3
/4 + 2*a**4*sin(x)*cos(x)
```

Giac [A] time = 1.13789, size = 58, normalized size = 0.98

$$\frac{35}{128}\,a^4x + \frac{1}{1024}\,a^4\sin(8\,x) + \frac{1}{96}\,a^4\sin(6\,x) + \frac{7}{128}\,a^4\sin(4\,x) + \frac{7}{32}\,a^4\sin(2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^4,x, algorithm="giac")
```

```
[Out] 35/128*a^4*x + 1/1024*a^4*sin(8*x) + 1/96*a^4*sin(6*x) + 7/128*a^4*sin(4*x)
 + 7/32*a^4*sin(2*x)
```

3.35 $\int \frac{\sin^7(c+dx)}{a-a\sin^2(c+dx)}\,dx$

Optimal. Leaf size=62

$$\frac{\cos^5(c+dx)}{5ad}-\frac{\cos^3(c+dx)}{ad}+\frac{3\cos(c+dx)}{ad}+\frac{\sec(c+dx)}{ad}$$

```
[Out] (3*Cos[c + d*x])/(a*d) - Cos[c + d*x]^3/(a*d) + Cos[c + d*x]^5/(5*a*d) + Se
c[c + d*x]/(a*d)
```

Rubi [A] time = 0.0870735, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 2590, 270}

$$\frac{\cos^5(c+dx)}{5ad}-\frac{\cos^3(c+dx)}{ad}+\frac{3\cos(c+dx)}{ad}+\frac{\sec(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^7/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] (3*Cos[c + d*x])/(a*d) - Cos[c + d*x]^3/(a*d) + Cos[c + d*x]^5/(5*a*d) + Se
c[c + d*x]/(a*d)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^7(c+dx)}{a-a\sin^2(c+dx)}\,dx &= \frac{\int \sin^5(c+dx)\tan^2(c+dx)\,dx}{a}\\
&= -\frac{\text{Subst}\left(\int \frac{(1-x^2)^3}{x^2}\,dx,x,\cos(c+dx)\right)}{ad}\\
&= -\frac{\text{Subst}\left(\int \left(-3+\frac{1}{x^2}+3x^2-x^4\right)dx,x,\cos(c+dx)\right)}{ad}\\
&= \frac{3\cos(c+dx)}{ad}-\frac{\cos^3(c+dx)}{ad}+\frac{\cos^5(c+dx)}{5ad}+\frac{\sec(c+dx)}{ad}
\end{aligned}$$

Mathematica [A] time = 0.0636532, size = 58, normalized size = 0.94

$$\frac{\frac{19\cos(c+dx)}{8d} - \frac{3\cos(3(c+dx))}{16d} + \frac{\cos(5(c+dx))}{80d} + \frac{\sec(c+dx)}{d}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^7/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] ((19*Cos[c + d*x])/(8*d) - (3*Cos[3*(c + d*x)])/(16*d) + Cos[5*(c + d*x)]/(
80*d) + Sec[c + d*x]/d)/a
```

Maple [A] time = 0.042, size = 45, normalized size = 0.7

$$\frac{1}{da}\left(\frac{(\cos(dx+c))^5}{5} - (\cos(dx+c))^3 + 3\cos(dx+c) + (\cos(dx+c))^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^7/(a-sin(d*x+c)^2*a),x)
```

```
[Out] 1/d/a*(1/5*cos(d*x+c)^5-cos(d*x+c)^3+3*cos(d*x+c)+1/cos(d*x+c))
```

Maxima [A] time = 0.966741, size = 68, normalized size = 1.1

$$\frac{\frac{\cos(dx+c)^5 - 5\cos(dx+c)^3 + 15\cos(dx+c)}{a} + \frac{5}{a\cos(dx+c)}}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/5*((cos(d*x + c)^5 - 5*cos(d*x + c)^3 + 15*cos(d*x + c))/a + 5/(a*cos(d*x
 + c)))/d
```

Fricas [A] time = 1.60437, size = 113, normalized size = 1.82

$$\frac{\cos(dx+c)^6 - 5\cos(dx+c)^4 + 15\cos(dx+c)^2 + 5}{5ad\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/5*(cos(d*x + c)^6 - 5*cos(d*x + c)^4 + 15*cos(d*x + c)^2 + 5)/(a*d*cos(d*
x + c))
```

Sympy [A] time = 125.581, size = 314, normalized size = 5.06

$$\begin{cases} -\frac{160\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{5ad\tan^{12}\left(\frac{c}{2}+\frac{dx}{2}\right)+20ad\tan^{10}\left(\frac{c}{2}+\frac{dx}{2}\right)+25ad\tan^8\left(\frac{c}{2}+\frac{dx}{2}\right)-25ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)-20ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-5ad} - \frac{}{5ad\tan^{12}\left(\frac{c}{2}+\frac{dx}{2}\right)+20ad\tan^{10}\left(\frac{c}{2}+\frac{dx}{2}\right)+2} \\ \frac{x\sin^7(c)}{-a\sin^2(c)+a} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**7/(a-a*sin(d*x+c)**2),x)
```

```
[Out] Piecewise((-160*tan(c/2 + d*x/2)**4/(5*a*d*tan(c/2 + d*x/2)**12 + 20*a*d*ta
n(c/2 + d*x/2)**10 + 25*a*d*tan(c/2 + d*x/2)**8 - 25*a*d*tan(c/2 + d*x/2)**
4 - 20*a*d*tan(c/2 + d*x/2)**2 - 5*a*d) - 128*tan(c/2 + d*x/2)**2/(5*a*d*ta
n(c/2 + d*x/2)**12 + 20*a*d*tan(c/2 + d*x/2)**10 + 25*a*d*tan(c/2 + d*x/2)*
*8 - 25*a*d*tan(c/2 + d*x/2)**4 - 20*a*d*tan(c/2 + d*x/2)**2 - 5*a*d) - 32/
(5*a*d*tan(c/2 + d*x/2)**12 + 20*a*d*tan(c/2 + d*x/2)**10 + 25*a*d*tan(c/2
+ d*x/2)**8 - 25*a*d*tan(c/2 + d*x/2)**4 - 20*a*d*tan(c/2 + d*x/2)**2 - 5*a
*d), Ne(d, 0)), (x*sin(c)**7/(-a*sin(c)**2 + a), True))
```

Giac [B] time = 1.16396, size = 201, normalized size = 3.24

$$\frac{2\left(\frac{5}{a\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)} + \frac{\frac{50(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{80(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} + \frac{30(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3} - \frac{5(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4} - 11}{a\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^5}\right)}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a-a*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 2/5*(5/(a*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) + (50*(cos(d*x + c)
- 1)/(cos(d*x + c) + 1) - 80*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 30
*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 5*(cos(d*x + c) - 1)^4/(cos(d*
x + c) + 1)^4 - 11)/(a*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^5))/d
```

3.36 $\int \frac{\sin^5(c+dx)}{a-a\sin^2(c+dx)}\,dx$

Optimal. Leaf size=46

$$-\frac{\cos^3(c+dx)}{3ad}+\frac{2\cos(c+dx)}{ad}+\frac{\sec(c+dx)}{ad}$$

```
[Out] (2*Cos[c + d*x])/(a*d) - Cos[c + d*x]^3/(3*a*d) + Sec[c + d*x]/(a*d)
```

Rubi [A] time = 0.0802002, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 2590, 270}

$$-\frac{\cos^3(c+dx)}{3ad}+\frac{2\cos(c+dx)}{ad}+\frac{\sec(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^5/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] (2*Cos[c + d*x])/(a*d) - Cos[c + d*x]^3/(3*a*d) + Sec[c + d*x]/(a*d)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^5(c+dx)}{a-a\sin^2(c+dx)}\,dx &= \frac{\int \sin^3(c+dx)\tan^2(c+dx)\,dx}{a}\\
&= -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x^2}\,dx, x, \cos(c+dx)\right)}{ad}\\
&= -\frac{\text{Subst}\left(\int \left(-2+\frac{1}{x^2}+x^2\right)dx, x, \cos(c+dx)\right)}{ad}\\
&= \frac{2\cos(c+dx)}{ad}-\frac{\cos^3(c+dx)}{3ad}+\frac{\sec(c+dx)}{ad}
\end{aligned}$$

Mathematica [A] time = 0.0436003, size = 43, normalized size = 0.93

$$\frac{\frac{7\cos(c+dx)}{4d}-\frac{\cos(3(c+dx))}{12d}+\frac{\sec(c+dx)}{d}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^5/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] ((7*Cos[c + d*x])/(4*d) - Cos[3*(c + d*x)]/(12*d) + Sec[c + d*x]/d)/a
```

Maple [A] time = 0.04, size = 35, normalized size = 0.8

$$\frac{1}{da}\left(-\frac{(\cos(dx+c))^3}{3}+2\cos(dx+c)+(\cos(dx+c))^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5/(a-sin(d*x+c)^2*a),x)
```

```
[Out] 1/d/a*(-1/3*cos(d*x+c)^3+2*cos(d*x+c)+1/cos(d*x+c))
```

Maxima [A] time = 0.941479, size = 54, normalized size = 1.17

$$-\frac{\frac{\cos(dx+c)^3-6\cos(dx+c)}{a}-\frac{3}{a\cos(dx+c)}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/3*((cos(d*x + c)^3 - 6*cos(d*x + c))/a - 3/(a*cos(d*x + c)))/d
```

Fricas [A] time = 1.86087, size = 88, normalized size = 1.91

$$-\frac{\cos(dx+c)^4-6\cos(dx+c)^2-3}{3\,ad\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/3*(cos(d*x + c)^4 - 6*cos(d*x + c)^2 - 3)/(a*d*cos(d*x + c))
```

Sympy [A] time = 42.9334, size = 143, normalized size = 3.11

$$\begin{cases} -\frac{32\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{3ad\tan^8\left(\frac{c}{2}+\frac{dx}{2}\right)+6ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)-6ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-3ad}-\frac{16}{3ad\tan^8\left(\frac{c}{2}+\frac{dx}{2}\right)+6ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)-6ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-3ad} & \text{for } d\neq 0 \\ \frac{x\sin^5(c)}{-a\sin^2(c)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5/(a-a*sin(d*x+c)**2),x)
```

```
[Out] Piecewise((-32*tan(c/2 + d*x/2)**2/(3*a*d*tan(c/2 + d*x/2)**8 + 6*a*d*tan(c
/2 + d*x/2)**6 - 6*a*d*tan(c/2 + d*x/2)**2 - 3*a*d) - 16/(3*a*d*tan(c/2 + d
*x/2)**8 + 6*a*d*tan(c/2 + d*x/2)**6 - 6*a*d*tan(c/2 + d*x/2)**2 - 3*a*d),
Ne(d, 0)), (x*sin(c)**5/(-a*sin(c)**2 + a), True))
```

Giac [B] time = 1.1816, size = 142, normalized size = 3.09

$$\frac{2\left(\frac{3}{a\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)}+\frac{\frac{12(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-5}{a\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^3}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a-a*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 2/3*(3/(a*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) + (12*(cos(d*x + c)
- 1)/(cos(d*x + c) + 1) - 3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 5)/
(a*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^3))/d
```

3.37 $\int \frac{\sin^3(c+dx)}{a-a\sin^2(c+dx)}\,dx$

Optimal. Leaf size=27

$$\frac{\cos(c+dx)}{ad}+\frac{\sec(c+dx)}{ad}$$

```
[Out] Cos[c + d*x]/(a*d) + Sec[c + d*x]/(a*d)
```

Rubi [A] time = 0.0654911, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 2590, 14}

$$\frac{\cos(c+dx)}{ad}+\frac{\sec(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] Cos[c + d*x]/(a*d) + Sec[c + d*x]/(a*d)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^3(c+dx)}{a-a\sin^2(c+dx)}\,dx &= \frac{\int \sin(c+dx)\tan^2(c+dx)\,dx}{a}\\
&= -\frac{\operatorname{Subst}\left(\int \frac{1-x^2}{x^2}\,dx, x, \cos(c+dx)\right)}{ad}\\
&= -\frac{\operatorname{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)dx, x, \cos(c+dx)\right)}{ad}\\
&= \frac{\cos(c+dx)}{ad}+\frac{\sec(c+dx)}{ad}
\end{aligned}$$

Mathematica [A] time = 0.0331787, size = 25, normalized size = 0.93

$$\frac{\frac{\cos(c+dx)}{d}+\frac{\sec(c+dx)}{d}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] (Cos[c + d*x]/d + Sec[c + d*x]/d)/a
```

Maple [A] time = 0.04, size = 23, normalized size = 0.9

$$\frac{\cos(dx+c)+(\cos(dx+c))^{-1}}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a-sin(d*x+c)^2*a),x)
```

```
[Out] 1/d/a*(cos(d*x+c)+1/cos(d*x+c))
```

Maxima [A] time = 0.949182, size = 36, normalized size = 1.33

$$\frac{\frac{\cos(dx+c)}{a}+\frac{1}{a\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] (cos(d*x + c)/a + 1/(a*cos(d*x + c)))/d
```

Fricas [A] time = 1.91117, size = 55, normalized size = 2.04

$$\frac{\cos(dx+c)^2+1}{ad\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] (cos(d*x + c)^2 + 1)/(a*d*cos(d*x + c))
```

Sympy [A] time = 15.0018, size = 36, normalized size = 1.33

$$\begin{cases} -\frac{4}{ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)-ad} & \text{for } d\neq 0 \\ \frac{x\sin^3(c)}{-a\sin^2(c)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a-a*sin(d*x+c)**2),x)
```

```
[Out] Piecewise((-4/(a*d*tan(c/2 + d*x/2)**4 - a*d), Ne(d, 0)), (x*sin(c)**3/(-a*
sin(c)**2 + a), True))
```

Giac [A] time = 1.10316, size = 39, normalized size = 1.44

$$\frac{\cos(dx+c)}{ad} + \frac{1}{ad\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a-a*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] cos(d*x + c)/(a*d) + 1/(a*d*cos(d*x + c))
```

3.38 $\int \frac{\sin(c+dx)}{a-a\sin^2(c+dx)} dx$

Optimal. Leaf size=13

$$\frac{\sec(c+dx)}{ad}$$

```
[Out] Sec[c + d*x]/(a*d)
```

Rubi [A] time = 0.0338714, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3175, 2606, 8}

$$\frac{\sec(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a - a*Sin[c + d*x]^2),x]

[Out] Sec[c + d*x]/(a*d)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(c+dx)}{a-a\sin^2(c+dx)}\,dx &= \frac{\int \sec(c+dx)\tan(c+dx)\,dx}{a}\\ &= \frac{\text{Subst}(\int 1\,dx, x, \sec(c+dx))}{ad}\\ &= \frac{\sec(c+dx)}{ad}\end{aligned}$$

Mathematica [A] time = 0.0127608, size = 13, normalized size = 1.

$$\frac{\sec(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] Sec[c + d*x]/(a*d)
```

Maple [A] time = 0.03, size = 16, normalized size = 1.2

$$\frac{1}{da\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a-sin(d*x+c)^2*a),x)
```

```
[Out] 1/d/a/cos(d*x+c)
```

Maxima [A] time = 0.949293, size = 20, normalized size = 1.54

$$\frac{1}{ad\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/(a*d*cos(d*x + c))
```

Fricas [A] time = 1.65355, size = 30, normalized size = 2.31

$$\frac{1}{ad\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/(a*d*cos(d*x + c))
```

Sympy [A] time = 3.1869, size = 34, normalized size = 2.62

$$\begin{cases} -\frac{2}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-ad} & \text{for } d \neq 0 \\ \frac{x\sin(c)}{-a\sin^2(c)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-a*sin(d*x+c)**2),x)
```

```
[Out] Piecewise((-2/(a*d*tan(c/2 + d*x/2)**2 - a*d), Ne(d, 0)), (x*sin(c)/(-a*sin
(c)**2 + a), True))
```

Giac [A] time = 1.11697, size = 20, normalized size = 1.54

$$\frac{1}{ad\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-a*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/(a*d*cos(d*x + c))
```

3.39 $\int \frac{\csc(c+dx)}{a-a\sin^2(c+dx)}\,dx$

Optimal. Leaf size=29

$$\frac{\sec(c+dx)}{ad}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a*d)) + Sec[c + d*x]/(a*d)
```

Rubi [A] time = 0.0575448, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3175, 2622, 321, 207}

$$\frac{\sec(c+dx)}{ad}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a*d)) + Sec[c + d*x]/(a*d)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc(c+dx)}{a-a\sin^2(c+dx)}\,dx = \frac{\int \csc(c+dx)\sec^2(c+dx)\,dx}{a}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{ad}$$
$$= \frac{\sec(c+dx)}{ad} + \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{ad}$$
$$= -\frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{\sec(c+dx)}{ad}$$

Mathematica [A] time = 0.0429316, size = 46, normalized size = 1.59

$$\frac{\frac{\sec(c+dx)}{d} + \frac{\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] (-(Log[Cos[(c + d*x)/2]]/d) + Log[Sin[(c + d*x)/2]]/d + Sec[c + d*x]/d)/a
```

Maple [A] time = 0.06, size = 51, normalized size = 1.8

$$\frac{1}{da\cos(dx+c)} + \frac{\ln(-1+\cos(dx+c))}{2\,da} - \frac{\ln(1+\cos(dx+c))}{2\,da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a-sin(d*x+c)^2*a),x)
```

```
[Out] 1/d/a/cos(d*x+c)+1/2/d/a*ln(-1+cos(d*x+c))-1/2/d/a*ln(1+cos(d*x+c))
```

Maxima [A] time = 0.948757, size = 62, normalized size = 2.14

$$-\frac{\frac{\log(\cos(dx+c)+1)}{a} - \frac{\log(\cos(dx+c)-1)}{a} - \frac{2}{a\cos(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(log(cos(d*x + c) + 1)/a - log(cos(d*x + c) - 1)/a - 2/(a*cos(d*x + c)
))/d
```

Fricas [A] time = 1.6274, size = 157, normalized size = 5.41

$$-\frac{\cos(dx+c)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right) - \cos(dx+c)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right) - 2}{2\,ad\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(cos(d*x + c)*log(1/2*cos(d*x + c) + 1/2) - cos(d*x + c)*log(-1/2*cos(
d*x + c) + 1/2) - 2)/(a*d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{\csc(c+dx)}{\sin^2(c+dx)-1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-a*sin(d*x+c)**2),x)
```

```
[Out] -Integral(csc(c + d*x)/(sin(c + d*x)**2 - 1), x)/a
```

Giac [B] time = 1.16671, size = 84, normalized size = 2.9

$$\frac{\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a}+\frac{4}{a\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-a*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*(log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a + 4/(a*((cos(d*x +
 c) - 1)/(cos(d*x + c) + 1) + 1)))/d
```

3.40 $\int \frac{\csc^3(c+dx)}{a-a\sin^2(c+dx)}\,dx$

Optimal. Leaf size=58

$$\frac{3\sec(c+dx)}{2ad}-\frac{3\tanh^{-1}(\cos(c+dx))}{2ad}-\frac{\csc^2(c+dx)\sec(c+dx)}{2ad}$$

```
[Out] (-3*ArcTanh[Cos[c + d*x]])/(2*a*d) + (3*Sec[c + d*x])/(2*a*d) - (Csc[c + d*
x]^2*Sec[c + d*x])/(2*a*d)
```

Rubi [A] time = 0.0959269, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3175, 2622, 288, 321, 207}

$$\frac{3\sec(c+dx)}{2ad}-\frac{3\tanh^{-1}(\cos(c+dx))}{2ad}-\frac{\csc^2(c+dx)\sec(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] (-3*ArcTanh[Cos[c + d*x]])/(2*a*d) + (3*Sec[c + d*x])/(2*a*d) - (Csc[c + d*
x]^2*Sec[c + d*x])/(2*a*d)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc^3(c+dx)}{a-a\sin^2(c+dx)}\,dx &= \frac{\int \csc^3(c+dx)\sec^2(c+dx)\,dx}{a}\\
&= \frac{\text{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^2}\,dx, x, \sec(c+dx)\right)}{ad}\\
&= -\frac{\csc^2(c+dx)\sec(c+dx)}{2ad}+\frac{3\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{2ad}\\
&= \frac{3\sec(c+dx)}{2ad}-\frac{\csc^2(c+dx)\sec(c+dx)}{2ad}+\frac{3\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{2ad}\\
&= -\frac{3\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{3\sec(c+dx)}{2ad}-\frac{\csc^2(c+dx)\sec(c+dx)}{2ad}
\end{aligned}
$$

Mathematica [B] time = 0.264649, size = 146, normalized size = 2.52

$$
\frac{\csc^4(c+dx)\left(-6\cos(2(c+dx))+2\cos(3(c+dx))+3\cos(3(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-3\cos(3(c+dx))\log\left(\sin\right.\right.}{2ad\left(\csc^2\left(\frac{1}{2}(c+dx)\right)-\sec^2\left(\frac{1}{2}(c+\right.\right.}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] (Csc[c + d*x]^4*(2 - 6*Cos[2*(c + d*x)] + 2*Cos[3*(c + d*x)] + 3*Cos[3*(c +
 d*x)]*Log[Cos[(c + d*x)/2]] - 3*Cos[3*(c + d*x)]*Log[Sin[(c + d*x)/2]] + C
os[c + d*x]*(-2 - 3*Log[Cos[(c + d*x)/2]] + 3*Log[Sin[(c + d*x)/2]])))/(2*a
*d*(Csc[(c + d*x)/2]^2 - Sec[(c + d*x)/2]^2))
```

Maple [A] time = 0.077, size = 87, normalized size = 1.5

$$
\frac{1}{4da(-1+\cos(dx+c))}+\frac{3\ln(-1+\cos(dx+c))}{4da}+\frac{1}{4da(1+\cos(dx+c))}-\frac{3\ln(1+\cos(dx+c))}{4da}+\frac{1}{da\cos(dx+c}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(a-sin(d*x+c)^2*a),x)
```

```
[Out] 1/4/d/a/(-1+cos(d*x+c))+3/4/d/a*ln(-1+cos(d*x+c))+1/4/a/d/(1+cos(d*x+c))-3/
4/d/a*ln(1+cos(d*x+c))+1/d/a/cos(d*x+c)
```

Maxima [A] time = 0.968064, size = 95, normalized size = 1.64

$$
\frac{\frac{2\left(3\cos(dx+c)^2-2\right)}{a\cos(dx+c)^3-a\cos(dx+c)}-\frac{3\log(\cos(dx+c)+1)}{a}+\frac{3\log(\cos(dx+c)-1)}{a}}{4d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*(3*cos(d*x + c)^2 - 2)/(a*cos(d*x + c)^3 - a*cos(d*x + c)) - 3*log(c
os(d*x + c) + 1)/a + 3*log(cos(d*x + c) - 1)/a)/d
```

Fricas [A] time = 1.84009, size = 266, normalized size = 4.59

$$\frac{6\cos(dx+c)^2 - 3\left(\cos(dx+c)^3 - \cos(dx+c)\right)\log\left(\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right) + 3\left(\cos(dx+c)^3 - \cos(dx+c)\right)\log\left(-\frac{1}{2}\cos(\right.}{4\left(ad\cos(dx+c)^3 - ad\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(6*cos(d*x + c)^2 - 3*(cos(d*x + c)^3 - cos(d*x + c))*log(1/2*cos(d*x +
 c) + 1/2) + 3*(cos(d*x + c)^3 - cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2)
 - 4)/(a*d*cos(d*x + c)^3 - a*d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{\csc^3(c+dx)}{\sin^2(c+dx)-1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a-a*sin(d*x+c)**2),x)
```

```
[Out] -Integral(csc(c + d*x)**3/(sin(c + d*x)**2 - 1), x)/a
```

Giac [B] time = 1.19287, size = 201, normalized size = 3.47

$$\frac{\frac{6\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a} + \frac{\frac{14(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} + 1}{a\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1} + \frac{(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)} - \frac{\cos(dx+c)-1}{a(\cos(dx+c)+1)}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a-a*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/8*(6*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a + (14*(cos(d*x +
 c) - 1)/(cos(d*x + c) + 1) - 3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 +
 1)/(a*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + (cos(d*x + c) - 1)^2/(cos(d
*x + c) + 1)^2)) - (cos(d*x + c) - 1)/(a*(cos(d*x + c) + 1)))/d
```

3.41 $\int \frac{\csc^5(c+dx)}{a-a\sin^2(c+dx)}\,dx$

Optimal. Leaf size=82

$$\frac{15\sec(c+dx)}{8ad}-\frac{15\tanh^{-1}(\cos(c+dx))}{8ad}-\frac{\csc^4(c+dx)\sec(c+dx)}{4ad}-\frac{5\csc^2(c+dx)\sec(c+dx)}{8ad}$$

```
[Out] (-15*ArcTanh[Cos[c + d*x]])/(8*a*d) + (15*Sec[c + d*x])/(8*a*d) - (5*Csc[c
+ d*x]^2*Sec[c + d*x])/(8*a*d) - (Csc[c + d*x]^4*Sec[c + d*x])/(4*a*d)
```

Rubi [A] time = 0.0945551, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3175, 2622, 288, 321, 207}

$$\frac{15\sec(c+dx)}{8ad}-\frac{15\tanh^{-1}(\cos(c+dx))}{8ad}-\frac{\csc^4(c+dx)\sec(c+dx)}{4ad}-\frac{5\csc^2(c+dx)\sec(c+dx)}{8ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] (-15*ArcTanh[Cos[c + d*x]])/(8*a*d) + (15*Sec[c + d*x])/(8*a*d) - (5*Csc[c
+ d*x]^2*Sec[c + d*x])/(8*a*d) - (Csc[c + d*x]^4*Sec[c + d*x])/(4*a*d)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc^5(c+dx)}{a-a\sin^2(c+dx)}\,dx = \frac{\int \csc^5(c+dx)\sec^2(c+dx)\,dx}{a}$$
$$= \frac{\text{Subst}\left(\int \frac{x^6}{\left(-1+x^2\right)^3}\,dx, x, \sec(c+dx)\right)}{ad}$$
$$= -\frac{\csc^4(c+dx)\sec(c+dx)}{4ad} + \frac{5\,\text{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^2}\,dx, x, \sec(c+dx)\right)}{4ad}$$
$$= -\frac{5\csc^2(c+dx)\sec(c+dx)}{8ad} - \frac{\csc^4(c+dx)\sec(c+dx)}{4ad} + \frac{15\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{8ad}$$
$$= \frac{15\sec(c+dx)}{8ad} - \frac{5\csc^2(c+dx)\sec(c+dx)}{8ad} - \frac{\csc^4(c+dx)\sec(c+dx)}{4ad} + \frac{15\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,d\right.}{8ad}$$
$$= -\frac{15\tanh^{-1}(\cos(c+dx))}{8ad} + \frac{15\sec(c+dx)}{8ad} - \frac{5\csc^2(c+dx)\sec(c+dx)}{8ad} - \frac{\csc^4(c+dx)\sec(c+}{4ad}$$

Mathematica [A] time = 4.47197, size = 132, normalized size = 1.61

$$-\frac{\csc^4\left(\frac{1}{2}(c+dx)\right) + 14\csc^2\left(\frac{1}{2}(c+dx)\right) + \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\left(-14\tan^2\left(\frac{1}{2}(c+dx)\right)+\cos(c+dx)\left(\sec^4\left(\frac{1}{2}(c+dx)\right)-8\left(-15\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+15\log\left(\cos\right.\right.\right.\right.}{\tan^2\left(\frac{1}{2}(c+dx)\right)-1}}{64ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^5/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] -(14*Csc[(c + d*x)/2]^2 + Csc[(c + d*x)/2]^4 + (Sec[(c + d*x)/2]^2*(78 + Co
s[c + d*x]*(-8*(8 + 15*Log[Cos[(c + d*x)/2]] - 15*Log[Sin[(c + d*x)/2]]) +
Sec[(c + d*x)/2]^4) - 14*Tan[(c + d*x)/2]^2))/(-1 + Tan[(c + d*x)/2]^2))/(6
4*a*d)
```

Maple [A] time = 0.08, size = 123, normalized size = 1.5

$$-\frac{1}{16\,da\left(-1+\cos(dx+c)\right)^2} + \frac{7}{16\,da\left(-1+\cos(dx+c)\right)} + \frac{15\,\ln\left(-1+\cos(dx+c)\right)}{16\,da} + \frac{1}{16\,da\left(1+\cos(dx+c)\right)^2} + \frac{}{16\,da\,(1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5/(a-sin(d*x+c)^2*a),x)
```

```
[Out] -1/16/d/a/(-1+cos(d*x+c))^2+7/16/d/a/(-1+cos(d*x+c))+15/16/d/a*ln(-1+cos(d*
x+c))+1/16/a/d/(1+cos(d*x+c))^2+7/16/a/d/(1+cos(d*x+c))-15/16/d/a*ln(1+cos(
d*x+c))+1/d/a/cos(d*x+c)
```

Maxima [A] time = 0.963531, size = 122, normalized size = 1.49

$$\frac{\frac{2\left(15\cos(dx+c)^4-25\cos(dx+c)^2+8\right)}{a\cos(dx+c)^5-2a\cos(dx+c)^3+a\cos(dx+c)}-\frac{15\log(\cos(dx+c)+1)}{a}+\frac{15\log(\cos(dx+c)-1)}{a}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/16*(2*(15*cos(d*x + c)^4 - 25*cos(d*x + c)^2 + 8)/(a*cos(d*x + c)^5 - 2*a
*cos(d*x + c)^3 + a*cos(d*x + c)) - 15*log(cos(d*x + c) + 1)/a + 15*log(cos
(d*x + c) - 1)/a)/d
```

Fricas [A] time = 1.79891, size = 382, normalized size = 4.66

$$\frac{30\cos(dx+c)^4-50\cos(dx+c)^2-15\left(\cos(dx+c)^5-2\cos(dx+c)^3+\cos(dx+c)\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+15}{16\left(ad\cos(dx+c)^5-2ad\cos(dx+c)^3+ad\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/16*(30*cos(d*x + c)^4 - 50*cos(d*x + c)^2 - 15*(cos(d*x + c)^5 - 2*cos(d*
x + c)^3 + cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 15*(cos(d*x + c)^5 -
 2*cos(d*x + c)^3 + cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 16)/(a*d*c
os(d*x + c)^5 - 2*a*d*cos(d*x + c)^3 + a*d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5/(a-a*sin(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.20958, size = 244, normalized size = 2.98

$$\frac{\frac{\left(\frac{16(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{90(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)^2}{a(\cos(dx+c)-1)^2}+\frac{60\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a}-\frac{\frac{16a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{a^2}+\frac{128}{a\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)}}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a-a*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/64*((16*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 90*(cos(d*x + c) - 1)^2/(
cos(d*x + c) + 1)^2 - 1)*(cos(d*x + c) + 1)^2/(a*(cos(d*x + c) - 1)^2) + 60
```

```
*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a - (16*a*(cos(d*x + c)
- 1)/(cos(d*x + c) + 1) - a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/a^2
+ 128/(a*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)))/d
```

3.42 $\int \frac{\sin^6(c+dx)}{a-a\sin^2(c+dx)}\,dx$

Optimal. Leaf size=73

$$\frac{15\tan(c+dx)}{8ad}-\frac{\sin^4(c+dx)\tan(c+dx)}{4ad}-\frac{5\sin^2(c+dx)\tan(c+dx)}{8ad}-\frac{15x}{8a}$$

```
[Out] (-15*x)/(8*a) + (15*Tan[c + d*x])/(8*a*d) - (5*Sin[c + d*x]^2*Tan[c + d*x])
/(8*a*d) - (Sin[c + d*x]^4*Tan[c + d*x])/(4*a*d)
```

Rubi [A] time = 0.0902474, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3175, 2591, 288, 321, 203}

$$\frac{15\tan(c+dx)}{8ad}-\frac{\sin^4(c+dx)\tan(c+dx)}{4ad}-\frac{5\sin^2(c+dx)\tan(c+dx)}{8ad}-\frac{15x}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^6/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] (-15*x)/(8*a) + (15*Tan[c + d*x])/(8*a*d) - (5*Sin[c + d*x]^2*Tan[c + d*x])
/(8*a*d) - (Sin[c + d*x]^4*Tan[c + d*x])/(4*a*d)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^6(c+dx)}{a-a\sin^2(c+dx)}\,dx &= \frac{\int \sin^4(c+dx)\tan^2(c+dx)\,dx}{a} \\
&= \frac{\mathrm{Subst}\left(\int \frac{x^6}{(1+x^2)^3}\,dx, x, \tan(c+dx)\right)}{ad} \\
&= -\frac{\sin^4(c+dx)\tan(c+dx)}{4ad} + \frac{5\,\mathrm{Subst}\left(\int \frac{x^4}{(1+x^2)^2}\,dx, x, \tan(c+dx)\right)}{4ad} \\
&= -\frac{5\sin^2(c+dx)\tan(c+dx)}{8ad} - \frac{\sin^4(c+dx)\tan(c+dx)}{4ad} + \frac{15\,\mathrm{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, \tan(c+dx)\right)}{8ad} \\
&= \frac{15\tan(c+dx)}{8ad} - \frac{5\sin^2(c+dx)\tan(c+dx)}{8ad} - \frac{\sin^4(c+dx)\tan(c+dx)}{4ad} - \frac{15\,\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(c+dx)\right)}{8ad} \\
&= -\frac{15x}{8a} + \frac{15\tan(c+dx)}{8ad} - \frac{5\sin^2(c+dx)\tan(c+dx)}{8ad} - \frac{\sin^4(c+dx)\tan(c+dx)}{4ad}
\end{aligned}
$$

Mathematica [A] time = 0.189153, size = 44, normalized size = 0.6

$$
-\frac{-16\sin(2(c+dx))+\sin(4(c+dx))-32\tan(c+dx)+60c+60dx}{32ad}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^6/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] -(60*c + 60*d*x - 16*Sin[2*(c + d*x)] + Sin[4*(c + d*x)] - 32*Tan[c + d*x])
/(32*a*d)
```

Maple [A] time = 0.046, size = 84, normalized size = 1.2

$$
\frac{\tan(dx+c)}{da} + \frac{9\,(\tan(dx+c))^3}{8da\left((\tan(dx+c))^2+1\right)^2} + \frac{7\tan(dx+c)}{8da\left((\tan(dx+c))^2+1\right)^2} - \frac{15\arctan(\tan(dx+c))}{8da}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^6/(a-sin(d*x+c)^2*a),x)
```

```
[Out] tan(d*x+c)/d/a+9/8/d/a/(tan(d*x+c)^2+1)^2*tan(d*x+c)^3+7/8/d/a/(tan(d*x+c)^
2+1)^2*tan(d*x+c)-15/8/d/a*arctan(tan(d*x+c))
```

Maxima [A] time = 1.45398, size = 97, normalized size = 1.33

$$
\frac{\frac{9\tan(dx+c)^3+7\tan(dx+c)}{a\tan(dx+c)^4+2a\tan(dx+c)^2+a} - \frac{15(dx+c)}{a} + \frac{8\tan(dx+c)}{a}}{8d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/8*((9*tan(d*x + c)^3 + 7*tan(d*x + c))/(a*tan(d*x + c)^4 + 2*a*tan(d*x +
c)^2 + a) - 15*(d*x + c)/a + 8*tan(d*x + c)/a)/d
```

Fricas [A] time = 1.64148, size = 140, normalized size = 1.92

$$-\frac{15\,dx\cos\left(dx+c\right)+\left(2\,\cos\left(dx+c\right)^{4}-9\,\cos\left(dx+c\right)^{2}-8\right)\sin\left(dx+c\right)}{8\,ad\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/8*(15*d*x*cos(d*x + c) + (2*cos(d*x + c)^4 - 9*cos(d*x + c)^2 - 8)*sin(d
*x + c))/(a*d*cos(d*x + c))
```

Sympy [A] time = 107.4, size = 1161, normalized size = 15.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**6/(a-a*sin(d*x+c)**2),x)
```

```
[Out] Piecewise((-15*d*x*tan(c/2 + d*x/2)**10/(8*a*d*tan(c/2 + d*x/2)**10 + 24*a*
d*tan(c/2 + d*x/2)**8 + 16*a*d*tan(c/2 + d*x/2)**6 - 16*a*d*tan(c/2 + d*x/2
)**4 - 24*a*d*tan(c/2 + d*x/2)**2 - 8*a*d) - 45*d*x*tan(c/2 + d*x/2)**8/(8*
a*d*tan(c/2 + d*x/2)**10 + 24*a*d*tan(c/2 + d*x/2)**8 + 16*a*d*tan(c/2 + d*
x/2)**6 - 16*a*d*tan(c/2 + d*x/2)**4 - 24*a*d*tan(c/2 + d*x/2)**2 - 8*a*d)
- 30*d*x*tan(c/2 + d*x/2)**6/(8*a*d*tan(c/2 + d*x/2)**10 + 24*a*d*tan(c/2 +
 d*x/2)**8 + 16*a*d*tan(c/2 + d*x/2)**6 - 16*a*d*tan(c/2 + d*x/2)**4 - 24*a
*d*tan(c/2 + d*x/2)**2 - 8*a*d) + 30*d*x*tan(c/2 + d*x/2)**4/(8*a*d*tan(c/2
 + d*x/2)**10 + 24*a*d*tan(c/2 + d*x/2)**8 + 16*a*d*tan(c/2 + d*x/2)**6 - 1
6*a*d*tan(c/2 + d*x/2)**4 - 24*a*d*tan(c/2 + d*x/2)**2 - 8*a*d) + 45*d*x*ta
n(c/2 + d*x/2)**2/(8*a*d*tan(c/2 + d*x/2)**10 + 24*a*d*tan(c/2 + d*x/2)**8
+ 16*a*d*tan(c/2 + d*x/2)**6 - 16*a*d*tan(c/2 + d*x/2)**4 - 24*a*d*tan(c/2
+ d*x/2)**2 - 8*a*d) + 15*d*x/(8*a*d*tan(c/2 + d*x/2)**10 + 24*a*d*tan(c/2
+ d*x/2)**8 + 16*a*d*tan(c/2 + d*x/2)**6 - 16*a*d*tan(c/2 + d*x/2)**4 - 24*
a*d*tan(c/2 + d*x/2)**2 - 8*a*d) - 30*tan(c/2 + d*x/2)**9/(8*a*d*tan(c/2 +
d*x/2)**10 + 24*a*d*tan(c/2 + d*x/2)**8 + 16*a*d*tan(c/2 + d*x/2)**6 - 16*a
*d*tan(c/2 + d*x/2)**4 - 24*a*d*tan(c/2 + d*x/2)**2 - 8*a*d) - 80*tan(c/2 +
 d*x/2)**7/(8*a*d*tan(c/2 + d*x/2)**10 + 24*a*d*tan(c/2 + d*x/2)**8 + 16*a*
d*tan(c/2 + d*x/2)**6 - 16*a*d*tan(c/2 + d*x/2)**4 - 24*a*d*tan(c/2 + d*x/2
)**2 - 8*a*d) - 36*tan(c/2 + d*x/2)**5/(8*a*d*tan(c/2 + d*x/2)**10 + 24*a*d
*tan(c/2 + d*x/2)**8 + 16*a*d*tan(c/2 + d*x/2)**6 - 16*a*d*tan(c/2 + d*x/2)
**4 - 24*a*d*tan(c/2 + d*x/2)**2 - 8*a*d) - 80*tan(c/2 + d*x/2)**3/(8*a*d*t
an(c/2 + d*x/2)**10 + 24*a*d*tan(c/2 + d*x/2)**8 + 16*a*d*tan(c/2 + d*x/2)*
*6 - 16*a*d*tan(c/2 + d*x/2)**4 - 24*a*d*tan(c/2 + d*x/2)**2 - 8*a*d) - 30*
tan(c/2 + d*x/2)/(8*a*d*tan(c/2 + d*x/2)**10 + 24*a*d*tan(c/2 + d*x/2)**8 +
 16*a*d*tan(c/2 + d*x/2)**6 - 16*a*d*tan(c/2 + d*x/2)**4 - 24*a*d*tan(c/2 +
```

```
d*x/2)**2 - 8*a*d), Ne(d, 0)), (x*sin(c)**6/(-a*sin(c)**2 + a), True))
```

Giac [A] time = 1.14889, size = 85, normalized size = 1.16

$$-\frac{\frac{15\,(dx+c)}{a}-\frac{8\,\tan(dx+c)}{a}-\frac{9\,\tan(dx+c)^{3}+7\,\tan(dx+c)}{\left(\tan(dx+c)^{2}+1\right)^{2}a}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a-a*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/8*(15*(d*x + c)/a - 8*tan(d*x + c)/a - (9*tan(d*x + c)^3 + 7*tan(d*x + c
))/((tan(d*x + c)^2 + 1)^2*a))/d
```

3.43 $\int \frac{\sin^4(c+dx)}{a-a\sin^2(c+dx)}\,dx$

Optimal. Leaf size=49

$$\frac{3\tan(c+dx)}{2ad}-\frac{\sin^2(c+dx)\tan(c+dx)}{2ad}-\frac{3x}{2a}$$

```
[Out] (-3*x)/(2*a) + (3*Tan[c + d*x])/(2*a*d) - (Sin[c + d*x]^2*Tan[c + d*x])/(2*
a*d)
```

Rubi [A] time = 0.0815679, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3175, 2591, 288, 321, 203}

$$\frac{3\tan(c+dx)}{2ad}-\frac{\sin^2(c+dx)\tan(c+dx)}{2ad}-\frac{3x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] (-3*x)/(2*a) + (3*Tan[c + d*x])/(2*a*d) - (Sin[c + d*x]^2*Tan[c + d*x])/(2*
a*d)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^4(c+dx)}{a-a\sin^2(c+dx)}\,dx &= \frac{\int \sin^2(c+dx)\tan^2(c+dx)\,dx}{a}\\
&= \frac{\text{Subst}\left(\int \frac{x^4}{\left(1+x^2\right)^2}\,dx,x,\tan(c+dx)\right)}{ad}\\
&= -\frac{\sin^2(c+dx)\tan(c+dx)}{2ad}+\frac{3\,\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx,x,\tan(c+dx)\right)}{2ad}\\
&= \frac{3\tan(c+dx)}{2ad}-\frac{\sin^2(c+dx)\tan(c+dx)}{2ad}-\frac{3\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(c+dx)\right)}{2ad}\\
&= -\frac{3x}{2a}+\frac{3\tan(c+dx)}{2ad}-\frac{\sin^2(c+dx)\tan(c+dx)}{2ad}
\end{aligned}$$

Mathematica [A] time = 0.126989, size = 34, normalized size = 0.69

$$\frac{-6(c+dx)+\sin(2(c+dx))+4\tan(c+dx)}{4ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] (-6*(c + d*x) + Sin[2*(c + d*x)] + 4*Tan[c + d*x])/(4*a*d)
```

Maple [A] time = 0.041, size = 56, normalized size = 1.1

$$\frac{\tan(dx+c)}{da}+\frac{\tan(dx+c)}{2\,da\left((\tan(dx+c))^2+1\right)}-\frac{3\arctan(\tan(dx+c))}{2\,da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a-sin(d*x+c)^2*a),x)
```

```
[Out] tan(d*x+c)/d/a+1/2/d/a*tan(d*x+c)/(tan(d*x+c)^2+1)-3/2/d/a*arctan(tan(d*x+c
))
```

Maxima [A] time = 1.43283, size = 66, normalized size = 1.35

$$-\frac{\frac{3\,(dx+c)}{a}-\frac{\tan(dx+c)}{a\tan(dx+c)^2+a}-\frac{2\tan(dx+c)}{a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(3*(d*x + c)/a - tan(d*x + c)/(a*tan(d*x + c)^2 + a) - 2*tan(d*x + c)/
a)/d
```

Fricas [A] time = 1.59544, size = 111, normalized size = 2.27

$$-\frac{3\,dx\cos\left(dx+c\right)-\left(\cos\left(dx+c\right)^{2}+2\right)\sin\left(dx+c\right)}{2\,ad\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(3*d*x*cos(d*x + c) - (cos(d*x + c)^2 + 2)*sin(d*x + c))/(a*d*cos(d*x
+ c))
```

Sympy [A] time = 39.793, size = 502, normalized size = 10.24

$$\begin{cases} -\frac{3dx\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)-2ad\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)-2ad}-\frac{3dx\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)-2ad\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)-2ad}+\frac{3}{2ad\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad} \\ \frac{x\sin^{4}(c)}{-a\sin^{2}(c)+a} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a-a*sin(d*x+c)**2),x)
```

```
[Out] Piecewise((-3*d*x*tan(c/2 + d*x/2)**6/(2*a*d*tan(c/2 + d*x/2)**6 + 2*a*d*ta
n(c/2 + d*x/2)**4 - 2*a*d*tan(c/2 + d*x/2)**2 - 2*a*d) - 3*d*x*tan(c/2 + d*
x/2)**4/(2*a*d*tan(c/2 + d*x/2)**6 + 2*a*d*tan(c/2 + d*x/2)**4 - 2*a*d*tan(
c/2 + d*x/2)**2 - 2*a*d) + 3*d*x*tan(c/2 + d*x/2)**2/(2*a*d*tan(c/2 + d*x/2
)**6 + 2*a*d*tan(c/2 + d*x/2)**4 - 2*a*d*tan(c/2 + d*x/2)**2 - 2*a*d) + 3*d
*x/(2*a*d*tan(c/2 + d*x/2)**6 + 2*a*d*tan(c/2 + d*x/2)**4 - 2*a*d*tan(c/2 +
 d*x/2)**2 - 2*a*d) - 6*tan(c/2 + d*x/2)**5/(2*a*d*tan(c/2 + d*x/2)**6 + 2*
a*d*tan(c/2 + d*x/2)**4 - 2*a*d*tan(c/2 + d*x/2)**2 - 2*a*d) - 4*tan(c/2 +
d*x/2)**3/(2*a*d*tan(c/2 + d*x/2)**6 + 2*a*d*tan(c/2 + d*x/2)**4 - 2*a*d*ta
n(c/2 + d*x/2)**2 - 2*a*d) - 6*tan(c/2 + d*x/2)/(2*a*d*tan(c/2 + d*x/2)**6
+ 2*a*d*tan(c/2 + d*x/2)**4 - 2*a*d*tan(c/2 + d*x/2)**2 - 2*a*d), Ne(d, 0))
, (x*sin(c)**4/(-a*sin(c)**2 + a), True))
```

Giac [A] time = 1.16521, size = 68, normalized size = 1.39

$$-\frac{\frac{3\,(dx+c)}{a}-\frac{2\,\tan(dx+c)}{a}-\frac{\tan(dx+c)}{\left(\tan(dx+c)^{2}+1\right)a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a-a*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/2*(3*(d*x + c)/a - 2*tan(d*x + c)/a - tan(d*x + c)/((tan(d*x + c)^2 + 1)
*a))/d
```

3.44 $\int \frac{\sin^2(c+dx)}{a-a\sin^2(c+dx)}\,dx$

Optimal. Leaf size=20

$$\frac{\tan(c+dx)}{ad}-\frac{x}{a}$$

```
[Out] -(x/a) + Tan[c + d*x]/(a*d)
```

Rubi [A] time = 0.0631069, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3171, 3175, 3767, 8}

$$\frac{\tan(c+dx)}{ad}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] -(x/a) + Tan[c + d*x]/(a*d)
```

Rule 3171

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(B*x)/b, x] + Dist[(A*b - a*B)/b, Int[1/(a + b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^2(c+dx)}{a-a\sin^2(c+dx)}\,dx &= -\frac{x}{a}+\int \frac{1}{a-a\sin^2(c+dx)}\,dx\\
&= -\frac{x}{a}+\frac{\int \sec^2(c+dx)\,dx}{a}\\
&= -\frac{x}{a}-\frac{\text{Subst}\left(\int 1\,dx,x,-\tan(c+dx)\right)}{ad}\\
&= -\frac{x}{a}+\frac{\tan(c+dx)}{ad}
\end{aligned}$$

Mathematica [A] time = 0.0136556, size = 27, normalized size = 1.35

$$\frac{\frac{\tan(c+dx)}{d}-\frac{\tan^{-1}(\tan(c+dx))}{d}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] (-(ArcTan[Tan[c + d*x]]/d) + Tan[c + d*x]/d)/a
```

Maple [A] time = 0.038, size = 30, normalized size = 1.5

$$\frac{\tan(dx+c)}{da}-\frac{\arctan(\tan(dx+c))}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a-sin(d*x+c)^2*a),x)
```

```
[Out] tan(d*x+c)/d/a-1/d/a*arctan(tan(d*x+c))
```

Maxima [A] time = 1.42814, size = 35, normalized size = 1.75

$$-\frac{\frac{dx+c}{a}-\frac{\tan(dx+c)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -((d*x + c)/a - tan(d*x + c)/a)/d
```

Fricas [A] time = 1.61839, size = 74, normalized size = 3.7

$$-\frac{dx\cos(dx+c)-\sin(dx+c)}{ad\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -(d*x*cos(d*x + c) - sin(d*x + c))/(a*d*cos(d*x + c))
```

Sympy [A] time = 7.73582, size = 100, normalized size = 5.

$$\begin{cases} -\frac{dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-ad}+\frac{dx}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-ad}-\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-ad} & \text{for } d\neq 0 \\ \frac{x\sin^2(c)}{-a\sin^2(c)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a-a*sin(d*x+c)**2),x)
```

```
[Out] Piecewise((-d*x*tan(c/2 + d*x/2)**2/(a*d*tan(c/2 + d*x/2)**2 - a*d) + d*x/(
a*d*tan(c/2 + d*x/2)**2 - a*d) - 2*tan(c/2 + d*x/2)/(a*d*tan(c/2 + d*x/2)**
2 - a*d), Ne(d, 0)), (x*sin(c)**2/(-a*sin(c)**2 + a), True))
```

Giac [A] time = 1.17535, size = 35, normalized size = 1.75

$$-\frac{\frac{dx+c}{a}-\frac{\tan(dx+c)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a-a*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -((d*x + c)/a - tan(d*x + c)/a)/d
```

3.45 $\int \frac{1}{a-a\sin^2(c+dx)}\,dx$

Optimal. Leaf size=13

$$\frac{\tan(c+dx)}{ad}$$

```
[Out] Tan[c + d*x]/(a*d)
```

Rubi [A] time = 0.0224128, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3175, 3767, 8}

$$\frac{\tan(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[c + d*x]^2)^(-1),x]
```

```
[Out] Tan[c + d*x]/(a*d)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a-a\sin^2(c+dx)}\,dx &= \frac{\int \sec^2(c+dx)\,dx}{a}\\ &= -\frac{\text{Subst}\left(\int 1\,dx, x, -\tan(c+dx)\right)}{ad}\\ &= \frac{\tan(c+dx)}{ad}\end{aligned}$$

Mathematica [A] time = 0.00612, size = 13, normalized size = 1.

$$\frac{\tan(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sin[c + d*x]^2)^(-1),x]
```

```
[Out] Tan[c + d*x]/(a*d)
```

Maple [A] time = 0.039, size = 14, normalized size = 1.1

$$\frac{\tan(dx+c)}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-sin(d*x+c)^2*a),x)
```

```
[Out] tan(d*x+c)/d/a
```

Maxima [A] time = 0.947621, size = 18, normalized size = 1.38

$$\frac{\tan(dx+c)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] tan(d*x + c)/(a*d)
```

Fricas [A] time = 1.59973, size = 45, normalized size = 3.46

$$\frac{\sin(dx+c)}{ad\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] sin(d*x + c)/(a*d*cos(d*x + c))
```

Sympy [A] time = 2.67555, size = 41, normalized size = 3.15

$$\begin{cases} -\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-ad} & \text{for } d\neq 0 \\ \frac{x}{-a\sin^2(c)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(d*x+c)**2),x)
```

```
[Out] Piecewise((-2*tan(c/2 + d*x/2)/(a*d*tan(c/2 + d*x/2)**2 - a*d), Ne(d, 0)),
(x/(-a*sin(c)**2 + a), True))
```

Giac [A] time = 1.12077, size = 18, normalized size = 1.38

$$\frac{\tan(dx + c)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] tan(d*x + c)/(a*d)
```

3.46 $\int \frac{\csc^2(c+dx)}{a-a\sin^2(c+dx)}\,dx$

Optimal. Leaf size=28

$$\frac{\tan(c+dx)}{ad}-\frac{\cot(c+dx)}{ad}$$

```
[Out] -(Cot[c + d*x]/(a*d)) + Tan[c + d*x]/(a*d)
```

Rubi [A] time = 0.07482, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 2620, 14}

$$\frac{\tan(c+dx)}{ad}-\frac{\cot(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] -(Cot[c + d*x]/(a*d)) + Tan[c + d*x]/(a*d)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^2(c+dx)}{a-a\sin^2(c+dx)}\,dx &= \frac{\int \csc^2(c+dx)\sec^2(c+dx)\,dx}{a}\\
&= \frac{\operatorname{Subst}\left(\int \frac{1+x^2}{x^2}\,dx, x, \tan(c+dx)\right)}{ad}\\
&= \frac{\operatorname{Subst}\left(\int \left(1+\frac{1}{x^2}\right)dx, x, \tan(c+dx)\right)}{ad}\\
&= -\frac{\cot(c+dx)}{ad}+\frac{\tan(c+dx)}{ad}
\end{aligned}$$

Mathematica [A] time = 0.0270954, size = 16, normalized size = 0.57

$$-\frac{2\cot(2(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] (-2*Cot[2*(c + d*x)])/(a*d)
```

Maple [A] time = 0.06, size = 25, normalized size = 0.9

$$\frac{1}{da}\left(\tan(dx+c)-(\tan(dx+c))^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a-sin(d*x+c)^2*a),x)
```

```
[Out] 1/d/a*(tan(d*x+c)-1/tan(d*x+c))
```

Maxima [A] time = 0.956545, size = 38, normalized size = 1.36

$$\frac{\frac{\tan(dx+c)}{a}-\frac{1}{a\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] (tan(d*x + c)/a - 1/(a*tan(d*x + c)))/d
```

Fricas [A] time = 1.58146, size = 77, normalized size = 2.75

$$-\frac{2\cos(dx+c)^2-1}{ad\cos(dx+c)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -(2*cos(d*x + c)^2 - 1)/(a*d*cos(d*x + c)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int\frac{\csc^2(c+dx)}{\sin^2(c+dx)-1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a-a*sin(d*x+c)**2),x)
```

```
[Out] -Integral(csc(c + d*x)**2/(sin(c + d*x)**2 - 1), x)/a
```

Giac [A] time = 1.15326, size = 26, normalized size = 0.93

$$-\frac{2}{ad\tan\left(2\,dx+2\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a-a*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -2/(a*d*tan(2*d*x + 2*c))
```

3.47 $\int \frac{\csc^4(c+dx)}{a-a\sin^2(c+dx)}\,dx$

Optimal. Leaf size=46

$$\frac{\tan(c+dx)}{ad}-\frac{\cot^3(c+dx)}{3ad}-\frac{2\cot(c+dx)}{ad}$$

```
[Out] (-2*Cot[c + d*x])/(a*d) - Cot[c + d*x]^3/(3*a*d) + Tan[c + d*x]/(a*d)
```

Rubi [A] time = 0.0801195, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 2620, 270}

$$\frac{\tan(c+dx)}{ad}-\frac{\cot^3(c+dx)}{3ad}-\frac{2\cot(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] (-2*Cot[c + d*x])/(a*d) - Cot[c + d*x]^3/(3*a*d) + Tan[c + d*x]/(a*d)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^4(c+dx)}{a-a\sin^2(c+dx)}\,dx &= \frac{\int \csc^4(c+dx)\sec^2(c+dx)\,dx}{a}\\
&= \frac{\operatorname{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^4}\,dx,x,\tan(c+dx)\right)}{ad}\\
&= \frac{\operatorname{Subst}\left(\int \left(1+\frac{1}{x^4}+\frac{2}{x^2}\right)dx,x,\tan(c+dx)\right)}{ad}\\
&= -\frac{2\cot(c+dx)}{ad}-\frac{\cot^3(c+dx)}{3ad}+\frac{\tan(c+dx)}{ad}
\end{aligned}$$

Mathematica [A] time = 0.0430301, size = 49, normalized size = 1.07

$$\frac{\frac{\tan(c+dx)}{d}-\frac{5\cot(c+dx)}{3d}-\frac{\cot(c+dx)\csc^2(c+dx)}{3d}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] ((-5*Cot[c + d*x])/(3*d) - (Cot[c + d*x]*Csc[c + d*x]^2)/(3*d) + Tan[c + d*
x]/d)/a
```

Maple [A] time = 0.075, size = 35, normalized size = 0.8

$$\frac{1}{da}\left(\tan(dx+c)-2\,(\tan(dx+c))^{-1}-\frac{1}{3\,(\tan(dx+c))^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4/(a-sin(d*x+c)^2*a),x)
```

```
[Out] 1/d/a*(tan(d*x+c)-2/tan(d*x+c)-1/3/tan(d*x+c)^3)
```

Maxima [A] time = 0.955211, size = 57, normalized size = 1.24

$$\frac{\frac{3\tan(dx+c)}{a}-\frac{6\tan(dx+c)^2+1}{a\tan(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*tan(d*x + c)/a - (6*tan(d*x + c)^2 + 1)/(a*tan(d*x + c)^3))/d
```

Fricas [A] time = 1.62205, size = 140, normalized size = 3.04

$$-\frac{8\cos(dx+c)^4-12\cos(dx+c)^2+3}{3\left(ad\cos(dx+c)^3-ad\cos(dx+c)\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/3*(8*cos(d*x + c)^4 - 12*cos(d*x + c)^2 + 3)/((a*d*cos(d*x + c)^3 - a*d*
cos(d*x + c))*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{\csc^4(c+dx)}{\sin^2(c+dx)-1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4/(a-a*sin(d*x+c)**2),x)
```

```
[Out] -Integral(csc(c + d*x)**4/(sin(c + d*x)**2 - 1), x)/a
```

Giac [A] time = 1.1818, size = 57, normalized size = 1.24

$$\frac{\frac{3\tan(dx+c)}{a} - \frac{6\tan(dx+c)^2+1}{a\tan(dx+c)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a-a*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*tan(d*x + c)/a - (6*tan(d*x + c)^2 + 1)/(a*tan(d*x + c)^3))/d
```

3.48 $\int \frac{\csc^6(c+dx)}{a-a\sin^2(c+dx)}\,dx$

Optimal. Leaf size=62

$$\frac{\tan(c+dx)}{ad}-\frac{\cot^5(c+dx)}{5ad}-\frac{\cot^3(c+dx)}{ad}-\frac{3\cot(c+dx)}{ad}$$

```
[Out] (-3*Cot[c + d*x])/(a*d) - Cot[c + d*x]^3/(a*d) - Cot[c + d*x]^5/(5*a*d) + T
an[c + d*x]/(a*d)
```

Rubi [A] time = 0.0844312, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 2620, 270}

$$\frac{\tan(c+dx)}{ad}-\frac{\cot^5(c+dx)}{5ad}-\frac{\cot^3(c+dx)}{ad}-\frac{3\cot(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] (-3*Cot[c + d*x])/(a*d) - Cot[c + d*x]^3/(a*d) - Cot[c + d*x]^5/(5*a*d) + T
an[c + d*x]/(a*d)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^6(c+dx)}{a-a\sin^2(c+dx)}\,dx &= \frac{\int \csc^6(c+dx)\sec^2(c+dx)\,dx}{a}\\
&= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{x^6}\,dx,x,\tan(c+dx)\right)}{ad}\\
&= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^6}+\frac{3}{x^4}+\frac{3}{x^2}\right)dx,x,\tan(c+dx)\right)}{ad}\\
&= -\frac{3\cot(c+dx)}{ad}-\frac{\cot^3(c+dx)}{ad}-\frac{\cot^5(c+dx)}{5ad}+\frac{\tan(c+dx)}{ad}
\end{aligned}$$

Mathematica [A] time = 0.0367788, size = 70, normalized size = 1.13

$$\frac{\frac{\tan(c+dx)}{d}-\frac{11\cot(c+dx)}{5d}-\frac{\cot(c+dx)\csc^4(c+dx)}{5d}-\frac{3\cot(c+dx)\csc^2(c+dx)}{5d}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] ((-11*Cot[c + d*x])/(5*d) - (3*Cot[c + d*x]*Csc[c + d*x]^2)/(5*d) - (Cot[c
+ d*x]*Csc[c + d*x]^4)/(5*d) + Tan[c + d*x]/d)/a
```

Maple [A] time = 0.078, size = 45, normalized size = 0.7

$$\frac{1}{da}\left(\tan(dx+c)-3\,(\tan(dx+c))^{-1}-\frac{1}{5\,(\tan(dx+c))^5}-(\tan(dx+c))^{-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6/(a-sin(d*x+c)^2*a),x)
```

```
[Out] 1/d/a*(tan(d*x+c)-3/tan(d*x+c)-1/5/tan(d*x+c)^5-1/tan(d*x+c)^3)
```

Maxima [A] time = 0.961835, size = 70, normalized size = 1.13

$$\frac{\frac{5\tan(dx+c)}{a}-\frac{15\tan(dx+c)^4+5\tan(dx+c)^2+1}{a\tan(dx+c)^5}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/5*(5*tan(d*x + c)/a - (15*tan(d*x + c)^4 + 5*tan(d*x + c)^2 + 1)/(a*tan(d
*x + c)^5))/d
```

Fricas [A] time = 1.57284, size = 200, normalized size = 3.23

$$-\frac{16\cos(dx+c)^6-40\cos(dx+c)^4+30\cos(dx+c)^2-5}{5\left(ad\cos(dx+c)^5-2\,ad\cos(dx+c)^3+ad\cos(dx+c)\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/5*(16*cos(d*x + c)^6 - 40*cos(d*x + c)^4 + 30*cos(d*x + c)^2 - 5)/((a*d*
cos(d*x + c)^5 - 2*a*d*cos(d*x + c)^3 + a*d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6/(a-a*sin(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22769, size = 70, normalized size = 1.13

$$\frac{\frac{5\tan(dx+c)}{a} - \frac{15\tan(dx+c)^4+5\tan(dx+c)^2+1}{a\tan(dx+c)^5}}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a-a*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/5*(5*tan(d*x + c)/a - (15*tan(d*x + c)^4 + 5*tan(d*x + c)^2 + 1)/(a*tan(d
*x + c)^5))/d
```

3.49 $\int \frac{\sin^7(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=65

$$\frac{\cos^3(c+dx)}{3a^2d}-\frac{3\cos(c+dx)}{a^2d}+\frac{\sec^3(c+dx)}{3a^2d}-\frac{3\sec(c+dx)}{a^2d}$$

```
[Out] (-3*Cos[c + d*x])/(a^2*d) + Cos[c + d*x]^3/(3*a^2*d) - (3*Sec[c + d*x])/(a^
2*d) + Sec[c + d*x]^3/(3*a^2*d)
```

Rubi [A] time = 0.0837538, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 2590, 270}

$$\frac{\cos^3(c+dx)}{3a^2d}-\frac{3\cos(c+dx)}{a^2d}+\frac{\sec^3(c+dx)}{3a^2d}-\frac{3\sec(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^7/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] (-3*Cos[c + d*x])/(a^2*d) + Cos[c + d*x]^3/(3*a^2*d) - (3*Sec[c + d*x])/(a^
2*d) + Sec[c + d*x]^3/(3*a^2*d)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^7(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx &= \frac{\int \sin^3(c+dx)\tan^4(c+dx)\,dx}{a^2}\\
&= -\frac{\operatorname{Subst}\left(\int \frac{\left(1-x^2\right)^3}{x^4}\,dx,x,\cos(c+dx)\right)}{a^2d}\\
&= -\frac{\operatorname{Subst}\left(\int \left(3+\frac{1}{x^4}-\frac{3}{x^2}-x^2\right)dx,x,\cos(c+dx)\right)}{a^2d}\\
&= -\frac{3\cos(c+dx)}{a^2d}+\frac{\cos^3(c+dx)}{3a^2d}-\frac{3\sec(c+dx)}{a^2d}+\frac{\sec^3(c+dx)}{3a^2d}
\end{aligned}$$

Mathematica [A] time = 0.0504638, size = 59, normalized size = 0.91

$$\frac{-\frac{11\cos(c+dx)}{4d}+\frac{\cos(3(c+dx))}{12d}+\frac{\sec^3(c+dx)}{3d}-\frac{3\sec(c+dx)}{d}}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^7/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] ((-11*Cos[c + d*x])/(4*d) + Cos[3*(c + d*x)]/(12*d) - (3*Sec[c + d*x])/d +
Sec[c + d*x]^3/(3*d))/a^2
```

Maple [A] time = 0.043, size = 47, normalized size = 0.7

$$\frac{1}{a^2d}\left(\frac{(\cos(dx+c))^3}{3}-3\cos(dx+c)-3(\cos(dx+c))^{-1}+\frac{1}{3(\cos(dx+c))^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^7/(a-sin(d*x+c)^2*a)^2,x)
```

```
[Out] 1/d/a^2*(1/3*cos(d*x+c)^3-3*cos(d*x+c)-3/cos(d*x+c)+1/3/cos(d*x+c)^3)
```

Maxima [A] time = 0.945454, size = 70, normalized size = 1.08

$$\frac{\frac{\cos(dx+c)^3-9\cos(dx+c)}{a^2}-\frac{9\cos(dx+c)^2-1}{a^2\cos(dx+c)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a-a*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*((cos(d*x + c)^3 - 9*cos(d*x + c))/a^2 - (9*cos(d*x + c)^2 - 1)/(a^2*co
s(d*x + c)^3))/d
```

Fricas [A] time = 1.62859, size = 117, normalized size = 1.8

$$\frac{\cos(dx+c)^6-9\cos(dx+c)^4-9\cos(dx+c)^2+1}{3a^2d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a-a*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(cos(d*x + c)^6 - 9*cos(d*x + c)^4 - 9*cos(d*x + c)^2 + 1)/(a^2*d*cos(d
*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**7/(a-a*sin(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16131, size = 77, normalized size = 1.18

$$-\frac{32\left(\frac{3\,(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-1\right)}{3\,a^2d\left(\frac{(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a-a*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -32/3*(3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 1)/(a^2*d*((cos(d*x +
c) - 1)^2/(cos(d*x + c) + 1)^2 - 1)^3)
```

3.50 $\int \frac{\sin^5(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=47

$$-\frac{\cos(c+dx)}{a^2d}+\frac{\sec^3(c+dx)}{3a^2d}-\frac{2\sec(c+dx)}{a^2d}$$

```
[Out] -(Cos[c + d*x]/(a^2*d)) - (2*Sec[c + d*x])/(a^2*d) + Sec[c + d*x]^3/(3*a^2*
d)
```

Rubi [A] time = 0.0694904, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 2590, 270}

$$-\frac{\cos(c+dx)}{a^2d}+\frac{\sec^3(c+dx)}{3a^2d}-\frac{2\sec(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^5/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] -(Cos[c + d*x]/(a^2*d)) - (2*Sec[c + d*x])/(a^2*d) + Sec[c + d*x]^3/(3*a^2*
d)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^5(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx &= \frac{\int \sin(c+dx)\tan^4(c+dx)\,dx}{a^2}\\
&= -\frac{\operatorname{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x^4}\,dx,x,\cos(c+dx)\right)}{a^2d}\\
&= -\frac{\operatorname{Subst}\left(\int \left(1+\frac{1}{x^4}-\frac{2}{x^2}\right)dx,x,\cos(c+dx)\right)}{a^2d}\\
&= -\frac{\cos(c+dx)}{a^2d}-\frac{2\sec(c+dx)}{a^2d}+\frac{\sec^3(c+dx)}{3a^2d}
\end{aligned}$$

Mathematica [A] time = 0.0367936, size = 42, normalized size = 0.89

$$\frac{-\frac{\cos(c+dx)}{d}+\frac{\sec^3(c+dx)}{3d}-\frac{2\sec(c+dx)}{d}}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^5/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] (-(Cos[c + d*x]/d) - (2*Sec[c + d*x])/d + Sec[c + d*x]^3/(3*d))/a^2
```

Maple [A] time = 0.042, size = 37, normalized size = 0.8

$$\frac{1}{a^2 d}\left(-\cos(dx+c)-2(\cos(dx+c))^{-1}+\frac{1}{3(\cos(dx+c))^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5/(a-sin(d*x+c)^2*a)^2,x)
```

```
[Out] 1/d/a^2*(-cos(d*x+c)-2/cos(d*x+c)+1/3/cos(d*x+c)^3)
```

Maxima [A] time = 0.950131, size = 55, normalized size = 1.17

$$-\frac{\frac{3\cos(dx+c)}{a^2}+\frac{6\cos(dx+c)^2-1}{a^2\cos(dx+c)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a-a*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(3*cos(d*x + c)/a^2 + (6*cos(d*x + c)^2 - 1)/(a^2*cos(d*x + c)^3))/d
```

Fricas [A] time = 1.6276, size = 96, normalized size = 2.04

$$-\frac{3\cos(dx+c)^4+6\cos(dx+c)^2-1}{3a^2 d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a-a*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*cos(d*x + c)^4 + 6*cos(d*x + c)^2 - 1)/(a^2*d*cos(d*x + c)^3)
```

Sympy [A] time = 93.1717, size = 156, normalized size = 3.32

$$\begin{cases} -\frac{32\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^2d\tan^8\left(\frac{c}{2}+\frac{dx}{2}\right)-6a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-3a^2d}+\frac{16}{3a^2d\tan^8\left(\frac{c}{2}+\frac{dx}{2}\right)-6a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-3a^2d} & \text{for } d\neq 0 \\ \frac{x\sin^5(c)}{\left(-a\sin^2(c)+a\right)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5/(a-a*sin(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((-32*tan(c/2 + d*x/2)**2/(3*a**2*d*tan(c/2 + d*x/2)**8 - 6*a**2*d
*tan(c/2 + d*x/2)**6 + 6*a**2*d*tan(c/2 + d*x/2)**2 - 3*a**2*d) + 16/(3*a**
2*d*tan(c/2 + d*x/2)**8 - 6*a**2*d*tan(c/2 + d*x/2)**6 + 6*a**2*d*tan(c/2 +
 d*x/2)**2 - 3*a**2*d), Ne(d, 0)), (x*sin(c)**5/(-a*sin(c)**2 + a)**2, True
))
```

Giac [B] time = 1.17277, size = 143, normalized size = 3.04

$$\frac{2\left(\frac{3}{a^2\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)}-\frac{\frac{12(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+5}{a^2\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^3}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a-a*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 2/3*(3/(a^2*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - (12*(cos(d*x + c
) - 1)/(cos(d*x + c) + 1) + 3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 5
)/(a^2*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^3))/d
```

3.51 $\int \frac{\sin^3(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2} dx$

Optimal. Leaf size=33

$$\frac{\sec^3(c+dx)}{3a^2d} - \frac{\sec(c+dx)}{a^2d}$$

```
[Out] -(Sec[c + d*x]/(a^2*d)) + Sec[c + d*x]^3/(3*a^2*d)
```

Rubi [A] time = 0.0615918, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3175, 2606}

$$\frac{\sec^3(c+dx)}{3a^2d} - \frac{\sec(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a - a*Sin[c + d*x]^2)^2,x]

[Out] -(Sec[c + d*x]/(a^2*d)) + Sec[c + d*x]^3/(3*a^2*d)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^3(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2} dx &= \frac{\int \sec(c+dx)\tan^3(c+dx)\,dx}{a^2} \\
&= \frac{\text{Subst}\left(\int \left(-1+x^2\right) dx, x, \sec(c+dx)\right)}{a^2d} \\
&= -\frac{\sec(c+dx)}{a^2d} + \frac{\sec^3(c+dx)}{3a^2d}
\end{aligned}$$

Mathematica [A] time = 0.0296034, size = 31, normalized size = 0.94

$$\frac{\frac{\sec^3(c+dx)}{3d} - \frac{\sec(c+dx)}{d}}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] (-(Sec[c + d*x]/d) + Sec[c + d*x]^3/(3*d))/a^2
```

Maple [A] time = 0.04, size = 29, normalized size = 0.9

$$\frac{1}{a^2 d}\left(-\left(\cos\left(dx+c\right)\right)^{-1}+\frac{1}{3\left(\cos\left(dx+c\right)\right)^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a-sin(d*x+c)^2*a)^2,x)
```

```
[Out] 1/d/a^2*(-1/cos(d*x+c)+1/3/cos(d*x+c)^3)
```

Maxima [A] time = 0.972657, size = 38, normalized size = 1.15

$$-\frac{3\cos\left(dx+c\right)^{2}-1}{3\,a^{2}d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a-a*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(3*cos(d*x + c)^2 - 1)/(a^2*d*cos(d*x + c)^3)
```

Fricas [A] time = 1.53299, size = 70, normalized size = 2.12

$$-\frac{3\cos\left(dx+c\right)^{2}-1}{3\,a^{2}d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a-a*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*cos(d*x + c)^2 - 1)/(a^2*d*cos(d*x + c)^3)
```

Sympy [A] time = 25.345, size = 309, normalized size = 9.36

$$\begin{cases} -\frac{4\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^{2}d\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)-9a^{2}d\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^{2}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)-3a^{2}d}+\frac{12\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^{2}d\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)-9a^{2}d\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^{2}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)-3a^{2}d}-\frac{\quad}{3a^{2}d\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right) \\ \frac{x\sin^{3}\left(c\right)}{\left(-a\sin^{2}\left(c\right)+a\right)^{2}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a-a*sin(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((-4*tan(c/2 + d*x/2)**6/(3*a**2*d*tan(c/2 + d*x/2)**6 - 9*a**2*d*
tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 - 3*a**2*d) + 12*tan(c/2
 + d*x/2)**4/(3*a**2*d*tan(c/2 + d*x/2)**6 - 9*a**2*d*tan(c/2 + d*x/2)**4 +
 9*a**2*d*tan(c/2 + d*x/2)**2 - 3*a**2*d) - 24*tan(c/2 + d*x/2)**2/(3*a**2*
d*tan(c/2 + d*x/2)**6 - 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d
*x/2)**2 - 3*a**2*d) + 8/(3*a**2*d*tan(c/2 + d*x/2)**6 - 9*a**2*d*tan(c/2 +
 d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 - 3*a**2*d), Ne(d, 0)), (x*sin(c)
**3/(-a*sin(c)**2 + a)**2, True))
```

Giac [A] time = 1.15713, size = 38, normalized size = 1.15

$$-\frac{3 \cos(dx+c)^2 - 1}{3\, a^2 d \cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a-a*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/3*(3*cos(d*x + c)^2 - 1)/(a^2*d*cos(d*x + c)^3)
```

3.52 $\int \frac{\sin(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=18

$$\frac{\sec^3(c+dx)}{3a^2d}$$

```
[Out] Sec[c + d*x]^3/(3*a^2*d)
```

Rubi [A] time = 0.0421728, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3175, 2606, 30}

$$\frac{\sec^3(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] Sec[c + d*x]^3/(3*a^2*d)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx &= \frac{\int \sec^3(c+dx)\tan(c+dx)\,dx}{a^2}\\ &= \frac{\text{Subst}\left(\int x^2\,dx, x, \sec(c+dx)\right)}{a^2d}\\ &= \frac{\sec^3(c+dx)}{3a^2d}\end{aligned}$$

Mathematica [A] time = 0.0111101, size = 18, normalized size = 1.

$$\frac{\sec^3(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] Sec[c + d*x]^3/(3*a^2*d)
```

Maple [A] time = 0.032, size = 17, normalized size = 0.9

$$\frac{1}{3\,a^2d\left(\cos\left(dx+c\right)\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a-sin(d*x+c)^2*a)^2,x)
```

```
[Out] 1/3/d/a^2/cos(d*x+c)^3
```

Maxima [A] time = 0.952217, size = 22, normalized size = 1.22

$$\frac{1}{3\,a^2d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-a*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3/(a^2*d*cos(d*x + c)^3)
```

Fricas [A] time = 1.59587, size = 38, normalized size = 2.11

$$\frac{1}{3\,a^2d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-a*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3/(a^2*d*cos(d*x + c)^3)
```

Sympy [A] time = 19.495, size = 156, normalized size = 8.67

$$\begin{cases} -\frac{6\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)-9a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-3a^2d} - \frac{2}{3a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)-9a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-3a^2d} & \text{for } d\neq 0 \\ \frac{x\sin\left(c\right)}{\left(-a\sin^2\left(c\right)+a\right)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-a*sin(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((-6*tan(c/2 + d*x/2)**4/(3*a**2*d*tan(c/2 + d*x/2)**6 - 9*a**2*d*
tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 - 3*a**2*d) - 2/(3*a**2*
d*tan(c/2 + d*x/2)**6 - 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d
*x/2)**2 - 3*a**2*d), Ne(d, 0)), (x*sin(c)/(-a*sin(c)**2 + a)**2, True))
```

Giac [A] time = 1.15784, size = 22, normalized size = 1.22

$$\frac{1}{3\,a^2 d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-a*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3/(a^2*d*cos(d*x + c)^3)
```

3.53 $\int \frac{\csc(c+dx)}{(a-a\sin^2(c+dx))^2}\,dx$

Optimal. Leaf size=47

$$\frac{\sec^3(c+dx)}{3a^2d}+\frac{\sec(c+dx)}{a^2d}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}$$

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a^2*d)) + Sec[c + d*x]/(a^2*d) + Sec[c + d*x]^3/(3
*a^2*d)
```

Rubi [A] time = 0.062176, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3175, 2622, 302, 207}

$$\frac{\sec^3(c+dx)}{3a^2d}+\frac{\sec(c+dx)}{a^2d}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a^2*d)) + Sec[c + d*x]/(a^2*d) + Sec[c + d*x]^3/(3
*a^2*d)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx = \frac{\int \csc(c+dx)\sec^4(c+dx)\,dx}{a^2}$$
$$= \frac{\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{a^2 d}$$
$$= \frac{\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)dx, x, \sec(c+dx)\right)}{a^2 d}$$
$$= \frac{\sec(c+dx)}{a^2 d} + \frac{\sec^3(c+dx)}{3a^2 d} + \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{a^2 d}$$
$$= -\frac{\tanh^{-1}(\cos(c+dx))}{a^2 d} + \frac{\sec(c+dx)}{a^2 d} + \frac{\sec^3(c+dx)}{3a^2 d}$$

Mathematica [A] time = 0.0411074, size = 61, normalized size = 1.3

$$\frac{\frac{\sec^3(c+dx)}{3d} + \frac{\sec(c+dx)}{d} + \frac{\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d}}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] (-(Log[Cos[(c + d*x)/2]]/d) + Log[Sin[(c + d*x)/2]]/d + Sec[c + d*x]/d + Se
c[c + d*x]^3/(3*d))/a^2
```

Maple [A] time = 0.065, size = 67, normalized size = 1.4

$$\frac{\ln(-1+\cos(dx+c))}{2a^2 d} - \frac{\ln(1+\cos(dx+c))}{2a^2 d} + \frac{1}{3a^2 d(\cos(dx+c))^3} + \frac{1}{a^2 d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a-sin(d*x+c)^2*a)^2,x)
```

```
[Out] 1/2/d/a^2*ln(-1+cos(d*x+c))-1/2/d/a^2*ln(1+cos(d*x+c))+1/3/d/a^2/cos(d*x+c)
^3+1/d/a^2/cos(d*x+c)
```

Maxima [A] time = 0.950793, size = 80, normalized size = 1.7

$$-\frac{\frac{3\log(\cos(dx+c)+1)}{a^2} - \frac{3\log(\cos(dx+c)-1)}{a^2} - \frac{2\left(3\cos(dx+c)^2+1\right)}{a^2\cos(dx+c)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-a*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(3*log(cos(d*x + c) + 1)/a^2 - 3*log(cos(d*x + c) - 1)/a^2 - 2*(3*cos(
d*x + c)^2 + 1)/(a^2*cos(d*x + c)^3))/d
```

Fricas [A] time = 1.69111, size = 198, normalized size = 4.21

$$-\frac{3\,\cos\left(dx+c\right)^{3}\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-3\,\cos\left(dx+c\right)^{3}\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-6\,\cos\left(dx+c\right)^{2}-2}{6\,a^{2}d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-a*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(3*cos(d*x + c)^3*log(1/2*cos(d*x + c) + 1/2) - 3*cos(d*x + c)^3*log(-
1/2*cos(d*x + c) + 1/2) - 6*cos(d*x + c)^2 - 2)/(a^2*d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\csc\left(c+dx\right)}{\sin^{4}\left(c+dx\right)-2\sin^{2}\left(c+dx\right)+1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-a*sin(d*x+c)**2)**2,x)
```

```
[Out] Integral(csc(c + d*x)/(sin(c + d*x)**4 - 2*sin(c + d*x)**2 + 1), x)/a**2
```

Giac [B] time = 1.17125, size = 144, normalized size = 3.06

$$\frac{\frac{3\,\log\left(\frac{\left|-\cos\left(dx+c\right)+1\right|}{\left|\cos\left(dx+c\right)+1\right|}\right)}{a^{2}}+\frac{8\left(\frac{3\left(\cos\left(dx+c\right)-1\right)}{\cos\left(dx+c\right)+1}+\frac{3\left(\cos\left(dx+c\right)-1\right)^{2}}{\left(\cos\left(dx+c\right)+1\right)^{2}}+2\right)}{a^{2}\left(\frac{\cos\left(dx+c\right)-1}{\cos\left(dx+c\right)+1}+1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-a*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/6*(3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^2 + 8*(3*(cos(d*
x + c) - 1)/(cos(d*x + c) + 1) + 3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^
2 + 2)/(a^2*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^3))/d
```

3.54 $\int \frac{\csc^3(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=78

$$\frac{5\sec^3(c+dx)}{6a^2d}+\frac{5\sec(c+dx)}{2a^2d}-\frac{5\tanh^{-1}(\cos(c+dx))}{2a^2d}-\frac{\csc^2(c+dx)\sec^3(c+dx)}{2a^2d}$$

```
[Out] (-5*ArcTanh[Cos[c + d*x]])/(2*a^2*d) + (5*Sec[c + d*x])/(2*a^2*d) + (5*Sec[
c + d*x]^3)/(6*a^2*d) - (Csc[c + d*x]^2*Sec[c + d*x]^3)/(2*a^2*d)
```

Rubi [A] time = 0.0847661, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3175, 2622, 288, 302, 207}

$$\frac{5\sec^3(c+dx)}{6a^2d}+\frac{5\sec(c+dx)}{2a^2d}-\frac{5\tanh^{-1}(\cos(c+dx))}{2a^2d}-\frac{\csc^2(c+dx)\sec^3(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/(a - a*Sin[c + d*x]^2)^2,x]
```


```
[Out] (-5*ArcTanh[Cos[c + d*x]])/(2*a^2*d) + (5*Sec[c + d*x])/(2*a^2*d) + (5*Sec[
c + d*x]^3)/(6*a^2*d) - (Csc[c + d*x]^2*Sec[c + d*x]^3)/(2*a^2*d)
```


Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc^3(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx = \frac{\int \csc^3(c+dx)\sec^4(c+dx)\,dx}{a^2}$$
$$= \frac{\text{Subst}\left(\int \frac{x^6}{\left(-1+x^2\right)^2}\,dx, x, \sec(c+dx)\right)}{a^2d}$$
$$= -\frac{\csc^2(c+dx)\sec^3(c+dx)}{2a^2d} + \frac{5\,\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{2a^2d}$$
$$= -\frac{\csc^2(c+dx)\sec^3(c+dx)}{2a^2d} + \frac{5\,\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)dx, x, \sec(c+dx)\right)}{2a^2d}$$
$$= \frac{5\sec(c+dx)}{2a^2d} + \frac{5\sec^3(c+dx)}{6a^2d} - \frac{\csc^2(c+dx)\sec^3(c+dx)}{2a^2d} + \frac{5\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \text{se}\right.}{2a^2d}$$
$$= -\frac{5\tanh^{-1}(\cos(c+dx))}{2a^2d} + \frac{5\sec(c+dx)}{2a^2d} + \frac{5\sec^3(c+dx)}{6a^2d} - \frac{\csc^2(c+dx)\sec^3(c+dx)}{2a^2d}$$

Mathematica [B] time = 0.491932, size = 208, normalized size = 2.67

$$\frac{2\csc^8(c+dx)\left(-40\cos(2(c+dx))+13\cos(3(c+dx))-30\cos(4(c+dx))+13\cos(5(c+dx))+15\cos(3(c+dx))\log\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] (2*Csc[c + d*x]^8*(22 - 40*Cos[2*(c + d*x)] + 13*Cos[3*(c + d*x)] - 30*Cos[
4*(c + d*x)] + 13*Cos[5*(c + d*x)] + 15*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/
2]] + 15*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2]] - 15*Cos[3*(c + d*x)]*Log[S
in[(c + d*x)/2]] - 15*Cos[5*(c + d*x)]*Log[Sin[(c + d*x)/2]] + Cos[c + d*x]
*(-26 - 30*Log[Cos[(c + d*x)/2]] + 30*Log[Sin[(c + d*x)/2]])))/(3*a^2*d*(Cs
c[(c + d*x)/2]^2 - Sec[(c + d*x)/2]^2)^3)
```

Maple [A] time = 0.086, size = 104, normalized size = 1.3

$$\frac{1}{4\,a^2d\,(-1+\cos(dx+c))} + \frac{5\,\ln(-1+\cos(dx+c))}{4\,a^2d} + \frac{1}{4\,a^2d\,(1+\cos(dx+c))} - \frac{5\,\ln(1+\cos(dx+c))}{4\,a^2d} + \frac{1}{3\,a^2d\,(\cos(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(a-sin(d*x+c)^2*a)^2,x)
```

```
[Out] 1/4/d/a^2/(-1+cos(d*x+c))+5/4/d/a^2*ln(-1+cos(d*x+c))+1/4/d/a^2/(1+cos(d*x+
c))-5/4/d/a^2*ln(1+cos(d*x+c))+1/3/d/a^2/cos(d*x+c)^3+2/d/a^2/cos(d*x+c)
```

Maxima [A] time = 0.956819, size = 116, normalized size = 1.49

$$\frac{\frac{2\left(15\cos(dx+c)^4-10\cos(dx+c)^2-2\right)}{a^2\cos(dx+c)^5-a^2\cos(dx+c)^3}-\frac{15\log(\cos(dx+c)+1)}{a^2}+\frac{15\log(\cos(dx+c)-1)}{a^2}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a-a*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(2*(15*cos(d*x + c)^4 - 10*cos(d*x + c)^2 - 2)/(a^2*cos(d*x + c)^5 - a
^2*cos(d*x + c)^3) - 15*log(cos(d*x + c) + 1)/a^2 + 15*log(cos(d*x + c) - 1
)/a^2)/d
```

Fricas [A] time = 1.76094, size = 312, normalized size = 4.

$$\frac{30\cos(dx+c)^4-20\cos(dx+c)^2-15\left(\cos(dx+c)^5-\cos(dx+c)^3\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+15\left(\cos(dx+c)^5-\cos\right.}{12\left(a^2d\cos(dx+c)^5-a^2d\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a-a*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(30*cos(d*x + c)^4 - 20*cos(d*x + c)^2 - 15*(cos(d*x + c)^5 - cos(d*x
+ c)^3)*log(1/2*cos(d*x + c) + 1/2) + 15*(cos(d*x + c)^5 - cos(d*x + c)^3)*
log(-1/2*cos(d*x + c) + 1/2) - 4)/(a^2*d*cos(d*x + c)^5 - a^2*d*cos(d*x + c
)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\csc^3(c+dx)}{\sin^4(c+dx)-2\sin^2(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a-a*sin(d*x+c)**2)**2,x)
```

```
[Out] Integral(csc(c + d*x)**3/(sin(c + d*x)**4 - 2*sin(c + d*x)**2 + 1), x)/a**2
```

Giac [B] time = 1.21886, size = 236, normalized size = 3.03

$$-\frac{\frac{3\left(\frac{10(\cos(dx+c)-1)}{\cos(dx+c)+1}-1\right)(\cos(dx+c)+1)}{a^2(\cos(dx+c)-1)}-\frac{30\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^2}+\frac{3(\cos(dx+c)-1)}{a^2(\cos(dx+c)+1)}-\frac{16\left(\frac{12(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{9(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+7\right)}{a^2\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a-a*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/24*(3*(10*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)*(cos(d*x + c) + 1)/
(a^2*(cos(d*x + c) - 1)) - 30*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) +
 1))/a^2 + 3*(cos(d*x + c) - 1)/(a^2*(cos(d*x + c) + 1)) - 16*(12*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) + 9*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2
+ 7)/(a^2*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^3))/d
```

3.55 $\int \frac{\sin^6(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=69

$$\frac{5\tan^3(c+dx)}{6a^2d}-\frac{5\tan(c+dx)}{2a^2d}-\frac{\sin^2(c+dx)\tan^3(c+dx)}{2a^2d}+\frac{5x}{2a^2}$$

```
[Out] (5*x)/(2*a^2) - (5*Tan[c + d*x])/(2*a^2*d) + (5*Tan[c + d*x]^3)/(6*a^2*d) -
 (Sin[c + d*x]^2*Tan[c + d*x]^3)/(2*a^2*d)
```

Rubi [A] time = 0.0818862, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3175, 2591, 288, 302, 203}

$$\frac{5\tan^3(c+dx)}{6a^2d}-\frac{5\tan(c+dx)}{2a^2d}-\frac{\sin^2(c+dx)\tan^3(c+dx)}{2a^2d}+\frac{5x}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^6/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] (5*x)/(2*a^2) - (5*Tan[c + d*x])/(2*a^2*d) + (5*Tan[c + d*x]^3)/(6*a^2*d) -
 (Sin[c + d*x]^2*Tan[c + d*x]^3)/(2*a^2*d)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^6(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx &= \frac{\int \sin^2(c+dx)\tan^4(c+dx)\,dx}{a^2}\\
&= \frac{\operatorname{Subst}\left(\int \frac{x^6}{\left(1+x^2\right)^2}\,dx, x, \tan(c+dx)\right)}{a^2 d}\\
&= -\frac{\sin^2(c+dx)\tan^3(c+dx)}{2a^2d}+\frac{5\operatorname{Subst}\left(\int \frac{x^4}{1+x^2}\,dx, x, \tan(c+dx)\right)}{2a^2d}\\
&= -\frac{\sin^2(c+dx)\tan^3(c+dx)}{2a^2d}+\frac{5\operatorname{Subst}\left(\int \left(-1+x^2+\frac{1}{1+x^2}\right)dx, x, \tan(c+dx)\right)}{2a^2d}\\
&= -\frac{5\tan(c+dx)}{2a^2d}+\frac{5\tan^3(c+dx)}{6a^2d}-\frac{\sin^2(c+dx)\tan^3(c+dx)}{2a^2d}+\frac{5\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, t\right.}{2a^2d}\\
&= \frac{5x}{2a^2}-\frac{5\tan(c+dx)}{2a^2d}+\frac{5\tan^3(c+dx)}{6a^2d}-\frac{\sin^2(c+dx)\tan^3(c+dx)}{2a^2d}
\end{aligned}
$$

Mathematica [A] time = 0.210398, size = 46, normalized size = 0.67

$$\frac{30(c+dx)-3\sin(2(c+dx))+4\tan(c+dx)\left(\sec^2(c+dx)-7\right)}{12a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^6/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] (30*(c + d*x) - 3*Sin[2*(c + d*x)] + 4*(-7 + Sec[c + d*x]^2)*Tan[c + d*x])/
(12*a^2*d)
```

Maple [A] time = 0.046, size = 73, normalized size = 1.1

$$\frac{(\tan(dx+c))^3}{3a^2d}-2\frac{\tan(dx+c)}{a^2d}-\frac{\tan(dx+c)}{2a^2d\left((\tan(dx+c))^2+1\right)}+\frac{5\arctan(\tan(dx+c))}{2a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^6/(a-sin(d*x+c)^2*a)^2,x)
```

```
[Out] 1/3*tan(d*x+c)^3/a^2/d-2*tan(d*x+c)/a^2/d-1/2/d/a^2*tan(d*x+c)/(tan(d*x+c)^
2+1)+5/2/d/a^2*arctan(tan(d*x+c))
```

Maxima [A] time = 1.46678, size = 86, normalized size = 1.25

$$-\frac{\frac{3\tan(dx+c)}{a^2\tan(dx+c)^2+a^2}-\frac{2\left(\tan(dx+c)^3-6\tan(dx+c)\right)}{a^2}-\frac{15(dx+c)}{a^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a-a*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(3*tan(d*x + c)/(a^2*tan(d*x + c)^2 + a^2) - 2*(tan(d*x + c)^3 - 6*tan
(d*x + c))/a^2 - 15*(d*x + c)/a^2)/d
```

Fricas [A] time = 1.70056, size = 149, normalized size = 2.16

$$\frac{15\,dx\cos\left(dx+c\right)^{3}-\left(3\,\cos\left(dx+c\right)^{4}+14\,\cos\left(dx+c\right)^{2}-2\right)\sin\left(dx+c\right)}{6\,a^{2}d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a-a*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(15*d*x*cos(d*x + c)^3 - (3*cos(d*x + c)^4 + 14*cos(d*x + c)^2 - 2)*sin
(d*x + c))/(a^2*d*cos(d*x + c)^3)
```

Sympy [A] time = 144.461, size = 1275, normalized size = 18.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**6/(a-a*sin(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((15*d*x*tan(c/2 + d*x/2)**10/(6*a**2*d*tan(c/2 + d*x/2)**10 - 6*a
**2*d*tan(c/2 + d*x/2)**8 - 12*a**2*d*tan(c/2 + d*x/2)**6 + 12*a**2*d*tan(c
/2 + d*x/2)**4 + 6*a**2*d*tan(c/2 + d*x/2)**2 - 6*a**2*d) - 15*d*x*tan(c/2
+ d*x/2)**8/(6*a**2*d*tan(c/2 + d*x/2)**10 - 6*a**2*d*tan(c/2 + d*x/2)**8 -
 12*a**2*d*tan(c/2 + d*x/2)**6 + 12*a**2*d*tan(c/2 + d*x/2)**4 + 6*a**2*d*t
an(c/2 + d*x/2)**2 - 6*a**2*d) - 30*d*x*tan(c/2 + d*x/2)**6/(6*a**2*d*tan(c
/2 + d*x/2)**10 - 6*a**2*d*tan(c/2 + d*x/2)**8 - 12*a**2*d*tan(c/2 + d*x/2)
**6 + 12*a**2*d*tan(c/2 + d*x/2)**4 + 6*a**2*d*tan(c/2 + d*x/2)**2 - 6*a**2
*d) + 30*d*x*tan(c/2 + d*x/2)**4/(6*a**2*d*tan(c/2 + d*x/2)**10 - 6*a**2*d*
tan(c/2 + d*x/2)**8 - 12*a**2*d*tan(c/2 + d*x/2)**6 + 12*a**2*d*tan(c/2 + d
*x/2)**4 + 6*a**2*d*tan(c/2 + d*x/2)**2 - 6*a**2*d) + 15*d*x*tan(c/2 + d*x/
2)**2/(6*a**2*d*tan(c/2 + d*x/2)**10 - 6*a**2*d*tan(c/2 + d*x/2)**8 - 12*a*
*2*d*tan(c/2 + d*x/2)**6 + 12*a**2*d*tan(c/2 + d*x/2)**4 + 6*a**2*d*tan(c/2
 + d*x/2)**2 - 6*a**2*d) - 15*d*x/(6*a**2*d*tan(c/2 + d*x/2)**10 - 6*a**2*d
*tan(c/2 + d*x/2)**8 - 12*a**2*d*tan(c/2 + d*x/2)**6 + 12*a**2*d*tan(c/2 +
d*x/2)**4 + 6*a**2*d*tan(c/2 + d*x/2)**2 - 6*a**2*d) + 30*tan(c/2 + d*x/2)*
*9/(6*a**2*d*tan(c/2 + d*x/2)**10 - 6*a**2*d*tan(c/2 + d*x/2)**8 - 12*a**2*
d*tan(c/2 + d*x/2)**6 + 12*a**2*d*tan(c/2 + d*x/2)**4 + 6*a**2*d*tan(c/2 +
d*x/2)**2 - 6*a**2*d) - 40*tan(c/2 + d*x/2)**7/(6*a**2*d*tan(c/2 + d*x/2)**
10 - 6*a**2*d*tan(c/2 + d*x/2)**8 - 12*a**2*d*tan(c/2 + d*x/2)**6 + 12*a**2
*d*tan(c/2 + d*x/2)**4 + 6*a**2*d*tan(c/2 + d*x/2)**2 - 6*a**2*d) - 44*tan(
c/2 + d*x/2)**5/(6*a**2*d*tan(c/2 + d*x/2)**10 - 6*a**2*d*tan(c/2 + d*x/2)*
*8 - 12*a**2*d*tan(c/2 + d*x/2)**6 + 12*a**2*d*tan(c/2 + d*x/2)**4 + 6*a**2
*d*tan(c/2 + d*x/2)**2 - 6*a**2*d) - 40*tan(c/2 + d*x/2)**3/(6*a**2*d*tan(c
/2 + d*x/2)**10 - 6*a**2*d*tan(c/2 + d*x/2)**8 - 12*a**2*d*tan(c/2 + d*x/2)
**6 + 12*a**2*d*tan(c/2 + d*x/2)**4 + 6*a**2*d*tan(c/2 + d*x/2)**2 - 6*a**2
*d) + 30*tan(c/2 + d*x/2)/(6*a**2*d*tan(c/2 + d*x/2)**10 - 6*a**2*d*tan(c/2
```

```
 + d*x/2)**8 - 12*a**2*d*tan(c/2 + d*x/2)**6 + 12*a**2*d*tan(c/2 + d*x/2)**
4 + 6*a**2*d*tan(c/2 + d*x/2)**2 - 6*a**2*d), Ne(d, 0)), (x*sin(c)**6/(-a*s
in(c)**2 + a)**2, True))
```

Giac [A] time = 1.16731, size = 92, normalized size = 1.33

$$\frac{\frac{15\,(dx+c)}{a^2} - \frac{3\,\tan(dx+c)}{\left(\tan(dx+c)^2+1\right)a^2} + \frac{2\left(a^4\tan(dx+c)^3-6\,a^4\tan(dx+c)\right)}{a^6}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a-a*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/6*(15*(d*x + c)/a^2 - 3*tan(d*x + c)/((tan(d*x + c)^2 + 1)*a^2) + 2*(a^4*
tan(d*x + c)^3 - 6*a^4*tan(d*x + c))/a^6)/d
```

3.56 $\int \frac{\sin^4(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=38

$$\frac{\tan^3(c+dx)}{3a^2d}-\frac{\tan(c+dx)}{a^2d}+\frac{x}{a^2}$$

```
[Out] x/a^2 - Tan[c + d*x]/(a^2*d) + Tan[c + d*x]^3/(3*a^2*d)
```

Rubi [A] time = 0.0556989, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 3473, 8}

$$\frac{\tan^3(c+dx)}{3a^2d}-\frac{\tan(c+dx)}{a^2d}+\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] x/a^2 - Tan[c + d*x]/(a^2*d) + Tan[c + d*x]^3/(3*a^2*d)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^4(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx &= \frac{\int \tan^4(c+dx)\,dx}{a^2}\\
&= \frac{\tan^3(c+dx)}{3a^2d}-\frac{\int \tan^2(c+dx)\,dx}{a^2}\\
&= -\frac{\tan(c+dx)}{a^2d}+\frac{\tan^3(c+dx)}{3a^2d}+\frac{\int 1\,dx}{a^2}\\
&= \frac{x}{a^2}-\frac{\tan(c+dx)}{a^2d}+\frac{\tan^3(c+dx)}{3a^2d}
\end{aligned}$$

Mathematica [A] time = 0.014339, size = 42, normalized size = 1.11

$$\frac{\frac{\tan^3(c+dx)}{3d}+\frac{\tan^{-1}(\tan(c+dx))}{d}-\frac{\tan(c+dx)}{d}}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] (ArcTan[Tan[c + d*x]]/d - Tan[c + d*x]/d + Tan[c + d*x]^3/(3*d))/a^2
```

Maple [A] time = 0.036, size = 46, normalized size = 1.2

$$\frac{(\tan(dx+c))^3}{3a^2d}-\frac{\tan(dx+c)}{a^2d}+\frac{\arctan(\tan(dx+c))}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a-sin(d*x+c)^2*a)^2,x)
```

```
[Out] 1/3*tan(d*x+c)^3/a^2/d-tan(d*x+c)/a^2/d+1/d/a^2*arctan(tan(d*x+c))
```

Maxima [A] time = 1.43872, size = 50, normalized size = 1.32

$$\frac{\frac{\tan(dx+c)^3-3\tan(dx+c)}{a^2}+\frac{3(dx+c)}{a^2}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a-a*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*((tan(d*x + c)^3 - 3*tan(d*x + c))/a^2 + 3*(d*x + c)/a^2)/d
```

Fricas [A] time = 1.5878, size = 120, normalized size = 3.16

$$\frac{3\,dx\cos(dx+c)^3-\left(4\cos(dx+c)^2-1\right)\sin(dx+c)}{3a^2d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a-a*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*d*x*cos(d*x + c)^3 - (4*cos(d*x + c)^2 - 1)*sin(d*x + c))/(a^2*d*cos
(d*x + c)^3)
```

Sympy [A] time = 61.3636, size = 551, normalized size = 14.5

$$\begin{cases}\frac{3dx\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)-9a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-3a^2d}-\frac{9dx\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)-9a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-3a^2d}+\frac{}{3a^2d\tan^6\left(\frac{c}{2}+\cdots\right.} \\ \frac{x\sin^4(c)}{\left(-a\sin^2(c)+a\right)^2}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a-a*sin(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((3*d*x*tan(c/2 + d*x/2)**6/(3*a**2*d*tan(c/2 + d*x/2)**6 - 9*a**2
*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 - 3*a**2*d) - 9*d*x*t
an(c/2 + d*x/2)**4/(3*a**2*d*tan(c/2 + d*x/2)**6 - 9*a**2*d*tan(c/2 + d*x/2
)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 - 3*a**2*d) + 9*d*x*tan(c/2 + d*x/2)**2
/(3*a**2*d*tan(c/2 + d*x/2)**6 - 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*ta
n(c/2 + d*x/2)**2 - 3*a**2*d) - 3*d*x/(3*a**2*d*tan(c/2 + d*x/2)**6 - 9*a**
2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 - 3*a**2*d) + 6*tan(
c/2 + d*x/2)**5/(3*a**2*d*tan(c/2 + d*x/2)**6 - 9*a**2*d*tan(c/2 + d*x/2)**
4 + 9*a**2*d*tan(c/2 + d*x/2)**2 - 3*a**2*d) - 20*tan(c/2 + d*x/2)**3/(3*a*
*2*d*tan(c/2 + d*x/2)**6 - 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2
+ d*x/2)**2 - 3*a**2*d) + 6*tan(c/2 + d*x/2)/(3*a**2*d*tan(c/2 + d*x/2)**6
- 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 - 3*a**2*d),
Ne(d, 0)), (x*sin(c)**4/(-a*sin(c)**2 + a)**2, True))
```

Giac [A] time = 1.15668, size = 59, normalized size = 1.55

$$\frac{\frac{3\,(dx+c)}{a^2} + \frac{a^4\tan(dx+c)^3 - 3\,a^4\tan(dx+c)}{a^6}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a-a*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)/a^2 + (a^4*tan(d*x + c)^3 - 3*a^4*tan(d*x + c))/a^6)/d
```

3.57 $\int \frac{\sin^2(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=18

$$\frac{\tan^3(c+dx)}{3a^2d}$$

```
[Out] Tan[c + d*x]^3/(3*a^2*d)
```

Rubi [A] time = 0.0674126, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 2607, 30}

$$\frac{\tan^3(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] Tan[c + d*x]^3/(3*a^2*d)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^2(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx &= \frac{\int \sec^2(c+dx)\tan^2(c+dx)\,dx}{a^2}\\ &= \frac{\text{Subst}\left(\int x^2\,dx, x, \tan(c+dx)\right)}{a^2d}\\ &= \frac{\tan^3(c+dx)}{3a^2d}\end{aligned}$$

Mathematica [A] time = 0.0176636, size = 18, normalized size = 1.

$$\frac{\tan^3(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] Tan[c + d*x]^3/(3*a^2*d)
```

Maple [A] time = 0.033, size = 17, normalized size = 0.9

$$\frac{(\tan(dx+c))^3}{3a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a-sin(d*x+c)^2*a)^2,x)
```

```
[Out] 1/3*tan(d*x+c)^3/a^2/d
```

Maxima [A] time = 1.13808, size = 22, normalized size = 1.22

$$\frac{\tan(dx+c)^3}{3a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a-a*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*tan(d*x + c)^3/(a^2*d)
```

Fricas [A] time = 1.53549, size = 85, normalized size = 4.72

$$-\frac{\left(\cos(dx+c)^2-1\right)\sin(dx+c)}{3a^2d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a-a*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(cos(d*x + c)^2 - 1)*sin(d*x + c)/(a^2*d*cos(d*x + c)^3)
```

Sympy [A] time = 31.7744, size = 94, normalized size = 5.22

$$\begin{cases} -\frac{8\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)-9a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-3a^2d} & \text{for } d \neq 0 \\ \frac{x\sin^2(c)}{\left(-a\sin^2(c)+a\right)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a-a*sin(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((-8*tan(c/2 + d*x/2)**3/(3*a**2*d*tan(c/2 + d*x/2)**6 - 9*a**2*d*
tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 - 3*a**2*d), Ne(d, 0)),
(x*sin(c)**2/(-a*sin(c)**2 + a)**2, True))
```

Giac [A] time = 1.1213, size = 22, normalized size = 1.22

$$\frac{\tan(dx+c)^3}{3\,a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a-a*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*tan(d*x + c)^3/(a^2*d)
```

3.58 $\int \frac{1}{\left(a-a\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=32

$$\frac{\tan^3(c+dx)}{3a^2d}+\frac{\tan(c+dx)}{a^2d}$$

```
[Out] Tan[c + d*x]/(a^2*d) + Tan[c + d*x]^3/(3*a^2*d)
```

Rubi [A] time = 0.0245773, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3175, 3767}

$$\frac{\tan^3(c+dx)}{3a^2d}+\frac{\tan(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[c + d*x]^2)^(-2),x]
```

```
[Out] Tan[c + d*x]/(a^2*d) + Tan[c + d*x]^3/(3*a^2*d)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a-a\sin^2(c+dx)\right)^2}\,dx &= \frac{\int \sec^4(c+dx)\,dx}{a^2}\\ &= -\frac{\text{Subst}\left(\int \left(1+x^2\right)\,dx, x, -\tan(c+dx)\right)}{a^2d}\\ &= \frac{\tan(c+dx)}{a^2d}+\frac{\tan^3(c+dx)}{3a^2d}\end{aligned}$$

Mathematica [A] time = 0.0382637, size = 26, normalized size = 0.81

$$\frac{\frac{1}{3}\tan^3(c+dx)+\tan(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sin[c + d*x]^2)^(-2),x]
```

```
[Out] (Tan[c + d*x] + Tan[c + d*x]^3/3)/(a^2*d)
```

Maple [A] time = 0.04, size = 25, normalized size = 0.8

$$\frac{1}{a^2 d}\left(\frac{(\tan(dx+c))^3}{3} + \tan(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-sin(d*x+c)^2*a)^2,x)
```

```
[Out] 1/d/a^2*(1/3*tan(d*x+c)^3+tan(d*x+c))
```

Maxima [A] time = 0.956355, size = 34, normalized size = 1.06

$$\frac{\tan(dx+c)^3 + 3\tan(dx+c)}{3\,a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(tan(d*x + c)^3 + 3*tan(d*x + c))/(a^2*d)
```

Fricas [A] time = 1.57837, size = 86, normalized size = 2.69

$$\frac{\left(2\cos(dx+c)^2 + 1\right)\sin(dx+c)}{3\,a^2 d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(2*cos(d*x + c)^2 + 1)*sin(d*x + c)/(a^2*d*cos(d*x + c)^3)
```

Sympy [A] time = 13.4509, size = 238, normalized size = 7.44

$$\begin{cases} -\frac{6\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)-9a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-3a^2d} + \frac{4\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)-9a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-3a^2d} - \frac{}{3a^2d\tan^6\left(\frac{c}{2}+\right.} \\ \frac{x}{\left(-a\sin^2(c)+a\right)^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((-6*tan(c/2 + d*x/2)**5/(3*a**2*d*tan(c/2 + d*x/2)**6 - 9*a**2*d*
tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 - 3*a**2*d) + 4*tan(c/2
+ d*x/2)**3/(3*a**2*d*tan(c/2 + d*x/2)**6 - 9*a**2*d*tan(c/2 + d*x/2)**4 +
```

```
9*a**2*d*tan(c/2 + d*x/2)**2 - 3*a**2*d) - 6*tan(c/2 + d*x/2)/(3*a**2*d*tan
(c/2 + d*x/2)**6 - 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)
**2 - 3*a**2*d), Ne(d, 0)), (x/(-a*sin(c)**2 + a)**2, True))
```

Giac [A] time = 1.129, size = 34, normalized size = 1.06

$$\frac{\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)}{3\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(tan(d*x + c)^3 + 3*tan(d*x + c))/(a^2*d)
```

3.59 $\int \frac{\csc^2(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2} \, dx$

Optimal. Leaf size=47

$$\frac{\tan^3(c+dx)}{3a^2d}+\frac{2\tan(c+dx)}{a^2d}-\frac{\cot(c+dx)}{a^2d}$$

```
[Out] -(Cot[c + d*x]/(a^2*d)) + (2*Tan[c + d*x])/(a^2*d) + Tan[c + d*x]^3/(3*a^2*
d)
```

Rubi [A] time = 0.0781224, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 2620, 270}

$$\frac{\tan^3(c+dx)}{3a^2d}+\frac{2\tan(c+dx)}{a^2d}-\frac{\cot(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] -(Cot[c + d*x]/(a^2*d)) + (2*Tan[c + d*x])/(a^2*d) + Tan[c + d*x]^3/(3*a^2*
d)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^2(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2} \, dx &= \frac{\int \csc^2(c+dx)\sec^4(c+dx)\,dx}{a^2}\\
&= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^2}\,dx, x, \tan(c+dx)\right)}{a^2d}\\
&= \frac{\text{Subst}\left(\int \left(2+\frac{1}{x^2}+x^2\right)dx, x, \tan(c+dx)\right)}{a^2d}\\
&= -\frac{\cot(c+dx)}{a^2d}+\frac{2\tan(c+dx)}{a^2d}+\frac{\tan^3(c+dx)}{3a^2d}
\end{aligned}$$

Mathematica [A] time = 0.038668, size = 50, normalized size = 1.06

$$\frac{\frac{5\tan(c+dx)}{3d}-\frac{\cot(c+dx)}{d}+\frac{\tan(c+dx)\sec^2(c+dx)}{3d}}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] (-(Cot[c + d*x]/d) + (5*Tan[c + d*x])/(3*d) + (Sec[c + d*x]^2*Tan[c + d*x])
/(3*d))/a^2
```

Maple [A] time = 0.066, size = 37, normalized size = 0.8

$$\frac{1}{a^2d}\left(\frac{(\tan(dx+c))^3}{3}+2\tan(dx+c)-(\tan(dx+c))^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a-sin(d*x+c)^2*a)^2,x)
```

```
[Out] 1/d/a^2*(1/3*tan(d*x+c)^3+2*tan(d*x+c)-1/tan(d*x+c))
```

Maxima [A] time = 0.959586, size = 54, normalized size = 1.15

$$\frac{\frac{\tan(dx+c)^3+6\tan(dx+c)}{a^2}-\frac{3}{a^2\tan(dx+c)}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a-a*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*((tan(d*x + c)^3 + 6*tan(d*x + c))/a^2 - 3/(a^2*tan(d*x + c)))/d
```

Fricas [A] time = 1.55082, size = 113, normalized size = 2.4

$$-\frac{8\cos(dx+c)^4-4\cos(dx+c)^2-1}{3a^2d\cos(dx+c)^3\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a-a*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(8*cos(d*x + c)^4 - 4*cos(d*x + c)^2 - 1)/(a^2*d*cos(d*x + c)^3*sin(d*
x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\csc^2(c+dx)}{\sin^4(c+dx)-2\sin^2(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a-a*sin(d*x+c)**2)**2,x)
```

```
[Out] Integral(csc(c + d*x)**2/(sin(c + d*x)**4 - 2*sin(c + d*x)**2 + 1), x)/a**2
```

Giac [A] time = 1.11487, size = 65, normalized size = 1.38

$$-\frac{\frac{3}{a^2\tan(dx+c)} - \frac{a^4\tan(dx+c)^3+6\,a^4\tan(dx+c)}{a^6}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a-a*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/3*(3/(a^2*tan(d*x + c)) - (a^4*tan(d*x + c)^3 + 6*a^4*tan(d*x + c))/a^6)
/d
```

3.60 $\int \frac{\csc^4(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=65

$$\frac{\tan^3(c+dx)}{3a^2d}+\frac{3\tan(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3a^2d}-\frac{3\cot(c+dx)}{a^2d}$$

```
[Out] (-3*Cot[c + d*x])/(a^2*d) - Cot[c + d*x]^3/(3*a^2*d) + (3*Tan[c + d*x])/(a^
2*d) + Tan[c + d*x]^3/(3*a^2*d)
```

Rubi [A] time = 0.0808019, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 2620, 270}

$$\frac{\tan^3(c+dx)}{3a^2d}+\frac{3\tan(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3a^2d}-\frac{3\cot(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] (-3*Cot[c + d*x])/(a^2*d) - Cot[c + d*x]^3/(3*a^2*d) + (3*Tan[c + d*x])/(a^
2*d) + Tan[c + d*x]^3/(3*a^2*d)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^4(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx &= \frac{\int \csc^4(c+dx)\sec^4(c+dx)\,dx}{a^2}\\
&= \frac{\operatorname{Subst}\left(\int \frac{\left(1+x^2\right)^3}{x^4}\,dx,x,\tan(c+dx)\right)}{a^2d}\\
&= \frac{\operatorname{Subst}\left(\int \left(3+\frac{1}{x^4}+\frac{3}{x^2}+x^2\right)dx,x,\tan(c+dx)\right)}{a^2d}\\
&= -\frac{3\cot(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{3\tan(c+dx)}{a^2d}+\frac{\tan^3(c+dx)}{3a^2d}
\end{aligned}$$

Mathematica [A] time = 0.0246865, size = 46, normalized size = 0.71

$$\frac{16\left(-\frac{\cot(2(c+dx))}{3d}-\frac{\cot(2(c+dx))\csc^2(2(c+dx))}{6d}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] (16*(-Cot[2*(c + d*x)]/(3*d) - (Cot[2*(c + d*x)]*Csc[2*(c + d*x)]^2)/(6*d))
)/a^2
```

Maple [A] time = 0.076, size = 47, normalized size = 0.7

$$\frac{1}{a^2 d}\left(\frac{(\tan(dx+c))^3}{3}+3\tan(dx+c)-3(\tan(dx+c))^{-1}-\frac{1}{3(\tan(dx+c))^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4/(a-sin(d*x+c)^2*a)^2,x)
```

```
[Out] 1/d/a^2*(1/3*tan(d*x+c)^3+3*tan(d*x+c)-3/tan(d*x+c)-1/3/tan(d*x+c)^3)
```

Maxima [A] time = 0.955788, size = 70, normalized size = 1.08

$$\frac{\frac{\tan(dx+c)^3+9\tan(dx+c)}{a^2}-\frac{9\tan(dx+c)^2+1}{a^2\tan(dx+c)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a-a*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*((tan(d*x + c)^3 + 9*tan(d*x + c))/a^2 - (9*tan(d*x + c)^2 + 1)/(a^2*ta
n(d*x + c)^3))/d
```

Fricas [A] time = 1.64036, size = 176, normalized size = 2.71

$$-\frac{16\cos(dx+c)^6-24\cos(dx+c)^4+6\cos(dx+c)^2+1}{3\left(a^2d\cos(dx+c)^5-a^2d\cos(dx+c)^3\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a-a*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(16*cos(d*x + c)^6 - 24*cos(d*x + c)^4 + 6*cos(d*x + c)^2 + 1)/((a^2*d
*cos(d*x + c)^5 - a^2*d*cos(d*x + c)^3)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\csc^4(c+dx)}{\sin^4(c+dx)-2\sin^2(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4/(a-a*sin(d*x+c)**2)**2,x)
```

```
[Out] Integral(csc(c + d*x)**4/(sin(c + d*x)**4 - 2*sin(c + d*x)**2 + 1), x)/a**2
```

Giac [A] time = 1.16087, size = 46, normalized size = 0.71

$$-\frac{8\left(3\tan\left(2\,dx+2\,c\right)^2+1\right)}{3\,a^2d\tan\left(2\,dx+2\,c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a-a*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -8/3*(3*tan(2*d*x + 2*c)^2 + 1)/(a^2*d*tan(2*d*x + 2*c)^3)
```

3.61 $\int \frac{1}{\left(a-a\sin^2(x)\right)^3}\,dx$

Optimal. Leaf size=29

$$\frac{\tan^5(x)}{5a^3}+\frac{2\tan^3(x)}{3a^3}+\frac{\tan(x)}{a^3}$$

```
[Out] Tan[x]/a^3 + (2*Tan[x]^3)/(3*a^3) + Tan[x]^5/(5*a^3)
```

Rubi [A] time = 0.0208021, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3175, 3767}

$$\frac{\tan^5(x)}{5a^3}+\frac{2\tan^3(x)}{3a^3}+\frac{\tan(x)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[x]^2)^(-3),x]
```

```
[Out] Tan[x]/a^3 + (2*Tan[x]^3)/(3*a^3) + Tan[x]^5/(5*a^3)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a-a\sin^2(x)\right)^3}\,dx &= \frac{\int \sec^6(x)\,dx}{a^3}\\
&= -\frac{\operatorname{Subst}\left(\int\left(1+2x^2+x^4\right)dx,x,-\tan(x)\right)}{a^3}\\
&= \frac{\tan(x)}{a^3}+\frac{2\tan^3(x)}{3a^3}+\frac{\tan^5(x)}{5a^3}
\end{aligned}$$

Mathematica [A] time = 0.0047576, size = 31, normalized size = 1.07

$$\frac{\frac{8\tan(x)}{15}+\frac{1}{5}\tan(x)\sec^4(x)+\frac{4}{15}\tan(x)\sec^2(x)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sin[x]^2)^(-3),x]
```

```
[Out] ((8*Tan[x])/15 + (4*Sec[x]^2*Tan[x])/15 + (Sec[x]^4*Tan[x])/5)/a^3
```

Maple [A] time = 0.03, size = 20, normalized size = 0.7

$$\frac{1}{a^3}\left(\frac{(\tan(x))^5}{5}+\frac{2\,(\tan(x))^3}{3}+\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sin(x)^2)^3,x)
```

```
[Out] 1/a^3*(1/5*tan(x)^5+2/3*tan(x)^3+tan(x))
```

Maxima [A] time = 0.951315, size = 30, normalized size = 1.03

$$\frac{3\,\tan(x)^5+10\,\tan(x)^3+15\,\tan(x)}{15\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/15*(3*tan(x)^5 + 10*tan(x)^3 + 15*tan(x))/a^3
```

Fricas [A] time = 1.56478, size = 78, normalized size = 2.69

$$\frac{\left(8\,\cos(x)^4+4\,\cos(x)^2+3\right)\sin(x)}{15\,a^3\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/15*(8*cos(x)^4 + 4*cos(x)^2 + 3)*sin(x)/(a^3*cos(x)^5)
```

Sympy [B] time = 24.0131, size = 362, normalized size = 12.48

$$-\frac{30\tan^9\left(\frac{x}{2}\right)}{15a^3\tan^{10}\left(\frac{x}{2}\right)-75a^3\tan^8\left(\frac{x}{2}\right)+150a^3\tan^6\left(\frac{x}{2}\right)-150a^3\tan^4\left(\frac{x}{2}\right)+75a^3\tan^2\left(\frac{x}{2}\right)-15a^3}+\frac{}{15a^3\tan^{10}\left(\frac{x}{2}\right)-75a^3\tan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)**2)**3,x)
```

```
[Out] -30*tan(x/2)**9/(15*a**3*tan(x/2)**10 - 75*a**3*tan(x/2)**8 + 150*a**3*tan(
x/2)**6 - 150*a**3*tan(x/2)**4 + 75*a**3*tan(x/2)**2 - 15*a**3) + 40*tan(x/
2)**7/(15*a**3*tan(x/2)**10 - 75*a**3*tan(x/2)**8 + 150*a**3*tan(x/2)**6 -
150*a**3*tan(x/2)**4 + 75*a**3*tan(x/2)**2 - 15*a**3) - 116*tan(x/2)**5/(15
```

```
*a**3*tan(x/2)**10 - 75*a**3*tan(x/2)**8 + 150*a**3*tan(x/2)**6 - 150*a**3*
tan(x/2)**4 + 75*a**3*tan(x/2)**2 - 15*a**3) + 40*tan(x/2)**3/(15*a**3*tan(
x/2)**10 - 75*a**3*tan(x/2)**8 + 150*a**3*tan(x/2)**6 - 150*a**3*tan(x/2)**
4 + 75*a**3*tan(x/2)**2 - 15*a**3) - 30*tan(x/2)/(15*a**3*tan(x/2)**10 - 75
*a**3*tan(x/2)**8 + 150*a**3*tan(x/2)**6 - 150*a**3*tan(x/2)**4 + 75*a**3*t
an(x/2)**2 - 15*a**3)
```

Giac [A] time = 1.10985, size = 30, normalized size = 1.03

$$\frac{3\tan(x)^5 + 10\tan(x)^3 + 15\tan(x)}{15\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^3,x, algorithm="giac")
```

```
[Out] 1/15*(3*tan(x)^5 + 10*tan(x)^3 + 15*tan(x))/a^3
```

3.62 $\int \frac{1}{\left(a-a\sin^2(x)\right)^4}\,dx$

Optimal. Leaf size=37

$$\frac{\tan^7(x)}{7a^4}+\frac{3\tan^5(x)}{5a^4}+\frac{\tan^3(x)}{a^4}+\frac{\tan(x)}{a^4}$$

```
[Out] Tan[x]/a^4 + Tan[x]^3/a^4 + (3*Tan[x]^5)/(5*a^4) + Tan[x]^7/(7*a^4)
```

Rubi [A] time = 0.02165, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3175, 3767}

$$\frac{\tan^7(x)}{7a^4}+\frac{3\tan^5(x)}{5a^4}+\frac{\tan^3(x)}{a^4}+\frac{\tan(x)}{a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[x]^2)^(-4),x]
```

```
[Out] Tan[x]/a^4 + Tan[x]^3/a^4 + (3*Tan[x]^5)/(5*a^4) + Tan[x]^7/(7*a^4)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a-a\sin^2(x)\right)^4}\,dx &= \frac{\int \sec^8(x)\,dx}{a^4}\\
&= -\frac{\text{Subst}\left(\int \left(1+3x^2+3x^4+x^6\right)\,dx,x,-\tan(x)\right)}{a^4}\\
&= \frac{\tan(x)}{a^4}+\frac{\tan^3(x)}{a^4}+\frac{3\tan^5(x)}{5a^4}+\frac{\tan^7(x)}{7a^4}
\end{aligned}$$

Mathematica [A] time = 0.0051274, size = 41, normalized size = 1.11

$$\frac{\frac{16\tan(x)}{35}+\frac{1}{7}\tan(x)\sec^6(x)+\frac{6}{35}\tan(x)\sec^4(x)+\frac{8}{35}\tan(x)\sec^2(x)}{a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sin[x]^2)^(-4),x]
```

```
[Out] ((16*Tan[x])/35 + (8*Sec[x]^2*Tan[x])/35 + (6*Sec[x]^4*Tan[x])/35 + (Sec[x]
^6*Tan[x])/7)/a^4
```

Maple [A] time = 0.032, size = 24, normalized size = 0.7

$$\frac{1}{a^4}\left(\frac{(\tan(x))^7}{7}+\frac{3\,(\tan(x))^5}{5}+(\tan(x))^3+\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sin(x)^2)^4,x)
```

```
[Out] 1/a^4*(1/7*tan(x)^7+3/5*tan(x)^5+tan(x)^3+tan(x))
```

Maxima [A] time = 0.941432, size = 38, normalized size = 1.03

$$\frac{5\,\tan(x)^7+21\,\tan(x)^5+35\,\tan(x)^3+35\,\tan(x)}{35\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^4,x, algorithm="maxima")
```

```
[Out] 1/35*(5*tan(x)^7 + 21*tan(x)^5 + 35*tan(x)^3 + 35*tan(x))/a^4
```

Fricas [A] time = 1.89384, size = 97, normalized size = 2.62

$$\frac{\left(16\,\cos(x)^6+8\,\cos(x)^4+6\,\cos(x)^2+5\right)\sin(x)}{35\,a^4\cos(x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^4,x, algorithm="fricas")
```

```
[Out] 1/35*(16*cos(x)^6 + 8*cos(x)^4 + 6*cos(x)^2 + 5)*sin(x)/(a^4*cos(x)^7)
```

Sympy [B] time = 108.262, size = 675, normalized size = 18.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)**2)**4,x)
```

```
[Out] -70*tan(x/2)**13/(35*a**4*tan(x/2)**14 - 245*a**4*tan(x/2)**12 + 735*a**4*t
an(x/2)**10 - 1225*a**4*tan(x/2)**8 + 1225*a**4*tan(x/2)**6 - 735*a**4*tan(
x/2)**4 + 245*a**4*tan(x/2)**2 - 35*a**4) + 140*tan(x/2)**11/(35*a**4*tan(x
/2)**14 - 245*a**4*tan(x/2)**12 + 735*a**4*tan(x/2)**10 - 1225*a**4*tan(x/2
)**8 + 1225*a**4*tan(x/2)**6 - 735*a**4*tan(x/2)**4 + 245*a**4*tan(x/2)**2
```

```
- 35*a**4) - 602*tan(x/2)**9/(35*a**4*tan(x/2)**14 - 245*a**4*tan(x/2)**12
+ 735*a**4*tan(x/2)**10 - 1225*a**4*tan(x/2)**8 + 1225*a**4*tan(x/2)**6 - 7
35*a**4*tan(x/2)**4 + 245*a**4*tan(x/2)**2 - 35*a**4) + 424*tan(x/2)**7/(35
*a**4*tan(x/2)**14 - 245*a**4*tan(x/2)**12 + 735*a**4*tan(x/2)**10 - 1225*a
**4*tan(x/2)**8 + 1225*a**4*tan(x/2)**6 - 735*a**4*tan(x/2)**4 + 245*a**4*t
an(x/2)**2 - 35*a**4) - 602*tan(x/2)**5/(35*a**4*tan(x/2)**14 - 245*a**4*ta
n(x/2)**12 + 735*a**4*tan(x/2)**10 - 1225*a**4*tan(x/2)**8 + 1225*a**4*tan(
x/2)**6 - 735*a**4*tan(x/2)**4 + 245*a**4*tan(x/2)**2 - 35*a**4) + 140*tan(
x/2)**3/(35*a**4*tan(x/2)**14 - 245*a**4*tan(x/2)**12 + 735*a**4*tan(x/2)**
10 - 1225*a**4*tan(x/2)**8 + 1225*a**4*tan(x/2)**6 - 735*a**4*tan(x/2)**4 +
 245*a**4*tan(x/2)**2 - 35*a**4) - 70*tan(x/2)/(35*a**4*tan(x/2)**14 - 245*
a**4*tan(x/2)**12 + 735*a**4*tan(x/2)**10 - 1225*a**4*tan(x/2)**8 + 1225*a*
*4*tan(x/2)**6 - 735*a**4*tan(x/2)**4 + 245*a**4*tan(x/2)**2 - 35*a**4)
```

Giac [A] time = 1.11095, size = 38, normalized size = 1.03

$$\frac{5\,\tan\left(x\right)^{7}+21\,\tan\left(x\right)^{5}+35\,\tan\left(x\right)^{3}+35\,\tan\left(x\right)}{35\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^4,x, algorithm="giac")
```

```
[Out] 1/35*(5*tan(x)^7 + 21*tan(x)^5 + 35*tan(x)^3 + 35*tan(x))/a^4
```

3.63 $\int \frac{1}{\left(a-a\sin^2(x)\right)^5}\,dx$

Optimal. Leaf size=51

$$\frac{\tan^9(x)}{9a^5}+\frac{4\tan^7(x)}{7a^5}+\frac{6\tan^5(x)}{5a^5}+\frac{4\tan^3(x)}{3a^5}+\frac{\tan(x)}{a^5}$$

```
[Out] Tan[x]/a^5 + (4*Tan[x]^3)/(3*a^5) + (6*Tan[x]^5)/(5*a^5) + (4*Tan[x]^7)/(7*
a^5) + Tan[x]^9/(9*a^5)
```

Rubi [A] time = 0.0263289, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3175, 3767}

$$\frac{\tan^9(x)}{9a^5}+\frac{4\tan^7(x)}{7a^5}+\frac{6\tan^5(x)}{5a^5}+\frac{4\tan^3(x)}{3a^5}+\frac{\tan(x)}{a^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[x]^2)^(-5),x]
```

```
[Out] Tan[x]/a^5 + (4*Tan[x]^3)/(3*a^5) + (6*Tan[x]^5)/(5*a^5) + (4*Tan[x]^7)/(7*
a^5) + Tan[x]^9/(9*a^5)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a-a\sin^2(x)\right)^5}\,dx &= \frac{\int \sec^{10}(x)\,dx}{a^5}\\ &= -\frac{\text{Subst}\left(\int \left(1+4x^2+6x^4+4x^6+x^8\right)\,dx,x,-\tan(x)\right)}{a^5}\\ &= \frac{\tan(x)}{a^5}+\frac{4\tan^3(x)}{3a^5}+\frac{6\tan^5(x)}{5a^5}+\frac{4\tan^7(x)}{7a^5}+\frac{\tan^9(x)}{9a^5}\end{aligned}$$

Mathematica [A] time = 0.0059237, size = 51, normalized size = 1.

$$\frac{\frac{128\tan(x)}{315}+\frac{1}{9}\tan(x)\sec^8(x)+\frac{8}{63}\tan(x)\sec^6(x)+\frac{16}{105}\tan(x)\sec^4(x)+\frac{64}{315}\tan(x)\sec^2(x)}{a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sin[x]^2)^(-5),x]
```

```
[Out] ((128*Tan[x])/315 + (64*Sec[x]^2*Tan[x])/315 + (16*Sec[x]^4*Tan[x])/105 + (
8*Sec[x]^6*Tan[x])/63 + (Sec[x]^8*Tan[x])/9)/a^5
```

Maple [A] time = 0.031, size = 32, normalized size = 0.6

$$\frac{1}{a^5}\left(\frac{(\tan(x))^9}{9}+\frac{4(\tan(x))^7}{7}+\frac{6(\tan(x))^5}{5}+\frac{4(\tan(x))^3}{3}+\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sin(x)^2)^5,x)
```

```
[Out] 1/a^5*(1/9*tan(x)^9+4/7*tan(x)^7+6/5*tan(x)^5+4/3*tan(x)^3+tan(x))
```

Maxima [A] time = 0.958202, size = 46, normalized size = 0.9

$$\frac{35\tan(x)^9+180\tan(x)^7+378\tan(x)^5+420\tan(x)^3+315\tan(x)}{315a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^5,x, algorithm="maxima")
```

```
[Out] 1/315*(35*tan(x)^9 + 180*tan(x)^7 + 378*tan(x)^5 + 420*tan(x)^3 + 315*tan(x
))/a^5
```

Fricas [A] time = 1.86624, size = 123, normalized size = 2.41

$$\frac{\left(128\cos(x)^8+64\cos(x)^6+48\cos(x)^4+40\cos(x)^2+35\right)\sin(x)}{315a^5\cos(x)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^5,x, algorithm="fricas")
```

```
[Out] 1/315*(128*cos(x)^8 + 64*cos(x)^6 + 48*cos(x)^4 + 40*cos(x)^2 + 35)*sin(x)/
(a^5*cos(x)^9)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)**2)**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15653, size = 46, normalized size = 0.9

$$\frac{35 \tan(x)^9 + 180 \tan(x)^7 + 378 \tan(x)^5 + 420 \tan(x)^3 + 315 \tan(x)}{315 a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^5,x, algorithm="giac")
```

```
[Out] 1/315*(35*tan(x)^9 + 180*tan(x)^7 + 378*tan(x)^5 + 420*tan(x)^3 + 315*tan(x
))/a^5
```

3.64 $\int \sin^3(c+dx)\left(a+b\sin^2(c+dx)\right)dx$

Optimal. Leaf size=51

$$\frac{(a+2b)\cos^3(c+dx)}{3d}-\frac{(a+b)\cos(c+dx)}{d}-\frac{b\cos^5(c+dx)}{5d}$$

```
[Out] -(((a + b)*Cos[c + d*x])/d) + ((a + 2*b)*Cos[c + d*x]^3)/(3*d) - (b*Cos[c +
 d*x]^5)/(5*d)
```

Rubi [A] time = 0.0451092, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3013, 373}

$$\frac{(a+2b)\cos^3(c+dx)}{3d}-\frac{(a+b)\cos(c+dx)}{d}-\frac{b\cos^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3*(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -(((a + b)*Cos[c + d*x])/d) + ((a + 2*b)*Cos[c + d*x]^3)/(3*d) - (b*Cos[c +
 d*x]^5)/(5*d)
```

Rule 3013

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^3(c+dx)\left(a+b\sin^2(c+dx)\right)dx &= -\frac{\text{Subst}\left(\int \left(1-x^2\right)\left(a+b-bx^2\right)dx,x,\cos(c+dx)\right)}{d}\\ &= -\frac{\text{Subst}\left(\int \left(a\left(1+\frac{b}{a}\right)-(a+2b)x^2+bx^4\right)dx,x,\cos(c+dx)\right)}{d}\\ &= -\frac{(a+b)\cos(c+dx)}{d}+\frac{(a+2b)\cos^3(c+dx)}{3d}-\frac{b\cos^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A] time = 0.0285599, size = 77, normalized size = 1.51

$$-\frac{3a\cos(c+dx)}{4d}+\frac{a\cos(3(c+dx))}{12d}-\frac{5b\cos(c+dx)}{8d}+\frac{5b\cos(3(c+dx))}{48d}-\frac{b\cos(5(c+dx))}{80d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3*(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (-3*a*Cos[c + d*x])/(4*d) - (5*b*Cos[c + d*x])/(8*d) + (a*Cos[3*(c + d*x)])
/(12*d) + (5*b*Cos[3*(c + d*x)])/(48*d) - (b*Cos[5*(c + d*x)])/(80*d)
```

Maple [A] time = 0.029, size = 54, normalized size = 1.1

$$\frac{1}{d}\left(-\frac{b\cos(dx+c)}{5}\left(\frac{8}{3}+(\sin(dx+c))^4+\frac{4(\sin(dx+c))^2}{3}\right)-\frac{a\left(2+(\sin(dx+c))^2\right)\cos(dx+c)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3*(a+sin(d*x+c)^2*b),x)
```

```
[Out] 1/d*(-1/5*b*(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)-1/3*a*(2+sin(d*x
+c)^2)*cos(d*x+c))
```

Maxima [A] time = 0.963426, size = 58, normalized size = 1.14

$$-\frac{3\,b\cos(dx+c)^5-5\,(a+2\,b)\cos(dx+c)^3+15\,(a+b)\cos(dx+c)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/15*(3*b*cos(d*x + c)^5 - 5*(a + 2*b)*cos(d*x + c)^3 + 15*(a + b)*cos(d*x
 + c))/d
```

Fricas [A] time = 1.92667, size = 115, normalized size = 2.25

$$-\frac{3\,b\cos(dx+c)^5-5\,(a+2\,b)\cos(dx+c)^3+15\,(a+b)\cos(dx+c)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/15*(3*b*cos(d*x + c)^5 - 5*(a + 2*b)*cos(d*x + c)^3 + 15*(a + b)*cos(d*x
 + c))/d
```

Sympy [A] time = 2.33809, size = 107, normalized size = 2.1

$$\begin{cases} -\frac{a\sin^2(c+dx)\cos(c+dx)}{d}-\frac{2a\cos^3(c+dx)}{3d}-\frac{b\sin^4(c+dx)\cos(c+dx)}{d}-\frac{4b\sin^2(c+dx)\cos^3(c+dx)}{3d}-\frac{8b\cos^5(c+dx)}{15d} & \text{for } d\neq 0 \\ x\left(a+b\sin^2(c)\right)\sin^3(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3*(a+b*sin(d*x+c)**2),x)
```

```
[Out] Piecewise((-a*sin(c + d*x)**2*cos(c + d*x)/d - 2*a*cos(c + d*x)**3/(3*d) -
b*sin(c + d*x)**4*cos(c + d*x)/d - 4*b*sin(c + d*x)**2*cos(c + d*x)**3/(3*d
) - 8*b*cos(c + d*x)**5/(15*d), Ne(d, 0)), (x*(a + b*sin(c)**2)*sin(c)**3,
True))
```

Giac [A] time = 1.10308, size = 90, normalized size = 1.76

$$-\frac{b\cos\left(dx+c\right)^{5}}{5\,d}+\frac{a\cos\left(dx+c\right)^{3}}{3\,d}+\frac{2\,b\cos\left(dx+c\right)^{3}}{3\,d}-\frac{a\cos\left(dx+c\right)}{d}-\frac{b\cos\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/5*b*cos(d*x + c)^5/d + 1/3*a*cos(d*x + c)^3/d + 2/3*b*cos(d*x + c)^3/d -
 a*cos(d*x + c)/d - b*cos(d*x + c)/d
```

3.65 $\int \sin(c+dx)\left(a+b\sin^2(c+dx)\right)dx$

Optimal. Leaf size=31

$$\frac{b\cos^3(c+dx)}{3d}-\frac{(a+b)\cos(c+dx)}{d}$$

```
[Out] -(((a + b)*Cos[c + d*x])/d) + (b*Cos[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0224598, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {3013}

$$\frac{b\cos^3(c+dx)}{3d}-\frac{(a+b)\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -(((a + b)*Cos[c + d*x])/d) + (b*Cos[c + d*x]^3)/(3*d)
```

Rule 3013

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sin(c+dx)\left(a+b\sin^2(c+dx)\right)dx&=-\frac{\text{Subst}\left(\int\left(a+b-bx^2\right)dx,x,\cos(c+dx)\right)}{d}\\&=-\frac{(a+b)\cos(c+dx)}{d}+\frac{b\cos^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.0194513, size = 54, normalized size = 1.74

$$\frac{a\sin(c)\sin(dx)}{d}-\frac{a\cos(c)\cos(dx)}{d}-\frac{3b\cos(c+dx)}{4d}+\frac{b\cos(3(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -((a*Cos[c]*Cos[d*x])/d) - (3*b*Cos[c + d*x])/(4*d) + (b*Cos[3*(c + d*x)])/
(12*d) + (a*Sin[c]*Sin[d*x])/d
```

Maple [A] time = 0.028, size = 34, normalized size = 1.1

$$\frac{1}{d}\left(-\frac{b\left(2+(\sin(dx+c))^2\right)\cos(dx+c)}{3}-\cos(dx+c)\,a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a+sin(d*x+c)^2*b),x)
```

```
[Out] 1/d*(-1/3*b*(2+sin(d*x+c)^2)*cos(d*x+c)-cos(d*x+c)*a)
```

Maxima [A] time = 0.936997, size = 46, normalized size = 1.48

$$\frac{\left(\cos\left(dx+c\right)^{3}-3\,\cos\left(dx+c\right)\right)b-3\,a\cos\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/3*((cos(d*x + c)^3 - 3*cos(d*x + c))*b - 3*a*cos(d*x + c))/d
```

Fricas [A] time = 1.70839, size = 69, normalized size = 2.23

$$\frac{b\cos\left(dx+c\right)^{3}-3\left(a+b\right)\cos\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/3*(b*cos(d*x + c)^3 - 3*(a + b)*cos(d*x + c))/d
```

Sympy [A] time = 0.901478, size = 58, normalized size = 1.87

$$\begin{cases} -\frac{a\cos\left(c+dx\right)}{d}-\frac{b\sin^{2}\left(c+dx\right)\cos\left(c+dx\right)}{d}-\frac{2b\cos^{3}\left(c+dx\right)}{3d} & \text{for } d\neq 0\\ x\left(a+b\sin^{2}\left(c\right)\right)\sin\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*sin(d*x+c)**2),x)
```

```
[Out] Piecewise((-a*cos(c + d*x)/d - b*sin(c + d*x)**2*cos(c + d*x)/d - 2*b*cos(c
 + d*x)**3/(3*d), Ne(d, 0)), (x*(a + b*sin(c)**2)*sin(c), True))
```

Giac [A] time = 1.13028, size = 54, normalized size = 1.74

$$\frac{1}{3}\left(\frac{\cos\left(dx+c\right)^{3}}{d}-\frac{3\,\cos\left(dx+c\right)}{d}\right)b-\frac{a\cos\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/3*(cos(d*x + c)^3/d - 3*cos(d*x + c)/d)*b - a*cos(d*x + c)/d
```

3.66 $\int \csc(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx$

Optimal. Leaf size=26

$$-\frac{a\tanh^{-1}(\cos(c+dx))}{d}-\frac{b\cos(c+dx)}{d}$$

```
[Out] -((a*ArcTanh[Cos[c + d*x]])/d) - (b*Cos[c + d*x])/d
```

Rubi [A] time = 0.0248953, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3014, 3770}

$$-\frac{a\tanh^{-1}(\cos(c+dx))}{d}-\frac{b\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -((a*ArcTanh[Cos[c + d*x]])/d) - (b*Cos[c + d*x])/d
```

Rule 3014

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*
(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m
, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx &= -\frac{b\cos(c+dx)}{d}+a\int \csc(c+dx)\,dx\\ &= -\frac{a\tanh^{-1}(\cos(c+dx))}{d}-\frac{b\cos(c+dx)}{d}\end{aligned}$$

Mathematica [B] time = 0.0319162, size = 63, normalized size = 2.42

$$\frac{a\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{a\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{b\sin(c)\sin(dx)}{d}-\frac{b\cos(c)\cos(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -((b*Cos[c]*Cos[d*x])/d) - (a*Log[Cos[c/2 + (d*x)/2]])/d + (a*Log[Sin[c/2 +
 (d*x)/2]])/d + (b*Sin[c]*Sin[d*x])/d
```

Maple [A] time = 0.046, size = 35, normalized size = 1.4

$$-\frac{b\cos(dx+c)}{d}+\frac{a\ln(\csc(dx+c)-\cot(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+sin(d*x+c)^2*b),x)
```

```
[Out] -b*cos(d*x+c)/d+1/d*a*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 0.94894, size = 51, normalized size = 1.96

$$-\frac{2\,b\cos(dx+c)+a\log(\cos(dx+c)+1)-a\log(\cos(dx+c)-1)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*cos(d*x + c) + a*log(cos(d*x + c) + 1) - a*log(cos(d*x + c) - 1))
/d
```

Fricas [A] time = 1.73244, size = 124, normalized size = 4.77

$$-\frac{2\,b\cos(dx+c)+a\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-a\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*cos(d*x + c) + a*log(1/2*cos(d*x + c) + 1/2) - a*log(-1/2*cos(d*x
 + c) + 1/2))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sin^2(c+dx)\right)\csc(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sin(c + d*x)**2)*csc(c + d*x), x)
```

Giac [B] time = 1.15154, size = 78, normalized size = 3.

$$\frac{a\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)+\frac{4b}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*(a*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) + 4*b/((cos(d*x +
c) - 1)/(cos(d*x + c) + 1) - 1))/d
```

3.67 $\int \csc^3(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx$

Optimal. Leaf size=40

$$-\frac{(a+2b)\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d}$$

```
[Out] -((a + 2*b)*ArcTanh[Cos[c + d*x]])/(2*d) - (a*Cot[c + d*x]*Csc[c + d*x])/(2
*d)
```

Rubi [A] time = 0.0308706, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3012, 3770}

$$-\frac{(a+2b)\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -((a + 2*b)*ArcTanh[Cos[c + d*x]])/(2*d) - (a*Cot[c + d*x]*Csc[c + d*x])/(2
*d)
```

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m
 + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc^3(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx &= -\frac{a\cot(c+dx)\csc(c+dx)}{2d}+\frac{1}{2}(a+2b)\int \csc(c+dx)\,dx\\ &= -\frac{(a+2b)\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d}\end{aligned}$$

Mathematica [B] time = 0.0373156, size = 118, normalized size = 2.95

$$-\frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+\frac{b\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{b}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -(a*Csc[(c + d*x)/2]^2)/(8*d) - (b*Log[Cos[c/2 + (d*x)/2]])/d - (a*Log[Cos[
(c + d*x)/2]])/(2*d) + (b*Log[Sin[c/2 + (d*x)/2]])/d + (a*Log[Sin[(c + d*x)
/2]])/(2*d) + (a*Sec[(c + d*x)/2]^2)/(8*d)
```

Maple [A] time = 0.055, size = 63, normalized size = 1.6

$$-\frac{\cot(dx+c)\,a\csc(dx+c)}{2d}+\frac{a\ln(\csc(dx+c)-\cot(dx+c))}{2d}+\frac{b\ln(\csc(dx+c)-\cot(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+sin(d*x+c)^2*b),x)
```

```
[Out] -1/2*a*cot(d*x+c)*csc(d*x+c)/d+1/2/d*a*ln(csc(d*x+c)-cot(d*x+c))+1/d*b*ln(c
sc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 0.936022, size = 78, normalized size = 1.95

$$-\frac{(a+2b)\log(\cos(dx+c)+1)-(a+2b)\log(\cos(dx+c)-1)-\frac{2a\cos(dx+c)}{\cos(dx+c)^2-1}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/4*((a + 2*b)*log(cos(d*x + c) + 1) - (a + 2*b)*log(cos(d*x + c) - 1) - 2
*a*cos(d*x + c)/(cos(d*x + c)^2 - 1))/d
```

Fricas [B] time = 1.70168, size = 246, normalized size = 6.15

$$\frac{2a\cos(dx+c)-\left((a+2b)\cos(dx+c)^2-a-2b\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+\left((a+2b)\cos(dx+c)^2-a-2b\right)\log\left(-\right.}{4\left(d\cos(dx+c)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*cos(d*x + c) - ((a + 2*b)*cos(d*x + c)^2 - a - 2*b)*log(1/2*cos(d*
x + c) + 1/2) + ((a + 2*b)*cos(d*x + c)^2 - a - 2*b)*log(-1/2*cos(d*x + c)
+ 1/2))/(d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(a+b*sin(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18255, size = 163, normalized size = 4.08

$$\frac{2\,(a+2\,b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)+\frac{\left(a-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{4\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{\cos(dx+c)-1}-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/8*(2*(a + 2*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) + (a - 2
*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 4*b*(cos(d*x + c) - 1)/(cos(d*x
+ c) + 1))*(cos(d*x + c) + 1)/(cos(d*x + c) - 1) - a*(cos(d*x + c) - 1)/(co
s(d*x + c) + 1))/d
```

3.68 $\int \sin^4(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx$

Optimal. Leaf size=89

$$-\frac{(6a+5b)\sin^3(c+dx)\cos(c+dx)}{24d}-\frac{(6a+5b)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}x(6a+5b)-\frac{b\sin^5(c+dx)\cos(c+dx)}{6d}$$

```
[Out] ((6*a + 5*b)*x)/16 - ((6*a + 5*b)*Cos[c + d*x]*Sin[c + d*x])/(16*d) - ((6*a
 + 5*b)*Cos[c + d*x]*Sin[c + d*x]^3)/(24*d) - (b*Cos[c + d*x]*Sin[c + d*x]^
5)/(6*d)
```

Rubi [A] time = 0.0528693, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3014, 2635, 8}

$$-\frac{(6a+5b)\sin^3(c+dx)\cos(c+dx)}{24d}-\frac{(6a+5b)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}x(6a+5b)-\frac{b\sin^5(c+dx)\cos(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4*(a + b*Sin[c + d*x]^2),x]
```

```
[Out] ((6*a + 5*b)*x)/16 - ((6*a + 5*b)*Cos[c + d*x]*Sin[c + d*x])/(16*d) - ((6*a
 + 5*b)*Cos[c + d*x]*Sin[c + d*x]^3)/(24*d) - (b*Cos[c + d*x]*Sin[c + d*x]^
5)/(6*d)
```

Rule 3014

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*
(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m
, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sin^4(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx &= -\frac{b\cos(c+dx)\sin^5(c+dx)}{6d}+\frac{1}{6}(6a+5b)\int \sin^4(c+dx)\,dx\\ &= -\frac{(6a+5b)\cos(c+dx)\sin^3(c+dx)}{24d}-\frac{b\cos(c+dx)\sin^5(c+dx)}{6d}+\frac{1}{8}(6a+5\\ &= -\frac{(6a+5b)\cos(c+dx)\sin(c+dx)}{16d}-\frac{(6a+5b)\cos(c+dx)\sin^3(c+dx)}{24d}-\frac{b}{}\\ &= \frac{1}{16}(6a+5b)x-\frac{(6a+5b)\cos(c+dx)\sin(c+dx)}{16d}-\frac{(6a+5b)\cos(c+dx)\sin}{24d}\end{aligned}$$

Mathematica [A] time = 0.111477, size = 70, normalized size = 0.79

$$\frac{-3(16a+15b)\sin(2(c+dx))+(6a+9b)\sin(4(c+dx))+72ac+72adx-b\sin(6(c+dx))+60bc+60bdx}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4*(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (72*a*c + 60*b*c + 72*a*d*x + 60*b*d*x - 3*(16*a + 15*b)*Sin[2*(c + d*x)] +
 (6*a + 9*b)*Sin[4*(c + d*x)] - b*Sin[6*(c + d*x)])/(192*d)
```

Maple [A] time = 0.029, size = 86, normalized size = 1.

$$\frac{1}{d}\left(b\left(-\frac{\cos(dx+c)}{6}\left((\sin(dx+c))^5+\frac{5(\sin(dx+c))^3}{4}+\frac{15\sin(dx+c)}{8}\right)+\frac{5dx}{16}+\frac{5c}{16}\right)+a\left(-\frac{\cos(dx+c)}{4}\left((\sin(dx+c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4*(a+sin(d*x+c)^2*b),x)
```

```
[Out] 1/d*(b*(-1/6*(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/1
6*d*x+5/16*c)+a*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*
c))
```

Maxima [A] time = 1.43799, size = 140, normalized size = 1.57

$$\frac{3(dx+c)(6a+5b)-\frac{3(10a+11b)\tan(dx+c)^5+8(6a+5b)\tan(dx+c)^3+3(6a+5b)\tan(dx+c)}{\tan(dx+c)^6+3\tan(dx+c)^4+3\tan(dx+c)^2+1}}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/48*(3*(d*x + c)*(6*a + 5*b) - (3*(10*a + 11*b)*tan(d*x + c)^5 + 8*(6*a +
5*b)*tan(d*x + c)^3 + 3*(6*a + 5*b)*tan(d*x + c))/(tan(d*x + c)^6 + 3*tan(d
*x + c)^4 + 3*tan(d*x + c)^2 + 1))/d
```

Fricas [A] time = 1.66861, size = 171, normalized size = 1.92

$$\frac{3(6a+5b)dx-\left(8b\cos(dx+c)^5-2(6a+13b)\cos(dx+c)^3+3(10a+11b)\cos(dx+c)\right)\sin(dx+c)}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/48*(3*(6*a + 5*b)*d*x - (8*b*cos(d*x + c)^5 - 2*(6*a + 13*b)*cos(d*x + c)
^3 + 3*(10*a + 11*b)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 10.6968, size = 258, normalized size = 2.9

$$\begin{cases} \frac{3ax\sin^4(c+dx)}{8} + \frac{3ax\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3ax\cos^4(c+dx)}{8} - \frac{5a\sin^3(c+dx)\cos(c+dx)}{8d} - \frac{3a\sin(c+dx)\cos^3(c+dx)}{8d} + \frac{5bx\sin^6(c+dx)}{16} + \frac{15b \\ x\left(a+b\sin^2(c)\right)\sin^4(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4*(a+b*sin(d*x+c)**2),x)
```

```
[Out] Piecewise((3*a*x*sin(c + d*x)**4/8 + 3*a*x*sin(c + d*x)**2*cos(c + d*x)**2/
4 + 3*a*x*cos(c + d*x)**4/8 - 5*a*sin(c + d*x)**3*cos(c + d*x)/(8*d) - 3*a*
sin(c + d*x)*cos(c + d*x)**3/(8*d) + 5*b*x*sin(c + d*x)**6/16 + 15*b*x*sin(
c + d*x)**4*cos(c + d*x)**2/16 + 15*b*x*sin(c + d*x)**2*cos(c + d*x)**4/16
+ 5*b*x*cos(c + d*x)**6/16 - 11*b*sin(c + d*x)**5*cos(c + d*x)/(16*d) - 5*b
*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) - 5*b*sin(c + d*x)*cos(c + d*x)**5/(
16*d), Ne(d, 0)), (x*(a + b*sin(c)**2)*sin(c)**4, True))
```

Giac [A] time = 1.12248, size = 92, normalized size = 1.03

$$\frac{1}{16}(6a+5b)x - \frac{b\sin(6dx+6c)}{192d} + \frac{(2a+3b)\sin(4dx+4c)}{64d} - \frac{(16a+15b)\sin(2dx+2c)}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/16*(6*a + 5*b)*x - 1/192*b*sin(6*d*x + 6*c)/d + 1/64*(2*a + 3*b)*sin(4*d*
x + 4*c)/d - 1/64*(16*a + 15*b)*sin(2*d*x + 2*c)/d
```

3.69 $\int \sin^2(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx$

Optimal. Leaf size=61

$$-\frac{(4a+3b)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x(4a+3b)-\frac{b\sin^3(c+dx)\cos(c+dx)}{4d}$$

```
[Out] ((4*a + 3*b)*x)/8 - ((4*a + 3*b)*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (b*Cos[
c + d*x]*Sin[c + d*x]^3)/(4*d)
```

Rubi [A] time = 0.0404287, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3014, 2635, 8}

$$-\frac{(4a+3b)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x(4a+3b)-\frac{b\sin^3(c+dx)\cos(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*(a + b*Sin[c + d*x]^2),x]
```

```
[Out] ((4*a + 3*b)*x)/8 - ((4*a + 3*b)*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (b*Cos[
c + d*x]*Sin[c + d*x]^3)/(4*d)
```

Rule 3014

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*
(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m
, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sin^2(c+dx)\left(a+b\sin^2(c+dx)\right)dx &= -\frac{b\cos(c+dx)\sin^3(c+dx)}{4d}+\frac{1}{4}(4a+3b)\int \sin^2(c+dx)\,dx\\ &= -\frac{(4a+3b)\cos(c+dx)\sin(c+dx)}{8d}-\frac{b\cos(c+dx)\sin^3(c+dx)}{4d}+\frac{1}{8}(4a+3b)\int\\ &= \frac{1}{8}(4a+3b)x-\frac{(4a+3b)\cos(c+dx)\sin(c+dx)}{8d}-\frac{b\cos(c+dx)\sin^3(c+dx)}{4d}\end{aligned}$$

Mathematica [A] time = 0.0938271, size = 45, normalized size = 0.74

$$\frac{4(4a+3b)(c+dx)-8(a+b)\sin(2(c+dx))+b\sin(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2*(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (4*(4*a + 3*b)*(c + d*x) - 8*(a + b)*Sin[2*(c + d*x)] + b*Sin[4*(c + d*x)])
/(32*d)
```

Maple [A] time = 0.027, size = 65, normalized size = 1.1

$$\frac{1}{d}\left(b\left(-\frac{\cos(dx+c)}{4}\left((\sin(dx+c))^3+\frac{3\sin(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+a\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2*(a+sin(d*x+c)^2*b),x)
```

```
[Out] 1/d*(b*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c)+a*(-1/
2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A] time = 1.43521, size = 100, normalized size = 1.64

$$\frac{(dx+c)(4a+3b)-\frac{(4a+5b)\tan(dx+c)^3+(4a+3b)\tan(dx+c)}{\tan(dx+c)^4+2\tan(dx+c)^2+1}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/8*((d*x + c)*(4*a + 3*b) - ((4*a + 5*b)*tan(d*x + c)^3 + (4*a + 3*b)*tan(
d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x + c)^2 + 1))/d
```

Fricas [A] time = 1.64944, size = 119, normalized size = 1.95

$$\frac{(4a+3b)dx+\left(2b\cos(dx+c)^3-(4a+5b)\cos(dx+c)\right)\sin(dx+c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/8*((4*a + 3*b)*d*x + (2*b*cos(d*x + c)^3 - (4*a + 5*b)*cos(d*x + c))*sin(
d*x + c))/d
```

Sympy [A] time = 2.12638, size = 158, normalized size = 2.59

$$\begin{cases}\frac{ax\sin^2(c+dx)}{2}+\frac{ax\cos^2(c+dx)}{2}-\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{3bx\sin^4(c+dx)}{8}+\frac{3bx\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{3bx\cos^4(c+dx)}{8}-\frac{5b\sin^3(c+dx)\cos}{8d}\\ x\left(a+b\sin^2(c)\right)\sin^2(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2*(a+b*sin(d*x+c)**2),x)
```

```
[Out] Piecewise((a*x*sin(c + d*x)**2/2 + a*x*cos(c + d*x)**2/2 - a*sin(c + d*x)*c
os(c + d*x)/(2*d) + 3*b*x*sin(c + d*x)**4/8 + 3*b*x*sin(c + d*x)**2*cos(c +
 d*x)**2/4 + 3*b*x*cos(c + d*x)**4/8 - 5*b*sin(c + d*x)**3*cos(c + d*x)/(8*
d) - 3*b*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a + b*sin(c)**2
)*sin(c)**2, True))
```

Giac [A] time = 1.11987, size = 58, normalized size = 0.95

$$\frac{1}{8}(4a + 3b)x + \frac{b\sin(4dx + 4c)}{32d} - \frac{(a + b)\sin(2dx + 2c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/8*(4*a + 3*b)*x + 1/32*b*sin(4*d*x + 4*c)/d - 1/4*(a + b)*sin(2*d*x + 2*c
)/d
```

3.70 $\int \left(a+b\sin^2(c+dx)\right) dx$

Optimal. Leaf size=30

$$ax-\frac{b\sin(c+dx)\cos(c+dx)}{2d}+\frac{bx}{2}$$

```
[Out] a*x + (b*x)/2 - (b*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.0148417, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2635, 8}

$$ax-\frac{b\sin(c+dx)\cos(c+dx)}{2d}+\frac{bx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Sin[c + d*x]^2,x]
```

```
[Out] a*x + (b*x)/2 - (b*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sin^2(c+dx)\right) dx &= ax+b\int \sin^2(c+dx)\,dx\\ &= ax-\frac{b\cos(c+dx)\sin(c+dx)}{2d}+\frac{1}{2}b\int 1\,dx\\ &= ax+\frac{bx}{2}-\frac{b\cos(c+dx)\sin(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.0282181, size = 33, normalized size = 1.1

$$ax+\frac{b(c+dx)}{2d}-\frac{b\sin(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Sin[c + d*x]^2,x]
```

```
[Out] a*x + (b*(c + d*x))/(2*d) - (b*Sin[2*(c + d*x)])/(4*d)
```

Maple [A] time = 0.022, size = 32, normalized size = 1.1

$$ax+\frac{b}{d}\left(-\frac{\cos\left(dx+c\right)\sin\left(dx+c\right)}{2}+\frac{dx}{2}+\frac{c}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+sin(d*x+c)^2*b,x)
```

```
[Out] a*x+b/d*(-1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)
```

Maxima [A] time = 0.944351, size = 39, normalized size = 1.3

$$ax+\frac{(2\,dx+2\,c-\sin\left(2\,dx+2\,c\right))b}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] a*x + 1/4*(2*d*x + 2*c - sin(2*d*x + 2*c))*b/d
```

Fricas [A] time = 1.57504, size = 72, normalized size = 2.4

$$\frac{(2\,a+b)dx-b\cos\left(dx+c\right)\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((2*a + b)*d*x - b*cos(d*x + c)*sin(d*x + c))/d
```

Sympy [A] time = 0.429219, size = 51, normalized size = 1.7

$$ax+b\left(\begin{cases}\frac{x\sin^2\left(c+dx\right)}{2}+\frac{x\cos^2\left(c+dx\right)}{2}-\frac{\sin\left(c+dx\right)\cos\left(c+dx\right)}{2d} & \text{for } d\neq 0\\ x\sin^2\left(c\right) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(d*x+c)**2,x)
```

```
[Out] a*x + b*Piecewise((x*sin(c + d*x)**2/2 + x*cos(c + d*x)**2/2 - sin(c + d*x)
*cos(c + d*x)/(2*d), Ne(d, 0)), (x*sin(c)**2, True))
```

Giac [A] time = 1.12648, size = 34, normalized size = 1.13

$$\frac{1}{4}\,b\left(2\,x-\frac{\sin\left(2\,dx+2\,c\right)}{d}\right)+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*b*(2*x - sin(2*d*x + 2*c)/d) + a*x
```

3.71 $\int \csc^2(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx$

Optimal. Leaf size=16

$$bx-\frac{a\cot(c+dx)}{d}$$

```
[Out] b*x - (a*Cot[c + d*x])/d
```

Rubi [A] time = 0.0232825, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3012, 8}

$$bx-\frac{a\cot(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + b*Sin[c + d*x]^2),x]

[Out] b*x - (a*Cot[c + d*x])/d
```

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m
 + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \csc^2(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx&=-\frac{a\cot(c+dx)}{d}+b\int 1\,dx\\&=bx-\frac{a\cot(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0182045, size = 16, normalized size = 1.

$$bx-\frac{a\cot(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*(a + b*Sin[c + d*x]^2),x]

[Out] b*x - (a*Cot[c + d*x])/d
```

Maple [A] time = 0.046, size = 22, normalized size = 1.4

$$\frac{-\cot(dx+c)\,a+(dx+c)\,b}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+sin(d*x+c)^2*b),x)
```

```
[Out] 1/d*(-cot(d*x+c)*a+(d*x+c)*b)
```

Maxima [A] time = 1.42677, size = 31, normalized size = 1.94

$$\frac{(dx+c)b-\frac{a}{\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] ((d*x + c)*b - a/tan(d*x + c))/d
```

Fricas [A] time = 1.5839, size = 76, normalized size = 4.75

$$\frac{bdx\sin(dx+c)-a\cos(dx+c)}{d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] (b*d*x*sin(d*x + c) - a*cos(d*x + c))/(d*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sin^2(c+dx)\right)\csc^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sin(c + d*x)**2)*csc(c + d*x)**2, x)
```

Giac [B] time = 1.15831, size = 53, normalized size = 3.31

$$\frac{2\,(dx+c)b+a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*(2*(d*x + c)*b + a*tan(1/2*d*x + 1/2*c) - a/tan(1/2*d*x + 1/2*c))/d
```

3.72 $\int \csc^4(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx$

Optimal. Leaf size=43

$$-\frac{(2a+3b)\cot(c+dx)}{3d}-\frac{a\cot(c+dx)\csc^2(c+dx)}{3d}$$

```
[Out] -((2*a + 3*b)*Cot[c + d*x])/(3*d) - (a*Cot[c + d*x]*Csc[c + d*x]^2)/(3*d)
```

Rubi [A] time = 0.0357116, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3012, 3767, 8}

$$-\frac{(2a+3b)\cot(c+dx)}{3d}-\frac{a\cot(c+dx)\csc^2(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -((2*a + 3*b)*Cot[c + d*x])/(3*d) - (a*Cot[c + d*x]*Csc[c + d*x]^2)/(3*d)
```

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m
 + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \csc^4(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx &= -\frac{a\cot(c+dx)\csc^2(c+dx)}{3d}+\frac{1}{3}(2a+3b)\int \csc^2(c+dx)\,dx\\
&= -\frac{a\cot(c+dx)\csc^2(c+dx)}{3d}-\frac{(2a+3b)\operatorname{Subst}\left(\int 1\,dx,x,\cot(c+dx)\right)}{3d}\\
&= -\frac{(2a+3b)\cot(c+dx)}{3d}-\frac{a\cot(c+dx)\csc^2(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] time = 0.0264045, size = 49, normalized size = 1.14

$$-\frac{2a\cot(c+dx)}{3d}-\frac{a\cot(c+dx)\csc^2(c+dx)}{3d}-\frac{b\cot(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (-2*a*Cot[c + d*x])/(3*d) - (b*Cot[c + d*x])/d - (a*Cot[c + d*x]*Csc[c + d*
x]^2)/(3*d)
```

Maple [A] time = 0.052, size = 35, normalized size = 0.8

$$\frac{1}{d}\left(a\left(-\frac{2}{3}-\frac{(\csc(dx+c))^2}{3}\right)\cot(dx+c)-b\cot(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+sin(d*x+c)^2*b),x)
```

```
[Out] 1/d*(a*(-2/3-1/3*csc(d*x+c)^2)*cot(d*x+c)-b*cot(d*x+c))
```

Maxima [A] time = 0.950237, size = 38, normalized size = 0.88

$$-\frac{3(a+b)\tan(dx+c)^2+a}{3d\tan(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*(a + b)*tan(d*x + c)^2 + a)/(d*tan(d*x + c)^3)
```

Fricas [A] time = 1.58397, size = 132, normalized size = 3.07

$$-\frac{(2a+3b)\cos(dx+c)^3-3(a+b)\cos(dx+c)}{3\left(d\cos(dx+c)^2-d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/3*((2*a + 3*b)*cos(d*x + c)^3 - 3*(a + b)*cos(d*x + c))/((d*cos(d*x + c)
^2 - d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+b*sin(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15288, size = 50, normalized size = 1.16

$$-\frac{3\,a\tan\left(dx+c\right)^{2}+3\,b\tan\left(dx+c\right)^{2}+a}{3\,d\tan\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/3*(3*a*tan(d*x + c)^2 + 3*b*tan(d*x + c)^2 + a)/(d*tan(d*x + c)^3)
```

3.73 $\int \csc^6(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx$

Optimal. Leaf size=65

$$-\frac{(4a+5b)\cot^3(c+dx)}{15d}-\frac{(4a+5b)\cot(c+dx)}{5d}-\frac{a\cot(c+dx)\csc^4(c+dx)}{5d}$$

```
[Out] -((4*a + 5*b)*Cot[c + d*x])/(5*d) - ((4*a + 5*b)*Cot[c + d*x]^3)/(15*d) - (
a*Cot[c + d*x]*Csc[c + d*x]^4)/(5*d)
```

Rubi [A] time = 0.042355, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3012, 3767}

$$-\frac{(4a+5b)\cot^3(c+dx)}{15d}-\frac{(4a+5b)\cot(c+dx)}{5d}-\frac{a\cot(c+dx)\csc^4(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6*(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -((4*a + 5*b)*Cot[c + d*x])/(5*d) - ((4*a + 5*b)*Cot[c + d*x]^3)/(15*d) - (
a*Cot[c + d*x]*Csc[c + d*x]^4)/(5*d)
```

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m
 + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \csc^6(c+dx)\left(a+b\sin^2(c+dx)\right)dx &= -\frac{a\cot(c+dx)\csc^4(c+dx)}{5d}+\frac{1}{5}(4a+5b)\int \csc^4(c+dx)\,dx\\ &= -\frac{a\cot(c+dx)\csc^4(c+dx)}{5d}-\frac{(4a+5b)\operatorname{Subst}\left(\int\left(1+x^2\right)dx,x,\cot(c+dx)\right)}{5d}\\ &= -\frac{(4a+5b)\cot(c+dx)}{5d}-\frac{(4a+5b)\cot^3(c+dx)}{15d}-\frac{a\cot(c+dx)\csc^4(c+dx)}{5d}\end{aligned}$$

Mathematica [A] time = 0.028352, size = 95, normalized size = 1.46

$$-\frac{8a\cot(c+dx)}{15d}-\frac{a\cot(c+dx)\csc^4(c+dx)}{5d}-\frac{4a\cot(c+dx)\csc^2(c+dx)}{15d}-\frac{2b\cot(c+dx)}{3d}-\frac{b\cot(c+dx)\csc^2(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6*(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (-8*a*Cot[c + d*x])/(15*d) - (2*b*Cot[c + d*x])/(3*d) - (4*a*Cot[c + d*x]*C
sc[c + d*x]^2)/(15*d) - (b*Cot[c + d*x]*Csc[c + d*x]^2)/(3*d) - (a*Cot[c +
d*x]*Csc[c + d*x]^4)/(5*d)
```

Maple [A] time = 0.053, size = 56, normalized size = 0.9

$$\frac{1}{d}\left(a\left(-\frac{8}{15}-\frac{(\csc(dx+c))^4}{5}-\frac{4\,(\csc(dx+c))^2}{15}\right)\cot(dx+c)+b\left(-\frac{2}{3}-\frac{(\csc(dx+c))^2}{3}\right)\cot(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6*(a+sin(d*x+c)^2*b),x)
```

```
[Out] 1/d*(a*(-8/15-1/5*csc(d*x+c)^4-4/15*csc(d*x+c)^2)*cot(d*x+c)+b*(-2/3-1/3*cs
c(d*x+c)^2)*cot(d*x+c))
```

Maxima [A] time = 0.943166, size = 61, normalized size = 0.94

$$-\frac{15\,(a+b)\tan(dx+c)^4+5\,(2\,a+b)\tan(dx+c)^2+3\,a}{15\,d\tan(dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*(a + b)*tan(d*x + c)^4 + 5*(2*a + b)*tan(d*x + c)^2 + 3*a)/(d*tan
(d*x + c)^5)
```

Fricas [A] time = 1.61107, size = 208, normalized size = 3.2

$$-\frac{2\,(4\,a+5\,b)\cos(dx+c)^5-5\,(4\,a+5\,b)\cos(dx+c)^3+15\,(a+b)\cos(dx+c)}{15\left(d\cos(dx+c)^4-2\,d\cos(dx+c)^2+d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/15*(2*(4*a + 5*b)*cos(d*x + c)^5 - 5*(4*a + 5*b)*cos(d*x + c)^3 + 15*(a
+ b)*cos(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c
))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6*(a+b*sin(d*x+c)**2),x)
```

[Out] Timed out

Giac [A] time = 1.1379, size = 82, normalized size = 1.26

$$-\frac{15\,a\tan\left(dx+c\right)^{4}+15\,b\tan\left(dx+c\right)^{4}+10\,a\tan\left(dx+c\right)^{2}+5\,b\tan\left(dx+c\right)^{2}+3\,a}{15\,d\tan\left(dx+c\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/15*(15*a*tan(d*x + c)^4 + 15*b*tan(d*x + c)^4 + 10*a*tan(d*x + c)^2 + 5*
b*tan(d*x + c)^2 + 3*a)/(d*tan(d*x + c)^5)
```

3.74 $\int \left(a+b\sin^2(x)\right)\,dx$

Optimal. Leaf size=19

$$ax+\frac{bx}{2}-\frac{1}{2}b\sin(x)\cos(x)$$

```
[Out] a*x + (b*x)/2 - (b*Cos[x]*Sin[x])/2
```

Rubi [A] time = 0.0092947, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2635, 8}

$$ax+\frac{bx}{2}-\frac{1}{2}b\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Sin[x]^2,x]
```

```
[Out] a*x + (b*x)/2 - (b*Cos[x]*Sin[x])/2
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\sin^2(x)\right)\,dx &= ax+b\int \sin^2(x)\,dx\\
&= ax-\frac{1}{2}b\cos(x)\sin(x)+\frac{1}{2}b\int 1\,dx\\
&= ax+\frac{bx}{2}-\frac{1}{2}b\cos(x)\sin(x)
\end{aligned}$$

Mathematica [A] time = 0.0033692, size = 19, normalized size = 1.

$$ax+\frac{bx}{2}-\frac{1}{4}b\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Sin[x]^2,x]
```

```
[Out] a*x + (b*x)/2 - (b*Sin[2*x])/4
```

Maple [A] time = 0.017, size = 17, normalized size = 0.9

$$ax + b\left(-\frac{\sin(x)\cos(x)}{2} + \frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*sin(x)^2,x)
```

```
[Out] a*x+b*(-1/2*sin(x)*cos(x)+1/2*x)
```

Maxima [A] time = 0.942653, size = 23, normalized size = 1.21

$$\frac{1}{4}\,b(2\,x - \sin(2\,x)) + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*b*(2*x - sin(2*x)) + a*x
```

Fricas [A] time = 1.58573, size = 54, normalized size = 2.84

$$-\frac{1}{2}\,b\cos(x)\sin(x) + \frac{1}{2}\,(2\,a + b)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*b*cos(x)*sin(x) + 1/2*(2*a + b)*x
```

Sympy [A] time = 0.099959, size = 15, normalized size = 0.79

$$ax + b\left(\frac{x}{2} - \frac{\sin(x)\cos(x)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(x)**2,x)
```

```
[Out] a*x + b*(x/2 - sin(x)*cos(x)/2)
```

Giac [A] time = 1.13251, size = 23, normalized size = 1.21

$$\frac{1}{4}\,b(2\,x - \sin(2\,x)) + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(x)^2,x, algorithm="giac")

[Out] 1/4*b*(2*x - sin(2*x)) + a*x
```

3.75 $\int \left(a+b\sin^2(x)\right)^2\,dx$

Optimal. Leaf size=50

$$\frac{1}{8}x\left(8a^2+8ab+3b^2\right)-\frac{1}{8}b(8a+3b)\sin(x)\cos(x)-\frac{1}{4}b^2\sin^3(x)\cos(x)$$

```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*x)/8 - (b*(8*a + 3*b)*Cos[x]*Sin[x])/8 - (b^2*Cos[
x]*Sin[x]^3)/4
```

Rubi [A] time = 0.0157437, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3179}

$$\frac{1}{8}x\left(8a^2+8ab+3b^2\right)-\frac{1}{8}b(8a+3b)\sin(x)\cos(x)-\frac{1}{4}b^2\sin^3(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[x]^2)^2,x]
```

```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*x)/8 - (b*(8*a + 3*b)*Cos[x]*Sin[x])/8 - (b^2*Cos[
x]*Sin[x]^3)/4
```

Rule 3179

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^2, x_Symbol] :> Simp[((8*a^2 +
 8*a*b + 3*b^2)*x)/8, x] + (-Simp[(b^2*Cos[e + f*x]*Sin[e + f*x]^3)/(4*f),
x] - Simp[(b*(8*a + 3*b)*Cos[e + f*x]*Sin[e + f*x])/(8*f), x]) /; FreeQ[{a,
 b, e, f}, x]
```

Rubi steps

$$\int \left(a+b\sin^2(x)\right)^2\,dx=\frac{1}{8}\left(8a^2+8ab+3b^2\right)x-\frac{1}{8}b(8a+3b)\cos(x)\sin(x)-\frac{1}{4}b^2\cos(x)\sin^3(x)$$

Mathematica [A] time = 0.0576988, size = 43, normalized size = 0.86

$$\frac{1}{32}\left(4x\left(8a^2+8ab+3b^2\right)-8b(2a+b)\sin(2x)+b^2\sin(4x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[x]^2)^2,x]
```

```
[Out] (4*(8*a^2 + 8*a*b + 3*b^2)*x - 8*b*(2*a + b)*Sin[2*x] + b^2*Sin[4*x])/32
```

Maple [A] time = 0.025, size = 42, normalized size = 0.8

$$b^2\left(-\frac{\cos(x)}{4}\left((\sin(x))^3+\frac{3\sin(x)}{2}\right)+\frac{3x}{8}\right)+2ab\left(-1/2\sin(x)\cos(x)+x/2\right)+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(x)^2)^2,x)
```

```
[Out] b^2*(-1/4*(sin(x)^3+3/2*sin(x))*cos(x)+3/8*x)+2*a*b*(-1/2*sin(x)*cos(x)+1/2
*x)+a^2*x
```

Maxima [A] time = 0.938432, size = 53, normalized size = 1.06

$$\frac{1}{32}\,b^2(12\,x+\sin(4\,x)-8\,\sin(2\,x))+\frac{1}{2}\,ab(2\,x-\sin(2\,x))+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/32*b^2*(12*x + sin(4*x) - 8*sin(2*x)) + 1/2*a*b*(2*x - sin(2*x)) + a^2*x
```

Fricas [A] time = 1.62652, size = 116, normalized size = 2.32

$$\frac{1}{8}\left(8\,a^2+8\,ab+3\,b^2\right)x+\frac{1}{8}\left(2\,b^2\cos(x)^3-\left(8\,ab+5\,b^2\right)\cos(x)\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(8*a^2 + 8*a*b + 3*b^2)*x + 1/8*(2*b^2*cos(x)^3 - (8*a*b + 5*b^2)*cos(x
))*sin(x)
```

Sympy [B] time = 1.28142, size = 110, normalized size = 2.2

$$a^2x+abx\sin^2(x)+abx\cos^2(x)-ab\sin(x)\cos(x)+\frac{3b^2x\sin^4(x)}{8}+\frac{3b^2x\sin^2(x)\cos^2(x)}{4}+\frac{3b^2x\cos^4(x)}{8}-\frac{5b^2\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)**2)**2,x)
```

```
[Out] a**2*x + a*b*x*sin(x)**2 + a*b*x*cos(x)**2 - a*b*sin(x)*cos(x) + 3*b**2*x*s
in(x)**4/8 + 3*b**2*x*sin(x)**2*cos(x)**2/4 + 3*b**2*x*cos(x)**4/8 - 5*b**2
*sin(x)**3*cos(x)/8 - 3*b**2*sin(x)*cos(x)**3/8
```

Giac [A] time = 1.13907, size = 57, normalized size = 1.14

$$\frac{1}{32}\,b^2\sin(4\,x)+\frac{1}{8}\left(8\,a^2+8\,ab+3\,b^2\right)x-\frac{1}{4}\left(2\,ab+b^2\right)\sin(2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/32*b^2*sin(4*x) + 1/8*(8*a^2 + 8*a*b + 3*b^2)*x - 1/4*(2*a*b + b^2)*sin(2
*x)
```

3.76 $\int \left(a+b\sin^2(x)\right)^3\,dx$

Optimal. Leaf size=87

$$\frac{1}{16}x(2a+b)\left(8a^2+8ab+5b^2\right)-\frac{1}{48}b\left(64a^2+54ab+15b^2\right)\sin(x)\cos(x)-\frac{5}{24}b^2(2a+b)\sin^3(x)\cos(x)-\frac{1}{6}b\sin(x)\cos$$

```
[Out] ((2*a + b)*(8*a^2 + 8*a*b + 5*b^2)*x)/16 - (b*(64*a^2 + 54*a*b + 15*b^2)*Co
s[x]*Sin[x])/48 - (5*b^2*(2*a + b)*Cos[x]*Sin[x]^3)/24 - (b*Cos[x]*Sin[x]*(
a + b*Sin[x]^2)^2)/6
```

Rubi [A] time = 0.0830855, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3180, 3169}

$$\frac{1}{16}x(2a+b)\left(8a^2+8ab+5b^2\right)-\frac{1}{48}b\left(64a^2+54ab+15b^2\right)\sin(x)\cos(x)-\frac{5}{24}b^2(2a+b)\sin^3(x)\cos(x)-\frac{1}{6}b\sin(x)\cos$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[x]^2)^3,x]
```

```
[Out] ((2*a + b)*(8*a^2 + 8*a*b + 5*b^2)*x)/16 - (b*(64*a^2 + 54*a*b + 15*b^2)*Co
s[x]*Sin[x])/48 - (5*b^2*(2*a + b)*Cos[x]*Sin[x]^3)/24 - (b*Cos[x]*Sin[x]*(
a + b*Sin[x]^2)^2)/6
```

Rule 3180

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p - 1))/(2*f*p), x] + Dist[
1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a + b
)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a +
 b, 0] && GtQ[p, 1]
```

Rule 3169

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)*((A_.) + (B_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[((4*A*(2*a + b) + B*(4*a + 3*b))*x)/8, x] +
(-Simp[(b*B*Cos[e + f*x]*Sin[e + f*x]^3)/(4*f), x] - Simp[((4*A*b + B*(4*a
+ 3*b))*Cos[e + f*x]*Sin[e + f*x])/(8*f), x]) /; FreeQ[{a, b, e, f, A, B},
x]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sin^2(x)\right)^3\,dx&=-\frac{1}{6}b\cos(x)\sin(x)\left(a+b\sin^2(x)\right)^2+\frac{1}{6}\int \left(a+b\sin^2(x)\right)\left(a(6a+b)+5b(2a+b)\sin^2(x)\right)\,dx\\&=\frac{1}{16}(2a+b)\left(8a^2+8ab+5b^2\right)x-\frac{1}{48}b\left(64a^2+54ab+15b^2\right)\cos(x)\sin(x)-\frac{5}{24}b^2(2a+b)\cos\end{aligned}$$

Mathematica [C] time = 0.10195, size = 80, normalized size = 0.92

$$\frac{1}{192}\left(12x(2a+b)\left(8a^2+8ab+5b^2\right)+9b^2(2a+b)\sin(4x)+9ib(4ia+(1+2i)b)(4a+(2+i)b)\sin(2x)+b^3(-\sin(6x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[x]^2)^3,x]
```

```
[Out] (12*(2*a + b)*(8*a^2 + 8*a*b + 5*b^2)*x + (9*I)*b*((4*I)*a + (1 + 2*I)*b)*(
4*a + (2 + I)*b)*Sin[2*x] + 9*b^2*(2*a + b)*Sin[4*x] - b^3*Sin[6*x])/192
```

Maple [A] time = 0.024, size = 73, normalized size = 0.8

$$b^3\left(-\frac{\cos(x)}{6}\left((\sin(x))^5+\frac{5\,(\sin(x))^3}{4}+\frac{15\sin(x)}{8}\right)+\frac{5\,x}{16}\right)+3\,ab^2\left(-1/4\,\left((\sin(x))^3+3/2\,\sin(x)\right)\cos(x)+3/8\,x\right)+3\,a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(x)^2)^3,x)
```

```
[Out] b^3*(-1/6*(sin(x)^5+5/4*sin(x)^3+15/8*sin(x))*cos(x)+5/16*x)+3*a*b^2*(-1/4*
(sin(x)^3+3/2*sin(x))*cos(x)+3/8*x)+3*a^2*b*(-1/2*sin(x)*cos(x)+1/2*x)+a^3*
x
```

Maxima [A] time = 0.93779, size = 96, normalized size = 1.1

$$\frac{1}{192}\left(4\,\sin(2\,x)^3+60\,x+9\,\sin(4\,x)-48\,\sin(2\,x)\right)b^3+\frac{3}{32}\,ab^2(12\,x+\sin(4\,x)-8\,\sin(2\,x))+\frac{3}{4}\,a^2b(2\,x-\sin(2\,x))+a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/192*(4*sin(2*x)^3 + 60*x + 9*sin(4*x) - 48*sin(2*x))*b^3 + 3/32*a*b^2*(12
*x + sin(4*x) - 8*sin(2*x)) + 3/4*a^2*b*(2*x - sin(2*x)) + a^3*x
```

Fricas [A] time = 1.66068, size = 207, normalized size = 2.38

$$\frac{1}{16}\left(16\,a^3+24\,a^2b+18\,ab^2+5\,b^3\right)x-\frac{1}{48}\left(8\,b^3\cos(x)^5-2\left(18\,ab^2+13\,b^3\right)\cos(x)^3+3\left(24\,a^2b+30\,ab^2+11\,b^3\right)\cos(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*x - 1/48*(8*b^3*cos(x)^5 - 2*(1
8*a*b^2 + 13*b^3)*cos(x)^3 + 3*(24*a^2*b + 30*a*b^2 + 11*b^3)*cos(x))*sin(x
)
```

Sympy [B] time = 5.62157, size = 246, normalized size = 2.83

$$a^3x+\frac{3a^2bx\sin^2(x)}{2}+\frac{3a^2bx\cos^2(x)}{2}-\frac{3a^2b\sin(x)\cos(x)}{2}+\frac{9ab^2x\sin^4(x)}{8}+\frac{9ab^2x\sin^2(x)\cos^2(x)}{4}+\frac{9ab^2x\cos^4(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)**2)**3,x)
```

```
[Out] a**3*x + 3*a**2*b*x*sin(x)**2/2 + 3*a**2*b*x*cos(x)**2/2 - 3*a**2*b*sin(x)*
cos(x)/2 + 9*a*b**2*x*sin(x)**4/8 + 9*a*b**2*x*sin(x)**2*cos(x)**2/4 + 9*a*
b**2*x*cos(x)**4/8 - 15*a*b**2*sin(x)**3*cos(x)/8 - 9*a*b**2*sin(x)*cos(x)*
*3/8 + 5*b**3*x*sin(x)**6/16 + 15*b**3*x*sin(x)**4*cos(x)**2/16 + 15*b**3*x
*sin(x)**2*cos(x)**4/16 + 5*b**3*x*cos(x)**6/16 - 11*b**3*sin(x)**5*cos(x)/
16 - 5*b**3*sin(x)**3*cos(x)**3/6 - 5*b**3*sin(x)*cos(x)**5/16
```

Giac [A] time = 1.10996, size = 103, normalized size = 1.18

$$-\frac{1}{192}\,b^3\sin(6\,x)+\frac{1}{16}\left(16\,a^3+24\,a^2b+18\,ab^2+5\,b^3\right)x+\frac{3}{64}\left(2\,ab^2+b^3\right)\sin(4\,x)-\frac{3}{64}\left(16\,a^2b+16\,ab^2+5\,b^3\right)\sin(2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)^3,x, algorithm="giac")
```

```
[Out] -1/192*b^3*sin(6*x) + 1/16*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*x + 3/64*
(2*a*b^2 + b^3)*sin(4*x) - 3/64*(16*a^2*b + 16*a*b^2 + 5*b^3)*sin(2*x)
```

3.77 $\int \left(a+b\sin^2(x)\right)^4\,dx$

Optimal. Leaf size=140

$$\frac{1}{128}x\left(288a^2b^2+256a^3b+128a^4+160ab^3+35b^4\right)-\frac{1}{192}b^2\left(104a^2+104ab+35b^2\right)\sin^3(x)\cos(x)-\frac{1}{384}b\left(808a^2b+60\right.$$

```
[Out] ((128*a^4 + 256*a^3*b + 288*a^2*b^2 + 160*a*b^3 + 35*b^4)*x)/128 - (b*(608*
a^3 + 808*a^2*b + 480*a*b^2 + 105*b^3)*Cos[x]*Sin[x])/384 - (b^2*(104*a^2 +
 104*a*b + 35*b^2)*Cos[x]*Sin[x]^3)/192 - (7*b*(2*a + b)*Cos[x]*Sin[x]*(a +
 b*Sin[x]^2)^2)/48 - (b*Cos[x]*Sin[x]*(a + b*Sin[x]^2)^3)/8
```

Rubi [A] time = 0.166894, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3180, 3170, 3169}

$$\frac{1}{128}x\left(288a^2b^2+256a^3b+128a^4+160ab^3+35b^4\right)-\frac{1}{192}b^2\left(104a^2+104ab+35b^2\right)\sin^3(x)\cos(x)-\frac{1}{384}b\left(808a^2b+60\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[x]^2)^4,x]
```

```
[Out] ((128*a^4 + 256*a^3*b + 288*a^2*b^2 + 160*a*b^3 + 35*b^4)*x)/128 - (b*(608*
a^3 + 808*a^2*b + 480*a*b^2 + 105*b^3)*Cos[x]*Sin[x])/384 - (b^2*(104*a^2 +
 104*a*b + 35*b^2)*Cos[x]*Sin[x]^3)/192 - (7*b*(2*a + b)*Cos[x]*Sin[x]*(a +
 b*Sin[x]^2)^2)/48 - (b*Cos[x]*Sin[x]*(a + b*Sin[x]^2)^3)/8
```

Rule 3180

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p - 1))/(2*f*p), x] + Dist[
1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a + b
)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a +
 b, 0] && GtQ[p, 1]
```

Rule 3170

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[(B*Cos[e + f*x]*Sin[e + f*x]*(a + b*Si
n[e + f*x]^2)^p)/(2*f*(p + 1)), x] + Dist[1/(2*(p + 1)), Int[(a + b*Sin[e +
 f*x]^2)^(p - 1)*Simp[a*B + 2*a*A*(p + 1) + (2*A*b*(p + 1) + B*(b + 2*a*p +
 2*b*p))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && GtQ[
p, 0]
```

Rule 3169

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)*((A_.) + (B_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[((4*A*(2*a + b) + B*(4*a + 3*b))*x)/8, x] +
(-Simp[(b*B*Cos[e + f*x]*Sin[e + f*x]^3)/(4*f), x] - Simp[((4*A*b + B*(4*a
+ 3*b))*Cos[e + f*x]*Sin[e + f*x])/(8*f), x]) /; FreeQ[{a, b, e, f, A, B},
x]
```

Rubi steps

$$\int \left(a+b\sin^2(x)\right)^4 dx = -\frac{1}{8}b\cos(x)\sin(x)\left(a+b\sin^2(x)\right)^3 + \frac{1}{8}\int \left(a+b\sin^2(x)\right)^2 \left(a(8a+b)+7b(2a+b)\sin^2(x)\right) dx$$
$$= -\frac{7}{48}b(2a+b)\cos(x)\sin(x)\left(a+b\sin^2(x)\right)^2 - \frac{1}{8}b\cos(x)\sin(x)\left(a+b\sin^2(x)\right)^3 + \frac{1}{48}\int \left(a+b\right.$$
$$= \frac{1}{128}\left(128a^4+256a^3b+288a^2b^2+160ab^3+35b^4\right)x - \frac{1}{384}b\left(608a^3+808a^2b+480ab^2+105\right.$$

Mathematica [A] time = 0.14971, size = 113, normalized size = 0.81

$$\frac{24x\left(288a^2b^2+256a^3b+128a^4+160ab^3+35b^4\right)+24b^2\left(24a^2+24ab+7b^2\right)\sin(4x)-96b(2a+b)\left(16a^2+16ab+7\right.}{3072}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[x]^2)^4,x]
```

```
[Out] (24*(128*a^4 + 256*a^3*b + 288*a^2*b^2 + 160*a*b^3 + 35*b^4)*x - 96*b*(2*a
+ b)*(16*a^2 + 16*a*b + 7*b^2)*Sin[2*x] + 24*b^2*(24*a^2 + 24*a*b + 7*b^2)*
Sin[4*x] - 32*b^3*(2*a + b)*Sin[6*x] + 3*b^4*Sin[8*x])/3072
```

Maple [A] time = 0.026, size = 110, normalized size = 0.8

$$b^4\left(-\frac{\cos(x)}{8}\left((\sin(x))^7+\frac{7(\sin(x))^5}{6}+\frac{35(\sin(x))^3}{24}+\frac{35\sin(x)}{16}\right)+\frac{35x}{128}\right)+4ab^3\left(-1/6\left((\sin(x))^5+5/4(\sin(x))^3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(x)^2)^4,x)
```

```
[Out] b^4*(-1/8*(sin(x)^7+7/6*sin(x)^5+35/24*sin(x)^3+35/16*sin(x))*cos(x)+35/128
*x)+4*a*b^3*(-1/6*(sin(x)^5+5/4*sin(x)^3+15/8*sin(x))*cos(x)+5/16*x)+6*a^2*
b^2*(-1/4*(sin(x)^3+3/2*sin(x))*cos(x)+3/8*x)+4*a^3*b*(-1/2*sin(x)*cos(x)+1
/2*x)+a^4*x
```

Maxima [A] time = 0.955661, size = 146, normalized size = 1.04

$$\frac{1}{48}\left(4\sin(2x)^3+60x+9\sin(4x)-48\sin(2x)\right)ab^3+\frac{1}{3072}\left(128\sin(2x)^3+840x+3\sin(8x)+168\sin(4x)-7\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)^4,x, algorithm="maxima")
```

```
[Out] 1/48*(4*sin(2*x)^3 + 60*x + 9*sin(4*x) - 48*sin(2*x))*a*b^3 + 1/3072*(128*s
in(2*x)^3 + 840*x + 3*sin(8*x) + 168*sin(4*x) - 768*sin(2*x))*b^4 + 3/16*a^
2*b^2*(12*x + sin(4*x) - 8*sin(2*x)) + a^3*b*(2*x - sin(2*x)) + a^4*x
```

Fricas [A] time = 1.74816, size = 323, normalized size = 2.31

$$\frac{1}{128}\left(128a^4+256a^3b+288a^2b^2+160ab^3+35b^4\right)x+\frac{1}{384}\left(48b^4\cos(x)^7-8\left(32ab^3+25b^4\right)\cos(x)^5+2\left(288a^2b^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)^4,x, algorithm="fricas")
```

```
[Out] 1/128*(128*a^4 + 256*a^3*b + 288*a^2*b^2 + 160*a*b^3 + 35*b^4)*x + 1/384*(4
8*b^4*cos(x)^7 - 8*(32*a*b^3 + 25*b^4)*cos(x)^5 + 2*(288*a^2*b^2 + 416*a*b^
3 + 163*b^4)*cos(x)^3 - 3*(256*a^3*b + 480*a^2*b^2 + 352*a*b^3 + 93*b^4)*co
s(x))*sin(x)
```

Sympy [B] time = 18.4127, size = 410, normalized size = 2.93

$$a^4x + 2a^3bx\sin^2(x) + 2a^3bx\cos^2(x) - 2a^3b\sin(x)\cos(x) + \frac{9a^2b^2x\sin^4(x)}{4} + \frac{9a^2b^2x\sin^2(x)\cos^2(x)}{2} + \frac{9a^2b^2x\cos^4(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)**2)**4,x)
```

```
[Out] a**4*x + 2*a**3*b*x*sin(x)**2 + 2*a**3*b*x*cos(x)**2 - 2*a**3*b*sin(x)*cos(
x) + 9*a**2*b**2*x*sin(x)**4/4 + 9*a**2*b**2*x*sin(x)**2*cos(x)**2/2 + 9*a*
*2*b**2*x*cos(x)**4/4 - 15*a**2*b**2*sin(x)**3*cos(x)/4 - 9*a**2*b**2*sin(x
)*cos(x)**3/4 + 5*a*b**3*x*sin(x)**6/4 + 15*a*b**3*x*sin(x)**4*cos(x)**2/4
+ 15*a*b**3*x*sin(x)**2*cos(x)**4/4 + 5*a*b**3*x*cos(x)**6/4 - 11*a*b**3*si
n(x)**5*cos(x)/4 - 10*a*b**3*sin(x)**3*cos(x)**3/3 - 5*a*b**3*sin(x)*cos(x)
**5/4 + 35*b**4*x*sin(x)**8/128 + 35*b**4*x*sin(x)**6*cos(x)**2/32 + 105*b*
*4*x*sin(x)**4*cos(x)**4/64 + 35*b**4*x*sin(x)**2*cos(x)**6/32 + 35*b**4*x*
cos(x)**8/128 - 93*b**4*sin(x)**7*cos(x)/128 - 511*b**4*sin(x)**5*cos(x)**3
/384 - 385*b**4*sin(x)**3*cos(x)**5/384 - 35*b**4*sin(x)*cos(x)**7/128
```

Giac [A] time = 1.10373, size = 159, normalized size = 1.14

$$\frac{1}{1024}b^4\sin(8x) + \frac{1}{128}\left(128a^4 + 256a^3b + 288a^2b^2 + 160ab^3 + 35b^4\right)x - \frac{1}{96}\left(2ab^3 + b^4\right)\sin(6x) + \frac{1}{128}\left(24a^2b^2 + 24\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)^4,x, algorithm="giac")
```

```
[Out] 1/1024*b^4*sin(8*x) + 1/128*(128*a^4 + 256*a^3*b + 288*a^2*b^2 + 160*a*b^3
+ 35*b^4)*x - 1/96*(2*a*b^3 + b^4)*sin(6*x) + 1/128*(24*a^2*b^2 + 24*a*b^3
+ 7*b^4)*sin(4*x) - 1/32*(32*a^3*b + 48*a^2*b^2 + 30*a*b^3 + 7*b^4)*sin(2*x
)
```

3.78 $\int \frac{\sin^7(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=106

$$-\frac{(a^2-ab+b^2)\cos(c+dx)}{b^3d}+\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{b^{7/2}d\sqrt{a+b}}-\frac{(a-2b)\cos^3(c+dx)}{3b^2d}-\frac{\cos^5(c+dx)}{5bd}$$

```
[Out] (a^3*ArcTanh[(Sqrt[b]*Cos[c + d*x])/Sqrt[a + b]])/(b^(7/2)*Sqrt[a + b]*d) -
 ((a^2 - a*b + b^2)*Cos[c + d*x])/(b^3*d) - ((a - 2*b)*Cos[c + d*x]^3)/(3*b
^2*d) - Cos[c + d*x]^5/(5*b*d)
```

Rubi [A] time = 0.11261, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3186, 390, 208}

$$-\frac{(a^2-ab+b^2)\cos(c+dx)}{b^3d}+\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{b^{7/2}d\sqrt{a+b}}-\frac{(a-2b)\cos^3(c+dx)}{3b^2d}-\frac{\cos^5(c+dx)}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^7/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (a^3*ArcTanh[(Sqrt[b]*Cos[c + d*x])/Sqrt[a + b]])/(b^(7/2)*Sqrt[a + b]*d) -
 ((a^2 - a*b + b^2)*Cos[c + d*x])/(b^3*d) - ((a - 2*b)*Cos[c + d*x]^3)/(3*b
^2*d) - Cos[c + d*x]^5/(5*b*d)
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sin^7(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{(1-x^2)^3}{a+b-bx^2}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{a^2-ab+b^2}{b^3}+\frac{(a-2b)x^2}{b^2}+\frac{x^4}{b}-\frac{a^3}{b^3(a+b-bx^2)}\right)dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{(a^2-ab+b^2)\cos(c+dx)}{b^3d}-\frac{(a-2b)\cos^3(c+dx)}{3b^2d}-\frac{\cos^5(c+dx)}{5bd}+\frac{a^3\,\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx,\right.}{b^3d}$$

$$= \frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{b^{7/2}\sqrt{a+b}d}-\frac{(a^2-ab+b^2)\cos(c+dx)}{b^3d}-\frac{(a-2b)\cos^3(c+dx)}{3b^2d}-\frac{\cos^5(c+dx)}{5bd}$$

Mathematica [C] time = 1.39507, size = 180, normalized size = 1.7

$$\frac{-2\sqrt{b}\sqrt{-a-b}\cos(c+dx)\left(120a^2+4b(5a-7b)\cos(2(c+dx))-100ab+3b^2\cos(4(c+dx))+89b^2\right)-240a^3\tan^{-1}\left(\frac{\sqrt{b}-i}{}\right.}{240b^{7/2}d\sqrt{-a-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^7/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (-240*a^3*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]] - 240
*a^3*ArcTan[(Sqrt[b] + I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]] - 2*Sqrt[-
a - b]*Sqrt[b]*Cos[c + d*x]*(120*a^2 - 100*a*b + 89*b^2 + 4*(5*a - 7*b)*b*C
os[2*(c + d*x)] + 3*b^2*Cos[4*(c + d*x)]))/(240*Sqrt[-a - b]*b^(7/2)*d)
```

Maple [A] time = 0.085, size = 110, normalized size = 1.

$$\frac{1}{d}\left(-\frac{1}{b^3}\left(\frac{(\cos(dx+c))^5 b^2}{5}+\frac{ab(\cos(dx+c))^3}{3}-\frac{2(\cos(dx+c))^3 b^2}{3}+\cos(dx+c)a^2-ab\cos(dx+c)+\cos(dx+c)b^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^7/(a+sin(d*x+c)^2*b),x)
```

```
[Out] 1/d*(-1/b^3*(1/5*cos(d*x+c)^5*b^2+1/3*a*b*cos(d*x+c)^3-2/3*cos(d*x+c)^3*b^2
+cos(d*x+c)*a^2-a*b*cos(d*x+c)+cos(d*x+c)*b^2)+a^3/b^3/((a+b)*b)^(1/2)*arct
anh(cos(d*x+c)*b/((a+b)*b)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.85589, size = 626, normalized size = 5.91

$$\left[-\frac{6\left(ab^3+b^4\right)\cos\left(dx+c\right)^5-15\sqrt{ab+b^2}a^3\log\left(\frac{b\cos(dx+c)^2+2\sqrt{ab+b^2}\cos(dx+c)+a+b}{b\cos(dx+c)^2-a-b}\right)+10\left(a^2b^2-ab^3-2b^4\right)\cos\left(dx+c\right.}{30\left(ab^4+b^5\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/30*(6*(a*b^3 + b^4)*cos(d*x + c)^5 - 15*sqrt(a*b + b^2)*a^3*log((b*cos(
d*x + c)^2 + 2*sqrt(a*b + b^2)*cos(d*x + c) + a + b)/(b*cos(d*x + c)^2 - a
- b)) + 10*(a^2*b^2 - a*b^3 - 2*b^4)*cos(d*x + c)^3 + 30*(a^3*b + b^4)*cos(
d*x + c))/((a*b^4 + b^5)*d), -1/15*(3*(a*b^3 + b^4)*cos(d*x + c)^5 + 15*sqr
t(-a*b - b^2)*a^3*arctan(sqrt(-a*b - b^2)*cos(d*x + c)/(a + b)) + 5*(a^2*b^
2 - a*b^3 - 2*b^4)*cos(d*x + c)^3 + 15*(a^3*b + b^4)*cos(d*x + c))/((a*b^4
+ b^5)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**7/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.15983, size = 448, normalized size = 4.23

$$-\frac{\frac{15\,a^3\arctan\left(\frac{b\cos(dx+c)+a+b}{\sqrt{-ab-b^2}\cos(dx+c)+\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}b^3}-\frac{2\left(15\,a^2-10\,ab+8\,b^2-\frac{60\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{50\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{40\,b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{90\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{70\,ab(\cos(d}{(\cos(dx+}\right.}{b^3\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/15*(15*a^3*arctan((b*cos(d*x + c) + a + b)/(sqrt(-a*b - b^2)*cos(d*x + c
) + sqrt(-a*b - b^2)))/(sqrt(-a*b - b^2)*b^3) - 2*(15*a^2 - 10*a*b + 8*b^2
- 60*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 50*a*b*(cos(d*x + c) - 1)/
(cos(d*x + c) + 1) - 40*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 90*a^2*
(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 70*a*b*(cos(d*x + c) - 1)^2/(co
s(d*x + c) + 1)^2 + 80*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 60*a
^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 30*a*b*(cos(d*x + c) - 1)^3/
(cos(d*x + c) + 1)^3 + 15*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4)/(b
^3*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^5))/d
```

3.79 $\int \frac{\sin^5(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=77

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{b^{5/2}d\sqrt{a+b}}+\frac{(a-b)\cos(c+dx)}{b^2d}+\frac{\cos^3(c+dx)}{3bd}$$

```
[Out] -((a^2*ArcTanh[(Sqrt[b]*Cos[c + d*x])/Sqrt[a + b]])/(b^(5/2)*Sqrt[a + b]*d)
) + ((a - b)*Cos[c + d*x])/(b^2*d) + Cos[c + d*x]^3/(3*b*d)
```

Rubi [A] time = 0.0916907, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3186, 390, 208}

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{b^{5/2}d\sqrt{a+b}}+\frac{(a-b)\cos(c+dx)}{b^2d}+\frac{\cos^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^5/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -((a^2*ArcTanh[(Sqrt[b]*Cos[c + d*x])/Sqrt[a + b]])/(b^(5/2)*Sqrt[a + b]*d)
) + ((a - b)*Cos[c + d*x])/(b^2*d) + Cos[c + d*x]^3/(3*b*d)
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sin^5(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{a+b-bx^2}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(-\frac{a-b}{b^2}-\frac{x^2}{b}+\frac{a^2}{b^2(a+b-bx^2)}\right)dx, x, \cos(c+dx)\right)}{d}$$
$$= \frac{(a-b)\cos(c+dx)}{b^2d}+\frac{\cos^3(c+dx)}{3bd}-\frac{a^2\,\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \cos(c+dx)\right)}{b^2d}$$
$$= -\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{b^{5/2}\sqrt{a+b}d}+\frac{(a-b)\cos(c+dx)}{b^2d}+\frac{\cos^3(c+dx)}{3bd}$$

Mathematica [C] time = 0.507072, size = 150, normalized size = 1.95

$$\frac{6a^2\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)+6a^2\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)+\sqrt{b}\sqrt{-a-b}\cos(c+dx)(6a+b\cos(2(c+dx))-5b)}{6b^{5/2}d\sqrt{-a-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^5/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (6*a^2*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]] + 6*a^2*
ArcTan[(Sqrt[b] + I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]] + Sqrt[-a - b]*
Sqrt[b]*Cos[c + d*x]*(6*a - 5*b + b*Cos[2*(c + d*x)]))/(6*Sqrt[-a - b]*b^(5
/2)*d)
```

Maple [A] time = 0.075, size = 70, normalized size = 0.9

$$\frac{1}{d}\left(\frac{1}{b^2}\left(\frac{b(\cos(dx+c))^3}{3}+\cos(dx+c)\,a-b\cos(dx+c)\right)-\frac{a^2}{b^2}\text{Artanh}\left(b\cos(dx+c)\frac{1}{\sqrt{(a+b)\,b}}\right)\frac{1}{\sqrt{(a+b)\,b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5/(a+sin(d*x+c)^2*b),x)
```

```
[Out] 1/d*(1/b^2*(1/3*b*cos(d*x+c)^3+cos(d*x+c)*a-b*cos(d*x+c))-a^2/b^2/((a+b)*b)
^(1/2)*arctanh(cos(d*x+c)*b/((a+b)*b)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.8727, size = 491, normalized size = 6.38

$$\left[\frac{2\left(ab^2+b^3\right)\cos\left(dx+c\right)^3+3\sqrt{ab+b^2}a^2\log\left(-\frac{b\cos(dx+c)^2-2\sqrt{ab+b^2}\cos(dx+c)+a+b}{b\cos(dx+c)^2-a-b}\right)+6\left(a^2b-b^3\right)\cos\left(dx+c\right)}{6\left(ab^3+b^4\right)d}, \frac{\left(ab^2+b^3\right)c\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/6*(2*(a*b^2 + b^3)*cos(d*x + c)^3 + 3*sqrt(a*b + b^2)*a^2*log(-(b*cos(d*
x + c)^2 - 2*sqrt(a*b + b^2)*cos(d*x + c) + a + b)/(b*cos(d*x + c)^2 - a -
b)) + 6*(a^2*b - b^3)*cos(d*x + c))/((a*b^3 + b^4)*d), 1/3*((a*b^2 + b^3)*c
os(d*x + c)^3 + 3*sqrt(-a*b - b^2)*a^2*arctan(sqrt(-a*b - b^2)*cos(d*x + c)
/(a + b)) + 3*(a^2*b - b^3)*cos(d*x + c))/((a*b^3 + b^4)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17106, size = 234, normalized size = 3.04

$$\frac{\frac{3\,a^2\arctan\left(\frac{b\cos(dx+c)+a+b}{\sqrt{-ab-b^2}\cos(dx+c)+\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}b^2}-\frac{2\left(3\,a-2\,b-\frac{6\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{6\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{3\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)}{b^2\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*a^2*arctan((b*cos(d*x + c) + a + b)/(sqrt(-a*b - b^2)*cos(d*x + c) +
 sqrt(-a*b - b^2)))/(sqrt(-a*b - b^2)*b^2) - 2*(3*a - 2*b - 6*a*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) + 6*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 3*
a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/(b^2*((cos(d*x + c) - 1)/(cos(
d*x + c) + 1) - 1)^3))/d
```

3.80 $\int \frac{\sin^3(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=52

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{b^{3/2}d\sqrt{a+b}}-\frac{\cos(c+dx)}{bd}$$

```
[Out] (a*ArcTanh[(Sqrt[b]*Cos[c + d*x])/Sqrt[a + b]])/(b^(3/2)*Sqrt[a + b]*d) - C
os[c + d*x]/(b*d)
```

Rubi [A] time = 0.0696408, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3186, 388, 208}

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{b^{3/2}d\sqrt{a+b}}-\frac{\cos(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (a*ArcTanh[(Sqrt[b]*Cos[c + d*x])/Sqrt[a + b]])/(b^(3/2)*Sqrt[a + b]*d) - C
os[c + d*x]/(b*d)
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{a+b-bx^2}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{\cos(c+dx)}{bd} + \frac{a\,\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \cos(c+dx)\right)}{bd}$$
$$= \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{b^{3/2}\sqrt{a+b}d} - \frac{\cos(c+dx)}{bd}$$

Mathematica [C] time = 0.257717, size = 125, normalized size = 2.4

$$-\frac{\sqrt{b}\sqrt{-a-b}\cos(c+dx) + a\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right) + a\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{b^{3/2}d\sqrt{-a-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -((a*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]] + a*ArcTan
[(Sqrt[b] + I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]] + Sqrt[-a - b]*Sqrt[b
]*Cos[c + d*x])/(Sqrt[-a - b]*b^(3/2)*d))
```

Maple [A] time = 0.069, size = 45, normalized size = 0.9

$$\frac{1}{d}\left(-\frac{\cos(dx+c)}{b} + \frac{a}{b}\text{Artanh}\left(b\cos(dx+c)\frac{1}{\sqrt{(a+b)b}}\right)\frac{1}{\sqrt{(a+b)b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a+sin(d*x+c)^2*b),x)
```

```
[Out] 1/d*(-1/b*cos(d*x+c)+a/b/((a+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/((a+b)*b)^(1/
2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.79428, size = 381, normalized size = 7.33

$$\left[\frac{\sqrt{ab+b^2}a\log\left(\frac{b\cos(dx+c)^2+2\sqrt{ab+b^2}\cos(dx+c)+a+b}{b\cos(dx+c)^2-a-b}\right) - 2\left(ab+b^2\right)\cos(dx+c)}{2\left(ab^2+b^3\right)d}, -\frac{\sqrt{-ab-b^2}a\arctan\left(\frac{\sqrt{-ab-b^2}\cos(dx+c)}{a+b}\right) + (ab}{\left(ab^2+b^3\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a*b + b^2)*a*log((b*cos(d*x + c)^2 + 2*sqrt(a*b + b^2)*cos(d*x +
 c) + a + b)/(b*cos(d*x + c)^2 - a - b)) - 2*(a*b + b^2)*cos(d*x + c))/((a*
b^2 + b^3)*d), -(sqrt(-a*b - b^2)*a*arctan(sqrt(-a*b - b^2)*cos(d*x + c)/(a
 + b)) + (a*b + b^2)*cos(d*x + c))/((a*b^2 + b^3)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16063, size = 77, normalized size = 1.48

$$-\frac{a \arctan\left(\frac{b \cos(dx+c)}{\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}bd} - \frac{\cos(dx+c)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -a*arctan(b*cos(d*x + c)/sqrt(-a*b - b^2))/(sqrt(-a*b - b^2)*b*d) - cos(d*x
 + c)/(b*d)
```

3.81 $\int \frac{\sin(c+dx)}{a+b\sin^2(c+dx)} dx$

Optimal. Leaf size=37

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{b}d\sqrt{a+b}}$$

```
[Out] -(ArcTanh[(Sqrt[b]*Cos[c + d*x])/Sqrt[a + b]]/(Sqrt[b]*Sqrt[a + b]*d))
```

Rubi [A] time = 0.0395803, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3186, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{b}d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -(ArcTanh[(Sqrt[b]*Cos[c + d*x])/Sqrt[a + b]]/(Sqrt[b]*Sqrt[a + b]*d))
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(c+dx)}{a+b\sin^2(c+dx)} dx &= -\frac{\text{Subst}\left(\int \frac{1}{a+b-bx^2} dx, x, \cos(c+dx)\right)}{d}\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{b}\sqrt{a+b}d}\end{aligned}$$

Mathematica [C] time = 0.156883, size = 97, normalized size = 2.62

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)+\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{\sqrt{b}d\sqrt{-a-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (ArcTan[(Sqrt[b] - I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]] + ArcTan[(Sqrt
[b] + I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]])/(Sqrt[-a - b]*Sqrt[b]*d)
```

Maple [A] time = 0.063, size = 29, normalized size = 0.8

$$-\frac{1}{d}\text{Artanh}\left(b\cos\left(dx+c\right)\frac{1}{\sqrt{\left(a+b\right)b}}\right)\frac{1}{\sqrt{\left(a+b\right)b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a+sin(d*x+c)^2*b),x)
```

```
[Out] -1/d/((a+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.77946, size = 273, normalized size = 7.38

$$\left[\frac{\log\left(-\frac{b\cos(dx+c)^2-2\sqrt{ab+b^2}\cos(dx+c)+a+b}{b\cos(dx+c)^2-a-b}\right)}{2\sqrt{ab+b^2}d}, \frac{\sqrt{-ab-b^2}\arctan\left(\frac{\sqrt{-ab-b^2}\cos(dx+c)}{a+b}\right)}{\left(ab+b^2\right)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(b*cos(d*x + c)^2 - 2*sqrt(a*b + b^2)*cos(d*x + c) + a + b)/(b*co
s(d*x + c)^2 - a - b))/(sqrt(a*b + b^2)*d), sqrt(-a*b - b^2)*arctan(sqrt(-a
*b - b^2)*cos(d*x + c)/(a + b))/((a*b + b^2)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1264, size = 50, normalized size = 1.35

$$\frac{\arctan\left(\frac{b\cos(dx+c)}{\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] arctan(b*cos(d*x + c)/sqrt(-a*b - b^2))/(sqrt(-a*b - b^2)*d)
```

3.82 $\int \frac{\csc(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=55

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{ad\sqrt{a+b}}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a*d)) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Cos[c + d*x])/Sq
rt[a + b]])/(a*Sqrt[a + b]*d)
```

Rubi [A] time = 0.0638404, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3186, 391, 206, 208}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{ad\sqrt{a+b}}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a*d)) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Cos[c + d*x])/Sq
rt[a + b]])/(a*Sqrt[a + b]*d)
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\csc(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a+b-bx^2)}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(c+dx)\right)}{ad} + \frac{b\,\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \cos(c+dx)\right)}{ad}$$
$$= -\frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}d}$$

Mathematica [C] time = 0.308812, size = 143, normalized size = 2.6

$$-\frac{\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}} + \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}} - \log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + \log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -(((Sqrt[b]*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]])/Sq
rt[-a - b] + (Sqrt[b]*ArcTan[(Sqrt[b] + I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a
 - b]])/Sqrt[-a - b] + Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]])/(a*d)
)
```

Maple [A] time = 0.102, size = 67, normalized size = 1.2

$$\frac{\ln(-1+\cos(dx+c))}{2da} - \frac{\ln(1+\cos(dx+c))}{2da} + \frac{b}{da}\text{Artanh}\left(b\cos(dx+c)\frac{1}{\sqrt{(a+b)b}}\right)\frac{1}{\sqrt{(a+b)b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a+sin(d*x+c)^2*b),x)
```

```
[Out] 1/2/d/a*ln(-1+cos(d*x+c))-1/2/d/a*ln(1+cos(d*x+c))+1/d/a*b/((a+b)*b)^(1/2)*
arctanh(cos(d*x+c)*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.90993, size = 440, normalized size = 8.

$$\left[\frac{\sqrt{\frac{b}{a+b}}\log\left(\frac{b\cos(dx+c)^2+2\,(a+b)\sqrt{\frac{b}{a+b}}\cos(dx+c)+a+b}{b\cos(dx+c)^2-a-b}\right)-\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\,ad},-\frac{2\sqrt{-\frac{b}{a+b}}\,\mathrm{arc}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b/(a + b))*log((b*cos(d*x + c)^2 + 2*(a + b)*sqrt(b/(a + b))*cos
(d*x + c) + a + b)/(b*cos(d*x + c)^2 - a - b)) - log(1/2*cos(d*x + c) + 1/2
) + log(-1/2*cos(d*x + c) + 1/2))/(a*d), -1/2*(2*sqrt(-b/(a + b))*arctan(sq
rt(-b/(a + b))*cos(d*x + c)) + log(1/2*cos(d*x + c) + 1/2) - log(-1/2*cos(d
*x + c) + 1/2))/(a*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(c+dx\right)}{a+b\sin^2\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral(csc(c + d*x)/(a + b*sin(c + d*x)**2), x)
```

Giac [B] time = 1.13179, size = 135, normalized size = 2.45

$$-\frac{\frac{2\,b\arctan\left(\frac{b\cos(dx+c)+a+b}{\sqrt{-ab-b^2}\cos(dx+c)+\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\,a}-\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/2*(2*b*arctan((b*cos(d*x + c) + a + b)/(sqrt(-a*b - b^2)*cos(d*x + c) +
sqrt(-a*b - b^2)))/(sqrt(-a*b - b^2)*a) - log(abs(-cos(d*x + c) + 1)/abs(co
s(d*x + c) + 1))/a)/d
```

3.83 $\int \frac{\csc^3(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=85

$$-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{a^2d\sqrt{a+b}}-\frac{(a-2b)\tanh^{-1}(\cos(c+dx))}{2a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

```
[Out] -((a - 2*b)*ArcTanh[Cos[c + d*x]])/(2*a^2*d) - (b^(3/2)*ArcTanh[(Sqrt[b]*Co
s[c + d*x])/Sqrt[a + b]])/(a^2*Sqrt[a + b]*d) - (Cot[c + d*x]*Csc[c + d*x])
/(2*a*d)
```

Rubi [A] time = 0.116098, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3186, 414, 522, 206, 208}

$$-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{a^2d\sqrt{a+b}}-\frac{(a-2b)\tanh^{-1}(\cos(c+dx))}{2a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -((a - 2*b)*ArcTanh[Cos[c + d*x]])/(2*a^2*d) - (b^(3/2)*ArcTanh[(Sqrt[b]*Co
s[c + d*x])/Sqrt[a + b]])/(a^2*Sqrt[a + b]*d) - (Cot[c + d*x]*Csc[c + d*x])
/(2*a*d)
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)^2(a+b-bx^2)}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{\cot(c+dx)\csc(c+dx)}{2ad} - \frac{\operatorname{Subst}\left(\int \frac{a-b-bx^2}{(1-x^2)(a+b-bx^2)}\,dx, x, \cos(c+dx)\right)}{2ad}$$
$$= -\frac{\cot(c+dx)\csc(c+dx)}{2ad} - \frac{(a-2b)\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(c+dx)\right)}{2a^2d} - \frac{b^2\operatorname{Subst}\left(\int \frac{1}{a+b-b\ldots}\right.}{\ldots}$$
$$= -\frac{(a-2b)\tanh^{-1}(\cos(c+dx))}{2a^2d} - \frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{a^2\sqrt{a+b}d} - \frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

Mathematica [C] time = 2.31014, size = 224, normalized size = 2.64

$$-\frac{\csc^2(c+dx)(2a-b\cos(2(c+dx))+b)\left(-8b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)-8b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)+\sqrt{-a-\ldots}\right.}{16a^2d\sqrt{-a-b}\left(a\csc^2(c\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -((2*a + b - b*Cos[2*(c + d*x)])*Csc[c + d*x]^2*(-8*b^(3/2)*ArcTan[(Sqrt[b]
 - I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]] - 8*b^(3/2)*ArcTan[(Sqrt[b] +
I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]] + Sqrt[-a - b]*(a*Csc[(c + d*x)/2
]^2 + 4*(a - 2*b)*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]]) - a*Sec[(
c + d*x)/2]^2)))/(16*a^2*Sqrt[-a - b]*d*(b + a*Csc[c + d*x]^2))
```

Maple [A] time = 0.122, size = 142, normalized size = 1.7

$$\frac{1}{4\,da\,(-1+\cos(dx+c))} + \frac{\ln(-1+\cos(dx+c))}{4\,da} - \frac{\ln(-1+\cos(dx+c))\,b}{2\,a^2d} + \frac{1}{4\,da\,(1+\cos(dx+c))} - \frac{\ln(1+\cos(dx\ldots}{4\,da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(a+sin(d*x+c)^2*b),x)
```

```
[Out] 1/4/d/a/(-1+cos(d*x+c))+1/4/d/a*ln(-1+cos(d*x+c))-1/2/d/a^2*ln(-1+cos(d*x+c
))*b+1/4/a/d/(1+cos(d*x+c))-1/4/d/a*ln(1+cos(d*x+c))+1/2/d/a^2*ln(1+cos(d*x
+c))*b-1/d/a^2*b^2/((a+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.94234, size = 824, normalized size = 9.69

$$\left[\frac{2\left(b\cos\left(dx+c\right)^2-b\right)\sqrt{\frac{b}{a+b}}\log\left(-\frac{b\cos(dx+c)^2-2\,(a+b)\sqrt{\frac{b}{a+b}}\cos(dx+c)+a+b}{b\cos(dx+c)^2-a-b}\right)+2\,a\cos\left(dx+c\right)-\left(\left(a-2\,b\right)\cos\left(dx+c\right)^2-a+2\right.}{4\left(a^2d\cos\left(dx+c\right)^2-a^2d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b*cos(d*x + c)^2 - b)*sqrt(b/(a + b))*log(-(b*cos(d*x + c)^2 - 2*(
a + b)*sqrt(b/(a + b))*cos(d*x + c) + a + b)/(b*cos(d*x + c)^2 - a - b)) +
2*a*cos(d*x + c) - ((a - 2*b)*cos(d*x + c)^2 - a + 2*b)*log(1/2*cos(d*x + c
) + 1/2) + ((a - 2*b)*cos(d*x + c)^2 - a + 2*b)*log(-1/2*cos(d*x + c) + 1/2
))/(a^2*d*cos(d*x + c)^2 - a^2*d), 1/4*(4*(b*cos(d*x + c)^2 - b)*sqrt(-b/(a
 + b))*arctan(sqrt(-b/(a + b))*cos(d*x + c)) + 2*a*cos(d*x + c) - ((a - 2*b
)*cos(d*x + c)^2 - a + 2*b)*log(1/2*cos(d*x + c) + 1/2) + ((a - 2*b)*cos(d*
x + c)^2 - a + 2*b)*log(-1/2*cos(d*x + c) + 1/2))/(a^2*d*cos(d*x + c)^2 - a
^2*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^3\left(c+dx\right)}{a+b\sin^2\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral(csc(c + d*x)**3/(a + b*sin(c + d*x)**2), x)
```

Giac [B] time = 1.20978, size = 265, normalized size = 3.12

$$\frac{\frac{8\,b^2\arctan\left(\frac{b\cos(dx+c)+a+b}{\sqrt{-ab-b^2}\cos(dx+c)+\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}a^2}+\frac{2\,(a-2\,b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^2}+\frac{\left(a-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{4\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{a^2(\cos(dx+c)-1)}-\frac{\cos(dx+c)-1}{a(\cos(dx+c)+1)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/8*(8*b^2*arctan((b*cos(d*x + c) + a + b)/(sqrt(-a*b - b^2)*cos(d*x + c) +
 sqrt(-a*b - b^2)))/(sqrt(-a*b - b^2)*a^2) + 2*(a - 2*b)*log(abs(-cos(d*x +
 c) + 1)/abs(cos(d*x + c) + 1))/a^2 + (a - 2*a*(cos(d*x + c) - 1)/(cos(d*x
+ c) + 1) + 4*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))*(cos(d*x + c) + 1)/(
a^2*(cos(d*x + c) - 1)) - (cos(d*x + c) - 1)/(a*(cos(d*x + c) + 1)))/d
```

3.84 $\int \frac{\csc^5(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=125

$$-\frac{(3a^2-4ab+8b^2)\tanh^{-1}(\cos(c+dx))}{8a^3d}+\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{a^3d\sqrt{a+b}}-\frac{(3a-4b)\cot(c+dx)\csc(c+dx)}{8a^2d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}$$

```
[Out] -((3*a^2 - 4*a*b + 8*b^2)*ArcTanh[Cos[c + d*x]])/(8*a^3*d) + (b^(5/2)*ArcTa
nh[(Sqrt[b]*Cos[c + d*x])/Sqrt[a + b]])/(a^3*Sqrt[a + b]*d) - ((3*a - 4*b)*
Cot[c + d*x]*Csc[c + d*x])/(8*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^3)/(4*a*d
)
```

Rubi [A] time = 0.188042, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3186, 414, 527, 522, 206, 208}

$$-\frac{(3a^2-4ab+8b^2)\tanh^{-1}(\cos(c+dx))}{8a^3d}+\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{a^3d\sqrt{a+b}}-\frac{(3a-4b)\cot(c+dx)\csc(c+dx)}{8a^2d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5/(a + b*Sin[c + d*x]^2),x]
```


```
[Out] -((3*a^2 - 4*a*b + 8*b^2)*ArcTanh[Cos[c + d*x]])/(8*a^3*d) + (b^(5/2)*ArcTa
nh[(Sqrt[b]*Cos[c + d*x])/Sqrt[a + b]])/(a^3*Sqrt[a + b]*d) - ((3*a - 4*b)*
Cot[c + d*x]*Csc[c + d*x])/(8*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^3)/(4*a*d
)
```


Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\csc^5(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^3(a+b-bx^2)}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\cot(c+dx)\csc^3(c+dx)}{4ad} - \frac{\text{Subst}\left(\int \frac{3a-b-3bx^2}{(1-x^2)^2(a+b-bx^2)}\,dx, x, \cos(c+dx)\right)}{4ad}$$

$$= -\frac{(3a-4b)\cot(c+dx)\csc(c+dx)}{8a^2d} - \frac{\cot(c+dx)\csc^3(c+dx)}{4ad} - \frac{\text{Subst}\left(\int \frac{3a^2-ab+4b^2-(3a-4b\ldots}{(1-x^2)(a+b-bx^2\ldots}\right.}{8a^2\ldots}$$

$$= -\frac{(3a-4b)\cot(c+dx)\csc(c+dx)}{8a^2d} - \frac{\cot(c+dx)\csc^3(c+dx)}{4ad} + \frac{b^3\,\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \ldots\right.}{a^3d}$$

$$= -\frac{(3a^2-4ab+8b^2)\tanh^{-1}(\cos(c+dx))}{8a^3d} + \frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{a^3\sqrt{a+bd}} - \frac{(3a-4b)\cot(c+d\ldots}{8a^2d}$$

Mathematica [C] time = 6.29067, size = 657, normalized size = 5.26

$$\frac{(3a^2-4ab+8b^2)\csc^2(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)(-2a+b\cos(2(c+dx))-b)}{16a^3d\left(a\csc^2(c+dx)+b\right)} + \frac{(-3a^2+4ab-8b^2)\csc^2(c+dx)\,\mathrm{l}\ldots}{16a^3\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^5/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (b^(5/2)*ArcTan[(Sec[(c + d*x)/2]*(Sqrt[b]*Cos[(c + d*x)/2] - I*Sqrt[a]*Sin
[(c + d*x)/2]))/Sqrt[-a - b]]*(-2*a - b + b*Cos[2*(c + d*x)])*Csc[c + d*x]^
2)/(2*a^3*Sqrt[-a - b]*d*(b + a*Csc[c + d*x]^2)) + (b^(5/2)*ArcTan[(Sec[(c
+ d*x)/2]*(Sqrt[b]*Cos[(c + d*x)/2] + I*Sqrt[a]*Sin[(c + d*x)/2]))/Sqrt[-a
- b]]*(-2*a - b + b*Cos[2*(c + d*x)])*Csc[c + d*x]^2)/(2*a^3*Sqrt[-a - b]*d
*(b + a*Csc[c + d*x]^2)) + ((3*a - 4*b)*(-2*a - b + b*Cos[2*(c + d*x)])*Csc
[(c + d*x)/2]^2*Csc[c + d*x]^2)/(64*a^2*d*(b + a*Csc[c + d*x]^2)) + ((-2*a
- b + b*Cos[2*(c + d*x)])*Csc[(c + d*x)/2]^4*Csc[c + d*x]^2)/(128*a*d*(b +
a*Csc[c + d*x]^2)) + ((3*a^2 - 4*a*b + 8*b^2)*(-2*a - b + b*Cos[2*(c + d*x)
```

```
])*Csc[c + d*x]^2*Log[Cos[(c + d*x)/2]])/(16*a^3*d*(b + a*Csc[c + d*x]^2))
+ ((-3*a^2 + 4*a*b - 8*b^2)*(-2*a - b + b*Cos[2*(c + d*x)])*Csc[c + d*x]^2*
Log[Sin[(c + d*x)/2]])/(16*a^3*d*(b + a*Csc[c + d*x]^2)) + ((-3*a + 4*b)*(-
2*a - b + b*Cos[2*(c + d*x)])*Csc[c + d*x]^2*Sec[(c + d*x)/2]^2)/(64*a^2*d*
(b + a*Csc[c + d*x]^2)) - ((-2*a - b + b*Cos[2*(c + d*x)])*Csc[c + d*x]^2*S
ec[(c + d*x)/2]^4)/(128*a*d*(b + a*Csc[c + d*x]^2))
```

Maple [B] time = 0.122, size = 255, normalized size = 2.

$$-\frac{1}{16\,da\,(-1+\cos(dx+c))^2}+\frac{3}{16\,da\,(-1+\cos(dx+c))}-\frac{b}{4\,a^2d\,(-1+\cos(dx+c))}+\frac{3\ln(-1+\cos(dx+c))}{16\,da}-\frac{\ln(-1+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5/(a+sin(d*x+c)^2*b),x)
```

```
[Out] -1/16/d/a/(-1+cos(d*x+c))^2+3/16/d/a/(-1+cos(d*x+c))-1/4/d/a^2/(-1+cos(d*x+
c))*b+3/16/d/a*ln(-1+cos(d*x+c))-1/4/d/a^2*ln(-1+cos(d*x+c))*b+1/2/d/a^3*ln
(-1+cos(d*x+c))*b^2+1/16/a/d/(1+cos(d*x+c))^2+3/16/a/d/(1+cos(d*x+c))-1/4/d
/a^2/(1+cos(d*x+c))*b-3/16/d/a*ln(1+cos(d*x+c))+1/4/d/a^2*ln(1+cos(d*x+c))*
b-1/2/d/a^3*ln(1+cos(d*x+c))*b^2+1/d*b^3/a^3/((a+b)*b)^(1/2)*arctanh(cos(d*
x+c)*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.11396, size = 1482, normalized size = 11.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*(3*a^2 - 4*a*b)*cos(d*x + c)^3 + 8*(b^2*cos(d*x + c)^4 - 2*b^2*cos
(d*x + c)^2 + b^2)*sqrt(b/(a + b))*log((b*cos(d*x + c)^2 + 2*(a + b)*sqrt(b
/(a + b))*cos(d*x + c) + a + b)/(b*cos(d*x + c)^2 - a - b)) - 2*(5*a^2 - 4*
a*b)*cos(d*x + c) - ((3*a^2 - 4*a*b + 8*b^2)*cos(d*x + c)^4 - 2*(3*a^2 - 4*
a*b + 8*b^2)*cos(d*x + c)^2 + 3*a^2 - 4*a*b + 8*b^2)*log(1/2*cos(d*x + c) +
 1/2) + ((3*a^2 - 4*a*b + 8*b^2)*cos(d*x + c)^4 - 2*(3*a^2 - 4*a*b + 8*b^2)
*cos(d*x + c)^2 + 3*a^2 - 4*a*b + 8*b^2)*log(-1/2*cos(d*x + c) + 1/2))/(a^3
*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^2 + a^3*d), 1/16*(2*(3*a^2 - 4*a*b
)*cos(d*x + c)^3 - 16*(b^2*cos(d*x + c)^4 - 2*b^2*cos(d*x + c)^2 + b^2)*sqr
t(-b/(a + b))*arctan(sqrt(-b/(a + b))*cos(d*x + c)) - 2*(5*a^2 - 4*a*b)*cos
(d*x + c) - ((3*a^2 - 4*a*b + 8*b^2)*cos(d*x + c)^4 - 2*(3*a^2 - 4*a*b + 8*
```

```
b^2)*cos(d*x + c)^2 + 3*a^2 - 4*a*b + 8*b^2)*log(1/2*cos(d*x + c) + 1/2) +
((3*a^2 - 4*a*b + 8*b^2)*cos(d*x + c)^4 - 2*(3*a^2 - 4*a*b + 8*b^2)*cos(d*x
 + c)^2 + 3*a^2 - 4*a*b + 8*b^2)*log(-1/2*cos(d*x + c) + 1/2))/(a^3*d*cos(d
*x + c)^4 - 2*a^3*d*cos(d*x + c)^2 + a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.26124, size = 451, normalized size = 3.61

$$-\frac{\frac{64\,b^3\arctan\left(\frac{b\cos(dx+c)+a+b}{\sqrt{-ab-b^2}\cos(dx+c)+\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}a^3}+\frac{\frac{8\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{8\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{a^2}-\frac{4\left(3\,a^2-4\,ab+8\,b^2\right)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^3}+\frac{\left(a^2-\frac{8\,a^2(\cos}{\cos(d}\right.}{}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/64*(64*b^3*arctan((b*cos(d*x + c) + a + b)/(sqrt(-a*b - b^2)*cos(d*x + c
) + sqrt(-a*b - b^2)))/(sqrt(-a*b - b^2)*a^3) + (8*a*(cos(d*x + c) - 1)/(co
s(d*x + c) + 1) - 8*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - a*(cos(d*x +
c) - 1)^2/(cos(d*x + c) + 1)^2)/a^2 - 4*(3*a^2 - 4*a*b + 8*b^2)*log(abs(-co
s(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^3 + (a^2 - 8*a^2*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) + 8*a*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 18*a^2
*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 24*a*b*(cos(d*x + c) - 1)^2/(c
os(d*x + c) + 1)^2 + 48*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)*(cos
(d*x + c) + 1)^2/(a^3*(cos(d*x + c) - 1)^2))/d
```

3.85 $\int \frac{\sin^8(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=163

$$\frac{a^{7/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{b^4d\sqrt{a+b}}-\frac{\left(8a^2-6ab+5b^2\right)\sin(c+dx)\cos(c+dx)}{16b^3d}-\frac{x\left(-8a^2b+16a^3+6ab^2-5b^3\right)}{16b^4}+\frac{(6a-5b)\sin^3}{}$$

```
[Out] -((16*a^3 - 8*a^2*b + 6*a*b^2 - 5*b^3)*x)/(16*b^4) + (a^(7/2)*ArcTan[(Sqrt[
a + b]*Tan[c + d*x])/Sqrt[a]])/(b^4*Sqrt[a + b]*d) - ((8*a^2 - 6*a*b + 5*b^
2)*Cos[c + d*x]*Sin[c + d*x])/(16*b^3*d) + ((6*a - 5*b)*Cos[c + d*x]*Sin[c
+ d*x]^3)/(24*b^2*d) - (Cos[c + d*x]*Sin[c + d*x]^5)/(6*b*d)
```

Rubi [A] time = 0.364735, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3187, 470, 578, 522, 203, 205}

$$\frac{a^{7/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{b^4d\sqrt{a+b}}-\frac{\left(8a^2-6ab+5b^2\right)\sin(c+dx)\cos(c+dx)}{16b^3d}-\frac{x\left(-8a^2b+16a^3+6ab^2-5b^3\right)}{16b^4}+\frac{(6a-5b)\sin^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^8/(a + b*Sin[c + d*x]^2),x]
```


```
[Out] -((16*a^3 - 8*a^2*b + 6*a*b^2 - 5*b^3)*x)/(16*b^4) + (a^(7/2)*ArcTan[(Sqrt[
a + b]*Tan[c + d*x])/Sqrt[a]])/(b^4*Sqrt[a + b]*d) - ((8*a^2 - 6*a*b + 5*b^
2)*Cos[c + d*x]*Sin[c + d*x])/(16*b^3*d) + ((6*a - 5*b)*Cos[c + d*x]*Sin[c
+ d*x]^3)/(24*b^2*d) - (Cos[c + d*x]*Sin[c + d*x]^5)/(6*b*d)
```


Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
```

```
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^8(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^8}{(1+x^2)^4(a+(a+b)x^2)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{\cos(c+dx)\sin^5(c+dx)}{6bd} + \frac{\text{Subst}\left(\int \frac{x^4(5a+(-a+5b)x^2)}{(1+x^2)^3(a+(a+b)x^2)}\,dx, x, \tan(c+dx)\right)}{6bd}$$

$$= \frac{(6a-5b)\cos(c+dx)\sin^3(c+dx)}{24b^2d} - \frac{\cos(c+dx)\sin^5(c+dx)}{6bd} - \frac{\text{Subst}\left(\int \frac{x^2(3a(6a-5b)-3(2a^2-}{(1+x^2)^2(a+(a+}\ldots\right)}{24\ldots}$$

$$= -\frac{(8a^2-6ab+5b^2)\cos(c+dx)\sin(c+dx)}{16b^3d} + \frac{(6a-5b)\cos(c+dx)\sin^3(c+dx)}{24b^2d} - \frac{\cos(c+\ldots}{\ldots}$$

$$= -\frac{(8a^2-6ab+5b^2)\cos(c+dx)\sin(c+dx)}{16b^3d} + \frac{(6a-5b)\cos(c+dx)\sin^3(c+dx)}{24b^2d} - \frac{\cos(c+\ldots}{\ldots}$$

$$= -\frac{(16a^3-8a^2b+6ab^2-5b^3)x}{16b^4} + \frac{a^{7/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{b^4\sqrt{a+b}d} - \frac{(8a^2-6ab+5b^2)\cos(c+d\ldots}{16b^3d}$$

Mathematica [A] time = 1.19546, size = 133, normalized size = 0.82

$$-\frac{12\left(-8a^2b+16a^3+6ab^2-5b^3\right)(c+dx)+3b\left(16a^2-16ab+15b^2\right)\sin(2(c+dx)) - \frac{192a^{7/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a+b}} + 3b^2(2\ldots}{192b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^8/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -(12*(16*a^3 - 8*a^2*b + 6*a*b^2 - 5*b^3)*(c + d*x) - (192*a^(7/2)*ArcTan[(
Sqrt[a + b]*Tan[c + d*x]])/Sqrt[a]])/Sqrt[a + b] + 3*b*(16*a^2 - 16*a*b + 15
*b^2)*Sin[2*(c + d*x)] + 3*(2*a - 3*b)*b^2*Sin[4*(c + d*x)] + b^3*Sin[6*(c
```

```
+ d*x)])/(192*b^4*d)
```

Maple [B] time = 0.102, size = 361, normalized size = 2.2

$$-\frac{(\tan(dx+c))^5 a^2}{2 db^3 \left((\tan(dx+c))^2+1\right)^3}+\frac{5 (\tan(dx+c))^5 a}{8 b^2 d \left((\tan(dx+c))^2+1\right)^3}-\frac{11 (\tan(dx+c))^5}{16 bd \left((\tan(dx+c))^2+1\right)^3}-\frac{(\tan(dx+c))^3 a^2}{db^3 \left((\tan(dx+c))^2+1\right)^3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^8/(a+sin(d*x+c)^2*b),x)
```

```
[Out] -1/2/d/b^3/(tan(d*x+c)^2+1)^3*tan(d*x+c)^5*a^2+5/8/d/b^2/(tan(d*x+c)^2+1)^3
*tan(d*x+c)^5*a-11/16/d/b/(tan(d*x+c)^2+1)^3*tan(d*x+c)^5-1/d/b^3/(tan(d*x+
c)^2+1)^3*tan(d*x+c)^3*a^2+1/d/b^2/(tan(d*x+c)^2+1)^3*tan(d*x+c)^3*a-5/6/d/
b/(tan(d*x+c)^2+1)^3*tan(d*x+c)^3-1/2/d/b^3/(tan(d*x+c)^2+1)^3*tan(d*x+c)*a
^2+3/8/d/b^2/(tan(d*x+c)^2+1)^3*tan(d*x+c)*a-5/16/d/b/(tan(d*x+c)^2+1)^3*ta
n(d*x+c)-1/d/b^4*arctan(tan(d*x+c))*a^3+1/2/d/b^3*arctan(tan(d*x+c))*a^2-3/
8/d/b^2*arctan(tan(d*x+c))*a+5/16/d/b*arctan(tan(d*x+c))+1/d*a^4/b^4/(a*(a+
b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.93242, size = 1077, normalized size = 6.61

$$\left[\frac{12 a^3 \sqrt{-\frac{a}{a+b}} \log\left(\frac{\left(8 a^2+8 ab+b^2\right)\cos(dx+c)^4-2\left(4 a^2+5 ab+b^2\right)\cos(dx+c)^2-4\left(\left(2 a^2+3 ab+b^2\right)\cos(dx+c)^3-\left(a^2+2 ab+b^2\right)\cos(dx+c)\right)\sqrt{-\frac{a}{a+b}}\sin(dx+c)+a^2}{b^2\cos(dx+c)^4-2\left(ab+b^2\right)\cos(dx+c)^2+a^2+2 ab+b^2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/48*(12*a^3*sqrt(-a/(a + b))*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 -
2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 - 4*((2*a^2 + 3*a*b + b^2)*cos(d*x +
 c)^3 - (a^2 + 2*a*b + b^2)*cos(d*x + c))*sqrt(-a/(a + b))*sin(d*x + c) + a
^2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2
+ 2*a*b + b^2)) - 3*(16*a^3 - 8*a^2*b + 6*a*b^2 - 5*b^3)*d*x - (8*b^3*cos(d
*x + c)^5 + 2*(6*a*b^2 - 13*b^3)*cos(d*x + c)^3 + 3*(8*a^2*b - 10*a*b^2 + 1
1*b^3)*cos(d*x + c))*sin(d*x + c))/(b^4*d), -1/48*(24*a^3*sqrt(a/(a + b))*a
rctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)*sqrt(a/(a + b))/(a*cos(d*x + c
)*sin(d*x + c))) + 3*(16*a^3 - 8*a^2*b + 6*a*b^2 - 5*b^3)*d*x + (8*b^3*cos(
```

```
d*x + c)^5 + 2*(6*a*b^2 - 13*b^3)*cos(d*x + c)^3 + 3*(8*a^2*b - 10*a*b^2 +
11*b^3)*cos(d*x + c))*sin(d*x + c))/(b^4*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**8/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17834, size = 315, normalized size = 1.93

$$\frac{\frac{48\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)a^4}{\sqrt{a^2+ab}b^4}-\frac{3\left(16a^3-8a^2b+6ab^2-5b^3\right)(dx+c)}{b^4}-\frac{24a^2\tan(dx+c)^5-30ab\tan(dx+c)^5+33b^2\tan(d}{}}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/48*(48*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x +
 c) + b*tan(d*x + c))/sqrt(a^2 + a*b)))*a^4/(sqrt(a^2 + a*b)*b^4) - 3*(16*a
^3 - 8*a^2*b + 6*a*b^2 - 5*b^3)*(d*x + c)/b^4 - (24*a^2*tan(d*x + c)^5 - 30
*a*b*tan(d*x + c)^5 + 33*b^2*tan(d*x + c)^5 + 48*a^2*tan(d*x + c)^3 - 48*a*
b*tan(d*x + c)^3 + 40*b^2*tan(d*x + c)^3 + 24*a^2*tan(d*x + c) - 18*a*b*tan
(d*x + c) + 15*b^2*tan(d*x + c))/((tan(d*x + c)^2 + 1)^3*b^3))/d
```

3.86 $\int \frac{\sin^6(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=117

$$-\frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{b^3d\sqrt{a+b}}+\frac{x\left(8a^2-4ab+3b^2\right)}{8b^3}+\frac{(4a-3b)\sin(c+dx)\cos(c+dx)}{8b^2d}-\frac{\sin^3(c+dx)\cos(c+dx)}{4bd}$$

```
[Out] ((8*a^2 - 4*a*b + 3*b^2)*x)/(8*b^3) - (a^(5/2)*ArcTan[(Sqrt[a + b]*Tan[c +
d*x])/Sqrt[a]])/(b^3*Sqrt[a + b]*d) + ((4*a - 3*b)*Cos[c + d*x]*Sin[c + d*x
])/(8*b^2*d) - (Cos[c + d*x]*Sin[c + d*x]^3)/(4*b*d)
```

Rubi [A] time = 0.223681, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3187, 470, 578, 522, 203, 205}

$$-\frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{b^3d\sqrt{a+b}}+\frac{x\left(8a^2-4ab+3b^2\right)}{8b^3}+\frac{(4a-3b)\sin(c+dx)\cos(c+dx)}{8b^2d}-\frac{\sin^3(c+dx)\cos(c+dx)}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^6/(a + b*Sin[c + d*x]^2),x]
```


```
[Out] ((8*a^2 - 4*a*b + 3*b^2)*x)/(8*b^3) - (a^(5/2)*ArcTan[(Sqrt[a + b]*Tan[c +
d*x])/Sqrt[a]])/(b^3*Sqrt[a + b]*d) + ((4*a - 3*b)*Cos[c + d*x]*Sin[c + d*x
])/(8*b^2*d) - (Cos[c + d*x]*Sin[c + d*x]^3)/(4*b*d)
```


Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^6(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)^3(a+(a+b)x^2)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{\cos(c+dx)\sin^3(c+dx)}{4bd} + \frac{\text{Subst}\left(\int \frac{x^2(3a+(-a+3b)x^2)}{(1+x^2)^2(a+(a+b)x^2)}\,dx, x, \tan(c+dx)\right)}{4bd}$$

$$= \frac{(4a-3b)\cos(c+dx)\sin(c+dx)}{8b^2d} - \frac{\cos(c+dx)\sin^3(c+dx)}{4bd} - \frac{\text{Subst}\left(\int \frac{a(4a-3b)+(-4a^2+ab-3}{(1+x^2)(a+(a+b)x^2}\right.}{8b^2d}$$

$$= \frac{(4a-3b)\cos(c+dx)\sin(c+dx)}{8b^2d} - \frac{\cos(c+dx)\sin^3(c+dx)}{4bd} - \frac{a^3\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x,\right.}{b^3d}$$

$$= \frac{(8a^2-4ab+3b^2)\,x}{8b^3} - \frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{b^3\sqrt{a+b}d} + \frac{(4a-3b)\cos(c+dx)\sin(c+dx)}{8b^2d} - \text{cos}$$

Mathematica [A] time = 0.445376, size = 95, normalized size = 0.81

$$\frac{4\left(8a^2-4ab+3b^2\right)(c+dx) - \frac{32a^{5/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a+b}} + 8b(a-b)\sin(2(c+dx)) + b^2\sin(4(c+dx))}{32b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^6/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (4*(8*a^2 - 4*a*b + 3*b^2)*(c + d*x) - (32*a^(5/2)*ArcTan[(Sqrt[a + b]*Tan[
c + d*x])/Sqrt[a]])/Sqrt[a + b] + 8*(a - b)*b*Sin[2*(c + d*x)] + b^2*Sin[4*
(c + d*x)])/(32*b^3*d)
```

Maple [A] time = 0.092, size = 196, normalized size = 1.7

$$\frac{(\tan(dx+c))^3 a}{2b^2d\left((\tan(dx+c))^2+1\right)^2} - \frac{5(\tan(dx+c))^3}{8bd\left((\tan(dx+c))^2+1\right)^2} + \frac{a\tan(dx+c)}{2b^2d\left((\tan(dx+c))^2+1\right)^2} - \frac{3\tan(dx+c)}{8bd\left((\tan(dx+c))^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^6/(a+sin(d*x+c)^2*b),x)
```

```
[Out] 1/2/d/b^2/(tan(d*x+c)^2+1)^2*tan(d*x+c)^3*a-5/8/d/b/(tan(d*x+c)^2+1)^2*tan(
d*x+c)^3+1/2/d/b^2/(tan(d*x+c)^2+1)^2*tan(d*x+c)*a-3/8/d/b/(tan(d*x+c)^2+1)
^2*tan(d*x+c)+1/d/b^3*arctan(tan(d*x+c))*a^2-1/2/d/b^2*arctan(tan(d*x+c))*a
+3/8/d/b*arctan(tan(d*x+c))-1/d*a^3/b^3/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*
x+c)/(a*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.92646, size = 884, normalized size = 7.56

$$\left[\frac{2\,a^2\sqrt{-\frac{a}{a+b}}\log\left(\frac{\left(8\,a^2+8\,ab+b^2\right)\cos(dx+c)^4-2\left(4\,a^2+5\,ab+b^2\right)\cos(dx+c)^2+4\left(\left(2\,a^2+3\,ab+b^2\right)\cos(dx+c)^3-\left(a^2+2\,ab+b^2\right)\cos(dx+c)\right)\sqrt{-\frac{a}{a+b}}\sin(dx+c)+a^2+}{b^2\cos(dx+c)^4-2\left(ab+b^2\right)\cos(dx+c)^2+a^2+2\,ab+b^2}\right.}{8\,b^3d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*a^2*sqrt(-a/(a + b))*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2*
(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a^2 + 3*a*b + b^2)*cos(d*x + c
)^3 - (a^2 + 2*a*b + b^2)*cos(d*x + c))*sqrt(-a/(a + b))*sin(d*x + c) + a^2
 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 +
2*a*b + b^2)) + (8*a^2 - 4*a*b + 3*b^2)*d*x + (2*b^2*cos(d*x + c)^3 + (4*a*
b - 5*b^2)*cos(d*x + c))*sin(d*x + c))/(b^3*d), 1/8*(4*a^2*sqrt(a/(a + b))*
arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)*sqrt(a/(a + b))/(a*cos(d*x +
c)*sin(d*x + c))) + (8*a^2 - 4*a*b + 3*b^2)*d*x + (2*b^2*cos(d*x + c)^3 + (
4*a*b - 5*b^2)*cos(d*x + c))*sin(d*x + c))/(b^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**6/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2068, size = 212, normalized size = 1.81

$$-\frac{\frac{8\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)a^3}{\sqrt{a^2+ab}b^3}-\frac{(8\,a^2-4\,ab+3\,b^2)(dx+c)}{b^3}-\frac{4\,a\tan(dx+c)^3-5\,b\tan(dx+c)^3+4\,a\tan(dx+c)-3\,b\tan(dx}{\left(\tan(dx+c)^2+1\right)^2b^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/8*(8*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x +
c) + b*tan(d*x + c))/sqrt(a^2 + a*b)))*a^3/(sqrt(a^2 + a*b)*b^3) - (8*a^2 -
 4*a*b + 3*b^2)*(d*x + c)/b^3 - (4*a*tan(d*x + c)^3 - 5*b*tan(d*x + c)^3 +
4*a*tan(d*x + c) - 3*b*tan(d*x + c))/((tan(d*x + c)^2 + 1)^2*b^2))/d
```

3.87 $\int \frac{\sin^4(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=77

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{b^2d\sqrt{a+b}}-\frac{x(2a-b)}{2b^2}-\frac{\sin(c+dx)\cos(c+dx)}{2bd}$$

```
[Out] -((2*a - b)*x)/(2*b^2) + (a^(3/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]
])/(b^2*Sqrt[a + b]*d) - (Cos[c + d*x]*Sin[c + d*x])/(2*b*d)
```

Rubi [A] time = 0.114091, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3187, 470, 522, 203, 205}

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{b^2d\sqrt{a+b}}-\frac{x(2a-b)}{2b^2}-\frac{\sin(c+dx)\cos(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -((2*a - b)*x)/(2*b^2) + (a^(3/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]
])/(b^2*Sqrt[a + b]*d) - (Cos[c + d*x]*Sin[c + d*x])/(2*b*d)
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^4(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)^2(a+(a+b)x^2)}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= -\frac{\cos(c+dx)\sin(c+dx)}{2bd} + \frac{\text{Subst}\left(\int \frac{a+(-a+b)x^2}{(1+x^2)(a+(a+b)x^2)}\,dx, x, \tan(c+dx)\right)}{2bd}$$
$$= -\frac{\cos(c+dx)\sin(c+dx)}{2bd} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \tan(c+dx)\right)}{b^2d} - \frac{(2a-b)\,\text{Subst}\left(\int \frac{1}{\ldots}\right.}{2\ldots}$$
$$= -\frac{(2a-b)x}{2b^2} + \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{b^2\sqrt{a+b}d} - \frac{\cos(c+dx)\sin(c+dx)}{2bd}$$

Mathematica [A] time = 0.310148, size = 69, normalized size = 0.9

$$-\frac{-\frac{4a^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a+b}} + 2(2a-b)(c+dx) + b\sin(2(c+dx))}{4b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -(2*(2*a - b)*(c + d*x) - (4*a^(3/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt
[a]])/Sqrt[a + b] + b*Sin[2*(c + d*x)])/(4*b^2*d)
```

Maple [A] time = 0.079, size = 94, normalized size = 1.2

$$-\frac{\tan(dx+c)}{2\,bd\left((\tan(dx+c))^2+1\right)} + \frac{\arctan(\tan(dx+c))}{2\,bd} - \frac{\arctan(\tan(dx+c))\,a}{b^2d} + \frac{a^2}{b^2d}\arctan\left((a+b)\tan(dx+c)\frac{1}{\sqrt{a\ldots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a+sin(d*x+c)^2*b),x)
```

```
[Out] -1/2/d/b*tan(d*x+c)/(tan(d*x+c)^2+1)+1/2/d/b*arctan(tan(d*x+c))-1/d/b^2*arc
tan(tan(d*x+c))*a+1/d*a^2/b^2/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a
+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.87633, size = 738, normalized size = 9.58

$$\left[-\frac{2\,(2\,a-b)dx+2\,b\cos\left(dx+c\right)\sin\left(dx+c\right)-a\sqrt{-\frac{a}{a+b}}\log\left(\frac{\left(8\,a^2+8\,ab+b^2\right)\cos(dx+c)^4-2\left(4\,a^2+5\,ab+b^2\right)\cos(dx+c)^2-4\left(\left(2\,a^2+3\,ab+b^2\right)\cos\ldots\right.}{b^2\cos(dx+c)^4-2\left(ab+b^2\right)\cos(d\ldots}\right.}{4\,b^2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(2*a - b)*d*x + 2*b*cos(d*x + c)*sin(d*x + c) - a*sqrt(-a/(a + b))
*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*
x + c)^2 - 4*((2*a^2 + 3*a*b + b^2)*cos(d*x + c)^3 - (a^2 + 2*a*b + b^2)*co
s(d*x + c))*sqrt(-a/(a + b))*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x
 + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2)))/(b^2*d), -1/2
*((2*a - b)*d*x + b*cos(d*x + c)*sin(d*x + c) + a*sqrt(a/(a + b))*arctan(1/
2*((2*a + b)*cos(d*x + c)^2 - a - b)*sqrt(a/(a + b))/(a*cos(d*x + c)*sin(d*
x + c))))/(b^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19799, size = 154, normalized size = 2.

$$\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)a^2}{\sqrt{a^2+ab}b^2}-\frac{(dx+c)(2\,a-b)}{b^2}-\frac{\tan(dx+c)}{\left(\tan(dx+c)^2+1\right)b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*(2*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c
) + b*tan(d*x + c))/sqrt(a^2 + a*b)))*a^2/(sqrt(a^2 + a*b)*b^2) - (d*x + c)
*(2*a - b)/b^2 - tan(d*x + c)/((tan(d*x + c)^2 + 1)*b))/d
```

3.88 $\int \frac{\sin^2(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=46

$$\frac{x}{b}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{bd\sqrt{a+b}}$$

```
[Out] x/b - (Sqrt[a]*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(b*Sqrt[a + b]*d
)
```

Rubi [A] time = 0.0747304, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3171, 3181, 205}

$$\frac{x}{b}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{bd\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] x/b - (Sqrt[a]*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(b*Sqrt[a + b]*d
)
```

Rule 3171

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[(B*x)/b, x] + Dist[(A*b - a*B)/b, Int[1/(a +
 b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^2(c+dx)}{a+b\sin^2(c+dx)}\,dx &= \frac{x}{b}-\frac{a\int \frac{1}{a+b\sin^2(c+dx)}\,dx}{b}\\
&= \frac{x}{b}-\frac{a\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx,x,\tan(c+dx)\right)}{bd}\\
&= \frac{x}{b}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{b\sqrt{a+b}d}
\end{aligned}$$

Mathematica [A] time = 0.145377, size = 46, normalized size = 1.

$$\frac{-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a+b}}+c+dx}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (c + d*x - (Sqrt[a]*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/Sqrt[a + b]
)/(b*d)
```

Maple [A] time = 0.075, size = 50, normalized size = 1.1

$$\frac{\arctan(\tan(dx+c))}{bd}-\frac{a}{bd}\arctan\left((a+b)\tan(dx+c)\frac{1}{\sqrt{a(a+b)}}\right)\frac{1}{\sqrt{a(a+b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+sin(d*x+c)^2*b),x)
```

```
[Out] 1/d/b*arctan(tan(d*x+c))-1/d*a/b/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a
*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.85861, size = 617, normalized size = 13.41

$$\left[\frac{4\,dx+\sqrt{-\frac{a}{a+b}}\log\left(\frac{\left(8\,a^2+8\,ab+b^2\right)\cos(dx+c)^4-2\left(4\,a^2+5\,ab+b^2\right)\cos(dx+c)^2+4\left(\left(2\,a^2+3\,ab+b^2\right)\cos(dx+c)^3-\left(a^2+2\,ab+b^2\right)\cos(dx+c)\right)\sqrt{-\frac{a}{a+b}}\sin(dx+c)+}{b^2\cos(dx+c)^4-2\left(ab+b^2\right)\cos(dx+c)^2+a^2+2\,ab+b^2}\right.}{4\,bd}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*d*x + sqrt(-a/(a + b))*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 -
2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a^2 + 3*a*b + b^2)*cos(d*x +
 c)^3 - (a^2 + 2*a*b + b^2)*cos(d*x + c))*sqrt(-a/(a + b))*sin(d*x + c) + a
^2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2
```

```
+ 2*a*b + b^2)))/(b*d), 1/2*(2*d*x + sqrt(a/(a + b))*arctan(1/2*((2*a + b)*
cos(d*x + c)^2 - a - b)*sqrt(a/(a + b))/(a*cos(d*x + c)*sin(d*x + c))))/(b*
d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.19646, size = 109, normalized size = 2.37

$$-\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)a}{\sqrt{a^2+ab}b}-\frac{dx+c}{b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -((pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c) + b
*tan(d*x + c))/sqrt(a^2 + a*b)))*a/(sqrt(a^2 + a*b)*b) - (d*x + c)/b)/d
```

3.89 $\int \frac{1}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=36

$$\frac{\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d\sqrt{a+b}}$$

```
[Out] ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a + b]*d)
```

Rubi [A] time = 0.0239375, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3181, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x]^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a + b]*d)
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+b\sin^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \tan(c+dx)\right)}{d}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a+b}d}\end{aligned}$$

Mathematica [A] time = 0.0778624, size = 36, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x]^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a + b]*d)
```

Maple [A] time = 0.075, size = 30, normalized size = 0.8

$$\frac{1}{d}\arctan\left((a+b)\tan(dx+c)\frac{1}{\sqrt{a(a+b)}}\right)\frac{1}{\sqrt{a(a+b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+sin(d*x+c)^2*b),x)
```

```
[Out] 1/d/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.79461, size = 568, normalized size = 15.78

$$\left[-\frac{\sqrt{-a^2-ab}\log\left(\frac{(8a^2+8ab+b^2)\cos(dx+c)^4-2(4a^2+5ab+b^2)\cos(dx+c)^2+4((2a+b)\cos(dx+c)^3-(a+b)\cos(dx+c))\sqrt{-a^2-ab}\sin(dx+c)+a^2+2ab+b^2}{b^2\cos(dx+c)^4-2(ab+b^2)\cos(dx+c)^2+a^2+2ab+b^2}\right.}{4(a^2+ab)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4*a^2
 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a + b)*cos(d*x + c)^3 - (a + b)*cos(
d*x + c))*sqrt(-a^2 - a*b)*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x +
 c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2))/((a^2 + a*b)*d),
 -1/2*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)/(sqrt(a^2 + a*b)*cos(d*
x + c)*sin(d*x + c)))/(sqrt(a^2 + a*b)*d)]
```

Sympy [A] time = 49.8561, size = 3907, normalized size = 108.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Piecewise((zoo*x/sin(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((tan(c/2 + d*
x/2)/(2*d) - 1/(2*d*tan(c/2 + d*x/2)))/b, Eq(a, 0)), (2*tan(c/2 + d*x/2)/(b
*d*tan(c/2 + d*x/2)**2 - b*d), Eq(a, -b)), (x/(a + b*sin(c)**2), Eq(d, 0)),
 (-a**3*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*log(-sqrt(-1 - 2*b/a - 2*sq
rt(a*b + b**2)/a) + tan(c/2 + d*x/2))/(2*a**4*d + 18*a**3*b*d - 8*a**3*d*sq
rt(a*b + b**2) + 32*a**2*b**2*d - 24*a**2*b*d*sqrt(a*b + b**2) + 16*a*b**3*
d - 16*a*b**2*d*sqrt(a*b + b**2)) + a**3*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**
2)/a)*log(sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a) + tan(c/2 + d*x/2))/(2*a*
*4*d + 18*a**3*b*d - 8*a**3*d*sqrt(a*b + b**2) + 32*a**2*b**2*d - 24*a**2*b
*d*sqrt(a*b + b**2) + 16*a*b**3*d - 16*a*b**2*d*sqrt(a*b + b**2)) - a**3*sq
rt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a)*log(-sqrt(-1 - 2*b/a + 2*sqrt(a*b + b
**2)/a) + tan(c/2 + d*x/2))/(2*a**4*d + 18*a**3*b*d - 8*a**3*d*sqrt(a*b + b
**2) + 32*a**2*b**2*d - 24*a**2*b*d*sqrt(a*b + b**2) + 16*a*b**3*d - 16*a*b
**2*d*sqrt(a*b + b**2)) + a**3*sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a)*log(
sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a) + tan(c/2 + d*x/2))/(2*a**4*d + 18*
a**3*b*d - 8*a**3*d*sqrt(a*b + b**2) + 32*a**2*b**2*d - 24*a**2*b*d*sqrt(a*
b + b**2) + 16*a*b**3*d - 16*a*b**2*d*sqrt(a*b + b**2)) - 13*a**2*b*sqrt(-1
 - 2*b/a - 2*sqrt(a*b + b**2)/a)*log(-sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/
a) + tan(c/2 + d*x/2))/(2*a**4*d + 18*a**3*b*d - 8*a**3*d*sqrt(a*b + b**2)
+ 32*a**2*b**2*d - 24*a**2*b*d*sqrt(a*b + b**2) + 16*a*b**3*d - 16*a*b**2*d
*sqrt(a*b + b**2)) + 13*a**2*b*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*log(
sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a) + tan(c/2 + d*x/2))/(2*a**4*d + 18*
a**3*b*d - 8*a**3*d*sqrt(a*b + b**2) + 32*a**2*b**2*d - 24*a**2*b*d*sqrt(a*
b + b**2) + 16*a*b**3*d - 16*a*b**2*d*sqrt(a*b + b**2)) - 5*a**2*b*sqrt(-1
- 2*b/a + 2*sqrt(a*b + b**2)/a)*log(-sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a
) + tan(c/2 + d*x/2))/(2*a**4*d + 18*a**3*b*d - 8*a**3*d*sqrt(a*b + b**2) +
 32*a**2*b**2*d - 24*a**2*b*d*sqrt(a*b + b**2) + 16*a*b**3*d - 16*a*b**2*d*
sqrt(a*b + b**2)) + 5*a**2*b*sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a)*log(sq
rt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a) + tan(c/2 + d*x/2))/(2*a**4*d + 18*a*
*3*b*d - 8*a**3*d*sqrt(a*b + b**2) + 32*a**2*b**2*d - 24*a**2*b*d*sqrt(a*b
+ b**2) + 16*a*b**3*d - 16*a*b**2*d*sqrt(a*b + b**2)) + 5*a**2*sqrt(a*b + b
**2)*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*log(-sqrt(-1 - 2*b/a - 2*sqrt(
a*b + b**2)/a) + tan(c/2 + d*x/2))/(2*a**4*d + 18*a**3*b*d - 8*a**3*d*sqrt(
a*b + b**2) + 32*a**2*b**2*d - 24*a**2*b*d*sqrt(a*b + b**2) + 16*a*b**3*d -
 16*a*b**2*d*sqrt(a*b + b**2)) - 5*a**2*sqrt(a*b + b**2)*sqrt(-1 - 2*b/a -
2*sqrt(a*b + b**2)/a)*log(sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a) + tan(c/2
 + d*x/2))/(2*a**4*d + 18*a**3*b*d - 8*a**3*d*sqrt(a*b + b**2) + 32*a**2*b*
*2*d - 24*a**2*b*d*sqrt(a*b + b**2) + 16*a*b**3*d - 16*a*b**2*d*sqrt(a*b +
b**2)) + 3*a**2*sqrt(a*b + b**2)*sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a)*lo
g(-sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a) + tan(c/2 + d*x/2))/(2*a**4*d +
18*a**3*b*d - 8*a**3*d*sqrt(a*b + b**2) + 32*a**2*b**2*d - 24*a**2*b*d*sqrt
(a*b + b**2) + 16*a*b**3*d - 16*a*b**2*d*sqrt(a*b + b**2)) - 3*a**2*sqrt(a*
b + b**2)*sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a)*log(sqrt(-1 - 2*b/a + 2*s
qrt(a*b + b**2)/a) + tan(c/2 + d*x/2))/(2*a**4*d + 18*a**3*b*d - 8*a**3*d*s
qrt(a*b + b**2) + 32*a**2*b**2*d - 24*a**2*b*d*sqrt(a*b + b**2) + 16*a*b**3
*d - 16*a*b**2*d*sqrt(a*b + b**2)) - 28*a*b**2*sqrt(-1 - 2*b/a - 2*sqrt(a*b
 + b**2)/a)*log(-sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a) + tan(c/2 + d*x/2)
)/(2*a**4*d + 18*a**3*b*d - 8*a**3*d*sqrt(a*b + b**2) + 32*a**2*b**2*d - 24
*a**2*b*d*sqrt(a*b + b**2) + 16*a*b**3*d - 16*a*b**2*d*sqrt(a*b + b**2)) +
28*a*b**2*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*log(sqrt(-1 - 2*b/a - 2*s
qrt(a*b + b**2)/a) + tan(c/2 + d*x/2))/(2*a**4*d + 18*a**3*b*d - 8*a**3*d*s
qrt(a*b + b**2) + 32*a**2*b**2*d - 24*a**2*b*d*sqrt(a*b + b**2) + 16*a*b**3
*d - 16*a*b**2*d*sqrt(a*b + b**2)) - 4*a*b**2*sqrt(-1 - 2*b/a + 2*sqrt(a*b
+ b**2)/a)*log(-sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a) + tan(c/2 + d*x/2))
/(2*a**4*d + 18*a**3*b*d - 8*a**3*d*sqrt(a*b + b**2) + 32*a**2*b**2*d - 24*
a**2*b*d*sqrt(a*b + b**2) + 16*a*b**3*d - 16*a*b**2*d*sqrt(a*b + b**2)) + 4
*a*b**2*sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a)*log(sqrt(-1 - 2*b/a + 2*sqr
t(a*b + b**2)/a) + tan(c/2 + d*x/2))/(2*a**4*d + 18*a**3*b*d - 8*a**3*d*sqr
t(a*b + b**2) + 32*a**2*b**2*d - 24*a**2*b*d*sqrt(a*b + b**2) + 16*a*b**3*d
```

```
 - 16*a*b**2*d*sqrt(a*b + b**2)) + 20*a*b*sqrt(a*b + b**2)*sqrt(-1 - 2*b/a
- 2*sqrt(a*b + b**2)/a)*log(-sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a) + tan(
c/2 + d*x/2))/(2*a**4*d + 18*a**3*b*d - 8*a**3*d*sqrt(a*b + b**2) + 32*a**2
*b**2*d - 24*a**2*b*d*sqrt(a*b + b**2) + 16*a*b**3*d - 16*a*b**2*d*sqrt(a*b
 + b**2)) - 20*a*b*sqrt(a*b + b**2)*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)
*log(sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a) + tan(c/2 + d*x/2))/(2*a**4*d
+ 18*a**3*b*d - 8*a**3*d*sqrt(a*b + b**2) + 32*a**2*b**2*d - 24*a**2*b*d*sq
rt(a*b + b**2) + 16*a*b**3*d - 16*a*b**2*d*sqrt(a*b + b**2)) + 4*a*b*sqrt(a
*b + b**2)*sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a)*log(-sqrt(-1 - 2*b/a + 2
*sqrt(a*b + b**2)/a) + tan(c/2 + d*x/2))/(2*a**4*d + 18*a**3*b*d - 8*a**3*d
*sqrt(a*b + b**2) + 32*a**2*b**2*d - 24*a**2*b*d*sqrt(a*b + b**2) + 16*a*b*
*3*d - 16*a*b**2*d*sqrt(a*b + b**2)) - 4*a*b*sqrt(a*b + b**2)*sqrt(-1 - 2*b
/a + 2*sqrt(a*b + b**2)/a)*log(sqrt(-1 - 2*b/a + 2*sqrt(a*b + b**2)/a) + ta
n(c/2 + d*x/2))/(2*a**4*d + 18*a**3*b*d - 8*a**3*d*sqrt(a*b + b**2) + 32*a*
*2*b**2*d - 24*a**2*b*d*sqrt(a*b + b**2) + 16*a*b**3*d - 16*a*b**2*d*sqrt(a
*b + b**2)) - 16*b**3*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*log(-sqrt(-1
- 2*b/a - 2*sqrt(a*b + b**2)/a) + tan(c/2 + d*x/2))/(2*a**4*d + 18*a**3*b*d
 - 8*a**3*d*sqrt(a*b + b**2) + 32*a**2*b**2*d - 24*a**2*b*d*sqrt(a*b + b**2
) + 16*a*b**3*d - 16*a*b**2*d*sqrt(a*b + b**2)) + 16*b**3*sqrt(-1 - 2*b/a -
 2*sqrt(a*b + b**2)/a)*log(sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a) + tan(c/
2 + d*x/2))/(2*a**4*d + 18*a**3*b*d - 8*a**3*d*sqrt(a*b + b**2) + 32*a**2*b
**2*d - 24*a**2*b*d*sqrt(a*b + b**2) + 16*a*b**3*d - 16*a*b**2*d*sqrt(a*b +
 b**2)) + 16*b**2*sqrt(a*b + b**2)*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*
log(-sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a) + tan(c/2 + d*x/2))/(2*a**4*d
+ 18*a**3*b*d - 8*a**3*d*sqrt(a*b + b**2) + 32*a**2*b**2*d - 24*a**2*b*d*sq
rt(a*b + b**2) + 16*a*b**3*d - 16*a*b**2*d*sqrt(a*b + b**2)) - 16*b**2*sqrt
(a*b + b**2)*sqrt(-1 - 2*b/a - 2*sqrt(a*b + b**2)/a)*log(sqrt(-1 - 2*b/a -
2*sqrt(a*b + b**2)/a) + tan(c/2 + d*x/2))/(2*a**4*d + 18*a**3*b*d - 8*a**3*
d*sqrt(a*b + b**2) + 32*a**2*b**2*d - 24*a**2*b*d*sqrt(a*b + b**2) + 16*a*b
**3*d - 16*a*b**2*d*sqrt(a*b + b**2)), True))
```

Giac [B] time = 1.17185, size = 86, normalized size = 2.39

$$\frac{\pi \left\lfloor \frac{dx+c}{\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}\left(2\,a + 2\,b\right) + \arctan\left(\frac{a\tan\left(dx+c\right) + b\tan\left(dx+c\right)}{\sqrt{a^2+ab}}\right)}{\sqrt{a^2+ab}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] (pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c) + b*t
an(d*x + c))/sqrt(a^2 + a*b)))/(sqrt(a^2 + a*b)*d)
```

3.90 $\int \frac{\csc^2(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=53

$$-\frac{b\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{3/2}d\sqrt{a+b}}-\frac{\cot(c+dx)}{ad}$$

```
[Out] -((b*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a^(3/2)*Sqrt[a + b]*d)) -
 Cot[c + d*x]/(a*d)
```

Rubi [A] time = 0.073055, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3187, 453, 205}

$$-\frac{b\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{3/2}d\sqrt{a+b}}-\frac{\cot(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -((b*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a^(3/2)*Sqrt[a + b]*d)) -
 Cot[c + d*x]/(a*d)
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{x^2\left(a+(a+b)x^2\right)}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= -\frac{\cot(c+dx)}{ad} - \frac{b\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \tan(c+dx)\right)}{ad}$$
$$= -\frac{b\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a+b}d} - \frac{\cot(c+dx)}{ad}$$

Mathematica [A] time = 0.295625, size = 53, normalized size = 1.

$$\frac{-\frac{b\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a+b}} - \sqrt{a}\cot(c+dx)}{a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (-((b*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/Sqrt[a + b]) - Sqrt[a]*Co
t[c + d*x])/(a^(3/2)*d)
```

Maple [A] time = 0.109, size = 52, normalized size = 1.

$$-\frac{1}{da\tan(dx+c)} - \frac{b}{da}\arctan\left((a+b)\tan(dx+c)\frac{1}{\sqrt{a(a+b)}}\right)\frac{1}{\sqrt{a(a+b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+sin(d*x+c)^2*b),x)
```

```
[Out] -1/d/a/tan(d*x+c)-1/d/a*b/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))
^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.78419, size = 748, normalized size = 14.11

$$\left[-\frac{\sqrt{-a^2-ab}b\log\left(\frac{\left(8a^2+8ab+b^2\right)\cos(dx+c)^4-2\left(4a^2+5ab+b^2\right)\cos(dx+c)^2-4\left((2a+b)\cos(dx+c)^3-(a+b)\cos(dx+c)\right)\sqrt{-a^2-ab}\sin(dx+c)+a^2+2ab+b}{b^2\cos(dx+c)^4-2\left(ab+b^2\right)\cos(dx+c)^2+a^2+2ab+b^2}\right.}{4\left(a^3+a^2b\right)d\sin(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(-a^2 - a*b)*b*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4*
a^2 + 5*a*b + b^2)*cos(d*x + c)^2 - 4*((2*a + b)*cos(d*x + c)^3 - (a + b)*c
os(d*x + c))*sqrt(-a^2 - a*b)*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*
x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2))*sin(d*x + c)
+ 4*(a^2 + a*b)*cos(d*x + c))/((a^3 + a^2*b)*d*sin(d*x + c)), 1/2*(sqrt(a^2
 + a*b)*b*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)/(sqrt(a^2 + a*b)*co
s(d*x + c)*sin(d*x + c)))*sin(d*x + c) - 2*(a^2 + a*b)*cos(d*x + c))/((a^3
+ a^2*b)*d*sin(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^2(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral(csc(c + d*x)**2/(a + b*sin(c + d*x)**2), x)
```

Giac [A] time = 1.18442, size = 112, normalized size = 2.11

$$-\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)b}{\sqrt{a^2+ab}a}+\frac{1}{a\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -((pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c) + b
*tan(d*x + c))/sqrt(a^2 + a*b)))*b/(sqrt(a^2 + a*b)*a) + 1/(a*tan(d*x + c))
)/d
```

3.91 $\int \frac{\csc^4(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=77

$$\frac{b^2\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{5/2}d\sqrt{a+b}}-\frac{(a-b)\cot(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3ad}$$

```
[Out] (b^2*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a^(5/2)*Sqrt[a + b]*d) -
((a - b)*Cot[c + d*x])/(a^2*d) - Cot[c + d*x]^3/(3*a*d)
```

Rubi [A] time = 0.105693, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3187, 461, 205}

$$\frac{b^2\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{5/2}d\sqrt{a+b}}-\frac{(a-b)\cot(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (b^2*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a^(5/2)*Sqrt[a + b]*d) -
((a - b)*Cot[c + d*x])/(a^2*d) - Cot[c + d*x]^3/(3*a*d)
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\csc^4(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^4\left(a+(a+b)x^2\right)}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^4}+\frac{a-b}{a^2x^2}+\frac{b^2}{a^2\left(a+(a+b)x^2\right)}\right)dx, x, \tan(c+dx)\right)}{d}$$
$$= -\frac{(a-b)\cot(c+dx)}{a^2d} - \frac{\cot^3(c+dx)}{3ad} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \tan(c+dx)\right)}{a^2d}$$
$$= \frac{b^2\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{5/2}\sqrt{a+b}d} - \frac{(a-b)\cot(c+dx)}{a^2d} - \frac{\cot^3(c+dx)}{3ad}$$

Mathematica [A] time = 0.654628, size = 119, normalized size = 1.55

$$-\frac{\csc^2(c+dx)(2a-b\cos(2(c+dx))+b)\left(\sqrt{a}\sqrt{a+b}\cot(c+dx)\left(a\csc^2(c+dx)+2a-3b\right)-3b^2\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)\right)}{6a^{5/2}d\sqrt{a+b}\left(a\csc^2(c+dx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -((2*a + b - b*Cos[2*(c + d*x)])*Csc[c + d*x]^2*(-3*b^2*ArcTan[(Sqrt[a + b]
*Tan[c + d*x])/Sqrt[a]] + Sqrt[a]*Sqrt[a + b]*Cot[c + d*x]*(2*a - 3*b + a*C
sc[c + d*x]^2)))/(6*a^(5/2)*Sqrt[a + b]*d*(b + a*Csc[c + d*x]^2))
```

Maple [A] time = 0.116, size = 85, normalized size = 1.1

$$\frac{b^2}{a^2d}\arctan\left((a+b)\tan(dx+c)\frac{1}{\sqrt{a(a+b)}}\right)\frac{1}{\sqrt{a(a+b)}} - \frac{1}{3da(\tan(dx+c))^3} - \frac{1}{da\tan(dx+c)} + \frac{b}{a^2d\tan(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4/(a+sin(d*x+c)^2*b),x)
```

```
[Out] 1/d/a^2*b^2/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))-1/3/d/
a/tan(d*x+c)^3-1/d/a/tan(d*x+c)+1/d/a^2/tan(d*x+c)*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.85966, size = 1046, normalized size = 13.58

$$\left[-\frac{4\left(2a^3-a^2b-3ab^2\right)\cos(dx+c)^3+3\left(b^2\cos(dx+c)^2-b^2\right)\sqrt{-a^2-ab}\log\left(\frac{\left(8a^2+8ab+b^2\right)\cos(dx+c)^4-2\left(4a^2+5ab+b^2\right)\cos(\ldots}{b^2\cos(dx\ldots}\right.}{12\left(\left(a^4+a^3b\right)d\cos(dx+c)^2-\left(a^4\ldots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/12*(4*(2*a^3 - a^2*b - 3*a*b^2)*cos(d*x + c)^3 + 3*(b^2*cos(d*x + c)^2
- b^2)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4*a^
2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a + b)*cos(d*x + c)^3 - (a + b)*cos
(d*x + c))*sqrt(-a^2 - a*b)*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x
+ c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2))*sin(d*x + c) -
12*(a^3 - a*b^2)*cos(d*x + c))/(((a^4 + a^3*b)*d*cos(d*x + c)^2 - (a^4 + a^
3*b)*d)*sin(d*x + c)), -1/6*(2*(2*a^3 - a^2*b - 3*a*b^2)*cos(d*x + c)^3 + 3
*(b^2*cos(d*x + c)^2 - b^2)*sqrt(a^2 + a*b)*arctan(1/2*((2*a + b)*cos(d*x +
 c)^2 - a - b)/(sqrt(a^2 + a*b)*cos(d*x + c)*sin(d*x + c)))*sin(d*x + c) -
6*(a^3 - a*b^2)*cos(d*x + c))/(((a^4 + a^3*b)*d*cos(d*x + c)^2 - (a^4 + a^3
*b)*d)*sin(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15074, size = 150, normalized size = 1.95

$$\frac{\frac{3\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)b^2}{\sqrt{a^2+ab}a^2}-\frac{3a\tan(dx+c)^2-3b\tan(dx+c)^2+a}{a^2\tan(dx+c)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c
) + b*tan(d*x + c))/sqrt(a^2 + a*b)))*b^2/(sqrt(a^2 + a*b)*a^2) - (3*a*tan(
d*x + c)^2 - 3*b*tan(d*x + c)^2 + a)/(a^2*tan(d*x + c)^3))/d
```

3.92 $\int \frac{\csc^6(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=109

$$-\frac{b^3\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{7/2}d\sqrt{a+b}}-\frac{\left(a^2-ab+b^2\right)\cot(c+dx)}{a^3d}-\frac{(2a-b)\cot^3(c+dx)}{3a^2d}-\frac{\cot^5(c+dx)}{5ad}$$

```
[Out] -((b^3*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a^(7/2)*Sqrt[a + b]*d))
 - ((a^2 - a*b + b^2)*Cot[c + d*x])/(a^3*d) - ((2*a - b)*Cot[c + d*x]^3)/(3
*a^2*d) - Cot[c + d*x]^5/(5*a*d)
```

Rubi [A] time = 0.124797, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3187, 461, 205}

$$-\frac{b^3\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{7/2}d\sqrt{a+b}}-\frac{\left(a^2-ab+b^2\right)\cot(c+dx)}{a^3d}-\frac{(2a-b)\cot^3(c+dx)}{3a^2d}-\frac{\cot^5(c+dx)}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6/(a + b*Sin[c + d*x]^2),x]
```


```
[Out] -((b^3*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a^(7/2)*Sqrt[a + b]*d))
 - ((a^2 - a*b + b^2)*Cot[c + d*x])/(a^3*d) - ((2*a - b)*Cot[c + d*x]^3)/(3
*a^2*d) - Cot[c + d*x]^5/(5*a*d)
```


Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\csc^6(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{x^6(a+(a+b)x^2)}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^6}+\frac{2a-b}{a^2x^4}+\frac{a^2-ab+b^2}{a^3x^2}+\frac{b^3}{a^3(-a-(a+b)x^2)}\right)dx, x, \tan(c+dx)\right)}{d}$$
$$= -\frac{(a^2-ab+b^2)\cot(c+dx)}{a^3d} - \frac{(2a-b)\cot^3(c+dx)}{3a^2d} - \frac{\cot^5(c+dx)}{5ad} + \frac{b^3\,\text{Subst}\left(\int \frac{1}{-a-(a+b)\ldots}\right.}{a\ldots}$$
$$= -\frac{b^3\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{7/2}\sqrt{a+b}d} - \frac{(a^2-ab+b^2)\cot(c+dx)}{a^3d} - \frac{(2a-b)\cot^3(c+dx)}{3a^2d} - \frac{\cot^5(c+\ldots}{5ad}$$

Mathematica [A] time = 1.54195, size = 147, normalized size = 1.35

$$-\frac{\csc^2(c+dx)(2a-b\cos(2(c+dx))+b)\left(\sqrt{a}\sqrt{a+b}\cot(c+dx)\left(3a^2\csc^4(c+dx)+8a^2+a(4a-5b)\csc^2(c+dx)-1\ldots\right.\right.}{30a^{7/2}d\sqrt{a+b}\left(a\csc^2(c+dx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -((2*a + b - b*Cos[2*(c + d*x)])*Csc[c + d*x]^2*(15*b^3*ArcTan[(Sqrt[a + b]
*Tan[c + d*x])/Sqrt[a]] + Sqrt[a]*Sqrt[a + b]*Cot[c + d*x]*(8*a^2 - 10*a*b
+ 15*b^2 + a*(4*a - 5*b)*Csc[c + d*x]^2 + 3*a^2*Csc[c + d*x]^4)))/(30*a^(7/
2)*Sqrt[a + b]*d*(b + a*Csc[c + d*x]^2))
```

Maple [A] time = 0.123, size = 138, normalized size = 1.3

$$-\frac{b^3}{da^3}\arctan\left((a+b)\tan(dx+c)\frac{1}{\sqrt{a(a+b)}}\right)\frac{1}{\sqrt{a(a+b)}} - \frac{1}{5da(\tan(dx+c))^5} - \frac{2}{3da(\tan(dx+c))^3} + \frac{b}{3a^2d(\tan(\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6/(a+sin(d*x+c)^2*b),x)
```

```
[Out] -1/d*b^3/a^3/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))-1/5/d
/a/tan(d*x+c)^5-2/3/d/a/tan(d*x+c)^3+1/3/d/a^2/tan(d*x+c)^3*b-1/d/a/tan(d*x
+c)+1/d/a^2/tan(d*x+c)*b-1/d/a^3/tan(d*x+c)*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.88241, size = 1400, normalized size = 12.84

$$\left[-\frac{4\left(8\,a^4-2\,a^3b+5\,a^2b^2+15\,ab^3\right)\cos\left(dx+c\right)^5-20\left(4\,a^4-a^3b+a^2b^2+6\,ab^3\right)\cos\left(dx+c\right)^3+15\left(b^3\cos\left(dx+c\right)^4-2\right.}{60\left(\left(a^5+a^4b\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/60*(4*(8*a^4 - 2*a^3*b + 5*a^2*b^2 + 15*a*b^3)*cos(d*x + c)^5 - 20*(4*a
^4 - a^3*b + a^2*b^2 + 6*a*b^3)*cos(d*x + c)^3 + 15*(b^3*cos(d*x + c)^4 - 2
*b^3*cos(d*x + c)^2 + b^3)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(
d*x + c)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 - 4*((2*a + b)*cos(d*x
+ c)^3 - (a + b)*cos(d*x + c))*sqrt(-a^2 - a*b)*sin(d*x + c) + a^2 + 2*a*b
+ b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b
^2))*sin(d*x + c) + 60*(a^4 + a*b^3)*cos(d*x + c))/(((a^5 + a^4*b)*d*cos(d*
x + c)^4 - 2*(a^5 + a^4*b)*d*cos(d*x + c)^2 + (a^5 + a^4*b)*d)*sin(d*x + c)
), -1/30*(2*(8*a^4 - 2*a^3*b + 5*a^2*b^2 + 15*a*b^3)*cos(d*x + c)^5 - 10*(4
*a^4 - a^3*b + a^2*b^2 + 6*a*b^3)*cos(d*x + c)^3 - 15*(b^3*cos(d*x + c)^4 -
 2*b^3*cos(d*x + c)^2 + b^3)*sqrt(a^2 + a*b)*arctan(1/2*((2*a + b)*cos(d*x
+ c)^2 - a - b)/(sqrt(a^2 + a*b)*cos(d*x + c)*sin(d*x + c)))*sin(d*x + c) +
 30*(a^4 + a*b^3)*cos(d*x + c))/(((a^5 + a^4*b)*d*cos(d*x + c)^4 - 2*(a^5 +
 a^4*b)*d*cos(d*x + c)^2 + (a^5 + a^4*b)*d)*sin(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16892, size = 209, normalized size = 1.92

$$-\frac{\frac{15\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)b^3}{\sqrt{a^2+ab}a^3}+\frac{15\,a^2\tan(dx+c)^4-15\,ab\tan(dx+c)^4+15\,b^2\tan(dx+c)^4+10\,a^2\tan(dx+c)^2-5\,ab\tan(dx+c}{a^3\tan(dx+c)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/15*(15*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x
+ c) + b*tan(d*x + c))/sqrt(a^2 + a*b)))*b^3/(sqrt(a^2 + a*b)*a^3) + (15*a^
2*tan(d*x + c)^4 - 15*a*b*tan(d*x + c)^4 + 15*b^2*tan(d*x + c)^4 + 10*a^2*t
an(d*x + c)^2 - 5*a*b*tan(d*x + c)^2 + 3*a^2)/(a^3*tan(d*x + c)^5))/d
```

3.93 $\int \frac{\csc^8(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=140

$$\frac{b^4\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{9/2}d\sqrt{a+b}}-\frac{\left(3a^2-2ab+b^2\right)\cot^3(c+dx)}{3a^3d}-\frac{(a-b)\left(a^2+b^2\right)\cot(c+dx)}{a^4d}-\frac{(3a-b)\cot^5(c+dx)}{5a^2d}-\text{co}$$

```
[Out] (b^4*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a^(9/2)*Sqrt[a + b]*d) -
((a - b)*(a^2 + b^2)*Cot[c + d*x])/(a^4*d) - ((3*a^2 - 2*a*b + b^2)*Cot[c +
 d*x]^3)/(3*a^3*d) - ((3*a - b)*Cot[c + d*x]^5)/(5*a^2*d) - Cot[c + d*x]^7/
(7*a*d)
```

Rubi [A] time = 0.155528, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3187, 461, 205}

$$\frac{b^4\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{9/2}d\sqrt{a+b}}-\frac{\left(3a^2-2ab+b^2\right)\cot^3(c+dx)}{3a^3d}-\frac{(a-b)\left(a^2+b^2\right)\cot(c+dx)}{a^4d}-\frac{(3a-b)\cot^5(c+dx)}{5a^2d}-\text{co}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^8/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (b^4*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a^(9/2)*Sqrt[a + b]*d) -
((a - b)*(a^2 + b^2)*Cot[c + d*x])/(a^4*d) - ((3*a^2 - 2*a*b + b^2)*Cot[c +
 d*x]^3)/(3*a^3*d) - ((3*a - b)*Cot[c + d*x]^5)/(5*a^2*d) - Cot[c + d*x]^7/
(7*a*d)
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\csc^8(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^4}{x^8(a+(a+b)x^2)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^8}+\frac{3a-b}{a^2x^6}+\frac{3a^2-2ab+b^2}{a^3x^4}+\frac{(a-b)(a^2+b^2)}{a^4x^2}+\frac{b^4}{a^4(a+(a+b)x^2)}\right)dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{(a-b)\left(a^2+b^2\right)\cot(c+dx)}{a^4d} - \frac{\left(3a^2-2ab+b^2\right)\cot^3(c+dx)}{3a^3d} - \frac{(3a-b)\cot^5(c+dx)}{5a^2d} - \frac{\cot^7}{}$$

$$= \frac{b^4\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{9/2}\sqrt{a+b}d} - \frac{(a-b)\left(a^2+b^2\right)\cot(c+dx)}{a^4d} - \frac{\left(3a^2-2ab+b^2\right)\cot^3(c+dx)}{3a^3d} - \frac{(3a}{}$$

Mathematica [A] time = 1.72937, size = 137, normalized size = 0.98

$$\frac{b^4\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{9/2}d\sqrt{a+b}} - \frac{\cot(c+dx)\left(a\left(24a^2-28ab+35b^2\right)\csc^2(c+dx)+3a^2(6a-7b)\csc^4(c+dx)-56a^2b+15a^3\right.}{105a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^8/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (b^4*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a^(9/2)*Sqrt[a + b]*d) -
(Cot[c + d*x]*(48*a^3 - 56*a^2*b + 70*a*b^2 - 105*b^3 + a*(24*a^2 - 28*a*b
+ 35*b^2)*Csc[c + d*x]^2 + 3*a^2*(6*a - 7*b)*Csc[c + d*x]^4 + 15*a^3*Csc[c
+ d*x]^6))/(105*a^4*d)
```

Maple [A] time = 0.158, size = 207, normalized size = 1.5

$$\frac{b^4}{da^4}\arctan\left((a+b)\tan(dx+c)\frac{1}{\sqrt{a(a+b)}}\right)\frac{1}{\sqrt{a(a+b)}} - \frac{1}{7da(\tan(dx+c))^7} - \frac{3}{5da(\tan(dx+c))^5} + \frac{b}{5a^2d(\tan(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^8/(a+sin(d*x+c)^2*b),x)
```

```
[Out] 1/d*b^4/a^4/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))-1/7/d/
a/tan(d*x+c)^7-3/5/d/a/tan(d*x+c)^5+1/5/d/a^2/tan(d*x+c)^5*b-1/d/a/tan(d*x+
c)^3+2/3/d/a^2/tan(d*x+c)^3*b-1/3/d/a^3/tan(d*x+c)^3*b^2-1/d/a/tan(d*x+c)+1
/d/a^2/tan(d*x+c)*b-1/d/a^3/tan(d*x+c)*b^2+1/d/a^4/tan(d*x+c)*b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.01844, size = 1839, normalized size = 13.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/420*(4*(48*a^5 - 8*a^4*b + 14*a^3*b^2 - 35*a^2*b^3 - 105*a*b^4)*cos(d*x
 + c)^7 - 28*(24*a^5 - 4*a^4*b + 7*a^3*b^2 - 10*a^2*b^3 - 45*a*b^4)*cos(d*x
 + c)^5 + 140*(6*a^5 - a^4*b + a^3*b^2 - a^2*b^3 - 9*a*b^4)*cos(d*x + c)^3
+ 105*(b^4*cos(d*x + c)^6 - 3*b^4*cos(d*x + c)^4 + 3*b^4*cos(d*x + c)^2 - b
^4)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4*a^2 +
 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a + b)*cos(d*x + c)^3 - (a + b)*cos(d*
x + c))*sqrt(-a^2 - a*b)*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x + c
)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2))*sin(d*x + c) - 420
*(a^5 - a*b^4)*cos(d*x + c))/(((a^6 + a^5*b)*d*cos(d*x + c)^6 - 3*(a^6 + a^
5*b)*d*cos(d*x + c)^4 + 3*(a^6 + a^5*b)*d*cos(d*x + c)^2 - (a^6 + a^5*b)*d)
*sin(d*x + c)), -1/210*(2*(48*a^5 - 8*a^4*b + 14*a^3*b^2 - 35*a^2*b^3 - 105
*a*b^4)*cos(d*x + c)^7 - 14*(24*a^5 - 4*a^4*b + 7*a^3*b^2 - 10*a^2*b^3 - 45
*a*b^4)*cos(d*x + c)^5 + 70*(6*a^5 - a^4*b + a^3*b^2 - a^2*b^3 - 9*a*b^4)*c
os(d*x + c)^3 + 105*(b^4*cos(d*x + c)^6 - 3*b^4*cos(d*x + c)^4 + 3*b^4*cos(
d*x + c)^2 - b^4)*sqrt(a^2 + a*b)*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a
- b)/(sqrt(a^2 + a*b)*cos(d*x + c)*sin(d*x + c)))*sin(d*x + c) - 210*(a^5 -
 a*b^4)*cos(d*x + c))/(((a^6 + a^5*b)*d*cos(d*x + c)^6 - 3*(a^6 + a^5*b)*d*
cos(d*x + c)^4 + 3*(a^6 + a^5*b)*d*cos(d*x + c)^2 - (a^6 + a^5*b)*d)*sin(d*
x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**8/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19436, size = 290, normalized size = 2.07

$$\frac{\frac{105\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)b^4}{\sqrt{a^2+ab}a^4}-\frac{105\,a^3\tan(dx+c)^6-105\,a^2b\tan(dx+c)^6+105\,ab^2\tan(dx+c)^6-105\,b^3\tan(dx+c)^6+105}{}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/105*(105*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x
 + c) + b*tan(d*x + c))/sqrt(a^2 + a*b)))*b^4/(sqrt(a^2 + a*b)*a^4) - (105*
```

```
a^3*tan(d*x + c)^6 - 105*a^2*b*tan(d*x + c)^6 + 105*a*b^2*tan(d*x + c)^6 -
105*b^3*tan(d*x + c)^6 + 105*a^3*tan(d*x + c)^4 - 70*a^2*b*tan(d*x + c)^4 +
 35*a*b^2*tan(d*x + c)^4 + 63*a^3*tan(d*x + c)^2 - 21*a^2*b*tan(d*x + c)^2
+ 15*a^3)/(a^4*tan(d*x + c)^7))/d
```

3.94 $\int \frac{\sin^7(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=128

$$\frac{a^3\cos(c+dx)}{2b^3d(a+b)\left(a-b\cos^2(c+dx)+b\right)}-\frac{a^2(5a+6b)\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2b^{7/2}d(a+b)^{3/2}}+\frac{(2a-b)\cos(c+dx)}{b^3d}+\frac{\cos^3(c+dx)}{3b^2d}$$

```
[Out] -(a^2*(5*a + 6*b)*ArcTanh[(Sqrt[b]*Cos[c + d*x])/Sqrt[a + b]])/(2*b^(7/2)*(
a + b)^(3/2)*d) + ((2*a - b)*Cos[c + d*x])/(b^3*d) + Cos[c + d*x]^3/(3*b^2*
d) + (a^3*Cos[c + d*x])/(2*b^3*(a + b)*d*(a + b - b*Cos[c + d*x]^2))
```

Rubi [A] time = 0.186128, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3186, 390, 385, 208}

$$\frac{a^3\cos(c+dx)}{2b^3d(a+b)\left(a-b\cos^2(c+dx)+b\right)}-\frac{a^2(5a+6b)\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2b^{7/2}d(a+b)^{3/2}}+\frac{(2a-b)\cos(c+dx)}{b^3d}+\frac{\cos^3(c+dx)}{3b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^7/(a + b*Sin[c + d*x]^2)^2,x]
```


```
[Out] -(a^2*(5*a + 6*b)*ArcTanh[(Sqrt[b]*Cos[c + d*x])/Sqrt[a + b]])/(2*b^(7/2)*(
a + b)^(3/2)*d) + ((2*a - b)*Cos[c + d*x])/(b^3*d) + Cos[c + d*x]^3/(3*b^2*
d) + (a^3*Cos[c + d*x])/(2*b^3*(a + b)*d*(a + b - b*Cos[c + d*x]^2))
```


Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sin^7(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{\left(a+b-bx^2\right)^2}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(-\frac{2a-b}{b^3}-\frac{x^2}{b^2}+\frac{a^2(2a+3b)-3a^2bx^2}{b^3\left(a+b-bx^2\right)^2}\right)dx, x, \cos(c+dx)\right)}{d}$$

$$= \frac{(2a-b)\cos(c+dx)}{b^3d}+\frac{\cos^3(c+dx)}{3b^2d}-\frac{\text{Subst}\left(\int \frac{a^2(2a+3b)-3a^2bx^2}{\left(a+b-bx^2\right)^2}\,dx, x, \cos(c+dx)\right)}{b^3d}$$

$$= \frac{(2a-b)\cos(c+dx)}{b^3d}+\frac{\cos^3(c+dx)}{3b^2d}+\frac{a^3\cos(c+dx)}{2b^3(a+b)d\left(a+b-b\cos^2(c+dx)\right)}-\frac{\left(a^2(5a+6b)\right.}{}$$

$$= -\frac{a^2(5a+6b)\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2b^{7/2}(a+b)^{3/2}d}+\frac{(2a-b)\cos(c+dx)}{b^3d}+\frac{\cos^3(c+dx)}{3b^2d}+\frac{a^3\,c}{2b^3(a+b)d\left(a\right.}$$

Mathematica [C] time = 1.57376, size = 194, normalized size = 1.52

$$\frac{\sqrt{b}\left(\cos(c+dx)\left(\frac{12a^3}{(a+b)(2a-b\cos(2(c+dx))+b)}+24a-9b\right)+b\cos(3(c+dx))\right)-\frac{6a^2(5a+6b)\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{(-a-b)^{3/2}}-\frac{6a^2(5a+6b)\tan^{-}}{(-}}{12b^{7/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^7/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] ((-6*a^2*(5*a + 6*b)*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a
- b]])/(-a - b)^(3/2) - (6*a^2*(5*a + 6*b)*ArcTan[(Sqrt[b] + I*Sqrt[a]*Tan[
(c + d*x)/2])/Sqrt[-a - b]])/(-a - b)^(3/2) + Sqrt[b]*(Cos[c + d*x]*(24*a -
 9*b + (12*a^3)/((a + b)*(2*a + b - b*Cos[2*(c + d*x)]))) + b*Cos[3*(c + d*
x)]))/(12*b^(7/2)*d)
```

Maple [A] time = 0.085, size = 118, normalized size = 0.9

$$\frac{1}{d}\left(\frac{1}{b^3}\left(\frac{b\left(\cos(dx+c)\right)^3}{3}+2\cos(dx+c)\,a-b\cos(dx+c)\right)+\frac{a^2}{b^3}\left(-\frac{\cos(dx+c)\,a}{(2a+2b)\left(b\left(\cos(dx+c)\right)^2-a-b\right)}-\frac{5a+6b}{2a+2b}\text{Arta}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^7/(a+sin(d*x+c)^2*b)^2,x)
```

```
[Out] 1/d*(1/b^3*(1/3*b*cos(d*x+c)^3+2*cos(d*x+c)*a-b*cos(d*x+c))+1/b^3*a^2*(-1/2
/(a+b)*a*cos(d*x+c)/(b*cos(d*x+c)^2-a-b)-1/2*(5*a+6*b)/(a+b)/((a+b)*b)^(1/2
)*arctanh(cos(d*x+c)*b/((a+b)*b)^(1/2))))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+b*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.07301, size = 1162, normalized size = 9.08

$$\left[\frac{4\left(a^2b^3+2\,ab^4+b^5\right)\cos\left(dx+c\right)^5+4\left(5\,a^3b^2+6\,a^2b^3-3\,ab^4-4\,b^5\right)\cos\left(dx+c\right)^3-3\left(5\,a^4+11\,a^3b+6\,a^2b^2-\left(5\,a^3\right.\right.}{12\left(\left(a^2b^5+2\,ab^6+b^7\right)d\cos\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+b*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*(a^2*b^3 + 2*a*b^4 + b^5)*cos(d*x + c)^5 + 4*(5*a^3*b^2 + 6*a^2*b^
3 - 3*a*b^4 - 4*b^5)*cos(d*x + c)^3 - 3*(5*a^4 + 11*a^3*b + 6*a^2*b^2 - (5*
a^3*b + 6*a^2*b^2)*cos(d*x + c)^2)*sqrt(a*b + b^2)*log(-(b*cos(d*x + c)^2 -
 2*sqrt(a*b + b^2)*cos(d*x + c) + a + b)/(b*cos(d*x + c)^2 - a - b)) - 6*(5
*a^4*b + 11*a^3*b^2 + 6*a^2*b^3 - 2*a*b^4 - 2*b^5)*cos(d*x + c))/((a^2*b^5
+ 2*a*b^6 + b^7)*d*cos(d*x + c)^2 - (a^3*b^4 + 3*a^2*b^5 + 3*a*b^6 + b^7)*d
), 1/6*(2*(a^2*b^3 + 2*a*b^4 + b^5)*cos(d*x + c)^5 + 2*(5*a^3*b^2 + 6*a^2*b
^3 - 3*a*b^4 - 4*b^5)*cos(d*x + c)^3 - 3*(5*a^4 + 11*a^3*b + 6*a^2*b^2 - (5
*a^3*b + 6*a^2*b^2)*cos(d*x + c)^2)*sqrt(-a*b - b^2)*arctan(sqrt(-a*b - b^2
)*cos(d*x + c)/(a + b)) - 3*(5*a^4*b + 11*a^3*b^2 + 6*a^2*b^3 - 2*a*b^4 - 2
*b^5)*cos(d*x + c))/((a^2*b^5 + 2*a*b^6 + b^7)*d*cos(d*x + c)^2 - (a^3*b^4
+ 3*a^2*b^5 + 3*a*b^6 + b^7)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**7/(a+b*sin(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18171, size = 435, normalized size = 3.4

$$\frac{\frac{3\left(5\,a^3+6\,a^2b\right)\arctan\left(\frac{b\cos(dx+c)+a+b}{\sqrt{-ab-b^2}\cos(dx+c)+\sqrt{-ab-b^2}}\right)}{\left(ab^3+b^4\right)\sqrt{-ab-b^2}}+\frac{6\left(a^3-\frac{a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{2\,a^2b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{\left(ab^3+b^4\right)\left(a-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{4\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)}-\frac{8\left(3\,a-b-\frac{6\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{3\,b(\cos(\ldots}{\cos(dx\ldots}\right.}{b^3\left(\frac{\cos(dx+c)-\ldots}{\cos(dx+c)+\ldots}\right.}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+b*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/6*(3*(5*a^3 + 6*a^2*b)*arctan((b*cos(d*x + c) + a + b)/(sqrt(-a*b - b^2)*
cos(d*x + c) + sqrt(-a*b - b^2)))/((a*b^3 + b^4)*sqrt(-a*b - b^2)) + 6*(a^3
 - a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*a^2*b*(cos(d*x + c) - 1)/(
cos(d*x + c) + 1))/((a*b^3 + b^4)*(a - 2*a*(cos(d*x + c) - 1)/(cos(d*x + c)
 + 1) - 4*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + a*(cos(d*x + c) - 1)^2/
(cos(d*x + c) + 1)^2)) - 8*(3*a - b - 6*a*(cos(d*x + c) - 1)/(cos(d*x + c)
+ 1) + 3*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 3*a*(cos(d*x + c) - 1)^2
/(cos(d*x + c) + 1)^2)/(b^3*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^3))
/d
```

3.95 $\int \frac{\sin^5(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=102

$$-\frac{a^2\cos(c+dx)}{2b^2d(a+b)\left(a-b\cos^2(c+dx)+b\right)}+\frac{a(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2b^{5/2}d(a+b)^{3/2}}-\frac{\cos(c+dx)}{b^2d}$$

```
[Out] (a*(3*a + 4*b)*ArcTanh[(Sqrt[b]*Cos[c + d*x])/Sqrt[a + b]])/(2*b^(5/2)*(a +
 b)^(3/2)*d) - Cos[c + d*x]/(b^2*d) - (a^2*Cos[c + d*x])/(2*b^2*(a + b)*d*(
a + b - b*Cos[c + d*x]^2))
```

Rubi [A] time = 0.145686, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3186, 390, 385, 208}

$$-\frac{a^2\cos(c+dx)}{2b^2d(a+b)\left(a-b\cos^2(c+dx)+b\right)}+\frac{a(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2b^{5/2}d(a+b)^{3/2}}-\frac{\cos(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^5/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] (a*(3*a + 4*b)*ArcTanh[(Sqrt[b]*Cos[c + d*x])/Sqrt[a + b]])/(2*b^(5/2)*(a +
 b)^(3/2)*d) - Cos[c + d*x]/(b^2*d) - (a^2*Cos[c + d*x])/(2*b^2*(a + b)*d*(
a + b - b*Cos[c + d*x]^2))
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sin^5(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\left(a+b-bx^2\right)^2}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\operatorname{Subst}\left(\int \left(\frac{1}{b^2}-\frac{a(a+2b)-2abx^2}{b^2\left(a+b-bx^2\right)^2}\right)dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\cos(c+dx)}{b^2d}+\frac{\operatorname{Subst}\left(\int \frac{a(a+2b)-2abx^2}{\left(a+b-bx^2\right)^2}\,dx, x, \cos(c+dx)\right)}{b^2d}$$

$$= -\frac{\cos(c+dx)}{b^2d}-\frac{a^2\cos(c+dx)}{2b^2(a+b)d\left(a+b-b\cos^2(c+dx)\right)}+\frac{(a(3a+4b))\operatorname{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x,\right.}{2b^2(a+b)d}$$

$$= \frac{a(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2b^{5/2}(a+b)^{3/2}d}-\frac{\cos(c+dx)}{b^2d}-\frac{a^2\cos(c+dx)}{2b^2(a+b)d\left(a+b-b\cos^2(c+dx)\right)}$$

Mathematica [C] time = 0.904157, size = 172, normalized size = 1.69

$$\frac{2\sqrt{b}\cos(c+dx)\left(-\frac{a^2}{(a+b)(2a-b\cos(2(c+dx))+b)}-1\right)+\frac{a(3a+4b)\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{(-a-b)^{3/2}}+\frac{a(3a+4b)\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{(-a-b)^{3/2}}}{2b^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^5/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] ((a*(3*a + 4*b)*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]]
)/(-a - b)^(3/2) + (a*(3*a + 4*b)*ArcTan[(Sqrt[b] + I*Sqrt[a]*Tan[(c + d*x)
/2])/Sqrt[-a - b]])/(-a - b)^(3/2) + 2*Sqrt[b]*Cos[c + d*x]*(-1 - a^2/((a +
 b)*(2*a + b - b*Cos[2*(c + d*x)]))))/(2*b^(5/2)*d)
```

Maple [A] time = 0.083, size = 94, normalized size = 0.9

$$\frac{1}{d}\left(-\frac{\cos(dx+c)}{b^2}-\frac{a}{b^2}\left(-\frac{\cos(dx+c)\,a}{(2a+2b)\left(b(\cos(dx+c))^2-a-b\right)}-\frac{3a+4b}{2a+2b}\operatorname{Artanh}\left(b\cos(dx+c)\frac{1}{\sqrt{(a+b)b}}\right)\frac{1}{\sqrt{(a+b)b}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5/(a+sin(d*x+c)^2*b)^2,x)
```

```
[Out] 1/d*(-1/b^2*cos(d*x+c)-1/b^2*a*(-1/2/(a+b)*a*cos(d*x+c)/(b*cos(d*x+c)^2-a-b
)-1/2*(3*a+4*b)/(a+b)/((a+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/((a+b)*b)^(1/2))
))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.97237, size = 941, normalized size = 9.23

$$\left[-\frac{4\left(a^{2}b^{2}+2\,ab^{3}+b^{4}\right)\cos\left(dx+c\right)^{3}+\left(3\,a^{3}+7\,a^{2}b+4\,ab^{2}-\left(3\,a^{2}b+4\,ab^{2}\right)\cos\left(dx+c\right)^{2}\right)\sqrt{ab+b^{2}}\log\left(\frac{b\cos(dx+c)^{2}+2}{b\cos}\right.}{4\left(\left(a^{2}b^{4}+2\,ab^{5}+b^{6}\right)d\cos\left(dx+c\right)^{2}-\left(a^{3}b^{3}+3\,a^{2}b^{4}+3\,a\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(a^2*b^2 + 2*a*b^3 + b^4)*cos(d*x + c)^3 + (3*a^3 + 7*a^2*b + 4*a*
b^2 - (3*a^2*b + 4*a*b^2)*cos(d*x + c)^2)*sqrt(a*b + b^2)*log((b*cos(d*x +
c)^2 + 2*sqrt(a*b + b^2)*cos(d*x + c) + a + b)/(b*cos(d*x + c)^2 - a - b))
- 2*(3*a^3*b + 7*a^2*b^2 + 6*a*b^3 + 2*b^4)*cos(d*x + c))/((a^2*b^4 + 2*a*b
^5 + b^6)*d*cos(d*x + c)^2 - (a^3*b^3 + 3*a^2*b^4 + 3*a*b^5 + b^6)*d), -1/2
*(2*(a^2*b^2 + 2*a*b^3 + b^4)*cos(d*x + c)^3 - (3*a^3 + 7*a^2*b + 4*a*b^2 -
 (3*a^2*b + 4*a*b^2)*cos(d*x + c)^2)*sqrt(-a*b - b^2)*arctan(sqrt(-a*b - b^
2)*cos(d*x + c)/(a + b)) - (3*a^3*b + 7*a^2*b^2 + 6*a*b^3 + 2*b^4)*cos(d*x
+ c))/((a^2*b^4 + 2*a*b^5 + b^6)*d*cos(d*x + c)^2 - (a^3*b^3 + 3*a^2*b^4 +
3*a*b^5 + b^6)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5/(a+b*sin(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.16373, size = 462, normalized size = 4.53

$$-\frac{\frac{\left(3\,a^{2}+4\,ab\right)\arctan\left(\frac{b\cos(dx+c)+a+b}{\sqrt{-ab-b^{2}}\cos(dx+c)+\sqrt{-ab-b^{2}}}\right)}{\left(ab^{2}+b^{3}\right)\sqrt{-ab-b^{2}}}+\frac{2\left(3\,a^{2}+2\,ab-\frac{6\,a^{2}(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{14\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{8\,b^{2}(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{3\,a^{2}(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{4\,ab(\cos(dx+c)-1}{(\cos(dx+c)+1)}\right.}{\left(ab^{2}+b^{3}\right)\left(a-\frac{3\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{4\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{3\,a(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{4\,b(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{a(\cos(dx+c)-1)^{3}}{(\cos(dx+c)+1)^{3}}\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((3*a^2 + 4*a*b)*arctan((b*cos(d*x + c) + a + b)/(sqrt(-a*b - b^2)*cos
(d*x + c) + sqrt(-a*b - b^2)))/((a*b^2 + b^3)*sqrt(-a*b - b^2)) + 2*(3*a^2
```

```
+ 2*a*b - 6*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 14*a*b*(cos(d*x + c
) - 1)/(cos(d*x + c) + 1) - 8*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 3
*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 4*a*b*(cos(d*x + c) - 1)^2
/(cos(d*x + c) + 1)^2)/((a*b^2 + b^3)*(a - 3*a*(cos(d*x + c) - 1)/(cos(d*x
+ c) + 1) - 4*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 3*a*(cos(d*x + c) -
 1)^2/(cos(d*x + c) + 1)^2 + 4*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2
- a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)))/d
```

3.96 $\int \frac{\sin^3(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=83

$$\frac{a\cos(c+dx)}{2bd(a+b)\left(a-b\cos^2(c+dx)+b\right)}-\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2b^{3/2}d(a+b)^{3/2}}$$

```
[Out] -((a + 2*b)*ArcTanh[(Sqrt[b]*Cos[c + d*x])/Sqrt[a + b]])/(2*b^(3/2)*(a + b)
^(3/2)*d) + (a*Cos[c + d*x])/(2*b*(a + b)*d*(a + b - b*Cos[c + d*x]^2))
```

Rubi [A] time = 0.0885921, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3186, 385, 208}

$$\frac{a\cos(c+dx)}{2bd(a+b)\left(a-b\cos^2(c+dx)+b\right)}-\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2b^{3/2}d(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] -((a + 2*b)*ArcTanh[(Sqrt[b]*Cos[c + d*x])/Sqrt[a + b]])/(2*b^(3/2)*(a + b)
^(3/2)*d) + (a*Cos[c + d*x])/(2*b*(a + b)*d*(a + b - b*Cos[c + d*x]^2))
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{(a+b-bx^2)^2}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= \frac{a\cos(c+dx)}{2b(a+b)d\left(a+b-b\cos^2(c+dx)\right)} - \frac{(a+2b)\,\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \cos(c+dx)\right)}{2b(a+b)d}$$
$$= -\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2b^{3/2}(a+b)^{3/2}d} + \frac{a\cos(c+dx)}{2b(a+b)d\left(a+b-b\cos^2(c+dx)\right)}$$

Mathematica [C] time = 0.49552, size = 160, normalized size = 1.93

$$\frac{\frac{2a\sqrt{b}\cos(c+dx)}{2a-b\cos(2(c+dx))+b} + \frac{(a+2b)\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}} + \frac{(a+2b)\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}}}{2b^{3/2}d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] (((a + 2*b)*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]])/Sq
rt[-a - b] + ((a + 2*b)*ArcTan[(Sqrt[b] + I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[
-a - b]])/Sqrt[-a - b] + (2*a*Sqrt[b]*Cos[c + d*x])/(2*a + b - b*Cos[2*(c +
 d*x)]))/(2*b^(3/2)*(a + b)*d)
```

Maple [A] time = 0.081, size = 80, normalized size = 1.

$$\frac{1}{d}\left(-\frac{\cos(dx+c)\,a}{(2a+2b)\,b\left(b(\cos(dx+c))^2-a-b\right)} - \frac{a+2b}{(2a+2b)\,b}\text{Artanh}\left(b\cos(dx+c)\frac{1}{\sqrt{(a+b)\,b}}\right)\frac{1}{\sqrt{(a+b)\,b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a+sin(d*x+c)^2*b)^2,x)
```

```
[Out] 1/d*(-1/2*a/(a+b)/b*cos(d*x+c)/(b*cos(d*x+c)^2-a-b)-1/2*(a+2*b)/(a+b)/b/((a
+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/((a+b)*b)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.87887, size = 721, normalized size = 8.69

$$\left[\frac{\left(\left(ab+2\,b^2\right)\cos\left(dx+c\right)^2-a^2-3\,ab-2\,b^2\right)\sqrt{ab+b^2}\log\left(-\frac{b\cos(dx+c)^2-2\,\sqrt{ab+b^2}\cos(dx+c)+a+b}{b\cos(dx+c)^2-a-b}\right)-2\left(a^2b+ab^2\right)\cos\left(dx\right.}{4\left(\left(a^2b^3+2\,ab^4+b^5\right)d\cos\left(dx+c\right)^2-\left(a^3b^2+3\,a^2b^3+3\,ab^4+b^5\right)d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(((a*b + 2*b^2)*cos(d*x + c)^2 - a^2 - 3*a*b - 2*b^2)*sqrt(a*b + b^2)*
log(-(b*cos(d*x + c)^2 - 2*sqrt(a*b + b^2)*cos(d*x + c) + a + b)/(b*cos(d*x
 + c)^2 - a - b)) - 2*(a^2*b + a*b^2)*cos(d*x + c))/((a^2*b^3 + 2*a*b^4 + b
^5)*d*cos(d*x + c)^2 - (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*d), 1/2*(((a*b
 + 2*b^2)*cos(d*x + c)^2 - a^2 - 3*a*b - 2*b^2)*sqrt(-a*b - b^2)*arctan(sqr
t(-a*b - b^2)*cos(d*x + c)/(a + b)) - (a^2*b + a*b^2)*cos(d*x + c))/((a^2*b
^3 + 2*a*b^4 + b^5)*d*cos(d*x + c)^2 - (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5
)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+b*sin(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13805, size = 126, normalized size = 1.52

$$\frac{(a+2\,b)\arctan\left(\frac{b\cos(dx+c)}{\sqrt{-ab-b^2}}\right)}{2\left(ab+b^2\right)\sqrt{-ab-b^2}d}-\frac{a\cos\left(dx+c\right)}{2\left(b\cos\left(dx+c\right)^2-a-b\right)\left(ab+b^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(a + 2*b)*arctan(b*cos(d*x + c)/sqrt(-a*b - b^2))/((a*b + b^2)*sqrt(-a*
b - b^2)*d) - 1/2*a*cos(d*x + c)/((b*cos(d*x + c)^2 - a - b)*(a*b + b^2)*d)
```

3.97 $\int \frac{\sin(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=74

$$-\frac{\cos(c+dx)}{2d(a+b)\left(a-b\cos^2(c+dx)+b\right)}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2\sqrt{b}d(a+b)^{3/2}}$$

```
[Out] -ArcTanh[(Sqrt[b]*Cos[c + d*x])/Sqrt[a + b]]/(2*Sqrt[b]*(a + b)^(3/2)*d) -
Cos[c + d*x]/(2*(a + b)*d*(a + b - b*Cos[c + d*x]^2))
```

Rubi [A] time = 0.0539592, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3186, 199, 208}

$$-\frac{\cos(c+dx)}{2d(a+b)\left(a-b\cos^2(c+dx)+b\right)}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2\sqrt{b}d(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] -ArcTanh[(Sqrt[b]*Cos[c + d*x])/Sqrt[a + b]]/(2*Sqrt[b]*(a + b)^(3/2)*d) -
Cos[c + d*x]/(2*(a + b)*d*(a + b - b*Cos[c + d*x]^2))
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{\left(a+b-bx^2\right)^2}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{\cos(c+dx)}{2(a+b)d\left(a+b-b\cos^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \cos(c+dx)\right)}{2(a+b)d}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2\sqrt{b}(a+b)^{3/2}d} - \frac{\cos(c+dx)}{2(a+b)d\left(a+b-b\cos^2(c+dx)\right)}$$

Mathematica [C] time = 0.274156, size = 149, normalized size = 2.01

$$\frac{-\frac{2\cos(c+dx)}{2a-b\cos(2(c+dx))+b} + \frac{\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{\sqrt{b}\sqrt{-a-b}} + \frac{\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{\sqrt{b}\sqrt{-a-b}}}{2d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] (ArcTan[(Sqrt[b] - I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]]/(Sqrt[-a - b]*
Sqrt[b]) + ArcTan[(Sqrt[b] + I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]]/(Sqr
t[-a - b]*Sqrt[b]) - (2*Cos[c + d*x])/(2*a + b - b*Cos[2*(c + d*x)]))/(2*(a
 + b)*d)
```

Maple [A] time = 0.072, size = 68, normalized size = 0.9

$$\frac{1}{d}\left(\frac{\cos(dx+c)}{(2a+2b)\left(b(\cos(dx+c))^2-a-b\right)} - \frac{1}{2a+2b}\text{Artanh}\left(b\cos(dx+c)\frac{1}{\sqrt{(a+b)b}}\right)\frac{1}{\sqrt{(a+b)b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a+sin(d*x+c)^2*b)^2,x)
```

```
[Out] 1/d*(1/2*cos(d*x+c)/(a+b)/(b*cos(d*x+c)^2-a-b)-1/2/(a+b)/((a+b)*b)^(1/2)*ar
ctanh(cos(d*x+c)*b/((a+b)*b)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.8257, size = 635, normalized size = 8.58

$$\left[\frac{\left(b\cos\left(dx+c\right)^2-a-b\right)\sqrt{ab+b^2}\log\left(-\frac{b\cos(dx+c)^2-2\sqrt{ab+b^2}\cos(dx+c)+a+b}{b\cos(dx+c)^2-a-b}\right)+2\left(ab+b^2\right)\cos\left(dx+c\right)}{4\left(\left(a^2b^2+2ab^3+b^4\right)d\cos\left(dx+c\right)^2-\left(a^3b+3a^2b^2+3ab^3+b^4\right)d\right)}, \frac{\left(b\cos\left(dx+c\right)^2-a-\cdots\right.}{2\left(\left(a^2b^2+2\cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*((b*cos(d*x + c)^2 - a - b)*sqrt(a*b + b^2)*log(-(b*cos(d*x + c)^2 - 2
*sqrt(a*b + b^2)*cos(d*x + c) + a + b)/(b*cos(d*x + c)^2 - a - b)) + 2*(a*b
 + b^2)*cos(d*x + c))/((a^2*b^2 + 2*a*b^3 + b^4)*d*cos(d*x + c)^2 - (a^3*b
+ 3*a^2*b^2 + 3*a*b^3 + b^4)*d), 1/2*((b*cos(d*x + c)^2 - a - b)*sqrt(-a*b
- b^2)*arctan(sqrt(-a*b - b^2)*cos(d*x + c)/(a + b)) + (a*b + b^2)*cos(d*x
+ c))/((a^2*b^2 + 2*a*b^3 + b^4)*d*cos(d*x + c)^2 - (a^3*b + 3*a^2*b^2 + 3*
a*b^3 + b^4)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13186, size = 107, normalized size = 1.45

$$\frac{\arctan\left(\frac{b\cos(dx+c)}{\sqrt{-ab-b^2}}\right)}{2\sqrt{-ab-b^2}(a+b)d}+\frac{\cos\left(dx+c\right)}{2\left(b\cos\left(dx+c\right)^2-a-b\right)(a+b)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*arctan(b*cos(d*x + c)/sqrt(-a*b - b^2))/(sqrt(-a*b - b^2)*(a + b)*d) +
1/2*cos(d*x + c)/((b*cos(d*x + c)^2 - a - b)*(a + b)*d)
```

3.98 $\int \frac{\csc(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=103

$$\frac{\sqrt{b}(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2a^2d(a+b)^{3/2}}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{b\cos(c+dx)}{2ad(a+b)\left(a-b\cos^2(c+dx)+b\right)}$$

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a^2*d)) + (Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Co
s[c + d*x])/Sqrt[a + b]])/(2*a^2*(a + b)^(3/2)*d) + (b*Cos[c + d*x])/(2*a*(
a + b)*d*(a + b - b*Cos[c + d*x]^2))
```

Rubi [A] time = 0.129107, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3186, 414, 522, 206, 208}

$$\frac{\sqrt{b}(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2a^2d(a+b)^{3/2}}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{b\cos(c+dx)}{2ad(a+b)\left(a-b\cos^2(c+dx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a^2*d)) + (Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Co
s[c + d*x])/Sqrt[a + b]])/(2*a^2*(a + b)^(3/2)*d) + (b*Cos[c + d*x])/(2*a*(
a + b)*d*(a + b - b*Cos[c + d*x]^2))
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\csc(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\left(a+b-bx^2\right)^2}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= \frac{b\cos(c+dx)}{2a(a+b)d\left(a+b-b\cos^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{-2a-b-bx^2}{(1-x^2)\left(a+b-bx^2\right)}\,dx, x, \cos(c+dx)\right)}{2a(a+b)d}$$
$$= \frac{b\cos(c+dx)}{2a(a+b)d\left(a+b-b\cos^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(c+dx)\right)}{a^2d} + \frac{(b(3a+2b))\,\text{Sub}}{}$$
$$= -\frac{\tanh^{-1}(\cos(c+dx))}{a^2d} + \frac{\sqrt{b}(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{3/2}d} + \frac{b\cos(c+dx)}{2a(a+b)d\left(a+b-b\cos^2(c\right.}$$

Mathematica [C] time = 0.768188, size = 194, normalized size = 1.88

$$\frac{\frac{\sqrt{b}(3a+2b)\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{(-a-b)^{3/2}} + \frac{\sqrt{b}(3a+2b)\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{(-a-b)^{3/2}} + 2\left(\frac{ab\cos(c+dx)}{(a+b)(2a-b\cos(2(c+dx))+b)} + \log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) - \log\right.}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] ((Sqrt[b]*(3*a + 2*b)*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tan[(c + d*x)/2]])/Sqrt[-a
 - b]])/(-a - b)^(3/2) + (Sqrt[b]*(3*a + 2*b)*ArcTan[(Sqrt[b] + I*Sqrt[a]*T
an[(c + d*x)/2]])/Sqrt[-a - b]])/(-a - b)^(3/2) + 2*((a*b*Cos[c + d*x])/((a
+ b)*(2*a + b - b*Cos[2*(c + d*x)])) - Log[Cos[(c + d*x)/2]] + Log[Sin[(c +
 d*x)/2]]))/(2*a^2*d)
```

Maple [A] time = 0.116, size = 150, normalized size = 1.5

$$\frac{\ln(-1+\cos(dx+c))}{2a^2d} - \frac{\ln(1+\cos(dx+c))}{2a^2d} - \frac{b\cos(dx+c)}{2da(a+b)\left(b(\cos(dx+c))^2-a-b\right)} + \frac{3b}{2da(a+b)}\text{Artanh}\left(b\cos(dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a+sin(d*x+c)^2*b)^2,x)
```

```
[Out] 1/2/d/a^2*ln(-1+cos(d*x+c))-1/2/d/a^2*ln(1+cos(d*x+c))-1/2/d/a*b/(a+b)*cos(
d*x+c)/(b*cos(d*x+c)^2-a-b)+3/2/d/a*b/(a+b)/((a+b)*b)^(1/2)*arctanh(cos(d*x
+c)*b/((a+b)*b)^(1/2))+1/d/a^2*b^2/(a+b)/((a+b)*b)^(1/2)*arctanh(cos(d*x+c)
```

```
*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.31169, size = 1081, normalized size = 10.5

$$\left[-\frac{2\,ab\cos\left(dx+c\right)-\left(\left(3\,ab+2\,b^{2}\right)\cos\left(dx+c\right)^{2}-3\,a^{2}-5\,ab-2\,b^{2}\right)\sqrt{\frac{b}{a+b}}\log\left(\frac{b\cos(dx+c)^{2}+2\,(a+b)\sqrt{\frac{b}{a+b}}\cos(dx+c)+a+b}{b\cos(dx+c)^{2}-a-b}\right)+2}{4\,\left(\left(a^{3}b+a^{2}b^{2}\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*b*cos(d*x + c) - ((3*a*b + 2*b^2)*cos(d*x + c)^2 - 3*a^2 - 5*a*b
 - 2*b^2)*sqrt(b/(a + b))*log((b*cos(d*x + c)^2 + 2*(a + b)*sqrt(b/(a + b))
*cos(d*x + c) + a + b)/(b*cos(d*x + c)^2 - a - b)) + 2*((a*b + b^2)*cos(d*x
 + c)^2 - a^2 - 2*a*b - b^2)*log(1/2*cos(d*x + c) + 1/2) - 2*((a*b + b^2)*c
os(d*x + c)^2 - a^2 - 2*a*b - b^2)*log(-1/2*cos(d*x + c) + 1/2))/((a^3*b +
a^2*b^2)*d*cos(d*x + c)^2 - (a^4 + 2*a^3*b + a^2*b^2)*d), -1/2*(a*b*cos(d*x
 + c) + ((3*a*b + 2*b^2)*cos(d*x + c)^2 - 3*a^2 - 5*a*b - 2*b^2)*sqrt(-b/(a
 + b))*arctan(sqrt(-b/(a + b))*cos(d*x + c)) + ((a*b + b^2)*cos(d*x + c)^2
- a^2 - 2*a*b - b^2)*log(1/2*cos(d*x + c) + 1/2) - ((a*b + b^2)*cos(d*x + c
)^2 - a^2 - 2*a*b - b^2)*log(-1/2*cos(d*x + c) + 1/2))/((a^3*b + a^2*b^2)*d
*cos(d*x + c)^2 - (a^4 + 2*a^3*b + a^2*b^2)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.16908, size = 332, normalized size = 3.22

$$-\frac{\frac{\left(3\,ab+2\,b^{2}\right)\arctan\left(\frac{b\cos(dx+c)+a+b}{\sqrt{-ab-b^{2}}\cos(dx+c)+\sqrt{-ab-b^{2}}}\right)}{\left(a^{3}+a^{2}b\right)\sqrt{-ab-b^{2}}}-\frac{2\left(ab-\frac{ab(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{2\,b^{2}(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{\left(a^{3}+a^{2}b\right)\left(a-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{4\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}\right)}-\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((3*a*b + 2*b^2)*arctan((b*cos(d*x + c) + a + b)/(sqrt(-a*b - b^2)*cos
(d*x + c) + sqrt(-a*b - b^2)))/((a^3 + a^2*b)*sqrt(-a*b - b^2)) - 2*(a*b -
a*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*b^2*(cos(d*x + c) - 1)/(cos(d
*x + c) + 1))/((a^3 + a^2*b)*(a - 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
 - 4*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + a*(cos(d*x + c) - 1)^2/(cos(
d*x + c) + 1)^2)) - log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^2)/
d
```

3.99 $\int \frac{\csc^3(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=153

$$-\frac{b^{3/2}(5a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2a^3d(a+b)^{3/2}}-\frac{b(a+2b)\cos(c+dx)}{2a^2d(a+b)\left(a-b\cos^2(c+dx)+b\right)}-\frac{(a-4b)\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{\cot(c}{2ad\left(a-\right.}$$

```
[Out] -((a - 4*b)*ArcTanh[Cos[c + d*x]])/(2*a^3*d) - (b^(3/2)*(5*a + 4*b)*ArcTanh
[(Sqrt[b]*Cos[c + d*x])/Sqrt[a + b]])/(2*a^3*(a + b)^(3/2)*d) - (b*(a + 2*b
)*Cos[c + d*x])/(2*a^2*(a + b)*d*(a + b - b*Cos[c + d*x]^2)) - (Cot[c + d*x
]*Csc[c + d*x])/(2*a*d*(a + b - b*Cos[c + d*x]^2))
```

Rubi [A] time = 0.243339, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3186, 414, 527, 522, 206, 208}

$$-\frac{b^{3/2}(5a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2a^3d(a+b)^{3/2}}-\frac{b(a+2b)\cos(c+dx)}{2a^2d(a+b)\left(a-b\cos^2(c+dx)+b\right)}-\frac{(a-4b)\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{\cot(c}{2ad\left(a-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/(a + b*Sin[c + d*x]^2)^2,x]
```


```
[Out] -((a - 4*b)*ArcTanh[Cos[c + d*x]])/(2*a^3*d) - (b^(3/2)*(5*a + 4*b)*ArcTanh
[(Sqrt[b]*Cos[c + d*x])/Sqrt[a + b]])/(2*a^3*(a + b)^(3/2)*d) - (b*(a + 2*b
)*Cos[c + d*x])/(2*a^2*(a + b)*d*(a + b - b*Cos[c + d*x]^2)) - (Cot[c + d*x
]*Csc[c + d*x])/(2*a*d*(a + b - b*Cos[c + d*x]^2))
```


Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2\left(a+b-bx^2\right)^2}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)}{2ad\left(a+b-b\cos^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{a-b-3bx^2}{\left(1-x^2\right)\left(a+b-bx^2\right)^2}\,dx, x, \cos(c+dx)\right)}{2ad}$$

$$= -\frac{b(a+2b)\cos(c+dx)}{2a^2(a+b)d\left(a+b-b\cos^2(c+dx)\right)} - \frac{\cot(c+dx)\csc(c+dx)}{2ad\left(a+b-b\cos^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{-2\left(a^2-2ab-\right.}{\left(1-x^2\right.}\right.}{}$$

$$= -\frac{b(a+2b)\cos(c+dx)}{2a^2(a+b)d\left(a+b-b\cos^2(c+dx)\right)} - \frac{\cot(c+dx)\csc(c+dx)}{2ad\left(a+b-b\cos^2(c+dx)\right)} - \frac{(a-4b)\,\text{Subst}\left(\int \right.}{2}$$

$$= -\frac{(a-4b)\tanh^{-1}(\cos(c+dx))}{2a^3d} - \frac{b^{3/2}(5a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2a^3(a+b)^{3/2}d} - \frac{b(a+2b)\cos}{2a^2(a+b)d\left(a+b-\right.}$$

Mathematica [C] time = 1.63596, size = 390, normalized size = 2.55

$$\frac{\csc^3(c+dx)(-2a+b\cos(2(c+dx))-b)\left(\frac{8ab^2\cot(c+dx)}{a+b} + \frac{4b^{3/2}(5a+4b)\csc(c+dx)(2a-b\cos(2(c+dx))+b)\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{(-a-b)^{3/2}} + \frac{4b^{3/2}(}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] ((-2*a - b + b*Cos[2*(c + d*x)])*Csc[c + d*x]^3*((8*a*b^2*Cot[c + d*x])/(a
+ b) + (4*b^(3/2)*(5*a + 4*b)*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tan[(c + d*x)/2])
/Sqrt[-a - b]]*(2*a + b - b*Cos[2*(c + d*x)])*Csc[c + d*x])/(-a - b)^(3/2)
+ (4*b^(3/2)*(5*a + 4*b)*ArcTan[(Sqrt[b] + I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt
[-a - b]]*(2*a + b - b*Cos[2*(c + d*x)])*Csc[c + d*x])/(-a - b)^(3/2) + a*(
```

```
2*a + b - b*Cos[2*(c + d*x)])*Csc[(c + d*x)/2]^2*Csc[c + d*x] + 4*(a - 4*b)
*(2*a + b - b*Cos[2*(c + d*x)])*Csc[c + d*x]*Log[Cos[(c + d*x)/2]] - 4*(a -
 4*b)*(2*a + b - b*Cos[2*(c + d*x)])*Csc[c + d*x]*Log[Sin[(c + d*x)/2]] - a
*(2*a + b - b*Cos[2*(c + d*x)])*Csc[c + d*x]*Sec[(c + d*x)/2]^2))/(32*a^3*d
*(b + a*Csc[c + d*x]^2)^2)
```

Maple [A] time = 0.14, size = 226, normalized size = 1.5

$$\frac{1}{4\,a^2d\,(-1+\cos(dx+c))}-\frac{\ln(-1+\cos(dx+c))\,b}{da^3}+\frac{\ln(-1+\cos(dx+c))}{4\,a^2d}+\frac{1}{4\,a^2d\,(1+\cos(dx+c))}+\frac{\ln(1+\cos(\ldots}{da^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(a+sin(d*x+c)^2*b)^2,x)
```

```
[Out] 1/4/d/a^2/(-1+cos(d*x+c))-1/d/a^3*ln(-1+cos(d*x+c))*b+1/4/d/a^2*ln(-1+cos(d
*x+c))+1/4/d/a^2/(1+cos(d*x+c))+1/d/a^3*ln(1+cos(d*x+c))*b-1/4/d/a^2*ln(1+c
os(d*x+c))+1/2/d*b^2/a^2/(a+b)*cos(d*x+c)/(b*cos(d*x+c)^2-a-b)-5/2/d/a^2*b^
2/(a+b)/((a+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/((a+b)*b)^(1/2))-2/d*b^3/a^3/(
a+b)/((a+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/((a+b)*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.56541, size = 1928, normalized size = 12.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a^2*b + 2*a*b^2)*cos(d*x + c)^3 + ((5*a*b^2 + 4*b^3)*cos(d*x + c)^
4 + 5*a^2*b + 9*a*b^2 + 4*b^3 - (5*a^2*b + 14*a*b^2 + 8*b^3)*cos(d*x + c)^2
)*sqrt(b/(a + b))*log(-(b*cos(d*x + c)^2 - 2*(a + b)*sqrt(b/(a + b))*cos(d*
x + c) + a + b)/(b*cos(d*x + c)^2 - a - b)) - 2*(a^3 + 2*a^2*b + 2*a*b^2)*c
os(d*x + c) - ((a^2*b - 3*a*b^2 - 4*b^3)*cos(d*x + c)^4 + a^3 - 2*a^2*b - 7
*a*b^2 - 4*b^3 - (a^3 - a^2*b - 10*a*b^2 - 8*b^3)*cos(d*x + c)^2)*log(1/2*c
os(d*x + c) + 1/2) + ((a^2*b - 3*a*b^2 - 4*b^3)*cos(d*x + c)^4 + a^3 - 2*a^
2*b - 7*a*b^2 - 4*b^3 - (a^3 - a^2*b - 10*a*b^2 - 8*b^3)*cos(d*x + c)^2)*lo
g(-1/2*cos(d*x + c) + 1/2))/((a^4*b + a^3*b^2)*d*cos(d*x + c)^4 - (a^5 + 3*
a^4*b + 2*a^3*b^2)*d*cos(d*x + c)^2 + (a^5 + 2*a^4*b + a^3*b^2)*d), 1/4*(2*
(a^2*b + 2*a*b^2)*cos(d*x + c)^3 + 2*((5*a*b^2 + 4*b^3)*cos(d*x + c)^4 + 5*
a^2*b + 9*a*b^2 + 4*b^3 - (5*a^2*b + 14*a*b^2 + 8*b^3)*cos(d*x + c)^2)*sqrt
(-b/(a + b))*arctan(sqrt(-b/(a + b))*cos(d*x + c)) - 2*(a^3 + 2*a^2*b + 2*a
```

```
*b^2)*cos(d*x + c) - ((a^2*b - 3*a*b^2 - 4*b^3)*cos(d*x + c)^4 + a^3 - 2*a^
2*b - 7*a*b^2 - 4*b^3 - (a^3 - a^2*b - 10*a*b^2 - 8*b^3)*cos(d*x + c)^2)*lo
g(1/2*cos(d*x + c) + 1/2) + ((a^2*b - 3*a*b^2 - 4*b^3)*cos(d*x + c)^4 + a^3
 - 2*a^2*b - 7*a*b^2 - 4*b^3 - (a^3 - a^2*b - 10*a*b^2 - 8*b^3)*cos(d*x + c
)^2)*log(-1/2*cos(d*x + c) + 1/2))/((a^4*b + a^3*b^2)*d*cos(d*x + c)^4 - (a
^5 + 3*a^4*b + 2*a^3*b^2)*d*cos(d*x + c)^2 + (a^5 + 2*a^4*b + a^3*b^2)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a+b*sin(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.23108, size = 691, normalized size = 4.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/24*(12*(5*a*b^2 + 4*b^3)*arctan((b*cos(d*x + c) + a + b)/(sqrt(-a*b - b^2
)*cos(d*x + c) + sqrt(-a*b - b^2)))/((a^4 + a^3*b)*sqrt(-a*b - b^2)) + (3*a
^3 + 3*a^2*b - 8*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 12*a^2*b*(cos(
d*x + c) - 1)/(cos(d*x + c) + 1) - 28*a*b^2*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1) + 7*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - a^2*b*(cos(d*x +
 c) - 1)^2/(cos(d*x + c) + 1)^2 - 16*a*b^2*(cos(d*x + c) - 1)^2/(cos(d*x +
c) + 1)^2 + 16*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 2*a^3*(cos(d
*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 6*a^2*b*(cos(d*x + c) - 1)^3/(cos(d*x
 + c) + 1)^3 + 8*a*b^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)/((a^4 + a
^3*b)*(a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*a*(cos(d*x + c) - 1)^2/(
cos(d*x + c) + 1)^2 - 4*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + a*(co
s(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)) + 6*(a - 4*b)*log(abs(-cos(d*x + c
) + 1)/abs(cos(d*x + c) + 1))/a^3 - 3*(cos(d*x + c) - 1)/(a^2*(cos(d*x + c)
 + 1)))/d
```

3.100 $\int \frac{\sin^6(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=148

$$\frac{a^{3/2}(4a+5b)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{2b^3d(a+b)^{3/2}}-\frac{a(2a+b)\tan(c+dx)}{2b^2d(a+b)\left((a+b)\tan^2(c+dx)+a\right)}-\frac{x(4a-b)}{2b^3}-\frac{\sin^2(c+dx)\tan(c+dx)}{2bd\left((a+b)\tan^2(c+dx)+a\right)}$$

```
[Out] -((4*a - b)*x)/(2*b^3) + (a^(3/2)*(4*a + 5*b)*ArcTan[(Sqrt[a + b]*Tan[c + d
*x])/Sqrt[a]])/(2*b^3*(a + b)^(3/2)*d) - (a*(2*a + b)*Tan[c + d*x])/(2*b^2*
(a + b)*d*(a + (a + b)*Tan[c + d*x]^2)) - (Sin[c + d*x]^2*Tan[c + d*x])/(2*
b*d*(a + (a + b)*Tan[c + d*x]^2))
```

Rubi [A] time = 0.291697, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3187, 470, 578, 522, 203, 205}

$$\frac{a^{3/2}(4a+5b)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{2b^3d(a+b)^{3/2}}-\frac{a(2a+b)\tan(c+dx)}{2b^2d(a+b)\left((a+b)\tan^2(c+dx)+a\right)}-\frac{x(4a-b)}{2b^3}-\frac{\sin^2(c+dx)\tan(c+dx)}{2bd\left((a+b)\tan^2(c+dx)+a\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^6/(a + b*Sin[c + d*x]^2)^2,x]
```


```
[Out] -((4*a - b)*x)/(2*b^3) + (a^(3/2)*(4*a + 5*b)*ArcTan[(Sqrt[a + b]*Tan[c + d
*x])/Sqrt[a]])/(2*b^3*(a + b)^(3/2)*d) - (a*(2*a + b)*Tan[c + d*x])/(2*b^2*
(a + b)*d*(a + (a + b)*Tan[c + d*x]^2)) - (Sin[c + d*x]^2*Tan[c + d*x])/(2*
b*d*(a + (a + b)*Tan[c + d*x]^2))
```


Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
```

```
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^6(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)^2\left(a+(a+b)x^2\right)^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{\sin^2(c+dx)\tan(c+dx)}{2bd\left(a+(a+b)\tan^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{x^2\left(3a+(-a+b)x^2\right)}{(1+x^2)\left(a+(a+b)x^2\right)^2}\,dx, x, \tan(c+dx)\right)}{2bd}$$

$$= -\frac{a(2a+b)\tan(c+dx)}{2b^2(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)} - \frac{\sin^2(c+dx)\tan(c+dx)}{2bd\left(a+(a+b)\tan^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{2a(2a\cdots}{(1\cdots}\right.}{\cdots}$$

$$= -\frac{a(2a+b)\tan(c+dx)}{2b^2(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)} - \frac{\sin^2(c+dx)\tan(c+dx)}{2bd\left(a+(a+b)\tan^2(c+dx)\right)} - \frac{(4a-b)\,\text{Subst}\cdots}{\cdots}$$

$$= -\frac{(4a-b)x}{2b^3} + \frac{a^{3/2}(4a+5b)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{2b^3(a+b)^{3/2}d} - \frac{a(2a+b)\tan(c+dx)}{2b^2(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)}$$

Mathematica [A] time = 1.47569, size = 106, normalized size = 0.72

$$\frac{\frac{2a^{3/2}(4a+5b)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{3/2}} + b\sin(2(c+dx))\left(-\frac{2a^2}{(a+b)(2a-b\cos(2(c+dx))+b)}-1\right) - 2(4a-b)(c+dx)}{4b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^6/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] (-2*(4*a - b)*(c + d*x) + (2*a^(3/2)*(4*a + 5*b)*ArcTan[(Sqrt[a + b]*Tan[c
+ d*x])/Sqrt[a]])/(a + b)^(3/2) + b*(-1 - (2*a^2)/((a + b)*(2*a + b - b*Cos
[2*(c + d*x)])))*Sin[2*(c + d*x)])/(4*b^3*d)
```

Maple [A] time = 0.099, size = 187, normalized size = 1.3

$$-\frac{\tan(dx+c)}{2\,b^2d\left((\tan(dx+c))^2+1\right)}+\frac{\arctan(\tan(dx+c))}{2\,b^2d}-2\,\frac{\arctan(\tan(dx+c))\,a}{db^3}-\frac{a^2\tan(dx+c}{2\,b^2d\,(a+b)\left(a\,(\tan(dx+c))^2+(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^6/(a+sin(d*x+c)^2*b)^2,x)
```

```
[Out] -1/2/d/b^2*tan(d*x+c)/(tan(d*x+c)^2+1)+1/2/d/b^2*arctan(tan(d*x+c))-2/d/b^3
*arctan(tan(d*x+c))*a-1/2/d*a^2/b^2/(a+b)*tan(d*x+c)/(a*tan(d*x+c)^2+tan(d*
x+c)^2*b+a)+2/d*a^3/b^3/(a+b)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a
+b))^(1/2))+5/2/d*a^2/b^2/(a+b)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*
(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.03464, size = 1413, normalized size = 9.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(4*a^2*b + 3*a*b^2 - b^3)*d*x*cos(d*x + c)^2 - 4*(4*a^3 + 7*a^2*b
+ 2*a*b^2 - b^3)*d*x + (4*a^3 + 9*a^2*b + 5*a*b^2 - (4*a^2*b + 5*a*b^2)*cos
(d*x + c)^2)*sqrt(-a/(a + b))*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2
*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 - 4*((2*a^2 + 3*a*b + b^2)*cos(d*x +
c)^3 - (a^2 + 2*a*b + b^2)*cos(d*x + c))*sqrt(-a/(a + b))*sin(d*x + c) + a^
2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 +
 2*a*b + b^2)) + 4*((a*b^2 + b^3)*cos(d*x + c)^3 - (2*a^2*b + 2*a*b^2 + b^3
)*cos(d*x + c))*sin(d*x + c))/((a*b^4 + b^5)*d*cos(d*x + c)^2 - (a^2*b^3 +
2*a*b^4 + b^5)*d), -1/4*(2*(4*a^2*b + 3*a*b^2 - b^3)*d*x*cos(d*x + c)^2 - 2
*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*d*x - (4*a^3 + 9*a^2*b + 5*a*b^2 - (4*a^
2*b + 5*a*b^2)*cos(d*x + c)^2)*sqrt(a/(a + b))*arctan(1/2*((2*a + b)*cos(d*
x + c)^2 - a - b)*sqrt(a/(a + b))/(a*cos(d*x + c)*sin(d*x + c))) + 2*((a*b^
2 + b^3)*cos(d*x + c)^3 - (2*a^2*b + 2*a*b^2 + b^3)*cos(d*x + c))*sin(d*x +
 c))/((a*b^4 + b^5)*d*cos(d*x + c)^2 - (a^2*b^3 + 2*a*b^4 + b^5)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**6/(a+b*sin(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16718, size = 301, normalized size = 2.03

$$\frac{\frac{\left(4\,a^3+5\,a^2b\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)}{\left(ab^3+b^4\right)\sqrt{a^2+ab}}-\frac{2\,a^2\tan(dx+c)^3+2\,ab\tan(dx+c)^3+b^2\tan(dx+c)^3+2\,a^2\tan(dx+c)+ab\tan(dx+c)}{\left(a\tan(dx+c)^4+b\tan(dx+c)^4+2\,a\tan(dx+c)^2+b\tan(dx+c)^2+a\right)\left(ab^2+b^3\right)}-}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((4*a^3 + 5*a^2*b)*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arcta
n((a*tan(d*x + c) + b*tan(d*x + c))/sqrt(a^2 + a*b)))/((a*b^3 + b^4)*sqrt(a
^2 + a*b)) - (2*a^2*tan(d*x + c)^3 + 2*a*b*tan(d*x + c)^3 + b^2*tan(d*x + c
)^3 + 2*a^2*tan(d*x + c) + a*b*tan(d*x + c))/((a*tan(d*x + c)^4 + b*tan(d*x
 + c)^4 + 2*a*tan(d*x + c)^2 + b*tan(d*x + c)^2 + a)*(a*b^2 + b^3)) - (d*x
+ c)*(4*a - b)/b^3)/d
```

3.101 $\int \frac{\sin^4(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2} dx$

Optimal. Leaf size=93

$$-\frac{\sqrt{a}(2a+3b)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{2b^2d(a+b)^{3/2}}+\frac{a\tan(c+dx)}{2bd(a+b)\left((a+b)\tan^2(c+dx)+a\right)}+\frac{x}{b^2}$$

```
[Out] x/b^2 - (Sqrt[a]*(2*a + 3*b)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(2
*b^2*(a + b)^(3/2)*d) + (a*Tan[c + d*x])/(2*b*(a + b)*d*(a + (a + b)*Tan[c
+ d*x]^2))
```

Rubi [A] time = 0.137644, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3187, 470, 522, 203, 205}

$$-\frac{\sqrt{a}(2a+3b)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{2b^2d(a+b)^{3/2}}+\frac{a\tan(c+dx)}{2bd(a+b)\left((a+b)\tan^2(c+dx)+a\right)}+\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] x/b^2 - (Sqrt[a]*(2*a + 3*b)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(2
*b^2*(a + b)^(3/2)*d) + (a*Tan[c + d*x])/(2*b*(a + b)*d*(a + (a + b)*Tan[c
+ d*x]^2))
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^4(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)\left(a+(a+b)x^2\right)^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{a\tan(c+dx)}{2b(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{a+(-a-2b)x^2}{(1+x^2)\left(a+(a+b)x^2\right)}\,dx, x, \tan(c+dx)\right)}{2b(a+b)d}$$

$$= \frac{a\tan(c+dx)}{2b(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(c+dx)\right)}{b^2d} - \frac{(a(2a+3b))\,\text{S}}{}$$

$$= \frac{x}{b^2} - \frac{\sqrt{a}(2a+3b)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{2b^2(a+b)^{3/2}d} + \frac{a\tan(c+dx)}{2b(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)}$$

Mathematica [A] time = 0.869192, size = 93, normalized size = 1.

$$\frac{-\frac{\sqrt{a}(2a+3b)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{3/2}} + \frac{ab\sin(2(c+dx))}{(a+b)(2a-b\cos(2(c+dx))+b)} + 2(c+dx)}{2b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] (2*(c + d*x) - (Sqrt[a]*(2*a + 3*b)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[
a]])/(a + b)^(3/2) + (a*b*Sin[2*(c + d*x)])/((a + b)*(2*a + b - b*Cos[2*(c
+ d*x)])))/(2*b^2*d)
```

Maple [A] time = 0.086, size = 140, normalized size = 1.5

$$\frac{\arctan(\tan(dx+c))}{b^2d} + \frac{a\tan(dx+c)}{2\,bd\,(a+b)\left(a\,(\tan(dx+c))^2+(\tan(dx+c))^2\,b+a\right)} - \frac{a^2}{b^2d\,(a+b)}\arctan\left((a+b)\tan(dx+c)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a+sin(d*x+c)^2*b)^2,x)
```

```
[Out] 1/d/b^2*arctan(tan(d*x+c))+1/2/d*a/b/(a+b)*tan(d*x+c)/(a*tan(d*x+c)^2+tan(d
*x+c)^2*b+a)-1/d*a^2/b^2/(a+b)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(
a+b))^(1/2))-3/2/d*a/b/(a+b)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+
```

```
b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.95181, size = 1165, normalized size = 12.53

$$\left[\frac{8\left(ab+b^2\right)dx\cos\left(dx+c\right)^2-4\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)-8\left(a^2+2\,ab+b^2\right)dx+\left(\left(2\,ab+3\,b^2\right)\cos\left(dx+c\right)^2-2\,a^2\right.}{8\left(\left(ab^3+b^4\right)d\,c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(8*(a*b + b^2)*d*x*cos(d*x + c)^2 - 4*a*b*cos(d*x + c)*sin(d*x + c) -
8*(a^2 + 2*a*b + b^2)*d*x + ((2*a*b + 3*b^2)*cos(d*x + c)^2 - 2*a^2 - 5*a*b
 - 3*b^2)*sqrt(-a/(a + b))*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4
*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a^2 + 3*a*b + b^2)*cos(d*x + c)^
3 - (a^2 + 2*a*b + b^2)*cos(d*x + c))*sqrt(-a/(a + b))*sin(d*x + c) + a^2 +
 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*
a*b + b^2)))/((a*b^3 + b^4)*d*cos(d*x + c)^2 - (a^2*b^2 + 2*a*b^3 + b^4)*d)
, 1/4*(4*(a*b + b^2)*d*x*cos(d*x + c)^2 - 2*a*b*cos(d*x + c)*sin(d*x + c) -
 4*(a^2 + 2*a*b + b^2)*d*x + ((2*a*b + 3*b^2)*cos(d*x + c)^2 - 2*a^2 - 5*a*
b - 3*b^2)*sqrt(a/(a + b))*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)*sq
rt(a/(a + b))/(a*cos(d*x + c)*sin(d*x + c))))/((a*b^3 + b^4)*d*cos(d*x + c)
^2 - (a^2*b^2 + 2*a*b^3 + b^4)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a+b*sin(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16786, size = 189, normalized size = 2.03

$$-\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)\left(2\,a^2+3\,ab\right)}{\left(ab^2+b^3\right)\sqrt{a^2+ab}}-\frac{a\tan(dx+c)}{\left(a\tan(dx+c)^2+b\tan(dx+c)^2+a\right)\left(ab+b^2\right)}-\frac{2\,(dx+c)}{b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c)
 + b*tan(d*x + c))/sqrt(a^2 + a*b)))*(2*a^2 + 3*a*b)/((a*b^2 + b^3)*sqrt(a^
2 + a*b)) - a*tan(d*x + c)/((a*tan(d*x + c)^2 + b*tan(d*x + c)^2 + a)*(a*b
+ b^2)) - 2*(d*x + c)/b^2)/d
```

3.102 $\int \frac{\sin^2(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=78

$$\frac{\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}d(a+b)^{3/2}}-\frac{\sin(c+dx)\cos(c+dx)}{2d(a+b)\left(a+b\sin^2(c+dx)\right)}$$

```
[Out] ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]]/(2*Sqrt[a]*(a + b)^(3/2)*d) - (C
os[c + d*x]*Sin[c + d*x])/(2*(a + b)*d*(a + b*Sin[c + d*x]^2))
```

Rubi [A] time = 0.0883624, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3173, 12, 3181, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}d(a+b)^{3/2}}-\frac{\sin(c+dx)\cos(c+dx)}{2d(a+b)\left(a+b\sin^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]]/(2*Sqrt[a]*(a + b)^(3/2)*d) - (C
os[c + d*x]*Sin[c + d*x])/(2*(a + b)*d*(a + b*Sin[c + d*x]^2))
```

Rule 3173

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b - a*B)*Cos[e + f*x]*Sin[e + f*x]
*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(a + b)*(p + 1)), x] - Dist[1/(2*a*
(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p +
 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = -\frac{\cos(c+dx)\sin(c+dx)}{2(a+b)d\left(a+b\sin^2(c+dx)\right)} + \frac{\int \frac{a}{a+b\sin^2(c+dx)}\,dx}{2a(a+b)}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{2(a+b)d\left(a+b\sin^2(c+dx)\right)} + \frac{\int \frac{1}{a+b\sin^2(c+dx)}\,dx}{2(a+b)}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{2(a+b)d\left(a+b\sin^2(c+dx)\right)} + \frac{\mathrm{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \tan(c+dx)\right)}{2(a+b)d}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}(a+b)^{3/2}d} - \frac{\cos(c+dx)\sin(c+dx)}{2(a+b)d\left(a+b\sin^2(c+dx)\right)}$$

Mathematica [A] time = 0.517016, size = 74, normalized size = 0.95

$$\frac{\frac{\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{3/2}} - \frac{\sin(2(c+dx))}{(a+b)(2a-b\cos(2(c+dx))+b)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] (ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]]/(Sqrt[a]*(a + b)^(3/2)) - Sin[2
*(c + d*x)]/((a + b)*(2*a + b - b*Cos[2*(c + d*x)])))/(2*d)
```

Maple [A] time = 0.088, size = 77, normalized size = 1.

$$-\frac{\tan(dx+c)}{2d(a+b)\left(a(\tan(dx+c))^2+(\tan(dx+c))^2b+a\right)} + \frac{1}{2d(a+b)}\arctan\left((a+b)\tan(dx+c)\frac{1}{\sqrt{a(a+b)}}\right)\frac{1}{\sqrt{a(a+b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+sin(d*x+c)^2*b)^2,x)
```

```
[Out] -1/2/d/(a+b)*tan(d*x+c)/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)+1/2/d/(a+b)/(a*(a
+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.8503, size = 965, normalized size = 12.37

$$\left[\frac{4\left(a^2+ab\right)\cos\left(dx+c\right)\sin\left(dx+c\right)-\left(b\cos\left(dx+c\right)^2-a-b\right)\sqrt{-a^2-ab}\log\left(\frac{\left(8a^2+8ab+b^2\right)\cos(dx+c)^4-2\left(4a^2+5ab+b^2\right)\cos(d}{b^2\cos(dx+}\right.}{8\left(\left(a^3b+2a^2b^2+ab^3\right)d\cos\left(dx+c\right)^2-\left(a^4+3a^3b+3a^2b^2+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a^2 + a*b)*cos(d*x + c)*sin(d*x + c) - (b*cos(d*x + c)^2 - a - b)*
sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4*a^2 + 5*a
*b + b^2)*cos(d*x + c)^2 + 4*((2*a + b)*cos(d*x + c)^3 - (a + b)*cos(d*x +
c))*sqrt(-a^2 - a*b)*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4
- 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2)))/((a^3*b + 2*a^2*b^2 +
 a*b^3)*d*cos(d*x + c)^2 - (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d), 1/4*(2*(
a^2 + a*b)*cos(d*x + c)*sin(d*x + c) - (b*cos(d*x + c)^2 - a - b)*sqrt(a^2
+ a*b)*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)/(sqrt(a^2 + a*b)*cos(d
*x + c)*sin(d*x + c))))/((a^3*b + 2*a^2*b^2 + a*b^3)*d*cos(d*x + c)^2 - (a^
4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+b*sin(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12825, size = 147, normalized size = 1.88

$$\frac{\frac{\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)}{\sqrt{a^2+ab}(a+b)}-\frac{\tan(dx+c)}{\left(a\tan(dx+c)^2+b\tan(dx+c)^2+a\right)(a+b)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c)
+ b*tan(d*x + c))/sqrt(a^2 + a*b)))/(sqrt(a^2 + a*b)*(a + b)) - tan(d*x + c
)/((a*tan(d*x + c)^2 + b*tan(d*x + c)^2 + a)*(a + b)))/d
```

3.103 $\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=87

$$\frac{(2a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a+b)^{3/2}}+\frac{b\sin(c+dx)\cos(c+dx)}{2ad(a+b)\left(a+b\sin^2(c+dx)\right)}$$

```
[Out] ((2*a + b)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a + b)^(
3/2)*d) + (b*Cos[c + d*x]*Sin[c + d*x])/(2*a*(a + b)*d*(a + b*Sin[c + d*x]^
2))
```

Rubi [A] time = 0.0625083, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3184, 12, 3181, 205}

$$\frac{(2a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a+b)^{3/2}}+\frac{b\sin(c+dx)\cos(c+dx)}{2ad(a+b)\left(a+b\sin^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x]^2)^(-2),x]
```

```
[Out] ((2*a + b)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a + b)^(
3/2)*d) + (b*Cos[c + d*x]*Sin[c + d*x])/(2*a*(a + b)*d*(a + b*Sin[c + d*x]^
2))
```

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a +
 b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)
*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = \frac{b\cos(c+dx)\sin(c+dx)}{2a(a+b)d\left(a+b\sin^2(c+dx)\right)} - \frac{\int \frac{-2a-b}{a+b\sin^2(c+dx)}\,dx}{2a(a+b)}$$

$$= \frac{b\cos(c+dx)\sin(c+dx)}{2a(a+b)d\left(a+b\sin^2(c+dx)\right)} + \frac{(2a+b)\int \frac{1}{a+b\sin^2(c+dx)}\,dx}{2a(a+b)}$$

$$= \frac{b\cos(c+dx)\sin(c+dx)}{2a(a+b)d\left(a+b\sin^2(c+dx)\right)} + \frac{(2a+b)\,\mathrm{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \tan(c+dx)\right)}{2a(a+b)d}$$

$$= \frac{(2a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{3/2}d} + \frac{b\cos(c+dx)\sin(c+dx)}{2a(a+b)d\left(a+b\sin^2(c+dx)\right)}$$

Mathematica [A] time = 0.41682, size = 84, normalized size = 0.97

$$\frac{\frac{(2a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{3/2}} + \frac{\sqrt{a}b\sin(2(c+dx))}{(a+b)(2a-b\cos(2(c+dx))+b)}}{2a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x]^2)^(-2),x]
```

```
[Out] (((2*a + b)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a + b)^(3/2) + (Sq
rt[a]*b*Sin[2*(c + d*x)])/((a + b)*(2*a + b - b*Cos[2*(c + d*x)])))/(2*a^(3
/2)*d)
```

Maple [A] time = 0.089, size = 119, normalized size = 1.4

$$\frac{b\tan(dx+c)}{2da(a+b)\left(a(\tan(dx+c))^2+(\tan(dx+c))^2b+a\right)} + \frac{1}{d(a+b)}\arctan\left((a+b)\tan(dx+c)\frac{1}{\sqrt{a(a+b)}}\right)\frac{1}{\sqrt{a(a+b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+sin(d*x+c)^2*b)^2,x)
```

```
[Out] 1/2/d*b/a/(a+b)*tan(d*x+c)/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)+1/d/(a+b)/(a*(
a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))+1/2/d/a/(a+b)/(a*(a+b)
)^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.88266, size = 1060, normalized size = 12.18

$$\left[-\frac{4\left(a^{2}b+ab^{2}\right)\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(\left(2\,ab+b^{2}\right)\cos\left(dx+c\right)^{2}-2\,a^{2}-3\,ab-b^{2}\right)\sqrt{-a^{2}-ab}\log\left(\frac{\left(8\,a^{2}+8\,ab+b^{2}\right)\cos(dx+}{}\right.}{8\left(\left(a^{4}b+2\,a^{3}b^{2}+a^{2}b^{3}\right)d\cos\left(dx+c\right)^{2}-\left(a^{5}+3\,a^{4}b+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(a^2*b + a*b^2)*cos(d*x + c)*sin(d*x + c) + ((2*a*b + b^2)*cos(d*x
 + c)^2 - 2*a^2 - 3*a*b - b^2)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*
cos(d*x + c)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a + b)*cos(
d*x + c)^3 - (a + b)*cos(d*x + c))*sqrt(-a^2 - a*b)*sin(d*x + c) + a^2 + 2*
a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b
 + b^2)))/((a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cos(d*x + c)^2 - (a^5 + 3*a^4*b
+ 3*a^3*b^2 + a^2*b^3)*d), -1/4*(2*(a^2*b + a*b^2)*cos(d*x + c)*sin(d*x + c
) + ((2*a*b + b^2)*cos(d*x + c)^2 - 2*a^2 - 3*a*b - b^2)*sqrt(a^2 + a*b)*ar
ctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)/(sqrt(a^2 + a*b)*cos(d*x + c)*s
in(d*x + c))))/((a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cos(d*x + c)^2 - (a^5 + 3*a
^4*b + 3*a^3*b^2 + a^2*b^3)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10842, size = 153, normalized size = 1.76

$$\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^{2}+ab}}\right)\right)(2\,a+b)}{\left(a^{2}+ab\right)^{\frac{3}{2}}}+\frac{b\tan(dx+c)}{\left(a\tan(dx+c)^{2}+b\tan(dx+c)^{2}+a\right)\left(a^{2}+ab\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c)
+ b*tan(d*x + c))/sqrt(a^2 + a*b)))*(2*a + b)/(a^2 + a*b)^(3/2) + b*tan(d*x
 + c)/((a*tan(d*x + c)^2 + b*tan(d*x + c)^2 + a)*(a^2 + a*b)))/d
```

3.104 $\int \frac{\csc^2(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=127

$$-\frac{(2ab+3b^2)\sin(c+dx)\cos(c+dx)}{2a^2d(a+b)\left(a+b\sin^2(c+dx)\right)}-\frac{b(4a+3b)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{5/2}d(a+b)^{3/2}}-\frac{\cot(c+dx)}{ad\left(a+b\sin^2(c+dx)\right)}$$

```
[Out] -(b*(4*a + 3*b)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(2*a^(5/2)*(a +
 b)^(3/2)*d) - Cot[c + d*x]/(a*d*(a + b*Sin[c + d*x]^2)) - ((2*a*b + 3*b^2)
*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*(a + b)*d*(a + b*Sin[c + d*x]^2))
```

Rubi [A] time = 0.146421, antiderivative size = 130, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3187, 462, 385, 205}

$$-\frac{(2a^2+4ab+3b^2)\tan(c+dx)}{2a^2d(a+b)\left((a+b)\tan^2(c+dx)+a\right)}-\frac{b(4a+3b)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{5/2}d(a+b)^{3/2}}-\frac{\cot(c+dx)}{ad\left((a+b)\tan^2(c+dx)+a\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] -(b*(4*a + 3*b)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(2*a^(5/2)*(a +
 b)^(3/2)*d) - Cot[c + d*x]/(a*d*(a + (a + b)*Tan[c + d*x]^2)) - ((2*a^2 +
4*a*b + 3*b^2)*Tan[c + d*x])/(2*a^2*(a + b)*d*(a + (a + b)*Tan[c + d*x]^2))
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^2\left(a+(a+b)x^2\right)^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{\cot(c+dx)}{ad\left(a+(a+b)\tan^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{-a-3b+ax^2}{\left(a+(a+b)x^2\right)^2}\,dx, x, \tan(c+dx)\right)}{ad}$$

$$= -\frac{\cot(c+dx)}{ad\left(a+(a+b)\tan^2(c+dx)\right)} - \frac{\left(2a^2+4ab+3b^2\right)\tan(c+dx)}{2a^2(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)} - \frac{(b(4a+3b))\,\text{Sub}}{}$$

$$= -\frac{b(4a+3b)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{5/2}(a+b)^{3/2}d} - \frac{\cot(c+dx)}{ad\left(a+(a+b)\tan^2(c+dx)\right)} - \frac{\left(2a^2+4ab+3b^2\right)}{2a^2(a+b)d\left(a+(a+\right.}$$

Mathematica [A] time = 1.15663, size = 155, normalized size = 1.22

$$-\frac{\csc^4(c+dx)(2a-b\cos(2(c+dx))+b)\left(\sqrt{a}\sqrt{a+b}\cot(c+dx)\left(4a^2-b(2a+3b)\cos(2(c+dx))+6ab+3b^2\right)+b(4a+3\right.}{8a^{5/2}d(a+b)^{3/2}\left(a\csc^2(c+dx)+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] -((2*a + b - b*Cos[2*(c + d*x)])*(b*(4*a + 3*b)*ArcTan[(Sqrt[a + b]*Tan[c +
 d*x])/Sqrt[a]]*(2*a + b - b*Cos[2*(c + d*x)]) + Sqrt[a]*Sqrt[a + b]*(4*a^2
 + 6*a*b + 3*b^2 - b*(2*a + 3*b)*Cos[2*(c + d*x)])*Cot[c + d*x])*Csc[c + d*
x]^4)/(8*a^(5/2)*(a + b)^(3/2)*d*(b + a*Csc[c + d*x]^2)^2)
```

Maple [A] time = 0.118, size = 144, normalized size = 1.1

$$-\frac{b^2\tan(dx+c)}{2a^2d(a+b)\left(a(\tan(dx+c))^2+(\tan(dx+c))^2b+a\right)} - 2\frac{b}{da(a+b)\sqrt{a(a+b)}}\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right) - \frac{3b}{2a^2d(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+sin(d*x+c)^2*b)^2,x)
```

```
[Out] -1/2/d/a^2*b^2/(a+b)*tan(d*x+c)/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)-2/d/a/(a+
b)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))*b-3/2/d/a^2*b^2
/(a+b)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))-1/d/a^2/tan
(d*x+c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.95988, size = 1330, normalized size = 10.47

$$\left[-\frac{4\left(2a^{3}b+5a^{2}b^{2}+3ab^{3}\right)\cos\left(dx+c\right)^{3}-\left(4a^{2}b+7ab^{2}+3b^{3}-\left(4ab^{2}+3b^{3}\right)\cos\left(dx+c\right)^{2}\right)\sqrt{-a^{2}-ab}\log\left(\frac{\left(8a^{2}+8a\cdots\right.}{\cdots}\right.}{8\left(\left(a^{5}b+2a^{4}b^{2}+a^{3}b^{3}\cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(2*a^3*b + 5*a^2*b^2 + 3*a*b^3)*cos(d*x + c)^3 - (4*a^2*b + 7*a*b^
2 + 3*b^3 - (4*a*b^2 + 3*b^3)*cos(d*x + c)^2)*sqrt(-a^2 - a*b)*log(((8*a^2
+ 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 - 4*
((2*a + b)*cos(d*x + c)^3 - (a + b)*cos(d*x + c))*sqrt(-a^2 - a*b)*sin(d*x
+ c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^
2 + a^2 + 2*a*b + b^2))*sin(d*x + c) - 4*(2*a^4 + 6*a^3*b + 7*a^2*b^2 + 3*a
*b^3)*cos(d*x + c))/(((a^5*b + 2*a^4*b^2 + a^3*b^3)*d*cos(d*x + c)^2 - (a^6
 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d)*sin(d*x + c)), -1/4*(2*(2*a^3*b + 5*a^
2*b^2 + 3*a*b^3)*cos(d*x + c)^3 + (4*a^2*b + 7*a*b^2 + 3*b^3 - (4*a*b^2 + 3
*b^3)*cos(d*x + c)^2)*sqrt(a^2 + a*b)*arctan(1/2*((2*a + b)*cos(d*x + c)^2
- a - b)/(sqrt(a^2 + a*b)*cos(d*x + c)*sin(d*x + c)))*sin(d*x + c) - 2*(2*a
^4 + 6*a^3*b + 7*a^2*b^2 + 3*a*b^3)*cos(d*x + c))/(((a^5*b + 2*a^4*b^2 + a^
3*b^3)*d*cos(d*x + c)^2 - (a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d)*sin(d*x
+ c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+b*sin(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20885, size = 242, normalized size = 1.91

$$-\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^{2}+ab}}\right)\right)\left(4ab+3b^{2}\right)}{\left(a^{3}+a^{2}b\right)\sqrt{a^{2}+ab}}+\frac{2a^{2}\tan(dx+c)^{2}+4ab\tan(dx+c)^{2}+3b^{2}\tan(dx+c)^{2}+2a^{2}+2ab}{\left(a\tan(dx+c)^{3}+b\tan(dx+c)^{3}+a\tan(dx+c)\right)\left(a^{3}+a^{2}b\right)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c)
 + b*tan(d*x + c))/sqrt(a^2 + a*b)))*(4*a*b + 3*b^2)/((a^3 + a^2*b)*sqrt(a^
2 + a*b)) + (2*a^2*tan(d*x + c)^2 + 4*a*b*tan(d*x + c)^2 + 3*b^2*tan(d*x +
c)^2 + 2*a^2 + 2*a*b)/((a*tan(d*x + c)^3 + b*tan(d*x + c)^3 + a*tan(d*x + c
))*(a^3 + a^2*b)))/d
```

3.105 $\int \frac{\csc^4(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2} \, dx$

Optimal. Leaf size=162

$$\frac{b^2(6a+5b)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{7/2}d(a+b)^{3/2}}-\frac{\left(2a^2-ab-5b^2\right)\cot(c+dx)}{2a^3d(a+b)}-\frac{(2a+5b)\cot^3(c+dx)}{6a^2d(a+b)}+\frac{b\csc^3(c+dx)\sec(}{2ad(a+b)\left((a+b)\tan^2\right.}$$

```
[Out] (b^2*(6*a + 5*b)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(2*a^(7/2)*(a
+ b)^(3/2)*d) - ((2*a^2 - a*b - 5*b^2)*Cot[c + d*x])/(2*a^3*(a + b)*d) - ((
2*a + 5*b)*Cot[c + d*x]^3)/(6*a^2*(a + b)*d) + (b*Csc[c + d*x]^3*Sec[c + d*
x])/(2*a*(a + b)*d*(a + (a + b)*Tan[c + d*x]^2))
```

Rubi [A] time = 0.202876, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3187, 468, 570, 205}

$$\frac{b^2(6a+5b)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{7/2}d(a+b)^{3/2}}-\frac{\left(2a^2-ab-5b^2\right)\cot(c+dx)}{2a^3d(a+b)}-\frac{(2a+5b)\cot^3(c+dx)}{6a^2d(a+b)}+\frac{b\csc^3(c+dx)\sec(}{2ad(a+b)\left((a+b)\tan^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] (b^2*(6*a + 5*b)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(2*a^(7/2)*(a
+ b)^(3/2)*d) - ((2*a^2 - a*b - 5*b^2)*Cot[c + d*x])/(2*a^3*(a + b)*d) - ((
2*a + 5*b)*Cot[c + d*x]^3)/(6*a^2*(a + b)*d) + (b*Csc[c + d*x]^3*Sec[c + d*
x])/(2*a*(a + b)*d*(a + (a + b)*Tan[c + d*x]^2))
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 570

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\csc^4(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{x^4\left(a+(a+b)x^2\right)^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{b\csc^3(c+dx)\sec(c+dx)}{2a(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)\left(-2a-5b+(-2a-b)x^2\right)}{x^4\left(a+(a+b)x^2\right)}\,dx, x, \tan(c+dx)\right)}{2a(a+b)d}$$

$$= \frac{b\csc^3(c+dx)\sec(c+dx)}{2a(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \left(\frac{-2a-5b}{ax^4} + \frac{-2a^2+ab+5b^2}{a^2x^2} + \frac{(-6a-5b)b^2}{a^2\left(a+(a+b)x^2\right)}\right) dx,$$

$$= -\frac{\left(2a^2-ab-5b^2\right)\cot(c+dx)}{2a^3(a+b)d} - \frac{(2a+5b)\cot^3(c+dx)}{6a^2(a+b)d} + \frac{b\csc^3(c+dx)\sec(c+dx)}{2a(a+b)d\left(a+(a+b)\tan^2(c+\right.}$$

$$= \frac{b^2(6a+5b)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{7/2}(a+b)^{3/2}d} - \frac{\left(2a^2-ab-5b^2\right)\cot(c+dx)}{2a^3(a+b)d} - \frac{(2a+5b)\cot^3(c+dx)}{6a^2(a+b)d}$$

Mathematica [A] time = 1.24527, size = 202, normalized size = 1.25

$$\frac{\csc^4(c+dx)(-2a+b\cos(2(c+dx))-b)\left(2a^{3/2}\cot(c+dx)\csc^2(c+dx)(2a-b\cos(2(c+dx))+b) - \frac{3\sqrt{a}b^3\sin(2(c+dx))}{a+b} + \frac{3b}{}\right.}{24a^{7/2}d\left(a\csc^2(c+dx)+b\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] ((-2*a - b + b*Cos[2*(c + d*x)])*Csc[c + d*x]^4*((3*b^2*(6*a + 5*b)*ArcTan[
(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]]*(-2*a - b + b*Cos[2*(c + d*x)]))/(a + b
)^(3/2) + 4*Sqrt[a]*(a - 3*b)*(2*a + b - b*Cos[2*(c + d*x)])*Cot[c + d*x] +
 2*a^(3/2)*(2*a + b - b*Cos[2*(c + d*x)])*Cot[c + d*x]*Csc[c + d*x]^2 - (3*
Sqrt[a]*b^3*Sin[2*(c + d*x)])/(a + b)))/(24*a^(7/2)*d*(b + a*Csc[c + d*x]^2
)^2)
```

Maple [A] time = 0.147, size = 179, normalized size = 1.1

$$\frac{b^3\tan(dx+c)}{2da^3(a+b)\left(a(\tan(dx+c))^2+(\tan(dx+c))^2b+a\right)} + 3\frac{b^2}{a^2d(a+b)\sqrt{a(a+b)}}\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right) + \frac{5b}{2da^3(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4/(a+sin(d*x+c)^2*b)^2,x)
```

```
[Out] 1/2/d*b^3/a^3/(a+b)*tan(d*x+c)/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)+3/d/a^2*b^
2/(a+b)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))+5/2/d*b^3/
a^3/(a+b)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))-1/3/d/a^
2/tan(d*x+c)^3-1/d/a^2/tan(d*x+c)+2/d/a^3/tan(d*x+c)*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.07046, size = 1912, normalized size = 11.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/24*(4*(4*a^4*b - 4*a^3*b^2 - 23*a^2*b^3 - 15*a*b^4)*cos(d*x + c)^5 - 8*
(2*a^5 + 3*a^4*b - 12*a^3*b^2 - 28*a^2*b^3 - 15*a*b^4)*cos(d*x + c)^3 + 3*(
(6*a*b^3 + 5*b^4)*cos(d*x + c)^4 + 6*a^2*b^2 + 11*a*b^3 + 5*b^4 - (6*a^2*b^
2 + 17*a*b^3 + 10*b^4)*cos(d*x + c)^2)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b
 + b^2)*cos(d*x + c)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a +
 b)*cos(d*x + c)^3 - (a + b)*cos(d*x + c))*sqrt(-a^2 - a*b)*sin(d*x + c) +
a^2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2
 + 2*a*b + b^2))*sin(d*x + c) + 12*(2*a^5 + 2*a^4*b - 6*a^3*b^2 - 11*a^2*b^
3 - 5*a*b^4)*cos(d*x + c))/(((a^6*b + 2*a^5*b^2 + a^4*b^3)*d*cos(d*x + c)^4
 - (a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d*cos(d*x + c)^2 + (a^7 + 3*a^6*
b + 3*a^5*b^2 + a^4*b^3)*d)*sin(d*x + c)), -1/12*(2*(4*a^4*b - 4*a^3*b^2 -
23*a^2*b^3 - 15*a*b^4)*cos(d*x + c)^5 - 4*(2*a^5 + 3*a^4*b - 12*a^3*b^2 - 2
8*a^2*b^3 - 15*a*b^4)*cos(d*x + c)^3 + 3*((6*a*b^3 + 5*b^4)*cos(d*x + c)^4
+ 6*a^2*b^2 + 11*a*b^3 + 5*b^4 - (6*a^2*b^2 + 17*a*b^3 + 10*b^4)*cos(d*x +
c)^2)*sqrt(a^2 + a*b)*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)/(sqrt(a
^2 + a*b)*cos(d*x + c)*sin(d*x + c)))*sin(d*x + c) + 6*(2*a^5 + 2*a^4*b - 6
*a^3*b^2 - 11*a^2*b^3 - 5*a*b^4)*cos(d*x + c))/(((a^6*b + 2*a^5*b^2 + a^4*b
^3)*d*cos(d*x + c)^4 - (a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d*cos(d*x +
c)^2 + (a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d)*sin(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4/(a+b*sin(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19397, size = 235, normalized size = 1.45

$$\frac{\frac{3 b^3 \tan(dx+c)}{(a^4+a^3b)\left(a\tan(dx+c)^2+b\tan(dx+c)^2+a\right)}+\frac{3\left(6ab^2+5b^3\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)}{(a^4+a^3b)\sqrt{a^2+ab}}-\frac{2\left(3a\tan(dx+c)^2-6b\tan(dx+c)^2+a\right)}{a^3\tan(dx+c)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/6*(3*b^3*tan(d*x + c)/((a^4 + a^3*b)*(a*tan(d*x + c)^2 + b*tan(d*x + c)^2
 + a)) + 3*(6*a*b^2 + 5*b^3)*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) +
 arctan((a*tan(d*x + c) + b*tan(d*x + c))/sqrt(a^2 + a*b)))/((a^4 + a^3*b)*
sqrt(a^2 + a*b)) - 2*(3*a*tan(d*x + c)^2 - 6*b*tan(d*x + c)^2 + a)/(a^3*tan
(d*x + c)^3))/d
```

3.106 $\int \frac{\sin^6(c+dx)}{\left(a+b\sin^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=148

$$-\frac{\sqrt{a}\left(8a^2+20ab+15b^2\right)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{8b^3d(a+b)^{5/2}}+\frac{a(4a+7b)\tan(c+dx)}{8b^2d(a+b)^2\left((a+b)\tan^2(c+dx)+a\right)}+\frac{a\tan^3(c+dx}{4bd(a+b)\left((a+b)\tan^2(\right.}$$

```
[Out] x/b^3 - (Sqrt[a]*(8*a^2 + 20*a*b + 15*b^2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x]
)/Sqrt[a]])/(8*b^3*(a + b)^(5/2)*d) + (a*Tan[c + d*x]^3)/(4*b*(a + b)*d*(a
+ (a + b)*Tan[c + d*x]^2)^2) + (a*(4*a + 7*b)*Tan[c + d*x])/(8*b^2*(a + b)^
2*d*(a + (a + b)*Tan[c + d*x]^2))
```

Rubi [A] time = 0.278846, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3187, 470, 578, 522, 203, 205}

$$-\frac{\sqrt{a}\left(8a^2+20ab+15b^2\right)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{8b^3d(a+b)^{5/2}}+\frac{a(4a+7b)\tan(c+dx)}{8b^2d(a+b)^2\left((a+b)\tan^2(c+dx)+a\right)}+\frac{a\tan^3(c+dx}{4bd(a+b)\left((a+b)\tan^2(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^6/(a + b*Sin[c + d*x]^2)^3,x]
```


```
[Out] x/b^3 - (Sqrt[a]*(8*a^2 + 20*a*b + 15*b^2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x]
)/Sqrt[a]])/(8*b^3*(a + b)^(5/2)*d) + (a*Tan[c + d*x]^3)/(4*b*(a + b)*d*(a
+ (a + b)*Tan[c + d*x]^2)^2) + (a*(4*a + 7*b)*Tan[c + d*x])/(8*b^2*(a + b)^
2*d*(a + (a + b)*Tan[c + d*x]^2))
```


Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
```

```
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^6(c+dx)}{\left(a+b\sin^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^6}{(1+x^2)\left(a+(a+b)x^2\right)^3}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{a\tan^3(c+dx)}{4b(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)^2} - \frac{\operatorname{Subst}\left(\int \frac{x^2\left(3a+(-a-4b)x^2\right)}{(1+x^2)\left(a+(a+b)x^2\right)^2}\,dx, x, \tan(c+dx)\right)}{4b(a+b)d}$$

$$= \frac{a\tan^3(c+dx)}{4b(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)^2} + \frac{a(4a+7b)\tan(c+dx)}{8b^2(a+b)^2d\left(a+(a+b)\tan^2(c+dx)\right)} - \frac{\operatorname{Subst}\left(\right.}{}$$

$$= \frac{a\tan^3(c+dx)}{4b(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)^2} + \frac{a(4a+7b)\tan(c+dx)}{8b^2(a+b)^2d\left(a+(a+b)\tan^2(c+dx)\right)} + \frac{\operatorname{Subst}\left(\right.}{}$$

$$= \frac{x}{b^3} - \frac{\sqrt{a}\left(8a^2+20ab+15b^2\right)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{8b^3(a+b)^{5/2}d} + \frac{a\tan^3(c+dx)}{4b(a+b)d\left(a+(a+b)\tan^2(c+dx)\right.}$$

Mathematica [A] time = 2.58449, size = 134, normalized size = 0.91

$$\frac{-\frac{\sqrt{a}\left(8a^2+20ab+15b^2\right)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{5/2}} + \frac{ab\sin(2(c+dx))\left(8a^2-3b(2a+3b)\cos(2(c+dx))+20ab+9b^2\right)}{(a+b)^2(2a-b\cos(2(c+dx))+b)^2} + 8(c+dx)}{8b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^6/(a + b*Sin[c + d*x]^2)^3,x]
```

```
[Out] (8*(c + d*x) - (Sqrt[a]*(8*a^2 + 20*a*b + 15*b^2)*ArcTan[(Sqrt[a + b]*Tan[c
 + d*x])/Sqrt[a]])/(a + b)^(5/2) + (a*b*(8*a^2 + 20*a*b + 9*b^2 - 3*b*(2*a
```

```
+ 3*b)*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])/((a + b)^2*(2*a + b - b*Cos[2*(c
 + d*x)])^2))/(8*b^3*d)
```

Maple [B] time = 0.096, size = 363, normalized size = 2.5

$$\frac{\arctan(\tan(dx+c))}{db^3}+\frac{a^2(\tan(dx+c))^3}{2b^2d\left(a(\tan(dx+c))^2+(\tan(dx+c))^2b+a\right)^2(a+b)}+\frac{9a(\tan(dx+c)}{8bd\left(a(\tan(dx+c))^2+(\tan(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^6/(a+sin(d*x+c)^2*b)^3,x)
```

```
[Out] 1/d/b^3*arctan(tan(d*x+c))+1/2/d*a^2/b^2/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)^
2/(a+b)*tan(d*x+c)^3+9/8/d*a/b/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)^2/(a+b)*ta
n(d*x+c)^3+1/2/d*a^3/b^2/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)^2/(a^2+2*a*b+b^2
)*tan(d*x+c)+7/8/d*a^2/b/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)^2/(a^2+2*a*b+b^2
)*tan(d*x+c)-1/d*a^3/b^3/(a^2+2*a*b+b^2)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d
*x+c)/(a*(a+b))^(1/2))-5/2/d*a^2/b^2/(a^2+2*a*b+b^2)/(a*(a+b))^(1/2)*arctan
((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))-15/8/d*a/b/(a^2+2*a*b+b^2)/(a*(a+b))^(1/
2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sin(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.22013, size = 2175, normalized size = 14.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sin(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(32*(a^2*b^2 + 2*a*b^3 + b^4)*d*x*cos(d*x + c)^4 - 64*(a^3*b + 3*a^2*
b^2 + 3*a*b^3 + b^4)*d*x*cos(d*x + c)^2 + 32*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4
*a*b^3 + b^4)*d*x + ((8*a^2*b^2 + 20*a*b^3 + 15*b^4)*cos(d*x + c)^4 + 8*a^4
 + 36*a^3*b + 63*a^2*b^2 + 50*a*b^3 + 15*b^4 - 2*(8*a^3*b + 28*a^2*b^2 + 35
*a*b^3 + 15*b^4)*cos(d*x + c)^2)*sqrt(-a/(a + b))*log(((8*a^2 + 8*a*b + b^2
)*cos(d*x + c)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a^2 + 3*a
*b + b^2)*cos(d*x + c)^3 - (a^2 + 2*a*b + b^2)*cos(d*x + c))*sqrt(-a/(a + b
))*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*co
s(d*x + c)^2 + a^2 + 2*a*b + b^2)) - 4*(3*(2*a^2*b^2 + 3*a*b^3)*cos(d*x + c
)^3 - (4*a^3*b + 13*a^2*b^2 + 9*a*b^3)*cos(d*x + c))*sin(d*x + c))/((a^2*b^
5 + 2*a*b^6 + b^7)*d*cos(d*x + c)^4 - 2*(a^3*b^4 + 3*a^2*b^5 + 3*a*b^6 + b^
7)*d*cos(d*x + c)^2 + (a^4*b^3 + 4*a^3*b^4 + 6*a^2*b^5 + 4*a*b^6 + b^7)*d),
```

```
1/16*(16*(a^2*b^2 + 2*a*b^3 + b^4)*d*x*cos(d*x + c)^4 - 32*(a^3*b + 3*a^2*
b^2 + 3*a*b^3 + b^4)*d*x*cos(d*x + c)^2 + 16*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4
*a*b^3 + b^4)*d*x + ((8*a^2*b^2 + 20*a*b^3 + 15*b^4)*cos(d*x + c)^4 + 8*a^4
 + 36*a^3*b + 63*a^2*b^2 + 50*a*b^3 + 15*b^4 - 2*(8*a^3*b + 28*a^2*b^2 + 35
*a*b^3 + 15*b^4)*cos(d*x + c)^2)*sqrt(a/(a + b))*arctan(1/2*((2*a + b)*cos(
d*x + c)^2 - a - b)*sqrt(a/(a + b))/(a*cos(d*x + c)*sin(d*x + c))) - 2*(3*(
2*a^2*b^2 + 3*a*b^3)*cos(d*x + c)^3 - (4*a^3*b + 13*a^2*b^2 + 9*a*b^3)*cos(
d*x + c))*sin(d*x + c))/((a^2*b^5 + 2*a*b^6 + b^7)*d*cos(d*x + c)^4 - 2*(a^
3*b^4 + 3*a^2*b^5 + 3*a*b^6 + b^7)*d*cos(d*x + c)^2 + (a^4*b^3 + 4*a^3*b^4
+ 6*a^2*b^5 + 4*a*b^6 + b^7)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**6/(a+b*sin(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2044, size = 302, normalized size = 2.04

$$-\frac{\frac{\left(8 a^3+20 a^2 b+15 a b^2\right)\left(\pi \left\lfloor \frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor \operatorname{sgn}(2 a+2 b)+\arctan\left(\frac{a \tan(dx+c)+b \tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)}{\left(a^2 b^3+2 a b^4+b^5\right)\sqrt{a^2+ab}}-\frac{4 a^3 \tan(dx+c)^3+13 a^2 b \tan(dx+c)^3+9 a b^2 \tan(dx+c)^3+4 a^3 \tan(dx+c)+}{\left(a^2 b^2+2 a b^3+b^4\right)\left(a \tan(dx+c)^2+b \tan(dx+c)^2+a\right)^2}}{8 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sin(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((8*a^3 + 20*a^2*b + 15*a*b^2)*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a +
 2*b) + arctan((a*tan(d*x + c) + b*tan(d*x + c))/sqrt(a^2 + a*b)))/((a^2*b^
3 + 2*a*b^4 + b^5)*sqrt(a^2 + a*b)) - (4*a^3*tan(d*x + c)^3 + 13*a^2*b*tan(
d*x + c)^3 + 9*a*b^2*tan(d*x + c)^3 + 4*a^3*tan(d*x + c) + 7*a^2*b*tan(d*x
+ c))/((a^2*b^2 + 2*a*b^3 + b^4)*(a*tan(d*x + c)^2 + b*tan(d*x + c)^2 + a)^
2) - 8*(d*x + c)/b^3)/d
```

3.107 $\int \frac{\sin^4(c+dx)}{\left(a+b\sin^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=110

$$-\frac{\tan^3(c+dx)}{4d(a+b)\left((a+b)\tan^2(c+dx)+a\right)^2}-\frac{3\tan(c+dx)}{8d(a+b)^2\left((a+b)\tan^2(c+dx)+a\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{8\sqrt{a}d(a+b)^{5/2}}$$

```
[Out] (3*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(8*Sqrt[a]*(a + b)^(5/2)*d)
- Tan[c + d*x]^3/(4*(a + b)*d*(a + (a + b)*Tan[c + d*x]^2)^2) - (3*Tan[c +
d*x])/(8*(a + b)^2*d*(a + (a + b)*Tan[c + d*x]^2))
```

Rubi [A] time = 0.0956382, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3187, 288, 205}

$$-\frac{\tan^3(c+dx)}{4d(a+b)\left((a+b)\tan^2(c+dx)+a\right)^2}-\frac{3\tan(c+dx)}{8d(a+b)^2\left((a+b)\tan^2(c+dx)+a\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{8\sqrt{a}d(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a + b*Sin[c + d*x]^2)^3,x]
```


```
[Out] (3*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(8*Sqrt[a]*(a + b)^(5/2)*d)
- Tan[c + d*x]^3/(4*(a + b)*d*(a + (a + b)*Tan[c + d*x]^2)^2) - (3*Tan[c +
d*x])/(8*(a + b)^2*d*(a + (a + b)*Tan[c + d*x]^2))
```


Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^4(c+dx)}{\left(a+b\sin^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(a+(a+b)x^2\right)^3}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{\tan^3(c+dx)}{4(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)^2} + \frac{3\,\text{Subst}\left(\int \frac{x^2}{\left(a+(a+b)x^2\right)^2}\,dx, x, \tan(c+dx)\right)}{4(a+b)d}$$

$$= -\frac{\tan^3(c+dx)}{4(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)^2} - \frac{3\tan(c+dx)}{8(a+b)^2d\left(a+(a+b)\tan^2(c+dx)\right)} + \frac{3\,\text{Subst}\left(\right.}{}$$

$$= \frac{3\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{8\sqrt{a}(a+b)^{5/2}d} - \frac{\tan^3(c+dx)}{4(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)^2} - \frac{3\tan(c+dx}{8(a+b)^2d\left(a+(a+b)\,\text{ta}\right.}$$

Mathematica [A] time = 1.24913, size = 97, normalized size = 0.88

$$\frac{\frac{3\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{5/2}} + \frac{\sin(2(c+dx))((2a+5b)\cos(2(c+dx))-8a-5b)}{(a+b)^2(2a-b\cos(2(c+dx))+b)^2}}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a + b*Sin[c + d*x]^2)^3,x]
```

```
[Out] ((3*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(5/2)) + (
(-8*a - 5*b + (2*a + 5*b)*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])/((a + b)^2*(2
*a + b - b*Cos[2*(c + d*x)])^2))/(8*d)
```

Maple [A] time = 0.089, size = 136, normalized size = 1.2

$$-\frac{5\,(\tan(dx+c))^3}{8\,d\left(a\,(\tan(dx+c))^2+(\tan(dx+c))^2\,b+a\right)^2(a+b)} - \frac{3\,a\tan(dx+c)}{8\,d\left(a\,(\tan(dx+c))^2+(\tan(dx+c))^2\,b+a\right)^2\left(a^2+2\,ab+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a+sin(d*x+c)^2*b)^3,x)
```

```
[Out] -5/8/d/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)^2/(a+b)*tan(d*x+c)^3-3/8/d/(a*tan(
d*x+c)^2+tan(d*x+c)^2*b+a)^2*a/(a^2+2*a*b+b^2)*tan(d*x+c)+3/8/d/(a^2+2*a*b+
b^2)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sin(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.97294, size = 1567, normalized size = 14.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sin(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(3*(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b
+ b^2)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4*a^
2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a + b)*cos(d*x + c)^3 - (a + b)*cos
(d*x + c))*sqrt(-a^2 - a*b)*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x
+ c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2)) - 4*((2*a^3 + 7
*a^2*b + 5*a*b^2)*cos(d*x + c)^3 - 5*(a^3 + 2*a^2*b + a*b^2)*cos(d*x + c))*
sin(d*x + c))/((a^4*b^2 + 3*a^3*b^3 + 3*a^2*b^4 + a*b^5)*d*cos(d*x + c)^4 -
 2*(a^5*b + 4*a^4*b^2 + 6*a^3*b^3 + 4*a^2*b^4 + a*b^5)*d*cos(d*x + c)^2 + (
a^6 + 5*a^5*b + 10*a^4*b^2 + 10*a^3*b^3 + 5*a^2*b^4 + a*b^5)*d), -1/16*(3*(
b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2)*sqrt
(a^2 + a*b)*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)/(sqrt(a^2 + a*b)*
cos(d*x + c)*sin(d*x + c))) - 2*((2*a^3 + 7*a^2*b + 5*a*b^2)*cos(d*x + c)^3
 - 5*(a^3 + 2*a^2*b + a*b^2)*cos(d*x + c))*sin(d*x + c))/((a^4*b^2 + 3*a^3*
b^3 + 3*a^2*b^4 + a*b^5)*d*cos(d*x + c)^4 - 2*(a^5*b + 4*a^4*b^2 + 6*a^3*b^
3 + 4*a^2*b^4 + a*b^5)*d*cos(d*x + c)^2 + (a^6 + 5*a^5*b + 10*a^4*b^2 + 10*
a^3*b^3 + 5*a^2*b^4 + a*b^5)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a+b*sin(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17635, size = 205, normalized size = 1.86

$$\frac{\frac{3\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)}{(a^2+2ab+b^2)\sqrt{a^2+ab}}-\frac{5a\tan(dx+c)^3+5b\tan(dx+c)^3+3a\tan(dx+c)}{\left(a\tan(dx+c)^2+b\tan(dx+c)^2+a\right)^2(a^2+2ab+b^2)}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sin(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c
) + b*tan(d*x + c))/sqrt(a^2 + a*b)))/((a^2 + 2*a*b + b^2)*sqrt(a^2 + a*b))
 - (5*a*tan(d*x + c)^3 + 5*b*tan(d*x + c)^3 + 3*a*tan(d*x + c))/((a*tan(d*x
 + c)^2 + b*tan(d*x + c)^2 + a)^2*(a^2 + 2*a*b + b^2)))/d
```

3.108 $\int \frac{\sin^2(c+dx)}{\left(a+b\sin^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=131

$$\frac{(4a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{8a^{3/2}d(a+b)^{5/2}}-\frac{(2a-b)\sin(c+dx)\cos(c+dx)}{8ad(a+b)^2\left(a+b\sin^2(c+dx)\right)}-\frac{\sin(c+dx)\cos(c+dx)}{4d(a+b)\left(a+b\sin^2(c+dx)\right)^2}$$

```
[Out] ((4*a + b)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(8*a^(3/2)*(a + b)^(
5/2)*d) - (Cos[c + d*x]*Sin[c + d*x])/(4*(a + b)*d*(a + b*Sin[c + d*x]^2)^2
) - ((2*a - b)*Cos[c + d*x]*Sin[c + d*x])/(8*a*(a + b)^2*d*(a + b*Sin[c + d
*x]^2))
```

Rubi [A] time = 0.14898, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3173, 12, 3181, 205}

$$\frac{(4a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{8a^{3/2}d(a+b)^{5/2}}-\frac{(2a-b)\sin(c+dx)\cos(c+dx)}{8ad(a+b)^2\left(a+b\sin^2(c+dx)\right)}-\frac{\sin(c+dx)\cos(c+dx)}{4d(a+b)\left(a+b\sin^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + b*Sin[c + d*x]^2)^3,x]
```


```
[Out] ((4*a + b)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(8*a^(3/2)*(a + b)^(
5/2)*d) - (Cos[c + d*x]*Sin[c + d*x])/(4*(a + b)*d*(a + b*Sin[c + d*x]^2)^2
) - ((2*a - b)*Cos[c + d*x]*Sin[c + d*x])/(8*a*(a + b)^2*d*(a + b*Sin[c + d
*x]^2))
```


Rule 3173

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b - a*B)*Cos[e + f*x]*Sin[e + f*x]
*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(a + b)*(p + 1)), x] - Dist[1/(2*a*
(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p +
 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{\left(a+b\sin^2(c+dx)\right)^3}\,dx = -\frac{\cos(c+dx)\sin(c+dx)}{4(a+b)d\left(a+b\sin^2(c+dx)\right)^2} + \frac{\int \frac{a+2a\sin^2(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx}{4a(a+b)}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{4(a+b)d\left(a+b\sin^2(c+dx)\right)^2} - \frac{(2a-b)\cos(c+dx)\sin(c+dx)}{8a(a+b)^2d\left(a+b\sin^2(c+dx)\right)} + \frac{\int \frac{a(4a+b)}{a+b\sin^2(c+dx)}\,dx}{8a^2(a+b)^2}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{4(a+b)d\left(a+b\sin^2(c+dx)\right)^2} - \frac{(2a-b)\cos(c+dx)\sin(c+dx)}{8a(a+b)^2d\left(a+b\sin^2(c+dx)\right)} + \frac{(4a+b)\int \frac{1}{a+b\sin^2}}{8a(a+b)^2}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{4(a+b)d\left(a+b\sin^2(c+dx)\right)^2} - \frac{(2a-b)\cos(c+dx)\sin(c+dx)}{8a(a+b)^2d\left(a+b\sin^2(c+dx)\right)} + \frac{(4a+b)\,\mathrm{Subst}\left(\int\right.}{}$$

$$= \frac{(4a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{8a^{3/2}(a+b)^{5/2}d} - \frac{\cos(c+dx)\sin(c+dx)}{4(a+b)d\left(a+b\sin^2(c+dx)\right)^2} - \frac{(2a-b)\cos(c+dx)\,s}{8a(a+b)^2d\left(a+b\sin\right.}$$

Mathematica [A] time = 1.37209, size = 112, normalized size = 0.85

$$\frac{\frac{(4a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{3/2}(a+b)^{5/2}} - \frac{\sin(2(c+dx))\left(8a^2+b(b-2a)\cos(2(c+dx))+4ab-b^2\right)}{a(a+b)^2(2a-b\cos(2(c+dx))+b)^2}}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + b*Sin[c + d*x]^2)^3,x]
```

```
[Out] (((4*a + b)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a^(3/2)*(a + b)^(5
/2)) - ((8*a^2 + 4*a*b - b^2 + b*(-2*a + b)*Cos[2*(c + d*x)])*Sin[2*(c + d*
x)])/(a*(a + b)^2*(2*a + b - b*Cos[2*(c + d*x)])^2))/(8*d)
```

Maple [B] time = 0.088, size = 278, normalized size = 2.1

$$-\frac{(\tan(dx+c))^3}{2\,d\left(a(\tan(dx+c))^2+(\tan(dx+c))^2b+a\right)^2(a+b)} + \frac{(\tan(dx+c))^3\,b}{8\,d\left(a(\tan(dx+c))^2+(\tan(dx+c))^2b+a\right)^2a(a+b)} - \frac{}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+sin(d*x+c)^2*b)^3,x)
```

```
[Out] -1/2/d/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)^2/(a+b)*tan(d*x+c)^3+1/8/d/(a*tan(
d*x+c)^2+tan(d*x+c)^2*b+a)^2/a/(a+b)*tan(d*x+c)^3*b-1/2/d/(a*tan(d*x+c)^2+t
an(d*x+c)^2*b+a)^2*a/(a^2+2*a*b+b^2)*tan(d*x+c)-1/8/d/(a*tan(d*x+c)^2+tan(d
*x+c)^2*b+a)^2/(a^2+2*a*b+b^2)*tan(d*x+c)*b+1/2/d/(a^2+2*a*b+b^2)/(a*(a+b))
^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))+1/8/d/(a^2+2*a*b+b^2)/a/(a*
(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.18566, size = 1719, normalized size = 13.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(((4*a*b^2 + b^3)*cos(d*x + c)^4 + 4*a^3 + 9*a^2*b + 6*a*b^2 + b^3 -
 2*(4*a^2*b + 5*a*b^2 + b^3)*cos(d*x + c)^2)*sqrt(-a^2 - a*b)*log(((8*a^2 +
 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*(
(2*a + b)*cos(d*x + c)^3 - (a + b)*cos(d*x + c))*sqrt(-a^2 - a*b)*sin(d*x +
 c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2
 + a^2 + 2*a*b + b^2)) - 4*((2*a^3*b + a^2*b^2 - a*b^3)*cos(d*x + c)^3 - (4
*a^4 + 7*a^3*b + 2*a^2*b^2 - a*b^3)*cos(d*x + c))*sin(d*x + c))/((a^5*b^2 +
 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*d*cos(d*x + c)^4 - 2*(a^6*b + 4*a^5*b^2 +
 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*d*cos(d*x + c)^2 + (a^7 + 5*a^6*b + 10*a^
5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*d), -1/16*(((4*a*b^2 + b^3)*cos(d
*x + c)^4 + 4*a^3 + 9*a^2*b + 6*a*b^2 + b^3 - 2*(4*a^2*b + 5*a*b^2 + b^3)*c
os(d*x + c)^2)*sqrt(a^2 + a*b)*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b
)/(sqrt(a^2 + a*b)*cos(d*x + c)*sin(d*x + c))) - 2*((2*a^3*b + a^2*b^2 - a*
b^3)*cos(d*x + c)^3 - (4*a^4 + 7*a^3*b + 2*a^2*b^2 - a*b^3)*cos(d*x + c))*s
in(d*x + c))/((a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*d*cos(d*x + c)^4
- 2*(a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*d*cos(d*x + c)^2
+ (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+b*sin(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20015, size = 258, normalized size = 1.97

$$\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)(4a+b)}{(a^3+2a^2b+ab^2)\sqrt{a^2+ab}}-\frac{4a^2\tan(dx+c)^3+3ab\tan(dx+c)^3-b^2\tan(dx+c)^3+4a^2\tan(dx+c)+ab\tan(dx+c)}{(a^3+2a^2b+ab^2)\left(a\tan(dx+c)^2+b\tan(dx+c)^2+a\right)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*((pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c)
+ b*tan(d*x + c))/sqrt(a^2 + a*b)))*(4*a + b)/((a^3 + 2*a^2*b + a*b^2)*sqrt
(a^2 + a*b)) - (4*a^2*tan(d*x + c)^3 + 3*a*b*tan(d*x + c)^3 - b^2*tan(d*x +
 c)^3 + 4*a^2*tan(d*x + c) + a*b*tan(d*x + c))/((a^3 + 2*a^2*b + a*b^2)*(a*
tan(d*x + c)^2 + b*tan(d*x + c)^2 + a)^2))/d
```

3.109 $\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=144

$$\frac{(8a^2+8ab+3b^2)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a+b)^{5/2}}+\frac{3b(2a+b)\sin(c+dx)\cos(c+dx)}{8a^2d(a+b)^2\left(a+b\sin^2(c+dx)\right)}+\frac{b\sin(c+dx)\cos(c+dx)}{4ad(a+b)\left(a+b\sin^2(c+dx)\right)^2}$$

```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(8*a^(
5/2)*(a + b)^(5/2)*d) + (b*Cos[c + d*x]*Sin[c + d*x])/(4*a*(a + b)*d*(a + b
*Sin[c + d*x]^2)^2) + (3*b*(2*a + b)*Cos[c + d*x]*Sin[c + d*x])/(8*a^2*(a +
 b)^2*d*(a + b*Sin[c + d*x]^2))
```

Rubi [A] time = 0.146423, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3184, 3173, 12, 3181, 205}

$$\frac{(8a^2+8ab+3b^2)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a+b)^{5/2}}+\frac{3b(2a+b)\sin(c+dx)\cos(c+dx)}{8a^2d(a+b)^2\left(a+b\sin^2(c+dx)\right)}+\frac{b\sin(c+dx)\cos(c+dx)}{4ad(a+b)\left(a+b\sin^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x]^2)^(-3),x]
```

```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(8*a^(
5/2)*(a + b)^(5/2)*d) + (b*Cos[c + d*x]*Sin[c + d*x])/(4*a*(a + b)*d*(a + b
*Sin[c + d*x]^2)^2) + (3*b*(2*a + b)*Cos[c + d*x]*Sin[c + d*x])/(8*a^2*(a +
 b)^2*d*(a + b*Sin[c + d*x]^2))
```

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a +
 b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)
*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rule 3173

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b - a*B)*Cos[e + f*x]*Sin[e + f*x]
*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(a + b)*(p + 1)), x] - Dist[1/(2*a*
(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p +
 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
```

```
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^3}\,dx = \frac{b\cos(c+dx)\sin(c+dx)}{4a(a+b)d\left(a+b\sin^2(c+dx)\right)^2} - \frac{\int \frac{-4a-3b+2b\sin^2(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx}{4a(a+b)}$$

$$= \frac{b\cos(c+dx)\sin(c+dx)}{4a(a+b)d\left(a+b\sin^2(c+dx)\right)^2} + \frac{3b(2a+b)\cos(c+dx)\sin(c+dx)}{8a^2(a+b)^2d\left(a+b\sin^2(c+dx)\right)} - \frac{\int \frac{-8a^2-8ab-3b^2}{a+b\sin^2(c+dx)}\,d}{8a^2(a+b)^2}$$

$$= \frac{b\cos(c+dx)\sin(c+dx)}{4a(a+b)d\left(a+b\sin^2(c+dx)\right)^2} + \frac{3b(2a+b)\cos(c+dx)\sin(c+dx)}{8a^2(a+b)^2d\left(a+b\sin^2(c+dx)\right)} + \frac{\left(8a^2+8ab+3b\right.}{8a^2}$$

$$= \frac{b\cos(c+dx)\sin(c+dx)}{4a(a+b)d\left(a+b\sin^2(c+dx)\right)^2} + \frac{3b(2a+b)\cos(c+dx)\sin(c+dx)}{8a^2(a+b)^2d\left(a+b\sin^2(c+dx)\right)} + \frac{\left(8a^2+8ab+3b\right.}{}$$

$$= \frac{\left(8a^2+8ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^{5/2}d} + \frac{b\cos(c+dx)\sin(c+dx)}{4a(a+b)d\left(a+b\sin^2(c+dx)\right)^2} + \frac{3b(2a+b}{8a^2(a+b}$$

Mathematica [A] time = 1.24192, size = 125, normalized size = 0.87

$$\frac{\frac{\left(8a^2+8ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{5/2}} + \frac{\sqrt{a}b\sin(2(c+dx))\left(16a^2-3b(2a+b)\cos(2(c+dx))+16ab+3b^2\right)}{(a+b)^2(2a-b\cos(2(c+dx))+b)^2}}{8a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x]^2)^(-3),x]
```

```
[Out] (((8*a^2 + 8*a*b + 3*b^2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a +
b)^(5/2) + (Sqrt[a]*b*(16*a^2 + 16*a*b + 3*b^2 - 3*b*(2*a + b)*Cos[2*(c + d
*x)])*Sin[2*(c + d*x)])/((a + b)^2*(2*a + b - b*Cos[2*(c + d*x)])^2))/(8*a^
(5/2)*d)
```

Maple [B] time = 0.093, size = 334, normalized size = 2.3

$$\frac{(\tan(dx+c))^3 b}{d\left(a(\tan(dx+c))^2+(\tan(dx+c))^2b+a\right)^2 a(a+b)} + \frac{3b^2(\tan(dx+c))^3}{8d\left(a(\tan(dx+c))^2+(\tan(dx+c))^2b+a\right)^2 a^2(a+b)} + \frac{}{d(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+sin(d*x+c)^2*b)^3,x)
```

```
[Out] 1/d/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)^2/a/(a+b)*tan(d*x+c)^3*b+3/8/d/(a*tan
(d*x+c)^2+tan(d*x+c)^2*b+a)^2/a^2*b^2/(a+b)*tan(d*x+c)^3+1/d/(a*tan(d*x+c)^
```

```
2+tan(d*x+c)^2*b+a)^2/(a^2+2*a*b+b^2)*tan(d*x+c)*b+5/8/d/(a*tan(d*x+c)^2+ta
n(d*x+c)^2*b+a)^2*b^2/a/(a^2+2*a*b+b^2)*tan(d*x+c)+1/d/(a^2+2*a*b+b^2)/(a*(
a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))+1/d/(a^2+2*a*b+b^2)/a/
(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))*b+3/8/d/a^2/(a^2+2
*a*b+b^2)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.3178, size = 1883, normalized size = 13.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(((8*a^2*b^2 + 8*a*b^3 + 3*b^4)*cos(d*x + c)^4 + 8*a^4 + 24*a^3*b +
27*a^2*b^2 + 14*a*b^3 + 3*b^4 - 2*(8*a^3*b + 16*a^2*b^2 + 11*a*b^3 + 3*b^4)
*cos(d*x + c)^2)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4
 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a + b)*cos(d*x + c)^3 - (
a + b)*cos(d*x + c))*sqrt(-a^2 - a*b)*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^
2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2)) + 4*(
3*(2*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(d*x + c)^3 - (8*a^4*b + 19*a^3*b^2 +
14*a^2*b^3 + 3*a*b^4)*cos(d*x + c))*sin(d*x + c))/((a^6*b^2 + 3*a^5*b^3 + 3
*a^4*b^4 + a^3*b^5)*d*cos(d*x + c)^4 - 2*(a^7*b + 4*a^6*b^2 + 6*a^5*b^3 + 4
*a^4*b^4 + a^3*b^5)*d*cos(d*x + c)^2 + (a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5
*b^3 + 5*a^4*b^4 + a^3*b^5)*d), -1/16*(((8*a^2*b^2 + 8*a*b^3 + 3*b^4)*cos(d
*x + c)^4 + 8*a^4 + 24*a^3*b + 27*a^2*b^2 + 14*a*b^3 + 3*b^4 - 2*(8*a^3*b +
 16*a^2*b^2 + 11*a*b^3 + 3*b^4)*cos(d*x + c)^2)*sqrt(a^2 + a*b)*arctan(1/2*
((2*a + b)*cos(d*x + c)^2 - a - b)/(sqrt(a^2 + a*b)*cos(d*x + c)*sin(d*x +
c))) + 2*(3*(2*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(d*x + c)^3 - (8*a^4*b + 19*
a^3*b^2 + 14*a^2*b^3 + 3*a*b^4)*cos(d*x + c))*sin(d*x + c))/((a^6*b^2 + 3*a
^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*cos(d*x + c)^4 - 2*(a^7*b + 4*a^6*b^2 + 6*a
^5*b^3 + 4*a^4*b^4 + a^3*b^5)*d*cos(d*x + c)^2 + (a^8 + 5*a^7*b + 10*a^6*b^
2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16285, size = 285, normalized size = 1.98

$$\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)(8a^2+8ab+3b^2)}{(a^4+2a^3b+a^2b^2)\sqrt{a^2+ab}}+\frac{8a^2b\tan(dx+c)^3+11ab^2\tan(dx+c)^3+3b^3\tan(dx+c)^3+8a^2b\tan(dx+c)+5}{(a^4+2a^3b+a^2b^2)\left(a\tan(dx+c)^2+b\tan(dx+c)^2+a\right)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*((pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c)
+ b*tan(d*x + c))/sqrt(a^2 + a*b)))*(8*a^2 + 8*a*b + 3*b^2)/((a^4 + 2*a^3*b
 + a^2*b^2)*sqrt(a^2 + a*b)) + (8*a^2*b*tan(d*x + c)^3 + 11*a*b^2*tan(d*x +
 c)^3 + 3*b^3*tan(d*x + c)^3 + 8*a^2*b*tan(d*x + c) + 5*a*b^2*tan(d*x + c))
/((a^4 + 2*a^3*b + a^2*b^2)*(a*tan(d*x + c)^2 + b*tan(d*x + c)^2 + a)^2))/d
```

3.110 $\int \frac{\csc^2(c+dx)}{\left(a+b\sin^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=196

$$-\frac{3b\left(8a^2+12ab+5b^2\right)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{8a^{7/2}d(a+b)^{5/2}}-\frac{(2a+3b)(4a+5b)\cot(c+dx)}{8a^3d(a+b)^2}+\frac{b\cot(c+dx)\left((4a+b)\tan^2(c+dx)+\right.}{8a^2d(a+b)^2\left((a+b)\tan^2(c+dx)\right.}$$

```
[Out] (-3*b*(8*a^2 + 12*a*b + 5*b^2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/
(8*a^(7/2)*(a + b)^(5/2)*d) - ((2*a + 3*b)*(4*a + 5*b)*Cot[c + d*x])/(8*a^3
*(a + b)^2*d) + (b*Csc[c + d*x]*Sec[c + d*x]^3)/(4*a*(a + b)*d*(a + (a + b)
*Tan[c + d*x]^2)^2) + (b*Cot[c + d*x]*(4*a + 5*b + (4*a + b)*Tan[c + d*x]^2
))/(8*a^2*(a + b)^2*d*(a + (a + b)*Tan[c + d*x]^2))
```

Rubi [A] time = 0.254987, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3187, 468, 577, 453, 205}

$$-\frac{3b\left(8a^2+12ab+5b^2\right)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{8a^{7/2}d(a+b)^{5/2}}-\frac{(2a+3b)(4a+5b)\cot(c+dx)}{8a^3d(a+b)^2}+\frac{b\cot(c+dx)\left((4a+b)\tan^2(c+dx)+\right.}{8a^2d(a+b)^2\left((a+b)\tan^2(c+dx)\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + b*Sin[c + d*x]^2)^3,x]
```

```
[Out] (-3*b*(8*a^2 + 12*a*b + 5*b^2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/
(8*a^(7/2)*(a + b)^(5/2)*d) - ((2*a + 3*b)*(4*a + 5*b)*Cot[c + d*x])/(8*a^3
*(a + b)^2*d) + (b*Csc[c + d*x]*Sec[c + d*x]^3)/(4*a*(a + b)*d*(a + (a + b)
*Tan[c + d*x]^2)^2) + (b*Cot[c + d*x]*(4*a + 5*b + (4*a + b)*Tan[c + d*x]^2
))/(8*a^2*(a + b)^2*d*(a + (a + b)*Tan[c + d*x]^2))
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 577

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*b*g*n*(p + 1)), x] + Dist[1/(a*
```

```
b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b
*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m + n
*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n, 0
] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a
*f])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{\left(a+b\sin^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{x^2\left(a+(a+b)x^2\right)^3}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{b\csc(c+dx)\sec^3(c+dx)}{4a(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)\left(-4a-5b+(-4a-b)x^2\right)}{x^2\left(a+(a+b)x^2\right)^2}\,dx, x, \tan(c+\ldots\right.}{4a(a+b)d}$$

$$= \frac{b\csc(c+dx)\sec^3(c+dx)}{4a(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)^2} + \frac{b\cot(c+dx)\left(4a+5b+(4a+b)\tan^2(c+dx)\right)}{8a^2(a+b)^2d\left(a+(a+b)\tan^2(c+dx)\right)} \ldots$$

$$= -\frac{(2a+3b)(4a+5b)\cot(c+dx)}{8a^3(a+b)^2d} + \frac{b\csc(c+dx)\sec^3(c+dx)}{4a(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)^2} + \frac{b\cot(c+d\ldots}{8a^2(a+\ldots}$$

$$= -\frac{3b\left(8a^2+12ab+5b^2\right)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{8a^{7/2}(a+b)^{5/2}d} - \frac{(2a+3b)(4a+5b)\cot(c+dx)}{8a^3(a+b)^2d} + \frac{\ldots}{4a(a\ldots}$$

Mathematica [A] time = 1.70502, size = 214, normalized size = 1.09

$$\frac{\csc^6(c+dx)(-2a+b\cos(2(c+dx))-b)\left(\frac{4a^{3/2}b^2\sin(2(c+dx))}{a+b} + \frac{3b\left(8a^2+12ab+5b^2\right)(2a-b\cos(2(c+dx))+b)^2\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{5/2}} + \frac{\sqrt{ab^2}\ldots}{\ldots}\right.}{64a^{7/2}d\left(a\csc^2(c+dx)+b\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a + b*Sin[c + d*x]^2)^3,x]
```

```
[Out] ((-2*a - b + b*Cos[2*(c + d*x)])*Csc[c + d*x]^6*((3*b*(8*a^2 + 12*a*b + 5*b
^2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]]*(2*a + b - b*Cos[2*(c + d*x)
])^2)/(a + b)^(5/2) + 8*Sqrt[a]*(2*a + b - b*Cos[2*(c + d*x)])^2*Cot[c + d*
```

```
x] + (4*a^(3/2)*b^2*Sin[2*(c + d*x)])/(a + b) + (Sqrt[a]*b^2*(10*a + 7*b)*(
2*a + b - b*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])/(a + b)^2))/(64*a^(7/2)*d*(
b + a*Csc[c + d*x]^2)^3)
```

Maple [B] time = 0.13, size = 367, normalized size = 1.9

$$-\frac{3\,b^{2}\left(\tan\left(dx+c\right)\right)^{3}}{2\,d\left(a\left(\tan\left(dx+c\right)\right)^{2}+\left(\tan\left(dx+c\right)\right)^{2}b+a\right)^{2}a^{2}\left(a+b\right)}-\frac{7\,b^{3}\left(\tan\left(dx+c\right)\right)^{3}}{8\,da^{3}\left(a\left(\tan\left(dx+c\right)\right)^{2}+\left(\tan\left(dx+c\right)\right)^{2}b+a\right)^{2}\left(a+b\right)}-\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+sin(d*x+c)^2*b)^3,x)
```

```
[Out] -3/2/d/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)^2/a^2*b^2/(a+b)*tan(d*x+c)^3-7/8/d
*b^3/a^3/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)^2/(a+b)*tan(d*x+c)^3-3/2/d/(a*ta
n(d*x+c)^2+tan(d*x+c)^2*b+a)^2*b^2/a/(a^2+2*a*b+b^2)*tan(d*x+c)-9/8/d*b^3/a
^2/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)^2/(a^2+2*a*b+b^2)*tan(d*x+c)-3/d/(a^2+
2*a*b+b^2)/a/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))*b-9/2
/d/a^2/(a^2+2*a*b+b^2)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1
/2))*b^2-15/8/d*b^3/a^3/(a^2+2*a*b+b^2)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*
x+c)/(a*(a+b))^(1/2))-1/d/a^3/tan(d*x+c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.24403, size = 2275, normalized size = 11.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(4*(8*a^4*b^2 + 34*a^3*b^3 + 41*a^2*b^4 + 15*a*b^5)*cos(d*x + c)^5 -
 4*(16*a^5*b + 76*a^4*b^2 + 137*a^3*b^3 + 107*a^2*b^4 + 30*a*b^5)*cos(d*x +
 c)^3 + 3*(8*a^4*b + 28*a^3*b^2 + 37*a^2*b^3 + 22*a*b^4 + 5*b^5 + (8*a^2*b^
3 + 12*a*b^4 + 5*b^5)*cos(d*x + c)^4 - 2*(8*a^3*b^2 + 20*a^2*b^3 + 17*a*b^4
 + 5*b^5)*cos(d*x + c)^2)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(d
*x + c)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 - 4*((2*a + b)*cos(d*x +
 c)^3 - (a + b)*cos(d*x + c))*sqrt(-a^2 - a*b)*sin(d*x + c) + a^2 + 2*a*b +
 b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^
2))*sin(d*x + c) + 4*(8*a^6 + 40*a^5*b + 92*a^4*b^2 + 111*a^3*b^3 + 66*a^2*
b^4 + 15*a*b^5)*cos(d*x + c))/(((a^7*b^2 + 3*a^6*b^3 + 3*a^5*b^4 + a^4*b^5)
*d*cos(d*x + c)^4 - 2*(a^8*b + 4*a^7*b^2 + 6*a^6*b^3 + 4*a^5*b^4 + a^4*b^5)
```

```
*d*cos(d*x + c)^2 + (a^9 + 5*a^8*b + 10*a^7*b^2 + 10*a^6*b^3 + 5*a^5*b^4 +
a^4*b^5)*d)*sin(d*x + c)), -1/16*(2*(8*a^4*b^2 + 34*a^3*b^3 + 41*a^2*b^4 +
15*a*b^5)*cos(d*x + c)^5 - 2*(16*a^5*b + 76*a^4*b^2 + 137*a^3*b^3 + 107*a^2
*b^4 + 30*a*b^5)*cos(d*x + c)^3 - 3*(8*a^4*b + 28*a^3*b^2 + 37*a^2*b^3 + 22
*a*b^4 + 5*b^5 + (8*a^2*b^3 + 12*a*b^4 + 5*b^5)*cos(d*x + c)^4 - 2*(8*a^3*b
^2 + 20*a^2*b^3 + 17*a*b^4 + 5*b^5)*cos(d*x + c)^2)*sqrt(a^2 + a*b)*arctan(
1/2*((2*a + b)*cos(d*x + c)^2 - a - b)/(sqrt(a^2 + a*b)*cos(d*x + c)*sin(d*
x + c)))*sin(d*x + c) + 2*(8*a^6 + 40*a^5*b + 92*a^4*b^2 + 111*a^3*b^3 + 66
*a^2*b^4 + 15*a*b^5)*cos(d*x + c))/(((a^7*b^2 + 3*a^6*b^3 + 3*a^5*b^4 + a^4
*b^5)*d*cos(d*x + c)^4 - 2*(a^8*b + 4*a^7*b^2 + 6*a^6*b^3 + 4*a^5*b^4 + a^4
*b^5)*d*cos(d*x + c)^2 + (a^9 + 5*a^8*b + 10*a^7*b^2 + 10*a^6*b^3 + 5*a^5*b
^4 + a^4*b^5)*d)*sin(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+b*sin(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18756, size = 313, normalized size = 1.6

$$-\frac{\frac{3\left(8a^2b+12ab^2+5b^3\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)}{\left(a^5+2a^4b+a^3b^2\right)\sqrt{a^2+ab}}+\frac{12a^2b^2\tan(dx+c)^3+19ab^3\tan(dx+c)^3+7b^4\tan(dx+c)^3+12a^2b^2\tan(dx+c)+9ab^3\tan(dx+c)}{\left(a^5+2a^4b+a^3b^2\right)\left(a\tan(dx+c)^2+b\tan(dx+c)^2+a\right)^2}+\frac{8}{a^3\tan(dx+c)}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*(3*(8*a^2*b + 12*a*b^2 + 5*b^3)*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a
+ 2*b) + arctan((a*tan(d*x + c) + b*tan(d*x + c))/sqrt(a^2 + a*b)))/((a^5 +
 2*a^4*b + a^3*b^2)*sqrt(a^2 + a*b)) + (12*a^2*b^2*tan(d*x + c)^3 + 19*a*b^
3*tan(d*x + c)^3 + 7*b^4*tan(d*x + c)^3 + 12*a^2*b^2*tan(d*x + c) + 9*a*b^3
*tan(d*x + c))/((a^5 + 2*a^4*b + a^3*b^2)*(a*tan(d*x + c)^2 + b*tan(d*x + c
)^2 + a)^2) + 8/(a^3*tan(d*x + c)))/d
```

3.111 $\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^4}\,dx$

Optimal. Leaf size=206

$$\frac{(2a+b)\left(8a^2+8ab+5b^2\right)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{16a^{7/2}d(a+b)^{7/2}}+\frac{b\left(44a^2+44ab+15b^2\right)\sin(c+dx)\cos(c+dx)}{48a^3d(a+b)^3\left(a+b\sin^2(c+dx)\right)}+\frac{5b(2a+b)\sin(c+}{24a^2d(a+b)^2\left(a+\right.}$$

```
[Out] ((2*a + b)*(8*a^2 + 8*a*b + 5*b^2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a
]])/(16*a^(7/2)*(a + b)^(7/2)*d) + (b*Cos[c + d*x]*Sin[c + d*x])/(6*a*(a +
b)*d*(a + b*Sin[c + d*x]^2)^3) + (5*b*(2*a + b)*Cos[c + d*x]*Sin[c + d*x])/
(24*a^2*(a + b)^2*d*(a + b*Sin[c + d*x]^2)^2) + (b*(44*a^2 + 44*a*b + 15*b^
2)*Cos[c + d*x]*Sin[c + d*x])/(48*a^3*(a + b)^3*d*(a + b*Sin[c + d*x]^2))
```

Rubi [A] time = 0.296316, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3184, 3173, 12, 3181, 205}

$$\frac{(2a+b)\left(8a^2+8ab+5b^2\right)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{16a^{7/2}d(a+b)^{7/2}}+\frac{b\left(44a^2+44ab+15b^2\right)\sin(c+dx)\cos(c+dx)}{48a^3d(a+b)^3\left(a+b\sin^2(c+dx)\right)}+\frac{5b(2a+b)\sin(c+}{24a^2d(a+b)^2\left(a+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x]^2)^(-4),x]
```

```
[Out] ((2*a + b)*(8*a^2 + 8*a*b + 5*b^2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a
]])/(16*a^(7/2)*(a + b)^(7/2)*d) + (b*Cos[c + d*x]*Sin[c + d*x])/(6*a*(a +
b)*d*(a + b*Sin[c + d*x]^2)^3) + (5*b*(2*a + b)*Cos[c + d*x]*Sin[c + d*x])/
(24*a^2*(a + b)^2*d*(a + b*Sin[c + d*x]^2)^2) + (b*(44*a^2 + 44*a*b + 15*b^
2)*Cos[c + d*x]*Sin[c + d*x])/(48*a^3*(a + b)^3*d*(a + b*Sin[c + d*x]^2))
```

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a +
 b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)
*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rule 3173

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b - a*B)*Cos[e + f*x]*Sin[e + f*x]
*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(a + b)*(p + 1)), x] - Dist[1/(2*a*
(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p +
 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^4}\,dx = \frac{b\cos(c+dx)\sin(c+dx)}{6a(a+b)d\left(a+b\sin^2(c+dx)\right)^3} - \frac{\int \frac{-6a-5b+4b\sin^2(c+dx)}{\left(a+b\sin^2(c+dx)\right)^3}\,dx}{6a(a+b)}$$

$$= \frac{b\cos(c+dx)\sin(c+dx)}{6a(a+b)d\left(a+b\sin^2(c+dx)\right)^3} + \frac{5b(2a+b)\cos(c+dx)\sin(c+dx)}{24a^2(a+b)^2d\left(a+b\sin^2(c+dx)\right)^2} - \frac{\int \frac{-24a^2-34ab-15}{(a+b}}{24}$$

$$= \frac{b\cos(c+dx)\sin(c+dx)}{6a(a+b)d\left(a+b\sin^2(c+dx)\right)^3} + \frac{5b(2a+b)\cos(c+dx)\sin(c+dx)}{24a^2(a+b)^2d\left(a+b\sin^2(c+dx)\right)^2} + \frac{b\left(44a^2+44ab}{48a^3(a}$$

$$= \frac{b\cos(c+dx)\sin(c+dx)}{6a(a+b)d\left(a+b\sin^2(c+dx)\right)^3} + \frac{5b(2a+b)\cos(c+dx)\sin(c+dx)}{24a^2(a+b)^2d\left(a+b\sin^2(c+dx)\right)^2} + \frac{b\left(44a^2+44ab}{48a^3(a}$$

$$= \frac{b\cos(c+dx)\sin(c+dx)}{6a(a+b)d\left(a+b\sin^2(c+dx)\right)^3} + \frac{5b(2a+b)\cos(c+dx)\sin(c+dx)}{24a^2(a+b)^2d\left(a+b\sin^2(c+dx)\right)^2} + \frac{b\left(44a^2+44ab}{48a^3(a}$$

$$= \frac{(2a+b)\left(8a^2+8ab+5b^2\right)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{16a^{7/2}(a+b)^{7/2}d} + \frac{b\cos(c+dx)\sin(c+dx)}{6a(a+b)d\left(a+b\sin^2(c+dx)\right)^3} + \frac{5}{2}$$

Mathematica [A] time = 1.39787, size = 201, normalized size = 0.98

$$\frac{\frac{3\left(24a^2b+16a^3+18ab^2+5b^3\right)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{7/2}} + \frac{\sqrt{a}b\left(44a^2+44ab+15b^2\right)\sin(2(c+dx))}{(a+b)^3(2a-b\cos(2(c+dx))+b)} + \frac{32a^{5/2}b\sin(2(c+dx))}{(a+b)(2a-b\cos(2(c+dx))+b)^3} + \frac{20a^{3/2}b(2a+b)\sin(2(c+d}{(a+b)^2(2a-b\cos(2(c+dx))}}{48a^{7/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x]^2)^(-4),x]
```

```
[Out] ((3*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*ArcTan[(Sqrt[a + b]*Tan[c + d*x]
)/Sqrt[a]])/(a + b)^(7/2) + (32*a^(5/2)*b*Sin[2*(c + d*x)])/((a + b)*(2*a +
 b - b*Cos[2*(c + d*x)])^3) + (20*a^(3/2)*b*(2*a + b)*Sin[2*(c + d*x)])/((a
 + b)^2*(2*a + b - b*Cos[2*(c + d*x)])^2) + (Sqrt[a]*b*(44*a^2 + 44*a*b + 1
5*b^2)*Sin[2*(c + d*x)])/((a + b)^3*(2*a + b - b*Cos[2*(c + d*x)])))/(48*a^
(7/2)*d)
```

Maple [B] time = 0.099, size = 705, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+sin(d*x+c)^2*b)^4,x)
```

```
[Out] 3/2/d/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)^3/a*b/(a+b)*tan(d*x+c)^5+9/8/d/(a*t
an(d*x+c)^2+tan(d*x+c)^2*b+a)^3/a^2*b^2/(a+b)*tan(d*x+c)^5+5/16/d/(a*tan(d*
x+c)^2+tan(d*x+c)^2*b+a)^3/a^3*b^3/(a+b)*tan(d*x+c)^5+3/d/(a*tan(d*x+c)^2+t
an(d*x+c)^2*b+a)^3*b/(a^2+2*a*b+b^2)*tan(d*x+c)^3+3/d/(a*tan(d*x+c)^2+tan(d
*x+c)^2*b+a)^3/a*b^2/(a^2+2*a*b+b^2)*tan(d*x+c)^3+5/6/d/(a*tan(d*x+c)^2+tan
(d*x+c)^2*b+a)^3/a^2*b^3/(a^2+2*a*b+b^2)*tan(d*x+c)^3+3/2/d/(a*tan(d*x+c)^2
+tan(d*x+c)^2*b+a)^3*b*a/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(d*x+c)+15/8/d/(a*tan
(d*x+c)^2+tan(d*x+c)^2*b+a)^3*b^2/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(d*x+c)+11/1
6/d/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)^3*b^3/a/(a^3+3*a^2*b+3*a*b^2+b^3)*tan
(d*x+c)+1/d/(a^3+3*a^2*b+3*a*b^2+b^3)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+
c)/(a*(a+b))^(1/2))+3/2/d/a/(a^3+3*a^2*b+3*a*b^2+b^3)/(a*(a+b))^(1/2)*arcta
n((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))*b+9/8/d/a^2/(a^3+3*a^2*b+3*a*b^2+b^3)/(
a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))*b^2+5/16/d/a^3/(a^3
+3*a^2*b+3*a*b^2+b^3)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/
2))*b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^2)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.42058, size = 3070, normalized size = 14.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^2)^4,x, algorithm="fricas")
```

```
[Out] [-1/192*(3*((16*a^3*b^3 + 24*a^2*b^4 + 18*a*b^5 + 5*b^6)*cos(d*x + c)^6 - 1
6*a^6 - 72*a^5*b - 138*a^4*b^2 - 147*a^3*b^3 - 93*a^2*b^4 - 33*a*b^5 - 5*b^
6 - 3*(16*a^4*b^2 + 40*a^3*b^3 + 42*a^2*b^4 + 23*a*b^5 + 5*b^6)*cos(d*x + c
)^4 + 3*(16*a^5*b + 56*a^4*b^2 + 82*a^3*b^3 + 65*a^2*b^4 + 28*a*b^5 + 5*b^6
)*cos(d*x + c)^2)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^
4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a + b)*cos(d*x + c)^3 -
(a + b)*cos(d*x + c))*sqrt(-a^2 - a*b)*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b
^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2)) + 4*
((44*a^4*b^3 + 88*a^3*b^4 + 59*a^2*b^5 + 15*a*b^6)*cos(d*x + c)^5 - 2*(54*a
^5*b^2 + 157*a^4*b^3 + 167*a^3*b^4 + 79*a^2*b^5 + 15*a*b^6)*cos(d*x + c)^3
+ 3*(24*a^6*b + 90*a^5*b^2 + 131*a^4*b^3 + 93*a^3*b^4 + 33*a^2*b^5 + 5*a*b^
6)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 + 4*a^7*b^4 + 6*a^6*b^5 + 4*a^5*b^
6 + a^4*b^7)*d*cos(d*x + c)^6 - 3*(a^9*b^2 + 5*a^8*b^3 + 10*a^7*b^4 + 10*a^
6*b^5 + 5*a^5*b^6 + a^4*b^7)*d*cos(d*x + c)^4 + 3*(a^10*b + 6*a^9*b^2 + 15*
a^8*b^3 + 20*a^7*b^4 + 15*a^6*b^5 + 6*a^5*b^6 + a^4*b^7)*d*cos(d*x + c)^2 -
 (a^11 + 7*a^10*b + 21*a^9*b^2 + 35*a^8*b^3 + 35*a^7*b^4 + 21*a^6*b^5 + 7*a
```

```
^5*b^6 + a^4*b^7)*d), -1/96*(3*((16*a^3*b^3 + 24*a^2*b^4 + 18*a*b^5 + 5*b^6
)*cos(d*x + c)^6 - 16*a^6 - 72*a^5*b - 138*a^4*b^2 - 147*a^3*b^3 - 93*a^2*b
^4 - 33*a*b^5 - 5*b^6 - 3*(16*a^4*b^2 + 40*a^3*b^3 + 42*a^2*b^4 + 23*a*b^5
+ 5*b^6)*cos(d*x + c)^4 + 3*(16*a^5*b + 56*a^4*b^2 + 82*a^3*b^3 + 65*a^2*b^
4 + 28*a*b^5 + 5*b^6)*cos(d*x + c)^2)*sqrt(a^2 + a*b)*arctan(1/2*((2*a + b)
*cos(d*x + c)^2 - a - b)/(sqrt(a^2 + a*b)*cos(d*x + c)*sin(d*x + c))) + 2*(
(44*a^4*b^3 + 88*a^3*b^4 + 59*a^2*b^5 + 15*a*b^6)*cos(d*x + c)^5 - 2*(54*a^
5*b^2 + 157*a^4*b^3 + 167*a^3*b^4 + 79*a^2*b^5 + 15*a*b^6)*cos(d*x + c)^3 +
 3*(24*a^6*b + 90*a^5*b^2 + 131*a^4*b^3 + 93*a^3*b^4 + 33*a^2*b^5 + 5*a*b^6
)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 + 4*a^7*b^4 + 6*a^6*b^5 + 4*a^5*b^6
 + a^4*b^7)*d*cos(d*x + c)^6 - 3*(a^9*b^2 + 5*a^8*b^3 + 10*a^7*b^4 + 10*a^6
*b^5 + 5*a^5*b^6 + a^4*b^7)*d*cos(d*x + c)^4 + 3*(a^10*b + 6*a^9*b^2 + 15*a
^8*b^3 + 20*a^7*b^4 + 15*a^6*b^5 + 6*a^5*b^6 + a^4*b^7)*d*cos(d*x + c)^2 -
(a^11 + 7*a^10*b + 21*a^9*b^2 + 35*a^8*b^3 + 35*a^7*b^4 + 21*a^6*b^5 + 7*a^
5*b^6 + a^4*b^7)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)**2)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12537, size = 464, normalized size = 2.25

$$\frac{\frac{3\left(16a^3+24a^2b+18ab^2+5b^3\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)}{\left(a^6+3a^5b+3a^4b^2+a^3b^3\right)\sqrt{a^2+ab}}+\frac{72a^4b\tan(dx+c)^5+198a^3b^2\tan(dx+c)^5+195a^2b^3\tan(dx+c)^5}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^2)^4,x, algorithm="giac")
```

```
[Out] 1/48*(3*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*(pi*floor((d*x + c)/pi + 1/2
)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c) + b*tan(d*x + c))/sqrt(a^2 + a*b)
))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*sqrt(a^2 + a*b)) + (72*a^4*b*tan(
d*x + c)^5 + 198*a^3*b^2*tan(d*x + c)^5 + 195*a^2*b^3*tan(d*x + c)^5 + 84*a
*b^4*tan(d*x + c)^5 + 15*b^5*tan(d*x + c)^5 + 144*a^4*b*tan(d*x + c)^3 + 28
8*a^3*b^2*tan(d*x + c)^3 + 184*a^2*b^3*tan(d*x + c)^3 + 40*a*b^4*tan(d*x +
c)^3 + 72*a^4*b*tan(d*x + c) + 90*a^3*b^2*tan(d*x + c) + 33*a^2*b^3*tan(d*x
 + c))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*(a*tan(d*x + c)^2 + b*tan(d*x
 + c)^2 + a)^3))/d
```

3.112 $\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^5}\,dx$

Optimal. Leaf size=279

$$\frac{\left(288a^2b^2+256a^3b+128a^4+160ab^3+35b^4\right)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{128a^{9/2}d(a+b)^{9/2}}+\frac{5b(2a+b)\left(40a^2+40ab+21b^2\right)\sin(c+dx)\cos(c}{384a^4d(a+b)^4\left(a+b\sin^2(c+dx)\right)}$$

```
[Out] ((128*a^4 + 256*a^3*b + 288*a^2*b^2 + 160*a*b^3 + 35*b^4)*ArcTan[(Sqrt[a +
b]*Tan[c + d*x])/Sqrt[a]])/(128*a^(9/2)*(a + b)^(9/2)*d) + (b*Cos[c + d*x]*
Sin[c + d*x])/(8*a*(a + b)*d*(a + b*Sin[c + d*x]^2)^4) + (7*b*(2*a + b)*Cos
[c + d*x]*Sin[c + d*x])/(48*a^2*(a + b)^2*d*(a + b*Sin[c + d*x]^2)^3) + (b*
(104*a^2 + 104*a*b + 35*b^2)*Cos[c + d*x]*Sin[c + d*x])/(192*a^3*(a + b)^3*
d*(a + b*Sin[c + d*x]^2)^2) + (5*b*(2*a + b)*(40*a^2 + 40*a*b + 21*b^2)*Cos
[c + d*x]*Sin[c + d*x])/(384*a^4*(a + b)^4*d*(a + b*Sin[c + d*x]^2))
```

Rubi [A] time = 0.530857, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3184, 3173, 12, 3181, 205}

$$\frac{\left(288a^2b^2+256a^3b+128a^4+160ab^3+35b^4\right)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{128a^{9/2}d(a+b)^{9/2}}+\frac{5b(2a+b)\left(40a^2+40ab+21b^2\right)\sin(c+dx)\cos(c}{384a^4d(a+b)^4\left(a+b\sin^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x]^2)^(-5),x]
```

```
[Out] ((128*a^4 + 256*a^3*b + 288*a^2*b^2 + 160*a*b^3 + 35*b^4)*ArcTan[(Sqrt[a +
b]*Tan[c + d*x])/Sqrt[a]])/(128*a^(9/2)*(a + b)^(9/2)*d) + (b*Cos[c + d*x]*
Sin[c + d*x])/(8*a*(a + b)*d*(a + b*Sin[c + d*x]^2)^4) + (7*b*(2*a + b)*Cos
[c + d*x]*Sin[c + d*x])/(48*a^2*(a + b)^2*d*(a + b*Sin[c + d*x]^2)^3) + (b*
(104*a^2 + 104*a*b + 35*b^2)*Cos[c + d*x]*Sin[c + d*x])/(192*a^3*(a + b)^3*
d*(a + b*Sin[c + d*x]^2)^2) + (5*b*(2*a + b)*(40*a^2 + 40*a*b + 21*b^2)*Cos
[c + d*x]*Sin[c + d*x])/(384*a^4*(a + b)^4*d*(a + b*Sin[c + d*x]^2))
```

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a +
 b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)
*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rule 3173

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b - a*B)*Cos[e + f*x]*Sin[e + f*x]
*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(a + b)*(p + 1)), x] - Dist[1/(2*a*
(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p +
 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^5}\,dx = \frac{b\cos(c+dx)\sin(c+dx)}{8a(a+b)d\left(a+b\sin^2(c+dx)\right)^4} - \frac{\int \frac{-8a-7b+6b\sin^2(c+dx)}{\left(a+b\sin^2(c+dx)\right)^4}\,dx}{8a(a+b)}$$

$$= \frac{b\cos(c+dx)\sin(c+dx)}{8a(a+b)d\left(a+b\sin^2(c+dx)\right)^4} + \frac{7b(2a+b)\cos(c+dx)\sin(c+dx)}{48a^2(a+b)^2d\left(a+b\sin^2(c+dx)\right)^3} - \frac{\int \frac{-48a^2-76ab-35}{(a+}}{48}$$

$$= \frac{b\cos(c+dx)\sin(c+dx)}{8a(a+b)d\left(a+b\sin^2(c+dx)\right)^4} + \frac{7b(2a+b)\cos(c+dx)\sin(c+dx)}{48a^2(a+b)^2d\left(a+b\sin^2(c+dx)\right)^3} + \frac{b\left(104a^2+104\right.}{192a^3(a}$$

$$= \frac{b\cos(c+dx)\sin(c+dx)}{8a(a+b)d\left(a+b\sin^2(c+dx)\right)^4} + \frac{7b(2a+b)\cos(c+dx)\sin(c+dx)}{48a^2(a+b)^2d\left(a+b\sin^2(c+dx)\right)^3} + \frac{b\left(104a^2+104\right.}{192a^3(a}$$

$$= \frac{b\cos(c+dx)\sin(c+dx)}{8a(a+b)d\left(a+b\sin^2(c+dx)\right)^4} + \frac{7b(2a+b)\cos(c+dx)\sin(c+dx)}{48a^2(a+b)^2d\left(a+b\sin^2(c+dx)\right)^3} + \frac{b\left(104a^2+104\right.}{192a^3(a}$$

$$= \frac{b\cos(c+dx)\sin(c+dx)}{8a(a+b)d\left(a+b\sin^2(c+dx)\right)^4} + \frac{7b(2a+b)\cos(c+dx)\sin(c+dx)}{48a^2(a+b)^2d\left(a+b\sin^2(c+dx)\right)^3} + \frac{b\left(104a^2+104\right.}{192a^3(a}$$

$$= \frac{\left(128a^4+256a^3b+288a^2b^2+160ab^3+35b^4\right)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{128a^{9/2}(a+b)^{9/2}d} + \frac{b\cos(c+dx)}{8a(a+b)d\left(a+b\right.}$$

Mathematica [A] time = 1.89236, size = 312, normalized size = 1.12

$$\frac{\frac{24\left(288a^2b^2+256a^3b+128a^4+160ab^3+35b^4\right)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{9/2}} + \frac{2\sqrt{a}b\sin(2(c+dx))\left(-400a^3b^3\cos(6(c+dx))-600a^2b^4\cos(6(c+dx))-b\left(73616a^3b^2+413\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x]^2)^(-5),x]
```

```
[Out] ((24*(128*a^4 + 256*a^3*b + 288*a^2*b^2 + 160*a*b^3 + 35*b^4)*ArcTan[(Sqrt[
a + b]*Tan[c + d*x])/Sqrt[a]])/(a + b)^(9/2) + (2*Sqrt[a]*b*(24576*a^6 + 73
728*a^5*b + 97280*a^4*b^2 + 71680*a^3*b^3 + 32272*a^2*b^4 + 8720*a*b^5 + 10
50*b^6 - b*(27648*a^5 + 69120*a^4*b + 73616*a^3*b^2 + 41304*a^2*b^3 + 12310
```

```
*a*b^4 + 1575*b^5)*Cos[2*(c + d*x)] + 2*b^2*(2816*a^4 + 5632*a^3*b + 4816*a
^2*b^2 + 2000*a*b^3 + 315*b^4)*Cos[4*(c + d*x)] - 400*a^3*b^3*Cos[6*(c + d*
x)] - 600*a^2*b^4*Cos[6*(c + d*x)] - 410*a*b^5*Cos[6*(c + d*x)] - 105*b^6*C
os[6*(c + d*x)])*Sin[2*(c + d*x)])/((a + b)^4*(2*a + b - b*Cos[2*(c + d*x)]
)^4))/(3072*a^(9/2)*d)
```

Maple [B] time = 0.098, size = 1249, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+sin(d*x+c)^2*b)^5,x)
```

```
[Out] 2/d/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)^4*b/a/(a+b)*tan(d*x+c)^7+9/4/d/(a*tan
(d*x+c)^2+tan(d*x+c)^2*b+a)^4*b^2/a^2/(a+b)*tan(d*x+c)^7+5/4/d/(a*tan(d*x+c
)^2+tan(d*x+c)^2*b+a)^4*b^3/a^3/(a+b)*tan(d*x+c)^7+35/128/d/(a*tan(d*x+c)^2
+tan(d*x+c)^2*b+a)^4*b^4/a^4/(a+b)*tan(d*x+c)^7+6/d/(a*tan(d*x+c)^2+tan(d*x
+c)^2*b+a)^4*b/(a^2+2*a*b+b^2)*tan(d*x+c)^5+33/4/d/(a*tan(d*x+c)^2+tan(d*x+
c)^2*b+a)^4/a*b^2/(a^2+2*a*b+b^2)*tan(d*x+c)^5+55/12/d/(a*tan(d*x+c)^2+tan(
d*x+c)^2*b+a)^4/a^2*b^3/(a^2+2*a*b+b^2)*tan(d*x+c)^5+385/384/d/(a*tan(d*x+c
)^2+tan(d*x+c)^2*b+a)^4/a^3*b^4/(a^2+2*a*b+b^2)*tan(d*x+c)^5+6/d/(a*tan(d*x
+c)^2+tan(d*x+c)^2*b+a)^4*a*b/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(d*x+c)^3+39/4/d
/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)^4*b^2/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(d*x+
c)^3+73/12/d/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)^4/a*b^3/(a^3+3*a^2*b+3*a*b^2
+b^3)*tan(d*x+c)^3+511/384/d/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)^4/a^2*b^4/(a
^3+3*a^2*b+3*a*b^2+b^3)*tan(d*x+c)^3+2/d/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)^
4*b*a^2/(a^4+4*a^3*b+6*a^2*b^2+4*a*b^3+b^4)*tan(d*x+c)+15/4/d/(a*tan(d*x+c)
^2+tan(d*x+c)^2*b+a)^4*b^2*a/(a^4+4*a^3*b+6*a^2*b^2+4*a*b^3+b^4)*tan(d*x+c)
+11/4/d/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)^4*b^3/(a^4+4*a^3*b+6*a^2*b^2+4*a*
b^3+b^4)*tan(d*x+c)+93/128/d/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)^4*b^4/a/(a^4
+4*a^3*b+6*a^2*b^2+4*a*b^3+b^4)*tan(d*x+c)+1/d/(a^4+4*a^3*b+6*a^2*b^2+4*a*b
^3+b^4)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))+2/d/a/(a^4
+4*a^3*b+6*a^2*b^2+4*a*b^3+b^4)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*
(a+b))^(1/2))*b+9/4/d/a^2/(a^4+4*a^3*b+6*a^2*b^2+4*a*b^3+b^4)/(a*(a+b))^(1/
2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))*b^2+5/4/d/a^3/(a^4+4*a^3*b+6*a^
2*b^2+4*a*b^3+b^4)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))
*b^3+35/128/d/a^4/(a^4+4*a^3*b+6*a^2*b^2+4*a*b^3+b^4)/(a*(a+b))^(1/2)*arcta
n((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))*b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^2)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.9847, size = 4771, normalized size = 17.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^2)^5,x, algorithm="fricas")
```

```
[Out] [-1/1536*(3*((128*a^4*b^4 + 256*a^3*b^5 + 288*a^2*b^6 + 160*a*b^7 + 35*b^8)
*cos(d*x + c)^8 + 128*a^8 + 768*a^7*b + 2080*a^6*b^2 + 3360*a^5*b^3 + 3555*
a^4*b^4 + 2508*a^3*b^5 + 1138*a^2*b^6 + 300*a*b^7 + 35*b^8 - 4*(128*a^5*b^3
 + 384*a^4*b^4 + 544*a^3*b^5 + 448*a^2*b^6 + 195*a*b^7 + 35*b^8)*cos(d*x +
c)^6 + 6*(128*a^6*b^2 + 512*a^5*b^3 + 928*a^4*b^4 + 992*a^3*b^5 + 643*a^2*b
^6 + 230*a*b^7 + 35*b^8)*cos(d*x + c)^4 - 4*(128*a^7*b + 640*a^6*b^2 + 1440
*a^5*b^3 + 1920*a^4*b^4 + 1635*a^3*b^5 + 873*a^2*b^6 + 265*a*b^7 + 35*b^8)*
cos(d*x + c)^2)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4
- 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a + b)*cos(d*x + c)^3 - (a
 + b)*cos(d*x + c))*sqrt(-a^2 - a*b)*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2
*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2)) + 4*(5
*(80*a^5*b^4 + 200*a^4*b^5 + 202*a^3*b^6 + 103*a^2*b^7 + 21*a*b^8)*cos(d*x
+ c)^7 - (1408*a^6*b^3 + 4824*a^5*b^4 + 6724*a^4*b^5 + 4923*a^3*b^6 + 1930*
a^2*b^7 + 315*a*b^8)*cos(d*x + c)^5 + (1728*a^7*b^2 + 7456*a^6*b^3 + 13370*
a^5*b^4 + 12969*a^4*b^5 + 7327*a^3*b^6 + 2315*a^2*b^7 + 315*a*b^8)*cos(d*x
+ c)^3 - 3*(256*a^8*b + 1312*a^7*b^2 + 2848*a^6*b^3 + 3427*a^5*b^4 + 2508*a
^4*b^5 + 1138*a^3*b^6 + 300*a^2*b^7 + 35*a*b^8)*cos(d*x + c))*sin(d*x + c))
/((a^10*b^4 + 5*a^9*b^5 + 10*a^8*b^6 + 10*a^7*b^7 + 5*a^6*b^8 + a^5*b^9)*d*
cos(d*x + c)^8 - 4*(a^11*b^3 + 6*a^10*b^4 + 15*a^9*b^5 + 20*a^8*b^6 + 15*a^
7*b^7 + 6*a^6*b^8 + a^5*b^9)*d*cos(d*x + c)^6 + 6*(a^12*b^2 + 7*a^11*b^3 +
21*a^10*b^4 + 35*a^9*b^5 + 35*a^8*b^6 + 21*a^7*b^7 + 7*a^6*b^8 + a^5*b^9)*d
*cos(d*x + c)^4 - 4*(a^13*b + 8*a^12*b^2 + 28*a^11*b^3 + 56*a^10*b^4 + 70*a
^9*b^5 + 56*a^8*b^6 + 28*a^7*b^7 + 8*a^6*b^8 + a^5*b^9)*d*cos(d*x + c)^2 +
(a^14 + 9*a^13*b + 36*a^12*b^2 + 84*a^11*b^3 + 126*a^10*b^4 + 126*a^9*b^5 +
 84*a^8*b^6 + 36*a^7*b^7 + 9*a^6*b^8 + a^5*b^9)*d), -1/768*(3*((128*a^4*b^4
 + 256*a^3*b^5 + 288*a^2*b^6 + 160*a*b^7 + 35*b^8)*cos(d*x + c)^8 + 128*a^8
 + 768*a^7*b + 2080*a^6*b^2 + 3360*a^5*b^3 + 3555*a^4*b^4 + 2508*a^3*b^5 +
1138*a^2*b^6 + 300*a*b^7 + 35*b^8 - 4*(128*a^5*b^3 + 384*a^4*b^4 + 544*a^3*
b^5 + 448*a^2*b^6 + 195*a*b^7 + 35*b^8)*cos(d*x + c)^6 + 6*(128*a^6*b^2 + 5
12*a^5*b^3 + 928*a^4*b^4 + 992*a^3*b^5 + 643*a^2*b^6 + 230*a*b^7 + 35*b^8)*
cos(d*x + c)^4 - 4*(128*a^7*b + 640*a^6*b^2 + 1440*a^5*b^3 + 1920*a^4*b^4 +
 1635*a^3*b^5 + 873*a^2*b^6 + 265*a*b^7 + 35*b^8)*cos(d*x + c)^2)*sqrt(a^2
+ a*b)*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)/(sqrt(a^2 + a*b)*cos(d
*x + c)*sin(d*x + c))) + 2*(5*(80*a^5*b^4 + 200*a^4*b^5 + 202*a^3*b^6 + 103
*a^2*b^7 + 21*a*b^8)*cos(d*x + c)^7 - (1408*a^6*b^3 + 4824*a^5*b^4 + 6724*a
^4*b^5 + 4923*a^3*b^6 + 1930*a^2*b^7 + 315*a*b^8)*cos(d*x + c)^5 + (1728*a^
7*b^2 + 7456*a^6*b^3 + 13370*a^5*b^4 + 12969*a^4*b^5 + 7327*a^3*b^6 + 2315*
a^2*b^7 + 315*a*b^8)*cos(d*x + c)^3 - 3*(256*a^8*b + 1312*a^7*b^2 + 2848*a^
6*b^3 + 3427*a^5*b^4 + 2508*a^4*b^5 + 1138*a^3*b^6 + 300*a^2*b^7 + 35*a*b^8
)*cos(d*x + c))*sin(d*x + c))/((a^10*b^4 + 5*a^9*b^5 + 10*a^8*b^6 + 10*a^7*
b^7 + 5*a^6*b^8 + a^5*b^9)*d*cos(d*x + c)^8 - 4*(a^11*b^3 + 6*a^10*b^4 + 15
*a^9*b^5 + 20*a^8*b^6 + 15*a^7*b^7 + 6*a^6*b^8 + a^5*b^9)*d*cos(d*x + c)^6
+ 6*(a^12*b^2 + 7*a^11*b^3 + 21*a^10*b^4 + 35*a^9*b^5 + 35*a^8*b^6 + 21*a^7
*b^7 + 7*a^6*b^8 + a^5*b^9)*d*cos(d*x + c)^4 - 4*(a^13*b + 8*a^12*b^2 + 28*
a^11*b^3 + 56*a^10*b^4 + 70*a^9*b^5 + 56*a^8*b^6 + 28*a^7*b^7 + 8*a^6*b^8 +
 a^5*b^9)*d*cos(d*x + c)^2 + (a^14 + 9*a^13*b + 36*a^12*b^2 + 84*a^11*b^3 +
 126*a^10*b^4 + 126*a^9*b^5 + 84*a^8*b^6 + 36*a^7*b^7 + 9*a^6*b^8 + a^5*b^9
)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)**2)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.16196, size = 707, normalized size = 2.53

$$\frac{\frac{3\left(128\,a^4+256\,a^3b+288\,a^2b^2+160\,ab^3+35\,b^4\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)}{\left(a^8+4\,a^7b+6\,a^6b^2+4\,a^5b^3+a^4b^4\right)\sqrt{a^2+ab}}+\frac{768\,a^6b\tan(dx+c)^7+3168\,a^5b^2\tan(dx+c)^7+5376\,a^4}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^2)^5,x, algorithm="giac")
```

```
[Out] 1/384*(3*(128*a^4 + 256*a^3*b + 288*a^2*b^2 + 160*a*b^3 + 35*b^4)*(pi*floor
((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c) + b*tan(d*x +
c))/sqrt(a^2 + a*b)))/((a^8 + 4*a^7*b + 6*a^6*b^2 + 4*a^5*b^3 + a^4*b^4)*sq
rt(a^2 + a*b)) + (768*a^6*b*tan(d*x + c)^7 + 3168*a^5*b^2*tan(d*x + c)^7 +
5376*a^4*b^3*tan(d*x + c)^7 + 4905*a^3*b^4*tan(d*x + c)^7 + 2619*a^2*b^5*ta
n(d*x + c)^7 + 795*a*b^6*tan(d*x + c)^7 + 105*b^7*tan(d*x + c)^7 + 2304*a^6
*b*tan(d*x + c)^5 + 7776*a^5*b^2*tan(d*x + c)^5 + 10400*a^4*b^3*tan(d*x + c
)^5 + 7073*a^3*b^4*tan(d*x + c)^5 + 2530*a^2*b^5*tan(d*x + c)^5 + 385*a*b^6
*tan(d*x + c)^5 + 2304*a^6*b*tan(d*x + c)^3 + 6048*a^5*b^2*tan(d*x + c)^3 +
 6080*a^4*b^3*tan(d*x + c)^3 + 2847*a^3*b^4*tan(d*x + c)^3 + 511*a^2*b^5*ta
n(d*x + c)^3 + 768*a^6*b*tan(d*x + c) + 1440*a^5*b^2*tan(d*x + c) + 1056*a^
4*b^3*tan(d*x + c) + 279*a^3*b^4*tan(d*x + c))/((a^8 + 4*a^7*b + 6*a^6*b^2
+ 4*a^5*b^3 + a^4*b^4)*(a*tan(d*x + c)^2 + b*tan(d*x + c)^2 + a)^4))/d
```

3.113 $\int \frac{\sin(x)}{\sqrt{1+\sin^2(x)}} \, dx$

Optimal. Leaf size=11

$$-\sin^{-1}\left(\frac{\cos(x)}{\sqrt{2}}\right)$$

```
[Out] -ArcSin[Cos[x]/Sqrt[2]]
```

Rubi [A] time = 0.0263391, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3186, 216}

$$-\sin^{-1}\left(\frac{\cos(x)}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/Sqrt[1 + Sin[x]^2],x]

[Out] -ArcSin[Cos[x]/Sqrt[2]]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(x)}{\sqrt{1+\sin^2(x)}} \, dx &= -\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}} \, dx, x, \cos(x)\right)\\ &= -\sin^{-1}\left(\frac{\cos(x)}{\sqrt{2}}\right)\end{aligned}$$

Mathematica [C] time = 0.0637318, size = 29, normalized size = 2.64

$$i \log\left(\sqrt{3-\cos(2x)}+i\sqrt{2}\cos(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/Sqrt[1 + Sin[x]^2],x]

[Out] I*Log[I*Sqrt[2]*Cos[x] + Sqrt[3 - Cos[2*x]]]
```

Maple [B] time = 0.463, size = 33, normalized size = 3.

$$\frac{\arcsin\left((\sin(x))^2\right)}{2\cos(x)}\sqrt{\left(1+(\sin(x))^2\right)(\cos(x))^2}\frac{1}{\sqrt{1+(\sin(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(1+sin(x)^2)^(1/2),x)
```

```
[Out] 1/2*((1+sin(x)^2)*cos(x)^2)^(1/2)*arcsin(sin(x)^2)/cos(x)/(1+sin(x)^2)^(1/2
)
```

Maxima [A] time = 1.41358, size = 14, normalized size = 1.27

$$-\arcsin\left(\frac{1}{2}\sqrt{2}\cos(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+sin(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(1/2*sqrt(2)*cos(x))
```

Fricas [B] time = 1.63381, size = 173, normalized size = 15.73

$$\frac{1}{2}\arctan\left(-\frac{\cos(x)\sin(x)-\left(\cos(x)^3-\cos(x)\right)\sqrt{-\cos(x)^2+2}}{\cos(x)^4-3\cos(x)^2+1}\right)-\frac{1}{2}\arctan\left(\frac{\sin(x)}{\cos(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+sin(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*arctan(-(cos(x)*sin(x) - (cos(x)^3 - cos(x))*sqrt(-cos(x)^2 + 2))/(cos(
x)^4 - 3*cos(x)^2 + 1)) - 1/2*arctan(sin(x)/cos(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+sin(x)**2)**(1/2),x)
```

```
[Out] Timed out
```

<u>Giac</u> [A] time = 1.1363, size = 14, normalized size = 1.27

$$-\arcsin\left(\frac{1}{2}\sqrt{2}\cos\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+sin(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -arcsin(1/2*sqrt(2)*cos(x))
```

3.114 $\int \sin(x)\sqrt{1+\sin^2(x)}\,dx$

Optimal. Leaf size=30

$$-\frac{1}{2}\cos(x)\sqrt{2-\cos^2(x)}-\sin^{-1}\left(\frac{\cos(x)}{\sqrt{2}}\right)$$

```
[Out] -ArcSin[Cos[x]/Sqrt[2]] - (Cos[x]*Sqrt[2 - Cos[x]^2])/2
```

Rubi [A] time = 0.02998, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3186, 195, 216}

$$-\frac{1}{2}\cos(x)\sqrt{2-\cos^2(x)}-\sin^{-1}\left(\frac{\cos(x)}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*Sqrt[1 + Sin[x]^2],x]

[Out] -ArcSin[Cos[x]/Sqrt[2]] - (Cos[x]*Sqrt[2 - Cos[x]^2])/2
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \sin(x)\sqrt{1+\sin^2(x)}\,dx &= -\text{Subst}\left(\int \sqrt{2-x^2}\,dx,x,\cos(x)\right)\\ &= -\frac{1}{2}\cos(x)\sqrt{2-\cos^2(x)}-\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}}\,dx,x,\cos(x)\right)\\ &= -\sin^{-1}\left(\frac{\cos(x)}{\sqrt{2}}\right)-\frac{1}{2}\cos(x)\sqrt{2-\cos^2(x)}\end{aligned}$$

Mathematica [C] time = 0.048358, size = 53, normalized size = 1.77

$$-\frac{\cos(x)\sqrt{3-\cos(2x)}}{2\sqrt{2}}+i\log\left(\sqrt{3-\cos(2x)}+i\sqrt{2}\cos(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Sqrt[1 + Sin[x]^2],x]
```

```
[Out] -(Cos[x]*Sqrt[3 - Cos[2*x]])/(2*Sqrt[2]) + I*Log[I*Sqrt[2]*Cos[x] + Sqrt[3
- Cos[2*x]]]
```

Maple [A] time = 0.938, size = 51, normalized size = 1.7

$$-\frac{1}{2\cos(x)}\sqrt{\left(1+(\sin(x))^2\right)(\cos(x))^2}\left(\sqrt{-(\cos(x))^4+2(\cos(x))^2}+\arcsin\left((\cos(x))^2-1\right)\right)\frac{1}{\sqrt{1+(\sin(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*(1+sin(x)^2)^(1/2),x)
```

```
[Out] -1/2*((1+sin(x)^2)*cos(x)^2)^(1/2)*((-cos(x)^4+2*cos(x)^2)^(1/2)+arcsin(cos
(x)^2-1))/cos(x)/(1+sin(x)^2)^(1/2)
```

Maxima [A] time = 1.41041, size = 34, normalized size = 1.13

$$-\frac{1}{2}\sqrt{-\cos(x)^2+2}\cos(x)-\arcsin\left(\frac{1}{2}\sqrt{2}\cos(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*(1+sin(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-cos(x)^2 + 2)*cos(x) - arcsin(1/2*sqrt(2)*cos(x))
```

Fricas [B] time = 1.79444, size = 219, normalized size = 7.3

$$-\frac{1}{2}\sqrt{-\cos(x)^2+2}\cos(x)+\frac{1}{2}\arctan\left(-\frac{\cos(x)\sin(x)-\left(\cos(x)^3-\cos(x)\right)\sqrt{-\cos(x)^2+2}}{\cos(x)^4-3\cos(x)^2+1}\right)-\frac{1}{2}\arctan\left(\frac{\sin(x)}{\cos(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*(1+sin(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(-cos(x)^2 + 2)*cos(x) + 1/2*arctan(-(cos(x)*sin(x) - (cos(x)^3 -
cos(x))*sqrt(-cos(x)^2 + 2))/(cos(x)^4 - 3*cos(x)^2 + 1)) - 1/2*arctan(sin(
x)/cos(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sin^2(x)+1}\sin(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*(1+sin(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(sin(x)**2 + 1)*sin(x), x)
```

Giac [A] time = 1.19713, size = 34, normalized size = 1.13

$$-\frac{1}{2}\sqrt{-\cos(x)^2+2}\cos(x)-\arcsin\left(\frac{1}{2}\sqrt{2}\cos(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*(1+sin(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-cos(x)^2 + 2)*cos(x) - arcsin(1/2*sqrt(2)*cos(x))
```

3.115 $\int \frac{\sin(7+3x)}{\sqrt{3+\sin^2(7+3x)}}\,dx$

Optimal. Leaf size=15

$$-\frac{1}{3}\sin^{-1}\left(\frac{1}{2}\cos(3x+7)\right)$$

```
[Out] -ArcSin[Cos[7 + 3*x]/2]/3
```

Rubi [A] time = 0.029909, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3186, 216}

$$-\frac{1}{3}\sin^{-1}\left(\frac{1}{2}\cos(3x+7)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[7 + 3*x]/Sqrt[3 + Sin[7 + 3*x]^2],x]

[Out] -ArcSin[Cos[7 + 3*x]/2]/3
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(7+3x)}{\sqrt{3+\sin^2(7+3x)}}\,dx &= -\left(\frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt{4-x^2}}\,dx, x, \cos(7+3x)\right)\right)\\ &= -\frac{1}{3}\sin^{-1}\left(\frac{1}{2}\cos(7+3x)\right)\end{aligned}$$

Mathematica [C] time = 0.0933488, size = 39, normalized size = 2.6

$$\frac{1}{3}i\log\left(\sqrt{7-\cos(2(3x+7))}+i\sqrt{2}\cos(3x+7)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[7 + 3*x]/Sqrt[3 + Sin[7 + 3*x]^2],x]

[Out] (I/3)*Log[I*Sqrt[2]*Cos[7 + 3*x] + Sqrt[7 - Cos[2*(7 + 3*x)]]]
```

Maple [B] time = 0.797, size = 57, normalized size = 3.8

$$-\frac{1}{6\cos\left(7+3x\right)}\sqrt{\left(3+\left(\sin\left(7+3x\right)\right)^{2}\right)\left(\cos\left(7+3x\right)\right)^{2}}\arcsin\left(-1+\frac{\left(\cos\left(7+3x\right)\right)^{2}}{2}\right)\frac{1}{\sqrt{3+\left(\sin\left(7+3x\right)\right)^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(7+3*x)/(3+sin(7+3*x)^2)^(1/2),x)
```

```
[Out] -1/6*((3+sin(7+3*x)^2)*cos(7+3*x)^2)^(1/2)*arcsin(-1+1/2*cos(7+3*x)^2)/cos(
7+3*x)/(3+sin(7+3*x)^2)^(1/2)
```

Maxima [A] time = 1.41911, size = 15, normalized size = 1.

$$-\frac{1}{3}\arcsin\left(\frac{1}{2}\cos\left(3x+7\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(7+3*x)/(3+sin(7+3*x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*arcsin(1/2*cos(3*x + 7))
```

Fricas [B] time = 1.81811, size = 251, normalized size = 16.73

$$\frac{1}{6}\arctan\left(-\frac{4\cos\left(3x+7\right)\sin\left(3x+7\right)-\left(\cos\left(3x+7\right)^{3}-2\cos\left(3x+7\right)\right)\sqrt{-\cos\left(3x+7\right)^{2}+4}}{\cos\left(3x+7\right)^{4}-8\cos\left(3x+7\right)^{2}+4}\right)-\frac{1}{6}\arctan\left(\frac{\sin\left(3x\right.}{\cos\left(3x\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(7+3*x)/(3+sin(7+3*x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*arctan(-(4*cos(3*x + 7)*sin(3*x + 7) - (cos(3*x + 7)^3 - 2*cos(3*x + 7)
)*sqrt(-cos(3*x + 7)^2 + 4))/(cos(3*x + 7)^4 - 8*cos(3*x + 7)^2 + 4)) - 1/6
*arctan(sin(3*x + 7)/cos(3*x + 7))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(7+3*x)/(3+sin(7+3*x)**2)**(1/2),x)
```

```
[Out] Timed out
```

<u>**Giac**</u> **[A]** time = 1.30528, size = 15, normalized size = 1.

$$-\frac{1}{3} \arcsin\left(\frac{1}{2} \cos\left(3x + 7\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(7+3*x)/(3+sin(7+3*x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*arcsin(1/2*cos(3*x + 7))
```

3.116 $\int \left(a - a\sin^2(x)\right)^{5/2}\,dx$

Optimal. Leaf size=53

$$\frac{8}{15}a^2\tan(x)\sqrt{a\cos^2(x)}+\frac{1}{5}\tan(x)\left(a\cos^2(x)\right)^{5/2}+\frac{4}{15}a\tan(x)\left(a\cos^2(x)\right)^{3/2}$$

```
[Out] (8*a^2*Sqrt[a*Cos[x]^2]*Tan[x])/15 + (4*a*(a*Cos[x]^2)^(3/2)*Tan[x])/15 + (
(a*Cos[x]^2)^(5/2)*Tan[x])/5
```

Rubi [A] time = 0.0540351, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3176, 3203, 3207, 2637}

$$\frac{8}{15}a^2\tan(x)\sqrt{a\cos^2(x)}+\frac{1}{5}\tan(x)\left(a\cos^2(x)\right)^{5/2}+\frac{4}{15}a\tan(x)\left(a\cos^2(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[x]^2)^(5/2),x]
```

```
[Out] (8*a^2*Sqrt[a*Cos[x]^2]*Tan[x])/15 + (4*a*(a*Cos[x]^2)^(3/2)*Tan[x])/15 + (
(a*Cos[x]^2)^(5/2)*Tan[x])/5
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3203

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(Cot[e + f*x
]*(b*Sin[e + f*x]^2)^p)/(2*f*p), x] + Dist[(b*(2*p - 1))/(2*p), Int[(b*Sin[
e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] && !IntegerQ[p] && GtQ[
p, 1]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \left(a-a\sin^2(x)\right)^{5/2}\,dx &= \int \left(a\cos^2(x)\right)^{5/2}\,dx\\
&= \frac{1}{5}\left(a\cos^2(x)\right)^{5/2}\tan(x)+\frac{1}{5}(4a)\int \left(a\cos^2(x)\right)^{3/2}\,dx\\
&= \frac{4}{15}a\left(a\cos^2(x)\right)^{3/2}\tan(x)+\frac{1}{5}\left(a\cos^2(x)\right)^{5/2}\tan(x)+\frac{1}{15}\left(8a^2\right)\int \sqrt{a\cos^2(x)}\,dx\\
&= \frac{4}{15}a\left(a\cos^2(x)\right)^{3/2}\tan(x)+\frac{1}{5}\left(a\cos^2(x)\right)^{5/2}\tan(x)+\frac{1}{15}\left(8a^2\sqrt{a\cos^2(x)}\sec(x)\right)\int \cos(x)\,d\\
&= \frac{8}{15}a^2\sqrt{a\cos^2(x)}\tan(x)+\frac{4}{15}a\left(a\cos^2(x)\right)^{3/2}\tan(x)+\frac{1}{5}\left(a\cos^2(x)\right)^{5/2}\tan(x)\end{aligned}$$

Mathematica [A] time = 0.0215734, size = 36, normalized size = 0.68

$$\frac{1}{240}a^2(150\sin(x)+25\sin(3x)+3\sin(5x))\sec(x)\sqrt{a\cos^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sin[x]^2)^(5/2),x]
```

```
[Out] (a^2*Sqrt[a*Cos[x]^2]*Sec[x]*(150*Sin[x] + 25*Sin[3*x] + 3*Sin[5*x]))/240
```

Maple [A] time = 0.677, size = 32, normalized size = 0.6

$$\frac{a^3\cos(x)\sin(x)\left(3\,(\cos(x))^4+4\,(\cos(x))^2+8\right)}{15}\frac{1}{\sqrt{a\,(\cos(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(x)^2)^(5/2),x)
```

```
[Out] 1/15*a^3*cos(x)*sin(x)*(3*cos(x)^4+4*cos(x)^2+8)/(a*cos(x)^2)^(1/2)
```

Maxima [A] time = 1.6172, size = 42, normalized size = 0.79

$$\frac{1}{240}\left(3\,a^2\sin(5\,x)+25\,a^2\sin(3\,x)+150\,a^2\sin(x)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/240*(3*a^2*sin(5*x) + 25*a^2*sin(3*x) + 150*a^2*sin(x))*sqrt(a)
```

Fricas [A] time = 1.67462, size = 107, normalized size = 2.02

$$\frac{\left(3\,a^2\cos(x)^4+4\,a^2\cos(x)^2+8\,a^2\right)\sqrt{a\cos(x)^2}\sin(x)}{15\,\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*a^2*cos(x)^4 + 4*a^2*cos(x)^2 + 8*a^2)*sqrt(a*cos(x)^2)*sin(x)/cos(
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.26189, size = 113, normalized size = 2.13

$$-\frac{2\left(15\,a^{\frac{5}{2}}\left(\frac{1}{\tan\left(\frac{1}{2}\,x\right)}+\tan\left(\frac{1}{2}\,x\right)\right)^{4}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,x\right)^{4}-1\right)-40\,a^{\frac{5}{2}}\left(\frac{1}{\tan\left(\frac{1}{2}\,x\right)}+\tan\left(\frac{1}{2}\,x\right)\right)^{2}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,x\right)^{4}-1\right)+48\,a^{\frac{5}{2}}\mathrm{sgn}\left(\mathrm{ta}\right.\right.}{15\left(\frac{1}{\tan\left(\frac{1}{2}\,x\right)}+\tan\left(\frac{1}{2}\,x\right)\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -2/15*(15*a^(5/2)*(1/tan(1/2*x) + tan(1/2*x))^4*sgn(tan(1/2*x)^4 - 1) - 40*
a^(5/2)*(1/tan(1/2*x) + tan(1/2*x))^2*sgn(tan(1/2*x)^4 - 1) + 48*a^(5/2)*sg
n(tan(1/2*x)^4 - 1))/(1/tan(1/2*x) + tan(1/2*x))^5
```

3.117 $\int \left(a-a\sin^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=34

$$\frac{1}{3}\tan(x)\left(a\cos^2(x)\right)^{3/2}+\frac{2}{3}a\tan(x)\sqrt{a\cos^2(x)}$$

```
[Out] (2*a*Sqrt[a*Cos[x]^2]*Tan[x])/3 + ((a*Cos[x]^2)^(3/2)*Tan[x])/3
```

Rubi [A] time = 0.0355551, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3176, 3203, 3207, 2637}

$$\frac{1}{3}\tan(x)\left(a\cos^2(x)\right)^{3/2}+\frac{2}{3}a\tan(x)\sqrt{a\cos^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[x]^2)^(3/2),x]
```

```
[Out] (2*a*Sqrt[a*Cos[x]^2]*Tan[x])/3 + ((a*Cos[x]^2)^(3/2)*Tan[x])/3
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3203

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(Cot[e + f*x
]*(b*Sin[e + f*x]^2)^p)/(2*f*p), x] + Dist[(b*(2*p - 1))/(2*p), Int[(b*Sin[
e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && GtQ[
p, 1]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \left(a-a\sin^2(x)\right)^{3/2}\,dx &= \int \left(a\cos^2(x)\right)^{3/2}\,dx\\ &= \frac{1}{3}\left(a\cos^2(x)\right)^{3/2}\tan(x)+\frac{1}{3}(2a)\int \sqrt{a\cos^2(x)}\,dx\\ &= \frac{1}{3}\left(a\cos^2(x)\right)^{3/2}\tan(x)+\frac{1}{3}\left(2a\sqrt{a\cos^2(x)}\sec(x)\right)\int \cos(x)\,dx\\ &= \frac{2}{3}a\sqrt{a\cos^2(x)}\tan(x)+\frac{1}{3}\left(a\cos^2(x)\right)^{3/2}\tan(x)\end{aligned}$$

Mathematica [A] time = 0.0094959, size = 26, normalized size = 0.76

$$\frac{1}{12}a(9\sin(x)+\sin(3x))\sec(x)\sqrt{a\cos^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sin[x]^2)^(3/2),x]
```

```
[Out] (a*Sqrt[a*Cos[x]^2]*Sec[x]*(9*Sin[x] + Sin[3*x]))/12
```

Maple [A] time = 0.541, size = 24, normalized size = 0.7

$$\frac{a^2\cos(x)\sin(x)\left((\cos(x))^2+2\right)}{3}\frac{1}{\sqrt{a(\cos(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(x)^2)^(3/2),x)
```

```
[Out] 1/3*a^2*cos(x)*sin(x)*(cos(x)^2+2)/(a*cos(x)^2)^(1/2)
```

Maxima [A] time = 1.61046, size = 23, normalized size = 0.68

$$\frac{1}{12}\left(a\sin(3x)+9a\sin(x)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*(a*sin(3*x) + 9*a*sin(x))*sqrt(a)
```

Fricas [A] time = 1.66836, size = 74, normalized size = 2.18

$$\frac{\left(a\cos(x)^2+2a\right)\sqrt{a\cos(x)^2}\sin(x)}{3\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(a*cos(x)^2 + 2*a)*sqrt(a*cos(x)^2)*sin(x)/cos(x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.21744, size = 77, normalized size = 2.26

$$-\frac{2\left(3\,a^{\frac{3}{2}}\left(\frac{1}{\tan\left(\frac{1}{2}\,x\right)}+\tan\left(\frac{1}{2}\,x\right)\right)^{2}\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,x\right)^{4}-1\right)-4\,a^{\frac{3}{2}}\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,x\right)^{4}-1\right)\right)}{3\left(\frac{1}{\tan\left(\frac{1}{2}\,x\right)}+\tan\left(\frac{1}{2}\,x\right)\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -2/3*(3*a^(3/2)*(1/tan(1/2*x) + tan(1/2*x))^2*sgn(tan(1/2*x)^4 - 1) - 4*a^(
3/2)*sgn(tan(1/2*x)^4 - 1))/(1/tan(1/2*x) + tan(1/2*x))^3
```

3.118 $\int \sqrt{a-a\sin^2(x)}\,dx$

Optimal. Leaf size=13

$$\tan(x)\sqrt{a\cos^2(x)}$$

```
[Out] Sqrt[a*Cos[x]^2]*Tan[x]
```

Rubi [A] time = 0.0259045, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3176, 3207, 2637}

$$\tan(x)\sqrt{a\cos^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Sin[x]^2],x]

[Out] Sqrt[a*Cos[x]^2]*Tan[x]
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a-a\sin^2(x)}\,dx &= \int \sqrt{a\cos^2(x)}\,dx\\ &= \left(\sqrt{a\cos^2(x)}\sec(x)\right)\int \cos(x)\,dx\\ &= \sqrt{a\cos^2(x)}\tan(x)\end{aligned}$$

Mathematica [A] time = 0.0042642, size = 13, normalized size = 1.

$$\tan(x)\sqrt{a\cos^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Sin[x]^2],x]
```

```
[Out] Sqrt[a*Cos[x]^2]*Tan[x]
```

Maple [A] time = 0.392, size = 15, normalized size = 1.2

$$a\cos(x)\sin(x)\frac{1}{\sqrt{a(\cos(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(x)^2)^(1/2),x)
```

```
[Out] a*cos(x)*sin(x)/(a*cos(x)^2)^(1/2)
```

Maxima [A] time = 1.59161, size = 8, normalized size = 0.62

$$\sqrt{a}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(a)*sin(x)
```

Fricas [A] time = 1.62571, size = 43, normalized size = 3.31

$$\frac{\sqrt{a\cos(x)^2}\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(a*cos(x)^2)*sin(x)/cos(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-a\sin^2(x)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-a*sin(x)**2 + a), x)
```

Giac [B] time = 1.14487, size = 36, normalized size = 2.77

$$-\frac{2\sqrt{a}\operatorname{sgn}\left(\tan\left(\frac{1}{2}x\right)^4-1\right)}{\frac{1}{\tan\left(\frac{1}{2}x\right)}+\tan\left(\frac{1}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(a)*sgn(tan(1/2*x)^4 - 1)/(1/tan(1/2*x) + tan(1/2*x))
```

3.119 $\int \frac{1}{\sqrt{a-a\sin^2(x)}}\,dx$

Optimal. Leaf size=16

$$\frac{\cos(x)\tanh^{-1}(\sin(x))}{\sqrt{a\cos^2(x)}}$$

```
[Out] (ArcTanh[Sin[x]]*Cos[x])/Sqrt[a*Cos[x]^2]
```

Rubi [A] time = 0.0306739, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3176, 3207, 3770}

$$\frac{\cos(x)\tanh^{-1}(\sin(x))}{\sqrt{a\cos^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a - a*Sin[x]^2],x]
```

```
[Out] (ArcTanh[Sin[x]]*Cos[x])/Sqrt[a*Cos[x]^2]
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[ActivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p] && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a-a\sin^2(x)}}\,dx &= \int \frac{1}{\sqrt{a\cos^2(x)}}\,dx\\ &= \frac{\cos(x)\int \sec(x)\,dx}{\sqrt{a\cos^2(x)}}\\ &= \frac{\tanh^{-1}(\sin(x))\cos(x)}{\sqrt{a\cos^2(x)}}\end{aligned}$$

Mathematica [B] time = 0.0194808, size = 46, normalized size = 2.88

$$\frac{\cos(x)\left(\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)\right)}{\sqrt{a\cos^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a - a*Sin[x]^2],x]
```

```
[Out] (Cos[x]*(-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]]))/Sqrt[a*Cos[
x]^2]
```

Maple [C] time = 0.11, size = 20, normalized size = 1.3

$$\frac{\cos(x)\,\mathrm{InverseJacobiAM}(x,1)}{\mathrm{csgn}(\cos(x))}\frac{1}{\sqrt{a(\cos(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sin(x)^2)^(1/2),x)
```

```
[Out] 1/(a*cos(x)^2)^(1/2)/csgn(cos(x))*cos(x)*InverseJacobiAM(x,1)
```

Maxima [B] time = 1.58033, size = 51, normalized size = 3.19

$$\frac{\log\left(\cos(x)^2+\sin(x)^2+2\sin(x)+1\right)-\log\left(\cos(x)^2+\sin(x)^2-2\sin(x)+1\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - log(cos(x)^2 + sin(x)^2 - 2*
sin(x) + 1))/sqrt(a)
```

Fricas [B] time = 1.65344, size = 182, normalized size = 11.38

$$\left[-\frac{\sqrt{a\cos(x)^2}\log\left(-\frac{\sin(x)-1}{\sin(x)+1}\right)}{2\,a\cos(x)},\,-\frac{\sqrt{-a}\arctan\left(\frac{\sqrt{a\cos(x)^2}\sqrt{-a}\sin(x)}{a\cos(x)}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(a*cos(x)^2)*log(-(sin(x) - 1)/(sin(x) + 1))/(a*cos(x)), -sqrt(-a
)*arctan(sqrt(a*cos(x)^2)*sqrt(-a)*sin(x)/(a*cos(x)))/a]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-a\sin^2(x)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-a*sin(x)**2 + a), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-a\sin(x)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-a*sin(x)^2 + a), x)
```

3.120 $\int \frac{1}{\left(a-a\sin^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=42

$$\frac{\tan(x)}{2a\sqrt{a\cos^2(x)}}+\frac{\cos(x)\tanh^{-1}(\sin(x))}{2a\sqrt{a\cos^2(x)}}$$

```
[Out] (ArcTanh[Sin[x]]*Cos[x])/(2*a*Sqrt[a*Cos[x]^2]) + Tan[x]/(2*a*Sqrt[a*Cos[x]
^2])
```

Rubi [A] time = 0.0374599, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3176, 3204, 3207, 3770}

$$\frac{\tan(x)}{2a\sqrt{a\cos^2(x)}}+\frac{\cos(x)\tanh^{-1}(\sin(x))}{2a\sqrt{a\cos^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[x]^2)^(-3/2),x]
```

```
[Out] (ArcTanh[Sin[x]]*Cos[x])/(2*a*Sqrt[a*Cos[x]^2]) + Tan[x]/(2*a*Sqrt[a*Cos[x]
^2])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3204

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(Cot[e + f*x]
*(b*Sin[e + f*x]^2)^(p + 1))/(b*f*(2*p + 1)), x] + Dist[(2*(p + 1))/(b*(2*p
 + 1)), Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&  !
IntegerQ[p] && LtQ[p, -1]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a - a\sin^2(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(a\cos^2(x)\right)^{3/2}}\,dx$$
$$= \frac{\tan(x)}{2a\sqrt{a\cos^2(x)}} + \frac{\int \frac{1}{\sqrt{a\cos^2(x)}}\,dx}{2a}$$
$$= \frac{\tan(x)}{2a\sqrt{a\cos^2(x)}} + \frac{\cos(x)\int \sec(x)\,dx}{2a\sqrt{a\cos^2(x)}}$$
$$= \frac{\tanh^{-1}(\sin(x))\cos(x)}{2a\sqrt{a\cos^2(x)}} + \frac{\tan(x)}{2a\sqrt{a\cos^2(x)}}$$

Mathematica [B] time = 0.0651455, size = 91, normalized size = 2.17

$$-\frac{\cos(x)\left(-2\sin(x) + \log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) + \cos(2x)\left(\log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) - \log\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right)\right) - \log\left(\sin\left(\right.\right.\right.}{4\left(a\cos^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sin[x]^2)^(-3/2),x]
```

```
[Out] -(Cos[x]*(Log[Cos[x/2] - Sin[x/2]] + Cos[2*x]*(Log[Cos[x/2] - Sin[x/2]] - L
og[Cos[x/2] + Sin[x/2]]) - Log[Cos[x/2] + Sin[x/2]] - 2*Sin[x]))/(4*(a*Cos[
x]^2)^(3/2))
```

Maple [B] time = 0.994, size = 70, normalized size = 1.7

$$\frac{1}{2\sin(x)\cos(x)}\sqrt{a(\sin(x))^2}\left(\ln\left(2\frac{\sqrt{a}\sqrt{a(\sin(x))^2}+a}{\cos(x)}\right)(\cos(x))^2 a + \sqrt{a}\sqrt{a(\sin(x))^2}\right)a^{-\frac{5}{2}}\frac{1}{\sqrt{a(\cos(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sin(x)^2)^(3/2),x)
```

```
[Out] 1/2/a^(5/2)/cos(x)*(a*sin(x)^2)^(1/2)*(ln(2/cos(x)*(a^(1/2)*(a*sin(x)^2)^(1
/2)+a))*cos(x)^2*a+a^(1/2)*(a*sin(x)^2)^(1/2))/sin(x)/(a*cos(x)^2)^(1/2)
```

Maxima [B] time = 1.6278, size = 410, normalized size = 9.76

$$\frac{4\left(\sin(3x) - \sin(x)\right)\cos(4x) + \left(2\left(2\cos(2x) + 1\right)\cos(4x) + \cos(4x)^2 + 4\cos(2x)^2 + \sin(4x)^2 + 4\sin(4x)\sin(2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(sin(3*x) - sin(x))*cos(4*x) + (2*(2*cos(2*x) + 1)*cos(4*x) + cos(4*
x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*x)*sin(2*x) + 4*sin(2*x)^2 + 4*c
os(2*x) + 1)*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - (2*(2*cos(2*x) + 1)*
```

```
cos(4*x) + cos(4*x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*x)*sin(2*x) + 4
*sin(2*x)^2 + 4*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1) - 4*(
cos(3*x) - cos(x))*sin(4*x) + 4*(2*cos(2*x) + 1)*sin(3*x) - 8*cos(3*x)*sin(
2*x) + 8*cos(x)*sin(2*x) - 8*cos(2*x)*sin(x) - 4*sin(x))/((a*cos(4*x)^2 + 4
*a*cos(2*x)^2 + a*sin(4*x)^2 + 4*a*sin(4*x)*sin(2*x) + 4*a*sin(2*x)^2 + 2*(
2*a*cos(2*x) + a)*cos(4*x) + 4*a*cos(2*x) + a)*sqrt(a))
```

Fricas [A] time = 1.75947, size = 124, normalized size = 2.95

$$-\frac{\sqrt{a\cos(x)^2}\left(\cos(x)^2\log\left(-\frac{\sin(x)-1}{\sin(x)+1}\right)-2\sin(x)\right)}{4a^2\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(a*cos(x)^2)*(cos(x)^2*log(-(sin(x) - 1)/(sin(x) + 1)) - 2*sin(x))
/(a^2*cos(x)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23198, size = 59, normalized size = 1.4

$$-\frac{\sqrt{a}\log\left(\left|-\sqrt{a}\tan(x)+\sqrt{a\tan(x)^2+a}\right|\right)-\sqrt{a\tan(x)^2+a}\,a\tan(x)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(a)*log(abs(-sqrt(a)*tan(x) + sqrt(a*tan(x)^2 + a))) - sqrt(a*tan
(x)^2 + a)*tan(x))/a^2
```

3.121 $\int \frac{1}{\left(a-a\sin^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=61

$$\frac{3\tan(x)}{8a^2\sqrt{a\cos^2(x)}}+\frac{3\cos(x)\tanh^{-1}(\sin(x))}{8a^2\sqrt{a\cos^2(x)}}+\frac{\tan(x)}{4a\left(a\cos^2(x)\right)^{3/2}}$$

```
[Out] (3*ArcTanh[Sin[x]]*Cos[x])/(8*a^2*Sqrt[a*Cos[x]^2]) + Tan[x]/(4*a*(a*Cos[x]
^2)^(3/2)) + (3*Tan[x])/(8*a^2*Sqrt[a*Cos[x]^2])
```

Rubi [A] time = 0.0498615, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3176, 3204, 3207, 3770}

$$\frac{3\tan(x)}{8a^2\sqrt{a\cos^2(x)}}+\frac{3\cos(x)\tanh^{-1}(\sin(x))}{8a^2\sqrt{a\cos^2(x)}}+\frac{\tan(x)}{4a\left(a\cos^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[x]^2)^(-5/2),x]
```

```
[Out] (3*ArcTanh[Sin[x]]*Cos[x])/(8*a^2*Sqrt[a*Cos[x]^2]) + Tan[x]/(4*a*(a*Cos[x]
^2)^(3/2)) + (3*Tan[x])/(8*a^2*Sqrt[a*Cos[x]^2])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3204

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(Cot[e + f*x]
*(b*Sin[e + f*x]^2)^(p + 1))/(b*f*(2*p + 1)), x] + Dist[(2*(p + 1))/(b*(2*p
 + 1)), Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&  !
IntegerQ[p] && LtQ[p, -1]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a-a\sin^2(x)\right)^{5/2}}\,dx=\int \frac{1}{\left(a\cos^2(x)\right)^{5/2}}\,dx$$
$$=\frac{\tan(x)}{4a\left(a\cos^2(x)\right)^{3/2}}+\frac{3\int \frac{1}{\left(a\cos^2(x)\right)^{3/2}}\,dx}{4a}$$
$$=\frac{\tan(x)}{4a\left(a\cos^2(x)\right)^{3/2}}+\frac{3\tan(x)}{8a^2\sqrt{a\cos^2(x)}}+\frac{3\int \frac{1}{\sqrt{a\cos^2(x)}}\,dx}{8a^2}$$
$$=\frac{\tan(x)}{4a\left(a\cos^2(x)\right)^{3/2}}+\frac{3\tan(x)}{8a^2\sqrt{a\cos^2(x)}}+\frac{(3\cos(x))\int \sec(x)\,dx}{8a^2\sqrt{a\cos^2(x)}}$$
$$=\frac{3\tanh^{-1}(\sin(x))\cos(x)}{8a^2\sqrt{a\cos^2(x)}}+\frac{\tan(x)}{4a\left(a\cos^2(x)\right)^{3/2}}+\frac{3\tan(x)}{8a^2\sqrt{a\cos^2(x)}}$$

Mathematica [A] time = 0.152077, size = 72, normalized size = 1.18

$$\frac{\cos^5(x)\left(\frac{1}{2}(11\sin(x)+3\sin(3x))\sec^4(x)-6\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+6\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\right)}{16\left(a\cos^2(x)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sin[x]^2)^(-5/2),x]
```

```
[Out] (Cos[x]^5*(-6*Log[Cos[x/2] - Sin[x/2]] + 6*Log[Cos[x/2] + Sin[x/2]] + (Sec[
x]^4*(11*Sin[x] + 3*Sin[3*x]))/2))/(16*(a*Cos[x]^2)^(5/2))
```

Maple [A] time = 1.084, size = 89, normalized size = 1.5

$$\frac{1}{8\,(\cos(x))^3\sin(x)}\sqrt{a\,(\sin(x))^2}\left(3\,\ln\left(2\,\frac{\sqrt{a}\sqrt{a\,(\sin(x))^2}+a}{\cos(x)}\right)(\cos(x))^4\,a+3\,\sqrt{a\,(\sin(x))^2}\,(\cos(x))^2\,\sqrt{a}+2\,\sqrt{a}\sqrt{a\,(\sin(x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sin(x)^2)^(5/2),x)
```

```
[Out] 1/8/a^(7/2)/cos(x)^3*(a*sin(x)^2)^(1/2)*(3*ln(2/cos(x)*(a^(1/2)*(a*sin(x)^2
)^(1/2)+a))*cos(x)^4*a+3*(a*sin(x)^2)^(1/2)*cos(x)^2*a^(1/2)+2*a^(1/2)*(a*s
in(x)^2)^(1/2))/sin(x)/(a*cos(x)^2)^(1/2)
```

Maxima [B] time = 2.56221, size = 1260, normalized size = 20.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/16*(4*(3*sin(7*x) + 11*sin(5*x) - 11*sin(3*x) - 3*sin(x))*cos(8*x) - 24*(
2*sin(6*x) + 3*sin(4*x) + 2*sin(2*x))*cos(7*x) + 16*(11*sin(5*x) - 11*sin(3
*x) - 3*sin(x))*cos(6*x) - 88*(3*sin(4*x) + 2*sin(2*x))*cos(5*x) - 24*(11*s
in(3*x) + 3*sin(x))*cos(4*x) + 3*(2*(4*cos(6*x) + 6*cos(4*x) + 4*cos(2*x) +
 1)*cos(8*x) + cos(8*x)^2 + 8*(6*cos(4*x) + 4*cos(2*x) + 1)*cos(6*x) + 16*c
os(6*x)^2 + 12*(4*cos(2*x) + 1)*cos(4*x) + 36*cos(4*x)^2 + 16*cos(2*x)^2 +
4*(2*sin(6*x) + 3*sin(4*x) + 2*sin(2*x))*sin(8*x) + sin(8*x)^2 + 16*(3*sin(
4*x) + 2*sin(2*x))*sin(6*x) + 16*sin(6*x)^2 + 36*sin(4*x)^2 + 48*sin(4*x)*s
in(2*x) + 16*sin(2*x)^2 + 8*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 + 2*sin(x
) + 1) - 3*(2*(4*cos(6*x) + 6*cos(4*x) + 4*cos(2*x) + 1)*cos(8*x) + cos(8*x
)^2 + 8*(6*cos(4*x) + 4*cos(2*x) + 1)*cos(6*x) + 16*cos(6*x)^2 + 12*(4*cos(
2*x) + 1)*cos(4*x) + 36*cos(4*x)^2 + 16*cos(2*x)^2 + 4*(2*sin(6*x) + 3*sin(
4*x) + 2*sin(2*x))*sin(8*x) + sin(8*x)^2 + 16*(3*sin(4*x) + 2*sin(2*x))*sin
(6*x) + 16*sin(6*x)^2 + 36*sin(4*x)^2 + 48*sin(4*x)*sin(2*x) + 16*sin(2*x)^
2 + 8*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1) - 4*(3*cos(7*x)
 + 11*cos(5*x) - 11*cos(3*x) - 3*cos(x))*sin(8*x) + 12*(4*cos(6*x) + 6*cos(
4*x) + 4*cos(2*x) + 1)*sin(7*x) - 16*(11*cos(5*x) - 11*cos(3*x) - 3*cos(x))
*sin(6*x) + 44*(6*cos(4*x) + 4*cos(2*x) + 1)*sin(5*x) + 24*(11*cos(3*x) + 3
*cos(x))*sin(4*x) - 44*(4*cos(2*x) + 1)*sin(3*x) + 176*cos(3*x)*sin(2*x) +
48*cos(x)*sin(2*x) - 48*cos(2*x)*sin(x) - 12*sin(x))/((a^2*cos(8*x)^2 + 16*
a^2*cos(6*x)^2 + 36*a^2*cos(4*x)^2 + 16*a^2*cos(2*x)^2 + a^2*sin(8*x)^2 + 1
6*a^2*sin(6*x)^2 + 36*a^2*sin(4*x)^2 + 48*a^2*sin(4*x)*sin(2*x) + 16*a^2*si
n(2*x)^2 + 8*a^2*cos(2*x) + a^2 + 2*(4*a^2*cos(6*x) + 6*a^2*cos(4*x) + 4*a^
2*cos(2*x) + a^2)*cos(8*x) + 8*(6*a^2*cos(4*x) + 4*a^2*cos(2*x) + a^2)*cos(
6*x) + 12*(4*a^2*cos(2*x) + a^2)*cos(4*x) + 4*(2*a^2*sin(6*x) + 3*a^2*sin(4
*x) + 2*a^2*sin(2*x))*sin(8*x) + 16*(3*a^2*sin(4*x) + 2*a^2*sin(2*x))*sin(6
*x))*sqrt(a))
```

Fricas [A] time = 1.76285, size = 151, normalized size = 2.48

$$-\frac{\left(3\cos\left(x\right)^{4}\log\left(-\frac{\sin\left(x\right)-1}{\sin\left(x\right)+1}\right)-2\left(3\cos\left(x\right)^{2}+2\right)\sin\left(x\right)\right)\sqrt{a\cos\left(x\right)^{2}}}{16\,a^{3}\cos\left(x\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/16*(3*cos(x)^4*log(-(sin(x) - 1)/(sin(x) + 1)) - 2*(3*cos(x)^2 + 2)*sin(
x))*sqrt(a*cos(x)^2)/(a^3*cos(x)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.28554, size = 174, normalized size = 2.85

$$-\frac{3\,\log\left(\left|\frac{1}{\tan\left(\frac{1}{2}\,x\right)}+\tan\left(\frac{1}{2}\,x\right)+2\right|\right)}{16\,a^{\frac{5}{2}}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,x\right)^{4}-1\right)}+\frac{3\,\log\left(\left|\frac{1}{\tan\left(\frac{1}{2}\,x\right)}+\tan\left(\frac{1}{2}\,x\right)-2\right|\right)}{16\,a^{\frac{5}{2}}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,x\right)^{4}-1\right)}-\frac{5\,\sqrt{a}\left(\frac{1}{\tan\left(\frac{1}{2}\,x\right)}+\tan\left(\frac{1}{2}\,x\right)\right)^{3}-12\,\sqrt{a}\left(\frac{1}{\tan\left(\frac{1}{2}\,x\right)}+\cdots\right.}{4\left(\left(\frac{1}{\tan\left(\frac{1}{2}\,x\right)}+\tan\left(\frac{1}{2}\,x\right)\right)^{2}-4\right)^{2}a^{3}\mathrm{sgn}\left(\tan\left(\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -3/16*log(abs(1/tan(1/2*x) + tan(1/2*x) + 2))/(a^(5/2)*sgn(tan(1/2*x)^4 - 1
)) + 3/16*log(abs(1/tan(1/2*x) + tan(1/2*x) - 2))/(a^(5/2)*sgn(tan(1/2*x)^4
 - 1)) - 1/4*(5*sqrt(a)*(1/tan(1/2*x) + tan(1/2*x))^3 - 12*sqrt(a)*(1/tan(1
/2*x) + tan(1/2*x)))/(((1/tan(1/2*x) + tan(1/2*x))^2 - 4)^2*a^3*sgn(tan(1/2
*x)^4 - 1))
```

3.122 $\int \sin^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=125

$$\frac{(a-3b)(a+b)\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{8b^{3/2}f}-\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}{4bf}+\frac{(a-3b)\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{8bf}$$

```
[Out] ((a - 3*b)*(a + b)*ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x
]^2]])/(8*b^(3/2)*f) + ((a - 3*b)*Cos[e + f*x]*Sqrt[a + b - b*Cos[e + f*x]^
2])/(8*b*f) - (Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2)^(3/2))/(4*b*f)
```

Rubi [A] time = 0.128017, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3186, 388, 195, 217, 203}

$$\frac{(a-3b)(a+b)\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{8b^{3/2}f}-\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}{4bf}+\frac{(a-3b)\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{8bf}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] ((a - 3*b)*(a + b)*ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x
]^2]])/(8*b^(3/2)*f) + ((a - 3*b)*Cos[e + f*x]*Sqrt[a + b - b*Cos[e + f*x]^
2])/(8*b*f) - (Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2)^(3/2))/(4*b*f)
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sin^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = -\frac{\text{Subst}\left(\int \left(1-x^2\right)\sqrt{a+b-bx^2}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^{3/2}}{4bf} + \frac{(a-3b)\,\text{Subst}\left(\int \sqrt{a+b-bx^2}\,dx, x\right.}{4bf}$$

$$= \frac{(a-3b)\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{8bf} - \frac{\cos(e+fx)\left(a+b-b\cos^2(e+f\right.}{4bf}$$

$$= \frac{(a-3b)\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{8bf} - \frac{\cos(e+fx)\left(a+b-b\cos^2(e+f\right.}{4bf}$$

$$= \frac{(a-3b)(a+b)\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{8b^{3/2}f} + \frac{(a-3b)\cos(e+fx)\sqrt{a+b-b\cos^2(e\right.}{8bf}$$

Mathematica [A] time = 0.420037, size = 119, normalized size = 0.95

$$\frac{\frac{\cos(e+fx)\sqrt{2a-b\cos(2(e+fx))+b}(-a+b\cos(2(e+fx))-4b)}{\sqrt{2}b} + \frac{(a+b)(3b-a)\log\left(\sqrt{2a-b\cos(2(e+fx))+b}+\sqrt{2}\sqrt{-b}\cos(e+fx)\right)}{(-b)^{3/2}}}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] ((Cos[e + f*x]*Sqrt[2*a + b - b*Cos[2*(e + f*x)]]*(-a - 4*b + b*Cos[2*(e +
f*x)]))/(Sqrt[2]*b) + ((a + b)*(-a + 3*b)*Log[Sqrt[2]*Sqrt[-b]*Cos[e + f*x]
 + Sqrt[2*a + b - b*Cos[2*(e + f*x)]]])/(-b)^(3/2))/(8*f)
```

Maple [B] time = 1.677, size = 311, normalized size = 2.5

$$-\frac{1}{16f\cos(fx+e)}\sqrt{\left(\cos(fx+e)\right)^2\left(a+b\left(\sin(fx+e)\right)^2\right)}\left(-4\sqrt{-b\left(\cos(fx+e)\right)^4+(a+b)\left(\cos(fx+e)\right)^2}b^{5/2}\left(\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/16*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*(-4*(-b*cos(f*x+e)^4+(a+b)*co
s(f*x+e)^2)^(1/2)*b^(5/2)*cos(f*x+e)^2+10*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)
^2)^(1/2)*b^(5/2)+2*a*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b^(3/2)+ar
ctan(1/2*(-2*b*cos(f*x+e)^2+a+b)/b^(1/2)/(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^
2)^(1/2))*a^2*b-2*a*arctan(1/2*(-2*b*cos(f*x+e)^2+a+b)/b^(1/2)/(-b*cos(f*x+
e)^4+(a+b)*cos(f*x+e)^2)^(1/2))*b^2-3*b^3*arctan(1/2*(-2*b*cos(f*x+e)^2+a+b
)/b^(1/2)/(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)))/b^(5/2)/cos(f*x+e)/(
```

```
a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 4.63161, size = 1214, normalized size = 9.71

$$\left[\frac{\left(a^2-2\,ab-3\,b^2\right)\sqrt{-b}\log\left(128\,b^4\cos\left(fx+e\right)^8-256\left(ab^3+b^4\right)\cos\left(fx+e\right)^6+160\left(a^2b^2+2\,ab^3+b^4\right)\cos\left(fx+e\right)\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/64*((a^2 - 2*a*b - 3*b^2)*sqrt(-b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b
^3 + b^4)*cos(f*x + e)^6 + 160*(a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 + a
^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4 - 32*(a^3*b + 3*a^2*b^2 + 3*a*b^3
+ b^4)*cos(f*x + e)^2 + 8*(16*b^3*cos(f*x + e)^7 - 24*(a*b^2 + b^3)*cos(f*x
 + e)^5 + 10*(a^2*b + 2*a*b^2 + b^3)*cos(f*x + e)^3 - (a^3 + 3*a^2*b + 3*a*
b^2 + b^3)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)) + 8*(2*b
^2*cos(f*x + e)^3 - (a*b + 5*b^2)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a
+ b))/(b^2*f), -1/32*((a^2 - 2*a*b - 3*b^2)*sqrt(b)*arctan(1/4*(8*b^2*cos(f
*x + e)^4 - 8*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(-b*cos(f
*x + e)^2 + a + b)*sqrt(b)/(2*b^3*cos(f*x + e)^5 - 3*(a*b^2 + b^3)*cos(f*x
+ e)^3 + (a^2*b + 2*a*b^2 + b^3)*cos(f*x + e))) - 4*(2*b^2*cos(f*x + e)^3 -
 (a*b + 5*b^2)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b))/(b^2*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.74229, size = 177, normalized size = 1.42

$$\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\left(2\cos\left(fx+e\right)^2-\frac{abf^4+5\,b^2f^4}{b^2f^4}\right)\cos\left(fx+e\right)}{8\,f}+\frac{\left(a^2-2\,ab-3\,b^2\right)\log\left(\left|\sqrt{-b\cos\left(fx+e\right)^2+a+\right.\right.}{8\,\sqrt{-b}b\left|f\right|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(-b*cos(f*x + e)^2 + a + b)*(2*cos(f*x + e)^2 - (a*b*f^4 + 5*b^2*f^
4)/(b^2*f^4))*cos(f*x + e)/f + 1/8*(a^2 - 2*a*b - 3*b^2)*log(abs(sqrt(-b*co
s(f*x + e)^2 + a + b) + sqrt(-b*f^2)*cos(f*x + e)/f))/(sqrt(-b)*b*abs(f))
```

3.123 $\int \sin(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=78

$$-\frac{\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{2f}-\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{2\sqrt{b}f}$$

```
[Out] -((a + b)*ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]])/(2
*Sqrt[b]*f) - (Cos[e + f*x]*Sqrt[a + b - b*Cos[e + f*x]^2])/(2*f)
```

Rubi [A] time = 0.0609011, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3186, 195, 217, 203}

$$-\frac{\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{2f}-\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{2\sqrt{b}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((a + b)*ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]])/(2
*Sqrt[b]*f) - (Cos[e + f*x]*Sqrt[a + b - b*Cos[e + f*x]^2])/(2*f)
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sin(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = -\frac{\text{Subst}\left(\int \sqrt{a+b-bx^2}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{2f} - \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b-bx^2}}\,dx, x, \cos(e+ \ldots\right.}{2f}$$
$$= -\frac{\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{2f} - \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{\cos(e+fx)}{\sqrt{a+b-b\cos^2(e\ldots}}\right.}{2f}$$
$$= -\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{2\sqrt{b}f} - \frac{\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{2f}$$

Mathematica [A] time = 0.253022, size = 93, normalized size = 1.19

$$-\frac{\sqrt{2}\cos(e+fx)\sqrt{2a-b\cos(2(e+fx))+b}+\frac{2(a+b)\log\left(\sqrt{2a-b\cos(2(e+fx))+b}+\sqrt{2}\sqrt{-b}\cos(e+fx)\right)}{\sqrt{-b}}}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -(Sqrt[2]*Cos[e + f*x]*Sqrt[2*a + b - b*Cos[2*(e + f*x)]] + (2*(a + b)*Log[
Sqrt[2]*Sqrt[-b]*Cos[e + f*x] + Sqrt[2*a + b - b*Cos[2*(e + f*x)]]])/Sqrt[-
b])/(4*f)
```

Maple [B] time = 1.282, size = 182, normalized size = 2.3

$$\frac{1}{4f\cos(fx+e)}\sqrt{(\cos(fx+e))^2\left(a+b(\sin(fx+e))^2\right)}\left(b\arctan\left(\frac{-2b(\cos(fx+e))^2+a+b}{2}\frac{1}{\sqrt{b}}\frac{1}{\sqrt{-b(\cos(fx+e))\ldots}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/4*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*(b*arctan(1/2*(-2*b*cos(f*x+e)^
2+a+b)/b^(1/2)/(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2))+a*arctan(1/2*(-2
*b*cos(f*x+e)^2+a+b)/b^(1/2)/(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2))-2*
b^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2))/b^(1/2)/cos(f*x+e)/(a+b
*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.89486, size = 1069, normalized size = 13.71

$$\left[-\frac{8\sqrt{-b\cos\left(fx+e\right)^2+a+b}b\cos\left(fx+e\right)+(a+b)\sqrt{-b}\log\left(128\,b^4\cos\left(fx+e\right)^8-256\left(ab^3+b^4\right)\cos\left(fx+e\right)^6+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(8*sqrt(-b*cos(f*x + e)^2 + a + b)*b*cos(f*x + e) + (a + b)*sqrt(-b)
*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + b^4)*cos(f*x + e)^6 + 160*(a^2*b
^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 +
b^4 - 32*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cos(f*x + e)^2 + 8*(16*b^3*cos
(f*x + e)^7 - 24*(a*b^2 + b^3)*cos(f*x + e)^5 + 10*(a^2*b + 2*a*b^2 + b^3)*
cos(f*x + e)^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e))*sqrt(-b*cos(
f*x + e)^2 + a + b)*sqrt(-b)))/(b*f), 1/8*((a + b)*sqrt(b)*arctan(1/4*(8*b^
2*cos(f*x + e)^4 - 8*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(-
b*cos(f*x + e)^2 + a + b)*sqrt(b)/(2*b^3*cos(f*x + e)^5 - 3*(a*b^2 + b^3)*c
os(f*x + e)^3 + (a^2*b + 2*a*b^2 + b^3)*cos(f*x + e))) - 4*sqrt(-b*cos(f*x
+ e)^2 + a + b)*b*cos(f*x + e))/(b*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\sin^2\left(e+fx\right)}\sin\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*sin(e + f*x), x)
```

Giac [A] time = 1.73559, size = 112, normalized size = 1.44

$$-\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\cos\left(fx+e\right)}{2f}-\frac{(a+b)\log\left(\left|\sqrt{-b\cos\left(fx+e\right)^2+a+b}+\frac{\sqrt{-bf^2}\cos\left(fx+e\right)}{f}\right|\right)}{2\sqrt{-b}|f|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-b*cos(f*x + e)^2 + a + b)*cos(f*x + e)/f - 1/2*(a + b)*log(abs(s
qrt(-b*cos(f*x + e)^2 + a + b) + sqrt(-b*f^2)*cos(f*x + e)/f))/(sqrt(-b)*ab
s(f))
```

3.124 $\int \csc(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=83

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{f}$$

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]])/f
) - (Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]]
)/f
```

Rubi [A] time = 0.0965129, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3186, 402, 217, 203, 377, 206}

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]])/f
) - (Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]]
)/f
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \csc(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{\sqrt{a+b-bx^2}}{1-x^2}\,dx, x, \cos(e+fx)\right)}{f}\\ &= -\frac{a\,\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+b-bx^2}}\,dx, x, \cos(e+fx)\right)}{f} - \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b-bx^2}}\,dx, x, \cos(e+fx)\right)}{f}\\ &= -\frac{a\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{f} - \frac{b\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{f}\\ &= -\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{f} - \frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{f}\end{aligned}$$

Mathematica [A] time = 0.116469, size = 99, normalized size = 1.19

$$\frac{\sqrt{-b}\log\left(\sqrt{2a-b\cos(2(e+fx))+b}+\sqrt{2}\sqrt{-b}\cos(e+fx)\right)-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cos(e+fx)}{\sqrt{2a-b\cos(2(e+fx))+b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-(Sqrt[a]*ArcTanh[(Sqrt[2]*Sqrt[a]*Cos[e + f*x])/Sqrt[2*a + b - b*Cos[2*(e
 + f*x)]]]) + Sqrt[-b]*Log[Sqrt[2]*Sqrt[-b]*Cos[e + f*x] + Sqrt[2*a + b - b
*Cos[2*(e + f*x)]]])/f
```

Maple [B] time = 1.883, size = 174, normalized size = 2.1

$$-\frac{1}{2f\cos(fx+e)}\sqrt{(\cos(fx+e))^2\left(a+b(\sin(fx+e))^2\right)}\left(\sqrt{a}\ln\left(\frac{1}{(\cos(fx+e))^2-1}\left(-(a-b)(\cos(fx+e))^2-2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/2*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*(a^(1/2)*ln((-(a-b)*cos(f*x+e)
^2-2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)-a-b)/(cos(f*x+e)^2-
1))-b^(1/2)*arctan(1/2*(-2*b*cos(f*x+e)^2+a+b)/b^(1/2)/(-b*cos(f*x+e)^4+(a+
b)*cos(f*x+e)^2)^(1/2)))/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.5495, size = 2862, normalized size = 34.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(-b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + b^4)*cos(f*x + e)^
6 + 160*(a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*b^
2 + 4*a*b^3 + b^4 - 32*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cos(f*x + e)^2 -
 8*(16*b^3*cos(f*x + e)^7 - 24*(a*b^2 + b^3)*cos(f*x + e)^5 + 10*(a^2*b + 2
*a*b^2 + b^3)*cos(f*x + e)^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)
)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)) + 2*sqrt(a)*log(2*((a^2 - 6*a*b
 + b^2)*cos(f*x + e)^4 + 2*(3*a^2 + 2*a*b - b^2)*cos(f*x + e)^2 - 4*((a - b
)*cos(f*x + e)^3 + (a + b)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sq
rt(a) + a^2 + 2*a*b + b^2)/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/f, 1/8
*(4*sqrt(-a)*arctan(-1/2*((a - b)*cos(f*x + e)^2 + a + b)*sqrt(-b*cos(f*x +
 e)^2 + a + b)*sqrt(-a)/(a*b*cos(f*x + e)^3 - (a^2 + a*b)*cos(f*x + e))) +
sqrt(-b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + b^4)*cos(f*x + e)^6 + 16
0*(a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*
a*b^3 + b^4 - 32*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cos(f*x + e)^2 - 8*(16
*b^3*cos(f*x + e)^7 - 24*(a*b^2 + b^3)*cos(f*x + e)^5 + 10*(a^2*b + 2*a*b^2
 + b^3)*cos(f*x + e)^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e))*sqrt
(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)))/f, 1/4*(sqrt(b)*arctan(1/4*(8*b^2*co
s(f*x + e)^4 - 8*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(-b*co
s(f*x + e)^2 + a + b)*sqrt(b)/(2*b^3*cos(f*x + e)^5 - 3*(a*b^2 + b^3)*cos(f
*x + e)^3 + (a^2*b + 2*a*b^2 + b^3)*cos(f*x + e))) + sqrt(a)*log(2*((a^2 -
6*a*b + b^2)*cos(f*x + e)^4 + 2*(3*a^2 + 2*a*b - b^2)*cos(f*x + e)^2 - 4*((
a - b)*cos(f*x + e)^3 + (a + b)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a +
b)*sqrt(a) + a^2 + 2*a*b + b^2)/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/f
, 1/4*(2*sqrt(-a)*arctan(-1/2*((a - b)*cos(f*x + e)^2 + a + b)*sqrt(-b*cos(
f*x + e)^2 + a + b)*sqrt(-a)/(a*b*cos(f*x + e)^3 - (a^2 + a*b)*cos(f*x + e)
)) + sqrt(b)*arctan(1/4*(8*b^2*cos(f*x + e)^4 - 8*(a*b + b^2)*cos(f*x + e)^
2 + a^2 + 2*a*b + b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(b)/(2*b^3*cos(f
*x + e)^5 - 3*(a*b^2 + b^3)*cos(f*x + e)^3 + (a^2*b + 2*a*b^2 + b^3)*cos(f*
x + e))))/f]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\sin^2\left(e + fx\right)}\csc\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*csc(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin\left(fx+e\right)^2+a}\,\csc\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*csc(f*x + e), x)
```

3.125 $\int \csc^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=84

$$-\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{2\sqrt{a}f}-\frac{\cot(e+fx)\csc(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{2f}$$

```
[Out] -((a + b)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]])/(
2*Sqrt[a]*f) - (Sqrt[a + b - b*Cos[e + f*x]^2]*Cot[e + f*x]*Csc[e + f*x])/(
2*f)
```

Rubi [A] time = 0.100486, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3186, 378, 377, 206}

$$-\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{2\sqrt{a}f}-\frac{\cot(e+fx)\csc(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((a + b)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]])/(
2*Sqrt[a]*f) - (Sqrt[a + b - b*Cos[e + f*x]^2]*Cot[e + f*x]*Csc[e + f*x])/(
2*f)
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \csc^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = -\frac{\text{Subst}\left(\int \frac{\sqrt{a+b-bx^2}}{(1-x^2)^2}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\sqrt{a+b-b\cos^2(e+fx)}\cot(e+fx)\csc(e+fx)}{2f} - \frac{(a+b)\,\text{Subst}\left(\int \frac{}{(1-x^2)\sqrt{}}\right.}{}$$

$$= -\frac{\sqrt{a+b-b\cos^2(e+fx)}\cot(e+fx)\csc(e+fx)}{2f} - \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,d\right.}{2f}$$

$$= -\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{2\sqrt{a}f} - \frac{\sqrt{a+b-b\cos^2(e+fx)}\cot(e+fx)\,\text{cs}}{2f}$$

Mathematica [A] time = 0.277023, size = 100, normalized size = 1.19

$$\frac{-2(a+b)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cos(e+fx)}{\sqrt{2a-b\cos(2(e+fx))+b}}\right) - \sqrt{2}\sqrt{a}\cot(e+fx)\csc(e+fx)\sqrt{2a-b\cos(2(e+fx))+b}}{4\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-2*(a + b)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cos[e + f*x])/Sqrt[2*a + b - b*Cos[2*(
e + f*x)]]] - Sqrt[2]*Sqrt[a]*Sqrt[2*a + b - b*Cos[2*(e + f*x)]]*Cot[e + f*
x]*Csc[e + f*x])/(4*Sqrt[a]*f)
```

Maple [B] time = 1.444, size = 227, normalized size = 2.7

$$-\frac{1}{4\left(\sin\left(fx+e\right)\right)^2\cos\left(fx+e\right)f}\sqrt{\left(\cos\left(fx+e\right)\right)^2\left(a+b\left(\sin\left(fx+e\right)\right)^2\right)}\left(a\ln\left(\frac{1}{\left(\sin\left(fx+e\right)\right)^2}\left((a-b)\left(\cos\left(fx+e\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/4*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*(a*ln(((a-b)*cos(f*x+e)^2+2*a^
(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)+a+b)/sin(f*x+e)^2)*sin(f*x
+e)^2+b*ln(((a-b)*cos(f*x+e)^2+2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^
2)^(1/2)+a+b)/sin(f*x+e)^2)*sin(f*x+e)^2+2*a^(1/2)*(cos(f*x+e)^2*(a+b*sin(f
*x+e)^2))^(1/2))/a^(1/2)/sin(f*x+e)^2/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin\left(fx+e\right)^2+a}\csc\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*csc(f*x + e)^3, x)
```

Fricas [A] time = 2.25762, size = 849, normalized size = 10.11

$$\left[\frac{4\sqrt{-b\cos\left(fx+e\right)^2+a+b}a\cos\left(fx+e\right)+\left((a+b)\cos\left(fx+e\right)^2-a-b\right)\sqrt{a}\log\left(\frac{2\left((a^2-6ab+b^2)\cos(fx+e)^4+2\left(3a^2+2ab-b^2\right)c\right.}{}\right.}{8\left(af\cos\left(fx+e\right)^2-af\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(-b*cos(f*x + e)^2 + a + b)*a*cos(f*x + e) + ((a + b)*cos(f*x +
 e)^2 - a - b)*sqrt(a)*log(2*((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 2*(3*a^2
 + 2*a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + (a + b)*cos(f*
x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) + a^2 + 2*a*b + b^2)/(cos(f
*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/(a*f*cos(f*x + e)^2 - a*f), 1/4*(((a +
b)*cos(f*x + e)^2 - a - b)*sqrt(-a)*arctan(-1/2*((a - b)*cos(f*x + e)^2 + a
 + b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a)/(a*b*cos(f*x + e)^3 - (a^2 +
 a*b)*cos(f*x + e))) + 2*sqrt(-b*cos(f*x + e)^2 + a + b)*a*cos(f*x + e))/(a
*f*cos(f*x + e)^2 - a*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a+b\sin^2\left(e+fx\right)}\csc^3\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*csc(e + f*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sin\left(fx+e\right)^2+a}\csc\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*csc(f*x + e)^3, x)
```

3.126 $\int \csc^5(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=143

$$-\frac{(3a-b)(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{8a^{3/2}f}-\frac{\cot(e+fx)\csc^3(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}{4af}-\frac{(3a-b)\cot(e+f}{}$$

```
[Out] -((3*a - b)*(a + b)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f
*x]^2]])/(8*a^(3/2)*f) - ((3*a - b)*Sqrt[a + b - b*Cos[e + f*x]^2]*Cot[e +
f*x]*Csc[e + f*x])/(8*a*f) - ((a + b - b*Cos[e + f*x]^2)^(3/2)*Cot[e + f*x]
*Csc[e + f*x]^3)/(4*a*f)
```

Rubi [A] time = 0.132382, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3186, 382, 378, 377, 206}

$$-\frac{(3a-b)(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{8a^{3/2}f}-\frac{\cot(e+fx)\csc^3(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}{4af}-\frac{(3a-b)\cot(e+f}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5*Sqrt[a + b*Sin[e + f*x]^2],x]
```


```
[Out] -((3*a - b)*(a + b)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f
*x]^2]])/(8*a^(3/2)*f) - ((3*a - b)*Sqrt[a + b - b*Cos[e + f*x]^2]*Cot[e +
f*x]*Csc[e + f*x])/(8*a*f) - ((a + b - b*Cos[e + f*x]^2)^(3/2)*Cot[e + f*x]
*Csc[e + f*x]^3)/(4*a*f)
```


Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \csc^5(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = -\frac{\text{Subst}\left(\int \frac{\sqrt{a+b-bx^2}}{\left(1-x^2\right)^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\left(a+b-b\cos^2(e+fx)\right)^{3/2}\cot(e+fx)\csc^3(e+fx)}{4af} - \frac{(3a-b)\,\text{Subst}\left(\int \frac{\sqrt{a+b-}}{(1-x^{}}\right.}{4a}$$

$$= -\frac{(3a-b)\sqrt{a+b-b\cos^2(e+fx)}\cot(e+fx)\csc(e+fx)}{8af} - \frac{\left(a+b-b\cos^2(e+\right.}{}$$

$$= -\frac{(3a-b)\sqrt{a+b-b\cos^2(e+fx)}\cot(e+fx)\csc(e+fx)}{8af} - \frac{\left(a+b-b\cos^2(e+\right.}{}$$

$$= -\frac{(3a-b)(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{8a^{3/2}f} - \frac{(3a-b)\sqrt{a+b-b\cos^2(e+fx)}\,\text{co}}{8af}$$

Mathematica [A] time = 0.517932, size = 127, normalized size = 0.89

$$\frac{\left(-6a^2-4ab+2b^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cos(e+fx)}{\sqrt{2a-b\cos(2(e+fx))+b}}\right) - \sqrt{2}\sqrt{a}\cot(e+fx)\csc(e+fx)\sqrt{2a-b\cos(2(e+fx))+b}\left(2a\csc^2(e+\right.}{16a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] ((-6*a^2 - 4*a*b + 2*b^2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cos[e + f*x])/Sqrt[2*a +
 b - b*Cos[2*(e + f*x)]]] - Sqrt[2]*Sqrt[a]*Sqrt[2*a + b - b*Cos[2*(e + f*x
)]]*Cot[e + f*x]*Csc[e + f*x]*(3*a + b + 2*a*Csc[e + f*x]^2))/(16*a^(3/2)*f
)
```

Maple [B] time = 1.583, size = 379, normalized size = 2.7

$$-\frac{1}{16\left(\sin\left(fx+e\right)\right)^4\cos\left(fx+e\right)f}\sqrt{\left(\cos\left(fx+e\right)\right)^2\left(a+b\left(\sin\left(fx+e\right)\right)^2\right)}\left(3a^3\ln\left(\frac{(a-b)\left(\cos\left(fx+e\right)\right)^2+2\sqrt{a}\sqrt{-b}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5*(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/16*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*(3*a^3*ln(((a-b)*cos(f*x+e)^2
+2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)+a+b)/sin(f*x+e)^2)*si
n(f*x+e)^4+2*b*ln(((a-b)*cos(f*x+e)^2+2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(
f*x+e)^2)^(1/2)+a+b)/sin(f*x+e)^2)*sin(f*x+e)^4*a^2-ln(((a-b)*cos(f*x+e)^2+
2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)+a+b)/sin(f*x+e)^2)*b^2
*sin(f*x+e)^4*a+6*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*sin(f*x+e)^2*a^(5
/2)+2*b*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*sin(f*x+e)^2*a^(3/2)+4*(cos
(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*a^(5/2))/sin(f*x+e)^4/a^(5/2)/cos(f*x+e
)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin\left(fx+e\right)^2 + a}\csc\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*csc(f*x + e)^5, x)
```

Fricas [A] time = 3.55323, size = 1239, normalized size = 8.66

$$\left[-\frac{\left(\left(3\,a^2+2\,ab-b^2\right)\cos\left(fx+e\right)^4-2\left(3\,a^2+2\,ab-b^2\right)\cos\left(fx+e\right)^2+3\,a^2+2\,ab-b^2\right)\sqrt{a}\log\left(\frac{2\left(\left(a^2-6\,ab+b^2\right)\cos\left(fx+e\right.\right.}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*(((3*a^2 + 2*a*b - b^2)*cos(f*x + e)^4 - 2*(3*a^2 + 2*a*b - b^2)*cos
(f*x + e)^2 + 3*a^2 + 2*a*b - b^2)*sqrt(a)*log(2*((a^2 - 6*a*b + b^2)*cos(f
*x + e)^4 + 2*(3*a^2 + 2*a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e
)^3 + (a + b)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) + a^2 +
 2*a*b + b^2)/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)) - 4*((3*a^2 + a*b)*c
os(f*x + e)^3 - (5*a^2 + a*b)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)
)/(a^2*f*cos(f*x + e)^4 - 2*a^2*f*cos(f*x + e)^2 + a^2*f), 1/16*(((3*a^2 +
2*a*b - b^2)*cos(f*x + e)^4 - 2*(3*a^2 + 2*a*b - b^2)*cos(f*x + e)^2 + 3*a^
2 + 2*a*b - b^2)*sqrt(-a)*arctan(-1/2*((a - b)*cos(f*x + e)^2 + a + b)*sqrt
(-b*cos(f*x + e)^2 + a + b)*sqrt(-a)/(a*b*cos(f*x + e)^3 - (a^2 + a*b)*cos(
f*x + e))) + 2*((3*a^2 + a*b)*cos(f*x + e)^3 - (5*a^2 + a*b)*cos(f*x + e))*
sqrt(-b*cos(f*x + e)^2 + a + b))/(a^2*f*cos(f*x + e)^4 - 2*a^2*f*cos(f*x +
e)^2 + a^2*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin\left(fx+e\right)^2+a}\csc\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*csc(f*x + e)^5, x)
```

3.127 $\int \sin^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=259

$$-\frac{\left(2a^2-3ab-8b^2\right)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}E\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{15b^2f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{2a(a-2b)(a+b)\sqrt{\mathrm{cc}}}{}$$

```
[Out] -((a + 4*b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(15*b*f)
- (Cos[e + f*x]*Sin[e + f*x]^3*Sqrt[a + b*Sin[e + f*x]^2])/(5*f) - ((2*a^2
- 3*a*b - 8*b^2)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a
)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(15*b^2*f*Sqrt[1 + (b*Sin[e + f
*x]^2)/a]) + (2*a*(a - 2*b)*(a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[S
in[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(15*b^2*
f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.315668, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3188, 478, 582, 524, 426, 424, 421, 419}

$$-\frac{\left(2a^2-3ab-8b^2\right)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}E\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{15b^2f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{2a(a-2b)(a+b)\sqrt{\mathrm{cc}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4*Sqrt[a + b*Sin[e + f*x]^2],x]
```


```
[Out] -((a + 4*b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(15*b*f)
- (Cos[e + f*x]*Sin[e + f*x]^3*Sqrt[a + b*Sin[e + f*x]^2])/(5*f) - ((2*a^2
- 3*a*b - 8*b^2)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a
)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(15*b^2*f*Sqrt[1 + (b*Sin[e + f
*x]^2)/a]) + (2*a*(a - 2*b)*(a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[S
in[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(15*b^2*
f*Sqrt[a + b*Sin[e + f*x]^2])
```


Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \sin^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^4\sqrt{a+bx^2}}{\sqrt{1-x^2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin^3(e+fx)\sqrt{a+b\sin^2(e+fx)}}{5f} + \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx\right)}{}$$

$$= -\frac{(a+4b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15bf} - \frac{\cos(e+fx)\sin^3(e+}{}$$

$$= -\frac{(a+4b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15bf} - \frac{\cos(e+fx)\sin^3(e+}{}$$

$$= -\frac{(a+4b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15bf} - \frac{\cos(e+fx)\sin^3(e+}{}$$

$$= -\frac{(a+4b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15bf} - \frac{\cos(e+fx)\sin^3(e+}{}$$

Mathematica [A] time = 1.4529, size = 199, normalized size = 0.77

$$\frac{-\sqrt{2}b\sin(2(e+fx))\left(8a^2-4b(4a+7b)\cos(2(e+fx))+48ab+3b^2\cos(4(e+fx))+25b^2\right)+32a\left(a^2-ab-2b^2\right)\sqrt{2}}{240b^2f\sqrt{2a-b\cos(2(e+fx))+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-16*a*(2*a^2 - 3*a*b - 8*b^2)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*Ellip
ticE[e + f*x, -(b/a)] + 32*a*(a^2 - a*b - 2*b^2)*Sqrt[(2*a + b - b*Cos[2*(e
 + f*x)])/a]*EllipticF[e + f*x, -(b/a)] - Sqrt[2]*b*(8*a^2 + 48*a*b + 25*b^
2 - 4*b*(4*a + 7*b)*Cos[2*(e + f*x)] + 3*b^2*Cos[4*(e + f*x)])*Sin[2*(e + f
*x)])/(240*b^2*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A] time = 1.037, size = 413, normalized size = 1.6

$$\frac{1}{15\,b^2\cos\left(fx+e\right)f}\left(3\,b^3\left(\sin\left(fx+e\right)\right)^7+4\,ab^2\left(\sin\left(fx+e\right)\right)^5+b^3\left(\sin\left(fx+e\right)\right)^5+2\sqrt{\left(\cos\left(fx+e\right)\right)^2}\sqrt{\frac{a+b\left(\mathrm{si}\right.}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/15*(3*b^3*sin(f*x+e)^7+4*a*b^2*sin(f*x+e)^5+b^3*sin(f*x+e)^5+2*(cos(f*x+e
)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2)
)*a^3-2*a^2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin
```

```
(f*x+e),(-1/a*b)^(1/2))*b-4*a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(
1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^2-2*(cos(f*x+e)^2)^(1/2)*((a+b*
sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3+3*(cos(f*x+
e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2
))*a^2*b+8*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(
f*x+e),(-1/a*b)^(1/2))*a*b^2+a^2*b*sin(f*x+e)^3-4*sin(f*x+e)^3*b^3-sin(f*x+
e)*a^2*b-4*a*b^2*sin(f*x+e))/b^2/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin\left(fx+e\right)^2+a}\sin\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*sin(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(\cos\left(fx+e\right)^4-2\,\cos\left(fx+e\right)^2+1\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-b*cos(f*x + e)^2 + a
 + b), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin\left(fx+e\right)^2+a}\sin\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*sin(f*x + e)^4, x)
```

3.128 $\int \sin^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=159

$$-\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(e+fx\middle|-\frac{b}{a}\right)}{3bf\sqrt{a+b\sin^2(e+fx)}}+\frac{(a+2b)\sqrt{a+b\sin^2(e+fx)}}{3bf\sqrt{\frac{b\sin^2(e+fx)}{a}}}$$

```
[Out] -(Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f) + ((a + 2*b)*
EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(3*b*f*Sqrt[1 + (b*S
in[e + f*x]^2)/a]) - (a*(a + b)*EllipticF[e + f*x, -(b/a)]*Sqrt[1 + (b*Sin[
e + f*x]^2)/a])/(3*b*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.191325, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3170, 3172, 3178, 3177, 3183, 3182}

$$-\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(e+fx\middle|-\frac{b}{a}\right)}{3bf\sqrt{a+b\sin^2(e+fx)}}+\frac{(a+2b)\sqrt{a+b\sin^2(e+fx)}}{3bf\sqrt{\frac{b\sin^2(e+fx)}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -(Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f) + ((a + 2*b)*
EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(3*b*f*Sqrt[1 + (b*S
in[e + f*x]^2)/a]) - (a*(a + b)*EllipticF[e + f*x, -(b/a)]*Sqrt[1 + (b*Sin[
e + f*x]^2)/a])/(3*b*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 3170

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[(B*Cos[e + f*x]*Sin[e + f*x]*(a + b*Si
n[e + f*x]^2)^p)/(2*f*(p + 1)), x] + Dist[1/(2*(p + 1)), Int[(a + b*Sin[e +
 f*x]^2)^(p - 1)*Simp[a*B + 2*a*A*(p + 1) + (2*A*b*(p + 1) + B*(b + 2*a*p +
 2*b*p))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && GtQ[
p, 0]
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rubi steps

$$\int \sin^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{1}{3}\int\frac{a+(a+2b)\sin^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,d$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{(a(a+b))\int\frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx}{3b}+$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{\left((a+2b)\sqrt{a+b\sin^2(e+fx)}\right)\int}{3b\sqrt{1+\frac{b\sin^2(e}{a}}}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{(a+2b)E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin}}{3bf\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

Mathematica [A] time = 0.812735, size = 159, normalized size = 1.

$$\frac{b\sin(2(e+fx))(-2a+b\cos(2(e+fx))-b)-2\sqrt{2}a(a+b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}F\left(e+fx\middle|-\frac{b}{a}\right)+2\sqrt{2}a(a+2b)\sqrt{\frac{2a-b\cos(2(e}{a}}}{6\sqrt{2}bf\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (2*Sqrt[2]*a*(a + 2*b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e +
 f*x, -(b/a)] - 2*Sqrt[2]*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*
EllipticF[e + f*x, -(b/a)] + b*(-2*a - b + b*Cos[2*(e + f*x)])*Sin[2*(e + f
*x)])/(6*Sqrt[2]*b*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A] time = 1.19, size = 266, normalized size = 1.7

$$-\frac{1}{3b\cos(fx+e)f}\left(-b^2(\sin(fx+e))^5+\sqrt{(\cos(fx+e))^2}\sqrt{\frac{a+b(\sin(fx+e))^2}{a}}\mathrm{EllipticF}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/3*(-b^2*sin(f*x+e)^5+(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*E
llipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2+a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*
x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-(cos(f*x+e)^2)^(1/2
)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2-2*(
cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a
*b)^(1/2))*a*b-a*b*sin(f*x+e)^3+b^2*sin(f*x+e)^3+sin(f*x+e)*a*b)/b/cos(f*x+
e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin\left(fx+e\right)^{2}+a}\sin\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*sin(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\left(\cos\left(fx+e\right)^{2}-1\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)*(cos(f*x + e)^2 - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin\left(fx+e\right)^{2}+a}\sin\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*sin(f*x + e)^2, x)
```

3.129 $\int \sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=51

$$\frac{\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\left|-\frac{b}{a}\right.\right)}{f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

```
[Out] (EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(f*Sqrt[1 + (b*Sin[
e + f*x]^2)/a])
```

Rubi [A] time = 0.0351473, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3178, 3177}

$$\frac{\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\left|-\frac{b}{a}\right.\right)}{f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(f*Sqrt[1 + (b*Sin[
e + f*x]^2)/a])
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+b\sin^2(e+fx)}\,dx &= \frac{\sqrt{a+b\sin^2(e+fx)}\int \sqrt{1+\frac{b\sin^2(e+fx)}{a}}\,dx}{\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}\\ &= \frac{E\left(e+fx\left|-\frac{b}{a}\right.\right)\sqrt{a+b\sin^2(e+fx)}}{f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}\end{aligned}$$

Mathematica [A] time = 0.0874134, size = 61, normalized size = 1.2

$$\frac{a\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}E\left(e+fx\left|-\frac{b}{a}\right.\right)}{f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)])/(f*Sq
rt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A] time = 0.609, size = 71, normalized size = 1.4

$$\frac{a}{f\cos\left(fx+e\right)}\sqrt{\left(\cos\left(fx+e\right)\right)^{2}}\sqrt{\frac{a+b\left(\sin\left(fx+e\right)\right)^{2}}{a}}\mathrm{EllipticE}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)\frac{1}{\sqrt{a+b\left(\sin\left(fx+e\right)\right)^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-
1/a*b)^(1/2))/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sin\left(fx+e\right)^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{-b\cos\left(fx+e\right)^{2}+a+b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a+b\sin^{2}\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sin \left(fx+e\right)^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a), x)
```

3.130 $\int \csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=174

$$-\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f}+\frac{(a+b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}-$$

```
[Out] -((Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/f) - (Sqrt[Cos[e + f*x]^2]*Elli
pticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2]
)/(f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + ((a + b)*Sqrt[Cos[e + f*x]^2]*Ellipt
icF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/
a])/(f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.162632, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3188, 475, 21, 423, 426, 424, 421, 419}

$$-\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f}+\frac{(a+b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}-$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2],x]
```


```
[Out] -((Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/f) - (Sqrt[Cos[e + f*x]^2]*Elli
pticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2]
)/(f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + ((a + b)*Sqrt[Cos[e + f*x]^2]*Ellipt
icF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/
a])/(f*Sqrt[a + b*Sin[e + f*x]^2])
```


Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])
```

Rule 423

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{1-x^2}}dx, x, \sin(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} + \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{b}{\sqrt{1-}}\right.}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} + \frac{\left(b\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{1}}{\sqrt{a}}\right.}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} - \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{a+}}{\sqrt{1-}}\right.}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} - \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+\right.}{f\sqrt{1+\frac{b}{}}}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} - \frac{\sqrt{\cos^2(e+fx)}E\left(\sin^{-1}(\sin(e+fx))\middle| -\frac{b}{a}\right)\text{s}}{f\sqrt{1+\frac{b\sin^2(e+}{a}}}$$

Mathematica [A] time = 0.598582, size = 137, normalized size = 0.79

$$\frac{-\sqrt{2}\cot(e+fx)(2a-b\cos(2(e+fx))+b)+2(a+b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}F\left(e+fx\middle|-\frac{b}{a}\right)-2a\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}E\left(e+fx\right.}{2f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-(Sqrt[2]*(2*a + b - b*Cos[2*(e + f*x)])*Cot[e + f*x]) - 2*a*Sqrt[(2*a + b
 - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] + 2*(a + b)*Sqrt[(2*a
+ b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)])/(2*f*Sqrt[2*a + b
- b*Cos[2*(e + f*x)]])
```

Maple [A] time = 1.209, size = 156, normalized size = 0.9

$$\frac{1}{\sin(fx+e)\cos(fx+e)f}\left(\sin(fx+e)\sqrt{(\cos(fx+e))^2}\sqrt{-\frac{b(\cos(fx+e))^2}{a}+\frac{a+b}{a}}\left(\text{EllipticF}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] (sin(f*x+e)*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(Ellipti
cF(sin(f*x+e),(-1/a*b)^(1/2))*a+EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-Elli
pticE(sin(f*x+e),(-1/a*b)^(1/2))*a)+b*cos(f*x+e)^4+(-a-b)*cos(f*x+e)^2)/sin
(f*x+e)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin\left(fx+e\right)^2 + a}\csc\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*csc(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-b\cos\left(fx+e\right)^2 + a + b}\csc\left(fx+e\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)*csc(f*x + e)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\sin^2\left(e+fx\right)}\csc^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*csc(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin\left(fx+e\right)^2 + a}\csc\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*csc(f*x + e)^2, x)
```

3.131 $\int \csc^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=234

$$-\frac{(2a+b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af}-\frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{2(a+b)\sqrt{\cos^2(e+fx)}\sec(e}{3}$$

```
[Out] -((2*a + b)*Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a*f) - (Cot[e + f*x
]*Csc[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2])/(3*f) - ((2*a + b)*Sqrt[Cos[e
+ f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*S
in[e + f*x]^2])/(3*a*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (2*(a + b)*Sqrt[Co
s[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 +
 (b*Sin[e + f*x]^2)/a])/(3*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.259876, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3188, 475, 583, 524, 426, 424, 421, 419}

$$-\frac{(2a+b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af}-\frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{2(a+b)\sqrt{\cos^2(e+fx)}\sec(e}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*Sqrt[a + b*Sin[e + f*x]^2],x]
```


```
[Out] -((2*a + b)*Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a*f) - (Cot[e + f*x
]*Csc[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2])/(3*f) - ((2*a + b)*Sqrt[Cos[e
+ f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*S
in[e + f*x]^2])/(3*a*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (2*(a + b)*Sqrt[Co
s[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 +
 (b*Sin[e + f*x]^2)/a])/(3*f*Sqrt[a + b*Sin[e + f*x]^2])
```


Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \csc^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^4\sqrt{1-x^2}}dx,x,\sin(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} + \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{S}}{}$$

$$= -\frac{(2a+b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af} - \frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin}}{3f}$$

$$= -\frac{(2a+b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af} - \frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin}}{3f}$$

$$= -\frac{(2a+b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af} - \frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin}}{3f}$$

$$= -\frac{(2a+b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af} - \frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin}}{3f}$$

Mathematica [A] time = 3.23071, size = 188, normalized size = 0.8

$$\frac{\frac{\cot(e+fx)\csc^2(e+fx)\left(4\left(2a^2+4ab+b^2\right)\cos(2(e+fx))-(2a+b)(8a+b\cos(4(e+fx))+3b)\right)}{2\sqrt{2}} + 4a(a+b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}F\left(e+fx\middle|-\frac{b}{a}\right) - 2a(2a+}{6af\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (((4*(2*a^2 + 4*a*b + b^2)*Cos[2*(e + f*x)] - (2*a + b)*(8*a + 3*b + b*Cos[
4*(e + f*x)]))*Cot[e + f*x]*Csc[e + f*x]^2)/(2*Sqrt[2]) - 2*a*(2*a + b)*Sqr
t[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] + 4*a*(a + b
)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)])/(6*a*f
*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A] time = 1.098, size = 342, normalized size = 1.5

$$\frac{1}{3a\left(\sin\left(fx+e\right)\right)^3\cos\left(fx+e\right)f}\left(2\,\text{EllipticF}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)\sqrt{\left(\cos\left(fx+e\right)\right)^2}\sqrt{\frac{a+b\left(\sin\left(fx+e\right)\right)^2}{a}}a^2\left(\sin\left(fx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3*(2*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*(cos(f*x+e)^2)^(1/2)*((a+b*sin(
f*x+e)^2)/a)^(1/2)*a^2*sin(f*x+e)^3+2*b*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+
e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*sin(f*x+e)^3-2*Ellipt
icE(sin(f*x+e),(-1/a*b)^(1/2))*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^
```

```
(1/2)*a^2*sin(f*x+e)^3-EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*(cos(f*x+e)^2)^
(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*a*b*sin(f*x+e)^3+2*a*b*sin(f*x+e)^6+b^2*
sin(f*x+e)^6+2*a^2*sin(f*x+e)^4-b^2*sin(f*x+e)^4-a^2*sin(f*x+e)^2-2*a*b*sin
(f*x+e)^2-a^2)/a/sin(f*x+e)^3/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin\left(fx+e\right)^2+a}\csc\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*csc(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{-b\cos\left(fx+e\right)^2+a+b}\csc\left(fx+e\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)*csc(f*x + e)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin\left(fx+e\right)^2+a}\csc\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*csc(f*x + e)^4, x)
```

3.132 $\int \sin^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=169

$$\frac{(a-5b)(a+b)^2\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{16b^{3/2}f}-\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{5/2}}{6bf}+\frac{(a-5b)\cos(e+fx)\left(a-b\cos^2(e+\right.}{24bf}$$

```
[Out] ((a - 5*b)*(a + b)^2*ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f
*x]^2]])/(16*b^(3/2)*f) + ((a - 5*b)*(a + b)*Cos[e + f*x]*Sqrt[a + b - b*Co
s[e + f*x]^2])/(16*b*f) + ((a - 5*b)*Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2
)^(3/2))/(24*b*f) - (Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2)^(5/2))/(6*b*f)
```

Rubi [A] time = 0.148433, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3186, 388, 195, 217, 203}

$$\frac{(a-5b)(a+b)^2\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{16b^{3/2}f}-\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{5/2}}{6bf}+\frac{(a-5b)\cos(e+fx)\left(a-b\cos^2(e+\right.}{24bf}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3*(a + b*Sin[e + f*x]^2)^(3/2),x]
```


```
[Out] ((a - 5*b)*(a + b)^2*ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f
*x]^2]])/(16*b^(3/2)*f) + ((a - 5*b)*(a + b)*Cos[e + f*x]*Sqrt[a + b - b*Co
s[e + f*x]^2])/(16*b*f) + ((a - 5*b)*Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2
)^(3/2))/(24*b*f) - (Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2)^(5/2))/(6*b*f)
```


Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sin^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = -\frac{\text{Subst}\left(\int\left(1-x^2\right)\left(a+b-bx^2\right)^{3/2}\,dx,x,\cos(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^{5/2}}{6bf}+\frac{(a-5b)\,\text{Subst}\left(\int\left(a+b-bx^2 \ldots\right.\right.}{6bf}$$

$$= \frac{(a-5b)\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^{3/2}}{24bf}-\frac{\cos(e+fx)\left(a+b-b\,c \ldots\right.}{6bf}$$

$$= \frac{(a-5b)(a+b)\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{16bf}+\frac{(a-5b)\cos(e+fx \ldots}{\ldots}$$

$$= \frac{(a-5b)(a+b)\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{16bf}+\frac{(a-5b)\cos(e+fx \ldots}{\ldots}$$

$$= \frac{(a-5b)(a+b)^2\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{16b^{3/2}f}+\frac{(a-5b)(a+b)\cos(e+fx)\sqrt{a \ldots}}{16bf}$$

Mathematica [A] time = 0.753845, size = 152, normalized size = 0.9

$$\frac{\frac{(a+b)^2(5b-a)\log\left(\sqrt{2a-b\cos(2(e+fx))+b}+\sqrt{2}\sqrt{-b}\cos(e+fx)\right)}{(-b)^{3/2}}-\frac{\cos(e+fx)\sqrt{2a-b\cos(2(e+fx))+b}\left(3a^2-b(7a+9b)\cos(2(e+fx))+29ab+b^2\cos(4(e+fx))+23 \ldots\right.}{3\sqrt{2}b}}{16f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^3*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-(Cos[e + f*x]*Sqrt[2*a + b - b*Cos[2*(e + f*x)]]*(3*a^2 + 29*a*b + 23*b^2
 - b*(7*a + 9*b)*Cos[2*(e + f*x)] + b^2*Cos[4*(e + f*x)]))/(3*Sqrt[2]*b) +
((a + b)^2*(-a + 5*b)*Log[Sqrt[2]*Sqrt[-b]*Cos[e + f*x] + Sqrt[2*a + b - b*
Cos[2*(e + f*x)]]])/(-b)^(3/2))/(16*f)
```

Maple [B] time = 1.565, size = 446, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/96*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*(16*b^(7/2)*(-b*cos(f*x+e)^4+
(a+b)*cos(f*x+e)^2)^(1/2)*cos(f*x+e)^4-4*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^
2)^(1/2)*b^(5/2)*(13*b+7*a)*cos(f*x+e)^2+66*b^(7/2)*(-b*cos(f*x+e)^4+(a+b)*
cos(f*x+e)^2)^(1/2)+72*a*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b^(5/2)
```

```
+6*a^2*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b^(3/2)+3*arctan(1/2*(-2*
b*cos(f*x+e)^2+a+b)/b^(1/2)/(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2))*a^3
*b-9*arctan(1/2*(-2*b*cos(f*x+e)^2+a+b)/b^(1/2)/(-b*cos(f*x+e)^4+(a+b)*cos(
f*x+e)^2)^(1/2))*a^2*b^2-27*b^3*a*arctan(1/2*(-2*b*cos(f*x+e)^2+a+b)/b^(1/2
)/(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2))-15*b^4*arctan(1/2*(-2*b*cos(f
*x+e)^2+a+b)/b^(1/2)/(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)))/b^(5/2)/c
os(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 12.4483, size = 1400, normalized size = 8.28

$$\left[\frac{3\left(a^3-3a^2b-9ab^2-5b^3\right)\sqrt{-b}\log\left(128b^4\cos\left(fx+e\right)^8-256\left(ab^3+b^4\right)\cos\left(fx+e\right)^6+160\left(a^2b^2+2ab^3+b^4\right)\cos\left(\ldots\right.\right.}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] [1/384*(3*(a^3 - 3*a^2*b - 9*a*b^2 - 5*b^3)*sqrt(-b)*log(128*b^4*cos(f*x +
e)^8 - 256*(a*b^3 + b^4)*cos(f*x + e)^6 + 160*(a^2*b^2 + 2*a*b^3 + b^4)*cos
(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4 - 32*(a^3*b + 3*a^2
*b^2 + 3*a*b^3 + b^4)*cos(f*x + e)^2 + 8*(16*b^3*cos(f*x + e)^7 - 24*(a*b^2
 + b^3)*cos(f*x + e)^5 + 10*(a^2*b + 2*a*b^2 + b^3)*cos(f*x + e)^3 - (a^3 +
 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqr
t(-b)) - 8*(8*b^3*cos(f*x + e)^5 - 2*(7*a*b^2 + 13*b^3)*cos(f*x + e)^3 + 3*
(a^2*b + 12*a*b^2 + 11*b^3)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b))/
(b^2*f), -1/192*(3*(a^3 - 3*a^2*b - 9*a*b^2 - 5*b^3)*sqrt(b)*arctan(1/4*(8*
b^2*cos(f*x + e)^4 - 8*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt
(-b*cos(f*x + e)^2 + a + b)*sqrt(b)/(2*b^3*cos(f*x + e)^5 - 3*(a*b^2 + b^3)
*cos(f*x + e)^3 + (a^2*b + 2*a*b^2 + b^3)*cos(f*x + e))) + 4*(8*b^3*cos(f*x
 + e)^5 - 2*(7*a*b^2 + 13*b^3)*cos(f*x + e)^3 + 3*(a^2*b + 12*a*b^2 + 11*b^
3)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b))/(b^2*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.7358, size = 266, normalized size = 1.57

$$-\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\left(\frac{2\left(4\,bf^2\cos\left(fx+e\right)^2-\frac{7\,ab^4f^{10}+13\,b^5f^{10}}{b^4f^8}\right)\cos\left(fx+e\right)^2}{f^2}+\frac{3\left(a^2b^3f^8+12\,ab^4f^8+11\,b^5f^8\right)}{b^4f^8}\right)\cos\left(fx+e\right)}{48\,f}+\frac{\left(a^3-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/48*sqrt(-b*cos(f*x + e)^2 + a + b)*(2*(4*b*f^2*cos(f*x + e)^2 - (7*a*b^4
*f^10 + 13*b^5*f^10)/(b^4*f^8))*cos(f*x + e)^2/f^2 + 3*(a^2*b^3*f^8 + 12*a*
b^4*f^8 + 11*b^5*f^8)/(b^4*f^8))*cos(f*x + e)/f + 1/16*(a^3 - 3*a^2*b - 9*a
*b^2 - 5*b^3)*log(abs(sqrt(-b*cos(f*x + e)^2 + a + b) + sqrt(-b*f^2)*cos(f*
x + e)/f))/(sqrt(-b)*b*abs(f))
```

3.133 $\int \sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=114

$$-\frac{3(a+b)\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{8f}-\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}{4f}-\frac{3(a+b)^2\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)}}\right.}{8\sqrt{b}f}$$

```
[Out] (-3*(a + b)^2*ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]]
)/(8*Sqrt[b]*f) - (3*(a + b)*Cos[e + f*x]*Sqrt[a + b - b*Cos[e + f*x]^2])/(
8*f) - (Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2)^(3/2))/(4*f)
```

Rubi [A] time = 0.075981, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3186, 195, 217, 203}

$$-\frac{3(a+b)\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{8f}-\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}{4f}-\frac{3(a+b)^2\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)}}\right.}{8\sqrt{b}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-3*(a + b)^2*ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]]
)/(8*Sqrt[b]*f) - (3*(a + b)*Cos[e + f*x]*Sqrt[a + b - b*Cos[e + f*x]^2])/(
8*f) - (Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2)^(3/2))/(4*f)
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx = -\frac{\text{Subst}\left(\int\left(a+b-bx^2\right)^{3/2}dx,x,\cos(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^{3/2}}{4f} - \frac{(3(a+b))\,\text{Subst}\left(\int\sqrt{a+b-b\right.}{4f}$$

$$= -\frac{3(a+b)\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{8f} - \frac{\cos(e+fx)\left(a+b-b\cos\right.}{4f}$$

$$= -\frac{3(a+b)\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{8f} - \frac{\cos(e+fx)\left(a+b-b\cos\right.}{4f}$$

$$= -\frac{3(a+b)^2\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{8\sqrt{b}f} - \frac{3(a+b)\cos(e+fx)\sqrt{a+b-b\cos^2}}{8f}$$

Mathematica [A] time = 0.400842, size = 113, normalized size = 0.99

$$-\frac{\frac{\cos(e+fx)\sqrt{2a-b\cos(2(e+fx))+b}(5a-b\cos(2(e+fx))+4b)}{\sqrt{2}}+\frac{3(a+b)^2\log\left(\sqrt{2a-b\cos(2(e+fx))+b}+\sqrt{2}\sqrt{-b}\cos(e+fx)\right)}{\sqrt{-b}}}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -((Cos[e + f*x]*Sqrt[2*a + b - b*Cos[2*(e + f*x)]]*(5*a + 4*b - b*Cos[2*(e
+ f*x)]))/Sqrt[2] + (3*(a + b)^2*Log[Sqrt[2]*Sqrt[-b]*Cos[e + f*x] + Sqrt[2
*a + b - b*Cos[2*(e + f*x)]]])/Sqrt[-b])/(8*f)
```

Maple [B] time = 1.507, size = 309, normalized size = 2.7

$$\frac{1}{16f\cos\left(fx+e\right)}\sqrt{\left(\cos\left(fx+e\right)\right)^2\left(a+b\left(\sin\left(fx+e\right)\right)^2\right)}\left(4b^{3/2}\sqrt{-b\left(\cos\left(fx+e\right)\right)^4+(a+b)\left(\cos\left(fx+e\right)\right)^2}\left(\cos\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/16*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*(4*b^(3/2)*(-b*cos(f*x+e)^4+(a
+b)*cos(f*x+e)^2)^(1/2)*cos(f*x+e)^2-10*b^(3/2)*(-b*cos(f*x+e)^4+(a+b)*cos(
f*x+e)^2)^(1/2)-10*a*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b^(1/2)+3*a
rctan(1/2*(-2*b*cos(f*x+e)^2+a+b)/b^(1/2)/(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)
^2)^(1/2))*a^2+6*b*a*arctan(1/2*(-2*b*cos(f*x+e)^2+a+b)/b^(1/2)/(-b*cos(f*x
+e)^4+(a+b)*cos(f*x+e)^2)^(1/2))+3*b^2*arctan(1/2*(-2*b*cos(f*x+e)^2+a+b)/b
^(1/2)/(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)))/b^(1/2)/cos(f*x+e)/(a+b
*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 4.64765, size = 1208, normalized size = 10.6

$$\left[-\frac{3\left(a^2+2\,ab+b^2\right)\sqrt{-b}\log\left(128\,b^4\cos\left(fx+e\right)^8-256\left(ab^3+b^4\right)\cos\left(fx+e\right)^6+160\left(a^2b^2+2\,ab^3+b^4\right)\cos\left(fx+e\right)^4\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(3*(a^2 + 2*a*b + b^2)*sqrt(-b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*
b^3 + b^4)*cos(f*x + e)^6 + 160*(a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 +
a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4 - 32*(a^3*b + 3*a^2*b^2 + 3*a*b^3
 + b^4)*cos(f*x + e)^2 + 8*(16*b^3*cos(f*x + e)^7 - 24*(a*b^2 + b^3)*cos(f*
x + e)^5 + 10*(a^2*b + 2*a*b^2 + b^3)*cos(f*x + e)^3 - (a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)) - 8*(2*
b^2*cos(f*x + e)^3 - 5*(a*b + b^2)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a
 + b))/(b*f), 1/32*(3*(a^2 + 2*a*b + b^2)*sqrt(b)*arctan(1/4*(8*b^2*cos(f*x
 + e)^4 - 8*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(-b*cos(f*x
 + e)^2 + a + b)*sqrt(b)/(2*b^3*cos(f*x + e)^5 - 3*(a*b^2 + b^3)*cos(f*x +
e)^3 + (a^2*b + 2*a*b^2 + b^3)*cos(f*x + e))) + 4*(2*b^2*cos(f*x + e)^3 - 5
*(a*b + b^2)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b))/(b*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.71035, size = 173, normalized size = 1.52

$$\frac{\left(2\,b\cos\left(fx+e\right)^2-\frac{5\left(ab^2f^4+b^3f^4\right)}{b^2f^4}\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}\cos\left(fx+e\right)}{8\,f}-\frac{3\left(a^2+2\,ab+b^2\right)\log\left(\left|\sqrt{-b\cos\left(fx+e\right)^2+\right.\right.}{8\sqrt{-b}|f|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*b*cos(f*x + e)^2 - 5*(a*b^2*f^4 + b^3*f^4)/(b^2*f^4))*sqrt(-b*cos(f*
x + e)^2 + a + b)*cos(f*x + e)/f - 3/8*(a^2 + 2*a*b + b^2)*log(abs(sqrt(-b*
cos(f*x + e)^2 + a + b) + sqrt(-b*f^2)*cos(f*x + e)/f))/(sqrt(-b)*abs(f))
```

3.134 $\int \csc(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=122

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{f}-\frac{b\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{2f}-\frac{\sqrt{b}(3a+b)\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{2f}$$

```
[Out] -(Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*
x]^2]])/(2*f) - (a^(3/2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[
e + f*x]^2]])/f - (b*Cos[e + f*x]*Sqrt[a + b - b*Cos[e + f*x]^2])/(2*f)
```

Rubi [A] time = 0.144398, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3186, 416, 523, 217, 203, 377, 206}

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{f}-\frac{b\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{2f}-\frac{\sqrt{b}(3a+b)\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*
x]^2]])/(2*f) - (a^(3/2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[
e + f*x]^2]])/f - (b*Cos[e + f*x]*Sqrt[a + b - b*Cos[e + f*x]^2])/(2*f)
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \csc(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx = -\frac{\text{Subst}\left(\int \frac{\left(a+b-bx^2\right)^{3/2}}{1-x^2}dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{b\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{2f} + \frac{\text{Subst}\left(\int \frac{-(a+b)(2a+b)+b(3a+b)x^2}{\left(1-x^2\right)\sqrt{a+b-bx^2}}dx\right.}{2f}$$

$$= -\frac{b\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{2f} - \frac{a^2\,\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\sqrt{a+b-bx^2}}dx, x,\right.}{f}$$

$$= -\frac{b\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{2f} - \frac{a^2\,\text{Subst}\left(\int \frac{1}{1-ax^2}dx, x, \frac{\cos(e}{\sqrt{a+b-b\cos}}\right.}{f}$$

$$= -\frac{\sqrt{b}(3a+b)\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{2f} - \frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{f} - \ldots$$

Mathematica [A] time = 0.898691, size = 141, normalized size = 1.16

$$-\frac{4a^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cos(e+fx)}{\sqrt{2a-b\cos(2(e+fx))+b}}\right)+\sqrt{2}b\cos(e+fx)\sqrt{2a-b\cos(2(e+fx))+b}-2\sqrt{-b}(3a+b)\log\left(\sqrt{2a-b\cos(2(\ldots\right.}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -(4*a^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cos[e + f*x])/Sqrt[2*a + b - b*Cos[2*(
e + f*x)]]] + Sqrt[2]*b*Cos[e + f*x]*Sqrt[2*a + b - b*Cos[2*(e + f*x)]] - 2
*Sqrt[-b]*(3*a + b)*Log[Sqrt[2]*Sqrt[-b]*Cos[e + f*x] + Sqrt[2*a + b - b*Co
s[2*(e + f*x)]]])/(4*f)
```

Maple [B] time = 1.888, size = 255, normalized size = 2.1

$$\frac{1}{4f\cos(fx+e)}\sqrt{(\cos(fx+e))^2\left(a+b(\sin(fx+e))^2\right)}\left(b^{\frac{3}{2}}\arctan\left(\frac{-2b(\cos(fx+e))^2+a+b}{2}\frac{1}{\sqrt{b}}\frac{1}{\sqrt{-b(\cos(fx+e)}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/4*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*(b^(3/2)*arctan(1/2*(-2*b*cos(f
*x+e)^2+a+b)/b^(1/2)/(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2))-2*a^(3/2)*
ln((-(a-b)*cos(f*x+e)^2-2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2
)-a-b)/(cos(f*x+e)^2-1))+3*b^(1/2)*a*arctan(1/2*(-2*b*cos(f*x+e)^2+a+b)/b^(
1/2)/(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2))-2*b*(-b*cos(f*x+e)^4+(a+b)
*cos(f*x+e)^2)^(1/2))/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.04739, size = 3204, normalized size = 26.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(8*sqrt(-b*cos(f*x + e)^2 + a + b)*b*cos(f*x + e) - (3*a + b)*sqrt(-
b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + b^4)*cos(f*x + e)^6 + 160*(a^2
*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3
+ b^4 - 32*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cos(f*x + e)^2 - 8*(16*b^3*c
os(f*x + e)^7 - 24*(a*b^2 + b^3)*cos(f*x + e)^5 + 10*(a^2*b + 2*a*b^2 + b^3
)*cos(f*x + e)^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e))*sqrt(-b*co
s(f*x + e)^2 + a + b)*sqrt(-b)) - 4*a^(3/2)*log(2*((a^2 - 6*a*b + b^2)*cos(
f*x + e)^4 + 2*(3*a^2 + 2*a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x +
e)^3 + (a + b)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) + a^2
+ 2*a*b + b^2)/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/f, 1/16*(8*sqrt(-a
)*a*arctan(-1/2*((a - b)*cos(f*x + e)^2 + a + b)*sqrt(-b*cos(f*x + e)^2 + a
 + b)*sqrt(-a)/(a*b*cos(f*x + e)^3 - (a^2 + a*b)*cos(f*x + e))) - 8*sqrt(-b
*cos(f*x + e)^2 + a + b)*b*cos(f*x + e) + (3*a + b)*sqrt(-b)*log(128*b^4*co
s(f*x + e)^8 - 256*(a*b^3 + b^4)*cos(f*x + e)^6 + 160*(a^2*b^2 + 2*a*b^3 +
b^4)*cos(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4 - 32*(a^3*b
 + 3*a^2*b^2 + 3*a*b^3 + b^4)*cos(f*x + e)^2 - 8*(16*b^3*cos(f*x + e)^7 - 2
4*(a*b^2 + b^3)*cos(f*x + e)^5 + 10*(a^2*b + 2*a*b^2 + b^3)*cos(f*x + e)^3
- (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a
+ b)*sqrt(-b)))/f, 1/8*((3*a + b)*sqrt(b)*arctan(1/4*(8*b^2*cos(f*x + e)^4
```

```
- 8*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(-b*cos(f*x + e)^2
+ a + b)*sqrt(b)/(2*b^3*cos(f*x + e)^5 - 3*(a*b^2 + b^3)*cos(f*x + e)^3 + (
a^2*b + 2*a*b^2 + b^3)*cos(f*x + e))) - 4*sqrt(-b*cos(f*x + e)^2 + a + b)*b
*cos(f*x + e) + 2*a^(3/2)*log(2*((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 2*(3*
a^2 + 2*a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + (a + b)*cos
(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) + a^2 + 2*a*b + b^2)/(co
s(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/f, 1/8*(4*sqrt(-a)*a*arctan(-1/2*((a
 - b)*cos(f*x + e)^2 + a + b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a)/(a*b
*cos(f*x + e)^3 - (a^2 + a*b)*cos(f*x + e))) + (3*a + b)*sqrt(b)*arctan(1/4
*(8*b^2*cos(f*x + e)^4 - 8*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*
sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(b)/(2*b^3*cos(f*x + e)^5 - 3*(a*b^2 +
b^3)*cos(f*x + e)^3 + (a^2*b + 2*a*b^2 + b^3)*cos(f*x + e))) - 4*sqrt(-b*co
s(f*x + e)^2 + a + b)*b*cos(f*x + e))/f]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*csc(f*x + e), x)
```

3.135 $\int \csc^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=128

$$-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{f}-\frac{\sqrt{a}(a+3b)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{2f}-\frac{a\cot(e+fx)\csc(e+fx)\sqrt{a-b\cos^2(e+fx)+}}{2f}$$

```
[Out] -((b^(3/2)*ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]])/f
) - (Sqrt[a]*(a + 3*b)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e
+ f*x]^2]])/(2*f) - (a*Sqrt[a + b - b*Cos[e + f*x]^2]*Cot[e + f*x]*Csc[e +
f*x])/(2*f)
```

Rubi [A] time = 0.151244, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3186, 413, 523, 217, 203, 377, 206}

$$-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{f}-\frac{\sqrt{a}(a+3b)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{2f}-\frac{a\cot(e+fx)\csc(e+fx)\sqrt{a-b\cos^2(e+fx)+}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -((b^(3/2)*ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]])/f
) - (Sqrt[a]*(a + 3*b)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e
+ f*x]^2]])/(2*f) - (a*Sqrt[a + b - b*Cos[e + f*x]^2]*Cot[e + f*x]*Csc[e +
f*x])/(2*f)
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \csc^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx = -\frac{\operatorname{Subst}\left(\int \frac{\left(a+b-bx^2\right)^{3/2}}{\left(1-x^2\right)^2}dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{a\sqrt{a+b-b\cos^2(e+fx)}\cot(e+fx)\csc(e+fx)}{2f} + \frac{\operatorname{Subst}\left(\int \frac{-(a+b)(a+2}{\left(1-x^2\right)\sqrt{a}}\right.}{}$$

$$= -\frac{a\sqrt{a+b-b\cos^2(e+fx)}\cot(e+fx)\csc(e+fx)}{2f} - \frac{b^2\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+b-b}}\right.}{}$$

$$= -\frac{a\sqrt{a+b-b\cos^2(e+fx)}\cot(e+fx)\csc(e+fx)}{2f} - \frac{b^2\operatorname{Subst}\left(\int \frac{1}{1+bx^2}d\right.}{}$$

$$= -\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{f} - \frac{\sqrt{a}(a+3b)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{2f} - $$

Mathematica [A] time = 1.21437, size = 147, normalized size = 1.15

$$-\frac{4(-b)^{3/2}\log\left(\sqrt{2a-b\cos(2(e+fx))+b}+\sqrt{2}\sqrt{-b}\cos(e+fx)\right)+2\sqrt{a}(a+3b)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cos(e+fx)}{\sqrt{2a-b\cos(2(e+fx))+b}}\right)+\sqrt{2}a\,c}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -(2*Sqrt[a]*(a + 3*b)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cos[e + f*x])/Sqrt[2*a + b -
 b*Cos[2*(e + f*x)]]] + Sqrt[2]*a*Sqrt[2*a + b - b*Cos[2*(e + f*x)]]*Cot[e
+ f*x]*Csc[e + f*x] + 4*(-b)^(3/2)*Log[Sqrt[2]*Sqrt[-b]*Cos[e + f*x] + Sqrt
[2*a + b - b*Cos[2*(e + f*x)]]])/(4*f)
```

Maple [B] time = 1.911, size = 287, normalized size = 2.2

$$\frac{1}{4\left(\sin\left(fx+e\right)\right)^{2}\cos\left(fx+e\right)f}\sqrt{\left(\cos\left(fx+e\right)\right)^{2}\left(a+b\left(\sin\left(fx+e\right)\right)^{2}\right)}\left(2b^{3/2}\arctan\left(1/2\frac{2b\left(\sin\left(fx+e\right)\right)^{2}}{\sqrt{b}\sqrt{\left(\cos\left(fx+e\right)\right)^{2}\left(a+b\right.}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/4*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*(2*b^(3/2)*arctan(1/2/b^(1/2)*(
2*b*sin(f*x+e)^2+a-b)/(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2))*sin(f*x+e)^2
-a^(3/2)*ln(((a-b)*cos(f*x+e)^2+2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)
^2)^(1/2)+a+b)/sin(f*x+e)^2)*sin(f*x+e)^2-3*a^(1/2)*b*ln(((a-b)*cos(f*x+e)^
2+2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)+a+b)/sin(f*x+e)^2)*s
in(f*x+e)^2-2*a*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2))/sin(f*x+e)^2/cos(f
*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^3, x)
```

Fricas [B] time = 5.7878, size = 3572, normalized size = 27.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(-b*cos(f*x + e)^2 + a + b)*a*cos(f*x + e) + (b*cos(f*x + e)^2
- b)*sqrt(-b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + b^4)*cos(f*x + e)^6
 + 160*(a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*b^2
 + 4*a*b^3 + b^4 - 32*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cos(f*x + e)^2 -
8*(16*b^3*cos(f*x + e)^7 - 24*(a*b^2 + b^3)*cos(f*x + e)^5 + 10*(a^2*b + 2*
a*b^2 + b^3)*cos(f*x + e)^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e))
*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)) + ((a + 3*b)*cos(f*x + e)^2 - a
- 3*b)*sqrt(a)*log(2*((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 2*(3*a^2 + 2*a*b
 - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + (a + b)*cos(f*x + e))*
sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) + a^2 + 2*a*b + b^2)/(cos(f*x + e)^
4 - 2*cos(f*x + e)^2 + 1)))/(f*cos(f*x + e)^2 - f), 1/8*(2*((a + 3*b)*cos(f
*x + e)^2 - a - 3*b)*sqrt(-a)*arctan(-1/2*((a - b)*cos(f*x + e)^2 + a + b)*
sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a)/(a*b*cos(f*x + e)^3 - (a^2 + a*b)*
cos(f*x + e))) + 4*sqrt(-b*cos(f*x + e)^2 + a + b)*a*cos(f*x + e) + (b*cos(
f*x + e)^2 - b)*sqrt(-b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + b^4)*cos
(f*x + e)^6 + 160*(a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 + a^4 + 4*a^3*b
```

```
+ 6*a^2*b^2 + 4*a*b^3 + b^4 - 32*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cos(f*
x + e)^2 - 8*(16*b^3*cos(f*x + e)^7 - 24*(a*b^2 + b^3)*cos(f*x + e)^5 + 10*
(a^2*b + 2*a*b^2 + b^3)*cos(f*x + e)^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*co
s(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)))/(f*cos(f*x + e)^2 -
f), 1/8*(2*(b*cos(f*x + e)^2 - b)*sqrt(b)*arctan(1/4*(8*b^2*cos(f*x + e)^4
- 8*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(-b*cos(f*x + e)^2
+ a + b)*sqrt(b)/(2*b^3*cos(f*x + e)^5 - 3*(a*b^2 + b^3)*cos(f*x + e)^3 + (
a^2*b + 2*a*b^2 + b^3)*cos(f*x + e))) + 4*sqrt(-b*cos(f*x + e)^2 + a + b)*a
*cos(f*x + e) + ((a + 3*b)*cos(f*x + e)^2 - a - 3*b)*sqrt(a)*log(2*((a^2 -
6*a*b + b^2)*cos(f*x + e)^4 + 2*(3*a^2 + 2*a*b - b^2)*cos(f*x + e)^2 - 4*((
a - b)*cos(f*x + e)^3 + (a + b)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a +
b)*sqrt(a) + a^2 + 2*a*b + b^2)/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/(
f*cos(f*x + e)^2 - f), 1/4*(((a + 3*b)*cos(f*x + e)^2 - a - 3*b)*sqrt(-a)*a
rctan(-1/2*((a - b)*cos(f*x + e)^2 + a + b)*sqrt(-b*cos(f*x + e)^2 + a + b)
*sqrt(-a)/(a*b*cos(f*x + e)^3 - (a^2 + a*b)*cos(f*x + e))) + (b*cos(f*x + e
)^2 - b)*sqrt(b)*arctan(1/4*(8*b^2*cos(f*x + e)^4 - 8*(a*b + b^2)*cos(f*x +
 e)^2 + a^2 + 2*a*b + b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(b)/(2*b^3*c
os(f*x + e)^5 - 3*(a*b^2 + b^3)*cos(f*x + e)^3 + (a^2*b + 2*a*b^2 + b^3)*co
s(f*x + e))) + 2*sqrt(-b*cos(f*x + e)^2 + a + b)*a*cos(f*x + e))/(f*cos(f*x
 + e)^2 - f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^3, x)
```

3.136 $\int \csc^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=128

$$-\frac{3(a+b)^2\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{8\sqrt{a}f}-\frac{\cot(e+fx)\csc^3(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}{4f}-\frac{3(a+b)\cot(e+fx)\csc(e\cdot}{}$$

```
[Out] (-3*(a + b)^2*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]
])/(8*Sqrt[a]*f) - (3*(a + b)*Sqrt[a + b - b*Cos[e + f*x]^2]*Cot[e + f*x]*C
sc[e + f*x])/(8*f) - ((a + b - b*Cos[e + f*x]^2)^(3/2)*Cot[e + f*x]*Csc[e +
 f*x]^3)/(4*f)
```

Rubi [A] time = 0.125199, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3186, 378, 377, 206}

$$-\frac{3(a+b)^2\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{8\sqrt{a}f}-\frac{\cot(e+fx)\csc^3(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}{4f}-\frac{3(a+b)\cot(e+fx)\csc(e\cdot}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-3*(a + b)^2*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]
])/(8*Sqrt[a]*f) - (3*(a + b)*Sqrt[a + b - b*Cos[e + f*x]^2]*Cot[e + f*x]*C
sc[e + f*x])/(8*f) - ((a + b - b*Cos[e + f*x]^2)^(3/2)*Cot[e + f*x]*Csc[e +
 f*x]^3)/(4*f)
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \csc^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(a+b-bx^2\right)^{3/2}}{\left(1-x^2\right)^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\left(a+b-b\cos^2(e+fx)\right)^{3/2}\cot(e+fx)\csc^3(e+fx)}{4f} - \frac{(3(a+b))\,\text{Subst}}{}$$

$$= -\frac{3(a+b)\sqrt{a+b-b\cos^2(e+fx)}\cot(e+fx)\csc(e+fx)}{8f} - \frac{\left(a+b-b\,\text{co}\right.}{}$$

$$= -\frac{3(a+b)\sqrt{a+b-b\cos^2(e+fx)}\cot(e+fx)\csc(e+fx)}{8f} - \frac{\left(a+b-b\,\text{co}\right.}{}$$

$$= -\frac{3(a+b)^2\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{8\sqrt{a}f} - \frac{3(a+b)\sqrt{a+b-b\cos^2(e+fx)}\,\text{c}}{8f}$$

Mathematica [A] time = 0.703615, size = 114, normalized size = 0.89

$$-\frac{\frac{6(a+b)^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cos(e+fx)}{\sqrt{2a-b\cos(2(e+fx))+b}}\right)}{\sqrt{a}} + \sqrt{2}\cot(e+fx)\csc(e+fx)\sqrt{2a-b\cos(2(e+fx))+b}\left(2a\csc^2(e+fx)+3a+5b\right)}{16f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -((6*(a + b)^2*ArcTanh[(Sqrt[2]*Sqrt[a]*Cos[e + f*x])/Sqrt[2*a + b - b*Cos[
2*(e + f*x)]]])/Sqrt[a] + Sqrt[2]*Sqrt[2*a + b - b*Cos[2*(e + f*x)]]*Cot[e
+ f*x]*Csc[e + f*x]*(3*a + 5*b + 2*a*Csc[e + f*x]^2))/(16*f)
```

Maple [B] time = 1.58, size = 376, normalized size = 2.9

$$-\frac{1}{16\left(\sin\left(fx+e\right)\right)^4\cos\left(fx+e\right)f}\sqrt{\left(\cos\left(fx+e\right)\right)^2\left(a+b\left(\sin\left(fx+e\right)\right)^2\right)}\left(3a^2\ln\left(\frac{(a-b)\left(\cos\left(fx+e\right)\right)^2+2\sqrt{a}\sqrt{}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5*(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/16*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*(3*a^2*ln(((a-b)*cos(f*x+e)^2
+2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)+a+b)/sin(f*x+e)^2)*si
n(f*x+e)^4+6*a*b*ln(((a-b)*cos(f*x+e)^2+2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*co
s(f*x+e)^2)^(1/2)+a+b)/sin(f*x+e)^2)*sin(f*x+e)^4+3*b^2*ln(((a-b)*cos(f*x+e
)^2+2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)+a+b)/sin(f*x+e)^2)
*sin(f*x+e)^4+6*a^(3/2)*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*sin(f*x+e)^
2+10*b*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*a^(1/2)*sin(f*x+e)^2+4*a^(3/
2)*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2))/a^(1/2)/sin(f*x+e)^4/cos(f*x+e)
/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^5, x)
```

Fricas [A] time = 4.33269, size = 1216, normalized size = 9.5

$$\left[\frac{3\left(\left(a^{2}+2\,ab+b^{2}\right)\cos\left(fx+e\right)^{4}-2\left(a^{2}+2\,ab+b^{2}\right)\cos\left(fx+e\right)^{2}+a^{2}+2\,ab+b^{2}\right)\sqrt{a}\log\left(\frac{2\left(\left(a^{2}-6\,ab+b^{2}\right)\cos\left(fx+e\right)^{4}+2\left(3\,a^{2}\right.\right.}{}\right.}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(3*((a^2 + 2*a*b + b^2)*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f*
x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(a)*log(2*((a^2 - 6*a*b + b^2)*cos(f*x +
e)^4 + 2*(3*a^2 + 2*a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 +
 (a + b)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) + a^2 + 2*a*
b + b^2)/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)) + 4*((3*a^2 + 5*a*b)*cos(
f*x + e)^3 - 5*(a^2 + a*b)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b))/(
a*f*cos(f*x + e)^4 - 2*a*f*cos(f*x + e)^2 + a*f), 1/16*(3*((a^2 + 2*a*b + b
^2)*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b
^2)*sqrt(-a)*arctan(-1/2*((a - b)*cos(f*x + e)^2 + a + b)*sqrt(-b*cos(f*x +
 e)^2 + a + b)*sqrt(-a)/(a*b*cos(f*x + e)^3 - (a^2 + a*b)*cos(f*x + e))) +
2*((3*a^2 + 5*a*b)*cos(f*x + e)^3 - 5*(a^2 + a*b)*cos(f*x + e))*sqrt(-b*cos
(f*x + e)^2 + a + b))/(a*f*cos(f*x + e)^4 - 2*a*f*cos(f*x + e)^2 + a*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^5, x)
```

3.137 $\int \csc^7(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=197

$$-\frac{(5a-b)(a+b)^2\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{16a^{3/2}f}-\frac{\cot(e+fx)\csc^5(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{5/2}}{6af}-\frac{(5a-b)\cot(e+fx)}{}$$

```
[Out] -((5*a - b)*(a + b)^2*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e +
 f*x]^2]])/(16*a^(3/2)*f) - ((5*a - b)*(a + b)*Sqrt[a + b - b*Cos[e + f*x]^
2]*Cot[e + f*x]*Csc[e + f*x])/(16*a*f) - ((5*a - b)*(a + b - b*Cos[e + f*x]
^2)^(3/2)*Cot[e + f*x]*Csc[e + f*x]^3)/(24*a*f) - ((a + b - b*Cos[e + f*x]^
2)^(5/2)*Cot[e + f*x]*Csc[e + f*x]^5)/(6*a*f)
```

Rubi [A] time = 0.174983, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3186, 382, 378, 377, 206}

$$-\frac{(5a-b)(a+b)^2\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{16a^{3/2}f}-\frac{\cot(e+fx)\csc^5(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{5/2}}{6af}-\frac{(5a-b)\cot(e+fx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^7*(a + b*Sin[e + f*x]^2)^(3/2),x]
```


```
[Out] -((5*a - b)*(a + b)^2*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e +
 f*x]^2]])/(16*a^(3/2)*f) - ((5*a - b)*(a + b)*Sqrt[a + b - b*Cos[e + f*x]^
2]*Cot[e + f*x]*Csc[e + f*x])/(16*a*f) - ((5*a - b)*(a + b - b*Cos[e + f*x]
^2)^(3/2)*Cot[e + f*x]*Csc[e + f*x]^3)/(24*a*f) - ((a + b - b*Cos[e + f*x]^
2)^(5/2)*Cot[e + f*x]*Csc[e + f*x]^5)/(6*a*f)
```


Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \csc^7(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx = -\frac{\text{Subst}\left(\int \frac{\left(a+b-bx^2\right)^{3/2}}{\left(1-x^2\right)^4}dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\left(a+b-b\cos^2(e+fx)\right)^{5/2}\cot(e+fx)\csc^5(e+fx)}{6af} - \frac{(5a-b)\,\text{Subst}\Big(\int$$

$$= -\frac{(5a-b)\left(a+b-b\cos^2(e+fx)\right)^{3/2}\cot(e+fx)\csc^3(e+fx)}{24af} - \frac{\Big(a+b-$$

$$= -\frac{(5a-b)(a+b)\sqrt{a+b-b\cos^2(e+fx)}\cot(e+fx)\csc(e+fx)}{16af} - \frac{(5a-$$

$$= -\frac{(5a-b)(a+b)\sqrt{a+b-b\cos^2(e+fx)}\cot(e+fx)\csc(e+fx)}{16af} - \frac{(5a-$$

$$= -\frac{(5a-b)(a+b)^2\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{16a^{3/2}f} - \frac{(5a-b)(a+b)\sqrt{a+b-b\,\text{c}$$

Mathematica [A] time = 1.25992, size = 161, normalized size = 0.82

$$\frac{-\sqrt{2}\sqrt{a}\csc^2(e+fx)\sqrt{2a-b\cos(2(e+fx))+b}\left(\left(15a^2+22ab+3b^2\right)\cos(e+fx)+2a\cot(e+fx)\csc(e+fx)\left(4a\,\text{cs}\right.\right.}{96a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^7*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-6*(5*a - b)*(a + b)^2*ArcTanh[(Sqrt[2]*Sqrt[a]*Cos[e + f*x])/Sqrt[2*a + b
 - b*Cos[2*(e + f*x)]]] - Sqrt[2]*Sqrt[a]*Sqrt[2*a + b - b*Cos[2*(e + f*x)]
]*Csc[e + f*x]^2*((15*a^2 + 22*a*b + 3*b^2)*Cos[e + f*x] + 2*a*Cot[e + f*x]
*Csc[e + f*x]*(5*a + 7*b + 4*a*Csc[e + f*x]^2)))/(96*a^(3/2)*f)
```

Maple [B] time = 2.063, size = 565, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^7*(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/96*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*(15*a^4*ln(((a-b)*cos(f*x+e)^
2+2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)+a+b)/sin(f*x+e)^2)*s
in(f*x+e)^6+27*a^3*b*ln(((a-b)*cos(f*x+e)^2+2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b
)*cos(f*x+e)^2)^(1/2)+a+b)/sin(f*x+e)^2)*sin(f*x+e)^6+9*b^2*ln(((a-b)*cos(f
*x+e)^2+2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)+a+b)/sin(f*x+e
)^2)*sin(f*x+e)^6*a^2-3*b^3*ln(((a-b)*cos(f*x+e)^2+2*a^(1/2)*(-b*cos(f*x+e)
^4+(a+b)*cos(f*x+e)^2)^(1/2)+a+b)/sin(f*x+e)^2)*sin(f*x+e)^6*a+30*a^(7/2)*(
cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*sin(f*x+e)^4+44*b*(cos(f*x+e)^2*(a+b
*sin(f*x+e)^2))^(1/2)*sin(f*x+e)^4*a^(5/2)+6*b^2*(cos(f*x+e)^2*(a+b*sin(f*x
+e)^2))^(1/2)*sin(f*x+e)^4*a^(3/2)+20*a^(7/2)*(cos(f*x+e)^2*(a+b*sin(f*x+e)
^2))^(1/2)*sin(f*x+e)^2+28*b*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*sin(f*
x+e)^2*a^(5/2)+16*a^(7/2)*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2))/sin(f*x+
e)^6/a^(5/2)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^{7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^7*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^7, x)
```

Fricas [A] time = 10.5865, size = 1773, normalized size = 9.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^7*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*((5*a^3 + 9*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^6 - 3*(5*a^3 + 9
*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^4 - 5*a^3 - 9*a^2*b - 3*a*b^2 + b^3 +
3*(5*a^3 + 9*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^2)*sqrt(a)*log(2*((a^2 - 6
*a*b + b^2)*cos(f*x + e)^4 + 2*(3*a^2 + 2*a*b - b^2)*cos(f*x + e)^2 + 4*((a
 - b)*cos(f*x + e)^3 + (a + b)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b
)*sqrt(a) + a^2 + 2*a*b + b^2)/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)) - 4
*((15*a^3 + 22*a^2*b + 3*a*b^2)*cos(f*x + e)^5 - 2*(20*a^3 + 29*a^2*b + 3*a
*b^2)*cos(f*x + e)^3 + 3*(11*a^3 + 12*a^2*b + a*b^2)*cos(f*x + e))*sqrt(-b*
cos(f*x + e)^2 + a + b))/(a^2*f*cos(f*x + e)^6 - 3*a^2*f*cos(f*x + e)^4 + 3
*a^2*f*cos(f*x + e)^2 - a^2*f), 1/96*(3*((5*a^3 + 9*a^2*b + 3*a*b^2 - b^3)*
cos(f*x + e)^6 - 3*(5*a^3 + 9*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^4 - 5*a^3
 - 9*a^2*b - 3*a*b^2 + b^3 + 3*(5*a^3 + 9*a^2*b + 3*a*b^2 - b^3)*cos(f*x +
e)^2)*sqrt(-a)*arctan(-1/2*((a - b)*cos(f*x + e)^2 + a + b)*sqrt(-b*cos(f*x
 + e)^2 + a + b)*sqrt(-a)/(a*b*cos(f*x + e)^3 - (a^2 + a*b)*cos(f*x + e)))
+ 2*((15*a^3 + 22*a^2*b + 3*a*b^2)*cos(f*x + e)^5 - 2*(20*a^3 + 29*a^2*b +
3*a*b^2)*cos(f*x + e)^3 + 3*(11*a^3 + 12*a^2*b + a*b^2)*cos(f*x + e))*sqrt(
-b*cos(f*x + e)^2 + a + b))/(a^2*f*cos(f*x + e)^6 - 3*a^2*f*cos(f*x + e)^4
+ 3*a^2*f*cos(f*x + e)^2 - a^2*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**7*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^7\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^7*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^7, x)
```

3.138 $\int \sin^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=325

$$-\frac{\left(a^2+11ab+8b^2\right)\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{35bf}+\frac{a(a+b)\left(2a^2-5ab-8b^2\right)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{}}{35b^2f\sqrt{a+b\sin^2(}}$$

```
[Out] -((a^2 + 11*a*b + 8*b^2)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^
2])/(35*b*f) - (2*(4*a + 3*b)*Cos[e + f*x]*Sin[e + f*x]^3*Sqrt[a + b*Sin[e
+ f*x]^2])/(35*f) - (b*Cos[e + f*x]*Sin[e + f*x]^5*Sqrt[a + b*Sin[e + f*x]^
2])/(7*f) - (2*(a + 2*b)*(a^2 - 4*a*b - 4*b^2)*Sqrt[Cos[e + f*x]^2]*Ellipti
cE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(
35*b^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (a*(a + b)*(2*a^2 - 5*a*b - 8*b^
2)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x
]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(35*b^2*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.482311, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3188, 477, 582, 524, 426, 424, 421, 419}

$$-\frac{\left(a^2+11ab+8b^2\right)\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{35bf}+\frac{a(a+b)\left(2a^2-5ab-8b^2\right)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{}}{35b^2f\sqrt{a+b\sin^2(}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a^2 + 11*a*b + 8*b^2)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^
2])/(35*b*f) - (2*(4*a + 3*b)*Cos[e + f*x]*Sin[e + f*x]^3*Sqrt[a + b*Sin[e
+ f*x]^2])/(35*f) - (b*Cos[e + f*x]*Sin[e + f*x]^5*Sqrt[a + b*Sin[e + f*x]^
2])/(7*f) - (2*(a + 2*b)*(a^2 - 4*a*b - 4*b^2)*Sqrt[Cos[e + f*x]^2]*Ellipti
cE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(
35*b^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (a*(a + b)*(2*a^2 - 5*a*b - 8*b^
2)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x
]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(35*b^2*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] && !IntegerQ[p]
```

Rule 477

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)
^(q - 1))/(b*e*(m + n*(p + q) + 1)), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
```

```
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \sin^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^4\left(a+bx^2\right)^{3/2}}{\sqrt{1-x^2}}dx,x,\sin(e+fx)\right)}{f}$$

$$= -\frac{b\cos(e+fx)\sin^5(e+fx)\sqrt{a+b\sin^2(e+fx)}}{7f} - \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+f\right.}{}$$

$$= -\frac{2(4a+3b)\cos(e+fx)\sin^3(e+fx)\sqrt{a+b\sin^2(e+fx)}}{35f} - \frac{b\cos(e+fx)\sin}{}$$

$$= -\frac{\left(a^2+11ab+8b^2\right)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{35bf} - \frac{2(4a+3b)}{}$$

$$= -\frac{\left(a^2+11ab+8b^2\right)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{35bf} - \frac{2(4a+3b)}{}$$

$$= -\frac{\left(a^2+11ab+8b^2\right)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{35bf} - \frac{2(4a+3b)}{}$$

$$= -\frac{\left(a^2+11ab+8b^2\right)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{35bf} - \frac{2(4a+3b)}{}$$

Mathematica [A] time = 2.74001, size = 249, normalized size = 0.77

$$\frac{\sqrt{2}b\sin(2(e+fx))\left(b\left(144a^2+480ab+299b^2\right)\cos(2(e+fx))-496a^2b-32a^3-2b^2(26a+27b)\cos(4(e+fx))-684ab\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-128*a*(a^3 - 2*a^2*b - 12*a*b^2 - 8*b^3)*Sqrt[(2*a + b - b*Cos[2*(e + f*x
)])/a]*EllipticE[e + f*x, -(b/a)] + 64*a*(2*a^3 - 3*a^2*b - 13*a*b^2 - 8*b^
3)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)] + Sqrt
[2]*b*(-32*a^3 - 496*a^2*b - 684*a*b^2 - 250*b^3 + b*(144*a^2 + 480*a*b + 2
99*b^2)*Cos[2*(e + f*x)] - 2*b^2*(26*a + 27*b)*Cos[4*(e + f*x)] + 5*b^3*Cos
[6*(e + f*x)])*Sin[2*(e + f*x)])/(2240*b^2*f*Sqrt[2*a + b - b*Cos[2*(e + f*
x)]])
```

Maple [B] time = 1.324, size = 602, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/35*(5*b^4*sin(f*x+e)^9+13*a*b^3*sin(f*x+e)^7+b^4*sin(f*x+e)^7+9*a^2*b^2*s
in(f*x+e)^5+4*a*b^3*sin(f*x+e)^5+2*b^4*sin(f*x+e)^5+2*(cos(f*x+e)^2)^(1/2)*
((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^4-3*(co
s(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b
)^(1/2))*a^3*b-13*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*Ellipti
cF(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b^2-8*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+
e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b^3-2*(cos(f*x+e)^2)^
(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^4
+4*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(
-1/a*b)^(1/2))*a^3*b+24*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*E
llipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b^2+16*(cos(f*x+e)^2)^(1/2)*((a+b*s
in(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^3+a^3*b*sin(
f*x+e)^3+2*a^2*b^2*sin(f*x+e)^3-9*a*b^3*sin(f*x+e)^3-8*b^4*sin(f*x+e)^3-sin
(f*x+e)*a^3*b-11*a^2*b^2*sin(f*x+e)-8*a*b^3*sin(f*x+e))/b^2/cos(f*x+e)/(a+b
*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(b\cos\left(fx+e\right)^{6}-(a+3b)\cos\left(fx+e\right)^{4}+(2a+3b)\cos\left(fx+e\right)^{2}-a-b\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(b*cos(f*x + e)^6 - (a + 3*b)*cos(f*x + e)^4 + (2*a + 3*b)*cos(f*
x + e)^2 - a - b)*sqrt(-b*cos(f*x + e)^2 + a + b), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^4, x)
```

3.139 $\int \sin^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=218

$$\frac{\left(3a^2+13ab+8b^2\right)\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{15bf\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{\sin(e+fx)\cos(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{5f}-\frac{(3a+4b)\,\mathrm{s}}{}$$

```
[Out] -((3*a + 4*b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(15*f)
- (Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2))/(5*f) + ((3*a^2
+ 13*a*b + 8*b^2)*EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(1
5*b*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) - (a*(a + b)*(3*a + 4*b)*EllipticF[e
+ f*x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(15*b*f*Sqrt[a + b*Sin[e + f
*x]^2])
```

Rubi [A] time = 0.303901, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3170, 3172, 3178, 3177, 3183, 3182}

$$\frac{\left(3a^2+13ab+8b^2\right)\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{15bf\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{\sin(e+fx)\cos(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{5f}-\frac{(3a+4b)\,\mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -((3*a + 4*b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(15*f)
- (Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2))/(5*f) + ((3*a^2
+ 13*a*b + 8*b^2)*EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(1
5*b*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) - (a*(a + b)*(3*a + 4*b)*EllipticF[e
+ f*x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(15*b*f*Sqrt[a + b*Sin[e + f
*x]^2])
```

Rule 3170

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[(B*Cos[e + f*x]*Sin[e + f*x]*(a + b*Si
n[e + f*x]^2)^p)/(2*f*(p + 1)), x] + Dist[1/(2*(p + 1)), Int[(a + b*Sin[e +
 f*x]^2)^(p - 1)*Simp[a*B + 2*a*A*(p + 1) + (2*A*b*(p + 1) + B*(b + 2*a*p +
 2*b*p))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && GtQ[
p, 0]
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
```

```
f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rubi steps

$$\int \sin^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = -\frac{\cos(e+fx)\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{5f}+\frac{1}{5}\int\sqrt{a+b\sin^2(e+fx)}$$

$$= -\frac{(3a+4b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15f}-\frac{\cos(e+fx)\sin(e+}{}$$

$$= -\frac{(3a+4b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15f}-\frac{\cos(e+fx)\sin(e+}{}$$

$$= -\frac{(3a+4b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15f}-\frac{\cos(e+fx)\sin(e+}{}$$

$$= -\frac{(3a+4b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15f}-\frac{\cos(e+fx)\sin(e+}{}$$

Mathematica [A] time = 1.37974, size = 201, normalized size = 0.92

$$\frac{-\sqrt{2}b\sin(2(e+fx))\left(48a^2-4b(9a+7b)\cos(2(e+fx))+68ab+3b^2\cos(4(e+fx))+25b^2\right)-16a\left(3a^2+7ab+4b^2\right)\sqrt{}}{240bf\sqrt{2a-b\cos(2(e+fx))+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (16*a*(3*a^2 + 13*a*b + 8*b^2)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*Ellip
ticE[e + f*x, -(b/a)] - 16*a*(3*a^2 + 7*a*b + 4*b^2)*Sqrt[(2*a + b - b*Cos[
2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)] - Sqrt[2]*b*(48*a^2 + 68*a*b +
25*b^2 - 4*b*(9*a + 7*b)*Cos[2*(e + f*x)] + 3*b^2*Cos[4*(e + f*x)])*Sin[2*(
e + f*x)])/(240*b*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A] time = 1.236, size = 429, normalized size = 2.

$$-\frac{1}{15\,b\cos\left(fx+e\right)f}\left(-3\,b^{3}\left(\sin\left(fx+e\right)\right)^{7}-9\,ab^{2}\left(\sin\left(fx+e\right)\right)^{5}-b^{3}\left(\sin\left(fx+e\right)\right)^{5}+3\,\sqrt{\left(\cos\left(fx+e\right)\right)^{2}}\sqrt{\frac{a+b(}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/15*(-3*b^3*sin(f*x+e)^7-9*a*b^2*sin(f*x+e)^5-b^3*sin(f*x+e)^5+3*(cos(f*x
+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/
2))*a^3+7*a^2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(s
in(f*x+e),(-1/a*b)^(1/2))*b+4*a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)
^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^2-3*(cos(f*x+e)^2)^(1/2)*((a+
b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3-13*(cos(f
*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(
1/2))*a^2*b-8*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(s
in(f*x+e),(-1/a*b)^(1/2))*a*b^2-6*a^2*b*sin(f*x+e)^3+5*a*b^2*sin(f*x+e)^3+4
*sin(f*x+e)^3*b^3+6*sin(f*x+e)*a^2*b+4*a*b^2*sin(f*x+e))/b/cos(f*x+e)/(a+b*
sin(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\cos\left(fx+e\right)^{4}-(a+2\,b)\cos\left(fx+e\right)^{2}+a+b\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(f*x + e)^4 - (a + 2*b)*cos(f*x + e)^2 + a + b)*sqrt(-b*cos(
f*x + e)^2 + a + b), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^2, x)
```

3.140 $\int \left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=154

$$-\frac{b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(e+fx\middle|-\frac{b}{a}\right)}{3f\sqrt{a+b\sin^2(e+fx)}}+\frac{2(2a+b)\sqrt{a+b\sin^2(e+\cdots}}{3f\sqrt{\frac{b\sin^2(e+f\cdots}{a}}}$$

```
[Out] -(b*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f) + (2*(2*a +
 b)*EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f*Sqrt[1 + (b
*Sin[e + f*x]^2)/a]) - (a*(a + b)*EllipticF[e + f*x, -(b/a)]*Sqrt[1 + (b*Si
n[e + f*x]^2)/a])/(3*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.163589, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3180, 3172, 3178, 3177, 3183, 3182}

$$-\frac{b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(e+fx\middle|-\frac{b}{a}\right)}{3f\sqrt{a+b\sin^2(e+fx)}}+\frac{2(2a+b)\sqrt{a+b\sin^2(e+\cdots}}{3f\sqrt{\frac{b\sin^2(e+f\cdots}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -(b*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f) + (2*(2*a +
 b)*EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f*Sqrt[1 + (b
*Sin[e + f*x]^2)/a]) - (a*(a + b)*EllipticF[e + f*x, -(b/a)]*Sqrt[1 + (b*Si
n[e + f*x]^2)/a])/(3*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 3180

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p - 1))/(2*f*p), x] + Dist[
1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a + b
)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a +
 b, 0] && GtQ[p, 1]
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rubi steps

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}dx = -\frac{b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{1}{3}\int \frac{a(3a+b)+2b(2a+b)\sin^2(e+fx}{\sqrt{a+b\sin^2(e+fx)}}$$

$$= -\frac{b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{1}{3}(a(a+b))\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}dx +$$

$$= -\frac{b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{\left(2(2a+b)\sqrt{a+b\sin^2(e+fx)}\right)\int\sqrt{1+}}{3\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

$$= -\frac{b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{2(2a+b)E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e}}{3f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

Mathematica [A] time = 0.779802, size = 156, normalized size = 1.01

$$\frac{b\sin(2(e+fx))(-2a+b\cos(2(e+fx))-b)-2\sqrt{2}a(a+b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}F\left(e+fx\middle|-\frac{b}{a}\right)+4\sqrt{2}a(2a+b)\sqrt{\frac{2a-b\cos(2(e}{a}}}{6\sqrt{2}f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (4*Sqrt[2]*a*(2*a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e +
 f*x, -(b/a)] - 2*Sqrt[2]*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*
EllipticF[e + f*x, -(b/a)] + b*(-2*a - b + b*Cos[2*(e + f*x)])*Sin[2*(e + f
*x)])/(6*Sqrt[2]*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A] time = 1.211, size = 266, normalized size = 1.7

$$\frac{1}{f\cos\left(fx+e\right)}\left(-\frac{a^2}{3}\sqrt{\left(\cos\left(fx+e\right)\right)^2}\sqrt{\frac{a+b\left(\sin\left(fx+e\right)\right)^2}{a}}\mathrm{EllipticF}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)-\frac{ab}{3}\sqrt{\left(\cos\left(fx+e\right)\right)^2}\sqrt{\frac{a}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] (-1/3*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e
),(-1/a*b)^(1/2))*a^2-1/3*a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/
2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b+4/3*(cos(f*x+e)^2)^(1/2)*((a+b*si
n(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2+2/3*(cos(f*x+
e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2
))*a*b+1/3*b^2*sin(f*x+e)^5+1/3*a*b*sin(f*x+e)^3-1/3*b^2*sin(f*x+e)^3-1/3*s
in(f*x+e)*a*b)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-b\cos\left(fx+e\right)^{2}+a+b\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((-b*cos(f*x + e)^2 + a + b)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.141 $\int \csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=181

$$-\frac{a\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f}+\frac{a(a+b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}-$$

```
[Out] -((a*Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/f) - ((a - b)*Sqrt[Cos[e + f*
x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e
 + f*x]^2])/(f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (a*(a + b)*Sqrt[Cos[e + f*
x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[
e + f*x]^2)/a])/(f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.192046, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3188, 474, 524, 426, 424, 421, 419}

$$-\frac{a\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f}+\frac{a(a+b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}-$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a*Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/f) - ((a - b)*Sqrt[Cos[e + f*
x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e
 + f*x]^2])/(f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (a*(a + b)*Sqrt[Cos[e + f*
x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[
e + f*x]^2)/a])/(f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
```

```
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x^2\sqrt{1-x^2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= -\frac{a\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} + \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int\right.}{f}$$

$$= -\frac{a\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} + \frac{\left((-a+b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{S}}{f}$$

$$= -\frac{a\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} + \frac{\left((-a+b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\right.}{f}$$

$$= -\frac{a\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} - \frac{(a-b)\sqrt{\cos^2(e+fx)}E\left(\sin^{-1}(\sin(e+\right.}{f\sqrt{1+}}$$

Mathematica [A] time = 1.32456, size = 141, normalized size = 0.78

$$-\frac{a\left(\sqrt{2}\cot(e+fx)(2a-b\cos(2(e+fx))+b)-2(a+b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}F\left(e+fx\middle|-\frac{b}{a}\right)+2(a-b)\sqrt{\frac{2a-b\cos(2(e+fx))\cdot}{a}}\right.}{2f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -(a*(Sqrt[2]*(2*a + b - b*Cos[2*(e + f*x)])*Cot[e + f*x] + 2*(a - b)*Sqrt[(
2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] - 2*(a + b)*Sqr
t[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)]))/(2*f*Sqrt[
2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A] time = 1.124, size = 174, normalized size = 1.

$$\frac{a}{\sin\left(fx+e\right)\cos\left(fx+e\right)f}\left(\sin\left(fx+e\right)\sqrt{-\frac{b\left(\cos\left(fx+e\right)\right)^{2}}{a}+\frac{a+b}{a}}\sqrt{\left(\cos\left(fx+e\right)\right)^{2}}\left(\mathrm{EllipticF}\left(\sin\left(fx+e\right),\sqrt{\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] a*(sin(f*x+e)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(cos(f*x+e)^2)^(1/2)*(Ellip
ticF(sin(f*x+e),(-1/a*b)^(1/2))*a+EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-El
lipticE(sin(f*x+e),(-1/a*b)^(1/2))*a+EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*b
)+b*cos(f*x+e)^4+(-a-b)*cos(f*x+e)^2)/sin(f*x+e)/cos(f*x+e)/(a+b*sin(f*x+e)
^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(b\cos\left(fx+e\right)^{2}-a-b\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\csc\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(b*cos(f*x + e)^2 - a - b)*sqrt(-b*cos(f*x + e)^2 + a + b)*csc(f*
x + e)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^2, x)
```

3.142 $\int \csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=236

$$-\frac{2(a+2b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{a\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{(a+b)(2a+3b)\sqrt{\cos^2}}{}$$

```
[Out] (-2*(a + 2*b)*Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f) - (a*Cot[e + f
*x]*Csc[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2])/(3*f) - (2*(a + 2*b)*Sqrt[Co
s[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a +
 b*Sin[e + f*x]^2])/(3*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + ((a + b)*(2*a +
3*b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f
*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.292907, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3188, 474, 583, 524, 426, 424, 421, 419}

$$-\frac{2(a+2b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{a\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{(a+b)(2a+3b)\sqrt{\cos^2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2),x]
```


```
[Out] (-2*(a + 2*b)*Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f) - (a*Cot[e + f
*x]*Csc[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2])/(3*f) - (2*(a + 2*b)*Sqrt[Co
s[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a +
 b*Sin[e + f*x]^2])/(3*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + ((a + b)*(2*a +
3*b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f
*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*f*Sqrt[a + b*Sin[e + f*x]^2])
```


Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x^4\sqrt{1-x^2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= -\frac{a\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} + \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e\right.}{}$$

$$= -\frac{2(a+2b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} - \frac{a\cot(e+fx)\csc^2(e+fx)\sqrt{}}{3f}$$

$$= -\frac{2(a+2b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} - \frac{a\cot(e+fx)\csc^2(e+fx)\sqrt{}}{3f}$$

$$= -\frac{2(a+2b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} - \frac{a\cot(e+fx)\csc^2(e+fx)\sqrt{}}{3f}$$

$$= -\frac{2(a+2b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} - \frac{a\cot(e+fx)\csc^2(e+fx)\sqrt{}}{3f}$$

Mathematica [A] time = 4.50498, size = 201, normalized size = 0.85

$$\frac{2\left(2a^2+5ab+3b^2\right)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}F\left(e+fx\middle|-\frac{b}{a}\right) + \frac{\cot(e+fx)\csc^2(e+fx)\left(2\left(2a^2+7ab+4b^2\right)\cos(2(e+fx))-8a^2-b(a+2b)\cos(4(e+fx))\right.}{\sqrt{2}}}{6f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (((-8*a^2 - 13*a*b - 6*b^2 + 2*(2*a^2 + 7*a*b + 4*b^2)*Cos[2*(e + f*x)] - b
*(a + 2*b)*Cos[4*(e + f*x)])*Cot[e + f*x]*Csc[e + f*x]^2)/Sqrt[2] - 4*a*(a
+ 2*b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] +
2*(2*a^2 + 5*a*b + 3*b^2)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[
e + f*x, -(b/a)])/(6*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A] time = 1.169, size = 408, normalized size = 1.7

$$\frac{1}{3\left(\sin\left(fx+e\right)\right)^3\cos\left(fx+e\right)f}\left(2\,\text{EllipticF}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)\sqrt{\left(\cos\left(fx+e\right)\right)^2}\sqrt{\frac{a+b\left(\sin\left(fx+e\right)\right)^2}{a}}a^2\left(\sin\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/3*(2*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*(cos(f*x+e)^2)^(1/2)*((a+b*sin(
f*x+e)^2)/a)^(1/2)*a^2*sin(f*x+e)^3+5*b*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+
e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*sin(f*x+e)^3+3*b^2*(c
```

```
os(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*
b)^(1/2))*sin(f*x+e)^3-2*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*(cos(f*x+e)^2
)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*a^2*sin(f*x+e)^3-4*EllipticE(sin(f*x+e
),(-1/a*b)^(1/2))*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*a*b*sin
(f*x+e)^3+2*a*b*sin(f*x+e)^6+4*b^2*sin(f*x+e)^6+2*a^2*sin(f*x+e)^4+3*a*b*si
n(f*x+e)^4-4*b^2*sin(f*x+e)^4-a^2*sin(f*x+e)^2-5*a*b*sin(f*x+e)^2-a^2)/sin(
f*x+e)^3/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(b\cos\left(fx+e\right)^2-a-b\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}\csc\left(fx+e\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(b*cos(f*x + e)^2 - a - b)*sqrt(-b*cos(f*x + e)^2 + a + b)*csc(f*
x + e)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^4, x)
```

3.143 $\int \left(a+b\sin^2(c+dx)\right)^{5/2}\,dx$

Optimal. Leaf size=210

$$\frac{(23a^2+23ab+8b^2)\sqrt{a+b\sin^2(c+dx)}E\left(c+dx\middle|-\frac{b}{a}\right)}{15d\sqrt{\frac{b\sin^2(c+dx)}{a}+1}}-\frac{b\sin(c+dx)\cos(c+dx)\left(a+b\sin^2(c+dx)\right)^{3/2}}{5d}-\frac{4b(2a+}{}$$

```
[Out] (-4*b*(2*a + b)*Cos[c + d*x]*Sin[c + d*x]*Sqrt[a + b*Sin[c + d*x]^2])/(15*d
) - (b*Cos[c + d*x]*Sin[c + d*x]*(a + b*Sin[c + d*x]^2)^(3/2))/(5*d) + ((23
*a^2 + 23*a*b + 8*b^2)*EllipticE[c + d*x, -(b/a)]*Sqrt[a + b*Sin[c + d*x]^2
])/(15*d*Sqrt[1 + (b*Sin[c + d*x]^2)/a]) - (4*a*(a + b)*(2*a + b)*EllipticF
[c + d*x, -(b/a)]*Sqrt[1 + (b*Sin[c + d*x]^2)/a])/(15*d*Sqrt[a + b*Sin[c +
d*x]^2])
```

Rubi [A] time = 0.281686, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3180, 3170, 3172, 3178, 3177, 3183, 3182}

$$\frac{(23a^2+23ab+8b^2)\sqrt{a+b\sin^2(c+dx)}E\left(c+dx\middle|-\frac{b}{a}\right)}{15d\sqrt{\frac{b\sin^2(c+dx)}{a}+1}}-\frac{b\sin(c+dx)\cos(c+dx)\left(a+b\sin^2(c+dx)\right)^{3/2}}{5d}-\frac{4b(2a+}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x]^2)^(5/2),x]
```

```
[Out] (-4*b*(2*a + b)*Cos[c + d*x]*Sin[c + d*x]*Sqrt[a + b*Sin[c + d*x]^2])/(15*d
) - (b*Cos[c + d*x]*Sin[c + d*x]*(a + b*Sin[c + d*x]^2)^(3/2))/(5*d) + ((23
*a^2 + 23*a*b + 8*b^2)*EllipticE[c + d*x, -(b/a)]*Sqrt[a + b*Sin[c + d*x]^2
])/(15*d*Sqrt[1 + (b*Sin[c + d*x]^2)/a]) - (4*a*(a + b)*(2*a + b)*EllipticF
[c + d*x, -(b/a)]*Sqrt[1 + (b*Sin[c + d*x]^2)/a])/(15*d*Sqrt[a + b*Sin[c +
d*x]^2])
```

Rule 3180

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p - 1))/(2*f*p), x] + Dist[
1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a + b
)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a +
 b, 0] && GtQ[p, 1]
```

Rule 3170

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[(B*Cos[e + f*x]*Sin[e + f*x]*(a + b*Si
n[e + f*x]^2)^p)/(2*f*(p + 1)), x] + Dist[1/(2*(p + 1)), Int[(a + b*Sin[e +
 f*x]^2)^(p - 1)*Simp[a*B + 2*a*A*(p + 1) + (2*A*b*(p + 1) + B*(b + 2*a*p +
 2*b*p))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && GtQ[
p, 0]
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rubi steps

$$\int \left(a+b\sin^2(c+dx)\right)^{5/2}\,dx = -\frac{b\cos(c+dx)\sin(c+dx)\left(a+b\sin^2(c+dx)\right)^{3/2}}{5d}+\frac{1}{5}\int \sqrt{a+b\sin^2(c+dx)}\,\left(a(5a+b)+\right.$$

$$= -\frac{4b(2a+b)\cos(c+dx)\sin(c+dx)\sqrt{a+b\sin^2(c+dx)}}{15d}-\frac{b\cos(c+dx)\sin(c+dx)\left(a+b\right.}{5d}$$

$$= -\frac{4b(2a+b)\cos(c+dx)\sin(c+dx)\sqrt{a+b\sin^2(c+dx)}}{15d}-\frac{b\cos(c+dx)\sin(c+dx)\left(a+b\right.}{5d}$$

$$= -\frac{4b(2a+b)\cos(c+dx)\sin(c+dx)\sqrt{a+b\sin^2(c+dx)}}{15d}-\frac{b\cos(c+dx)\sin(c+dx)\left(a+b\right.}{5d}$$

$$= -\frac{4b(2a+b)\cos(c+dx)\sin(c+dx)\sqrt{a+b\sin^2(c+dx)}}{15d}-\frac{b\cos(c+dx)\sin(c+dx)\left(a+b\right.}{5d}$$

Mathematica [A] time = 1.44033, size = 194, normalized size = 0.92

$$\frac{-\sqrt{2}b\sin(2(c+dx))\left(88a^2-28b(2a+b)\cos(2(c+dx))+88ab+3b^2\cos(4(c+dx))+25b^2\right)-64a\left(2a^2+3ab+b^2\right)\sqrt{\frac{2a}{}}}{240d\sqrt{2a-b\cos(2(c+dx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x]^2)^(5/2),x]
```

```
[Out] (16*a*(23*a^2 + 23*a*b + 8*b^2)*Sqrt[(2*a + b - b*Cos[2*(c + d*x)])/a]*Elli
pticE[c + d*x, -(b/a)] - 64*a*(2*a^2 + 3*a*b + b^2)*Sqrt[(2*a + b - b*Cos[2
*(c + d*x)])/a]*EllipticF[c + d*x, -(b/a)] - Sqrt[2]*b*(88*a^2 + 88*a*b + 2
5*b^2 - 28*b*(2*a + b)*Cos[2*(c + d*x)] + 3*b^2*Cos[4*(c + d*x)])*Sin[2*(c
+ d*x)])/(240*d*Sqrt[2*a + b - b*Cos[2*(c + d*x)]])
```

Maple [A] time = 1.256, size = 437, normalized size = 2.1

$$\frac{1}{d\cos(dx+c)}\left(-\frac{b^3\sin(dx+c)(\cos(dx+c))^6}{5}+\frac{\left(14\,ab^2+10\,b^3\right)(\cos(dx+c))^4\sin(dx+c)}{15}+\frac{\left(-11\,a^2b-18\,ab^2-7\,b\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+sin(d*x+c)^2*b)^(5/2),x)
```

```
[Out] (-1/5*b^3*sin(d*x+c)*cos(d*x+c)^6+1/15*(14*a*b^2+10*b^3)*cos(d*x+c)^4*sin(d
*x+c)+1/15*(-11*a^2*b-18*a*b^2-7*b^3)*cos(d*x+c)^2*sin(d*x+c)-8/15*(cos(d*x
+c)^2)^(1/2)*(-b/a*cos(d*x+c)^2+(a+b)/a)^(1/2)*EllipticF(sin(d*x+c),(-1/a*b
)^(1/2))*a^3-4/5*a^2*(cos(d*x+c)^2)^(1/2)*(-b/a*cos(d*x+c)^2+(a+b)/a)^(1/2)
*EllipticF(sin(d*x+c),(-1/a*b)^(1/2))*b-4/15*a*(cos(d*x+c)^2)^(1/2)*(-b/a*c
os(d*x+c)^2+(a+b)/a)^(1/2)*EllipticF(sin(d*x+c),(-1/a*b)^(1/2))*b^2+23/15*(
cos(d*x+c)^2)^(1/2)*(-b/a*cos(d*x+c)^2+(a+b)/a)^(1/2)*EllipticE(sin(d*x+c),
(-1/a*b)^(1/2))*a^3+23/15*(cos(d*x+c)^2)^(1/2)*(-b/a*cos(d*x+c)^2+(a+b)/a)^
(1/2)*EllipticE(sin(d*x+c),(-1/a*b)^(1/2))*a^2*b+8/15*(cos(d*x+c)^2)^(1/2)*
(-b/a*cos(d*x+c)^2+(a+b)/a)^(1/2)*EllipticE(sin(d*x+c),(-1/a*b)^(1/2))*a*b^
2)/cos(d*x+c)/(a+sin(d*x+c)^2*b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sin(dx+c)^2+a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2\cos(dx+c)^4-2\left(ab+b^2\right)\cos(dx+c)^2+a^2+2\,ab+b^2\right)\sqrt{-b\cos(dx+c)^2+a+b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b +
 b^2)*sqrt(-b*cos(d*x + c)^2 + a + b), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(dx+c\right)^{2}+a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^2 + a)^(5/2), x)
```

3.144 $\int \frac{\sin^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=83

$$\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{2b^{3/2}f}-\frac{\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{2bf}$$

```
[Out] ((a - b)*ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]])/(2*
b^(3/2)*f) - (Cos[e + f*x]*Sqrt[a + b - b*Cos[e + f*x]^2])/(2*b*f)
```

Rubi [A] time = 0.0955315, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3186, 388, 217, 203}

$$\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{2b^{3/2}f}-\frac{\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{2bf}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] ((a - b)*ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]])/(2*
b^(3/2)*f) - (Cos[e + f*x]*Sqrt[a + b - b*Cos[e + f*x]^2])/(2*b*f)
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{\sqrt{a+b-bx^2}}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{2bf} + \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b-bx^2}}\,dx, x, \cos(e+fx)\right)}{2bf}$$

$$= -\frac{\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{2bf} + \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{2bf}$$

$$= \frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{2b^{3/2}f} - \frac{\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{2bf}$$

Mathematica [A] time = 0.278719, size = 105, normalized size = 1.27

$$\frac{(a-b)\log\left(\sqrt{2a-b\cos(2(e+fx))+b}+\sqrt{2}\sqrt{-b}\cos(e+fx)\right)}{2\sqrt{-b}bf} - \frac{\cos(e+fx)\sqrt{2a-b\cos(2(e+fx))+b}}{2\sqrt{2}bf}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -(Cos[e + f*x]*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])/(2*Sqrt[2]*b*f) + ((a -
b)*Log[Sqrt[2]*Sqrt[-b]*Cos[e + f*x] + Sqrt[2*a + b - b*Cos[2*(e + f*x)]]])
/(2*Sqrt[-b]*b*f)
```

Maple [B] time = 1.256, size = 186, normalized size = 2.2

$$-\frac{1}{4f\cos(fx+e)}\sqrt{(\cos(fx+e))^2\left(a+b(\sin(fx+e))^2\right)}\left(2b^{3/2}\sqrt{-b(\cos(fx+e))^4+(a+b)(\cos(fx+e))^2}+ba\,\text{arc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/4*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*(2*b^(3/2)*(-b*cos(f*x+e)^4+(a
+b)*cos(f*x+e)^2)^(1/2)+b*a*arctan(1/2*(-2*b*cos(f*x+e)^2+a+b)/b^(1/2)/(-b*
cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2))-b^2*arctan(1/2*(-2*b*cos(f*x+e)^2+a
+b)/b^(1/2)/(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)))/b^(5/2)/cos(f*x+e)
/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.32528, size = 1076, normalized size = 12.96

$$\left[-\frac{8\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}b\cos\left(fx+e\right)-(a-b)\sqrt{-b}\log\left(128\,b^{4}\cos\left(fx+e\right)^{8}-256\left(ab^{3}+b^{4}\right)\cos\left(fx+e\right)^{6}+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(8*sqrt(-b*cos(f*x + e)^2 + a + b)*b*cos(f*x + e) - (a - b)*sqrt(-b)
*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + b^4)*cos(f*x + e)^6 + 160*(a^2*b
^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 +
b^4 - 32*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cos(f*x + e)^2 + 8*(16*b^3*cos
(f*x + e)^7 - 24*(a*b^2 + b^3)*cos(f*x + e)^5 + 10*(a^2*b + 2*a*b^2 + b^3)*
cos(f*x + e)^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e))*sqrt(-b*cos(
f*x + e)^2 + a + b)*sqrt(-b)))/(b^2*f), -1/8*((a - b)*sqrt(b)*arctan(1/4*(8
*b^2*cos(f*x + e)^4 - 8*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqr
t(-b*cos(f*x + e)^2 + a + b)*sqrt(b)/(2*b^3*cos(f*x + e)^5 - 3*(a*b^2 + b^3
)*cos(f*x + e)^3 + (a^2*b + 2*a*b^2 + b^3)*cos(f*x + e))) + 4*sqrt(-b*cos(f
*x + e)^2 + a + b)*b*cos(f*x + e))/(b^2*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.83821, size = 126, normalized size = 1.52

$$-\frac{\sqrt{-\left(\cos\left(fx+e\right)^{2}-1\right)b+a}\cos\left(fx+e\right)}{2\,bf}+\frac{(a-b)\log\left(\left|\sqrt{-\left(\cos\left(fx+e\right)^{2}-1\right)b+a}+\frac{\sqrt{-bf^{2}}\cos(fx+e)}{f}\right|\right)}{2\sqrt{-b}b|f|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-(cos(f*x + e)^2 - 1)*b + a)*cos(f*x + e)/(b*f) + 1/2*(a - b)*log
(abs(sqrt(-(cos(f*x + e)^2 - 1)*b + a) + sqrt(-b*f^2)*cos(f*x + e)/f))/(sqr
t(-b)*b*abs(f))
```

3.145 $\int \frac{\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=41

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{\sqrt{b}f}$$

```
[Out] -(ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]]/(Sqrt[b]*f)
)
```

Rubi [A] time = 0.0472215, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3186, 217, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{\sqrt{b}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -(ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]]/(Sqrt[b]*f)
)
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b-bx^2}}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{f}$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{\sqrt{b}f}$$

Mathematica [A] time = 0.113035, size = 53, normalized size = 1.29

$$-\frac{\log\left(\sqrt{2a-b\cos(2(e+fx))+b}+\sqrt{2}\sqrt{-b}\cos(e+fx)\right)}{\sqrt{-b}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -(Log[Sqrt[2]*Sqrt[-b]*Cos[e + f*x] + Sqrt[2*a + b - b*Cos[2*(e + f*x)]]]/(
Sqrt[-b]*f))
```

Maple [B] time = 0.861, size = 99, normalized size = 2.4

$$\frac{1}{2f\cos\left(fx+e\right)}\sqrt{\left(\cos\left(fx+e\right)\right)^2\left(a+b\left(\sin\left(fx+e\right)\right)^2\right)}\arctan\left(\frac{2b\left(\sin\left(fx+e\right)\right)^2+a-b}{2}\frac{1}{\sqrt{b}}\frac{1}{\sqrt{\left(\cos\left(fx+e\right)\right)^2\left(a\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/2*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)/b^(1/2)*arctan(1/2/b^(1/2)*(2*b
*sin(f*x+e)^2+a-b)/(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2))/cos(f*x+e)/(a+b
*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.94815, size = 900, normalized size = 21.95

$$\left[-\frac{\sqrt{-b}\log\left(128\,b^4\cos\left(fx+e\right)^8 - 256\left(ab^3+b^4\right)\cos\left(fx+e\right)^6 + 160\left(a^2b^2+2\,ab^3+b^4\right)\cos\left(fx+e\right)^4 + a^4 + 4\,a^3b + 6\,a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*sqrt(-b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + b^4)*cos(f*x + e)^
6 + 160*(a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*b^
2 + 4*a*b^3 + b^4 - 32*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cos(f*x + e)^2 +
 8*(16*b^3*cos(f*x + e)^7 - 24*(a*b^2 + b^3)*cos(f*x + e)^5 + 10*(a^2*b + 2
*a*b^2 + b^3)*cos(f*x + e)^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)
)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b))/(b*f), 1/4*arctan(1/4*(8*b^2*co
s(f*x + e)^4 - 8*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(-b*co
s(f*x + e)^2 + a + b)*sqrt(b)/(2*b^3*cos(f*x + e)^5 - 3*(a*b^2 + b^3)*cos(f
*x + e)^3 + (a^2*b + 2*a*b^2 + b^3)*cos(f*x + e)))/(sqrt(b)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.76948, size = 68, normalized size = 1.66

$$-\frac{\log\left(\left|\sqrt{-b\cos\left(fx+e\right)^2+a+b}+\frac{\sqrt{-bf^2}\cos\left(fx+e\right)}{f}\right|\right)}{\sqrt{-b}\left|f\right|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(sqrt(-b*cos(f*x + e)^2 + a + b) + sqrt(-b*f^2)*cos(f*x + e)/f))/(s
qrt(-b)*abs(f))
```

3.146 $\int \frac{\csc(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=41

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{\sqrt{a}f}$$

```
[Out] -(ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]]/(Sqrt[a]*f
))
```

Rubi [A] time = 0.0759755, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3186, 377, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]]/(Sqrt[a]*f
))
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+b-bx^2}}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{f}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{\sqrt{a}f}$$

Mathematica [A] time = 0.149938, size = 48, normalized size = 1.17

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cos(e+fx)}{\sqrt{2a-b\cos(2(e+fx))+b}}\right)}{\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[(Sqrt[2]*Sqrt[a]*Cos[e + f*x])/Sqrt[2*a + b - b*Cos[2*(e + f*x)]]
]/(Sqrt[a]*f))
```

Maple [B] time = 0.875, size = 112, normalized size = 2.7

$$-\frac{1}{2f\cos(fx+e)}\sqrt{(\cos(fx+e))^2\left(a+b(\sin(fx+e))^2\right)}\ln\left(\frac{1}{(\sin(fx+e))^2}\left((a-b)(\cos(fx+e))^2+2\sqrt{a}\sqrt{-b(\cos(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/2*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)/a^(1/2)*ln(((a-b)*cos(f*x+e)^2
+2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)+a+b)/sin(f*x+e)^2)/co
s(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.77761, size = 558, normalized size = 13.61

$$\left[\frac{\log\left(\frac{2\left(\left(a^2-6ab+b^2\right)\cos\left(fx+e\right)^4+2\left(3a^2+2ab-b^2\right)\cos\left(fx+e\right)^2-4\left(\left(a-b\right)\cos\left(fx+e\right)^3+\left(a+b\right)\cos\left(fx+e\right)\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}\sqrt{a}+a^2+2ab+b^2\right)}{\cos\left(fx+e\right)^4-2\cos\left(fx+e\right)^2+1}\right)}{4\sqrt{a}f}, \sqrt{-\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(2*((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 2*(3*a^2 + 2*a*b - b^2)*co
s(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + (a + b)*cos(f*x + e))*sqrt(-b*co
s(f*x + e)^2 + a + b)*sqrt(a) + a^2 + 2*a*b + b^2)/(cos(f*x + e)^4 - 2*cos(
f*x + e)^2 + 1))/(sqrt(a)*f), 1/2*sqrt(-a)*arctan(-1/2*((a - b)*cos(f*x + e
)^2 + a + b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a)/(a*b*cos(f*x + e)^3 -
 (a^2 + a*b)*cos(f*x + e)))/(a*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(e+fx\right)}{\sqrt{a+b\sin^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)/sqrt(a + b*sin(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)/sqrt(b*sin(f*x + e)^2 + a), x)
```

3.147 $\int \frac{\csc^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=89

$$-\frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{2a^{3/2}f}-\frac{\cot(e+fx)\csc(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{2af}$$

```
[Out] -((a - b)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]])/(
2*a^(3/2)*f) - (Sqrt[a + b - b*Cos[e + f*x]^2]*Cot[e + f*x]*Csc[e + f*x])/(
2*a*f)
```

Rubi [A] time = 0.104088, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3186, 382, 377, 206}

$$-\frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{2a^{3/2}f}-\frac{\cot(e+fx)\csc(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((a - b)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]])/(
2*a^(3/2)*f) - (Sqrt[a + b - b*Cos[e + f*x]^2]*Cot[e + f*x]*Csc[e + f*x])/(
2*a*f)
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2\sqrt{a+b-bx^2}}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\sqrt{a+b-b\cos^2(e+fx)}\cot(e+fx)\csc(e+fx)}{2af} - \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+b-bx^2}}\,dx, x\right.}{2af}$$

$$= -\frac{\sqrt{a+b-b\cos^2(e+fx)}\cot(e+fx)\csc(e+fx)}{2af} - \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\cos(}{\sqrt{a+b-b}}\right.}{2af}$$

$$= -\frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{2a^{3/2}f} - \frac{\sqrt{a+b-b\cos^2(e+fx)}\cot(e+fx)\csc(e+fx)}{2af}$$

Mathematica [A] time = 0.309953, size = 102, normalized size = 1.15

$$\frac{-2(a-b)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cos(e+fx)}{\sqrt{2a-b\cos(2(e+fx))+b}}\right)-\sqrt{2}\sqrt{a}\cot(e+fx)\csc(e+fx)\sqrt{2a-b\cos(2(e+fx))+b}}{4a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-2*(a - b)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cos[e + f*x])/Sqrt[2*a + b - b*Cos[2*(
e + f*x)]]] - Sqrt[2]*Sqrt[a]*Sqrt[2*a + b - b*Cos[2*(e + f*x)]]*Cot[e + f*
x]*Csc[e + f*x])/(4*a^(3/2)*f)
```

Maple [B] time = 1.43, size = 231, normalized size = 2.6

$$-\frac{1}{4\left(\sin\left(fx+e\right)\right)^2\cos\left(fx+e\right)f}\sqrt{\left(\cos\left(fx+e\right)\right)^2\left(a+b\left(\sin\left(fx+e\right)\right)^2\right)}\left(\ln\left(\frac{1}{\left(\sin\left(fx+e\right)\right)^2}\left((a-b)\left(\cos\left(fx+e\right)\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/4*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*(ln(((a-b)*cos(f*x+e)^2+2*a^(1
/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)+a+b)/sin(f*x+e)^2)*sin(f*x+e
)^2*a^2-ln(((a-b)*cos(f*x+e)^2+2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^
2)^(1/2)+a+b)/sin(f*x+e)^2)*b*sin(f*x+e)^2*a+2*a^(3/2)*(cos(f*x+e)^2*(a+b*s
in(f*x+e)^2))^(1/2))/sin(f*x+e)^2/a^(5/2)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/
2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^3}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^3/sqrt(b*sin(f*x + e)^2 + a), x)
```

Fricas [A] time = 2.06539, size = 860, normalized size = 9.66

$$\left[\frac{4\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,a\cos\left(fx+e\right)-\left(\left(a-b\right)\cos\left(fx+e\right)^{2}-a+b\right)\sqrt{a}\log\left(\frac{2\left(\left(a^{2}-6ab+b^{2}\right)\cos\left(fx+e\right)^{4}+2\left(3a^{2}+2ab-b^{2}\right)c\cdots}{\cdots}\right.}{8\left(a^{2}f\cos\left(fx+e\right)^{2}-a^{2}f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(-b*cos(f*x + e)^2 + a + b)*a*cos(f*x + e) - ((a - b)*cos(f*x +
 e)^2 - a + b)*sqrt(a)*log(2*((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 2*(3*a^2
 + 2*a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + (a + b)*cos(f*
x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) + a^2 + 2*a*b + b^2)/(cos(f
*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/(a^2*f*cos(f*x + e)^2 - a^2*f), 1/4*(((
a - b)*cos(f*x + e)^2 - a + b)*sqrt(-a)*arctan(-1/2*((a - b)*cos(f*x + e)^2
 + a + b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a)/(a*b*cos(f*x + e)^3 - (a
^2 + a*b)*cos(f*x + e))) + 2*sqrt(-b*cos(f*x + e)^2 + a + b)*a*cos(f*x + e)
)/(a^2*f*cos(f*x + e)^2 - a^2*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^{3}\left(e+fx\right)}{\sqrt{a+b\sin^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)**3/sqrt(a + b*sin(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^{3}}{\sqrt{b\sin\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^3/sqrt(b*sin(f*x + e)^2 + a), x)
```

3.148 $\int \frac{\sin^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=206

$$\frac{a(2a-b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{3b^2f\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a-b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a-}}{3b^2f\sqrt{\frac{b}{}}}$$

```
[Out] -(Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*b*f) - (2*(a - b
)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]
*Sqrt[a + b*Sin[e + f*x]^2])/(3*b^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (a*
(2*a - b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[
e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*b^2*f*Sqrt[a + b*Sin[e + f*x]^2
])
```

Rubi [A] time = 0.198534, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3188, 479, 524, 426, 424, 421, 419}

$$\frac{a(2a-b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{3b^2f\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a-b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a-}}{3b^2f\sqrt{\frac{b}{}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/Sqrt[a + b*Sin[e + f*x]^2],x]
```


```
[Out] -(Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*b*f) - (2*(a - b
)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]
*Sqrt[a + b*Sin[e + f*x]^2])/(3*b^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (a*
(2*a - b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[
e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*b^2*f*Sqrt[a + b*Sin[e + f*x]^2
])
```


Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] && !IntegerQ[p]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{1-x^2}\sqrt{a+bx^2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3bf} + \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \right.}{3bf}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3bf} - \frac{\left(2(a-b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Sul}}{3b^2f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3bf} - \frac{\left(2(a-b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a-}\right.}{3b^2f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3bf} - \frac{2(a-b)\sqrt{\cos^2(e+fx)}E\left(\sin^{-1}(\sin(e+\right.}{3b^2f\sqrt{1+}}$$

Mathematica [A] time = 0.977425, size = 163, normalized size = 0.79

$$\frac{b\sin(2(e+fx))(-2a+b\cos(2(e+fx))-b)+2\sqrt{2}a(2a-b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}F\left(e+fx\middle|-\frac{b}{a}\right)-4\sqrt{2}a(a-b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}}{6\sqrt{2}b^2f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-4*Sqrt[2]*a*(a - b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e +
f*x, -(b/a)] + 2*Sqrt[2]*a*(2*a - b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]
*EllipticF[e + f*x, -(b/a)] + b*(-2*a - b + b*Cos[2*(e + f*x)])*Sin[2*(e +
f*x)])/(6*Sqrt[2]*b^2*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A] time = 1.089, size = 268, normalized size = 1.3

$$\frac{1}{3b^2\cos\left(fx+e\right)f}\left(b^2\left(\sin\left(fx+e\right)\right)^5+2\sqrt{\left(\cos\left(fx+e\right)\right)^2}\sqrt{\frac{a+b\left(\sin\left(fx+e\right)\right)^2}{a}}\mathrm{EllipticF}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)a^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3*(b^2*sin(f*x+e)^5+2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*E
llipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2-a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*
x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-2*(cos(f*x+e)^2)^(1
/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2+2
*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1
/a*b)^(1/2))*a*b+a*b*sin(f*x+e)^3-b^2*sin(f*x+e)^3-sin(f*x+e)*a*b)/b^2/cos(
f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^4}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^4/sqrt(b*sin(f*x + e)^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(\cos\left(fx+e\right)^4-2\cos\left(fx+e\right)^2+1\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}}{b\cos\left(fx+e\right)^2-a-b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-b*cos(f*x + e)^2 +
a + b)/(b*cos(f*x + e)^2 - a - b), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^4}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^4/sqrt(b*sin(f*x + e)^2 + a), x)
```

3.149 $\int \frac{\sin^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=111

$$\frac{\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{bf\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(e+fx\middle|-\frac{b}{a}\right)}{bf\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] (EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(b*f*Sqrt[1 + (b*Si
n[e + f*x]^2)/a]) - (a*EllipticF[e + f*x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^
2)/a])/(b*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.126082, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3172, 3178, 3177, 3183, 3182}

$$\frac{\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{bf\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(e+fx\middle|-\frac{b}{a}\right)}{bf\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(b*f*Sqrt[1 + (b*Si
n[e + f*x]^2)/a]) - (a*EllipticF[e + f*x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^
2)/a])/(b*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\int \sqrt{a+b\sin^2(e+fx)}\,dx}{b} - \frac{a\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx}{b}$$

$$= \frac{\sqrt{a+b\sin^2(e+fx)}\int \sqrt{1+\frac{b\sin^2(e+fx)}{a}}\,dx}{b\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} - \frac{\left(a\sqrt{1+\frac{b\sin^2(e+fx)}{a}}\right)\int \frac{1}{\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}\,dx}{b\sqrt{a+b\sin^2(e+fx)}}$$

$$= \frac{E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{bf\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} - \frac{aF\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{bf\sqrt{a+b\sin^2(e+fx)}}$$

Mathematica [A] time = 0.221119, size = 78, normalized size = 0.7

$$\frac{\sqrt{2a-b\cos(2(e+fx))+b}\left(E\left(e+fx\middle|-\frac{b}{a}\right)-F\left(e+fx\middle|-\frac{b}{a}\right)\right)}{bf\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (Sqrt[2*a + b - b*Cos[2*(e + f*x)]]*(EllipticE[e + f*x, -(b/a)] - EllipticF
[e + f*x, -(b/a)]))/(b*f*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a])
```

Maple [A] time = 0.835, size = 93, normalized size = 0.8

$$-\frac{a}{b\cos\left(fx+e\right)f}\sqrt{\left(\cos\left(fx+e\right)\right)^2}\sqrt{\frac{a+b\left(\sin\left(fx+e\right)\right)^2}{a}}\left(\mathrm{EllipticF}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)-\mathrm{EllipticE}\left(\sin\left(fx+e\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] -a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)/b*(EllipticF(sin(f*x+e
),(-1/a*b)^(1/2))-EllipticE(sin(f*x+e),(-1/a*b)^(1/2)))/cos(f*x+e)/(a+b*sin
(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^2}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^2/sqrt(b*sin(f*x + e)^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\left(\cos\left(fx+e\right)^2-1\right)}{b\cos\left(fx+e\right)^2-a-b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)*(cos(f*x + e)^2 - 1)/(b*cos(f*x +
e)^2 - a - b), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^2}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^2/sqrt(b*sin(f*x + e)^2 + a), x)
```

3.150 $\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=51

$$\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] (EllipticF[e + f*x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(f*Sqrt[a + b*S
in[e + f*x]^2])
```

Rubi [A] time = 0.0328417, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3183, 3182}

$$\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (EllipticF[e + f*x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(f*Sqrt[a + b*S
in[e + f*x]^2])
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx &= \frac{\sqrt{1+\frac{b\sin^2(e+fx)}{a}}\int \frac{1}{\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}\,dx}{\sqrt{a+b\sin^2(e+fx)}}\\ &= \frac{F\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{f\sqrt{a+b\sin^2(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.0846094, size = 60, normalized size = 1.18

$$\frac{\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}F\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)])/(f*Sqrt
[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [C] time = 0.266, size = 60, normalized size = 1.2

$$\frac{1}{f}\sqrt{-\frac{b\left(\cos\left(fx+e\right)\right)^{2}-a-b}{a}}\text{InverseJacobiAM}\left(fx+e,i\sqrt{b}\frac{1}{\sqrt{a}}\right)\frac{1}{\sqrt{a+b-b\left(\cos\left(fx+e\right)\right)^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/f/(a+b-b*cos(f*x+e)^2)^(1/2)*(-(b*cos(f*x+e)^2-a-b)/a)^(1/2)*InverseJacob
iAM(f*x+e,I/a^(1/2)*b^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sin\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sin(f*x + e)^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}}{b\cos\left(fx+e\right)^{2}-a-b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)/(b*cos(f*x + e)^2 - a - b), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + b\sin^2\left(e + fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sin(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sin\left(fx + e\right)^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sin(f*x + e)^2 + a), x)
```

3.151 $\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=177

$$-\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{af}+\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}-\frac{\sqrt{\cos^2(e+}}{}$$

```
[Out] -((Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(a*f)) - (Sqrt[Cos[e + f*x]^2]*
EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x
]^2])/(a*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (Sqrt[Cos[e + f*x]^2]*Elliptic
F[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a]
)/(f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.178687, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3188, 480, 12, 493, 426, 424, 421, 419}

$$-\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{af}+\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}-\frac{\sqrt{\cos^2(e+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(a*f)) - (Sqrt[Cos[e + f*x]^2]*
EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x
]^2])/(a*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (Sqrt[Cos[e + f*x]^2]*Elliptic
F[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a]
)/(f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 493

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) &&  !(EqQ[n, 2] && Simp
lerSqrtQ[-(b/a), -(d/c)])
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-x^2}\sqrt{a+bx^2}}\,dx,x,\sin(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{af} - \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{bx^2}{\sqrt{1-x^2}\sqrt{a+bx^2}}\,dx,\right.}{af}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{af} - \frac{\left(b\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-x^2}\sqrt{a+bx^2}}\,dx,\right.}{af}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{af} + \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{a+bx^2}}\,dx,\right.}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{af} - \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\right)\text{Subst}}{af\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{af} - \frac{\sqrt{\cos^2(e+fx)}E\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)\sec(e+fx)\sqrt{}}{af\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

Mathematica [A] time = 0.500522, size = 138, normalized size = 0.78

$$\frac{-\sqrt{2}\cot(e+fx)(2a-b\cos(2(e+fx))+b)+2a\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}F\left(e+fx\middle|-\frac{b}{a}\right)-2a\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)}{2af\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-(Sqrt[2]*(2*a + b - b*Cos[2*(e + f*x)])*Cot[e + f*x]) - 2*a*Sqrt[(2*a + b
 - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] + 2*a*Sqrt[(2*a + b -
b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)])/(2*a*f*Sqrt[2*a + b - b*
Cos[2*(e + f*x)]])
```

Maple [A] time = 1.098, size = 140, normalized size = 0.8

$$\frac{1}{a\sin(fx+e)\cos(fx+e)f}\left(\sin(fx+e)\sqrt{-\frac{b(\cos(fx+e))^2}{a}+\frac{a+b}{a}}\sqrt{(\cos(fx+e))^2}a\left(\text{EllipticF}\left(\sin(fx+e),\sqrt{-}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] (sin(f*x+e)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(cos(f*x+e)^2)^(1/2)*a*(Ellip
ticF(sin(f*x+e),(-1/a*b)^(1/2))-EllipticE(sin(f*x+e),(-1/a*b)^(1/2)))+b*cos
(f*x+e)^4+(-a-b)*cos(f*x+e)^2)/a/sin(f*x+e)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(
1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^2}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^2/sqrt(b*sin(f*x + e)^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\csc\left(fx+e\right)^2}{b\cos\left(fx+e\right)^2-a-b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)*csc(f*x + e)^2/(b*cos(f*x + e)^2
- a - b), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^2\left(e+fx\right)}{\sqrt{a+b\sin^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)**2/sqrt(a + b*sin(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^2}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^2/sqrt(b*sin(f*x + e)^2 + a), x)
```

3.152 $\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=244

$$-\frac{2(a-b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^2f}-\frac{2(a-b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}E\left(\sin^{-1}(\sin(e+fx))|\right.}{3a^2f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

```
[Out] (-2*(a - b)*Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a^2*f) - (Cot[e + f
*x]*Csc[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2])/(3*a*f) - (2*(a - b)*Sqrt[Co
s[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a +
 b*Sin[e + f*x]^2])/(3*a^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + ((2*a - b)*S
qrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sq
rt[1 + (b*Sin[e + f*x]^2)/a])/(3*a*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.266694, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3188, 480, 583, 524, 426, 424, 421, 419}

$$-\frac{2(a-b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^2f}-\frac{2(a-b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}E\left(\sin^{-1}(\sin(e+fx))|\right.}{3a^2f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4/Sqrt[a + b*Sin[e + f*x]^2],x]
```


```
[Out] (-2*(a - b)*Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a^2*f) - (Cot[e + f
*x]*Csc[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2])/(3*a*f) - (2*(a - b)*Sqrt[Co
s[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a +
 b*Sin[e + f*x]^2])/(3*a^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + ((2*a - b)*S
qrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sq
rt[1 + (b*Sin[e + f*x]^2)/a])/(3*a*f*Sqrt[a + b*Sin[e + f*x]^2])
```


Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{1}{x^4\sqrt{1-x^2}\sqrt{a+bx^2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af} + \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{2}{x^2\sqrt{}}\right.}{3af}$$

$$= -\frac{2(a-b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^2f} - \frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af} -$$

$$= -\frac{2(a-b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^2f} - \frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af} -$$

$$= -\frac{2(a-b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^2f} - \frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af} -$$

$$= -\frac{2(a-b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^2f} - \frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af} -$$

Mathematica [A] time = 3.92908, size = 195, normalized size = 0.8

$$\frac{\frac{\cot(e+fx)\csc^2(e+fx)\left(2\left(2a^2+ab-2b^2\right)\cos(2(e+fx))-8a^2+b(b-a)\cos(4(e+fx))-ab+3b^2\right)}{\sqrt{2}} + 2a(2a-b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}F\left(e+fx\middle|-\frac{b}{a}\right) - 4a(a}{6a^2f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (((-8*a^2 - a*b + 3*b^2 + 2*(2*a^2 + a*b - 2*b^2)*Cos[2*(e + f*x)] + b*(-a
+ b)*Cos[4*(e + f*x)])*Cot[e + f*x]*Csc[e + f*x]^2)/Sqrt[2] - 4*a*(a - b)*S
qrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] + 2*a*(2*a
 - b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)])/(6
*a^2*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A] time = 1.183, size = 354, normalized size = 1.5

$$\frac{1}{3a^2\left(\sin\left(fx+e\right)\right)^3\cos\left(fx+e\right)f}\left(2\,\text{EllipticF}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)\sqrt{\left(\cos\left(fx+e\right)\right)^2}\sqrt{\frac{a+b\left(\sin\left(fx+e\right)\right)^2}{a}}a^2\left(\sin\left(f\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3*(2*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*(cos(f*x+e)^2)^(1/2)*((a+b*sin(
f*x+e)^2)/a)^(1/2)*a^2*sin(f*x+e)^3-b*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)
^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*sin(f*x+e)^3-2*Elliptic
```

```
E(sin(f*x+e),(-1/a*b)^(1/2))*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1
/2)*a^2*sin(f*x+e)^3+2*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*(cos(f*x+e)^2)^
(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*a*b*sin(f*x+e)^3+2*a*b*sin(f*x+e)^6-2*b^
2*sin(f*x+e)^6+2*a^2*sin(f*x+e)^4-3*a*b*sin(f*x+e)^4+2*b^2*sin(f*x+e)^4-a^2
*sin(f*x+e)^2+a*b*sin(f*x+e)^2-a^2)/a^2/sin(f*x+e)^3/cos(f*x+e)/(a+b*sin(f*
x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(fx+e)^4}{\sqrt{b\sin(fx+e)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^4/sqrt(b*sin(f*x + e)^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-b\cos(fx+e)^2+a+b}\csc(fx+e)^4}{b\cos(fx+e)^2-a-b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)*csc(f*x + e)^4/(b*cos(f*x + e)^2
- a - b), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(fx+e)^4}{\sqrt{b\sin(fx+e)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^4/sqrt(b*sin(f*x + e)^2 + a), x)
```

3.153 $\int \frac{\sin^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=79

$$\frac{a\cos(e+fx)}{bf(a+b)\sqrt{a-b\cos^2(e+fx)+b}}-\frac{\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{b^{3/2}f}$$

```
[Out] -(ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]]/(b^(3/2)*f)
) + (a*Cos[e + f*x])/(b*(a + b)*f*Sqrt[a + b - b*Cos[e + f*x]^2])
```

Rubi [A] time = 0.0988078, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3186, 385, 217, 203}

$$\frac{a\cos(e+fx)}{bf(a+b)\sqrt{a-b\cos^2(e+fx)+b}}-\frac{\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{b^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]]/(b^(3/2)*f)
) + (a*Cos[e + f*x])/(b*(a + b)*f*Sqrt[a + b - b*Cos[e + f*x]^2])
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{\left(a+b-bx^2\right)^{3/2}}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= \frac{a\cos(e+fx)}{b(a+b)f\sqrt{a+b-b\cos^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b-bx^2}}\,dx, x, \cos(e+fx)\right)}{bf}$$
$$= \frac{a\cos(e+fx)}{b(a+b)f\sqrt{a+b-b\cos^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{bf}$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{b^{3/2}f} + \frac{a\cos(e+fx)}{b(a+b)f\sqrt{a+b-b\cos^2(e+fx)}}$$

Mathematica [A] time = 0.4146, size = 96, normalized size = 1.22

$$\frac{\frac{\sqrt{2}ab\cos(e+fx)}{(a+b)\sqrt{2a-b\cos(2(e+fx))+b}} + \sqrt{-b}\log\left(\sqrt{2a-b\cos(2(e+fx))+b} + \sqrt{2}\sqrt{-b}\cos(e+fx)\right)}{b^2 f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] ((Sqrt[2]*a*b*Cos[e + f*x])/((a + b)*Sqrt[2*a + b - b*Cos[2*(e + f*x)]]) +
Sqrt[-b]*Log[Sqrt[2]*Sqrt[-b]*Cos[e + f*x] + Sqrt[2*a + b - b*Cos[2*(e + f*
x)]]])/(b^2*f)
```

Maple [B] time = 1.646, size = 156, normalized size = 2.

$$\frac{1}{f\cos\left(fx+e\right)}\sqrt{-\left(-b\left(\sin\left(fx+e\right)\right)^2-a\right)\left(\cos\left(fx+e\right)\right)^2}\left(\frac{1}{2}\arctan\left(\sqrt{b}\left(\left(\sin\left(fx+e\right)\right)^2-\frac{-a+b}{2b}\right)\frac{1}{\sqrt{-\left(-b\left(\sin\left(fx+\right.\right.\right.}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] (-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2)*(1/2/b^(3/2)*arctan(b^(1/2)*(sin(
f*x+e)^2-1/2*(-a+b)/b)/(-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2))+a/b*cos(f
*x+e)^2/(a+b)/(-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2))/cos(f*x+e)/(a+b*si
n(f*x+e)^2)^(1/2)/f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.62754, size = 1339, normalized size = 16.95

$$\left[-\frac{8\sqrt{-b\cos\left(fx+e\right)^2+a+b}ab\cos\left(fx+e\right)+\left(\left(ab+b^2\right)\cos\left(fx+e\right)^2-a^2-2ab-b^2\right)\sqrt{-b}\log\left(128b^4\cos\left(fx+e\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(8*sqrt(-b*cos(f*x + e)^2 + a + b)*a*b*cos(f*x + e) + ((a*b + b^2)*co
s(f*x + e)^2 - a^2 - 2*a*b - b^2)*sqrt(-b)*log(128*b^4*cos(f*x + e)^8 - 256
*(a*b^3 + b^4)*cos(f*x + e)^6 + 160*(a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^
4 + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4 - 32*(a^3*b + 3*a^2*b^2 + 3*a
*b^3 + b^4)*cos(f*x + e)^2 + 8*(16*b^3*cos(f*x + e)^7 - 24*(a*b^2 + b^3)*co
s(f*x + e)^5 + 10*(a^2*b + 2*a*b^2 + b^3)*cos(f*x + e)^3 - (a^3 + 3*a^2*b +
 3*a*b^2 + b^3)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)))/((
a*b^3 + b^4)*f*cos(f*x + e)^2 - (a^2*b^2 + 2*a*b^3 + b^4)*f), -1/4*(4*sqrt(
-b*cos(f*x + e)^2 + a + b)*a*b*cos(f*x + e) - ((a*b + b^2)*cos(f*x + e)^2 -
 a^2 - 2*a*b - b^2)*sqrt(b)*arctan(1/4*(8*b^2*cos(f*x + e)^4 - 8*(a*b + b^2
)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(
b)/(2*b^3*cos(f*x + e)^5 - 3*(a*b^2 + b^3)*cos(f*x + e)^3 + (a^2*b + 2*a*b^
2 + b^3)*cos(f*x + e))))/((a*b^3 + b^4)*f*cos(f*x + e)^2 - (a^2*b^2 + 2*a*b
^3 + b^4)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.92863, size = 154, normalized size = 1.95

$$-\frac{\sqrt{-\left(\cos\left(fx+e\right)^2-1\right)b+a}a\cos\left(fx+e\right)}{\left(\left(\cos\left(fx+e\right)^2-1\right)b-a\right)\left(ab+b^2\right)f}-\frac{\log\left(\left|\sqrt{-\left(\cos\left(fx+e\right)^2-1\right)b+a}+\frac{\sqrt{-bf^2}\cos\left(fx+e\right)}{f}\right|\right)}{\sqrt{-b}b\left|f\right|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -sqrt(-(cos(f*x + e)^2 - 1)*b + a)*a*cos(f*x + e)/(((cos(f*x + e)^2 - 1)*b
 - a)*(a*b + b^2)*f) - log(abs(sqrt(-(cos(f*x + e)^2 - 1)*b + a) + sqrt(-b*f
^2)*cos(f*x + e)/f))/(sqrt(-b)*b*abs(f))
```

3.154 $\int \frac{\sin(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=34

$$-\frac{\cos(e+fx)}{f(a+b)\sqrt{a-b\cos^2(e+fx)+b}}$$

```
[Out] -(Cos[e + f*x]/((a + b)*f*Sqrt[a + b - b*Cos[e + f*x]^2]))
```

Rubi [A] time = 0.045294, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3186, 191}

$$-\frac{\cos(e+fx)}{f(a+b)\sqrt{a-b\cos^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Cos[e + f*x]/((a + b)*f*Sqrt[a + b - b*Cos[e + f*x]^2]))
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{1}{\left(a+b-bx^2\right)^{3/2}}\,dx, x, \cos(e+fx)\right)}{f}\\ &= -\frac{\cos(e+fx)}{(a+b)f\sqrt{a+b-b\cos^2(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.114126, size = 41, normalized size = 1.21

$$-\frac{\sqrt{2}\cos(e+fx)}{f(a+b)\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -((Sqrt[2]*Cos[e + f*x])/((a + b)*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]]))
```

Maple [A] time = 0.839, size = 31, normalized size = 0.9

$$-\frac{\cos\left(fx+e\right)}{\left(a+b\right)f}\frac{1}{\sqrt{a+b\left(\sin\left(fx+e\right)\right)^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] -cos(f*x+e)/(a+b)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [A] time = 0.945119, size = 43, normalized size = 1.26

$$-\frac{\cos\left(fx+e\right)}{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,(a+b)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -cos(f*x + e)/(sqrt(-b*cos(f*x + e)^2 + a + b)*(a + b)*f)
```

Fricas [A] time = 1.55985, size = 136, normalized size = 4.

$$\frac{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\cos\left(fx+e\right)}{\left(ab+b^{2}\right)f\cos\left(fx+e\right)^{2}-\left(a^{2}+2\,ab+b^{2}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(-b*cos(f*x + e)^2 + a + b)*cos(f*x + e)/((a*b + b^2)*f*cos(f*x + e)^2
- (a^2 + 2*a*b + b^2)*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3584, size = 72, normalized size = 2.12

$$\frac{\sqrt{-\left(\cos\left(fx+e\right)^{2}-1\right)b+a}\cos\left(fx+e\right)}{\left(\left(\cos\left(fx+e\right)^{2}-1\right)b-a\right)(a+b)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] sqrt(-(cos(f*x + e)^2 - 1)*b + a)*cos(f*x + e)/(((cos(f*x + e)^2 - 1)*b - a
)*(a + b)*f)
```

3.155 $\int \frac{\csc(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=79

$$\frac{b\cos(e+fx)}{af(a+b)\sqrt{a-b\cos^2(e+fx)+b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{a^{3/2}f}$$

```
[Out] -(ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]]/(a^(3/2)*f
)) + (b*Cos[e + f*x])/(a*(a + b)*f*Sqrt[a + b - b*Cos[e + f*x]^2])
```

Rubi [A] time = 0.0938341, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3186, 382, 377, 206}

$$\frac{b\cos(e+fx)}{af(a+b)\sqrt{a-b\cos^2(e+fx)+b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]]/(a^(3/2)*f
)) + (b*Cos[e + f*x])/(a*(a + b)*f*Sqrt[a + b - b*Cos[e + f*x]^2])
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\left(a+b-bx^2\right)^{3/2}}\,dx,x,\cos(e+fx)\right)}{f}$$

$$= \frac{b\cos(e+fx)}{a(a+b)f\sqrt{a+b-b\cos^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+b-bx^2}}\,dx,x,\cos(e+fx)\right)}{af}$$

$$= \frac{b\cos(e+fx)}{a(a+b)f\sqrt{a+b-b\cos^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{af}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{a^{3/2}f} + \frac{b\cos(e+fx)}{a(a+b)f\sqrt{a+b-b\cos^2(e+fx)}}$$

Mathematica [A] time = 0.338754, size = 93, normalized size = 1.18

$$\frac{\frac{\sqrt{2}\sqrt{a}b\cos(e+fx)}{(a+b)\sqrt{2a-b\cos(2(e+fx))+b}} - \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cos(e+fx)}{\sqrt{2a-b\cos(2(e+fx))+b}}\right)}{a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-ArcTanh[(Sqrt[2]*Sqrt[a]*Cos[e + f*x])/Sqrt[2*a + b - b*Cos[2*(e + f*x)]]
] + (Sqrt[2]*Sqrt[a]*b*Cos[e + f*x])/((a + b)*Sqrt[2*a + b - b*Cos[2*(e + f
*x)]]))/(a^(3/2)*f)
```

Maple [B] time = 2.155, size = 165, normalized size = 2.1

$$\frac{1}{f\cos\left(fx+e\right)}\sqrt{-\left(-b\left(\sin\left(fx+e\right)\right)^2-a\right)\left(\cos\left(fx+e\right)\right)^2}\left(-\frac{1}{2}\ln\left(\frac{1}{\left(\sin\left(fx+e\right)\right)^2}\left(2a+(-a+b)\left(\sin\left(fx+e\right)\right)^2+2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] (-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2)*(-1/2/a^(3/2)*ln((2*a+(-a+b)*sin(
f*x+e)^2+2*a^(1/2)*(-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2))/sin(f*x+e)^2)
+1/a*b*cos(f*x+e)^2/(a+b)/(-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2))/cos(f*
x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.28678, size = 1008, normalized size = 12.76

$$\left[-\frac{4\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,ab\cos\left(fx+e\right)-\left(\left(ab+b^{2}\right)\cos\left(fx+e\right)^{2}-a^{2}-2\,ab-b^{2}\right)\sqrt{a}\log\left(\frac{2\left(\left(a^{2}-6\,ab+b^{2}\right)\cos\left(fx+e\right)^{4}+\cdots}{\cdots}\right)}{4\left(\left(a^{3}b+a^{2}b^{2}\right)f\cos\left(fx+e\right)^{2}-\left(a^{4}+2\,a^{3}b\cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*sqrt(-b*cos(f*x + e)^2 + a + b)*a*b*cos(f*x + e) - ((a*b + b^2)*co
s(f*x + e)^2 - a^2 - 2*a*b - b^2)*sqrt(a)*log(2*((a^2 - 6*a*b + b^2)*cos(f*
x + e)^4 + 2*(3*a^2 + 2*a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)
^3 + (a + b)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) + a^2 +
2*a*b + b^2)/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/((a^3*b + a^2*b^2)*f
*cos(f*x + e)^2 - (a^4 + 2*a^3*b + a^2*b^2)*f), -1/2*(2*sqrt(-b*cos(f*x + e
)^2 + a + b)*a*b*cos(f*x + e) - ((a*b + b^2)*cos(f*x + e)^2 - a^2 - 2*a*b -
 b^2)*sqrt(-a)*arctan(-1/2*((a - b)*cos(f*x + e)^2 + a + b)*sqrt(-b*cos(f*x
 + e)^2 + a + b)*sqrt(-a)/(a*b*cos(f*x + e)^3 - (a^2 + a*b)*cos(f*x + e))))
/((a^3*b + a^2*b^2)*f*cos(f*x + e)^2 - (a^4 + 2*a^3*b + a^2*b^2)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(e+fx\right)}{\left(a+b\sin^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)/(a + b*sin(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.156 $\int \frac{\csc^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=134

$$-\frac{b(a+3b)\cos(e+fx)}{2a^2f(a+b)\sqrt{a-b\cos^2(e+fx)+b}}-\frac{(a-3b)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{2a^{5/2}f}-\frac{\cot(e+fx)\csc(e+fx)}{2af\sqrt{a-b\cos^2(e+fx)+b}}$$

```
[Out] -((a - 3*b)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]])
/(2*a^(5/2)*f) - (b*(a + 3*b)*Cos[e + f*x])/(2*a^2*(a + b)*f*Sqrt[a + b - b
*Cos[e + f*x]^2]) - (Cot[e + f*x]*Csc[e + f*x])/(2*a*f*Sqrt[a + b - b*Cos[e
 + f*x]^2])
```

Rubi [A] time = 0.164457, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3186, 414, 527, 12, 377, 206}

$$-\frac{b(a+3b)\cos(e+fx)}{2a^2f(a+b)\sqrt{a-b\cos^2(e+fx)+b}}-\frac{(a-3b)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{2a^{5/2}f}-\frac{\cot(e+fx)\csc(e+fx)}{2af\sqrt{a-b\cos^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a - 3*b)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]])
/(2*a^(5/2)*f) - (b*(a + 3*b)*Cos[e + f*x])/(2*a^2*(a + b)*f*Sqrt[a + b - b
*Cos[e + f*x]^2]) - (Cot[e + f*x]*Csc[e + f*x])/(2*a*f*Sqrt[a + b - b*Cos[e
 + f*x]^2])
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2\left(a+b-bx^2\right)^{3/2}}\,dx,x,\cos(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af\sqrt{a+b-b\cos^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{a-b-2bx^2}{\left(1-x^2\right)\left(a+b-bx^2\right)^{3/2}}\,dx,x,\cos(e+fx)\right)}{2af}$$

$$= -\frac{b(a+3b)\cos(e+fx)}{2a^2(a+b)f\sqrt{a+b-b\cos^2(e+fx)}} - \frac{\cot(e+fx)\csc(e+fx)}{2af\sqrt{a+b-b\cos^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{(a-3b)(}{(1-x^2)\sqrt{a}}\right.}{2a}$$

$$= -\frac{b(a+3b)\cos(e+fx)}{2a^2(a+b)f\sqrt{a+b-b\cos^2(e+fx)}} - \frac{\cot(e+fx)\csc(e+fx)}{2af\sqrt{a+b-b\cos^2(e+fx)}} - \frac{(a-3b)\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{b(a+3b)\cos(e+fx)}{2a^2(a+b)f\sqrt{a+b-b\cos^2(e+fx)}} - \frac{\cot(e+fx)\csc(e+fx)}{2af\sqrt{a+b-b\cos^2(e+fx)}} - \frac{(a-3b)\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{(a-3b)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{2a^{5/2}f} - \frac{b(a+3b)\cos(e+fx)}{2a^2(a+b)f\sqrt{a+b-b\cos^2(e+fx)}} - \frac{\cot(e+f}{2af\sqrt{a+b}}$$

Mathematica [A] time = 0.659131, size = 134, normalized size = 1.

$$\frac{\frac{\cot(e+fx)\csc(e+fx)\left(-2a^2+b(a+3b)\cos(2(e+fx))-3ab-3b^2\right)}{\sqrt{2}a^2(a+b)\sqrt{2a-b\cos(2(e+fx))+b}} - \frac{(a-3b)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cos(e+fx)}{\sqrt{2a-b\cos(2(e+fx))+b}}\right)}{a^{5/2}}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-(((a - 3*b)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cos[e + f*x])/Sqrt[2*a + b - b*Cos[2
*(e + f*x)]]])/a^(5/2)) + ((-2*a^2 - 3*a*b - 3*b^2 + b*(a + 3*b)*Cos[2*(e +
 f*x)])*Cot[e + f*x]*Csc[e + f*x])/(Sqrt[2]*a^2*(a + b)*Sqrt[2*a + b - b*Co
s[2*(e + f*x)]]))/(2*f)
```

Maple [B] time = 2.581, size = 274, normalized size = 2.

$$\frac{1}{f\cos\left(fx+e\right)}\sqrt{-\left(-b\left(\sin\left(fx+e\right)\right)^{2}-a\right)\left(\cos\left(fx+e\right)\right)^{2}}\left(\frac{3b}{4}\ln\left(\frac{1}{\left(\sin\left(fx+e\right)\right)^{2}}\left(2a+(-a+b)\left(\sin\left(fx+e\right)\right)^{2}+2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] (-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2)*(3/4/a^(5/2)*b*ln((2*a+(-a+b)*sin
(f*x+e)^2+2*a^(1/2)*(-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2))/sin(f*x+e)^2
)-1/a^2*b^2*cos(f*x+e)^2/(a+b)/(-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2)-1/
2/a^2/sin(f*x+e)^2*(-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2)-1/4/a^(3/2)*ln
((2*a+(-a+b)*sin(f*x+e)^2+2*a^(1/2)*(-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/
2))/sin(f*x+e)^2))/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 3.83483, size = 1474, normalized size = 11.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(((a^2*b - 2*a*b^2 - 3*b^3)*cos(f*x + e)^4 + a^3 - a^2*b - 5*a*b^2 -
3*b^3 - (a^3 - 7*a*b^2 - 6*b^3)*cos(f*x + e)^2)*sqrt(a)*log(2*((a^2 - 6*a*b
 + b^2)*cos(f*x + e)^4 + 2*(3*a^2 + 2*a*b - b^2)*cos(f*x + e)^2 + 4*((a - b
)*cos(f*x + e)^3 + (a + b)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sq
rt(a) + a^2 + 2*a*b + b^2)/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)) - 4*((a
^2*b + 3*a*b^2)*cos(f*x + e)^3 - (a^3 + 2*a^2*b + 3*a*b^2)*cos(f*x + e))*sq
rt(-b*cos(f*x + e)^2 + a + b))/((a^4*b + a^3*b^2)*f*cos(f*x + e)^4 - (a^5 +
 3*a^4*b + 2*a^3*b^2)*f*cos(f*x + e)^2 + (a^5 + 2*a^4*b + a^3*b^2)*f), 1/4*
(((a^2*b - 2*a*b^2 - 3*b^3)*cos(f*x + e)^4 + a^3 - a^2*b - 5*a*b^2 - 3*b^3
- (a^3 - 7*a*b^2 - 6*b^3)*cos(f*x + e)^2)*sqrt(-a)*arctan(-1/2*((a - b)*cos
(f*x + e)^2 + a + b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a)/(a*b*cos(f*x
+ e)^3 - (a^2 + a*b)*cos(f*x + e))) + 2*((a^2*b + 3*a*b^2)*cos(f*x + e)^3 -
 (a^3 + 2*a^2*b + 3*a*b^2)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b))/(
(a^4*b + a^3*b^2)*f*cos(f*x + e)^4 - (a^5 + 3*a^4*b + 2*a^3*b^2)*f*cos(f*x
+ e)^2 + (a^5 + 2*a^4*b + a^3*b^2)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)**3/(a + b*sin(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(fx+e)^3}{\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^3/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.157 $\int \frac{\sin^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=274

$$-\frac{\left(8a^2+3ab-2b^2\right)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{3b^3f(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{(4a+b)\sin(e+fx)\,\mathrm{c}}{3}$$

```
[Out] (a*Cos[e + f*x]*Sin[e + f*x]^3)/(b*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2]) -
((4*a + b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*b^2*(a
+ b)*f) - ((8*a^2 + 3*a*b - 2*b^2)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Si
n[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*b^3*(a + b
)*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (a*(8*a - b)*Sqrt[Cos[e + f*x]^2]*Ell
ipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^
2)/a])/(3*b^3*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.31424, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3188, 470, 582, 524, 426, 424, 421, 419}

$$-\frac{\left(8a^2+3ab-2b^2\right)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{3b^3f(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{(4a+b)\sin(e+fx)\,\mathrm{c}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^6/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (a*Cos[e + f*x]*Sin[e + f*x]^3)/(b*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2]) -
((4*a + b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*b^2*(a
+ b)*f) - ((8*a^2 + 3*a*b - 2*b^2)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Si
n[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*b^3*(a + b
)*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (a*(8*a - b)*Sqrt[Cos[e + f*x]^2]*Ell
ipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^
2)/a])/(3*b^3*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
```

```
p, q, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\sin^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^6}{\sqrt{1-x^2}\left(a+bx^2\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{a\cos(e+fx)\sin^3(e+fx)}{b(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^2\left(3a+(-4a-b)x^2\right)}{\sqrt{1-x^2}\sqrt{a+bx^2}}\right.}{b(a+b)f}$$

$$= \frac{a\cos(e+fx)\sin^3(e+fx)}{b(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{(4a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3b^2(a+b)f} -$$

$$= \frac{a\cos(e+fx)\sin^3(e+fx)}{b(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{(4a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3b^2(a+b)f} -$$

$$= \frac{a\cos(e+fx)\sin^3(e+fx)}{b(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{(4a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3b^2(a+b)f} -$$

$$= \frac{a\cos(e+fx)\sin^3(e+fx)}{b(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{(4a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3b^2(a+b)f} -$$

Mathematica [A] time = 1.26239, size = 197, normalized size = 0.72

$$\frac{b\sin(2(e+fx))\left(-8a^2+b(a+b)\cos(2(e+fx))-3ab-b^2\right)+2\sqrt{2}a\left(8a^2+7ab-b^2\right)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}F\left(e+fx\middle|-\frac{b}{a}\right.}{6\sqrt{2}b^3f(a+b)\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^6/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[2]*a*(8*a^2 + 3*a*b - 2*b^2)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a
]*EllipticE[e + f*x, -(b/a)] + 2*Sqrt[2]*a*(8*a^2 + 7*a*b - b^2)*Sqrt[(2*a
+ b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)] + b*(-8*a^2 - 3*a*b
 - b^2 + b*(a + b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/(6*Sqrt[2]*b^3*(a +
b)*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A] time = 1.309, size = 405, normalized size = 1.5

$$\frac{1}{3b^3(a+b)\cos(fx+e)f}\left(ab^2\left(\sin(fx+e)\right)^5+b^3\left(\sin(fx+e)\right)^5+8\sqrt{\left(\cos(fx+e)\right)^2}\sqrt{\frac{a+b\left(\sin(fx+e)\right)^2}{a}}\text{Ellip}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^6/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/3*(a*b^2*sin(f*x+e)^5+b^3*sin(f*x+e)^5+8*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f
*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3+7*a^2*(cos(f*x+e
)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2)
)*b-a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e
),(-1/a*b)^(1/2))*b^2-8*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*E
llipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3-3*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*
x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b+2*(cos(f*x+e)^2
)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a
*b^2+4*a^2*b*sin(f*x+e)^3-sin(f*x+e)^3*b^3-4*sin(f*x+e)*a^2*b-a*b^2*sin(f*x
+e))/b^3/(a+b)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^{6}}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^6/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\left(\cos\left(fx+e\right)^{6}-3\cos\left(fx+e\right)^{4}+3\cos\left(fx+e\right)^{2}-1\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}}{b^{2}\cos\left(fx+e\right)^{4}-2\left(ab+b^{2}\right)\cos\left(fx+e\right)^{2}+a^{2}+2ab+b^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^6 - 3*cos(f*x + e)^4 + 3*cos(f*x + e)^2 - 1)*sqrt(-
b*cos(f*x + e)^2 + a + b)/(b^2*cos(f*x + e)^4 - 2*(a*b + b^2)*cos(f*x + e)^
2 + a^2 + 2*a*b + b^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**6/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^{6}}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^6/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.158 $\int \frac{\sin^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=202

$$-\frac{2a\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{b^2f\sqrt{a+b\sin^2(e+fx)}}+\frac{(2a+b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin}}{b^2f(a+b)\sqrt{\frac{b\sin}{}}}$$

```
[Out] (a*Cos[e + f*x]*Sin[e + f*x])/(b*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2]) + ((
2*a + b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e
 + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(b^2*(a + b)*f*Sqrt[1 + (b*Sin[e + f*x]
^2)/a]) - (2*a*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]
*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(b^2*f*Sqrt[a + b*Sin[e + f*x
]^2])
```

Rubi [A] time = 0.199586, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3188, 470, 524, 426, 424, 421, 419}

$$-\frac{2a\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{b^2f\sqrt{a+b\sin^2(e+fx)}}+\frac{(2a+b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin}}{b^2f(a+b)\sqrt{\frac{b\sin}{}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(3/2),x]
```


```
[Out] (a*Cos[e + f*x]*Sin[e + f*x])/(b*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2]) + ((
2*a + b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e
 + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(b^2*(a + b)*f*Sqrt[1 + (b*Sin[e + f*x]
^2)/a]) - (2*a*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]
*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(b^2*f*Sqrt[a + b*Sin[e + f*x
]^2])
```


Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{1-x^2}\left(a+bx^2\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{a\cos(e+fx)\sin(e+fx)}{b(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{a+(-2a-b)x^2}{\sqrt{1-x^2}\sqrt{a+bx^2}}\,dx, x,\right.}{b(a+b)f}$$

$$= \frac{a\cos(e+fx)\sin(e+fx)}{b(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(2a\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{a+bx^2}}\,dx,\right.}{b^2 f}$$

$$= \frac{a\cos(e+fx)\sin(e+fx)}{b(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left((-2a-b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\right.}{b^2(a+b)f\sqrt{1+\frac{b\sin^2(e}{a}}}$$

$$= \frac{a\cos(e+fx)\sin(e+fx)}{b(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{(2a+b)\sqrt{\cos^2(e+fx)}E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)\sec(e-}{b^2(a+b)f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

Mathematica [A] time = 0.67261, size = 136, normalized size = 0.67

$$\frac{a\left(-4(a+b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}F\left(e+fx\middle|-\frac{b}{a}\right)+2(2a+b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)+\sqrt{2}b\sin(2(e+fx))\right)}{2b^2f(a+b)\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (a*(2*(2*a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -
(b/a)] - 4*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x
, -(b/a)] + Sqrt[2]*b*Sin[2*(e + f*x)]))/(2*b^2*(a + b)*f*Sqrt[2*a + b - b*
Cos[2*(e + f*x)]])
```

Maple [A] time = 1.25, size = 241, normalized size = 1.2

$$-\frac{a}{b^2(a+b)\cos(fx+e)f}\left(2a\sqrt{(\cos(fx+e))^2}\sqrt{\frac{a+b(\sin(fx+e))^2}{a}}\text{EllipticF}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)+2\sqrt{(\cos(fx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] -a*(2*a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x
+e),(-1/a*b)^(1/2))+2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*Ell
ipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-2*a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+
e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))-(cos(f*x+e)^2)^(1/2)*((
a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*b+b*sin(f*x
```

```
+e)^3-b*sin(f*x+e))/b^2/(a+b)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^4}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^4/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\cos\left(fx+e\right)^4-2\,\cos\left(fx+e\right)^2+1\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}}{b^2\cos\left(fx+e\right)^4-2\left(ab+b^2\right)\cos\left(fx+e\right)^2+a^2+2\,ab+b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-b*cos(f*x + e)^2 + a
 + b)/(b^2*cos(f*x + e)^4 - 2*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^
2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^4}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^4/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.159 $\int \frac{\sin^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=153

$$-\frac{\sin(e+fx)\cos(e+fx)}{f(a+b)\sqrt{a+b\sin^2(e+fx)}}+\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(e+fx\middle|-\frac{b}{a}\right)}{bf\sqrt{a+b\sin^2(e+fx)}}-\frac{\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{bf(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

```
[Out] -((Cos[e + f*x]*Sin[e + f*x])/((a + b)*f*Sqrt[a + b*Sin[e + f*x]^2])) - (El
lipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(b*(a + b)*f*Sqrt[1 +
(b*Sin[e + f*x]^2)/a]) + (EllipticF[e + f*x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*
x]^2)/a])/(b*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.191641, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3173, 3172, 3178, 3177, 3183, 3182}

$$-\frac{\sin(e+fx)\cos(e+fx)}{f(a+b)\sqrt{a+b\sin^2(e+fx)}}+\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(e+fx\middle|-\frac{b}{a}\right)}{bf\sqrt{a+b\sin^2(e+fx)}}-\frac{\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{bf(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -((Cos[e + f*x]*Sin[e + f*x])/((a + b)*f*Sqrt[a + b*Sin[e + f*x]^2])) - (El
lipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(b*(a + b)*f*Sqrt[1 +
(b*Sin[e + f*x]^2)/a]) + (EllipticF[e + f*x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*
x]^2)/a])/(b*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 3173

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b - a*B)*Cos[e + f*x]*Sin[e + f*x]
*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(a + b)*(p + 1)), x] - Dist[1/(2*a*
(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p +
 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = -\frac{\cos(e+fx)\sin(e+fx)}{(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\int \frac{a-a\sin^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx}{a(a+b)}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx}{b} - \frac{\int \sqrt{a+b\sin^2(e+fx)}\,dx}{b(a+b)}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\sqrt{a+b\sin^2(e+fx)}\int \sqrt{1+\frac{b\sin^2(e+fx)}{a}}\,dx}{b(a+b)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} + \frac{\sqrt{1+\frac{b}{}}}{b}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{b(a+b)f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} + \frac{F\left(e+fx\middle|-\frac{b}{a}\right)}{bf\sqrt{a+b}}$$

Mathematica [A] time = 0.442664, size = 138, normalized size = 0.9

$$\frac{\sqrt{2}(a+b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}F\left(e+fx\middle|-\frac{b}{a}\right)-\sqrt{2}a\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)-b\sin(2(e+fx))}{\sqrt{2}bf(a+b)\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-(Sqrt[2]*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/
a)]) + Sqrt[2]*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e +
 f*x, -(b/a)] - b*Sin[2*(e + f*x)])/(Sqrt[2]*b*(a + b)*f*Sqrt[2*a + b - b*C
os[2*(e + f*x)]])
```

Maple [A] time = 1.262, size = 191, normalized size = 1.3

$$\frac{1}{(a+b)\,b\cos\left(fx+e\right)f}\left(a\sqrt{\left(\cos\left(fx+e\right)\right)^{2}}\sqrt{\frac{a+b\left(\sin\left(fx+e\right)\right)^{2}}{a}}\mathrm{EllipticF}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)+\sqrt{\left(\cos\left(fx+e\right)\right)^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] (a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(
-1/a*b)^(1/2))+(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(
sin(f*x+e),(-1/a*b)^(1/2))*b-a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^
(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))+b*sin(f*x+e)^3-b*sin(f*x+e))/b/(
a+b)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^{2}}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\left(\cos\left(fx+e\right)^{2}-1\right)}{b^{2}\cos\left(fx+e\right)^{4}-2\left(ab+b^{2}\right)\cos\left(fx+e\right)^{2}+a^{2}+2\,ab+b^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)*(cos(f*x + e)^2 - 1)/(b^2*cos(f*x
 + e)^4 - 2*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^2}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.160 $\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=101

$$\frac{b\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}+\frac{\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{af(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

```
[Out] (b*Cos[e + f*x]*Sin[e + f*x])/(a*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2]) + (E
llipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(a*(a + b)*f*Sqrt[1 +
 (b*Sin[e + f*x]^2)/a])
```

Rubi [A] time = 0.0573837, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3184, 21, 3178, 3177}

$$\frac{b\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}+\frac{\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{af(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x]^2)^(-3/2),x]
```

```
[Out] (b*Cos[e + f*x]*Sin[e + f*x])/(a*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2]) + (E
llipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(a*(a + b)*f*Sqrt[1 +
 (b*Sin[e + f*x]^2)/a])
```

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a +
 b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)
*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{b\cos(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\int \frac{-a-b\sin^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx}{a(a+b)}$$

$$= \frac{b\cos(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\int \sqrt{a+b\sin^2(e+fx)}\,dx}{a(a+b)}$$

$$= \frac{b\cos(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\sqrt{a+b\sin^2(e+fx)}\int \sqrt{1+\frac{b\sin^2(e+fx)}{a}}\,dx}{a(a+b)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

$$= \frac{b\cos(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{a(a+b)f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

Mathematica [A] time = 0.147582, size = 90, normalized size = 0.89

$$\frac{2a\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)+\sqrt{2}b\sin(2(e+fx))}{2af(a+b)\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)^(-3/2),x]
```

```
[Out] (2*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] + Sq
rt[2]*b*Sin[2*(e + f*x)])/(2*a*(a + b)*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]]
)
```

Maple [A] time = 1.217, size = 103, normalized size = 1.

$$\frac{1}{a\,(a+b)\cos\left(fx+e\right)f}\left(\sqrt{\left(\cos\left(fx+e\right)\right)^2}\sqrt{-\frac{b\left(\cos\left(fx+e\right)\right)^2}{a}+\frac{a+b}{a}}a\mathrm{EllipticE}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)+\sin\left(fx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] ((cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*a*EllipticE(sin(f*x
+e),(-1/a*b)^(1/2))+sin(f*x+e)*cos(f*x+e)^2*b)/a/(a+b)/cos(f*x+e)/(a+b*sin(
f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}}{b^{2}\cos\left(fx+e\right)^{4}-2\left(ab+b^{2}\right)\cos\left(fx+e\right)^{2}+a^{2}+2\,ab+b^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)/(b^2*cos(f*x + e)^4 - 2*(a*b + b^2
)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(-3/2), x)
```

3.161 $\int \frac{\csc^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=235

$$-\frac{(a+2b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a^2f(a+b)}-\frac{(a+2b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}E\left(\sin^{-1}(\sin(e+f\right.}{a^2f(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

```
[Out] (b*Cot[e + f*x])/(a*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2]) - ((a + 2*b)*Cot[
e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(a^2*(a + b)*f) - ((a + 2*b)*Sqrt[Cos[
e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b
*Sin[e + f*x]^2])/(a^2*(a + b)*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (Sqrt[Co
s[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 +
 (b*Sin[e + f*x]^2)/a])/(a*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.268003, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3188, 472, 583, 524, 426, 424, 421, 419}

$$-\frac{(a+2b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a^2f(a+b)}-\frac{(a+2b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}E\left(\sin^{-1}(\sin(e+f\right.}{a^2f(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (b*Cot[e + f*x])/(a*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2]) - ((a + 2*b)*Cot[
e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(a^2*(a + b)*f) - ((a + 2*b)*Sqrt[Cos[
e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b
*Sin[e + f*x]^2])/(a^2*(a + b)*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (Sqrt[Co
s[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 +
 (b*Sin[e + f*x]^2)/a])/(a*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{1-x^2}\left(a+bx^2\right)^{3/2}}dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{b\cot(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\operatorname{Subst}\left(\int \frac{-a-2b+bx^2}{x^2\sqrt{1-x^2}\sqrt{a+bx^2}}\right.}{a(a+b)f}$$

$$= \frac{b\cot(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{(a+2b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a^2(a+b)f} + \frac{\left(\sqrt{\cos^2(e\right.}}{}$$

$$= \frac{b\cot(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{(a+2b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a^2(a+b)f} + \frac{\left(\sqrt{\cos^2(e\right.}}{}$$

$$= \frac{b\cot(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{(a+2b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a^2(a+b)f} + \frac{\left((-a-2b\right.}{}$$

$$= \frac{b\cot(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{(a+2b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a^2(a+b)f} - \frac{(a+2b)\sqrt{}}{}$$

Mathematica [A] time = 1.19775, size = 170, normalized size = 0.72

$$\frac{\cot(e+fx)\left(-2a^2+b(a+2b)\cos(2(e+fx))-3ab-2b^2\right)+\sqrt{2}a(a+b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}F\left(e+fx\middle|-\frac{b}{a}\right)-\sqrt{2}a(a+}{\sqrt{2}a^2f(a+b)\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] ((-2*a^2 - 3*a*b - 2*b^2 + b*(a + 2*b)*Cos[2*(e + f*x)])*Cot[e + f*x] - Sqr
t[2]*a*(a + 2*b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x,
-(b/a)] + Sqrt[2]*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*Elliptic
F[e + f*x, -(b/a)])/(Sqrt[2]*a^2*(a + b)*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)
]])
```

Maple [A] time = 1.267, size = 199, normalized size = 0.9

$$\frac{1}{a^2\sin\left(fx+e\right)(a+b)\cos\left(fx+e\right)f}\left(\sin\left(fx+e\right)\sqrt{-\frac{b\left(\cos\left(fx+e\right)\right)^2}{a}+\frac{a+b}{a}}\sqrt{\left(\cos\left(fx+e\right)\right)^2}a\left(\operatorname{EllipticF}\left(\sin\left(\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] (sin(f*x+e)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(cos(f*x+e)^2)^(1/2)*a*(Ellip
ticF(sin(f*x+e),(-1/a*b)^(1/2))*a+EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-El
```

```
lipticE(sin(f*x+e),(-1/a*b)^(1/2))*a-2*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))
*b)+(a*b+2*b^2)*cos(f*x+e)^4+(-a^2-2*a*b-2*b^2)*cos(f*x+e)^2)/a^2/sin(f*x+e
)/(a+b)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^2}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\csc\left(fx+e\right)^2}{b^2\cos\left(fx+e\right)^4-2\left(ab+b^2\right)\cos\left(fx+e\right)^2+a^2+2\,ab+b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)*csc(f*x + e)^2/(b^2*cos(f*x + e)^4
 - 2*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^2\left(e+fx\right)}{\left(a+b\sin^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)**2/(a + b*sin(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^2}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.162 $\int \frac{\sin^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=137

$$\frac{a(3a+5b)\cos(e+fx)}{3b^2f(a+b)^2\sqrt{a-b\cos^2(e+fx)+b}}-\frac{\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{b^{5/2}f}+\frac{a\sin^2(e+fx)\cos(e+fx)}{3bf(a+b)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}$$

```
[Out] -(ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]]/(b^(5/2)*f)
) + (a*(3*a + 5*b)*Cos[e + f*x])/(3*b^2*(a + b)^2*f*Sqrt[a + b - b*Cos[e +
f*x]^2]) + (a*Cos[e + f*x]*Sin[e + f*x]^2)/(3*b*(a + b)*f*(a + b - b*Cos[e
+ f*x]^2)^(3/2))
```

Rubi [A] time = 0.136832, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3186, 413, 385, 217, 203}

$$\frac{a(3a+5b)\cos(e+fx)}{3b^2f(a+b)^2\sqrt{a-b\cos^2(e+fx)+b}}-\frac{\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{b^{5/2}f}+\frac{a\sin^2(e+fx)\cos(e+fx)}{3bf(a+b)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5/(a + b*Sin[e + f*x]^2)^(5/2),x]
```


```
[Out] -(ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]]/(b^(5/2)*f)
) + (a*(3*a + 5*b)*Cos[e + f*x])/(3*b^2*(a + b)^2*f*Sqrt[a + b - b*Cos[e +
f*x]^2]) + (a*Cos[e + f*x]*Sin[e + f*x]^2)/(3*b*(a + b)*f*(a + b - b*Cos[e
+ f*x]^2)^(3/2))
```


Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\left(a+b-bx^2\right)^{5/2}}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= \frac{a\cos(e+fx)\sin^2(e+fx)}{3b(a+b)f\left(a+b-b\cos^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-a-3b+3(a+b)x^2}{\left(a+b-bx^2\right)^{3/2}}\,dx, x, \cos(e+fx)\right)}{3b(a+b)f}$$

$$= \frac{a(3a+5b)\cos(e+fx)}{3b^2(a+b)^2f\sqrt{a+b-b\cos^2(e+fx)}} + \frac{a\cos(e+fx)\sin^2(e+fx)}{3b(a+b)f\left(a+b-b\cos^2(e+fx)\right)^{3/2}} - \text{Subs}$$

$$= \frac{a(3a+5b)\cos(e+fx)}{3b^2(a+b)^2f\sqrt{a+b-b\cos^2(e+fx)}} + \frac{a\cos(e+fx)\sin^2(e+fx)}{3b(a+b)f\left(a+b-b\cos^2(e+fx)\right)^{3/2}} - \text{Subs}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{b^{5/2}f} + \frac{a(3a+5b)\cos(e+fx)}{3b^2(a+b)^2f\sqrt{a+b-b\cos^2(e+fx)}} + \frac{a\cos(e+f}{3b(a+b)f\left(a+b\right.}$$

Mathematica [A] time = 0.754213, size = 133, normalized size = 0.97

$$\frac{\frac{2\sqrt{2}a\cos(e+fx)\left(3a^2-b(2a+3b)\cos(2(e+fx))+7ab+3b^2\right)}{(a+b)^2(2a-b\cos(2(e+fx))+b)^{3/2}}-\frac{3\log\left(\sqrt{2a-b\cos(2(e+fx))+b}+\sqrt{2}\sqrt{-b}\cos(e+fx)\right)}{\sqrt{-b}}}{3b^2f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^5/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] ((2*Sqrt[2]*a*Cos[e + f*x]*(3*a^2 + 7*a*b + 3*b^2 - b*(2*a + 3*b)*Cos[2*(e
+ f*x)]))/((a + b)^2*(2*a + b - b*Cos[2*(e + f*x)])^(3/2)) - (3*Log[Sqrt[2]
*Sqrt[-b]*Cos[e + f*x] + Sqrt[2*a + b - b*Cos[2*(e + f*x)]]])/Sqrt[-b])/(3*
b^2*f)
```

Maple [A] time = 2.746, size = 243, normalized size = 1.8

$$\frac{1}{f\cos\left(fx+e\right)}\sqrt{-\left(-b\left(\sin\left(fx+e\right)\right)^2-a\right)\left(\cos\left(fx+e\right)\right)^2}\left(\frac{1}{2}\arctan\left(\sqrt{b}\left(\left(\sin\left(fx+e\right)\right)^2-\frac{-a+b}{2b}\right)\frac{}{\sqrt{-\left(-b\left(\sin\left(f\right.\right.\right.}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] (-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2)*(1/2/b^(5/2)*arctan(b^(1/2)*(sin(
f*x+e)^2-1/2*(-a+b)/b)/(-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2))-1/3*a^2/b
^2*(2*b*sin(f*x+e)^2+3*a+b)*cos(f*x+e)^2/(-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2
)^(1/2)/(a+b*sin(f*x+e)^2)/(a^2+2*a*b+b^2)+2*a/b^2*cos(f*x+e)^2/(a+b)/(-(-b
*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2))/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 6.0517, size = 2045, normalized size = 14.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2
*b^2 + 4*a*b^3 + b^4 - 2*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cos(f*x + e)^2
)*sqrt(-b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + b^4)*cos(f*x + e)^6 +
160*(a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*b^2 +
4*a*b^3 + b^4 - 32*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cos(f*x + e)^2 + 8*(
16*b^3*cos(f*x + e)^7 - 24*(a*b^2 + b^3)*cos(f*x + e)^5 + 10*(a^2*b + 2*a*b
^2 + b^3)*cos(f*x + e)^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e))*sq
rt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)) + 8*(2*(2*a^2*b^2 + 3*a*b^3)*cos(f*
x + e)^3 - 3*(a^3*b + 3*a^2*b^2 + 2*a*b^3)*cos(f*x + e))*sqrt(-b*cos(f*x +
e)^2 + a + b))/((a^2*b^5 + 2*a*b^6 + b^7)*f*cos(f*x + e)^4 - 2*(a^3*b^4 + 3
*a^2*b^5 + 3*a*b^6 + b^7)*f*cos(f*x + e)^2 + (a^4*b^3 + 4*a^3*b^4 + 6*a^2*b
^5 + 4*a*b^6 + b^7)*f), 1/12*(3*((a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 +
 a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4 - 2*(a^3*b + 3*a^2*b^2 + 3*a*b^3
 + b^4)*cos(f*x + e)^2)*sqrt(b)*arctan(1/4*(8*b^2*cos(f*x + e)^4 - 8*(a*b +
 b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*s
qrt(b)/(2*b^3*cos(f*x + e)^5 - 3*(a*b^2 + b^3)*cos(f*x + e)^3 + (a^2*b + 2*
a*b^2 + b^3)*cos(f*x + e))) - 4*(2*(2*a^2*b^2 + 3*a*b^3)*cos(f*x + e)^3 - 3
*(a^3*b + 3*a^2*b^2 + 2*a*b^3)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b
))/((a^2*b^5 + 2*a*b^6 + b^7)*f*cos(f*x + e)^4 - 2*(a^3*b^4 + 3*a^2*b^5 + 3
*a*b^6 + b^7)*f*cos(f*x + e)^2 + (a^4*b^3 + 4*a^3*b^4 + 6*a^2*b^5 + 4*a*b^6
 + b^7)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^5}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^5/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

3.163 $\int \frac{\sin^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=81

$$-\frac{2\cos(e+fx)}{3f(a+b)^2\sqrt{a-b\cos^2(e+fx)+b}}-\frac{\sin^2(e+fx)\cos(e+fx)}{3f(a+b)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}$$

```
[Out] (-2*Cos[e + f*x])/(3*(a + b)^2*f*Sqrt[a + b - b*Cos[e + f*x]^2]) - (Cos[e +
 f*x]*Sin[e + f*x]^2)/(3*(a + b)*f*(a + b - b*Cos[e + f*x]^2)^(3/2))
```

Rubi [A] time = 0.0951856, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3186, 378, 191}

$$-\frac{2\cos(e+fx)}{3f(a+b)^2\sqrt{a-b\cos^2(e+fx)+b}}-\frac{\sin^2(e+fx)\cos(e+fx)}{3f(a+b)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (-2*Cos[e + f*x])/(3*(a + b)^2*f*Sqrt[a + b - b*Cos[e + f*x]^2]) - (Cos[e +
 f*x]*Sin[e + f*x]^2)/(3*(a + b)*f*(a + b - b*Cos[e + f*x]^2)^(3/2))
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{\left(a+b-bx^2\right)^{5/2}}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{\cos(e+fx)\sin^2(e+fx)}{3(a+b)f\left(a+b-b\cos^2(e+fx)\right)^{3/2}} - \frac{2\,\text{Subst}\left(\int \frac{1}{\left(a+b-bx^2\right)^{3/2}}\,dx, x, \cos(e+fx)\right)}{3(a+b)f}$$
$$= -\frac{2\cos(e+fx)}{3(a+b)^2f\sqrt{a+b-b\cos^2(e+fx)}} - \frac{\cos(e+fx)\sin^2(e+fx)}{3(a+b)f\left(a+b-b\cos^2(e+fx)\right)^{3/2}}$$

Mathematica [A] time = 0.310118, size = 64, normalized size = 0.79

$$\frac{\sqrt{2}\cos(e+fx)((a+3b)\cos(2(e+fx))-5a-3b)}{3f(a+b)^2(2a-b\cos(2(e+fx))+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (Sqrt[2]*Cos[e + f*x]*(-5*a - 3*b + (a + 3*b)*Cos[2*(e + f*x)]))/(3*(a + b)
^2*f*(2*a + b - b*Cos[2*(e + f*x)])^(3/2))
```

Maple [A] time = 1.161, size = 56, normalized size = 0.7

$$-\frac{\left(a\left(\sin\left(fx+e\right)\right)^2+3\,b\left(\sin\left(fx+e\right)\right)^2+2\,a\right)\cos\left(fx+e\right)}{3\,(a+b)^2\,f}\left(a+b\left(\sin\left(fx+e\right)\right)^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/3*(a*sin(f*x+e)^2+3*b*sin(f*x+e)^2+2*a)*cos(f*x+e)/(a+b)^2/(a+b*sin(f*x+
e)^2)^(3/2)/f
```

Maxima [A] time = 0.99422, size = 163, normalized size = 2.01

$$-\frac{\frac{2\cos\left(fx+e\right)}{\sqrt{-b\cos\left(fx+e\right)^2+a+b}(a+b)^2}+\frac{\cos\left(fx+e\right)}{\left(-b\cos\left(fx+e\right)^2+a+b\right)^{\frac{3}{2}}(a+b)}-\frac{\cos\left(fx+e\right)}{\left(-b\cos\left(fx+e\right)^2+a+b\right)^{\frac{3}{2}}b}+\frac{\cos\left(fx+e\right)}{\sqrt{-b\cos\left(fx+e\right)^2+a+b}(a+b)b}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(2*cos(f*x + e)/(sqrt(-b*cos(f*x + e)^2 + a + b)*(a + b)^2) + cos(f*x
+ e)/((-b*cos(f*x + e)^2 + a + b)^(3/2)*(a + b)) - cos(f*x + e)/((-b*cos(f*
x + e)^2 + a + b)^(3/2)*b) + cos(f*x + e)/(sqrt(-b*cos(f*x + e)^2 + a + b)*
```

```
(a + b)*b))/f
```

Fricas [A] time = 2.67331, size = 323, normalized size = 3.99

$$\frac{\left((a+3b)\cos\left(fx+e\right)^3-3(a+b)\cos\left(fx+e\right)\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}}{3\left(\left(a^2b^2+2ab^3+b^4\right)f\cos\left(fx+e\right)^4-2\left(a^3b+3a^2b^2+3ab^3+b^4\right)f\cos\left(fx+e\right)^2+\left(a^4+4a^3b+6a^2b^2+4ab^3+b^4\right)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*((a + 3*b)*cos(f*x + e)^3 - 3*(a + b)*cos(f*x + e))*sqrt(-b*cos(f*x + e
)^2 + a + b)/((a^2*b^2 + 2*a*b^3 + b^4)*f*cos(f*x + e)^4 - 2*(a^3*b + 3*a^2
*b^2 + 3*a*b^3 + b^4)*f*cos(f*x + e)^2 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b
^3 + b^4)*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.35355, size = 201, normalized size = 2.48

$$-\frac{\sqrt{-\left(\cos\left(fx+e\right)^2-1\right)b+a}\left(\frac{3\left(abf^2+b^2f^2\right)}{a^2bf^2+2ab^2f^2+b^3f^2}-\frac{\left(abf^4+3b^2f^4\right)\cos\left(fx+e\right)^2}{\left(a^2bf^2+2ab^2f^2+b^3f^2\right)f^2}\right)\cos\left(fx+e\right)}{3\left(\left(\cos\left(fx+e\right)^2-1\right)b-a\right)^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(-(cos(f*x + e)^2 - 1)*b + a)*(3*(a*b*f^2 + b^2*f^2)/(a^2*b*f^2 +
2*a*b^2*f^2 + b^3*f^2) - (a*b*f^4 + 3*b^2*f^4)*cos(f*x + e)^2/((a^2*b*f^2 +
 2*a*b^2*f^2 + b^3*f^2)*f^2))*cos(f*x + e)/(((cos(f*x + e)^2 - 1)*b - a)^2*
f)
```

3.164 $\int \frac{\sin(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=73

$$-\frac{2\cos(e+fx)}{3f(a+b)^2\sqrt{a-b\cos^2(e+fx)+b}}-\frac{\cos(e+fx)}{3f(a+b)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}$$

```
[Out] -Cos[e + f*x]/(3*(a + b)*f*(a + b - b*Cos[e + f*x]^2)^(3/2)) - (2*Cos[e + f
*x])/(3*(a + b)^2*f*Sqrt[a + b - b*Cos[e + f*x]^2])
```

Rubi [A] time = 0.0580009, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3186, 192, 191}

$$-\frac{2\cos(e+fx)}{3f(a+b)^2\sqrt{a-b\cos^2(e+fx)+b}}-\frac{\cos(e+fx)}{3f(a+b)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] -Cos[e + f*x]/(3*(a + b)*f*(a + b - b*Cos[e + f*x]^2)^(3/2)) - (2*Cos[e + f
*x])/(3*(a + b)^2*f*Sqrt[a + b - b*Cos[e + f*x]^2])
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\sin(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{\left(a+b-bx^2\right)^{5/2}}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)}{3(a+b)f\left(a+b-b\cos^2(e+fx)\right)^{3/2}} - \frac{2\,\text{Subst}\left(\int \frac{1}{\left(a+b-bx^2\right)^{3/2}}\,dx, x, \cos(e+fx)\right)}{3(a+b)f}$$

$$= -\frac{\cos(e+fx)}{3(a+b)f\left(a+b-b\cos^2(e+fx)\right)^{3/2}} - \frac{2\cos(e+fx)}{3(a+b)^2f\sqrt{a+b-b\cos^2(e+fx)}}$$

Mathematica [A] time = 0.171712, size = 60, normalized size = 0.82

$$\frac{2\sqrt{2}\cos(e+fx)(-3a+b\cos(2(e+fx))-2b)}{3f(a+b)^2(2a-b\cos(2(e+fx))+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (2*Sqrt[2]*Cos[e + f*x]*(-3*a - 2*b + b*Cos[2*(e + f*x)]))/(3*(a + b)^2*f*(
2*a + b - b*Cos[2*(e + f*x)])^(3/2))
```

Maple [A] time = 1.041, size = 55, normalized size = 0.8

$$-\frac{\left(2\,b\left(\sin\left(fx+e\right)\right)^2+3\,a+b\right)\cos\left(fx+e\right)}{\left(3\,a^2+6\,ab+3\,b^2\right)f}\left(a+b\left(\sin\left(fx+e\right)\right)^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/3*(2*b*sin(f*x+e)^2+3*a+b)*cos(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2)/(a^2+2*a*
b+b^2)/f
```

Maxima [A] time = 0.958838, size = 85, normalized size = 1.16

$$-\frac{\frac{2\cos\left(fx+e\right)}{\sqrt{-b\cos\left(fx+e\right)^2+a+b}(a+b)^2}+\frac{\cos\left(fx+e\right)}{\left(-b\cos\left(fx+e\right)^2+a+b\right)^{\frac{3}{2}}(a+b)}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(2*cos(f*x + e)/(sqrt(-b*cos(f*x + e)^2 + a + b)*(a + b)^2) + cos(f*x
+ e)/((-b*cos(f*x + e)^2 + a + b)^(3/2)*(a + b)))/f
```

Fricas [B] time = 2.53603, size = 315, normalized size = 4.32

$$\frac{\left(2\,b\cos\left(fx+e\right)^{3}-3\,(a+b)\cos\left(fx+e\right)\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}}{3\,\big(\left(a^{2}b^{2}+2\,ab^{3}+b^{4}\right)f\cos\left(fx+e\right)^{4}-2\left(a^{3}b+3\,a^{2}b^{2}+3\,ab^{3}+b^{4}\right)f\cos\left(fx+e\right)^{2}+\big(a^{4}+4\,a^{3}b+6\,a^{2}b^{2}+4\,ab^{3}+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*b*cos(f*x + e)^3 - 3*(a + b)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 +
a + b)/((a^2*b^2 + 2*a*b^3 + b^4)*f*cos(f*x + e)^4 - 2*(a^3*b + 3*a^2*b^2 +
 3*a*b^3 + b^4)*f*cos(f*x + e)^2 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b
^4)*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.34598, size = 185, normalized size = 2.53

$$\frac{\left(\frac{2\,b^{2}f^{2}\cos\left(fx+e\right)^{2}}{a^{2}bf^{2}+2\,ab^{2}f^{2}+b^{3}f^{2}}-\frac{3\left(abf^{2}+b^{2}f^{2}\right)}{a^{2}bf^{2}+2\,ab^{2}f^{2}+b^{3}f^{2}}\right)\sqrt{-\left(\cos\left(fx+e\right)^{2}-1\right)b+a}\,\cos\left(fx+e\right)}{3\left(\left(\cos\left(fx+e\right)^{2}-1\right)b-a\right)^{2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(2*b^2*f^2*cos(f*x + e)^2/(a^2*b*f^2 + 2*a*b^2*f^2 + b^3*f^2) - 3*(a*b*
f^2 + b^2*f^2)/(a^2*b*f^2 + 2*a*b^2*f^2 + b^3*f^2))*sqrt(-(cos(f*x + e)^2 -
 1)*b + a)*cos(f*x + e)/(((cos(f*x + e)^2 - 1)*b - a)^2*f)
```

3.165 $\int \frac{\csc(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=129

$$\frac{b(5a+3b)\cos(e+fx)}{3a^2f(a+b)^2\sqrt{a-b\cos^2(e+fx)+b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{a^{5/2}f}+\frac{b\cos(e+fx)}{3af(a+b)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}$$

```
[Out] -(ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]]/(a^(5/2)*f
)) + (b*Cos[e + f*x])/(3*a*(a + b)*f*(a + b - b*Cos[e + f*x]^2)^(3/2)) + (b
*(5*a + 3*b)*Cos[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[a + b - b*Cos[e + f*x]^2
])
```

Rubi [A] time = 0.152596, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3186, 414, 527, 12, 377, 206}

$$\frac{b(5a+3b)\cos(e+fx)}{3a^2f(a+b)^2\sqrt{a-b\cos^2(e+fx)+b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{a^{5/2}f}+\frac{b\cos(e+fx)}{3af(a+b)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]]/(a^(5/2)*f
)) + (b*Cos[e + f*x])/(3*a*(a + b)*f*(a + b - b*Cos[e + f*x]^2)^(3/2)) + (b
*(5*a + 3*b)*Cos[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[a + b - b*Cos[e + f*x]^2
])
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\left(a+b-bx^2\right)^{5/2}}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= \frac{b\cos(e+fx)}{3a(a+b)f\left(a+b-b\cos^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-3a-b-2bx^2}{(1-x^2)\left(a+b-bx^2\right)^{3/2}}\,dx, x, \cos(e+fx)\right)}{3a(a+b)f}$$

$$= \frac{b\cos(e+fx)}{3a(a+b)f\left(a+b-b\cos^2(e+fx)\right)^{3/2}} + \frac{b(5a+3b)\cos(e+fx)}{3a^2(a+b)^2f\sqrt{a+b-b\cos^2(e+fx)}} - \frac{\text{Subs}}{}$$

$$= \frac{b\cos(e+fx)}{3a(a+b)f\left(a+b-b\cos^2(e+fx)\right)^{3/2}} + \frac{b(5a+3b)\cos(e+fx)}{3a^2(a+b)^2f\sqrt{a+b-b\cos^2(e+fx)}} - \frac{\text{Subs}}{}$$

$$= \frac{b\cos(e+fx)}{3a(a+b)f\left(a+b-b\cos^2(e+fx)\right)^{3/2}} + \frac{b(5a+3b)\cos(e+fx)}{3a^2(a+b)^2f\sqrt{a+b-b\cos^2(e+fx)}} - \frac{\text{Subs}}{}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{a^{5/2}f} + \frac{b\cos(e+fx)}{3a(a+b)f\left(a+b-b\cos^2(e+fx)\right)^{3/2}} + \frac{b(5a+3}{3a^2(a+b)^2f\sqrt{}}$$

Mathematica [A] time = 0.590195, size = 127, normalized size = 0.98

$$\frac{\frac{\sqrt{2}b\cos(e+fx)\left(12a^2-b(5a+3b)\cos(2(e+fx))+13ab+3b^2\right)}{3a^2(a+b)^2(2a-b\cos(2(e+fx))+b)^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cos(e+fx)}{\sqrt{2a-b\cos(2(e+fx))+b}}\right)}{a^{5/2}}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (-(ArcTanh[(Sqrt[2]*Sqrt[a]*Cos[e + f*x])/Sqrt[2*a + b - b*Cos[2*(e + f*x)]
]]/a^(5/2)) + (Sqrt[2]*b*Cos[e + f*x]*(12*a^2 + 13*a*b + 3*b^2 - b*(5*a + 3
```

```
*b)*Cos[2*(e + f*x)]))/(3*a^2*(a + b)^2*(2*a + b - b*Cos[2*(e + f*x)])^(3/2
)))/f
```

Maple [B] time = 2.726, size = 249, normalized size = 1.9

$$\frac{1}{f\cos\left(fx+e\right)}\sqrt{-\left(-b\left(\sin\left(fx+e\right)\right)^{2}-a\right)\left(\cos\left(fx+e\right)\right)^{2}}\left(-\frac{1}{2}\ln\left(\frac{1}{\left(\sin\left(fx+e\right)\right)^{2}}\left(2a+\left(-a+b\right)\left(\sin\left(fx+e\right)\right)^{2}+2\sqrt{a}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] (-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2)*(-1/2/a^(5/2)*ln((2*a+(-a+b)*sin(
f*x+e)^2+2*a^(1/2)*(-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2))/sin(f*x+e)^2)
+1/3/a*b*(2*b*sin(f*x+e)^2+3*a+b)*cos(f*x+e)^2/(-(-b*sin(f*x+e)^2-a)*cos(f*
x+e)^2)^(1/2)/(a+b*sin(f*x+e)^2)/(a^2+2*a*b+b^2)+1/a^2*b*cos(f*x+e)^2/(a+b)
/(-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2))/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(
1/2)/f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.16613, size = 1723, normalized size = 13.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*
b^2 + 4*a*b^3 + b^4 - 2*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cos(f*x + e)^2)
*sqrt(a)*log(2*((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 2*(3*a^2 + 2*a*b - b^2
)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + (a + b)*cos(f*x + e))*sqrt(-
b*cos(f*x + e)^2 + a + b)*sqrt(a) + a^2 + 2*a*b + b^2)/(cos(f*x + e)^4 - 2*
cos(f*x + e)^2 + 1)) - 4*((5*a^2*b^2 + 3*a*b^3)*cos(f*x + e)^3 - 3*(2*a^3*b
 + 3*a^2*b^2 + a*b^3)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b))/((a^5*
b^2 + 2*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^4 - 2*(a^6*b + 3*a^5*b^2 + 3*a^4*
b^3 + a^3*b^4)*f*cos(f*x + e)^2 + (a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 +
a^3*b^4)*f), 1/6*(3*((a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 + a^4 + 4*a^3
*b + 6*a^2*b^2 + 4*a*b^3 + b^4 - 2*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cos(
f*x + e)^2)*sqrt(-a)*arctan(-1/2*((a - b)*cos(f*x + e)^2 + a + b)*sqrt(-b*c
os(f*x + e)^2 + a + b)*sqrt(-a)/(a*b*cos(f*x + e)^3 - (a^2 + a*b)*cos(f*x +
```

```
 e))) - 2*((5*a^2*b^2 + 3*a*b^3)*cos(f*x + e)^3 - 3*(2*a^3*b + 3*a^2*b^2 +
a*b^3)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b))/((a^5*b^2 + 2*a^4*b^3
 + a^3*b^4)*f*cos(f*x + e)^4 - 2*(a^6*b + 3*a^5*b^2 + 3*a^4*b^3 + a^3*b^4)*
f*cos(f*x + e)^2 + (a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

3.166 $\int \frac{\sin^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=285

$$\frac{\left(8a^2+13ab+3b^2\right)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}E\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{3b^3f(a+b)^2\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{2a(2a+3b)\sin(e+fx)\,c}{3b^2f(a+b)^2\sqrt{a+b\sin}}$$

```
[Out] (a*Cos[e + f*x]*Sin[e + f*x]^3)/(3*b*(a + b)*f*(a + b*Sin[e + f*x]^2)^(3/2)
) + (2*a*(2*a + 3*b)*Cos[e + f*x]*Sin[e + f*x])/(3*b^2*(a + b)^2*f*Sqrt[a +
 b*Sin[e + f*x]^2]) + ((8*a^2 + 13*a*b + 3*b^2)*Sqrt[Cos[e + f*x]^2]*Ellipt
icE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/
(3*b^3*(a + b)^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) - (a*(8*a + 9*b)*Sqrt[Co
s[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 +
 (b*Sin[e + f*x]^2)/a])/(3*b^3*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.332362, antiderivative size = 285, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3188, 470, 578, 524, 426, 424, 421, 419}

$$\frac{\left(8a^2+13ab+3b^2\right)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}E\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{3b^3f(a+b)^2\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{2a(2a+3b)\sin(e+fx)\,c}{3b^2f(a+b)^2\sqrt{a+b\sin}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^6/(a + b*Sin[e + f*x]^2)^(5/2),x]
```


```
[Out] (a*Cos[e + f*x]*Sin[e + f*x]^3)/(3*b*(a + b)*f*(a + b*Sin[e + f*x]^2)^(3/2)
) + (2*a*(2*a + 3*b)*Cos[e + f*x]*Sin[e + f*x])/(3*b^2*(a + b)^2*f*Sqrt[a +
 b*Sin[e + f*x]^2]) + ((8*a^2 + 13*a*b + 3*b^2)*Sqrt[Cos[e + f*x]^2]*Ellipt
icE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/
(3*b^3*(a + b)^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) - (a*(8*a + 9*b)*Sqrt[Co
s[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 +
 (b*Sin[e + f*x]^2)/a])/(3*b^3*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2])
```


Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
```

```
p, q, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\sin^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^6}{\sqrt{1-x^2}\left(a+bx^2\right)^{5/2}}\,dx,x,\sin(e+fx)\right)}{f}$$

$$= \frac{a\cos(e+fx)\sin^3(e+fx)}{3b(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^2\left(3a+(-4a-3b)x\right.}{\sqrt{1-x^2}\left(a+bx^2\right)^{3/}}\right.}{3b(a+b)f}$$

$$= \frac{a\cos(e+fx)\sin^3(e+fx)}{3b(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2a(2a+3b)\cos(e+fx)\sin(e+fx)}{3b^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(\sqrt{\cos^2(e+f\right.}{}$$

$$= \frac{a\cos(e+fx)\sin^3(e+fx)}{3b(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2a(2a+3b)\cos(e+fx)\sin(e+fx)}{3b^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(a(8a+9b)\sqrt{\right.}{}$$

$$= \frac{a\cos(e+fx)\sin^3(e+fx)}{3b(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2a(2a+3b)\cos(e+fx)\sin(e+fx)}{3b^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(\left(-8a^2-13ab\right.\right.}{}$$

$$= \frac{a\cos(e+fx)\sin^3(e+fx)}{3b(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2a(2a+3b)\cos(e+fx)\sin(e+fx)}{3b^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\left(8a^2+13ab+\right.}{}$$

Mathematica [A] time = 2.01392, size = 192, normalized size = 0.67

$$-\frac{a\left(\sqrt{2}b\sin(2(e+fx))\left(-8a^2+b(5a+7b)\cos(2(e+fx))-17ab-7b^2\right)+2a\left(8a^2+17ab+9b^2\right)\left(\frac{2a-b\cos(2(e+fx))+b}{a}\right)^{3/2}F\right.}{6b^3f(a+b)^2(2a-b\cos(2(e+fx))+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^6/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] -(a*(-2*a*(8*a^2 + 13*a*b + 3*b^2)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)
*EllipticE[e + f*x, -(b/a)] + 2*a*(8*a^2 + 17*a*b + 9*b^2)*((2*a + b - b*Co
s[2*(e + f*x)])/a)^(3/2)*EllipticF[e + f*x, -(b/a)] + Sqrt[2]*b*(-8*a^2 - 1
7*a*b - 7*b^2 + b*(5*a + 7*b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)]))/(6*b^3*(
a + b)^2*f*(2*a + b - b*Cos[2*(e + f*x)])^(3/2))
```

Maple [B] time = 1.5, size = 698, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^6/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/3*((5*a*b^2+7*b^3)*sin(f*x+e)*cos(f*x+e)^4+(-4*a^2*b-11*a*b^2-7*b^3)*cos
(f*x+e)^2*sin(f*x+e)-(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(cos(f*x+e)^2)^(1/2)
```

```
*b*(8*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2+17*EllipticF(sin(f*x+e),(-1/
a*b)^(1/2))*a*b+9*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^2-8*EllipticE(sin(
f*x+e),(-1/a*b)^(1/2))*a^2-13*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b-3*El
lipticE(sin(f*x+e),(-1/a*b)^(1/2))*b^2)*cos(f*x+e)^2+8*(cos(f*x+e)^2)^(1/2)
*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3
+25*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*
x+e),(-1/a*b)^(1/2))*a^2*b+26*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)
/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2+9*(cos(f*x+e)^2)^(1/2)
*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^3
-8*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x
+e),(-1/a*b)^(1/2))*a^3-21*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)
^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b-16*(cos(f*x+e)^2)^(1/2)*(
-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2
-3*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x
+e),(-1/a*b)^(1/2))*b^3)*a/(a+b*sin(f*x+e)^2)^(3/2)/(a+b)^2/b^3/cos(f*x+e)/
f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^{6}}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^6/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(\cos\left(fx+e\right)^{6}-3\,\cos\left(fx+e\right)^{4}+3\,\cos\left(fx+e\right)^{2}-1\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}}{b^{3}\cos\left(fx+e\right)^{6}-3\left(ab^{2}+b^{3}\right)\cos\left(fx+e\right)^{4}-a^{3}-3\,a^{2}b-3\,ab^{2}-b^{3}+3\left(a^{2}b+2\,ab^{2}+b^{3}\right)\cos\left(fx+e\right)^{2}},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((cos(f*x + e)^6 - 3*cos(f*x + e)^4 + 3*cos(f*x + e)^2 - 1)*sqrt(-b
*cos(f*x + e)^2 + a + b)/(b^3*cos(f*x + e)^6 - 3*(a*b^2 + b^3)*cos(f*x + e)
^4 - a^3 - 3*a^2*b - 3*a*b^2 - b^3 + 3*(a^2*b + 2*a*b^2 + b^3)*cos(f*x + e)
^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**6/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^{6}}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^6/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

3.167 $\int \frac{\sin^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=269

$$\frac{(2a+3b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{3b^2f(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a+2b)\sqrt{\cos^2(e+fx)}\sec(e+fx)}{3b^2f(a+}$$

```
[Out] (a*Cos[e + f*x]*Sin[e + f*x])/(3*b*(a + b)*f*(a + b*Sin[e + f*x]^2)^(3/2))
- (2*(a + 2*b)*Cos[e + f*x]*Sin[e + f*x])/(3*b*(a + b)^2*f*Sqrt[a + b*Sin[e
 + f*x]^2]) - (2*(a + 2*b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*
x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*b^2*(a + b)^2*f*Sq
rt[1 + (b*Sin[e + f*x]^2)/a]) + ((2*a + 3*b)*Sqrt[Cos[e + f*x]^2]*EllipticF
[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])
/(3*b^2*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.275601, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3188, 470, 527, 524, 426, 424, 421, 419}

$$\frac{(2a+3b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{3b^2f(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a+2b)\sqrt{\cos^2(e+fx)}\sec(e+fx)}{3b^2f(a+}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(5/2),x]
```


```
[Out] (a*Cos[e + f*x]*Sin[e + f*x])/(3*b*(a + b)*f*(a + b*Sin[e + f*x]^2)^(3/2))
- (2*(a + 2*b)*Cos[e + f*x]*Sin[e + f*x])/(3*b*(a + b)^2*f*Sqrt[a + b*Sin[e
 + f*x]^2]) - (2*(a + 2*b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*
x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*b^2*(a + b)^2*f*Sq
rt[1 + (b*Sin[e + f*x]^2)/a]) + ((2*a + 3*b)*Sqrt[Cos[e + f*x]^2]*EllipticF
[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])
/(3*b^2*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2])
```


Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
```

p, q, x]

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\operatorname{Subst}\left(\int \frac{x^4}{\sqrt{1-x^2}\left(a+bx^2\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{a\cos(e+fx)\sin(e+fx)}{3b(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\operatorname{Subst}\left(\int \frac{a+(-2a-3b)}{\sqrt{1-x^2}\left(a+bx^2\right.}\right.}{3b(a+b)f}$$

$$= \frac{a\cos(e+fx)\sin(e+fx)}{3b(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{2(a+2b)\cos(e+fx)\sin(e+fx)}{3b(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\left(\sqrt{\cos^2(e+f\right.}{}$$

$$= \frac{a\cos(e+fx)\sin(e+fx)}{3b(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{2(a+2b)\cos(e+fx)\sin(e+fx)}{3b(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(2(a+2b)\sqrt{c\right.}{}$$

$$= \frac{a\cos(e+fx)\sin(e+fx)}{3b(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{2(a+2b)\cos(e+fx)\sin(e+fx)}{3b(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(2(a+2b)\sqrt{c\right.}{}$$

$$= \frac{a\cos(e+fx)\sin(e+fx)}{3b(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{2(a+2b)\cos(e+fx)\sin(e+fx)}{3b(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{2(a+2b)\sqrt{co}{}$$

Mathematica [A] time = 1.63713, size = 182, normalized size = 0.68

$$-\frac{-\sqrt{2}b\sin(2(e+fx))\left(-a^2+b(a+2b)\cos(2(e+fx))-4ab-2b^2\right)-a\left(2a^2+5ab+3b^2\right)\left(\frac{2a-b\cos(2(e+fx))+b}{a}\right)^{3/2}F\left(e+\right.}{3b^2f(a+b)^2(2a-b\cos(2(e+fx))+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] -(2*a^2*(a + 2*b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticE[e + f*
x, -(b/a)] - a*(2*a^2 + 5*a*b + 3*b^2)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(
3/2)*EllipticF[e + f*x, -(b/a)] - Sqrt[2]*b*(-a^2 - 4*a*b - 2*b^2 + b*(a +
2*b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/(3*b^2*(a + b)^2*f*(2*a + b - b*Co
s[2*(e + f*x)])^(3/2))
```

Maple [B] time = 1.527, size = 623, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3*((2*a*b^2+4*b^3)*sin(f*x+e)*cos(f*x+e)^4+(-a^2*b-5*a*b^2-4*b^3)*cos(f*x
+e)^2*sin(f*x+e)-(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*b*(
2*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2+5*EllipticF(sin(f*x+e),(-1/a*b)^
```

```
(1/2))*a*b+3*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^2-2*EllipticE(sin(f*x+e
),(-1/a*b)^(1/2))*a^2-4*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b)*cos(f*x+e
)^2+2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(
f*x+e),(-1/a*b)^(1/2))*a^3+7*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/
a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b+8*(cos(f*x+e)^2)^(1/2)*
(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b^
2+3*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*
x+e),(-1/a*b)^(1/2))*b^3-2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)
^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3-6*(cos(f*x+e)^2)^(1/2)*(-b/
a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b-4*
(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e)
,(-1/a*b)^(1/2))*a*b^2)/(a+b*sin(f*x+e)^2)^(3/2)/(a+b)^2/b^2/cos(f*x+e)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^4}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^4/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(\cos\left(fx+e\right)^4-2\cos\left(fx+e\right)^2+1\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}}{b^3\cos\left(fx+e\right)^6-3\left(ab^2+b^3\right)\cos\left(fx+e\right)^4-a^3-3a^2b-3ab^2-b^3+3\left(a^2b+2ab^2+b^3\right)\cos\left(fx+e\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-b*cos(f*x + e)^2 +
a + b)/(b^3*cos(f*x + e)^6 - 3*(a*b^2 + b^3)*cos(f*x + e)^4 - a^3 - 3*a^2*b
 - 3*a*b^2 - b^3 + 3*(a^2*b + 2*a*b^2 + b^3)*cos(f*x + e)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^4}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^4/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

3.168 $\int \frac{\sin^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=221

$$-\frac{(a-b)\sin(e+fx)\cos(e+fx)}{3af(a+b)^2\sqrt{a+b\sin^2(e+fx)}}-\frac{\sin(e+fx)\cos(e+fx)}{3f(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(e+fx\middle|-\frac{b}{a}\right)}{3bf(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{(a-b)\sqrt{a+b\text{s}}}{3abf(a+}$$

```
[Out] -(Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)*f*(a + b*Sin[e + f*x]^2)^(3/2)) - (
(a - b)*Cos[e + f*x]*Sin[e + f*x])/(3*a*(a + b)^2*f*Sqrt[a + b*Sin[e + f*x]
^2]) - ((a - b)*EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a
*b*(a + b)^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (EllipticF[e + f*x, -(b/a)
]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*b*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2]
)
```

Rubi [A] time = 0.289666, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3173, 3172, 3178, 3177, 3183, 3182}

$$-\frac{(a-b)\sin(e+fx)\cos(e+fx)}{3af(a+b)^2\sqrt{a+b\sin^2(e+fx)}}-\frac{\sin(e+fx)\cos(e+fx)}{3f(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(e+fx\middle|-\frac{b}{a}\right)}{3bf(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{(a-b)\sqrt{a+b\text{s}}}{3abf(a+}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] -(Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)*f*(a + b*Sin[e + f*x]^2)^(3/2)) - (
(a - b)*Cos[e + f*x]*Sin[e + f*x])/(3*a*(a + b)^2*f*Sqrt[a + b*Sin[e + f*x]
^2]) - ((a - b)*EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a
*b*(a + b)^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (EllipticF[e + f*x, -(b/a)
]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*b*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2]
)
```

Rule 3173

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b - a*B)*Cos[e + f*x]*Sin[e + f*x]
*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(a + b)*(p + 1)), x] - Dist[1/(2*a*
(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p +
 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = -\frac{\cos(e+fx)\sin(e+fx)}{3(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{\int \frac{a+a\sin^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx}{3a(a+b)}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{3(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{(a-b)\cos(e+fx)\sin(e+fx)}{3a(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\int \frac{2a^2-a(a-b)\sin^{2}(e}{\sqrt{a+b\sin^2(e}}}{3a^2(a+}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{3(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{(a-b)\cos(e+fx)\sin(e+fx)}{3a(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{(a-b)\int\sqrt{a+}}{3ab}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{3(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{(a-b)\cos(e+fx)\sin(e+fx)}{3a(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left((a-b)\sqrt{a+}\right.}{3a}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{3(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{(a-b)\cos(e+fx)\sin(e+fx)}{3a(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{(a-b)E\left(e+f\right.}{3ab(a+}$$

Mathematica [A] time = 1.46077, size = 174, normalized size = 0.79

$$\frac{-\sqrt{2}b\sin(2(e+fx))\left(4a^2+b(b-a)\cos(2(e+fx))+ab-b^2\right)+2a^2(a+b)\left(\frac{2a-b\cos(2(e+fx))+b}{a}\right)^{3/2}F\left(e+fx\middle|-\frac{b}{a}\right)-2a^2}{6abf(a+b)^2(2a-b\cos(2(e+fx))+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (-2*a^2*(a - b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticE[e + f*x,
 -(b/a)] + 2*a^2*(a + b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticF
[e + f*x, -(b/a)] - Sqrt[2]*b*(4*a^2 + a*b - b^2 + b*(-a + b)*Cos[2*(e + f*
x)])*Sin[2*(e + f*x)])/(6*a*b*(a + b)^2*f*(2*a + b - b*Cos[2*(e + f*x)])^(3
/2))
```

Maple [A] time = 1.424, size = 483, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3*((a*b^2-b^3)*sin(f*x+e)*cos(f*x+e)^4+(-2*a^2*b-a*b^2+b^3)*cos(f*x+e)^2*
sin(f*x+e)-(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(cos(f*x+e)^2)^(1/2)*a*b*(Elli
pticF(sin(f*x+e),(-1/a*b)^(1/2))*a+EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-E
llipticE(sin(f*x+e),(-1/a*b)^(1/2))*a+EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*
b)*cos(f*x+e)^2+(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*Elli
pticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3+2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e
)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b+(cos(f*x+e)^2
)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/
2))*a*b^2-(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(
sin(f*x+e),(-1/a*b)^(1/2))*a^3+(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b
)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2)/(a+b*sin(f*x+e)^2)^(
3/2)/(a+b)^2/a/b/cos(f*x+e)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^2}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\left(\cos\left(fx+e\right)^2-1\right)}{b^3\cos\left(fx+e\right)^6-3\left(ab^2+b^3\right)\cos\left(fx+e\right)^4-a^3-3a^2b-3ab^2-b^3+3\left(a^2b+2ab^2+b^3\right)\cos\left(fx+e\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)*(cos(f*x + e)^2 - 1)/(b^3*cos(f*x
+ e)^6 - 3*(a*b^2 + b^3)*cos(f*x + e)^4 - a^3 - 3*a^2*b - 3*a*b^2 - b^3 + 3
```

```
*(a^2*b + 2*a*b^2 + b^3)*cos(f*x + e)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(fx+e\right)^2}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

3.169 $\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=223

$$\frac{2b(2a+b)\sin(e+fx)\cos(e+fx)}{3a^2f(a+b)^2\sqrt{a+b\sin^2(e+fx)}}+\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\left|-\frac{b}{a}\right.\right)}{3a^2f(a+b)^2\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{b\sin(e+fx)\cos(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}-$$

```
[Out] (b*Cos[e + f*x]*Sin[e + f*x])/(3*a*(a + b)*f*(a + b*Sin[e + f*x]^2)^(3/2))
+ (2*b*(2*a + b)*Cos[e + f*x]*Sin[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[a + b*S
in[e + f*x]^2]) + (2*(2*a + b)*EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e
+ f*x]^2])/(3*a^2*(a + b)^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) - (EllipticF[
e + f*x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*a*(a + b)*f*Sqrt[a + b*
Sin[e + f*x]^2])
```

Rubi [A] time = 0.255501, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3184, 3173, 3172, 3178, 3177, 3183, 3182}

$$\frac{2b(2a+b)\sin(e+fx)\cos(e+fx)}{3a^2f(a+b)^2\sqrt{a+b\sin^2(e+fx)}}+\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\left|-\frac{b}{a}\right.\right)}{3a^2f(a+b)^2\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{b\sin(e+fx)\cos(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x]^2)^(-5/2),x]
```

```
[Out] (b*Cos[e + f*x]*Sin[e + f*x])/(3*a*(a + b)*f*(a + b*Sin[e + f*x]^2)^(3/2))
+ (2*b*(2*a + b)*Cos[e + f*x]*Sin[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[a + b*S
in[e + f*x]^2]) + (2*(2*a + b)*EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e
+ f*x]^2])/(3*a^2*(a + b)^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) - (EllipticF[
e + f*x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*a*(a + b)*f*Sqrt[a + b*
Sin[e + f*x]^2])
```

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a +
 b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)
*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rule 3173

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b - a*B)*Cos[e + f*x]*Sin[e + f*x]
*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(a + b)*(p + 1)), x] - Dist[1/(2*a*
(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p +
 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{b\cos(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\int \frac{-3a-2b+b\sin^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx}{3a(a+b)}$$

$$= \frac{b\cos(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(2a+b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{\int \frac{-a(3a+b)-2b}{\sqrt{a+b}}}{3a^2}$$

$$= \frac{b\cos(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(2a+b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{\int \frac{1}{\sqrt{a+b\sin^2(e}}}{3a(a+}$$

$$= \frac{b\cos(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(2a+b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\left(2(2a+b)\sqrt{}\right.}{}$$

$$= \frac{b\cos(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(2a+b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{2(2a+b)E\left(\right.}{3a^2(}$$

Mathematica [A] time = 1.32157, size = 172, normalized size = 0.77

$$\frac{-\sqrt{2}b\sin(2(e+fx))\left(-5a^2+b(2a+b)\cos(2(e+fx))-5ab-b^2\right)-a^2(a+b)\left(\frac{2a-b\cos(2(e+fx))+b}{a}\right)^{3/2}F\left(e+fx\middle|-\frac{b}{a}\right)+2}{3a^2f(a+b)^2(2a-b\cos(2(e+fx))+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)^(-5/2),x]
```

```
[Out] (2*a^2*(2*a + b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticE[e + f*x
, -(b/a)] - a^2*(a + b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticF[
e + f*x, -(b/a)] - Sqrt[2]*b*(-5*a^2 - 5*a*b - b^2 + b*(2*a + b)*Cos[2*(e +
 f*x)])*Sin[2*(e + f*x)])/(3*a^2*(a + b)^2*f*(2*a + b - b*Cos[2*(e + f*x)])
^(3/2))
```

Maple [B] time = 1.27, size = 547, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/3*((cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e
),(-1/a*b)^(1/2))*a^2*b*sin(f*x+e)^2+(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^
2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2*sin(f*x+e)^2-4*(cos(
f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^
(1/2))*a^2*b*sin(f*x+e)^2-2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/
2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2*sin(f*x+e)^2+4*a*b^2*sin(f*x+
e)^5+2*b^3*sin(f*x+e)^5+(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*E
llipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3+a^2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(
f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-4*(cos(f*x+e)^2)^
(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3
-2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(
-1/a*b)^(1/2))*a^2*b+5*a^2*b*sin(f*x+e)^3-a*b^2*sin(f*x+e)^3-2*sin(f*x+e)^3
*b^3-5*sin(f*x+e)*a^2*b-3*a*b^2*sin(f*x+e))/(a+b*sin(f*x+e)^2)^(3/2)/a^2/(a
+b)^2/cos(f*x+e)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(-5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}}{b^{3}\cos\left(fx+e\right)^{6}-3\left(ab^{2}+b^{3}\right)\cos\left(fx+e\right)^{4}-a^{3}-3\,a^{2}b-3\,ab^{2}-b^{3}+3\left(a^{2}b+2\,ab^{2}+b^{3}\right)\cos\left(fx+e\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)/(b^3*cos(f*x + e)^6 - 3*(a*b^2 +
b^3)*cos(f*x + e)^4 - a^3 - 3*a^2*b - 3*a*b^2 - b^3 + 3*(a^2*b + 2*a*b^2 +
b^3)*cos(f*x + e)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(-5/2), x)
```

3.170 $\int \frac{\csc^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=322

$$-\frac{\left(3a^2+13ab+8b^2\right)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^3f(a+b)^2}-\frac{\left(3a^2+13ab+8b^2\right)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx}}{3a^3f(a+b)^2\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

```
[Out] (b*Cot[e + f*x])/(3*a*(a + b)*f*(a + b*Sin[e + f*x]^2)^(3/2)) + (2*b*(3*a +
 2*b)*Cot[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[a + b*Sin[e + f*x]^2]) - ((3*a^
2 + 13*a*b + 8*b^2)*Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a^3*(a + b)
^2*f) - ((3*a^2 + 13*a*b + 8*b^2)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin
[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a^3*(a + b)
^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + ((3*a + 4*b)*Sqrt[Cos[e + f*x]^2]*El
lipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]
^2)/a])/(3*a^2*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.411473, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {3188, 472, 579, 583, 524, 426, 424, 421, 419}

$$-\frac{\left(3a^2+13ab+8b^2\right)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^3f(a+b)^2}-\frac{\left(3a^2+13ab+8b^2\right)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx}}{3a^3f(a+b)^2\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (b*Cot[e + f*x])/(3*a*(a + b)*f*(a + b*Sin[e + f*x]^2)^(3/2)) + (2*b*(3*a +
 2*b)*Cot[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[a + b*Sin[e + f*x]^2]) - ((3*a^
2 + 13*a*b + 8*b^2)*Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a^3*(a + b)
^2*f) - ((3*a^2 + 13*a*b + 8*b^2)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin
[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a^3*(a + b)
^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + ((3*a + 4*b)*Sqrt[Cos[e + f*x]^2]*El
lipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]
^2)/a])/(3*a^2*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
```

```
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-x^2}\left(a+bx^2\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{b\cot(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{-3a-4b+3bx^2}{x^2\sqrt{1-x^2}\left(a+bx^2\right)^{3}}\right.}{3a(a+b)f}$$

$$= \frac{b\cot(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(3a+2b)\cot(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\left(\sqrt{\cos^2(e+fx)}\right.}{}$$

$$= \frac{b\cot(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(3a+2b)\cot(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(3a^2+13ab+8b\right.}{}$$

$$= \frac{b\cot(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(3a+2b)\cot(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(3a^2+13ab+8b\right.}{}$$

$$= \frac{b\cot(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(3a+2b)\cot(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(3a^2+13ab+8b\right.}{}$$

$$= \frac{b\cot(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(3a+2b)\cot(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(3a^2+13ab+8b\right.}{}$$

Mathematica [A] time = 2.28742, size = 214, normalized size = 0.66

$$\frac{4a^2\left(\frac{2a-b\cos(2(e+fx))+b}{a}\right)^{3/2}\left(\left(3a^2+7ab+4b^2\right)F\left(e+fx\middle|-\frac{b}{a}\right)-\left(3a^2+13ab+8b^2\right)E\left(e+fx\middle|-\frac{b}{a}\right)\right)-2\sqrt{2}\left(2ab^2(a+b)\sin\right.}{12a^3f(a+b)^2(2a-b\cos}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (4*a^2*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*(-((3*a^2 + 13*a*b + 8*b^2)
*EllipticE[e + f*x, -(b/a)]) + (3*a^2 + 7*a*b + 4*b^2)*EllipticF[e + f*x, -
(b/a)]) - 2*Sqrt[2]*(3*(a + b)^2*(2*a + b - b*Cos[2*(e + f*x)])^2*Cot[e + f
*x] + 2*a*b^2*(a + b)*Sin[2*(e + f*x)] + b^2*(7*a + 5*b)*(2*a + b - b*Cos[2
*(e + f*x)])*Sin[2*(e + f*x)]))/(12*a^3*(a + b)^2*f*(2*a + b - b*Cos[2*(e +
 f*x)])^(3/2))
```

Maple [A] time = 1.819, size = 527, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3*(-(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*a*b*(3*Ellipti
cF(sin(f*x+e),(-1/a*b)^(1/2))*a^2+7*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*
b+4*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^2-3*EllipticE(sin(f*x+e),(-1/a*b
)^(1/2))*a^2-13*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b-8*EllipticE(sin(f*
x+e),(-1/a*b)^(1/2))*b^2)*sin(f*x+e)*cos(f*x+e)^2+(cos(f*x+e)^2)^(1/2)*(-b/
a*cos(f*x+e)^2+(a+b)/a)^(1/2)*a*(3*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3
+10*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b+11*EllipticF(sin(f*x+e),(-1/
a*b)^(1/2))*a*b^2+4*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^3-3*EllipticE(si
n(f*x+e),(-1/a*b)^(1/2))*a^3-16*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b-
21*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2-8*EllipticE(sin(f*x+e),(-1/a*
b)^(1/2))*b^3)*sin(f*x+e)+(-3*a^2*b^2-13*a*b^3-8*b^4)*cos(f*x+e)^6+(6*a^3*b
+26*a^2*b^2+38*a*b^3+16*b^4)*cos(f*x+e)^4+(-3*a^4-12*a^3*b-26*a^2*b^2-25*a*
b^3-8*b^4)*cos(f*x+e)^2)/(a+b*sin(f*x+e)^2)^(3/2)/(a+b)^2/sin(f*x+e)/a^3/co
s(f*x+e)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\csc\left(fx+e\right)^2}{b^3\cos\left(fx+e\right)^6-3\left(ab^2+b^3\right)\cos\left(fx+e\right)^4-a^3-3a^2b-3ab^2-b^3+3\left(a^2b+2ab^2+b^3\right)\cos\left(fx+e\right)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)*csc(f*x + e)^2/(b^3*cos(f*x + e)^
6 - 3*(a*b^2 + b^3)*cos(f*x + e)^4 - a^3 - 3*a^2*b - 3*a*b^2 - b^3 + 3*(a^2
*b + 2*a*b^2 + b^3)*cos(f*x + e)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^{2}}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

3.171 $\int (d\sin(e+fx))^m \left(a+b\sin^2(e+fx)\right)^p \, dx$

Optimal. Leaf size=122

$$-\frac{d\cos(e+fx)\sin^2(e+fx)^{\frac{1-m}{2}}(d\sin(e+fx))^{m-1}\left(a-b\cos^2(e+fx)+b\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}F_1\left(\frac{1}{2};\frac{1-m}{2},-p;\frac{3}{2};\cos^2(e}{f}$$

```
[Out] -((d*AppellF1[1/2, (1 - m)/2, -p, 3/2, Cos[e + f*x]^2, (b*Cos[e + f*x]^2)/(
a + b)]*Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2)^p*(d*Sin[e + f*x])^(-1 + m)
*(Sin[e + f*x]^2)^((1 - m)/2))/(f*(1 - (b*Cos[e + f*x]^2)/(a + b))^p))
```

Rubi [A] time = 0.115666, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3189, 430, 429}

$$-\frac{d\cos(e+fx)\sin^2(e+fx)^{\frac{1-m}{2}}(d\sin(e+fx))^{m-1}\left(a-b\cos^2(e+fx)+b\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}F_1\left(\frac{1}{2};\frac{1-m}{2},-p;\frac{3}{2};\cos^2(e}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^m*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] -((d*AppellF1[1/2, (1 - m)/2, -p, 3/2, Cos[e + f*x]^2, (b*Cos[e + f*x]^2)/(
a + b)]*Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2)^p*(d*Sin[e + f*x])^(-1 + m)
*(Sin[e + f*x]^2)^((1 - m)/2))/(f*(1 - (b*Cos[e + f*x]^2)/(a + b))^p))
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int (d\sin(e+fx))^m \left(a+b\sin^2(e+fx)\right)^p\,dx = -\frac{\left(d(d\sin(e+fx))^{2\left(-\frac{1}{2}+\frac{m}{2}\right)}\sin^2(e+fx)^{\frac{1}{2}-\frac{m}{2}}\right)\text{Subst}\left(\int \left(1-x^2\right)^{\frac{1}{2}(-1+m)}\left(a+\right.\right.}{f}$$

$$= -\frac{\left(d\left(a+b-b\cos^2(e+fx)\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}(d\sin(e+fx))^{2\left(-\frac{1}{2}+\frac{m}{2}\right)}\sin\right.}{f}$$

$$= -\frac{dF_1\left(\frac{1}{2};\frac{1-m}{2},-p;\frac{3}{2};\cos^2(e+fx),\frac{b\cos^2(e+fx)}{a+b}\right)\cos(e+fx)\left(a+b-b\cos^2(e\right.}{f}$$

Mathematica [A] time = 0.44214, size = 113, normalized size = 0.93

$$\frac{\sqrt{\cos^2(e+fx)}\tan(e+fx)(d\sin(e+fx))^m\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2};\frac{1}{2},-p;\frac{m+3}{2};\sin^2(e+fx),-\frac{b\mathrm{s}}{}\right.}{f(m+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^m*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[(1 + m)/2, 1/2, -p, (3 + m)/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^
2)/a)]*Sqrt[Cos[e + f*x]^2]*(d*Sin[e + f*x])^m*(a + b*Sin[e + f*x]^2)^p*Tan
[e + f*x])/(f*(1 + m)*(1 + (b*Sin[e + f*x]^2)/a)^p)
```

Maple [F] time = 1.32, size = 0, normalized size = 0.

$$\int \left(d\sin\left(fx+e\right)\right)^m\left(a+b\left(\sin\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^m*(a+b*sin(f*x+e)^2)^p,x)
```

```
[Out] int((d*sin(f*x+e))^m*(a+b*sin(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p\left(d\sin\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*(d*sin(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-b\cos\left(fx+e\right)^2+a+b\right)^p\left(d\sin\left(fx+e\right)\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*(d*sin(f*x + e))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**m*(a+b*sin(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p \left(d\sin\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*(d*sin(f*x + e))^m, x)
```

3.172 $\int \sin^5(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=220

$$-\frac{\left(3a^2-4ab(p+1)+4b^2\left(p^2+3p+2\right)\right)\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{b\cos^2(e+fx)}{a+b}\right.}{b^2f(2p+3)(2p+5)}$$

```
[Out] ((3*a - 2*b*(2 + p))*Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2)^(1 + p))/(b^2*
f*(3 + 2*p)*(5 + 2*p)) - ((3*a^2 - 4*a*b*(1 + p) + 4*b^2*(2 + 3*p + p^2))*C
os[e + f*x]*(a + b - b*Cos[e + f*x]^2)^p*Hypergeometric2F1[1/2, -p, 3/2, (b
*Cos[e + f*x]^2)/(a + b)])/(b^2*f*(3 + 2*p)*(5 + 2*p)*(1 - (b*Cos[e + f*x]^
2)/(a + b))^p) - (Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2)^(1 + p)*Sin[e + f
*x]^2)/(b*f*(5 + 2*p))
```

Rubi [A] time = 0.224846, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3186, 416, 388, 246, 245}

$$-\frac{\left(3a^2-4ab(p+1)+4b^2\left(p^2+3p+2\right)\right)\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{b\cos^2(e+fx)}{a+b}\right.}{b^2f(2p+3)(2p+5)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] ((3*a - 2*b*(2 + p))*Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2)^(1 + p))/(b^2*
f*(3 + 2*p)*(5 + 2*p)) - ((3*a^2 - 4*a*b*(1 + p) + 4*b^2*(2 + 3*p + p^2))*C
os[e + f*x]*(a + b - b*Cos[e + f*x]^2)^p*Hypergeometric2F1[1/2, -p, 3/2, (b
*Cos[e + f*x]^2)/(a + b)])/(b^2*f*(3 + 2*p)*(5 + 2*p)*(1 - (b*Cos[e + f*x]^
2)/(a + b))^p) - (Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2)^(1 + p)*Sin[e + f
*x]^2)/(b*f*(5 + 2*p))
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \sin^5(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = -\frac{\text{Subst}\left(\int \left(1-x^2\right)^2\left(a+b-bx^2\right)^p\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^{1+p}\sin^2(e+fx)}{bf(5+2p)} + \frac{\text{Subst}\left(\int \left(a+b-\right.\right.}{}$$

$$= \frac{(3a-2b(2+p))\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)} - \frac{\cos(e+fx)\left(a+\right.}{}$$

$$= \frac{(3a-2b(2+p))\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)} - \frac{\cos(e+fx)\left(a+\right.}{}$$

$$= \frac{(3a-2b(2+p))\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)} - \frac{\left(3a^2-4ab(1+\right.}{}$$

Mathematica [C] time = 0.546825, size = 98, normalized size = 0.45

$$\frac{\sin^5(e+fx)\sqrt{\cos^2(e+fx)}\tan(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{a+b\sin^2(e+fx)}{a}\right)^{-p}F_1\left(3;\frac{1}{2},-p;4;\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right.}{6f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^5*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[3, 1/2, -p, 4, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sqrt[Cos[
e + f*x]^2]*Sin[e + f*x]^5*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x])/(6*f*((a
+ b*Sin[e + f*x]^2)/a)^p)
```

Maple [F] time = 1.431, size = 0, normalized size = 0.

$$\int \left(\sin\left(fx+e\right)\right)^5\left(a+b\left(\sin\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x)
```

```
[Out] int(sin(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p \sin\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sin(f*x + e)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\cos\left(fx+e\right)^4-2\,\cos\left(fx+e\right)^2+1\right)\left(-b\cos\left(fx+e\right)^2+a+b\right)^p\sin\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*(-b*cos(f*x + e)^2 + a + b
)^p*sin(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5*(a+b*sin(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p \sin\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sin(f*x + e)^5, x)
```

3.173 $\int \sin^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=131

$$\frac{(a-2b(p+1))\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{b\cos^2(e+fx)}{a+b}\right)}{bf(2p+3)}-\frac{\cos(e+fx)\left(a-b\right.}{bf(}$$

```
[Out] -((Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2)^(1 + p))/(b*f*(3 + 2*p))) + ((a
- 2*b*(1 + p))*Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2)^p*Hypergeometric2F1[
1/2, -p, 3/2, (b*Cos[e + f*x]^2)/(a + b)])/(b*f*(3 + 2*p)*(1 - (b*Cos[e + f
*x]^2)/(a + b))^p)
```

Rubi [A] time = 0.109882, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3186, 388, 246, 245}

$$\frac{(a-2b(p+1))\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{b\cos^2(e+fx)}{a+b}\right)}{bf(2p+3)}-\frac{\cos(e+fx)\left(a-b\right.}{bf(}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3*(a + b*Sin[e + f*x]^2)^p,x]
```



```
[Out] -((Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2)^(1 + p))/(b*f*(3 + 2*p))) + ((a
- 2*b*(1 + p))*Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2)^p*Hypergeometric2F1[
1/2, -p, 3/2, (b*Cos[e + f*x]^2)/(a + b)])/(b*f*(3 + 2*p)*(1 - (b*Cos[e + f
*x]^2)/(a + b))^p)
```


Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
```

```
GtQ[a, 0])
```

Rubi steps

$$\int \sin^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = -\frac{\text{Subst}\left(\int \left(1-x^2\right)\left(a+b-bx^2\right)^p\,dx,x,\cos(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^{1+p}}{bf(3+2p)} + \frac{(a-2b(1+p))\,\text{Subst}\left(\int \left(a+b-\right.\right.}{bf(3+2p}$$

$$= -\frac{\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^{1+p}}{bf(3+2p)} + \frac{\left((a-2b(1+p))\left(a+b-b\cos^2(e\right.\right.}{}$$

$$= -\frac{\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^{1+p}}{bf(3+2p)} + \frac{(a-2b(1+p))\cos(e+fx)\left(a+b\right.}{}$$

Mathematica [C] time = 0.37294, size = 98, normalized size = 0.75

$$\frac{\sin^3(e+fx)\sqrt{\cos^2(e+fx)}\tan(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{a+b\sin^2(e+fx)}{a}\right)^{-p}F_1\left(2;\frac{1}{2},-p;3;\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)}{4f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^3*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[2, 1/2, -p, 3, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sqrt[Cos[
e + f*x]^2]*Sin[e + f*x]^3*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x])/(4*f*((a
+ b*Sin[e + f*x]^2)/a)^p)
```

Maple [F] time = 2.264, size = 0, normalized size = 0.

$$\int \left(\sin\left(fx+e\right)\right)^3\left(a+b\left(\sin\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x)
```

```
[Out] int(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p\sin\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sin(f*x + e)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(\cos\left(fx+e\right)^2-1\right)\left(-b\cos\left(fx+e\right)^2+a+b\right)^p\sin\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^2 - 1)*(-b*cos(f*x + e)^2 + a + b)^p*sin(f*x + e),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(a+b*sin(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sin\left(fx+e\right)^2+a\right)^p\sin\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sin(f*x + e)^3, x)
```

3.174 $\int \sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=74

$$-\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{b\cos^2(e+fx)}{a+b}\right)}{f}$$

```
[Out] -((Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2)^p*Hypergeometric2F1[1/2, -p, 3/2
, (b*Cos[e + f*x]^2)/(a + b)])/(f*(1 - (b*Cos[e + f*x]^2)/(a + b))^p))
```

Rubi [A] time = 0.0461089, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3186, 246, 245}

$$-\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{b\cos^2(e+fx)}{a+b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] -((Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2)^p*Hypergeometric2F1[1/2, -p, 3/2
, (b*Cos[e + f*x]^2)/(a + b)])/(f*(1 - (b*Cos[e + f*x]^2)/(a + b))^p))
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = -\frac{\text{Subst}\left(\int\left(a+b-bx^2\right)^p\,dx,x,\cos(e+fx)\right)}{f}$$
$$= -\frac{\left(\left(a+b-b\cos^2(e+fx)\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int\left(1-\frac{bx^2}{a+b}\right)^p\,dx,x,c\right.}{f}$$
$$= -\frac{\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{b\cos}{}\right.}{f}$$

Mathematica [A] time = 0.174166, size = 74, normalized size = 1.

$$-\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{b\cos^2(e+fx)}{a+b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] -((Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2)^p*Hypergeometric2F1[1/2, -p, 3/2
, (b*Cos[e + f*x]^2)/(a + b)])/(f*(1 - (b*Cos[e + f*x]^2)/(a + b))^p))
```

Maple [F] time = 1.348, size = 0, normalized size = 0.

$$\int \sin\left(fx+e\right)\left(a+b\left(\sin\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)*(a+b*sin(f*x+e)^2)^p,x)
```

```
[Out] int(sin(f*x+e)*(a+b*sin(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p\sin\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sin(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-b\cos\left(fx+e\right)^2+a+b\right)^p\sin\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*sin(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^{2}+a\right)^{p}\sin\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sin(f*x + e), x)
```

3.175 $\int \csc(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=83

$$-\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\cos^2(e+fx),\frac{b\cos^2(e+fx)}{a+b}\right)}{f}$$

```
[Out] -((AppellF1[1/2, 1, -p, 3/2, Cos[e + f*x]^2, (b*Cos[e + f*x]^2)/(a + b)]*Co
s[e + f*x]*(a + b - b*Cos[e + f*x]^2)^p)/(f*(1 - (b*Cos[e + f*x]^2)/(a + b)
)^p))
```

Rubi [A] time = 0.0778559, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3186, 430, 429}

$$-\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\cos^2(e+fx),\frac{b\cos^2(e+fx)}{a+b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] -((AppellF1[1/2, 1, -p, 3/2, Cos[e + f*x]^2, (b*Cos[e + f*x]^2)/(a + b)]*Co
s[e + f*x]*(a + b - b*Cos[e + f*x]^2)^p)/(f*(1 - (b*Cos[e + f*x]^2)/(a + b)
)^p))
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \csc(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = -\frac{\text{Subst}\left(\int \frac{(a+b-bx^2)^p}{1-x^2}dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\left(\left(a+b-b\cos^2(e+fx)\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1-\frac{bx^2}{a+b}\right)^p}{1-x^2}dx, x, \cos(e+\right.}{f}$$

$$= -\frac{F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\cos^2(e+fx),\frac{b\cos^2(e+fx)}{a+b}\right)\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^p}{f}$$

Mathematica [F] time = 4.74459, size = 0, normalized size = 0.

$$\int \csc(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csc[e + f*x]*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] Integrate[Csc[e + f*x]*(a + b*Sin[e + f*x]^2)^p, x]
```

Maple [F] time = 0.892, size = 0, normalized size = 0.

$$\int \csc(fx+e)\left(a+b\left(\sin(fx+e)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(a+b*sin(f*x+e)^2)^p,x)
```

```
[Out] int(csc(f*x+e)*(a+b*sin(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin(fx+e)^2+a\right)^p\csc(fx+e)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*csc(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-b\cos(fx+e)^2+a+b\right)^p\csc(fx+e), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*csc(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^{2}+a\right)^{p}\csc\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*csc(f*x + e), x)
```

3.176 $\int \csc^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=83

$$-\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}F_1\left(\frac{1}{2};2,-p;\frac{3}{2};\cos^2(e+fx),\frac{b\cos^2(e+fx)}{a+b}\right)}{f}$$

```
[Out] -((AppellF1[1/2, 2, -p, 3/2, Cos[e + f*x]^2, (b*Cos[e + f*x]^2)/(a + b)]*Co
s[e + f*x]*(a + b - b*Cos[e + f*x]^2)^p)/(f*(1 - (b*Cos[e + f*x]^2)/(a + b)
)^p))
```

Rubi [A] time = 0.0835968, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3186, 430, 429}

$$-\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}F_1\left(\frac{1}{2};2,-p;\frac{3}{2};\cos^2(e+fx),\frac{b\cos^2(e+fx)}{a+b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] -((AppellF1[1/2, 2, -p, 3/2, Cos[e + f*x]^2, (b*Cos[e + f*x]^2)/(a + b)]*Co
s[e + f*x]*(a + b - b*Cos[e + f*x]^2)^p)/(f*(1 - (b*Cos[e + f*x]^2)/(a + b)
)^p))
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \csc^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = -\frac{\text{Subst}\left(\int \frac{(a+b-bx^2)^p}{(1-x^2)^2}dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\left(\left(a+b-b\cos^2(e+fx)\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1-\frac{bx^2}{a+b}\right)^p}{(1-x^2)^2}dx, x, \text{co}\right.}{f}$$

$$= -\frac{F_1\left(\frac{1}{2};2,-p;\frac{3}{2};\cos^2(e+fx),\frac{b\cos^2(e+fx)}{a+b}\right)\cos(e+fx)\left(a+b-b\cos^2(e+f\right.}{f}$$

Mathematica [F] time = 6.18479, size = 0, normalized size = 0.

$$\int \csc^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csc[e + f*x]^3*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] Integrate[Csc[e + f*x]^3*(a + b*Sin[e + f*x]^2)^p, x]
```

Maple [F] time = 0.895, size = 0, normalized size = 0.

$$\int \left(\csc\left(fx+e\right)\right)^3\left(a+b\left(\sin\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x)
```

```
[Out] int(csc(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*csc(f*x + e)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-b\cos\left(fx+e\right)^2+a+b\right)^p\csc\left(fx+e\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*csc(f*x + e)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(a+b*sin(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*csc(f*x + e)^3, x)
```

3.177 $\int \csc^5(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=83

$$-\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}F_1\left(\frac{1}{2};3,-p;\frac{3}{2};\cos^2(e+fx),\frac{b\cos^2(e+fx)}{a+b}\right)}{f}$$

```
[Out] -((AppellF1[1/2, 3, -p, 3/2, Cos[e + f*x]^2, (b*Cos[e + f*x]^2)/(a + b)]*Co
s[e + f*x]*(a + b - b*Cos[e + f*x]^2)^p)/(f*(1 - (b*Cos[e + f*x]^2)/(a + b)
)^p))
```

Rubi [A] time = 0.0852897, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3186, 430, 429}

$$-\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}F_1\left(\frac{1}{2};3,-p;\frac{3}{2};\cos^2(e+fx),\frac{b\cos^2(e+fx)}{a+b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] -((AppellF1[1/2, 3, -p, 3/2, Cos[e + f*x]^2, (b*Cos[e + f*x]^2)/(a + b)]*Co
s[e + f*x]*(a + b - b*Cos[e + f*x]^2)^p)/(f*(1 - (b*Cos[e + f*x]^2)/(a + b)
)^p))
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \csc^5(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = -\frac{\text{Subst}\left(\int \frac{(a+b-bx^2)^p}{(1-x^2)^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\left(\left(a+b-b\cos^2(e+fx)\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1-\frac{bx^2}{a+b}\right)^p}{(1-x^2)^3}\,dx, x, \cos(e\right.}{f}$$

$$= -\frac{F_1\left(\frac{1}{2};3,-p;\frac{3}{2};\cos^2(e+fx),\frac{b\cos^2(e+fx)}{a+b}\right)\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)}{f}$$

Mathematica [F] time = 8.75447, size = 0, normalized size = 0.

$$\int \csc^5(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csc[e + f*x]^5*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] Integrate[Csc[e + f*x]^5*(a + b*Sin[e + f*x]^2)^p, x]
```

Maple [F] time = 0.563, size = 0, normalized size = 0.

$$\int \left(\csc\left(fx+e\right)\right)^5\left(a+b\left(\sin\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x)
```

```
[Out] int(csc(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*csc(f*x + e)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-b\cos\left(fx+e\right)^2+a+b\right)^p\csc\left(fx+e\right)^5, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*csc(f*x + e)^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5*(a+b*sin(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*csc(f*x + e)^5, x)
```

3.178 $\int \sin^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=101

$$\frac{\sin^4(e+fx)\sqrt{\cos^2(e+fx)}\tan(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{5}{2};\frac{1}{2},-p;\frac{7}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)}{5f}$$

```
[Out] (AppellF1[5/2, 1/2, -p, 7/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sqrt[
Cos[e + f*x]^2]*Sin[e + f*x]^4*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x])/(5*f*
(1 + (b*Sin[e + f*x]^2)/a)^p)
```

Rubi [A] time = 0.103097, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3188, 511, 510}

$$\frac{\sin^4(e+fx)\sqrt{\cos^2(e+fx)}\tan(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{5}{2};\frac{1}{2},-p;\frac{7}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[5/2, 1/2, -p, 7/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sqrt[
Cos[e + f*x]^2]*Sin[e + f*x]^4*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x])/(5*f*
(1 + (b*Sin[e + f*x]^2)/a)^p)
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \sin^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int\frac{x^4\left(a+bx^2\right)^p}{\sqrt{1-x^2}}dx,x,\sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+fx)}{a}\right)^{-p}\right)\text{Subs}}{f}$$

$$= \frac{F_1\left(\frac{5}{2};\frac{1}{2},-p;\frac{7}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sqrt{\cos^2(e+fx)}\sin^4(e+fx)\left(a+b\right.}{5f}$$

Mathematica [A] time = 0.538984, size = 102, normalized size = 1.01

$$\frac{\sin^4(e+fx)\sqrt{\cos^2(e+fx)}\tan(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{a+b\sin^2(e+fx)}{a}\right)^{-p}F_1\left(\frac{5}{2};\frac{1}{2},-p;\frac{7}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right.}{5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[5/2, 1/2, -p, 7/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sqrt[
Cos[e + f*x]^2]*Sin[e + f*x]^4*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x])/(5*f*
((a + b*Sin[e + f*x]^2)/a)^p)
```

Maple [F] time = 1.232, size = 0, normalized size = 0.

$$\int\left(\sin\left(fx+e\right)\right)^4\left(a+b\left(\sin\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x)
```

```
[Out] int(sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sin\left(fx+e\right)^2+a\right)^p\sin\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sin(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\cos\left(fx+e\right)^4-2\cos\left(fx+e\right)^2+1\right)\left(-b\cos\left(fx+e\right)^2+a+b\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*(-b*cos(f*x + e)^2 + a + b
)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4*(a+b*sin(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^{2}+a\right)^{p}\sin\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sin(f*x + e)^4, x)
```

3.179 $\int \sin^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=99

$$\frac{\tan^3(e+fx)\sec^2(e+fx)^p\left(a+b\sin^2(e+fx)\right)^p\left(\frac{(a+b)\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{3}{2};p+2,-p;\frac{5}{2};-\tan^2(e+fx),-\frac{(a+b)\tan^2(e}{a}\right.}{3f}$$

```
[Out] (AppellF1[3/2, 2 + p, -p, 5/2, -Tan[e + f*x]^2, -(((a + b)*Tan[e + f*x]^2)/
a)]*(Sec[e + f*x]^2)^p*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x]^3)/(3*f*(1 + (
(a + b)*Tan[e + f*x]^2)/a)^p)
```

Rubi [A] time = 0.165888, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3174, 511, 510}

$$\frac{\tan^3(e+fx)\sec^2(e+fx)^p\left(a+b\sin^2(e+fx)\right)^p\left(\frac{(a+b)\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{3}{2};p+2,-p;\frac{5}{2};-\tan^2(e+fx),-\frac{(a+b)\tan^2(e}{a}\right.}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[3/2, 2 + p, -p, 5/2, -Tan[e + f*x]^2, -(((a + b)*Tan[e + f*x]^2)/
a)]*(Sec[e + f*x]^2)^p*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x]^3)/(3*f*(1 + (
(a + b)*Tan[e + f*x]^2)/a)^p)
```

Rule 3174

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[(ff*(a + b*Sin[e + f*x]^2)^p*(Sec[e + f*x]^2)^p)/(f*(a + (a + b)*Tan[e
+ f*x]^2)^p), Subst[Int[((a + (a + b)*ff^2*x^2)^p*(A + (A + B)*ff^2*x^2))/(
1 + ff^2*x^2)^(p + 2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, A
, B}, x] &&  !IntegerQ[p]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \sin^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \frac{\left(\sec^2(e+fx)^p\left(a+b\sin^2(e+fx)\right)^p\left(a+(a+b)\tan^2(e+fx)\right)^{-p}\right)\text{Subst}\left(\int x^2\right.}{f}$$

$$= \frac{\left(\sec^2(e+fx)^p\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{(a+b)\tan^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int x^2\left(1+\right.\right.}{f}$$

$$= \frac{F_1\left(\frac{3}{2};2+p,-p;\frac{5}{2};-\tan^2(e+fx),-\frac{(a+b)\tan^2(e+fx)}{a}\right)\sec^2(e+fx)^p\left(a+b\sin^2(e\right.}{3f}$$

Mathematica [B] time = 0.720841, size = 240, normalized size = 2.42

$$-\frac{2^{-p-2}\csc(2(e+fx))\sqrt{-\frac{b\sin^2(e+fx)}{a}}\sqrt{\frac{b\cos^2(e+fx)}{a+b}}(2a-b\cos(2(e+fx))+b)^{p+1}\left(2a(p+2)F_1\left(p+1;\frac{1}{2},\frac{1}{2};p+2;\frac{2a+b-b\cos(2}{2(a+b)}\right.\right.}{b^2f(p+1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^2*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] -((2^(-2 - p)*Sqrt[(b*Cos[e + f*x]^2)/(a + b)]*(2*a + b - b*Cos[2*(e + f*x)
])^(1 + p)*(2*a*(2 + p)*AppellF1[1 + p, 1/2, 1/2, 2 + p, (2*a + b - b*Cos[2
*(e + f*x)])/(2*(a + b)), (2*a + b - b*Cos[2*(e + f*x)])/(2*a)] - (1 + p)*A
ppellF1[2 + p, 1/2, 1/2, 3 + p, (2*a + b - b*Cos[2*(e + f*x)])/(2*(a + b)),
 (2*a + b - b*Cos[2*(e + f*x)])/(2*a)]*(2*a + b - b*Cos[2*(e + f*x)]))*Csc[
2*(e + f*x)]*Sqrt[-((b*Sin[e + f*x]^2)/a)])/(b^2*f*(1 + p)*(2 + p)))
```

Maple [F] time = 1.733, size = 0, normalized size = 0.

$$\int \left(\sin\left(fx+e\right)\right)^2\left(a+b\left(\sin\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x)
```

```
[Out] int(sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p\sin\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sin(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(\cos\left(fx+e\right)^{2}-1\right)\left(-b\cos\left(fx+e\right)^{2}+a+b\right)^{p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^2 - 1)*(-b*cos(f*x + e)^2 + a + b)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2*(a+b*sin(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sin\left(fx+e\right)^{2}+a\right)^{p}\sin\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sin(f*x + e)^2, x)
```

3.180 $\int \csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=97

$$-\frac{\sqrt{\cos^2(e+fx)}\csc(e+fx)\sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(-\frac{1}{2};\frac{1}{2},-p;\frac{1}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx}{a}\right.}{f}$$

```
[Out] -((AppellF1[-1/2, 1/2, -p, 1/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sq
rt[Cos[e + f*x]^2]*Csc[e + f*x]*Sec[e + f*x]*(a + b*Sin[e + f*x]^2)^p)/(f*(
1 + (b*Sin[e + f*x]^2)/a)^p))
```

Rubi [A] time = 0.103825, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3188, 511, 510}

$$-\frac{\sqrt{\cos^2(e+fx)}\csc(e+fx)\sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(-\frac{1}{2};\frac{1}{2},-p;\frac{1}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx}{a}\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] -((AppellF1[-1/2, 1/2, -p, 1/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sq
rt[Cos[e + f*x]^2]*Csc[e + f*x]*Sec[e + f*x]*(a + b*Sin[e + f*x]^2)^p)/(f*(
1 + (b*Sin[e + f*x]^2)/a)^p))
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] && !IntegerQ[p]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\mathrm{Subst}\left(\int \frac{\left(a+bx^2\right)^p}{x^2\sqrt{1-x^2}}\,dx,x,\sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+fx)}{a}\right)^{-p}\right)\mathrm{Subs}}{f}$$

$$= -\frac{F_1\left(-\frac{1}{2};\frac{1}{2},-p;\frac{1}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sqrt{\cos^2(e+fx)}\csc(e+fx)\sec}{f}$$

Mathematica [F] time = 4.69994, size = 0, normalized size = 0.

$$\int \csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csc[e + f*x]^2*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] Integrate[Csc[e + f*x]^2*(a + b*Sin[e + f*x]^2)^p, x]
```

Maple [F] time = 0.791, size = 0, normalized size = 0.

$$\int \left(\csc\left(fx+e\right)\right)^2\left(a+b\left(\sin\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x)
```

```
[Out] int(csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*csc(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(-b\cos\left(fx+e\right)^2+a+b\right)^p\csc\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*csc(f*x + e)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*sin(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^{2}+a\right)^{p}\csc\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*csc(f*x + e)^2, x)
```

3.181 $\int \csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=101

$$-\frac{\sqrt{\cos^2(e+fx)}\csc^3(e+fx)\sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(-\frac{3}{2};\frac{1}{2},-p;-\frac{1}{2};\sin^2(e+fx),-\frac{b\sin}{}\right.}{3f}$$

```
[Out] -(AppellF1[-3/2, 1/2, -p, -1/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sq
rt[Cos[e + f*x]^2]*Csc[e + f*x]^3*Sec[e + f*x]*(a + b*Sin[e + f*x]^2)^p)/(3
*f*(1 + (b*Sin[e + f*x]^2)/a)^p)
```

Rubi [A] time = 0.0996099, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3188, 511, 510}

$$-\frac{\sqrt{\cos^2(e+fx)}\csc^3(e+fx)\sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(-\frac{3}{2};\frac{1}{2},-p;-\frac{1}{2};\sin^2(e+fx),-\frac{b\sin}{}\right.}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] -(AppellF1[-3/2, 1/2, -p, -1/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sq
rt[Cos[e + f*x]^2]*Csc[e + f*x]^3*Sec[e + f*x]*(a + b*Sin[e + f*x]^2)^p)/(3
*f*(1 + (b*Sin[e + f*x]^2)/a)^p)
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int\frac{(a+bx^2)^p}{x^4\sqrt{1-x^2}}\,dx,x,\sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int\right.}{f}$$

$$= -\frac{F_1\left(-\frac{3}{2};\frac{1}{2},-p;-\frac{1}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sqrt{\cos^2(e+fx)}\csc^3(e+fx)\sec(}{3f}$$

Mathematica [F] time = 6.41609, size = 0, normalized size = 0.

$$\int \csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csc[e + f*x]^4*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] Integrate[Csc[e + f*x]^4*(a + b*Sin[e + f*x]^2)^p, x]
```

Maple [F] time = 0.605, size = 0, normalized size = 0.

$$\int \left(\csc\left(fx+e\right)\right)^4\left(a+b\left(\sin\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x)
```

```
[Out] int(csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*csc(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-b\cos\left(fx+e\right)^2+a+b\right)^p\csc\left(fx+e\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*csc(f*x + e)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(a+b*sin(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^{2}+a\right)^{p}\csc\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*csc(f*x + e)^4, x)
```

3.182 $\int \frac{\sin^7(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=335

$$\frac{2(-1)^{2/3}a^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3b^{7/3}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}-\frac{2a^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3b^{7/3}d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2\sqrt[3]{-1}a^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3b^{7/3}d\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}$$

```
[Out] (3*x)/(8*b) + (2*(-1)^(2/3)*a^(5/3)*ArcTan[((-1)^(1/3)*b^(1/3) - a^(1/3)*Ta
n[(c + d*x)/2])/Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*Sqrt[a^(2/3) - (-1)
^(2/3)*b^(2/3)]*b^(7/3)*d) - (2*a^(5/3)*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c +
d*x)/2])/Sqrt[a^(2/3) - b^(2/3)]])/(3*Sqrt[a^(2/3) - b^(2/3)]*b^(7/3)*d) +
(2*(-1)^(1/3)*a^(5/3)*ArcTan[((-1)^(2/3)*b^(1/3) + a^(1/3)*Tan[(c + d*x)/2]
)/Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]])/(3*Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)
]*b^(7/3)*d) + (a*Cos[c + d*x])/(b^2*d) - (3*Cos[c + d*x]*Sin[c + d*x])/(8*
b*d) - (Cos[c + d*x]*Sin[c + d*x]^3)/(4*b*d)
```

Rubi [A] time = 0.699105, antiderivative size = 335, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3220, 2638, 2635, 8, 2660, 618, 204}

$$\frac{2(-1)^{2/3}a^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3b^{7/3}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}-\frac{2a^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3b^{7/3}d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2\sqrt[3]{-1}a^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3b^{7/3}d\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^7/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (3*x)/(8*b) + (2*(-1)^(2/3)*a^(5/3)*ArcTan[((-1)^(1/3)*b^(1/3) - a^(1/3)*Ta
n[(c + d*x)/2])/Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*Sqrt[a^(2/3) - (-1)
^(2/3)*b^(2/3)]*b^(7/3)*d) - (2*a^(5/3)*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c +
d*x)/2])/Sqrt[a^(2/3) - b^(2/3)]])/(3*Sqrt[a^(2/3) - b^(2/3)]*b^(7/3)*d) +
(2*(-1)^(1/3)*a^(5/3)*ArcTan[((-1)^(2/3)*b^(1/3) + a^(1/3)*Tan[(c + d*x)/2]
)/Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]])/(3*Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)
]*b^(7/3)*d) + (a*Cos[c + d*x])/(b^2*d) - (3*Cos[c + d*x]*Sin[c + d*x])/(8*
b*d) - (Cos[c + d*x]*Sin[c + d*x]^3)/(4*b*d)
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin^7(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \left(-\frac{a\sin(c+dx)}{b^2}+\frac{\sin^4(c+dx)}{b}+\frac{a^2\sin(c+dx)}{b^2\left(a+b\sin^3(c+dx)\right)}\right)dx$$

$$= -\frac{a\int \sin(c+dx)\,dx}{b^2}+\frac{a^2\int \frac{\sin(c+dx)}{a+b\sin^3(c+dx)}\,dx}{b^2}+\frac{\int \sin^4(c+dx)\,dx}{b}$$

$$= \frac{a\cos(c+dx)}{b^2d}-\frac{\cos(c+dx)\sin^3(c+dx)}{4bd}+\frac{a^2\int\left(-\frac{1}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}-\frac{(-1)^{2/3}}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\ \cdots\right.}\cdots\right.}{b^2}\cdots$$

$$= \frac{a\cos(c+dx)}{b^2d}-\frac{3\cos(c+dx)\sin(c+dx)}{8bd}-\frac{\cos(c+dx)\sin^3(c+dx)}{4bd}-\frac{a^{5/3}\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\sin(c+d\cdots}}{3b^{7/3}}\cdots$$

$$= \frac{3x}{8b}+\frac{a\cos(c+dx)}{b^2d}-\frac{3\cos(c+dx)\sin(c+dx)}{8bd}-\frac{\cos(c+dx)\sin^3(c+dx)}{4bd}-\frac{\left(2a^{5/3}\right)\text{Subst}\cdots}{\cdots}$$

$$= \frac{3x}{8b}+\frac{a\cos(c+dx)}{b^2d}-\frac{3\cos(c+dx)\sin(c+dx)}{8bd}-\frac{\cos(c+dx)\sin^3(c+dx)}{4bd}+\frac{\left(4a^{5/3}\right)\text{Subst}\cdots}{\cdots}$$

$$= \frac{3x}{8b}+\frac{2(-1)^{2/3}a^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,b^{7/3}d}-\frac{2a^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3\sqrt{a^{2/3}-b^{2/3}}\,b^{7/3}d}+\frac{2\sqrt[3]{-\cdots}}{\cdots}$$

Mathematica [C] time = 0.487818, size = 219, normalized size = 0.65

$$\frac{-32a^2\text{RootSum}\left[8\#1^3a+i\#1^6b-3i\#1^4b+3i\#1^2b-ib\&,\frac{-i\#1^2\log\left(\#1^2-2\#1\cos(c+dx)+1\right)+i\log\left(\#1^2-2\#1\cos(c+dx)+1\right)+2\#1^2\tan^{-1}\left(\cdots\right.}{\#1^4b-2\#1^2b-4i\#1a+b}\cdots\right.}{96b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^7/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (96*a*Cos[c + d*x] - 32*a^2*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I
)*b*#1^4 + I*b*#1^6 & , (-2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + I*Lo
g[1 - 2*Cos[c + d*x]*#1 + #1^2] + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)
]*#1^2 - I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2)/(b - (4*I)*a*#1 - 2*b*#1
^2 + b*#1^4) & ] + 3*b*(12*(c + d*x) - 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x)
]))/(96*b^2*d)
```

Maple [C] time = 0.178, size = 366, normalized size = 1.1

$$\frac{2a^2}{3b^2d}\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\_R^3+\_R}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)+\frac{3}{4bd}\left(\tan\left(\frac{dx}{2}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^7/(a+b*sin(d*x+c)^3),x)
```

```
[Out] 2/3/d*a^2/b^2*sum((_R^3+_R)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+
1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))+3/4/d/b/(1+tan(1
/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7+2/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^4*
tan(1/2*d*x+1/2*c)^6*a+11/4/d/b/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*
c)^5+6/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^4*a-11/4/d/b/(1+
tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3+6/d/b^2/(1+tan(1/2*d*x+1/2*c)^
2)^4*tan(1/2*d*x+1/2*c)^2*a-3/4/d/b/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+
1/2*c)+2/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^4*a+3/4/d/b*arctan(tan(1/2*d*x+1/2*
c))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**7/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)^7}{b\sin(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^7/(b*sin(d*x + c)^3 + a), x)
```

3.183 $\int \frac{\sin^5(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=273

$$-\frac{2a\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3b^{5/3}d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2a\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}\right)}{3b^{5/3}d\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}+\frac{2a\tanh^{-1}\left(\frac{(-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}}\right)}{3b^{5/3}d\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}}-\frac{\sin(c+dx)}{2}$$

```
[Out] x/(2*b) - (2*a*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b
^(2/3)]])/(3*Sqrt[a^(2/3) - b^(2/3)]*b^(5/3)*d) + (2*a*ArcTanh[(b^(1/3) - (
-1)^(1/3)*a^(1/3)*Tan[(c + d*x)/2])/Sqrt[-((-1)^(2/3)*a^(2/3)) + b^(2/3)]])
/(3*Sqrt[-((-1)^(2/3)*a^(2/3)) + b^(2/3)]*b^(5/3)*d) + (2*a*ArcTanh[(b^(1/3
) + (-1)^(2/3)*a^(1/3)*Tan[(c + d*x)/2])/Sqrt[(-1)^(1/3)*a^(2/3) + b^(2/3)]
])/(3*Sqrt[(-1)^(1/3)*a^(2/3) + b^(2/3)]*b^(5/3)*d) - (Cos[c + d*x]*Sin[c +
 d*x])/(2*b*d)
```

Rubi [A] time = 0.566971, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3220, 2635, 8, 2660, 618, 204, 206}

$$-\frac{2a\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3b^{5/3}d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2a\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}\right)}{3b^{5/3}d\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}+\frac{2a\tanh^{-1}\left(\frac{(-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}}\right)}{3b^{5/3}d\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}}-\frac{\sin(c+dx)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^5/(a + b*Sin[c + d*x]^3),x]
```


```
[Out] x/(2*b) - (2*a*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b
^(2/3)]])/(3*Sqrt[a^(2/3) - b^(2/3)]*b^(5/3)*d) + (2*a*ArcTanh[(b^(1/3) - (
-1)^(1/3)*a^(1/3)*Tan[(c + d*x)/2])/Sqrt[-((-1)^(2/3)*a^(2/3)) + b^(2/3)]])
/(3*Sqrt[-((-1)^(2/3)*a^(2/3)) + b^(2/3)]*b^(5/3)*d) + (2*a*ArcTanh[(b^(1/3
) + (-1)^(2/3)*a^(1/3)*Tan[(c + d*x)/2])/Sqrt[(-1)^(1/3)*a^(2/3) + b^(2/3)]
])/(3*Sqrt[(-1)^(1/3)*a^(2/3) + b^(2/3)]*b^(5/3)*d) - (Cos[c + d*x]*Sin[c +
 d*x])/(2*b*d)
```


Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin^5(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \left(\frac{\sin^2(c+dx)}{b} - \frac{a\sin^2(c+dx)}{b\left(a+b\sin^3(c+dx)\right)}\right)dx$$

$$= \frac{\int \sin^2(c+dx)\,dx}{b} - \frac{a\int \frac{\sin^2(c+dx)}{a+b\sin^3(c+dx)}\,dx}{b}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{2bd} + \frac{\int 1\,dx}{2b} - \frac{a\int \left(\frac{1}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)} + \frac{1}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}\right.}{b}$$

$$= \frac{x}{2b} - \frac{\cos(c+dx)\sin(c+dx)}{2bd} - \frac{a\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)}\,dx}{3b^{5/3}} - \frac{a\int \frac{1}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)}\,dx}{3b^{5/3}} - \frac{a\int \frac{}{(}}{}$$

$$= \frac{x}{2b} - \frac{\cos(c+dx)\sin(c+dx)}{2bd} - \frac{(2a)\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a}+2\sqrt[3]{b}x+\sqrt[3]{a}x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{3b^{5/3}d} - \frac{(2a)}{}$$

$$= \frac{x}{2b} - \frac{\cos(c+dx)\sin(c+dx)}{2bd} + \frac{(4a)\operatorname{Subst}\left(\int \frac{1}{-4\left(a^{2/3}-b^{2/3}\right)-x^2}\,dx, x, 2\sqrt[3]{b}+2\sqrt[3]{a}\tan\left(\frac{1}{2}(c+\right.\right.}{3b^{5/3}d}$$

$$= \frac{x}{2b} - \frac{2a\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3\sqrt{a^{2/3}-b^{2/3}}b^{5/3}d} + \frac{2a\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}+b^{2/3}}}\right)}{3\sqrt{-(-1)^{2/3}a^{2/3}+b^{2/3}}b^{5/3}d} + \frac{2a\tanh^{-1}\left(\frac{\sqrt[3]{b}+}{}\right.}{3\sqrt{\sqrt[3]{-1}a}}$$

Mathematica [C] time = 0.277522, size = 255, normalized size = 0.93

$$\frac{-2ia\operatorname{RootSum}\left[8\#1^3a+i\#1^6b-3i\#1^4b+3i\#1^2b-ib\&, \frac{-i\#1^4\log\left(\#1^2-2\#1\cos(c+dx)+1\right)+2i\#1^2\log\left(\#1^2-2\#1\cos(c+dx)+1\right)-i\log\left(\#1^2\right.}{-4i\#}\right.}{12bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^5/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (6*(c + d*x) - (2*I)*a*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#
1^4 + I*b*#1^6 & , (2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - I*Log[1 -
2*Cos[c + d*x]*#1 + #1^2] - 4*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2
 + (2*I)*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + 2*ArcTan[Sin[c + d*x]/(Co
s[c + d*x] - #1)]*#1^4 - I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4)/(b*#1 -
(4*I)*a*#1^2 - 2*b*#1^3 + b*#1^5) & ] - 3*Sin[2*(c + d*x)])/(12*b*d)
```

Maple [C] time = 0.161, size = 163, normalized size = 0.6

$$-\frac{4a}{3bd}\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\_R^2}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)+\frac{1}{bd}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5/(a+b*sin(d*x+c)^3),x)
```

```
[Out] -4/3/d*a/b*sum(_R^2/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_
R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))+1/d/b/(1+tan(1/2*d*x+1/2
*c)^2)^2*tan(1/2*d*x+1/2*c)^3-1/d/b/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+
1/2*c)+1/d/b*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)^5}{b\sin(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^5/(b*sin(d*x + c)^3 + a), x)
```

3.184 $\int \frac{\sin^3(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=259

$$-\frac{2\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3bd\sqrt{a^{2/3}-b^{2/3}}}-\frac{2\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3bd\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}+\frac{2\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{-1}\left((-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3bd\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}+\frac{x}{b}$$

```
[Out] x/b - (2*a^(1/3)*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) -
 b^(2/3)]])/(3*Sqrt[a^(2/3) - b^(2/3)]*b*d) - (2*a^(1/3)*ArcTan[((-1)^(2/3)
*b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]])/(
3*Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*b*d) + (2*a^(1/3)*ArcTan[((-1)^(1/3)*(
b^(1/3) + (-1)^(2/3)*a^(1/3)*Tan[(c + d*x)/2]))/Sqrt[a^(2/3) - (-1)^(2/3)*b
^(2/3)]])/(3*Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]*b*d)
```

Rubi [A] time = 0.457509, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3220, 3213, 2660, 618, 204}

$$-\frac{2\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3bd\sqrt{a^{2/3}-b^{2/3}}}-\frac{2\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3bd\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}+\frac{2\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{-1}\left((-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3bd\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}+\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] x/b - (2*a^(1/3)*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) -
 b^(2/3)]])/(3*Sqrt[a^(2/3) - b^(2/3)]*b*d) - (2*a^(1/3)*ArcTan[((-1)^(2/3)
*b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]])/(
3*Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*b*d) + (2*a^(1/3)*ArcTan[((-1)^(1/3)*(
b^(1/3) + (-1)^(2/3)*a^(1/3)*Tan[(c + d*x)/2]))/Sqrt[a^(2/3) - (-1)^(2/3)*b
^(2/3)]])/(3*Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]*b*d)
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \left(\frac{1}{b}-\frac{a}{b\left(a+b\sin^3(c+dx)\right)}\right)dx$$
$$= \frac{x}{b}-\frac{a\int \frac{1}{a+b\sin^3(c+dx)}\,dx}{b}$$
$$= \frac{x}{b}-\frac{a\int \left(-\frac{1}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}\sin(c+dx)\right)}-\frac{1}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}\sin(c+dx)\right)}-\frac{1}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\sin(c+dx)\right)}\right) d}{b}$$
$$= \frac{x}{b}+\frac{\sqrt[3]{a}\int \frac{1}{-\sqrt[3]{a}-\sqrt[3]{b}\sin(c+dx)}\,dx}{3b}+\frac{\sqrt[3]{a}\int \frac{1}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}\sin(c+dx)}\,dx}{3b}+\frac{\sqrt[3]{a}\int \frac{1}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\sin(c+dx)}\,\text{c}}{3b}$$
$$= \frac{x}{b}+\frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{-\sqrt[3]{a}-2\sqrt[3]{b}x-\sqrt[3]{a}x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3bd}+\frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{-\sqrt[3]{a}+2\sqrt[3]{-1}\sqrt[3]{}}\right.}{}$$
$$= \frac{x}{b}-\frac{\left(4\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{-4\left(a^{2/3}-b^{2/3}\right)-x^2}\,dx,x,-2\sqrt[3]{b}-2\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3bd}-\frac{\left(4\sqrt[3]{a}\right)\text{Subst}\left(\int\right.}{}$$
$$= \frac{x}{b}+\frac{2\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,bd}-\frac{2\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3\sqrt{a^{2/3}-b^{2/3}}\,bd}-\frac{2\sqrt[3]{a}\tan^{-1}\left(\frac{(-}{}\right.}{3\sqrt{a^{2/}}}$$

Mathematica [C] time = 0.179957, size = 140, normalized size = 0.54

$$\frac{2ia\,\text{RootSum}\left[8\#1^3a+i\#1^6b-3i\#1^4b+3i\#1^2b-ib\&,\frac{2\#1\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)-i\#1\log\left(\#1^2-2\#1\cos(c+dx)+1\right)}{\#1^4b-2\#1^2b-4i\#1a+b}\&\right]+3c+3dx}{3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (3*c + 3*d*x + (2*I)*a*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#
1^4 + I*b*#1^6 & , (2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 - I*Log[1
 - 2*Cos[c + d*x]*#1 + #1^2]*#1)/(b - (4*I)*a*#1 - 2*b*#1^2 + b*#1^4) & ])/
(3*b*d)
```

Maple [C] time = 0.153, size = 106, normalized size = 0.4

$$-\frac{a}{3\,bd}\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\_R^4+2\_R^2+1}{\_R^5a+2\_R^3a+4\_R^2b+\_R\,a}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)+2\,\frac{\arctan\left(\tan\left(1/\right.\right.}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a+b*sin(d*x+c)^3),x)
```

```
[Out] -1/3/d*a/b*sum((_R^4+2*_R^2+1)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d
*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))+2/d/b*arctan(
tan(1/2*d*x+1/2*c))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin\left(dx+c\right)^3}{b\sin\left(dx+c\right)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)^3),x, algorithm="giac")

[Out] integrate(sin(d*x + c)^3/(b*sin(d*x + c)^3 + a), x)
```

3.185 $\int \frac{\sin(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=267

$$\frac{2(-1)^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt[3]{b}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt[3]{b}d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2\sqrt[3]{-1}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt[3]{b}d\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}$$

```
[Out] (2*(-1)^(2/3)*ArcTan[((-1)^(1/3)*b^(1/3) - a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a
^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(1/3)*Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)
]*b^(1/3)*d) - (2*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3)
- b^(2/3)]])/(3*a^(1/3)*Sqrt[a^(2/3) - b^(2/3)]*b^(1/3)*d) + (2*(-1)^(1/3)*
ArcTan[((-1)^(2/3)*b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) + (-1)^
(1/3)*b^(2/3)]])/(3*a^(1/3)*Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*b^(1/3)*d)
```

Rubi [A] time = 0.262249, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3220, 2660, 618, 204}

$$\frac{2(-1)^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt[3]{b}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt[3]{b}d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2\sqrt[3]{-1}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt[3]{b}d\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (2*(-1)^(2/3)*ArcTan[((-1)^(1/3)*b^(1/3) - a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a
^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(1/3)*Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)
]*b^(1/3)*d) - (2*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3)
- b^(2/3)]])/(3*a^(1/3)*Sqrt[a^(2/3) - b^(2/3)]*b^(1/3)*d) + (2*(-1)^(1/3)*
ArcTan[((-1)^(2/3)*b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) + (-1)^
(1/3)*b^(2/3)]])/(3*a^(1/3)*Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*b^(1/3)*d)
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int\left(-\frac{1}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)} - \frac{(-1)^{2/3}}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}\sin(c+dx)\right)} + \frac{\cdots}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}\cdots\right.}\right.$$

$$= -\frac{\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}} + \frac{\sqrt[3]{-1}\int \frac{1}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\sin(c+dx)}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}} - \frac{(-1)^{2/3}\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}\sin(c+dx)}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}}$$

$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+2\sqrt[3]{b}x+\sqrt[3]{a}x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{3\sqrt[3]{a}\sqrt[3]{b}d} + \frac{\left(2\sqrt[3]{-1}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+2(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}x^2\cdots}\right.}{3\sqrt[3]{a}\sqrt[3]{b}d}$$

$$= \frac{4\,\text{Subst}\left(\int \frac{1}{-4\left(a^{2/3}-b^{2/3}\right)-x^2}\,dx, x, 2\sqrt[3]{b}+2\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3\sqrt[3]{a}\sqrt[3]{b}d} - \frac{\left(4\sqrt[3]{-1}\right)\text{Subst}\left(\int \frac{\cdots}{-4\left(a^{2/3}+\sqrt[3]{\cdots}\right.}\right.}{\cdots}$$

$$= \frac{2(-1)^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\sqrt[3]{b}d} - \frac{2\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt{a^{2/3}-b^{2/3}}\sqrt[3]{b}d} + \frac{2\sqrt[3]{-1}\tan^{-1}\left(\frac{(-1)^{2\cdots}}{\cdots}\right.}{3\sqrt[3]{a}\sqrt{a^{2/3}+\cdots}}$$

Mathematica [C] time = 0.184182, size = 172, normalized size = 0.64

$$-\frac{\text{RootSum}\left[8\#1^3a+i\#1^6b-3i\#1^4b+3i\#1^2b-ib\&, \frac{-i\#1^2\log\left(\#1^2-2\#1\cos(c+dx)+1\right)+i\log\left(\#1^2-2\#1\cos(c+dx)+1\right)+2\#1^2\tan^{-1}\left(\frac{\sin(\cdots}{\cos(c+\cdots}\right.}{\#1^4b-2\#1^2b-4i\#1a+b}\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] -RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#1^6 & , (-2
*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + I*Log[1 - 2*Cos[c + d*x]*#1 + #
1^2] + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 - I*Log[1 - 2*Cos[c
+ d*x]*#1 + #1^2]*#1^2)/(b - (4*I)*a*#1 - 2*b*#1^2 + b*#1^4) & ]/(3*d)
```

Maple [C] time = 0.156, size = 78, normalized size = 0.3

$$\frac{2}{3d}\sum_{\_R=\text{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\_R^3+\_R}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a+b*sin(d*x+c)^3),x)
```

```
[Out] 2/3/d*sum((_R^3+_R)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_
R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{b\sin(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/(b*sin(d*x + c)^3 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Integral(sin(c + d*x)/(a + b*sin(c + d*x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{b\sin(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)/(b*sin(d*x + c)^3 + a), x)
```

3.186 $\int \frac{\csc(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=264

$$-\frac{2\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3ad\sqrt{a^{2/3}-b^{2/3}}}+\frac{2\sqrt[3]{b}\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}\right)}{3ad\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}+\frac{2\sqrt[3]{b}\tanh^{-1}\left(\frac{(-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}}\right)}{3ad\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}}-\text{ta}$$

```
[Out] (-2*b^(1/3)*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b^(2
/3)]])/(3*a*Sqrt[a^(2/3) - b^(2/3)]*d) - ArcTanh[Cos[c + d*x]]/(a*d) + (2*b
^(1/3)*ArcTanh[(b^(1/3) - (-1)^(1/3)*a^(1/3)*Tan[(c + d*x)/2])/Sqrt[-((-1)^
(2/3)*a^(2/3)) + b^(2/3)]])/(3*a*Sqrt[-((-1)^(2/3)*a^(2/3)) + b^(2/3)]*d) +
 (2*b^(1/3)*ArcTanh[(b^(1/3) + (-1)^(2/3)*a^(1/3)*Tan[(c + d*x)/2])/Sqrt[(-
1)^(1/3)*a^(2/3) + b^(2/3)]])/(3*a*Sqrt[(-1)^(1/3)*a^(2/3) + b^(2/3)]*d)
```

Rubi [A] time = 0.364646, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3220, 3770, 2660, 618, 204, 206}

$$-\frac{2\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3ad\sqrt{a^{2/3}-b^{2/3}}}+\frac{2\sqrt[3]{b}\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}\right)}{3ad\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}+\frac{2\sqrt[3]{b}\tanh^{-1}\left(\frac{(-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}}\right)}{3ad\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}}-\text{ta}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a + b*Sin[c + d*x]^3),x]
```


```
[Out] (-2*b^(1/3)*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b^(2
/3)]])/(3*a*Sqrt[a^(2/3) - b^(2/3)]*d) - ArcTanh[Cos[c + d*x]]/(a*d) + (2*b
^(1/3)*ArcTanh[(b^(1/3) - (-1)^(1/3)*a^(1/3)*Tan[(c + d*x)/2])/Sqrt[-((-1)^
(2/3)*a^(2/3)) + b^(2/3)]])/(3*a*Sqrt[-((-1)^(2/3)*a^(2/3)) + b^(2/3)]*d) +
 (2*b^(1/3)*ArcTanh[(b^(1/3) + (-1)^(2/3)*a^(1/3)*Tan[(c + d*x)/2])/Sqrt[(-
1)^(1/3)*a^(2/3) + b^(2/3)]])/(3*a*Sqrt[(-1)^(1/3)*a^(2/3) + b^(2/3)]*d)
```


Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \left(\frac{\csc(c+dx)}{a} - \frac{b\sin^2(c+dx)}{a\left(a+b\sin^3(c+dx)\right)}\right)dx$$

$$= \frac{\int \csc(c+dx)\,dx}{a} - \frac{b\int \frac{\sin^2(c+dx)}{a+b\sin^3(c+dx)}\,dx}{a}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{ad} - \frac{b\int\left(\frac{1}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)} + \frac{1}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)} + \frac{1}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\right.}\right.}{a}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{ad} - \frac{\sqrt[3]{b}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)}\,dx}{3a} - \frac{\sqrt[3]{b}\int \frac{1}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)}\,dx}{3a} - \frac{\sqrt[3]{b}\int \frac{}{(-1)^{2/3}}}{}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{ad} - \frac{\left(2\sqrt[3]{b}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+2\sqrt[3]{b}x+\sqrt[3]{a}x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{3ad} - \frac{\left(2\sqrt[3]{b}\right)\text{Subs}}{}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{\left(4\sqrt[3]{b}\right)\text{Subst}\left(\int \frac{1}{-4\left(a^{2/3}-b^{2/3}\right)-x^2}\,dx, x, 2\sqrt[3]{b}+2\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3ad} +$$

$$= -\frac{2\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a\sqrt{a^{2/3}-b^{2/3}}\,d} - \frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{2\sqrt[3]{b}\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}+b^{2/3}}}\right)}{3a\sqrt{-(-1)^{2/3}a^{2/3}+b^{2/3}}\,d}$$

Mathematica [C] time = 0.263686, size = 264, normalized size = 1.

$$-\frac{ib\,\text{RootSum}\left[8\#1^3a+i\#1^6b-3i\#1^4b+3i\#1^2b-ib\&, \frac{-i\#1^4\log\left(\#1^2-2\#1\cos(c+dx)+1\right)+2i\#1^2\log\left(\#1^2-2\#1\cos(c+dx)+1\right)-i\log\left(\#1^2-2\#1\,\mathrm{c}\right.}{-4i\#1^2a+}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] -(6*Log[Cos[(c + d*x)/2]] - 6*Log[Sin[(c + d*x)/2]] + I*b*RootSum[(-I)*b +
(3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#1^6 & , (2*ArcTan[Sin[c + d*x
]/(Cos[c + d*x] - #1)] - I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] - 4*ArcTan[Sin
```

```
[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 + (2*I)*Log[1 - 2*Cos[c + d*x]*#1 + #1^
2]*#1^2 + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - I*Log[1 - 2*Cos
[c + d*x]*#1 + #1^2]*#1^4)/(b*#1 - (4*I)*a*#1^2 - 2*b*#1^3 + b*#1^5) & ])/(
6*a*d)
```

Maple [C] time = 0.188, size = 98, normalized size = 0.4

$$-\frac{4b}{3da}\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\_R^2}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)+\frac{1}{da}\ln\left(\tan\left(\frac{d}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a+b*sin(d*x+c)^3),x)
```

```
[Out] -4/3/d/a*b*sum(_R^2/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_
R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))+1/d/a*ln(tan(1/2*d*x+1/2
*c))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(dx+c)}{b\sin(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(csc(d*x + c)/(b*sin(d*x + c)^3 + a), x)
```

3.187 $\int \frac{\csc^3(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=287

$$-\frac{2b\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{5/3}d\sqrt{a^{2/3}-b^{2/3}}}-\frac{2b\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3a^{5/3}d\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}+\frac{2b\tan^{-1}\left(\frac{\sqrt[3]{-1}\left((-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{5/3}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}-\tan$$

```
[Out] (-2*b*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b^(2/3)]])
/(3*a^(5/3)*Sqrt[a^(2/3) - b^(2/3)]*d) - (2*b*ArcTan[((-1)^(2/3)*b^(1/3) +
a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]])/(3*a^(5/3)*S
qrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*d) + (2*b*ArcTan[((-1)^(1/3)*(b^(1/3) + (
-1)^(2/3)*a^(1/3)*Tan[(c + d*x)/2]))/Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(
3*a^(5/3)*Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]*d) - ArcTanh[Cos[c + d*x]]/(2*
a*d) - (Cot[c + d*x]*Csc[c + d*x])/(2*a*d)
```

Rubi [A] time = 0.403905, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3220, 3768, 3770, 3213, 2660, 618, 204}

$$-\frac{2b\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{5/3}d\sqrt{a^{2/3}-b^{2/3}}}-\frac{2b\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3a^{5/3}d\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}+\frac{2b\tan^{-1}\left(\frac{\sqrt[3]{-1}\left((-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{5/3}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}-\tan$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (-2*b*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b^(2/3)]])
/(3*a^(5/3)*Sqrt[a^(2/3) - b^(2/3)]*d) - (2*b*ArcTan[((-1)^(2/3)*b^(1/3) +
a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]])/(3*a^(5/3)*S
qrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*d) + (2*b*ArcTan[((-1)^(1/3)*(b^(1/3) + (
-1)^(2/3)*a^(1/3)*Tan[(c + d*x)/2]))/Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(
3*a^(5/3)*Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]*d) - ArcTanh[Cos[c + d*x]]/(2*
a*d) - (Cot[c + d*x]*Csc[c + d*x])/(2*a*d)
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc^3(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \left(\frac{\csc^3(c+dx)}{a} - \frac{b}{a\left(a+b\sin^3(c+dx)\right)}\right)dx$$

$$= \frac{\int \csc^3(c+dx)\,dx}{a} - \frac{b\int \frac{1}{a+b\sin^3(c+dx)}\,dx}{a}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)}{2ad} + \frac{\int \csc(c+dx)\,dx}{2a} - \frac{b\int\left(-\frac{1}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}\sin(c+dx)\right)} - \frac{1}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}\,\mathrm{s}\right.}\right.}{a}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{2ad} - \frac{\cot(c+dx)\csc(c+dx)}{2ad} + \frac{b\int \frac{1}{-\sqrt[3]{a}-\sqrt[3]{b}\sin(c+dx)}\,dx}{3a^{5/3}} + \frac{b\int \frac{1}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}\,\mathrm{si}}}{3a^{5/3}}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{2ad} - \frac{\cot(c+dx)\csc(c+dx)}{2ad} + \frac{(2b)\,\mathrm{Subst}\left(\int \frac{1}{-\sqrt[3]{a}-2\sqrt[3]{b}x-\sqrt[3]{a}x^2}\,dx, x, \tan\left(\frac{1}{2}\right.\right.}{3a^{5/3}d}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{2ad} - \frac{\cot(c+dx)\csc(c+dx)}{2ad} - \frac{(4b)\,\mathrm{Subst}\left(\int \frac{1}{-4\left(a^{2/3}-b^{2/3}\right)-x^2}\,dx, x, -2\sqrt[3]{b}-\right.}{3a^{5/3}d}$$

$$= \frac{2b\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{5/3}\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,d} - \frac{2b\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{5/3}\sqrt{a^{2/3}-b^{2/3}}\,d} - \frac{2b\tan^{-1}\left(\frac{(-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\right.}{\sqrt{a^{2/3}+\sqrt[3]{-1}b}}\right.}{3a^{5/3}\sqrt{a^{2/3}+\sqrt[3]{-1}b^2}}$$

Mathematica [C] time = 0.396276, size = 181, normalized size = 0.63

$$\frac{-3\left(\csc^2\left(\frac{1}{2}(c+dx)\right)-\sec^2\left(\frac{1}{2}(c+dx)\right)-4\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)+16ib\text{RootSum}\left[-8i\#1^3\right.}{24ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] ((16*I)*b*RootSum[-b + 3*b*#1^2 - (8*I)*a*#1^3 - 3*b*#1^4 + b*#1^6 & , (2*A
rcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 - I*Log[1 - 2*Cos[c + d*x]*#1 +
#1^2]*#1)/(b - (4*I)*a*#1 - 2*b*#1^2 + b*#1^4) & ] - 3*(Csc[(c + d*x)/2]^2
+ 4*Log[Cos[(c + d*x)/2]] - 4*Log[Sin[(c + d*x)/2]] - Sec[(c + d*x)/2]^2))/
(24*a*d)
```

Maple [C] time = 0.213, size = 144, normalized size = 0.5

$$\frac{1}{8da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2-\frac{b}{3da}\sum_{\_R=\text{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\_R^4+2\_R^2+1}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(a+b*sin(d*x+c)^3),x)
```

```
[Out] 1/8/d*tan(1/2*d*x+1/2*c)^2/a-1/3/d/a*b*sum((_R^4+2*_R^2+1)/(_R^5*a+2*_R^3*a
+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*
b+3*_Z^2*a+a))-1/8/d/a/tan(1/2*d*x+1/2*c)^2+1/2/d/a*ln(tan(1/2*d*x+1/2*c))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

<u>**Sympy**</u> **[F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Timed out
```

<u>**Giac**</u> **[F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(dx+c)^3}{b\sin(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(csc(d*x + c)^3/(b*sin(d*x + c)^3 + a), x)
```

3.188 $\int \frac{\csc^5(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=344

$$\frac{2(-1)^{2/3}b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{7/3}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}-\frac{2b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{7/3}d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2\sqrt[3]{-1}b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3a^{7/3}d\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}$$

```
[Out] (2*(-1)^(2/3)*b^(5/3)*ArcTan[((-1)^(1/3)*b^(1/3) - a^(1/3)*Tan[(c + d*x)/2]
)/Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(7/3)*Sqrt[a^(2/3) - (-1)^(2/3)
*b^(2/3)]*d) - (2*b^(5/3)*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[
a^(2/3) - b^(2/3)]])/(3*a^(7/3)*Sqrt[a^(2/3) - b^(2/3)]*d) + (2*(-1)^(1/3)*
b^(5/3)*ArcTan[((-1)^(2/3)*b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3)
 + (-1)^(1/3)*b^(2/3)]])/(3*a^(7/3)*Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*d) -
 (3*ArcTanh[Cos[c + d*x]])/(8*a*d) + (b*Cot[c + d*x])/(a^2*d) - (3*Cot[c +
d*x]*Csc[c + d*x])/(8*a*d) - (Cot[c + d*x]*Csc[c + d*x]^3)/(4*a*d)
```

Rubi [A] time = 0.477802, antiderivative size = 344, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3220, 3767, 8, 3768, 3770, 2660, 618, 204}

$$\frac{2(-1)^{2/3}b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{7/3}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}-\frac{2b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{7/3}d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2\sqrt[3]{-1}b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3a^{7/3}d\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (2*(-1)^(2/3)*b^(5/3)*ArcTan[((-1)^(1/3)*b^(1/3) - a^(1/3)*Tan[(c + d*x)/2]
)/Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(7/3)*Sqrt[a^(2/3) - (-1)^(2/3)
*b^(2/3)]*d) - (2*b^(5/3)*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[
a^(2/3) - b^(2/3)]])/(3*a^(7/3)*Sqrt[a^(2/3) - b^(2/3)]*d) + (2*(-1)^(1/3)*
b^(5/3)*ArcTan[((-1)^(2/3)*b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3)
 + (-1)^(1/3)*b^(2/3)]])/(3*a^(7/3)*Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*d) -
 (3*ArcTanh[Cos[c + d*x]])/(8*a*d) + (b*Cot[c + d*x])/(a^2*d) - (3*Cot[c +
d*x]*Csc[c + d*x])/(8*a*d) - (Cot[c + d*x]*Csc[c + d*x]^3)/(4*a*d)
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc^5(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \left(-\frac{b\csc^2(c+dx)}{a^2}+\frac{\csc^5(c+dx)}{a}+\frac{b^2\sin(c+dx)}{a^2\left(a+b\sin^3(c+dx)\right)}\right)dx$$

$$= \frac{\int \csc^5(c+dx)\,dx}{a}-\frac{b\int \csc^2(c+dx)\,dx}{a^2}+\frac{b^2\int \frac{\sin(c+dx)}{a+b\sin^3(c+dx)}\,dx}{a^2}$$

$$= -\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}+\frac{3\int \csc^3(c+dx)\,dx}{4a}+\frac{b^2\int\left(-\frac{1}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}-\frac{}{3\sqrt[3]{a}\sqrt[3]{b}(}\right.}{}$$

$$= \frac{b\cot(c+dx)}{a^2d}-\frac{3\cot(c+dx)\csc(c+dx)}{8ad}-\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}+\frac{3\int\csc(c+dx)\,dx}{8a}$$

$$= -\frac{3\tanh^{-1}(\cos(c+dx))}{8ad}+\frac{b\cot(c+dx)}{a^2d}-\frac{3\cot(c+dx)\csc(c+dx)}{8ad}-\frac{\cot(c+dx)\csc^3(c-}{4ad}$$

$$= -\frac{3\tanh^{-1}(\cos(c+dx))}{8ad}+\frac{b\cot(c+dx)}{a^2d}-\frac{3\cot(c+dx)\csc(c+dx)}{8ad}-\frac{\cot(c+dx)\csc^3(c-}{4ad}$$

$$= \frac{2(-1)^{2/3}b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{7/3}\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,d}-\frac{2b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{7/3}\sqrt{a^{2/3}-b^{2/3}}\,d}+\frac{2\sqrt[3]{-1}b^{5/3}\,\mathrm{t}}{3}$$

Mathematica [C] time = 1.99384, size = 290, normalized size = 0.84

$$\frac{3\left(-a\csc^4\left(\frac{1}{2}(c+dx)\right)-6a\csc^2\left(\frac{1}{2}(c+dx)\right)+a\sec^4\left(\frac{1}{2}(c+dx)\right)+6a\sec^2\left(\frac{1}{2}(c+dx)\right)+24a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^5/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (-64*b^2*RootSum[-b + 3*b*#1^2 - (8*I)*a*#1^3 - 3*b*#1^4 + b*#1^6 & , (-2*A
rcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + I*Log[1 - 2*Cos[c + d*x]*#1 + #1^
2] + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 - I*Log[1 - 2*Cos[c +
d*x]*#1 + #1^2]*#1^2)/(b - (4*I)*a*#1 - 2*b*#1^2 + b*#1^4) & ] + 3*(32*b*Co
t[(c + d*x)/2] - 6*a*Csc[(c + d*x)/2]^2 - a*Csc[(c + d*x)/2]^4 - 24*a*Log[C
os[(c + d*x)/2]] + 24*a*Log[Sin[(c + d*x)/2]] + 6*a*Sec[(c + d*x)/2]^2 + a*
Sec[(c + d*x)/2]^4 - 32*b*Tan[(c + d*x)/2]))/(192*a^2*d)
```

Maple [C] time = 0.21, size = 217, normalized size = 0.6

$$\frac{1}{64da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4+\frac{1}{8da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2-\frac{b}{2a^2d}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{2b^2}{3a^2d}\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5/(a+b*sin(d*x+c)^3),x)
```

```
[Out] 1/64/d/a*tan(1/2*d*x+1/2*c)^4+1/8/d*tan(1/2*d*x+1/2*c)^2/a-1/2/d/a^2*tan(1/
2*d*x+1/2*c)*b+2/3/d/a^2*b^2*sum((_R^3+_R)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*
```

```
ln(tan(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))-1
/64/d/a/tan(1/2*d*x+1/2*c)^4-1/8/d/a/tan(1/2*d*x+1/2*c)^2+3/8/d/a*ln(tan(1/
2*d*x+1/2*c))+1/2/d*b/a^2/tan(1/2*d*x+1/2*c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+b*sin(d*x+c)^3),x, algorithm="maxima")

[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+b*sin(d*x+c)^3),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5/(a+b*sin(d*x+c)**3),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(dx+c)^5}{b\sin(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+b*sin(d*x+c)^3),x, algorithm="giac")

[Out] integrate(csc(d*x + c)^5/(b*sin(d*x + c)^3 + a), x)
```

3.189 $\int \frac{\sin^6(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=293

$$\frac{2a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3b^2d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3b^2d\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}-\frac{2a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\left((-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3b^2d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}$$

```
[Out] -((a*x)/b^2) + (2*a^(4/3)*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[
a^(2/3) - b^(2/3)]])/(3*Sqrt[a^(2/3) - b^(2/3)]*b^2*d) + (2*a^(4/3)*ArcTan[
((-1)^(2/3)*b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) + (-1)^(1/3)*b
^(2/3)]])/(3*Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*b^2*d) - (2*a^(4/3)*ArcTan[
((-1)^(1/3)*(b^(1/3) + (-1)^(2/3)*a^(1/3)*Tan[(c + d*x)/2]))/Sqrt[a^(2/3) -
 (-1)^(2/3)*b^(2/3)]])/(3*Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]*b^2*d) - Cos[c
 + d*x]/(b*d) + Cos[c + d*x]^3/(3*b*d)
```

Rubi [A] time = 0.39395, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3220, 2633, 3213, 2660, 618, 204}

$$\frac{2a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3b^2d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3b^2d\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}-\frac{2a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\left((-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3b^2d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^6/(a + b*Sin[c + d*x]^3),x]
```



```
[Out] -((a*x)/b^2) + (2*a^(4/3)*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[
a^(2/3) - b^(2/3)]])/(3*Sqrt[a^(2/3) - b^(2/3)]*b^2*d) + (2*a^(4/3)*ArcTan[
((-1)^(2/3)*b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) + (-1)^(1/3)*b
^(2/3)]])/(3*Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*b^2*d) - (2*a^(4/3)*ArcTan[
((-1)^(1/3)*(b^(1/3) + (-1)^(2/3)*a^(1/3)*Tan[(c + d*x)/2]))/Sqrt[a^(2/3) -
 (-1)^(2/3)*b^(2/3)]])/(3*Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]*b^2*d) - Cos[c
 + d*x]/(b*d) + Cos[c + d*x]^3/(3*b*d)
```


Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
```

```
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin^6(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \left(-\frac{a}{b^2}+\frac{\sin^3(c+dx)}{b}+\frac{a^2}{b^2\left(a+b\sin^3(c+dx)\right)}\right)dx$$

$$= -\frac{ax}{b^2}+\frac{a^2\int \frac{1}{a+b\sin^3(c+dx)}\,dx}{b^2}+\frac{\int \sin^3(c+dx)\,dx}{b}$$

$$= -\frac{ax}{b^2}+\frac{a^2\int \left(-\frac{1}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}\sin(c+dx)\right)}-\frac{1}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}\sin(c+dx)\right)}-\frac{1}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\sin(c+dx)\right)}\right) d}{b^2}$$

$$= -\frac{ax}{b^2}-\frac{\cos(c+dx)}{bd}+\frac{\cos^3(c+dx)}{3bd}-\frac{a^{4/3}\int \frac{1}{-\sqrt[3]{a}-\sqrt[3]{b}\sin(c+dx)}\,dx}{3b^2}-\frac{a^{4/3}\int \frac{1}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}\sin(c+dx)}\,dx}{3b^2}$$

$$= -\frac{ax}{b^2}-\frac{\cos(c+dx)}{bd}+\frac{\cos^3(c+dx)}{3bd}-\frac{\left(2a^{4/3}\right)\mathrm{Subst}\left(\int \frac{1}{-\sqrt[3]{a}-2\sqrt[3]{b}x-\sqrt[3]{a}x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3b^2d}$$

$$= -\frac{ax}{b^2}-\frac{\cos(c+dx)}{bd}+\frac{\cos^3(c+dx)}{3bd}+\frac{\left(4a^{4/3}\right)\mathrm{Subst}\left(\int \frac{1}{-4\left(a^{2/3}-b^{2/3}\right)-x^2}\,dx,x,-2\sqrt[3]{b}-2\sqrt[3]{a}\tan\left(\right.\right.}{3b^2d}$$

$$= -\frac{ax}{b^2}-\frac{2a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,b^2d}+\frac{2a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3\sqrt{a^{2/3}-b^{2/3}}\,b^2d}+\frac{2a^{4/3}\tan^{-1}\left(\right.}{3\sqrt{a^{2/}}}$$

Mathematica [C] time = 0.272872, size = 164, normalized size = 0.56

$$-\frac{8ia^2\mathrm{RootSum}\left[8\#1^3a+i\#1^6b-3i\#1^4b+3i\#1^2b-ib\&,\frac{2\#1\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)-i\#1\log\left(\#1^2-2\#1\cos(c+dx)+1\right)}{\#1^4b-2\#1^2b-4i\#1a+b}\&\right]+12ac+12adx}{12b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^6/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] -(12*a*c + 12*a*d*x + 9*b*Cos[c + d*x] - b*Cos[3*(c + d*x)] + (8*I)*a^2*Roo
tSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#1^6 & , (2*ArcT
an[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 - I*Log[1 - 2*Cos[c + d*x]*#1 + #1^
2]*#1)/(b - (4*I)*a*#1 - 2*b*#1^2 + b*#1^4) & ])/(12*b^2*d)
```

Maple [C] time = 0.161, size = 166, normalized size = 0.6

$$\frac{a^2}{3\,b^2d}\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3\,a\_Z^4+8\,b\_Z^3+3\,a\_Z^2+a\right)}\frac{\_R^4+2\_R^2+1}{\_R^5a+2\_R^3a+4\_R^2b+\_R\,a}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)-4\,\frac{(\tan(1/}{bd\left(1+(\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^6/(a+b*sin(d*x+c)^3),x)
```

```
[Out] 1/3/d*a^2/b^2*sum((_R^4+2*_R^2+1)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/
2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))-4/d/b/(1+t
an(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^2-4/3/d/b/(1+tan(1/2*d*x+1/2*c)^2
)^3-2/d/b^2*a*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**6/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)^6}{b\sin(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^6/(b*sin(d*x + c)^3 + a), x)
```

3.190 $\int \frac{\sin^4(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=281

$$-\frac{2(-1)^{2/3}a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3b^{4/3}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}+\frac{2a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3b^{4/3}d\sqrt{a^{2/3}-b^{2/3}}}-\frac{2\sqrt[3]{-1}a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3b^{4/3}d\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}$$

```
[Out] (-2*(-1)^(2/3)*a^(2/3)*ArcTan[((-1)^(1/3)*b^(1/3) - a^(1/3)*Tan[(c + d*x)/2
])/Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3
)]*b^(4/3)*d) + (2*a^(2/3)*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt
[a^(2/3) - b^(2/3)]])/(3*Sqrt[a^(2/3) - b^(2/3)]*b^(4/3)*d) - (2*(-1)^(1/3)
*a^(2/3)*ArcTan[((-1)^(2/3)*b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3
) + (-1)^(1/3)*b^(2/3)]])/(3*Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*b^(4/3)*d)
- Cos[c + d*x]/(b*d)
```

Rubi [A] time = 0.422213, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3220, 2638, 2660, 618, 204}

$$-\frac{2(-1)^{2/3}a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3b^{4/3}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}+\frac{2a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3b^{4/3}d\sqrt{a^{2/3}-b^{2/3}}}-\frac{2\sqrt[3]{-1}a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3b^{4/3}d\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (-2*(-1)^(2/3)*a^(2/3)*ArcTan[((-1)^(1/3)*b^(1/3) - a^(1/3)*Tan[(c + d*x)/2
])/Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3
)]*b^(4/3)*d) + (2*a^(2/3)*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt
[a^(2/3) - b^(2/3)]])/(3*Sqrt[a^(2/3) - b^(2/3)]*b^(4/3)*d) - (2*(-1)^(1/3)
*a^(2/3)*ArcTan[((-1)^(2/3)*b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3
) + (-1)^(1/3)*b^(2/3)]])/(3*Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*b^(4/3)*d)
- Cos[c + d*x]/(b*d)
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin^4(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \left(\frac{\sin(c+dx)}{b} - \frac{a\sin(c+dx)}{b\left(a+b\sin^3(c+dx)\right)}\right) dx$$

$$= \frac{\int \sin(c+dx)\,dx}{b} - \frac{a\int \frac{\sin(c+dx)}{a+b\sin^3(c+dx)}\,dx}{b}$$

$$= -\frac{\cos(c+dx)}{bd} - \frac{a\int \left(-\frac{1}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)} - \frac{(-1)^{2/3}}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}\sin(c+dx)\right)} + \frac{\sqrt[3]{-1}}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\right.}}{b}$$

$$= -\frac{\cos(c+dx)}{bd} + \frac{a^{2/3}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)}\,dx}{3b^{4/3}} - \frac{\left(\sqrt[3]{-1}a^{2/3}\right)\int \frac{1}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\sin(c+dx)}\,dx}{3b^{4/3}} + \frac{\left((-1)^{2/3}a^{2/3}\right.}{}$$

$$= -\frac{\cos(c+dx)}{bd} + \frac{\left(2a^{2/3}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{a}+2\sqrt[3]{b}x+\sqrt[3]{a}x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{3b^{4/3}d} - \frac{\left(2\sqrt[3]{-1}a^{2/3}\right)\mathrm{Subst}\left(\right.}{}$$

$$= -\frac{\cos(c+dx)}{bd} - \frac{\left(4a^{2/3}\right)\mathrm{Subst}\left(\int \frac{1}{-4\left(a^{2/3}-b^{2/3}\right)-x^2}\,dx, x, 2\sqrt[3]{b}+2\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3b^{4/3}d} + \frac{\left(4\sqrt[3]{-1}a\right.}{}$$

$$= -\frac{2(-1)^{2/3}a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,b^{4/3}d} + \frac{2a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3\sqrt{a^{2/3}-b^{2/3}}\,b^{4/3}d} - \frac{2\sqrt[3]{-1}a^{2/3}\tan}{3\sqrt{}}$$

Mathematica [C] time = 0.257005, size = 186, normalized size = 0.66

$$\frac{-3\cos(c+dx) + a\,\mathrm{RootSum}\left[8\#1^3a + i\#1^6b - 3i\#1^4b + 3i\#1^2b - ib\&, \frac{-i\#1^2\log\left(\#1^2-2\#1\cos(c+dx)+1\right)+i\log\left(\#1^2-2\#1\cos(c+dx)+1\right)+}{\#1^4b-2\#1^2b-4i\#1a}\right.}{3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (-3*Cos[c + d*x] + a*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^
4 + I*b*#1^6 & , (-2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + I*Log[1 - 2
*Cos[c + d*x]*#1 + #1^2] + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2
- I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2)/(b - (4*I)*a*#1 - 2*b*#1^2 + b*
#1^4) & ])/(3*b*d)
```

Maple [C] time = 0.155, size = 106, normalized size = 0.4

$$-\frac{2a}{3bd}\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\_R^3+\_R}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)-2\frac{}{bd\left(1+(\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a+b*sin(d*x+c)^3),x)
```

```
[Out] -2/3/d*a/b*sum((_R^3+_R)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2
*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))-2/d/b/(1+tan(1/2*d*
x+1/2*c)^2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin^4(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Integral(sin(c + d*x)**4/(a + b*sin(c + d*x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin(dx+c)^4}{b\sin(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^4/(b*sin(d*x + c)^3 + a), x)
```

3.191 $\int \frac{\sin^2(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=240

$$\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3b^{2/3}d\sqrt{a^{2/3}-b^{2/3}}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}\right)}{3b^{2/3}d\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}-\frac{2\tanh^{-1}\left(\frac{(-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}}\right)}{3b^{2/3}d\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}}$$

```
[Out] (2*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b^(2/3)]])/(3
*Sqrt[a^(2/3) - b^(2/3)]*b^(2/3)*d) - (2*ArcTanh[(b^(1/3) - (-1)^(1/3)*a^(1
/3)*Tan[(c + d*x)/2])/Sqrt[-((-1)^(2/3)*a^(2/3)) + b^(2/3)]])/(3*Sqrt[-((-1
)^(2/3)*a^(2/3)) + b^(2/3)]*b^(2/3)*d) - (2*ArcTanh[(b^(1/3) + (-1)^(2/3)*a
^(1/3)*Tan[(c + d*x)/2])/Sqrt[(-1)^(1/3)*a^(2/3) + b^(2/3)]])/(3*Sqrt[(-1)^
(1/3)*a^(2/3) + b^(2/3)]*b^(2/3)*d)
```

Rubi [A] time = 0.272057, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3220, 2660, 618, 204, 206}

$$\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3b^{2/3}d\sqrt{a^{2/3}-b^{2/3}}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}\right)}{3b^{2/3}d\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}-\frac{2\tanh^{-1}\left(\frac{(-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}}\right)}{3b^{2/3}d\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (2*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b^(2/3)]])/(3
*Sqrt[a^(2/3) - b^(2/3)]*b^(2/3)*d) - (2*ArcTanh[(b^(1/3) - (-1)^(1/3)*a^(1
/3)*Tan[(c + d*x)/2])/Sqrt[-((-1)^(2/3)*a^(2/3)) + b^(2/3)]])/(3*Sqrt[-((-1
)^(2/3)*a^(2/3)) + b^(2/3)]*b^(2/3)*d) - (2*ArcTanh[(b^(1/3) + (-1)^(2/3)*a
^(1/3)*Tan[(c + d*x)/2])/Sqrt[(-1)^(1/3)*a^(2/3) + b^(2/3)]])/(3*Sqrt[(-1)^
(1/3)*a^(2/3) + b^(2/3)]*b^(2/3)*d)
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \left(\frac{1}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}+\frac{1}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}+\frac{1}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\right.}\right.$$

$$= \frac{\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)}\,dx}{3b^{2/3}}+\frac{\int \frac{1}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)}\,dx}{3b^{2/3}}+\frac{\int \frac{1}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)}\,dx}{3b^{2/3}}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+2\sqrt[3]{b}x+\sqrt[3]{a}x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{3b^{2/3}d}+\frac{2\,\text{Subst}\left(\int \frac{1}{-\sqrt[3]{-1}\sqrt[3]{a}+2\sqrt[3]{b}x-\sqrt[3]{-1}\sqrt[3]{a}x^2}\,dx, x, \text{t}\right.}{3b^{2/3}d}$$

$$= -\frac{4\,\text{Subst}\left(\int \frac{1}{-4\left(a^{2/3}-b^{2/3}\right)-x^2}\,dx, x, 2\sqrt[3]{b}+2\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3b^{2/3}d}-\frac{4\,\text{Subst}\left(\int \frac{1}{-4\left((-1)^{2/3}a^{2/3}-b^{2/3}\right)-}\right.}{}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3\sqrt{a^{2/3}-b^{2/3}}b^{2/3}d}-\frac{2\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}+b^{2/3}}}\right)}{3\sqrt{-(-1)^{2/3}a^{2/3}+b^{2/3}}b^{2/3}d}-\frac{2\tanh^{-1}\left(\frac{\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}\tan\left(\right.}{\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}}\right.}{3\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}b^{2/}}$$

Mathematica [C] time = 0.18427, size = 231, normalized size = 0.96

$$\frac{i\text{RootSum}\left[8\#1^3a+i\#1^6b-3i\#1^4b+3i\#1^2b-ib\&, \frac{-i\#1^4\log\left(\#1^2-2\#1\cos(c+dx)+1\right)+2i\#1^2\log\left(\#1^2-2\#1\cos(c+dx)+1\right)-i\log\left(\#1^2-2\#1\cos(\right.}{-4i\#1^2a+\#1^5}\right.}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] ((I/6)*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#1^6 &
 , (2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - I*Log[1 - 2*Cos[c + d*x]*#
1 + #1^2] - 4*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 + (2*I)*Log[1 -
 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)
]*#1^4 - I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4)/(b*#1 - (4*I)*a*#1^2 - 2
*b*#1^3 + b*#1^5) & ])/d
```

Maple [C] time = 0.141, size = 76, normalized size = 0.3

$$\frac{4}{3d}\sum_{\_R=\text{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\_R^2}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+b*sin(d*x+c)^3),x)
```

```
[Out] 4/3/d*sum(_R^2/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_R),_R
=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)^2}{b\sin(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^2/(b*sin(d*x + c)^3 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Integral(sin(c + d*x)**2/(a + b*sin(c + d*x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)^2}{b\sin(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^2/(b*sin(d*x + c)^3 + a), x)
```

3.192 $\int \frac{1}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=245

$$\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{-1}\left((-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}$$

```
[Out] (2*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b^(2/3)]])/(3
*a^(2/3)*Sqrt[a^(2/3) - b^(2/3)]*d) + (2*ArcTan[((-1)^(2/3)*b^(1/3) + a^(1/
3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]])/(3*a^(2/3)*Sqrt[a
^(2/3) + (-1)^(1/3)*b^(2/3)]*d) - (2*ArcTan[((-1)^(1/3)*(b^(1/3) + (-1)^(2/
3)*a^(1/3)*Tan[(c + d*x)/2]))/Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(2/
3)*Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]*d)
```

Rubi [A] time = 0.260708, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3213, 2660, 618, 204}

$$\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{-1}\left((-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x]^3)^(-1),x]
```


```
[Out] (2*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b^(2/3)]])/(3
*a^(2/3)*Sqrt[a^(2/3) - b^(2/3)]*d) + (2*ArcTan[((-1)^(2/3)*b^(1/3) + a^(1/
3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]])/(3*a^(2/3)*Sqrt[a
^(2/3) + (-1)^(1/3)*b^(2/3)]*d) - (2*ArcTan[((-1)^(1/3)*(b^(1/3) + (-1)^(2/
3)*a^(1/3)*Tan[(c + d*x)/2]))/Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(2/
3)*Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]*d)
```


Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{a+b\sin^3(c+dx)}\,dx = \int \left(-\frac{1}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}\sin(c+dx)\right)} - \frac{1}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}\sin(c+dx)\right)} - \frac{1}{3a^{2/3}\left(-\sqrt[3]{a}-(-\right.}\right.$$

$$= -\frac{\int \frac{1}{-\sqrt[3]{a}-\sqrt[3]{b}\sin(c+dx)}\,dx}{3a^{2/3}} - \frac{\int \frac{1}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}\sin(c+dx)}\,dx}{3a^{2/3}} - \frac{\int \frac{1}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\sin(c+dx)}\,dx}{3a^{2/3}}$$

$$= -\frac{2\,\mathrm{Subst}\left(\int \frac{1}{-\sqrt[3]{a}-2\sqrt[3]{b}x-\sqrt[3]{a}x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{3a^{2/3}d} - \frac{2\,\mathrm{Subst}\left(\int \frac{1}{-\sqrt[3]{a}+2\sqrt[3]{-1}\sqrt[3]{b}x-\sqrt[3]{a}x^2}\,dx, x,\right.}{3a^{2/3}d}$$

$$= \frac{4\,\mathrm{Subst}\left(\int \frac{1}{-4\left(a^{2/3}-b^{2/3}\right)-x^2}\,dx, x, -2\sqrt[3]{b}-2\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3a^{2/3}d} + \frac{4\,\mathrm{Subst}\left(\int \frac{1}{-4\left(a^{2/3}+\sqrt[3]{-1}b^{2/3}\right.}\right.}{}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,d} + \frac{2\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-b^{2/3}}\,d} + \frac{2\tan^{-1}\left(\frac{(-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}\tan}{\sqrt{a^{2/3}+\sqrt[3]{-}}}\right.}{3a^{2/3}\sqrt{a^{2/3}+\sqrt[3]{-1}}}$$

Mathematica [C] time = 0.124832, size = 126, normalized size = 0.51

$$-\frac{2i\mathrm{RootSum}\left[8\#1^3a+i\#1^6b-3i\#1^4b+3i\#1^2b-ib\&, \frac{2\#1\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)-i\#1\log\left(\#1^2-2\#1\cos(c+dx)+1\right)}{\#1^4b-2\#1^2b-4i\#1a+b}\&\right]}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x]^3)^(-1),x]
```

```
[Out] (((-2*I)/3)*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#
1^6 & , (2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 - I*Log[1 - 2*Cos[c
+ d*x]*#1 + #1^2]*#1)/(b - (4*I)*a*#1 - 2*b*#1^2 + b*#1^4) & ])/d
```

Maple [C] time = 0.151, size = 83, normalized size = 0.3

$$\frac{1}{3d}\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)} \frac{\_R^4+2\_R^2+1}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(d*x+c)^3),x)
```

```
[Out] 1/3/d*sum((_R^4+2*_R^2+1)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/
2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b\sin(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*sin(d*x + c)^3 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{a+b\sin^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Integral(1/(a + b*sin(c + d*x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b\sin(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(1/(b*sin(d*x + c)^3 + a), x)
```

3.193 $\int \frac{\csc^2(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=281

$$-\frac{2(-1)^{2/3}b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{4/3}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}+\frac{2b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{4/3}d\sqrt{a^{2/3}-b^{2/3}}}-\frac{2\sqrt[3]{-1}b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\cdots}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3a^{4/3}d\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}$$

```
[Out] (-2*(-1)^(2/3)*b^(2/3)*ArcTan[((-1)^(1/3)*b^(1/3) - a^(1/3)*Tan[(c + d*x)/2
])/Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(4/3)*Sqrt[a^(2/3) - (-1)^(2/3
)*b^(2/3)]*d) + (2*b^(2/3)*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt
[a^(2/3) - b^(2/3)]])/(3*a^(4/3)*Sqrt[a^(2/3) - b^(2/3)]*d) - (2*(-1)^(1/3)
*b^(2/3)*ArcTan[((-1)^(2/3)*b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3
) + (-1)^(1/3)*b^(2/3)]])/(3*a^(4/3)*Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*d)
- Cot[c + d*x]/(a*d)
```

Rubi [A] time = 0.428511, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3220, 3767, 8, 2660, 618, 204}

$$-\frac{2(-1)^{2/3}b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{4/3}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}+\frac{2b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{4/3}d\sqrt{a^{2/3}-b^{2/3}}}-\frac{2\sqrt[3]{-1}b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\cdots}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3a^{4/3}d\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (-2*(-1)^(2/3)*b^(2/3)*ArcTan[((-1)^(1/3)*b^(1/3) - a^(1/3)*Tan[(c + d*x)/2
])/Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(4/3)*Sqrt[a^(2/3) - (-1)^(2/3
)*b^(2/3)]*d) + (2*b^(2/3)*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt
[a^(2/3) - b^(2/3)]])/(3*a^(4/3)*Sqrt[a^(2/3) - b^(2/3)]*d) - (2*(-1)^(1/3)
*b^(2/3)*ArcTan[((-1)^(2/3)*b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3
) + (-1)^(1/3)*b^(2/3)]])/(3*a^(4/3)*Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*d)
- Cot[c + d*x]/(a*d)
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \left(\frac{\csc^2(c+dx)}{a} - \frac{b\sin(c+dx)}{a\left(a+b\sin^3(c+dx)\right)}\right)dx$$

$$= \frac{\int \csc^2(c+dx)\,dx}{a} - \frac{b\int \frac{\sin(c+dx)}{a+b\sin^3(c+dx)}\,dx}{a}$$

$$= -\frac{b\int \left(-\frac{1}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)} - \frac{(-1)^{2/3}}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}\sin(c+dx)\right)} + \frac{\sqrt[3]{-1}}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\sin(c+dx)\right)}\right)dx}{a} -$$

$$= -\frac{\cot(c+dx)}{ad} + \frac{b^{2/3}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)}\,dx}{3a^{4/3}} - \frac{\left(\sqrt[3]{-1}b^{2/3}\right)\int \frac{1}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\sin(c+dx)}\,dx}{3a^{4/3}} + \frac{\left((-1)^{2/3}b^{2/3}\right)}{}$$

$$= -\frac{\cot(c+dx)}{ad} + \frac{\left(2b^{2/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+2\sqrt[3]{b}x+\sqrt[3]{a}x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{3a^{4/3}d} - \frac{\left(2\sqrt[3]{-1}b^{2/3}\right)\text{Subst}\left(\right.}{}$$

$$= -\frac{\cot(c+dx)}{ad} - \frac{\left(4b^{2/3}\right)\text{Subst}\left(\int \frac{1}{-4\left(a^{2/3}-b^{2/3}\right)-x^2}\,dx, x, 2\sqrt[3]{b}+2\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3a^{4/3}d} + \frac{\left(4\sqrt[3]{-1}b\right.}{}$$

$$= -\frac{2(-1)^{2/3}b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{4/3}\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,d} + \frac{2b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{4/3}\sqrt{a^{2/3}-b^{2/3}}\,d} - \frac{2\sqrt[3]{-1}b^{2/3}\tan}{3a^4}$$

Mathematica [C] time = 0.309986, size = 196, normalized size = 0.7

$$\frac{2b\,\text{RootSum}\left[-8i\#1^3a+\#1^6b-3\#1^4b+3\#1^2b-b\&, \frac{-i\#1^2\log\left(\#1^2-2\#1\cos(c+dx)+1\right)+i\log\left(\#1^2-2\#1\cos(c+dx)+1\right)+2\#1^2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-}\right.}{\#1^4b-2\#1^2b-4i\#1a+b}\right.}{6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (-3*Cot[(c + d*x)/2] + 2*b*RootSum[-b + 3*b*#1^2 - (8*I)*a*#1^3 - 3*b*#1^4
+ b*#1^6 & , (-2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + I*Log[1 - 2*Cos
```

```
[c + d*x]*#1 + #1^2] + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 - I*
Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2)/(b - (4*I)*a*#1 - 2*b*#1^2 + b*#1^4
) & ] + 3*Tan[(c + d*x)/2])/(6*a*d)
```

Maple [C] time = 0.193, size = 119, normalized size = 0.4

$$\frac{1}{2da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-1}-\frac{2b}{3da}\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\_R^3+\_R}{\_R^5a+2\_R^3a+4\_R^2b+\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+b*sin(d*x+c)^3),x)
```

```
[Out] 1/2/d/a*tan(1/2*d*x+1/2*c)-1/2/d/a/tan(1/2*d*x+1/2*c)-2/3/d/a*b*sum((_R^3+_
R)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6
*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(dx+c)^2}{b\sin(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(csc(d*x + c)^2/(b*sin(d*x + c)^3 + a), x)
```

3.194 $\int \frac{\csc^4(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=296

$$\frac{2b^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^2d\sqrt{a^{2/3}-b^{2/3}}}-\frac{2b^{4/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}\right)}{3a^2d\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}-\frac{2b^{4/3}\tanh^{-1}\left(\frac{(-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}}\right)}{3a^2d\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}}+\frac{b}{}$$

```
[Out] (2*b^(4/3)*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b^(2/
3)]])/(3*a^2*Sqrt[a^(2/3) - b^(2/3)]*d) + (b*ArcTanh[Cos[c + d*x]])/(a^2*d)
 - (2*b^(4/3)*ArcTanh[(b^(1/3) - (-1)^(1/3)*a^(1/3)*Tan[(c + d*x)/2])/Sqrt[
-((-1)^(2/3)*a^(2/3)) + b^(2/3)]])/(3*a^2*Sqrt[-((-1)^(2/3)*a^(2/3)) + b^(2
/3)]*d) - (2*b^(4/3)*ArcTanh[(b^(1/3) + (-1)^(2/3)*a^(1/3)*Tan[(c + d*x)/2]
)/Sqrt[(-1)^(1/3)*a^(2/3) + b^(2/3)]])/(3*a^2*Sqrt[(-1)^(1/3)*a^(2/3) + b^(
2/3)]*d) - Cot[c + d*x]/(a*d) - Cot[c + d*x]^3/(3*a*d)
```

Rubi [A] time = 0.393332, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3220, 3770, 3767, 2660, 618, 204, 206}

$$\frac{2b^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^2d\sqrt{a^{2/3}-b^{2/3}}}-\frac{2b^{4/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}\right)}{3a^2d\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}-\frac{2b^{4/3}\tanh^{-1}\left(\frac{(-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}}\right)}{3a^2d\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}}+\frac{b}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (2*b^(4/3)*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b^(2/
3)]])/(3*a^2*Sqrt[a^(2/3) - b^(2/3)]*d) + (b*ArcTanh[Cos[c + d*x]])/(a^2*d)
 - (2*b^(4/3)*ArcTanh[(b^(1/3) - (-1)^(1/3)*a^(1/3)*Tan[(c + d*x)/2])/Sqrt[
-((-1)^(2/3)*a^(2/3)) + b^(2/3)]])/(3*a^2*Sqrt[-((-1)^(2/3)*a^(2/3)) + b^(2
/3)]*d) - (2*b^(4/3)*ArcTanh[(b^(1/3) + (-1)^(2/3)*a^(1/3)*Tan[(c + d*x)/2]
)/Sqrt[(-1)^(1/3)*a^(2/3) + b^(2/3)]])/(3*a^2*Sqrt[(-1)^(1/3)*a^(2/3) + b^(
2/3)]*d) - Cot[c + d*x]/(a*d) - Cot[c + d*x]^3/(3*a*d)
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc^4(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \left(-\frac{b\csc(c+dx)}{a^2}+\frac{\csc^4(c+dx)}{a}+\frac{b^2\sin^2(c+dx)}{a^2\left(a+b\sin^3(c+dx)\right)}\right)dx$$

$$= \frac{\int \csc^4(c+dx)\,dx}{a}-\frac{b\int \csc(c+dx)\,dx}{a^2}+\frac{b^2\int \frac{\sin^2(c+dx)}{a+b\sin^3(c+dx)}\,dx}{a^2}$$

$$= \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{b^2\int \left(\frac{1}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}+\frac{1}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}+\frac{1}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}\right.}\right.}{a^2}$$

$$= \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{ad}-\frac{\cot^3(c+dx)}{3ad}+\frac{b^{4/3}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)}\,dx}{3a^2}+\frac{b^{4/3}\int \frac{}{-\sqrt[3]{-1}}}{}$$

$$= \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{ad}-\frac{\cot^3(c+dx)}{3ad}+\frac{\left(2b^{4/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+2\sqrt[3]{b}x+\sqrt[3]{a}x^2}\,dx,x,\right.}{3a^2d}$$

$$= \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{ad}-\frac{\cot^3(c+dx)}{3ad}-\frac{\left(4b^{4/3}\right)\text{Subst}\left(\int \frac{1}{-4\left(a^{2/3}-b^{2/3}\right)-x^2}\,dx,x,2\right.}{3a^2d}$$

$$= \frac{2b^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^2\sqrt{a^{2/3}-b^{2/3}}\,d}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{2b^{4/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx\right.}{\sqrt{-(-1)^{2/3}a^{2/3}+b^{2/3}}}\right.}{3a^2\sqrt{-(-1)^{2/3}a^{2/3}+b^{2/3}}\,d}$$

Mathematica [C] time = 2.05013, size = 333, normalized size = 1.12

$$\frac{4ib^2\text{RootSum}\left[-8i\#1^3a+\#1^6b-3\#1^4b+3\#1^2b-b\&,\frac{-i\#1^4\log\left(\#1^2-2\#1\cos(c+dx)+1\right)+2i\#1^2\log\left(\#1^2-2\#1\cos(c+dx)+1\right)-i\log\left(\#1^2-2\#1\right.}{-4i\#1^2a+}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (-8*a*Cot[(c + d*x)/2] + 24*b*Log[Cos[(c + d*x)/2]] - 24*b*Log[Sin[(c + d*x
)/2]] + (4*I)*b^2*RootSum[-b + 3*b*#1^2 - (8*I)*a*#1^3 - 3*b*#1^4 + b*#1^6
& , (2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - I*Log[1 - 2*Cos[c + d*x]*
#1 + #1^2] - 4*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 + (2*I)*Log[1
- 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1
)]*#1^4 - I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4)/(b*#1 - (4*I)*a*#1^2 -
2*b*#1^3 + b*#1^5) & ] + 8*a*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 - (a*Csc[(c
+ d*x)/2]^4*Sin[c + d*x])/2 + 8*a*Tan[(c + d*x)/2])/(24*a^2*d)
```

Maple [C] time = 0.207, size = 176, normalized size = 0.6

$$\frac{1}{24\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}+\frac{3}{8\,da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{4\,b^{2}}{3\,a^{2}d}\sum_{\_R=\mathrm{RootOf}\left(a\_Z^{6}+3\,a\_Z^{4}+8\,b\_Z^{3}+3\,a\_Z^{2}+a\right)}\frac{\_R^{2}}{\_R^{5}a+2\_R^{3}a+4\_R^{2}b+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4/(a+b*sin(d*x+c)^3),x)
```

```
[Out] 1/24/d/a*tan(1/2*d*x+1/2*c)^3+3/8/d/a*tan(1/2*d*x+1/2*c)+4/3/d/a^2*b^2*sum(
_R^2/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z
^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))-1/24/d/a/tan(1/2*d*x+1/2*c)^3-3/8/d/a/t
an(1/2*d*x+1/2*c)-1/d/a^2*b*ln(tan(1/2*d*x+1/2*c))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(dx+c)^4}{b\sin(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(csc(d*x + c)^4/(b*sin(d*x + c)^3 + a), x)
```

3.195 $\int \frac{\sin^9(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=177

$$-\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{9/4}d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{9/4}d\sqrt{\sqrt{a}+\sqrt{b}}}+\frac{(a+b)\cos(c+dx)}{b^2d}+\frac{\cos^5(c+dx)}{5bd}-\frac{2\cos^3(c+dx)}{3bd}$$

```
[Out] -(a^(3/2)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*Sqrt[S
qrt[a] - Sqrt[b]]*b^(9/4)*d) - (a^(3/2)*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt
[Sqrt[a] + Sqrt[b]]])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(9/4)*d) + ((a + b)*Cos[
c + d*x])/(b^2*d) - (2*Cos[c + d*x]^3)/(3*b*d) + Cos[c + d*x]^5/(5*b*d)
```

Rubi [A] time = 0.251947, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3215, 1170, 1093, 205, 208}

$$-\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{9/4}d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{9/4}d\sqrt{\sqrt{a}+\sqrt{b}}}+\frac{(a+b)\cos(c+dx)}{b^2d}+\frac{\cos^5(c+dx)}{5bd}-\frac{2\cos^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^9/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -(a^(3/2)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*Sqrt[S
qrt[a] - Sqrt[b]]*b^(9/4)*d) - (a^(3/2)*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt
[Sqrt[a] + Sqrt[b]]])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(9/4)*d) + ((a + b)*Cos[
c + d*x])/(b^2*d) - (2*Cos[c + d*x]^3)/(3*b*d) + Cos[c + d*x]^5/(5*b*d)
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sin^9(c+dx)}{a-b\sin^4(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^4}{a-b+2bx^2-bx^4}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(-\frac{a+b}{b^2}+\frac{2x^2}{b}-\frac{x^4}{b}+\frac{a^2}{b^2\left(a-b+2bx^2-bx^4\right)}\right)dx, x, \cos(c+dx)\right)}{d}$$

$$= \frac{(a+b)\cos(c+dx)}{b^2d} - \frac{2\cos^3(c+dx)}{3bd} + \frac{\cos^5(c+dx)}{5bd} - \frac{a^2\,\text{Subst}\left(\int \frac{1}{a-b+2bx^2-bx^4}\,dx, x, \cos(c+\right.}{b^2d}$$

$$= \frac{(a+b)\cos(c+dx)}{b^2d} - \frac{2\cos^3(c+dx)}{3bd} + \frac{\cos^5(c+dx)}{5bd} + \frac{a^{3/2}\,\text{Subst}\left(\int \frac{1}{-\sqrt{a}\sqrt{b}+b-bx^2}\,dx, x, \cos(c\right.}{2b^{3/2}d}$$

$$= -\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^{9/4}d} - \frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}\,b^{9/4}d} + \frac{(a+b)\cos(c+dx)}{b^2d} - \frac{2\cos^3(c+dx)}{3bd}$$

Mathematica [C] time = 0.470281, size = 228, normalized size = 1.29

$$\frac{\cos(c+dx)(120a-28b\cos(2(c+dx))+3b\cos(4(c+dx))+89b)+60ia^2\text{RootSum}\left[-16\#1^4a+\#1^8b-4\#1^6b+6\#1^4b-\right.}{120b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^9/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (Cos[c + d*x]*(120*a + 89*b - 28*b*Cos[2*(c + d*x)] + 3*b*Cos[4*(c + d*x)])
 + (60*I)*a^2*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1
^8 & , (-2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 + I*Log[1 - 2*Cos[c
+ d*x]*#1 + #1^2]*#1 + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^3 - I*
Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^3)/(-b - 8*a*#1^2 + 3*b*#1^2 - 3*b*#1^
4 + b*#1^6) & ])/(120*b^2*d)
```

Maple [A] time = 0.116, size = 159, normalized size = 0.9

$$\frac{(\cos(dx+c))^5}{5\,bd} - \frac{2\,(\cos(dx+c))^3}{3\,bd} + \frac{\cos(dx+c)\,a}{b^2d} + \frac{\cos(dx+c)}{bd} - \frac{a^2}{2\,bd}\arctan\left(b\cos(dx+c)\frac{1}{\sqrt{\left(\sqrt{ab}-b\right)b}}\right)\frac{1}{\sqrt{ab}}\frac{}{\sqrt{(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^9/(a-b*sin(d*x+c)^4),x)
```

```
[Out] 1/5*cos(d*x+c)^5/b/d-2/3*cos(d*x+c)^3/b/d+a*cos(d*x+c)/b^2/d+cos(d*x+c)/b/d
-1/2/d*a^2/b/(a*b)^(1/2)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a
*b)^(1/2)-b)*b)^(1/2))-1/2/d*a^2/b/(a*b)^(1/2)/(((a*b)^(1/2)+b)*b)^(1/2)*ar
ctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^9/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] 1/240*(240*b^2*d*integrate(8*(4*a^2*b*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) + 2
*(8*a^3 - 3*a^2*b)*cos(3*d*x + 3*c)*sin(4*d*x + 4*c) - 2*(8*a^3 - 3*a^2*b)*
cos(4*d*x + 4*c)*sin(3*d*x + 3*c) - (a^2*b*sin(5*d*x + 5*c) - a^2*b*sin(3*d
*x + 3*c))*cos(8*d*x + 8*c) + 4*(a^2*b*sin(5*d*x + 5*c) - a^2*b*sin(3*d*x +
 3*c))*cos(6*d*x + 6*c) - 2*(2*a^2*b*sin(2*d*x + 2*c) + (8*a^3 - 3*a^2*b)*s
in(4*d*x + 4*c))*cos(5*d*x + 5*c) + (a^2*b*cos(5*d*x + 5*c) - a^2*b*cos(3*d
*x + 3*c))*sin(8*d*x + 8*c) - 4*(a^2*b*cos(5*d*x + 5*c) - a^2*b*cos(3*d*x +
 3*c))*sin(6*d*x + 6*c) + (4*a^2*b*cos(2*d*x + 2*c) - a^2*b + 2*(8*a^3 - 3*
a^2*b)*cos(4*d*x + 4*c))*sin(5*d*x + 5*c) - (4*a^2*b*cos(2*d*x + 2*c) - a^2
*b)*sin(3*d*x + 3*c))/(b^4*cos(8*d*x + 8*c)^2 + 16*b^4*cos(6*d*x + 6*c)^2 +
 16*b^4*cos(2*d*x + 2*c)^2 + b^4*sin(8*d*x + 8*c)^2 + 16*b^4*sin(6*d*x + 6*
c)^2 + 16*b^4*sin(2*d*x + 2*c)^2 - 8*b^4*cos(2*d*x + 2*c) + b^4 + 4*(64*a^2
*b^2 - 48*a*b^3 + 9*b^4)*cos(4*d*x + 4*c)^2 + 4*(64*a^2*b^2 - 48*a*b^3 + 9*
b^4)*sin(4*d*x + 4*c)^2 + 16*(8*a*b^3 - 3*b^4)*sin(4*d*x + 4*c)*sin(2*d*x +
 2*c) - 2*(4*b^4*cos(6*d*x + 6*c) + 4*b^4*cos(2*d*x + 2*c) - b^4 + 2*(8*a*b
^3 - 3*b^4)*cos(4*d*x + 4*c))*cos(8*d*x + 8*c) + 8*(4*b^4*cos(2*d*x + 2*c)
- b^4 + 2*(8*a*b^3 - 3*b^4)*cos(4*d*x + 4*c))*cos(6*d*x + 6*c) - 4*(8*a*b^3
 - 3*b^4 - 4*(8*a*b^3 - 3*b^4)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 4*(2*b^
4*sin(6*d*x + 6*c) + 2*b^4*sin(2*d*x + 2*c) + (8*a*b^3 - 3*b^4)*sin(4*d*x +
 4*c))*sin(8*d*x + 8*c) + 16*(2*b^4*sin(2*d*x + 2*c) + (8*a*b^3 - 3*b^4)*si
n(4*d*x + 4*c))*sin(6*d*x + 6*c)), x) + 3*b*cos(5*d*x + 5*c) - 25*b*cos(3*d
*x + 3*c) + 30*(8*a + 5*b)*cos(d*x + c))/(b^2*d)
```

Fricas [B] time = 3.01022, size = 1758, normalized size = 9.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^9/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/60*(12*b*cos(d*x + c)^5 - 15*b^2*d*sqrt(-((a*b^4 - b^5)*sqrt(a^7/((a^2*b^
9 - 2*a*b^10 + b^11)*d^4))*d^2 + a^3)/((a*b^4 - b^5)*d^2))*log(a^5*cos(d*x
+ c) + (a^4*b^2*d - (a*b^7 - b^8)*sqrt(a^7/((a^2*b^9 - 2*a*b^10 + b^11)*d^4
))*d^3)*sqrt(-((a*b^4 - b^5)*sqrt(a^7/((a^2*b^9 - 2*a*b^10 + b^11)*d^4))*d^
2 + a^3)/((a*b^4 - b^5)*d^2))) + 15*b^2*d*sqrt(((a*b^4 - b^5)*sqrt(a^7/((a^
2*b^9 - 2*a*b^10 + b^11)*d^4))*d^2 - a^3)/((a*b^4 - b^5)*d^2))*log(a^5*cos(
d*x + c) - (a^4*b^2*d + (a*b^7 - b^8)*sqrt(a^7/((a^2*b^9 - 2*a*b^10 + b^11)
*d^4))*d^3)*sqrt(((a*b^4 - b^5)*sqrt(a^7/((a^2*b^9 - 2*a*b^10 + b^11)*d^4))
*d^2 - a^3)/((a*b^4 - b^5)*d^2))) + 15*b^2*d*sqrt(-((a*b^4 - b^5)*sqrt(a^7/
((a^2*b^9 - 2*a*b^10 + b^11)*d^4))*d^2 + a^3)/((a*b^4 - b^5)*d^2))*log(-a^5
*cos(d*x + c) + (a^4*b^2*d - (a*b^7 - b^8)*sqrt(a^7/((a^2*b^9 - 2*a*b^10 +
```

```
b^11)*d^4))*d^3)*sqrt(-((a*b^4 - b^5)*sqrt(a^7/((a^2*b^9 - 2*a*b^10 + b^11)
*d^4))*d^2 + a^3)/((a*b^4 - b^5)*d^2))) - 15*b^2*d*sqrt(((a*b^4 - b^5)*sqrt
(a^7/((a^2*b^9 - 2*a*b^10 + b^11)*d^4))*d^2 - a^3)/((a*b^4 - b^5)*d^2))*log
(-a^5*cos(d*x + c) - (a^4*b^2*d + (a*b^7 - b^8)*sqrt(a^7/((a^2*b^9 - 2*a*b^
10 + b^11)*d^4))*d^3)*sqrt(((a*b^4 - b^5)*sqrt(a^7/((a^2*b^9 - 2*a*b^10 + b
^11)*d^4))*d^2 - a^3)/((a*b^4 - b^5)*d^2))) - 40*b*cos(d*x + c)^3 + 60*(a +
 b)*cos(d*x + c))/(b^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**9/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^9/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.196 $\int \frac{\sin^7(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=148

$$-\frac{a\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{7/4}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{a\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{7/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\cos^3(c+dx)}{3bd}+\frac{\cos(c+dx)}{bd}$$

```
[Out] -(a*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*Sqrt[Sqrt[a]
 - Sqrt[b]]*b^(7/4)*d) + (a*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + S
qrt[b]]])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(7/4)*d) + Cos[c + d*x]/(b*d) - Cos[
c + d*x]^3/(3*b*d)
```

Rubi [A] time = 0.175588, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3215, 1170, 1166, 205, 208}

$$-\frac{a\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{7/4}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{a\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{7/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\cos^3(c+dx)}{3bd}+\frac{\cos(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^7/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -(a*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*Sqrt[Sqrt[a]
 - Sqrt[b]]*b^(7/4)*d) + (a*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + S
qrt[b]]])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(7/4)*d) + Cos[c + d*x]/(b*d) - Cos[
c + d*x]^3/(3*b*d)
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sin^7(c+dx)}{a-b\sin^4(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{a-b+2bx^2-bx^4}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(-\frac{1}{b}+\frac{x^2}{b}+\frac{a-ax^2}{b\left(a-b+2bx^2-bx^4\right)}\right)dx, x, \cos(c+dx)\right)}{d}$$

$$= \frac{\cos(c+dx)}{bd}-\frac{\cos^3(c+dx)}{3bd}-\frac{\text{Subst}\left(\int \frac{a-ax^2}{a-b+2bx^2-bx^4}\,dx, x, \cos(c+dx)\right)}{bd}$$

$$= \frac{\cos(c+dx)}{bd}-\frac{\cos^3(c+dx)}{3bd}+\frac{a\,\text{Subst}\left(\int \frac{1}{-\sqrt{a}\sqrt{b}+b-bx^2}\,dx, x, \cos(c+dx)\right)}{2bd}+\frac{a\,\text{Subst}\left(\int \frac{1}{\sqrt{a}\sqrt{b}+ \cdots}\right.}{\cdots}$$

$$= -\frac{a\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^{7/4}d}+\frac{a\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}\,b^{7/4}d}+\frac{\cos(c+dx)}{bd}-\frac{\cos^3(c+dx)}{3bd}$$

Mathematica [C] time = 0.283913, size = 310, normalized size = 2.09

$$-3ia\text{RootSum}\left[-16\#1^4a+\#1^8b-4\#1^6b+6\#1^4b-4\#1^2b+b\&, \frac{-i\#1^6\log\left(\#1^2-2\#1\cos(c+dx)+1\right)+3i\#1^4\log\left(\#1^2-2\#1\cos(c+dx)+1\right)-3i \cdots}{\cdots}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]^7/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (18*Cos[c + d*x] - 2*Cos[3*(c + d*x)] - (3*I)*a*RootSum[b - 4*b*#1^2 - 16*a
*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-2*ArcTan[Sin[c + d*x]/(Cos[c + d
*x] - #1)] + I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] + 6*ArcTan[Sin[c + d*x]/(C
os[c + d*x] - #1)]*#1^2 - (3*I)*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - 6*
ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 + (3*I)*Log[1 - 2*Cos[c + d*x
]*#1 + #1^2]*#1^4 + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 - I*Log
[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#
1^5 + b*#1^7) & ])/(24*b*d)
```

Maple [A] time = 0.09, size = 115, normalized size = 0.8

$$-\frac{(\cos(dx+c))^3}{3\,bd}+\frac{\cos(dx+c)}{bd}-\frac{a}{2\,bd}\arctan\left(b\cos(dx+c)\frac{1}{\sqrt{\left(\sqrt{ab}-b\right)b}}\right)\frac{1}{\sqrt{\left(\sqrt{ab}-b\right)b}}+\frac{a}{2\,bd}\text{Artanh}\left(b\cos(dx+ \cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^7/(a-b*sin(d*x+c)^4),x)
```

```
[Out] -1/3*cos(d*x+c)^3/b/d+cos(d*x+c)/b/d-1/2/d*a/b/(((a*b)^(1/2)-b)*b)^(1/2)*ar
ctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))+1/2/d*a/b/(((a*b)^(1/2)+b)*b)^
(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -1/12*(12*b*d*integrate(-2*(12*a*b*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) - 4*a*
b*cos(d*x + c)*sin(2*d*x + 2*c) + 4*a*b*cos(2*d*x + 2*c)*sin(d*x + c) - a*b
*sin(d*x + c) + (a*b*sin(7*d*x + 7*c) - 3*a*b*sin(5*d*x + 5*c) + 3*a*b*sin(
3*d*x + 3*c) - a*b*sin(d*x + c))*cos(8*d*x + 8*c) + 2*(2*a*b*sin(6*d*x + 6*
c) + 2*a*b*sin(2*d*x + 2*c) + (8*a^2 - 3*a*b)*sin(4*d*x + 4*c))*cos(7*d*x +
 7*c) + 4*(3*a*b*sin(5*d*x + 5*c) - 3*a*b*sin(3*d*x + 3*c) + a*b*sin(d*x +
c))*cos(6*d*x + 6*c) - 6*(2*a*b*sin(2*d*x + 2*c) + (8*a^2 - 3*a*b)*sin(4*d*
x + 4*c))*cos(5*d*x + 5*c) - 2*(3*(8*a^2 - 3*a*b)*sin(3*d*x + 3*c) - (8*a^2
 - 3*a*b)*sin(d*x + c))*cos(4*d*x + 4*c) - (a*b*cos(7*d*x + 7*c) - 3*a*b*co
s(5*d*x + 5*c) + 3*a*b*cos(3*d*x + 3*c) - a*b*cos(d*x + c))*sin(8*d*x + 8*c
) - (4*a*b*cos(6*d*x + 6*c) + 4*a*b*cos(2*d*x + 2*c) - a*b + 2*(8*a^2 - 3*a
*b)*cos(4*d*x + 4*c))*sin(7*d*x + 7*c) - 4*(3*a*b*cos(5*d*x + 5*c) - 3*a*b*
cos(3*d*x + 3*c) + a*b*cos(d*x + c))*sin(6*d*x + 6*c) + 3*(4*a*b*cos(2*d*x
+ 2*c) - a*b + 2*(8*a^2 - 3*a*b)*cos(4*d*x + 4*c))*sin(5*d*x + 5*c) + 2*(3*
(8*a^2 - 3*a*b)*cos(3*d*x + 3*c) - (8*a^2 - 3*a*b)*cos(d*x + c))*sin(4*d*x
+ 4*c) - 3*(4*a*b*cos(2*d*x + 2*c) - a*b)*sin(3*d*x + 3*c))/(b^3*cos(8*d*x
+ 8*c)^2 + 16*b^3*cos(6*d*x + 6*c)^2 + 16*b^3*cos(2*d*x + 2*c)^2 + b^3*sin(
8*d*x + 8*c)^2 + 16*b^3*sin(6*d*x + 6*c)^2 + 16*b^3*sin(2*d*x + 2*c)^2 - 8*
b^3*cos(2*d*x + 2*c) + b^3 + 4*(64*a^2*b - 48*a*b^2 + 9*b^3)*cos(4*d*x + 4*
c)^2 + 4*(64*a^2*b - 48*a*b^2 + 9*b^3)*sin(4*d*x + 4*c)^2 + 16*(8*a*b^2 - 3
*b^3)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) - 2*(4*b^3*cos(6*d*x + 6*c) + 4*b^3
*cos(2*d*x + 2*c) - b^3 + 2*(8*a*b^2 - 3*b^3)*cos(4*d*x + 4*c))*cos(8*d*x +
 8*c) + 8*(4*b^3*cos(2*d*x + 2*c) - b^3 + 2*(8*a*b^2 - 3*b^3)*cos(4*d*x + 4
*c))*cos(6*d*x + 6*c) - 4*(8*a*b^2 - 3*b^3 - 4*(8*a*b^2 - 3*b^3)*cos(2*d*x
+ 2*c))*cos(4*d*x + 4*c) - 4*(2*b^3*sin(6*d*x + 6*c) + 2*b^3*sin(2*d*x + 2*
c) + (8*a*b^2 - 3*b^3)*sin(4*d*x + 4*c))*sin(8*d*x + 8*c) + 16*(2*b^3*sin(2
*d*x + 2*c) + (8*a*b^2 - 3*b^3)*sin(4*d*x + 4*c))*sin(6*d*x + 6*c)), x) + c
os(3*d*x + 3*c) - 9*cos(d*x + c))/(b*d)
```

Fricas [B] time = 2.76169, size = 1662, normalized size = 11.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b*d*sqrt(-((a*b^3 - b^4)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^
4)) + a^2)/((a*b^3 - b^4)*d^2))*log(a^3*cos(d*x + c) + (a^2*b^2*d - (a*b^5
```

```
- b^6)*d^3*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)))*sqrt(-((a*b^3 - b^4)*
d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) + a^2)/((a*b^3 - b^4)*d^2)))
- 3*b*d*sqrt(((a*b^3 - b^4)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) -
 a^2)/((a*b^3 - b^4)*d^2))*log(a^3*cos(d*x + c) - (a^2*b^2*d + (a*b^5 - b^6
)*d^3*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)))*sqrt(((a*b^3 - b^4)*d^2*sq
rt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) - a^2)/((a*b^3 - b^4)*d^2))) - 3*b*
d*sqrt(-((a*b^3 - b^4)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) + a^2)
/((a*b^3 - b^4)*d^2))*log(-a^3*cos(d*x + c) + (a^2*b^2*d - (a*b^5 - b^6)*d^
3*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)))*sqrt(-((a*b^3 - b^4)*d^2*sqrt(
a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) + a^2)/((a*b^3 - b^4)*d^2))) + 3*b*d*s
qrt(((a*b^3 - b^4)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) - a^2)/((a
*b^3 - b^4)*d^2))*log(-a^3*cos(d*x + c) - (a^2*b^2*d + (a*b^5 - b^6)*d^3*sq
rt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)))*sqrt(((a*b^3 - b^4)*d^2*sqrt(a^5/(
(a^2*b^7 - 2*a*b^8 + b^9)*d^4)) - a^2)/((a*b^3 - b^4)*d^2))) - 4*cos(d*x +
c)^3 + 12*cos(d*x + c))/(b*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**7/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.197 $\int \frac{\sin^5(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=138

$$-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{5/4}d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{5/4}d\sqrt{\sqrt{a}+\sqrt{b}}}+\frac{\cos(c+dx)}{bd}$$

```
[Out] -(Sqrt[a]*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*Sqrt[S
qrt[a] - Sqrt[b]]*b^(5/4)*d) - (Sqrt[a]*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt
[Sqrt[a] + Sqrt[b]]])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(5/4)*d) + Cos[c + d*x]/
(b*d)
```

Rubi [A] time = 0.178527, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3215, 1170, 1093, 205, 208}

$$-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{5/4}d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{5/4}d\sqrt{\sqrt{a}+\sqrt{b}}}+\frac{\cos(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^5/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -(Sqrt[a]*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*Sqrt[S
qrt[a] - Sqrt[b]]*b^(5/4)*d) - (Sqrt[a]*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt
[Sqrt[a] + Sqrt[b]]])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(5/4)*d) + Cos[c + d*x]/
(b*d)
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sin^5(c+dx)}{a-b\sin^4(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{a-b+2bx^2-bx^4}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(-\frac{1}{b}+\frac{a}{b(a-b+2bx^2-bx^4)}\right)dx, x, \cos(c+dx)\right)}{d}$$
$$= \frac{\cos(c+dx)}{bd} - \frac{a\,\text{Subst}\left(\int \frac{1}{a-b+2bx^2-bx^4}\,dx, x, \cos(c+dx)\right)}{bd}$$
$$= \frac{\cos(c+dx)}{bd} + \frac{\sqrt{a}\,\text{Subst}\left(\int \frac{1}{-\sqrt{a}\sqrt{b}+b-bx^2}\,dx, x, \cos(c+dx)\right)}{2\sqrt{b}d} - \frac{\sqrt{a}\,\text{Subst}\left(\int \frac{1}{\sqrt{a}\sqrt{b}+b-bx^2}\,dx, x, c\right.}{2\sqrt{b}d}$$
$$= -\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}b^{5/4}d} - \frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}b^{5/4}d} + \frac{\cos(c+dx)}{bd}$$

Mathematica [C] time = 0.250087, size = 198, normalized size = 1.43

$$\frac{2\cos(c+dx) + ia\text{RootSum}\left[-16\#1^4a + \#1^8b - 4\#1^6b + 6\#1^4b - 4\#1^2b + b\&, \frac{-i\#1^3\log(\#1^2-2\#1\cos(c+dx)+1)+i\#1\log(\#1^2-2\#1\text{co}}{-8\#1^2a+\#}\right.}{2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^5/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (2*Cos[c + d*x] + I*a*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^
6 + b*#1^8 & , (-2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 + I*Log[1 -
2*Cos[c + d*x]*#1 + #1^2]*#1 + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#
1^3 - I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^3)/(-b - 8*a*#1^2 + 3*b*#1^2 -
 3*b*#1^4 + b*#1^6) & ])/(2*b*d)
```

Maple [A] time = 0.085, size = 103, normalized size = 0.8

$$\frac{\cos(dx+c)}{bd} - \frac{a}{2d}\arctan\left(b\cos(dx+c)\frac{1}{\sqrt{(\sqrt{ab}-b)b}}\right)\frac{1}{\sqrt{ab}}\frac{1}{\sqrt{(\sqrt{ab}-b)b}} - \frac{a}{2d}\text{Artanh}\left(b\cos(dx+c)\frac{1}{\sqrt{(\sqrt{ab}+b)b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5/(a-b*sin(d*x+c)^4),x)
```

```
[Out] cos(d*x+c)/b/d-1/2/d*a/(a*b)^(1/2)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x
+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))-1/2/d*a/(a*b)^(1/2)/(((a*b)^(1/2)+b)*b)^(1
/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] (b*d*integrate(8*(4*a*b*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) + 2*(8*a^2 - 3*a*
b)*cos(3*d*x + 3*c)*sin(4*d*x + 4*c) - 2*(8*a^2 - 3*a*b)*cos(4*d*x + 4*c)*s
in(3*d*x + 3*c) - (a*b*sin(5*d*x + 5*c) - a*b*sin(3*d*x + 3*c))*cos(8*d*x +
 8*c) + 4*(a*b*sin(5*d*x + 5*c) - a*b*sin(3*d*x + 3*c))*cos(6*d*x + 6*c) -
2*(2*a*b*sin(2*d*x + 2*c) + (8*a^2 - 3*a*b)*sin(4*d*x + 4*c))*cos(5*d*x + 5
*c) + (a*b*cos(5*d*x + 5*c) - a*b*cos(3*d*x + 3*c))*sin(8*d*x + 8*c) - 4*(a
*b*cos(5*d*x + 5*c) - a*b*cos(3*d*x + 3*c))*sin(6*d*x + 6*c) + (4*a*b*cos(2
*d*x + 2*c) - a*b + 2*(8*a^2 - 3*a*b)*cos(4*d*x + 4*c))*sin(5*d*x + 5*c) -
(4*a*b*cos(2*d*x + 2*c) - a*b)*sin(3*d*x + 3*c))/(b^3*cos(8*d*x + 8*c)^2 +
16*b^3*cos(6*d*x + 6*c)^2 + 16*b^3*cos(2*d*x + 2*c)^2 + b^3*sin(8*d*x + 8*c
)^2 + 16*b^3*sin(6*d*x + 6*c)^2 + 16*b^3*sin(2*d*x + 2*c)^2 - 8*b^3*cos(2*d
*x + 2*c) + b^3 + 4*(64*a^2*b - 48*a*b^2 + 9*b^3)*cos(4*d*x + 4*c)^2 + 4*(6
4*a^2*b - 48*a*b^2 + 9*b^3)*sin(4*d*x + 4*c)^2 + 16*(8*a*b^2 - 3*b^3)*sin(4
*d*x + 4*c)*sin(2*d*x + 2*c) - 2*(4*b^3*cos(6*d*x + 6*c) + 4*b^3*cos(2*d*x
+ 2*c) - b^3 + 2*(8*a*b^2 - 3*b^3)*cos(4*d*x + 4*c))*cos(8*d*x + 8*c) + 8*(
4*b^3*cos(2*d*x + 2*c) - b^3 + 2*(8*a*b^2 - 3*b^3)*cos(4*d*x + 4*c))*cos(6*
d*x + 6*c) - 4*(8*a*b^2 - 3*b^3 - 4*(8*a*b^2 - 3*b^3)*cos(2*d*x + 2*c))*cos
(4*d*x + 4*c) - 4*(2*b^3*sin(6*d*x + 6*c) + 2*b^3*sin(2*d*x + 2*c) + (8*a*b
^2 - 3*b^3)*sin(4*d*x + 4*c))*sin(8*d*x + 8*c) + 16*(2*b^3*sin(2*d*x + 2*c)
 + (8*a*b^2 - 3*b^3)*sin(4*d*x + 4*c))*sin(6*d*x + 6*c)), x) + cos(d*x + c)
)/(b*d)
```

Fricas [B] time = 2.91648, size = 1592, normalized size = 11.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/4*(b*d*sqrt(-((a*b^2 - b^3)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)
) + a)/((a*b^2 - b^3)*d^2))*log(a^2*cos(d*x + c) - ((a*b^4 - b^5)*d^3*sqrt(
a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - a^2*b*d)*sqrt(-((a*b^2 - b^3)*d^2*sq
rt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) + a)/((a*b^2 - b^3)*d^2))) - b*d*sq
rt(((a*b^2 - b^3)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - a)/((a*b^
2 - b^3)*d^2))*log(a^2*cos(d*x + c) - ((a*b^4 - b^5)*d^3*sqrt(a^3/((a^2*b^5
 - 2*a*b^6 + b^7)*d^4)) + a^2*b*d)*sqrt(((a*b^2 - b^3)*d^2*sqrt(a^3/((a^2*b
^5 - 2*a*b^6 + b^7)*d^4)) - a)/((a*b^2 - b^3)*d^2))) - b*d*sqrt(-((a*b^2 -
b^3)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) + a)/((a*b^2 - b^3)*d^2)
)*log(-a^2*cos(d*x + c) - ((a*b^4 - b^5)*d^3*sqrt(a^3/((a^2*b^5 - 2*a*b^6 +
 b^7)*d^4)) - a^2*b*d)*sqrt(-((a*b^2 - b^3)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^
6 + b^7)*d^4)) + a)/((a*b^2 - b^3)*d^2))) + b*d*sqrt(((a*b^2 - b^3)*d^2*sqr
t(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - a)/((a*b^2 - b^3)*d^2))*log(-a^2*c
```

```
os(d*x + c) - ((a*b^4 - b^5)*d^3*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4))
+ a^2*b*d)*sqrt(((a*b^2 - b^3)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)
) - a)/((a*b^2 - b^3)*d^2))) - 4*cos(d*x + c))/(b*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.198 $\int \frac{\sin^3(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=115

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{3/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{3/4}d\sqrt{\sqrt{a}-\sqrt{b}}}$$

```
[Out] -ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]]/(2*Sqrt[Sqrt[a] - S
qrt[b]]*b^(3/4)*d) + ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]
]/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(3/4)*d)
```

Rubi [A] time = 0.117166, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3215, 1166, 205, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{3/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{3/4}d\sqrt{\sqrt{a}-\sqrt{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]]/(2*Sqrt[Sqrt[a] - S
qrt[b]]*b^(3/4)*d) + ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]
]/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(3/4)*d)
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{a-b\sin^4(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{a-b+2bx^2-bx^4}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{-\sqrt{a}\sqrt{b}+b-bx^2}\,dx, x, \cos(c+dx)\right)}{2d} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a}\sqrt{b}+b-bx^2}\,dx, x, \cos(c+dx)\right)}{2d}$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^{3/4}d} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}\,b^{3/4}d}$$

Mathematica [C] time = 0.170779, size = 285, normalized size = 2.48

$$-\frac{i\text{RootSum}\left[-16\#1^4a+\#1^8b-4\#1^6b+6\#1^4b-4\#1^2b+b\&, \frac{-i\#1^6\log\left(\#1^2-2\#1\cos(c+dx)+1\right)+3i\#1^4\log\left(\#1^2-2\#1\cos(c+dx)+1\right)-3i\#1^2}{\ldots}\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] ((-I/8)*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & ,
 (-2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + I*Log[1 - 2*Cos[c + d*x]*#1
 + #1^2] + 6*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 - (3*I)*Log[1 -
2*Cos[c + d*x]*#1 + #1^2]*#1^2 - 6*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]
*#1^4 + (3*I)*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 + 2*ArcTan[Sin[c + d*x
]/(Cos[c + d*x] - #1)]*#1^6 - I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6)/(-(
b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/d
```

Maple [A] time = 0.085, size = 78, normalized size = 0.7

$$-\frac{1}{2d}\arctan\left(b\cos(dx+c)\frac{1}{\sqrt{\left(\sqrt{ab}-b\right)b}}\right)\frac{1}{\sqrt{\left(\sqrt{ab}-b\right)b}}+\frac{1}{2d}\text{Artanh}\left(b\cos(dx+c)\frac{1}{\sqrt{\left(\sqrt{ab}+b\right)b}}\right)\frac{1}{\sqrt{\left(\sqrt{ab}+b\right)b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a-b*sin(d*x+c)^4),x)
```

```
[Out] -1/2/d/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1
/2))+1/2/d/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*
b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sin(dx+c)^3}{b\sin(dx+c)^4-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -integrate(sin(d*x + c)^3/(b*sin(d*x + c)^4 - a), x)
```

Fricas [B] time = 2.41604, size = 1435, normalized size = 12.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(-((a*b - b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + 1)/((a
*b - b^2)*d^2))*log(-((a*b^2 - b^3)*d^3*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d
^4)) - b*d)*sqrt(-((a*b - b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4))
+ 1)/((a*b - b^2)*d^2)) + cos(d*x + c)) - 1/4*sqrt(-((a*b - b^2)*d^2*sqrt(a
/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + 1)/((a*b - b^2)*d^2))*log(-((a*b^2 - b^
3)*d^3*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - b*d)*sqrt(-((a*b - b^2)*d^
2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + 1)/((a*b - b^2)*d^2)) - cos(d*x
 + c)) - 1/4*sqrt(((a*b - b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4))
- 1)/((a*b - b^2)*d^2))*log(-((a*b^2 - b^3)*d^3*sqrt(a/((a^2*b^3 - 2*a*b^4
+ b^5)*d^4)) + b*d)*sqrt(((a*b - b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)
*d^4)) - 1)/((a*b - b^2)*d^2)) + cos(d*x + c)) + 1/4*sqrt(((a*b - b^2)*d^2*
sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - 1)/((a*b - b^2)*d^2))*log(-((a*b^
2 - b^3)*d^3*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + b*d)*sqrt(((a*b - b^
2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - 1)/((a*b - b^2)*d^2)) - co
s(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.199 $\int \frac{\sin(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=125

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{a}\sqrt[4]{b}d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{a}\sqrt[4]{b}d\sqrt{\sqrt{a}+\sqrt{b}}}$$

```
[Out] -ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]]/(2*Sqrt[a]*Sqrt[Sqr
t[a] - Sqrt[b]]*b^(1/4)*d) - ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] +
Sqrt[b]]]/(2*Sqrt[a]*Sqrt[Sqrt[a] + Sqrt[b]]*b^(1/4)*d)
```

Rubi [A] time = 0.102077, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3215, 1093, 205, 208}

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{a}\sqrt[4]{b}d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{a}\sqrt[4]{b}d\sqrt{\sqrt{a}+\sqrt{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]]/(2*Sqrt[a]*Sqrt[Sqr
t[a] - Sqrt[b]]*b^(1/4)*d) - ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] +
Sqrt[b]]]/(2*Sqrt[a]*Sqrt[Sqrt[a] + Sqrt[b]]*b^(1/4)*d)
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(c+dx)}{a-b\sin^4(c+dx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{a-b+2bx^2-bx^4}\,dx,x,\cos(c+dx)\right)}{d}\\ &= \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{1}{-\sqrt{a}\sqrt{b}+b-bx^2}\,dx,x,\cos(c+dx)\right)}{2\sqrt{a}d} - \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{1}{\sqrt{a}\sqrt{b}+b-bx^2}\,dx,x,\cos(c+dx)\right)}{2\sqrt{a}d}\\ &= -\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}-\sqrt{b}}\sqrt[4]{b}d} - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}+\sqrt{b}}\sqrt[4]{b}d}\end{aligned}$$

Mathematica [C] time = 0.162016, size = 183, normalized size = 1.46

$$\frac{i\text{RootSum}\left[-16\#1^4a+\#1^8b-4\#1^6b+6\#1^4b-4\#1^2b+b\&,\frac{-i\#1^3\log\left(\#1^2-2\#1\cos(c+dx)+1\right)+i\#1\log\left(\#1^2-2\#1\cos(c+dx)+1\right)+2\#1^3}{-8\#1^2a+\#1^6b-3\#1^4b+3\#1^2b}\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] ((I/2)*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & ,
(-2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 + I*Log[1 - 2*Cos[c + d*x]*
#1 + #1^2]*#1 + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^3 - I*Log[1 -
 2*Cos[c + d*x]*#1 + #1^2]*#1^3)/(-b - 8*a*#1^2 + 3*b*#1^2 - 3*b*#1^4 + b*#
1^6) & ])/d
```

Maple [A] time = 0.095, size = 90, normalized size = 0.7

$$-\frac{b}{2d}\arctan\left(b\cos(dx+c)\frac{1}{\sqrt{\left(\sqrt{ab}-b\right)b}}\right)\frac{1}{\sqrt{ab}}\frac{1}{\sqrt{\left(\sqrt{ab}-b\right)b}}-\frac{b}{2d}\text{Artanh}\left(b\cos(dx+c)\frac{1}{\sqrt{\left(\sqrt{ab}+b\right)b}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a-b*sin(d*x+c)^4),x)
```

```
[Out] -1/2*b/d/(a*b)^(1/2)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^
(1/2)-b)*b)^(1/2))-1/2*b/d/(a*b)^(1/2)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(co
s(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sin(dx+c)}{b\sin(dx+c)^4-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -integrate(sin(d*x + c)/(b*sin(d*x + c)^4 - a), x)
```

Fricas [B] time = 2.56133, size = 1480, normalized size = 11.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-((a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1)/
((a^2 - a*b)*d^2))*log(-((a^2*b - a*b^2)*d^3*sqrt(1/((a^3*b - 2*a^2*b^2 + a
*b^3)*d^4)) - a*d)*sqrt(-((a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^
3)*d^4)) + 1)/((a^2 - a*b)*d^2)) + cos(d*x + c)) + 1/4*sqrt(-((a^2 - a*b)*d
^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1)/((a^2 - a*b)*d^2))*log(-(
(a^2*b - a*b^2)*d^3*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - a*d)*sqrt(-
((a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1)/((a^2 - a*b
)*d^2)) - cos(d*x + c)) + 1/4*sqrt(((a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*
b^2 + a*b^3)*d^4)) - 1)/((a^2 - a*b)*d^2))*log(-((a^2*b - a*b^2)*d^3*sqrt(1
/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + a*d)*sqrt(((a^2 - a*b)*d^2*sqrt(1/((a
^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1)/((a^2 - a*b)*d^2)) + cos(d*x + c)) - 1
/4*sqrt(((a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1)/((a
^2 - a*b)*d^2))*log(-((a^2*b - a*b^2)*d^3*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^
3)*d^4)) + a*d)*sqrt(((a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d
^4)) - 1)/((a^2 - a*b)*d^2)) - cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.200 $\int \frac{\csc(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=136

$$-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2ad\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2ad\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

```
[Out] -(b^(1/4)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*a*Sqrt
[Sqrt[a] - Sqrt[b]]*d) - ArcTanh[Cos[c + d*x]]/(a*d) + (b^(1/4)*ArcTanh[(b^
(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*a*Sqrt[Sqrt[a] + Sqrt[b]]*
d)
```

Rubi [A] time = 0.178445, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3215, 1170, 207, 1166, 205, 208}

$$-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2ad\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2ad\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a - b*Sin[c + d*x]^4),x]
```


```
[Out] -(b^(1/4)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*a*Sqrt
[Sqrt[a] - Sqrt[b]]*d) - ArcTanh[Cos[c + d*x]]/(a*d) + (b^(1/4)*ArcTanh[(b^
(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*a*Sqrt[Sqrt[a] + Sqrt[b]]*
d)
```


Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\csc(c+dx)}{a-b\sin^4(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a-b+2bx^2-bx^4)}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(-\frac{1}{a(-1+x^2)}+\frac{b-bx^2}{a(a-b+2bx^2-bx^4)}\right)dx, x, \cos(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \cos(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int \frac{b-bx^2}{a-b+2bx^2-bx^4}\,dx, x, \cos(c+dx)\right)}{ad}$$
$$= -\frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{b\,\text{Subst}\left(\int \frac{1}{-\sqrt{a}\sqrt{b}+b-bx^2}\,dx, x, \cos(c+dx)\right)}{2ad} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a}\sqrt{b}+b-bx^2}\right.}{2ad}$$
$$= -\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a\sqrt{\sqrt{a}-\sqrt{b}}\,d} - \frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a\sqrt{\sqrt{a}+\sqrt{b}}\,d}$$

Mathematica [C] time = 0.250389, size = 318, normalized size = 2.34

$$-\frac{ib\text{RootSum}\left[-16\#1^4a+\#1^8b-4\#1^6b+6\#1^4b-4\#1^2b+b\&, \frac{-i\#1^6\log\left(\#1^2-2\#1\cos(c+dx)+1\right)+3i\#1^4\log\left(\#1^2-2\#1\cos(c+dx)+1\right)-3i\#1\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -(8*Log[Cos[(c + d*x)/2]] - 8*Log[Sin[(c + d*x)/2]] + I*b*RootSum[b - 4*b*#
1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-2*ArcTan[Sin[c + d*x]/
(Cos[c + d*x] - #1)] + I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] + 6*ArcTan[Sin[c
 + d*x]/(Cos[c + d*x] - #1)]*#1^2 - (3*I)*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]
*#1^2 - 6*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 + (3*I)*Log[1 - 2*C
os[c + d*x]*#1 + #1^2]*#1^4 + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1
^6 - I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1
^3 - 3*b*#1^5 + b*#1^7) & ])/(8*a*d)
```

Maple [A] time = 0.121, size = 120, normalized size = 0.9

$$\frac{\ln\left(-1+\cos\left(dx+c\right)\right)}{2\,da}-\frac{\ln\left(1+\cos\left(dx+c\right)\right)}{2\,da}-\frac{b}{2\,da}\arctan\left(b\cos\left(dx+c\right)\frac{1}{\sqrt{\left(\sqrt{ab}-b\right)b}}\right)\frac{1}{\sqrt{\left(\sqrt{ab}-b\right)b}}+\frac{b}{2\,da}\mathrm{Arta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a-b*sin(d*x+c)^4),x)
```

```
[Out] 1/2/d/a*ln(-1+cos(d*x+c))-1/2/d/a*ln(1+cos(d*x+c))-1/2/d/a*b/(((a*b)^(1/2)-
b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))+1/2/d/a*b/(((a*b
)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*d*integrate(-2*(12*b^2*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) - 4*b^2*
cos(d*x + c)*sin(2*d*x + 2*c) + 4*b^2*cos(2*d*x + 2*c)*sin(d*x + c) - b^2*s
in(d*x + c) + (b^2*sin(7*d*x + 7*c) - 3*b^2*sin(5*d*x + 5*c) + 3*b^2*sin(3*
d*x + 3*c) - b^2*sin(d*x + c))*cos(8*d*x + 8*c) + 2*(2*b^2*sin(6*d*x + 6*c)
 + 2*b^2*sin(2*d*x + 2*c) + (8*a*b - 3*b^2)*sin(4*d*x + 4*c))*cos(7*d*x + 7
*c) + 4*(3*b^2*sin(5*d*x + 5*c) - 3*b^2*sin(3*d*x + 3*c) + b^2*sin(d*x + c)
)*cos(6*d*x + 6*c) - 6*(2*b^2*sin(2*d*x + 2*c) + (8*a*b - 3*b^2)*sin(4*d*x
+ 4*c))*cos(5*d*x + 5*c) - 2*(3*(8*a*b - 3*b^2)*sin(3*d*x + 3*c) - (8*a*b -
 3*b^2)*sin(d*x + c))*cos(4*d*x + 4*c) - (b^2*cos(7*d*x + 7*c) - 3*b^2*cos(
5*d*x + 5*c) + 3*b^2*cos(3*d*x + 3*c) - b^2*cos(d*x + c))*sin(8*d*x + 8*c)
- (4*b^2*cos(6*d*x + 6*c) + 4*b^2*cos(2*d*x + 2*c) - b^2 + 2*(8*a*b - 3*b^2
)*cos(4*d*x + 4*c))*sin(7*d*x + 7*c) - 4*(3*b^2*cos(5*d*x + 5*c) - 3*b^2*co
s(3*d*x + 3*c) + b^2*cos(d*x + c))*sin(6*d*x + 6*c) + 3*(4*b^2*cos(2*d*x +
2*c) - b^2 + 2*(8*a*b - 3*b^2)*cos(4*d*x + 4*c))*sin(5*d*x + 5*c) + 2*(3*(8
*a*b - 3*b^2)*cos(3*d*x + 3*c) - (8*a*b - 3*b^2)*cos(d*x + c))*sin(4*d*x +
4*c) - 3*(4*b^2*cos(2*d*x + 2*c) - b^2)*sin(3*d*x + 3*c))/(a*b^2*cos(8*d*x
+ 8*c)^2 + 16*a*b^2*cos(6*d*x + 6*c)^2 + 16*a*b^2*cos(2*d*x + 2*c)^2 + a*b^
2*sin(8*d*x + 8*c)^2 + 16*a*b^2*sin(6*d*x + 6*c)^2 + 16*a*b^2*sin(2*d*x + 2
*c)^2 - 8*a*b^2*cos(2*d*x + 2*c) + a*b^2 + 4*(64*a^3 - 48*a^2*b + 9*a*b^2)*
cos(4*d*x + 4*c)^2 + 4*(64*a^3 - 48*a^2*b + 9*a*b^2)*sin(4*d*x + 4*c)^2 + 1
6*(8*a^2*b - 3*a*b^2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) - 2*(4*a*b^2*cos(6*
d*x + 6*c) + 4*a*b^2*cos(2*d*x + 2*c) - a*b^2 + 2*(8*a^2*b - 3*a*b^2)*cos(4
*d*x + 4*c))*cos(8*d*x + 8*c) + 8*(4*a*b^2*cos(2*d*x + 2*c) - a*b^2 + 2*(8*
a^2*b - 3*a*b^2)*cos(4*d*x + 4*c))*cos(6*d*x + 6*c) - 4*(8*a^2*b - 3*a*b^2
- 4*(8*a^2*b - 3*a*b^2)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 4*(2*a*b^2*sin
(6*d*x + 6*c) + 2*a*b^2*sin(2*d*x + 2*c) + (8*a^2*b - 3*a*b^2)*sin(4*d*x +
4*c))*sin(8*d*x + 8*c) + 16*(2*a*b^2*sin(2*d*x + 2*c) + (8*a^2*b - 3*a*b^2)
*sin(4*d*x + 4*c))*sin(6*d*x + 6*c)), x) + log(cos(d*x)^2 + 2*cos(d*x)*cos(
c) + cos(c)^2 + sin(d*x)^2 - 2*sin(d*x)*sin(c) + sin(c)^2) - log(cos(d*x)^2
 - 2*cos(d*x)*cos(c) + cos(c)^2 + sin(d*x)^2 + 2*sin(d*x)*sin(c) + sin(c)^2
))/(a*d)
```

Fricas [B] time = 3.18046, size = 1601, normalized size = 11.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/4*(a*d*sqrt(-((a^3 - a^2*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) +
 b)/((a^3 - a^2*b)*d^2))*log(b*cos(d*x + c) - ((a^4 - a^3*b)*d^3*sqrt(b/((a
^5 - 2*a^4*b + a^3*b^2)*d^4)) - a*b*d)*sqrt(-((a^3 - a^2*b)*d^2*sqrt(b/((a^
5 - 2*a^4*b + a^3*b^2)*d^4)) + b)/((a^3 - a^2*b)*d^2))) - a*d*sqrt(((a^3 -
a^2*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - b)/((a^3 - a^2*b)*d^2)
)*log(b*cos(d*x + c) - ((a^4 - a^3*b)*d^3*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)
*d^4)) + a*b*d)*sqrt(((a^3 - a^2*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d
^4)) - b)/((a^3 - a^2*b)*d^2))) - a*d*sqrt(-((a^3 - a^2*b)*d^2*sqrt(b/((a^5
 - 2*a^4*b + a^3*b^2)*d^4)) + b)/((a^3 - a^2*b)*d^2))*log(-b*cos(d*x + c) -
 ((a^4 - a^3*b)*d^3*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - a*b*d)*sqrt(-
((a^3 - a^2*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + b)/((a^3 - a^2
*b)*d^2))) + a*d*sqrt(((a^3 - a^2*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*
d^4)) - b)/((a^3 - a^2*b)*d^2))*log(-b*cos(d*x + c) - ((a^4 - a^3*b)*d^3*sq
rt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + a*b*d)*sqrt(((a^3 - a^2*b)*d^2*sqrt
(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - b)/((a^3 - a^2*b)*d^2))) - 2*log(1/2*
cos(d*x + c) + 1/2) + 2*log(-1/2*cos(d*x + c) + 1/2))/(a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.201 $\int \frac{\csc^3(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=184

$$-\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{3/2}d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{3/2}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{1}{4ad(1-\cos(c+dx))}+\frac{1}{4ad(\cos(c+dx)+1)}-\frac{\tanh^{-1}(\cos(c}{2ad}$$

```
[Out] -(b^(3/4)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*a^(3/2
)*Sqrt[Sqrt[a] - Sqrt[b]]*d) - ArcTanh[Cos[c + d*x]]/(2*a*d) - (b^(3/4)*Arc
Tanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*a^(3/2)*Sqrt[Sqrt[
a] + Sqrt[b]]*d) - 1/(4*a*d*(1 - Cos[c + d*x])) + 1/(4*a*d*(1 + Cos[c + d*x
]))
```

Rubi [A] time = 0.209986, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3215, 1170, 207, 1093, 205, 208}

$$-\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{3/2}d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{3/2}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{1}{4ad(1-\cos(c+dx))}+\frac{1}{4ad(\cos(c+dx)+1)}-\frac{\tanh^{-1}(\cos(c}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -(b^(3/4)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*a^(3/2
)*Sqrt[Sqrt[a] - Sqrt[b]]*d) - ArcTanh[Cos[c + d*x]]/(2*a*d) - (b^(3/4)*Arc
Tanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*a^(3/2)*Sqrt[Sqrt[
a] + Sqrt[b]]*d) - 1/(4*a*d*(1 - Cos[c + d*x])) + 1/(4*a*d*(1 + Cos[c + d*x
]))
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)}{a-b\sin^4(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2(a-b+2bx^2-bx^4)}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{4a(-1+x)^2}+\frac{1}{4a(1+x)^2}-\frac{1}{2a(-1+x^2)}+\frac{b}{a(a-b+2bx^2-bx^4)}\right) dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{1}{4ad(1-\cos(c+dx))}+\frac{1}{4ad(1+\cos(c+dx))}+\frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \cos(c+dx)\right)}{2ad} - \frac{b\,\text{Subs}\ldots}{\ldots}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{2ad}-\frac{1}{4ad(1-\cos(c+dx))}+\frac{1}{4ad(1+\cos(c+dx))}+\frac{b^{3/2}\,\text{Subst}\left(\int \frac{1}{-\sqrt{a}\sqrt{b}+\ldots}\right.}{2\ldots}$$

$$= -\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{3/2}\sqrt{\sqrt{a}-\sqrt{b}}\,d}-\frac{\tanh^{-1}(\cos(c+dx))}{2ad}-\frac{b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{3/2}\sqrt{\sqrt{a}+\sqrt{b}}\,d}-\frac{1}{4ad(1-\cos(c\ldots}$$

Mathematica [C] time = 0.316901, size = 242, normalized size = 1.32

$$\frac{4ib\text{RootSum}\left[-16\#1^4a+\#1^8b-4\#1^6b+6\#1^4b-4\#1^2b+b\&,\frac{-i\#1^3\log\left(\#1^2-2\#1\cos(c+dx)+1\right)+i\#1\log\left(\#1^2-2\#1\cos(c+dx)+1\right)+2\#1^3\,\text{t}\ldots}{-8\#1^2a+\#1^6b-3\#1^4b+3\#1^2b-\ldots}\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (-Csc[(c + d*x)/2]^2 - 4*Log[Cos[(c + d*x)/2]] + 4*Log[Sin[(c + d*x)/2]] +
(4*I)*b*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & ,
 (-2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 + I*Log[1 - 2*Cos[c + d*x]
*#1 + #1^2]*#1 + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^3 - I*Log[1
- 2*Cos[c + d*x]*#1 + #1^2]*#1^3)/(-b - 8*a*#1^2 + 3*b*#1^2 - 3*b*#1^4 + b*
#1^6) & ] + Sec[(c + d*x)/2]^2)/(8*a*d)
```

Maple [A] time = 0.132, size = 170, normalized size = 0.9

$$\frac{1}{4\,da\,(-1+\cos(dx+c))}+\frac{\ln(-1+\cos(dx+c))}{4\,da}+\frac{1}{4\,da\,(1+\cos(dx+c))}-\frac{\ln(1+\cos(dx+c))}{4\,da}-\frac{b^2}{2\,da}\arctan\left(b\,\text{co}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(a-b*sin(d*x+c)^4),x)
```

```
[Out] 1/4/d/a/(-1+cos(d*x+c))+1/4/d/a*ln(-1+cos(d*x+c))+1/4/a/d/(1+cos(d*x+c))-1/
4/d/a*ln(1+cos(d*x+c))-1/2/d/a*b^2/(a*b)^(1/2)/(((a*b)^(1/2)-b)*b)^(1/2)*ar
ctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))-1/2/d/a*b^2/(a*b)^(1/2)/(((a*b
)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(cos(3*d*x + 3*c) + cos(d*x + c))*cos(4*d*x + 4*c) - 4*(2*cos(2*d*x
+ 2*c) - 1)*cos(3*d*x + 3*c) - 8*cos(2*d*x + 2*c)*cos(d*x + c) + 4*(a*d*cos
(4*d*x + 4*c)^2 + 4*a*d*cos(2*d*x + 2*c)^2 + a*d*sin(4*d*x + 4*c)^2 - 4*a*d
*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*a*d*sin(2*d*x + 2*c)^2 - 4*a*d*cos(2
*d*x + 2*c) + a*d - 2*(2*a*d*cos(2*d*x + 2*c) - a*d)*cos(4*d*x + 4*c))*inte
grate(8*(4*b^2*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) + 2*(8*a*b - 3*b^2)*cos(3*
d*x + 3*c)*sin(4*d*x + 4*c) - 2*(8*a*b - 3*b^2)*cos(4*d*x + 4*c)*sin(3*d*x
+ 3*c) - (b^2*sin(5*d*x + 5*c) - b^2*sin(3*d*x + 3*c))*cos(8*d*x + 8*c) + 4
*(b^2*sin(5*d*x + 5*c) - b^2*sin(3*d*x + 3*c))*cos(6*d*x + 6*c) - 2*(2*b^2*
sin(2*d*x + 2*c) + (8*a*b - 3*b^2)*sin(4*d*x + 4*c))*cos(5*d*x + 5*c) + (b^
2*cos(5*d*x + 5*c) - b^2*cos(3*d*x + 3*c))*sin(8*d*x + 8*c) - 4*(b^2*cos(5*
d*x + 5*c) - b^2*cos(3*d*x + 3*c))*sin(6*d*x + 6*c) + (4*b^2*cos(2*d*x + 2*
c) - b^2 + 2*(8*a*b - 3*b^2)*cos(4*d*x + 4*c))*sin(5*d*x + 5*c) - (4*b^2*co
s(2*d*x + 2*c) - b^2)*sin(3*d*x + 3*c))/(a*b^2*cos(8*d*x + 8*c)^2 + 16*a*b^
2*cos(6*d*x + 6*c)^2 + 16*a*b^2*cos(2*d*x + 2*c)^2 + a*b^2*sin(8*d*x + 8*c)
^2 + 16*a*b^2*sin(6*d*x + 6*c)^2 + 16*a*b^2*sin(2*d*x + 2*c)^2 - 8*a*b^2*co
s(2*d*x + 2*c) + a*b^2 + 4*(64*a^3 - 48*a^2*b + 9*a*b^2)*cos(4*d*x + 4*c)^2
 + 4*(64*a^3 - 48*a^2*b + 9*a*b^2)*sin(4*d*x + 4*c)^2 + 16*(8*a^2*b - 3*a*b
^2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) - 2*(4*a*b^2*cos(6*d*x + 6*c) + 4*a*b
^2*cos(2*d*x + 2*c) - a*b^2 + 2*(8*a^2*b - 3*a*b^2)*cos(4*d*x + 4*c))*cos(8
*d*x + 8*c) + 8*(4*a*b^2*cos(2*d*x + 2*c) - a*b^2 + 2*(8*a^2*b - 3*a*b^2)*c
os(4*d*x + 4*c))*cos(6*d*x + 6*c) - 4*(8*a^2*b - 3*a*b^2 - 4*(8*a^2*b - 3*a
*b^2)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 4*(2*a*b^2*sin(6*d*x + 6*c) + 2*
a*b^2*sin(2*d*x + 2*c) + (8*a^2*b - 3*a*b^2)*sin(4*d*x + 4*c))*sin(8*d*x +
8*c) + 16*(2*a*b^2*sin(2*d*x + 2*c) + (8*a^2*b - 3*a*b^2)*sin(4*d*x + 4*c))
*sin(6*d*x + 6*c)), x) + (2*(2*cos(2*d*x + 2*c) - 1)*cos(4*d*x + 4*c) - cos
(4*d*x + 4*c)^2 - 4*cos(2*d*x + 2*c)^2 - sin(4*d*x + 4*c)^2 + 4*sin(4*d*x +
 4*c)*sin(2*d*x + 2*c) - 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) - 1)*log
(cos(d*x)^2 + 2*cos(d*x)*cos(c) + cos(c)^2 + sin(d*x)^2 - 2*sin(d*x)*sin(c)
 + sin(c)^2) - (2*(2*cos(2*d*x + 2*c) - 1)*cos(4*d*x + 4*c) - cos(4*d*x + 4
*c)^2 - 4*cos(2*d*x + 2*c)^2 - sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(
2*d*x + 2*c) - 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) - 1)*log(cos(d*x)^
2 - 2*cos(d*x)*cos(c) + cos(c)^2 + sin(d*x)^2 + 2*sin(d*x)*sin(c) + sin(c)^
```

```
2) + 4*(sin(3*d*x + 3*c) + sin(d*x + c))*sin(4*d*x + 4*c) - 8*sin(3*d*x + 3
*c)*sin(2*d*x + 2*c) - 8*sin(2*d*x + 2*c)*sin(d*x + c) + 4*cos(d*x + c))/(a
*d*cos(4*d*x + 4*c)^2 + 4*a*d*cos(2*d*x + 2*c)^2 + a*d*sin(4*d*x + 4*c)^2 -
 4*a*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*a*d*sin(2*d*x + 2*c)^2 - 4*a*d
*cos(2*d*x + 2*c) + a*d - 2*(2*a*d*cos(2*d*x + 2*c) - a*d)*cos(4*d*x + 4*c)
)
```

Fricas [B] time = 3.94465, size = 1905, normalized size = 10.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/4*((a*d*cos(d*x + c)^2 - a*d)*sqrt(-((a^4 - a^3*b)*d^2*sqrt(b^3/((a^7 -
2*a^6*b + a^5*b^2)*d^4)) + b^2)/((a^4 - a^3*b)*d^2))*log(b^2*cos(d*x + c) -
 ((a^5 - a^4*b)*d^3*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - a^2*b*d)*sq
rt(-((a^4 - a^3*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) + b^2)/((a
^4 - a^3*b)*d^2))) - (a*d*cos(d*x + c)^2 - a*d)*sqrt(((a^4 - a^3*b)*d^2*sqr
t(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - b^2)/((a^4 - a^3*b)*d^2))*log(b^2*
cos(d*x + c) - ((a^5 - a^4*b)*d^3*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4))
 + a^2*b*d)*sqrt(((a^4 - a^3*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4
)) - b^2)/((a^4 - a^3*b)*d^2))) - (a*d*cos(d*x + c)^2 - a*d)*sqrt(-((a^4 -
a^3*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) + b^2)/((a^4 - a^3*b)*
d^2))*log(-b^2*cos(d*x + c) - ((a^5 - a^4*b)*d^3*sqrt(b^3/((a^7 - 2*a^6*b +
 a^5*b^2)*d^4)) - a^2*b*d)*sqrt(-((a^4 - a^3*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*
b + a^5*b^2)*d^4)) + b^2)/((a^4 - a^3*b)*d^2))) + (a*d*cos(d*x + c)^2 - a*d
)*sqrt(((a^4 - a^3*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - b^2)/
((a^4 - a^3*b)*d^2))*log(-b^2*cos(d*x + c) - ((a^5 - a^4*b)*d^3*sqrt(b^3/((
a^7 - 2*a^6*b + a^5*b^2)*d^4)) + a^2*b*d)*sqrt(((a^4 - a^3*b)*d^2*sqrt(b^3/
((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - b^2)/((a^4 - a^3*b)*d^2))) + (cos(d*x +
c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2) - (cos(d*x + c)^2 - 1)*log(-1/2*cos(d
*x + c) + 1/2) - 2*cos(d*x + c))/(a*d*cos(d*x + c)^2 - a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.202 $\int \frac{\csc^5(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=229

$$-\frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^2d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{b^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^2d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{(3a+8b)\tanh^{-1}(\cos(c+dx))}{8a^2d}-\frac{3}{16ad(1-\cos(c+dx))}+\frac{}{16ad(}$$

```
[Out] -(b^(5/4)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*a^2*Sq
rt[Sqrt[a] - Sqrt[b]]*d) - ((3*a + 8*b)*ArcTanh[Cos[c + d*x]])/(8*a^2*d) +
(b^(5/4)*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*a^2*Sq
rt[Sqrt[a] + Sqrt[b]]*d) - 1/(16*a*d*(1 - Cos[c + d*x])^2) - 3/(16*a*d*(1 -
 Cos[c + d*x])) + 1/(16*a*d*(1 + Cos[c + d*x])^2) + 3/(16*a*d*(1 + Cos[c +
d*x]))
```

Rubi [A] time = 0.246567, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3215, 1170, 207, 1166, 205, 208}

$$-\frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^2d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{b^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^2d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{(3a+8b)\tanh^{-1}(\cos(c+dx))}{8a^2d}-\frac{3}{16ad(1-\cos(c+dx))}+\frac{}{16ad(}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5/(a - b*Sin[c + d*x]^4),x]
```


```
[Out] -(b^(5/4)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*a^2*Sq
rt[Sqrt[a] - Sqrt[b]]*d) - ((3*a + 8*b)*ArcTanh[Cos[c + d*x]])/(8*a^2*d) +
(b^(5/4)*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*a^2*Sq
rt[Sqrt[a] + Sqrt[b]]*d) - 1/(16*a*d*(1 - Cos[c + d*x])^2) - 3/(16*a*d*(1 -
 Cos[c + d*x])) + 1/(16*a*d*(1 + Cos[c + d*x])^2) + 3/(16*a*d*(1 + Cos[c +
d*x]))
```


Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\csc^5(c+dx)}{a-b\sin^4(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^3\left(a-b+2bx^2-bx^4\right)}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(-\frac{1}{8a(-1+x)^3}+\frac{3}{16a(-1+x)^2}+\frac{1}{8a(1+x)^3}+\frac{3}{16a(1+x)^2}+\frac{-3a-8b}{8a^2\left(-1+x^2\right)}-\frac{b^2\left(-1+x^2\right)}{a^2\left(a-b+2bx^2-bx^4\right)}\right) dx\right.}{d}$$

$$= -\frac{1}{16ad(1-\cos(c+dx))^2}-\frac{3}{16ad(1-\cos(c+dx))}+\frac{1}{16ad(1+\cos(c+dx))^2}+\frac{3}{16ad(1+\cos}$$

$$= -\frac{(3a+8b)\tanh^{-1}(\cos(c+dx))}{8a^2d}-\frac{1}{16ad(1-\cos(c+dx))^2}-\frac{3}{16ad(1-\cos(c+dx))}+\frac{}{16ad}$$

$$= -\frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^2\sqrt{\sqrt{a}-\sqrt{b}}\,d}-\frac{(3a+8b)\tanh^{-1}(\cos(c+dx))}{8a^2d}+\frac{b^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^2\sqrt{\sqrt{a}+\sqrt{b}}\,d}-\frac{}{1}$$

Mathematica [C] time = 1.12617, size = 409, normalized size = 1.79

$$-8ib^2\text{RootSum}\left[-16\#1^4a+\#1^8b-4\#1^6b+6\#1^4b-4\#1^2b+b\&,\frac{-i\#1^6\log\left(\#1^2-2\#1\cos(c+dx)+1\right)+3i\#1^4\log\left(\#1^2-2\#1\cos(c+dx)+\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^5/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (-6*a*Csc[(c + d*x)/2]^2 - a*Csc[(c + d*x)/2]^4 - 24*a*Log[Cos[(c + d*x)/2]
] - 64*b*Log[Cos[(c + d*x)/2]] + 24*a*Log[Sin[(c + d*x)/2]] + 64*b*Log[Sin[
(c + d*x)/2]] - (8*I)*b^2*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b
*#1^6 + b*#1^8 & , (-2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + I*Log[1 -
 2*Cos[c + d*x]*#1 + #1^2] + 6*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^
2 - (3*I)*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - 6*ArcTan[Sin[c + d*x]/(C
```

```
os[c + d*x] - #1)]*#1^4 + (3*I)*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 + 2*
ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 - I*Log[1 - 2*Cos[c + d*x]*#1
 + #1^2]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ] + 6*
a*Sec[(c + d*x)/2]^2 + a*Sec[(c + d*x)/2]^4)/(64*a^2*d)
```

Maple [A] time = 0.142, size = 232, normalized size = 1.

$$-\frac{1}{16\,da\,(-1+\cos(dx+c))^2}+\frac{3}{16\,da\,(-1+\cos(dx+c))}+\frac{3\,\ln(-1+\cos(dx+c))}{16\,da}+\frac{\ln(-1+\cos(dx+c))\,b}{2\,a^2d}+\frac{}{16\,da\,(1+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5/(a-b*sin(d*x+c)^4),x)
```

```
[Out] -1/16/d/a/(-1+cos(d*x+c))^2+3/16/d/a/(-1+cos(d*x+c))+3/16/d/a*ln(-1+cos(d*x
+c))+1/2/d/a^2*ln(-1+cos(d*x+c))*b+1/16/a/d/(1+cos(d*x+c))^2+3/16/a/d/(1+co
s(d*x+c))-3/16/d/a*ln(1+cos(d*x+c))-1/2/d/a^2*ln(1+cos(d*x+c))*b-1/2/d/a^2*
b^2/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2)
)+1/2/d/a^2*b^2/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2
)+b)*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -1/16*(48*a*cos(2*d*x + 2*c)*cos(d*x + c) - 176*a*sin(3*d*x + 3*c)*sin(2*d*
x + 2*c) + 48*a*sin(2*d*x + 2*c)*sin(d*x + c) - 4*(3*a*cos(7*d*x + 7*c) - 1
1*a*cos(5*d*x + 5*c) - 11*a*cos(3*d*x + 3*c) + 3*a*cos(d*x + c))*cos(8*d*x
+ 8*c) + 12*(4*a*cos(6*d*x + 6*c) - 6*a*cos(4*d*x + 4*c) + 4*a*cos(2*d*x +
2*c) - a)*cos(7*d*x + 7*c) - 16*(11*a*cos(5*d*x + 5*c) + 11*a*cos(3*d*x + 3
*c) - 3*a*cos(d*x + c))*cos(6*d*x + 6*c) + 44*(6*a*cos(4*d*x + 4*c) - 4*a*c
os(2*d*x + 2*c) + a)*cos(5*d*x + 5*c) + 24*(11*a*cos(3*d*x + 3*c) - 3*a*cos
(d*x + c))*cos(4*d*x + 4*c) - 44*(4*a*cos(2*d*x + 2*c) - a)*cos(3*d*x + 3*c
) - 12*a*cos(d*x + c) + 16*(a^2*d*cos(8*d*x + 8*c)^2 + 16*a^2*d*cos(6*d*x +
 6*c)^2 + 36*a^2*d*cos(4*d*x + 4*c)^2 + 16*a^2*d*cos(2*d*x + 2*c)^2 + a^2*d
*sin(8*d*x + 8*c)^2 + 16*a^2*d*sin(6*d*x + 6*c)^2 + 36*a^2*d*sin(4*d*x + 4*
c)^2 - 48*a^2*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a^2*d*sin(2*d*x + 2*
c)^2 - 8*a^2*d*cos(2*d*x + 2*c) + a^2*d - 2*(4*a^2*d*cos(6*d*x + 6*c) - 6*a
^2*d*cos(4*d*x + 4*c) + 4*a^2*d*cos(2*d*x + 2*c) - a^2*d)*cos(8*d*x + 8*c)
- 8*(6*a^2*d*cos(4*d*x + 4*c) - 4*a^2*d*cos(2*d*x + 2*c) + a^2*d)*cos(6*d*x
 + 6*c) - 12*(4*a^2*d*cos(2*d*x + 2*c) - a^2*d)*cos(4*d*x + 4*c) - 4*(2*a^2
*d*sin(6*d*x + 6*c) - 3*a^2*d*sin(4*d*x + 4*c) + 2*a^2*d*sin(2*d*x + 2*c))*
sin(8*d*x + 8*c) - 16*(3*a^2*d*sin(4*d*x + 4*c) - 2*a^2*d*sin(2*d*x + 2*c))
*sin(6*d*x + 6*c))*integrate(-2*(12*b^3*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) -
 4*b^3*cos(d*x + c)*sin(2*d*x + 2*c) + 4*b^3*cos(2*d*x + 2*c)*sin(d*x + c)
- b^3*sin(d*x + c) + (b^3*sin(7*d*x + 7*c) - 3*b^3*sin(5*d*x + 5*c) + 3*b^3
*sin(3*d*x + 3*c) - b^3*sin(d*x + c))*cos(8*d*x + 8*c) + 2*(2*b^3*sin(6*d*x
 + 6*c) + 2*b^3*sin(2*d*x + 2*c) + (8*a*b^2 - 3*b^3)*sin(4*d*x + 4*c))*cos(
7*d*x + 7*c) + 4*(3*b^3*sin(5*d*x + 5*c) - 3*b^3*sin(3*d*x + 3*c) + b^3*sin
```

```
(d*x + c))*cos(6*d*x + 6*c) - 6*(2*b^3*sin(2*d*x + 2*c) + (8*a*b^2 - 3*b^3)
*sin(4*d*x + 4*c))*cos(5*d*x + 5*c) - 2*(3*(8*a*b^2 - 3*b^3)*sin(3*d*x + 3*
c) - (8*a*b^2 - 3*b^3)*sin(d*x + c))*cos(4*d*x + 4*c) - (b^3*cos(7*d*x + 7*
c) - 3*b^3*cos(5*d*x + 5*c) + 3*b^3*cos(3*d*x + 3*c) - b^3*cos(d*x + c))*si
n(8*d*x + 8*c) - (4*b^3*cos(6*d*x + 6*c) + 4*b^3*cos(2*d*x + 2*c) - b^3 + 2
*(8*a*b^2 - 3*b^3)*cos(4*d*x + 4*c))*sin(7*d*x + 7*c) - 4*(3*b^3*cos(5*d*x
+ 5*c) - 3*b^3*cos(3*d*x + 3*c) + b^3*cos(d*x + c))*sin(6*d*x + 6*c) + 3*(4
*b^3*cos(2*d*x + 2*c) - b^3 + 2*(8*a*b^2 - 3*b^3)*cos(4*d*x + 4*c))*sin(5*d
*x + 5*c) + 2*(3*(8*a*b^2 - 3*b^3)*cos(3*d*x + 3*c) - (8*a*b^2 - 3*b^3)*cos
(d*x + c))*sin(4*d*x + 4*c) - 3*(4*b^3*cos(2*d*x + 2*c) - b^3)*sin(3*d*x +
3*c))/(a^2*b^2*cos(8*d*x + 8*c)^2 + 16*a^2*b^2*cos(6*d*x + 6*c)^2 + 16*a^2*
b^2*cos(2*d*x + 2*c)^2 + a^2*b^2*sin(8*d*x + 8*c)^2 + 16*a^2*b^2*sin(6*d*x
+ 6*c)^2 + 16*a^2*b^2*sin(2*d*x + 2*c)^2 - 8*a^2*b^2*cos(2*d*x + 2*c) + a^2
*b^2 + 4*(64*a^4 - 48*a^3*b + 9*a^2*b^2)*cos(4*d*x + 4*c)^2 + 4*(64*a^4 - 4
8*a^3*b + 9*a^2*b^2)*sin(4*d*x + 4*c)^2 + 16*(8*a^3*b - 3*a^2*b^2)*sin(4*d*
x + 4*c)*sin(2*d*x + 2*c) - 2*(4*a^2*b^2*cos(6*d*x + 6*c) + 4*a^2*b^2*cos(2
*d*x + 2*c) - a^2*b^2 + 2*(8*a^3*b - 3*a^2*b^2)*cos(4*d*x + 4*c))*cos(8*d*x
 + 8*c) + 8*(4*a^2*b^2*cos(2*d*x + 2*c) - a^2*b^2 + 2*(8*a^3*b - 3*a^2*b^2)
*cos(4*d*x + 4*c))*cos(6*d*x + 6*c) - 4*(8*a^3*b - 3*a^2*b^2 - 4*(8*a^3*b -
 3*a^2*b^2)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 4*(2*a^2*b^2*sin(6*d*x + 6
*c) + 2*a^2*b^2*sin(2*d*x + 2*c) + (8*a^3*b - 3*a^2*b^2)*sin(4*d*x + 4*c))*
sin(8*d*x + 8*c) + 16*(2*a^2*b^2*sin(2*d*x + 2*c) + (8*a^3*b - 3*a^2*b^2)*s
in(4*d*x + 4*c))*sin(6*d*x + 6*c)), x) + ((3*a + 8*b)*cos(8*d*x + 8*c)^2 +
16*(3*a + 8*b)*cos(6*d*x + 6*c)^2 + 36*(3*a + 8*b)*cos(4*d*x + 4*c)^2 + 16*
(3*a + 8*b)*cos(2*d*x + 2*c)^2 + (3*a + 8*b)*sin(8*d*x + 8*c)^2 + 16*(3*a +
 8*b)*sin(6*d*x + 6*c)^2 + 36*(3*a + 8*b)*sin(4*d*x + 4*c)^2 - 48*(3*a + 8*
b)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(3*a + 8*b)*sin(2*d*x + 2*c)^2 -
2*(4*(3*a + 8*b)*cos(6*d*x + 6*c) - 6*(3*a + 8*b)*cos(4*d*x + 4*c) + 4*(3*a
 + 8*b)*cos(2*d*x + 2*c) - 3*a - 8*b)*cos(8*d*x + 8*c) - 8*(6*(3*a + 8*b)*c
os(4*d*x + 4*c) - 4*(3*a + 8*b)*cos(2*d*x + 2*c) + 3*a + 8*b)*cos(6*d*x + 6
*c) - 12*(4*(3*a + 8*b)*cos(2*d*x + 2*c) - 3*a - 8*b)*cos(4*d*x + 4*c) - 8*
(3*a + 8*b)*cos(2*d*x + 2*c) - 4*(2*(3*a + 8*b)*sin(6*d*x + 6*c) - 3*(3*a +
 8*b)*sin(4*d*x + 4*c) + 2*(3*a + 8*b)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) -
 16*(3*(3*a + 8*b)*sin(4*d*x + 4*c) - 2*(3*a + 8*b)*sin(2*d*x + 2*c))*sin(6
*d*x + 6*c) + 3*a + 8*b)*log(cos(d*x)^2 + 2*cos(d*x)*cos(c) + cos(c)^2 + si
n(d*x)^2 - 2*sin(d*x)*sin(c) + sin(c)^2) - ((3*a + 8*b)*cos(8*d*x + 8*c)^2
+ 16*(3*a + 8*b)*cos(6*d*x + 6*c)^2 + 36*(3*a + 8*b)*cos(4*d*x + 4*c)^2 + 1
6*(3*a + 8*b)*cos(2*d*x + 2*c)^2 + (3*a + 8*b)*sin(8*d*x + 8*c)^2 + 16*(3*a
 + 8*b)*sin(6*d*x + 6*c)^2 + 36*(3*a + 8*b)*sin(4*d*x + 4*c)^2 - 48*(3*a +
8*b)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(3*a + 8*b)*sin(2*d*x + 2*c)^2
- 2*(4*(3*a + 8*b)*cos(6*d*x + 6*c) - 6*(3*a + 8*b)*cos(4*d*x + 4*c) + 4*(3
*a + 8*b)*cos(2*d*x + 2*c) - 3*a - 8*b)*cos(8*d*x + 8*c) - 8*(6*(3*a + 8*b)
*cos(4*d*x + 4*c) - 4*(3*a + 8*b)*cos(2*d*x + 2*c) + 3*a + 8*b)*cos(6*d*x +
 6*c) - 12*(4*(3*a + 8*b)*cos(2*d*x + 2*c) - 3*a - 8*b)*cos(4*d*x + 4*c) -
8*(3*a + 8*b)*cos(2*d*x + 2*c) - 4*(2*(3*a + 8*b)*sin(6*d*x + 6*c) - 3*(3*a
 + 8*b)*sin(4*d*x + 4*c) + 2*(3*a + 8*b)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c)
 - 16*(3*(3*a + 8*b)*sin(4*d*x + 4*c) - 2*(3*a + 8*b)*sin(2*d*x + 2*c))*sin
(6*d*x + 6*c) + 3*a + 8*b)*log(cos(d*x)^2 - 2*cos(d*x)*cos(c) + cos(c)^2 +
sin(d*x)^2 + 2*sin(d*x)*sin(c) + sin(c)^2) - 4*(3*a*sin(7*d*x + 7*c) - 11*a
*sin(5*d*x + 5*c) - 11*a*sin(3*d*x + 3*c) + 3*a*sin(d*x + c))*sin(8*d*x + 8
*c) + 24*(2*a*sin(6*d*x + 6*c) - 3*a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c
))*sin(7*d*x + 7*c) - 16*(11*a*sin(5*d*x + 5*c) + 11*a*sin(3*d*x + 3*c) - 3
*a*sin(d*x + c))*sin(6*d*x + 6*c) + 88*(3*a*sin(4*d*x + 4*c) - 2*a*sin(2*d*
x + 2*c))*sin(5*d*x + 5*c) + 24*(11*a*sin(3*d*x + 3*c) - 3*a*sin(d*x + c))*
sin(4*d*x + 4*c))/(a^2*d*cos(8*d*x + 8*c)^2 + 16*a^2*d*cos(6*d*x + 6*c)^2 +
 36*a^2*d*cos(4*d*x + 4*c)^2 + 16*a^2*d*cos(2*d*x + 2*c)^2 + a^2*d*sin(8*d*
x + 8*c)^2 + 16*a^2*d*sin(6*d*x + 6*c)^2 + 36*a^2*d*sin(4*d*x + 4*c)^2 - 48
*a^2*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a^2*d*sin(2*d*x + 2*c)^2 - 8*
a^2*d*cos(2*d*x + 2*c) + a^2*d - 2*(4*a^2*d*cos(6*d*x + 6*c) - 6*a^2*d*cos(
```

```
4*d*x + 4*c) + 4*a^2*d*cos(2*d*x + 2*c) - a^2*d)*cos(8*d*x + 8*c) - 8*(6*a^
2*d*cos(4*d*x + 4*c) - 4*a^2*d*cos(2*d*x + 2*c) + a^2*d)*cos(6*d*x + 6*c) -
 12*(4*a^2*d*cos(2*d*x + 2*c) - a^2*d)*cos(4*d*x + 4*c) - 4*(2*a^2*d*sin(6*
d*x + 6*c) - 3*a^2*d*sin(4*d*x + 4*c) + 2*a^2*d*sin(2*d*x + 2*c))*sin(8*d*x
 + 8*c) - 16*(3*a^2*d*sin(4*d*x + 4*c) - 2*a^2*d*sin(2*d*x + 2*c))*sin(6*d*
x + 6*c))
```

Fricas [B] time = 4.91023, size = 2292, normalized size = 10.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/16*(6*a*cos(d*x + c)^3 + 4*(a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2
 + a^2*d)*sqrt(-((a^5 - a^4*b)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)
) + b^3)/((a^5 - a^4*b)*d^2))*log(b^4*cos(d*x + c) + (a^2*b^3*d - (a^7 - a^
6*b)*d^3*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)))*sqrt(-((a^5 - a^4*b)*d^
2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) + b^3)/((a^5 - a^4*b)*d^2))) -
4*(a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2 + a^2*d)*sqrt(((a^5 - a^4*
b)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) - b^3)/((a^5 - a^4*b)*d^2)
)*log(b^4*cos(d*x + c) - (a^2*b^3*d + (a^7 - a^6*b)*d^3*sqrt(b^5/((a^9 - 2*
a^8*b + a^7*b^2)*d^4)))*sqrt(((a^5 - a^4*b)*d^2*sqrt(b^5/((a^9 - 2*a^8*b +
a^7*b^2)*d^4)) - b^3)/((a^5 - a^4*b)*d^2))) - 4*(a^2*d*cos(d*x + c)^4 - 2*a
^2*d*cos(d*x + c)^2 + a^2*d)*sqrt(-((a^5 - a^4*b)*d^2*sqrt(b^5/((a^9 - 2*a^
8*b + a^7*b^2)*d^4)) + b^3)/((a^5 - a^4*b)*d^2))*log(-b^4*cos(d*x + c) + (a
^2*b^3*d - (a^7 - a^6*b)*d^3*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)))*sqr
t(-((a^5 - a^4*b)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) + b^3)/((a^
5 - a^4*b)*d^2))) + 4*(a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2 + a^2*
d)*sqrt(((a^5 - a^4*b)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) - b^3)
/((a^5 - a^4*b)*d^2))*log(-b^4*cos(d*x + c) - (a^2*b^3*d + (a^7 - a^6*b)*d^
3*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)))*sqrt(((a^5 - a^4*b)*d^2*sqrt(b
^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) - b^3)/((a^5 - a^4*b)*d^2))) - 10*a*cos
(d*x + c) - ((3*a + 8*b)*cos(d*x + c)^4 - 2*(3*a + 8*b)*cos(d*x + c)^2 + 3*
a + 8*b)*log(1/2*cos(d*x + c) + 1/2) + ((3*a + 8*b)*cos(d*x + c)^4 - 2*(3*a
 + 8*b)*cos(d*x + c)^2 + 3*a + 8*b)*log(-1/2*cos(d*x + c) + 1/2))/(a^2*d*co
s(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2 + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.203 $\int \frac{\sin^8(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=184

$$\frac{a^{5/4}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2b^2d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{a^{5/4}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2b^2d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{x(a+b)}{b^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{4bd}+\frac{5\sin(c+dx)\cos(c}{8bd}$$

```
[Out] (5*x)/(8*b) - ((a + b)*x)/b^2 + (a^(5/4)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Ta
n[c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt[a] - Sqrt[b]]*b^2*d) + (a^(5/4)*ArcTan[(
Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*
b^2*d) + (5*Cos[c + d*x]*Sin[c + d*x])/(8*b*d) - (Cos[c + d*x]^3*Sin[c + d*
x])/(4*b*d)
```

Rubi [A] time = 0.292675, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3217, 1287, 199, 203, 1166, 205}

$$\frac{a^{5/4}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2b^2d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{a^{5/4}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2b^2d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{x(a+b)}{b^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{4bd}+\frac{5\sin(c+dx)\cos(c}{8bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^8/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (5*x)/(8*b) - ((a + b)*x)/b^2 + (a^(5/4)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Ta
n[c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt[a] - Sqrt[b]]*b^2*d) + (a^(5/4)*ArcTan[(
Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*
b^2*d) + (5*Cos[c + d*x]*Sin[c + d*x])/(8*b*d) - (Cos[c + d*x]^3*Sin[c + d*
x])/(4*b*d)
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
```

```
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^8(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^8}{(1+x^2)^3(a+2ax^2+(a-b)x^4)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{b(1+x^2)^3}+\frac{2}{b(1+x^2)^2}+\frac{-a-b}{b^2(1+x^2)}+\frac{a^2(1+x^2)}{b^2(a+2ax^2+(a-b)x^4)}\right) dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{a^2\,\text{Subst}\left(\int \frac{1+x^2}{a+2ax^2+(a-b)x^4}\,dx, x, \tan(c+dx)\right)}{b^2d} - \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^3}\,dx, x, \tan(c+dx)\right)}{bd} + \frac{2\,\text{S}}{}$$

$$= -\frac{(a+b)x}{b^2} + \frac{\cos(c+dx)\sin(c+dx)}{bd} - \frac{\cos^3(c+dx)\sin(c+dx)}{4bd} + \frac{\left(a^{3/2}\left(\sqrt{a}-\sqrt{b}\right)\right)\text{Subst}}{}$$

$$= \frac{x}{b} - \frac{(a+b)x}{b^2} + \frac{a^{5/4}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^2d} + \frac{a^{5/4}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}\,b^2d} + \frac{5\cos(c+d}{8}$$

$$= \frac{5x}{8b} - \frac{(a+b)x}{b^2} + \frac{a^{5/4}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^2d} + \frac{a^{5/4}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}\,b^2d} + \frac{5\cos(c+}{}$$

Mathematica [A] time = 0.891659, size = 172, normalized size = 0.93

$$-\frac{-\frac{16a^{3/2}\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}+a}}+\frac{16a^{3/2}\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}-a}}+4(8a+3b)(c+dx)-8b\sin(2(c+dx))+b\sin(4(c+dx))}{32b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^8/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -(4*(8*a + 3*b)*(c + d*x) - (16*a^(3/2)*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c +
 d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]] + (16*a^(3/2)*
ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/Sqr
t[-a + Sqrt[a]*Sqrt[b]] - 8*b*Sin[2*(c + d*x)] + b*Sin[4*(c + d*x)])/(32*b^
2*d)
```

Maple [B] time = 0.119, size = 605, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^8/(a-b*sin(d*x+c)^4),x)
```

```
[Out] 1/2/d*a^3/b/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*ta
n(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+1/2/d*a^3/b^2/(a-b)/(((a*b)^(1/2)+a
)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+1/2/d
*a^3/b^2/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a
*b)^(1/2)-a)*(a-b))^(1/2))-1/2/d*a^3/b/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(
a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-1/2/d*
a^2/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)
/(((a*b)^(1/2)+a)*(a-b))^(1/2))-1/2/d*a^2/b/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(
1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-1/2/d*a^2/b/(a-
b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)
*(a-b))^(1/2))+1/2/d*a^2/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*ar
ctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))+5/8/d/b/(tan(d*x+c)^
2+1)^2*tan(d*x+c)^3+3/8/d/b/(tan(d*x+c)^2+1)^2*tan(d*x+c)-3/8/d/b*arctan(ta
n(d*x+c))-1/d/b^2*arctan(tan(d*x+c))*a
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 3.42395, size = 2712, normalized size = 14.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/8*(b^2*d*sqrt(-((a*b^4 - b^5)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^
4)) + a^3)/((a*b^4 - b^5)*d^2))*log(1/4*a^3*cos(d*x + c)^2 - 1/4*a^3 - 1/4*
(2*(a^2*b^3 - a*b^4)*d^2*cos(d*x + c)^2 - (a^2*b^3 - a*b^4)*d^2)*sqrt(a^5/(
(a^2*b^7 - 2*a*b^8 + b^9)*d^4)) + 1/2*(a^2*b^2*d*cos(d*x + c)*sin(d*x + c)
- (a*b^5 - b^6)*d^3*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4))*cos(d*x + c)*
```

```
sin(d*x + c))*sqrt(-((a*b^4 - b^5)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*
d^4)) + a^3)/((a*b^4 - b^5)*d^2))) - b^2*d*sqrt(-((a*b^4 - b^5)*d^2*sqrt(a^
5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) + a^3)/((a*b^4 - b^5)*d^2))*log(1/4*a^3*
cos(d*x + c)^2 - 1/4*a^3 - 1/4*(2*(a^2*b^3 - a*b^4)*d^2*cos(d*x + c)^2 - (a
^2*b^3 - a*b^4)*d^2)*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) - 1/2*(a^2*b
^2*d*cos(d*x + c)*sin(d*x + c) - (a*b^5 - b^6)*d^3*sqrt(a^5/((a^2*b^7 - 2*a
*b^8 + b^9)*d^4))*cos(d*x + c)*sin(d*x + c))*sqrt(-((a*b^4 - b^5)*d^2*sqrt(
a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) + a^3)/((a*b^4 - b^5)*d^2))) - b^2*d*s
qrt(((a*b^4 - b^5)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) - a^3)/((a
*b^4 - b^5)*d^2))*log(-1/4*a^3*cos(d*x + c)^2 + 1/4*a^3 - 1/4*(2*(a^2*b^3 -
 a*b^4)*d^2*cos(d*x + c)^2 - (a^2*b^3 - a*b^4)*d^2)*sqrt(a^5/((a^2*b^7 - 2*
a*b^8 + b^9)*d^4)) + 1/2*(a^2*b^2*d*cos(d*x + c)*sin(d*x + c) + (a*b^5 - b^
6)*d^3*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4))*cos(d*x + c)*sin(d*x + c))
*sqrt(((a*b^4 - b^5)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) - a^3)/(
(a*b^4 - b^5)*d^2))) + b^2*d*sqrt(((a*b^4 - b^5)*d^2*sqrt(a^5/((a^2*b^7 - 2
*a*b^8 + b^9)*d^4)) - a^3)/((a*b^4 - b^5)*d^2))*log(-1/4*a^3*cos(d*x + c)^2
 + 1/4*a^3 - 1/4*(2*(a^2*b^3 - a*b^4)*d^2*cos(d*x + c)^2 - (a^2*b^3 - a*b^4
)*d^2)*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) - 1/2*(a^2*b^2*d*cos(d*x +
 c)*sin(d*x + c) + (a*b^5 - b^6)*d^3*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^
4))*cos(d*x + c)*sin(d*x + c))*sqrt(((a*b^4 - b^5)*d^2*sqrt(a^5/((a^2*b^7 -
 2*a*b^8 + b^9)*d^4)) - a^3)/((a*b^4 - b^5)*d^2))) + (8*a + 3*b)*d*x + (2*b
*cos(d*x + c)^3 - 5*b*cos(d*x + c))*sin(d*x + c))/(b^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**8/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.204 $\int \frac{\sin^6(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=155

$$\frac{a^{3/4}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2b^{3/2}d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{a^{3/4}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2b^{3/2}d\sqrt{\sqrt{a}+\sqrt{b}}}+\frac{\sin(c+dx)\cos(c+dx)}{2bd}-\frac{x}{2b}$$

```
[Out] -x/(2*b) + (a^(3/4)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])
/(2*Sqrt[Sqrt[a] - Sqrt[b]]*b^(3/2)*d) - (a^(3/4)*ArcTan[(Sqrt[Sqrt[a] + Sq
rt[b]]*Tan[c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(3/2)*d) + (Cos
[c + d*x]*Sin[c + d*x])/(2*b*d)
```

Rubi [A] time = 0.204312, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3217, 1287, 199, 203, 1130, 205}

$$\frac{a^{3/4}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2b^{3/2}d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{a^{3/4}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2b^{3/2}d\sqrt{\sqrt{a}+\sqrt{b}}}+\frac{\sin(c+dx)\cos(c+dx)}{2bd}-\frac{x}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^6/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -x/(2*b) + (a^(3/4)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])
/(2*Sqrt[Sqrt[a] - Sqrt[b]]*b^(3/2)*d) - (a^(3/4)*ArcTan[(Sqrt[Sqrt[a] + Sq
rt[b]]*Tan[c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(3/2)*d) + (Cos
[c + d*x]*Sin[c + d*x])/(2*b*d)
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^6(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)^2\left(a+2ax^2+(a-b)x^4\right)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{b(1+x^2)^2}-\frac{1}{b(1+x^2)}+\frac{ax^2}{b\left(a+2ax^2+(a-b)x^4\right)}\right)dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^2}\,dx, x, \tan(c+dx)\right)}{bd} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(c+dx)\right)}{bd} + \frac{a\,\text{Subst}\left(\int \frac{}{a+2}\right.}{}$$

$$= -\frac{x}{b}+\frac{\cos(c+dx)\sin(c+dx)}{2bd}+\frac{\left(a\left(\sqrt{a}+\sqrt{b}\right)\right)\text{Subst}\left(\int \frac{1}{a+\sqrt{a}\sqrt{b}+(a-b)x^2}\,dx, x, \tan(c+dx)\right)}{2b^{3/2}d}$$

$$= -\frac{x}{2b}+\frac{a^{3/4}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^{3/2}d}-\frac{a^{3/4}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}\,b^{3/2}d}+\frac{\cos(c+dx)\sin(c+}{2bd}$$

Mathematica [A] time = 0.805675, size = 157, normalized size = 1.01

$$\frac{-\frac{2a\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}+a}}-\frac{2a\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}-a}}-2\sqrt{b}(c+dx)+\sqrt{b}\sin(2(c+dx))}{4b^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^6/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (-2*Sqrt[b]*(c + d*x) - (2*a*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/Sqrt
[a + Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]] - (2*a*ArcTanh[((Sqrt[a]
- Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a + Sqrt[a]*Sqr
t[b]] + Sqrt[b]*Sin[2*(c + d*x)])/(4*b^(3/2)*d)
```

Maple [B] time = 0.097, size = 551, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^6/(a-b*sin(d*x+c)^4),x)
```

```
[Out] 1/2/d*a^2/b/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((
a*b)^(1/2)+a)*(a-b))^(1/2))+1/2/d*a^3/b/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*
(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+1/2/d*a
^2/b/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^
(1/2)-a)*(a-b))^(1/2))-1/2/d*a^3/b/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b)
)^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-1/2/d*a/(a
-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*
(a-b))^(1/2))-1/2/d*a^2/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arc
tan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-1/2/d*a/(a-b)/(((a*b)^(
1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2
))+1/2/d*a^2/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)
*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))+1/2/d/b*tan(d*x+c)/(tan(d*x+c)^2
+1)-1/2/d/b*arctan(tan(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] 1/4*(4*b*d*integrate(-4*(4*a*b*cos(6*d*x + 6*c)^2 + 4*a*b*cos(2*d*x + 2*c)^
2 + 4*a*b*sin(6*d*x + 6*c)^2 + 4*a*b*sin(2*d*x + 2*c)^2 - 4*(8*a^2 - 3*a*b)
*cos(4*d*x + 4*c)^2 - a*b*cos(2*d*x + 2*c) - 4*(8*a^2 - 3*a*b)*sin(4*d*x +
4*c)^2 + 2*(8*a^2 - 7*a*b)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) - (a*b*cos(6*d
*x + 6*c) - 2*a*b*cos(4*d*x + 4*c) + a*b*cos(2*d*x + 2*c))*cos(8*d*x + 8*c)
 + (8*a*b*cos(2*d*x + 2*c) - a*b + 2*(8*a^2 - 7*a*b)*cos(4*d*x + 4*c))*cos(
6*d*x + 6*c) + 2*(a*b + (8*a^2 - 7*a*b)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c)
- (a*b*sin(6*d*x + 6*c) - 2*a*b*sin(4*d*x + 4*c) + a*b*sin(2*d*x + 2*c))*si
n(8*d*x + 8*c) + 2*(4*a*b*sin(2*d*x + 2*c) + (8*a^2 - 7*a*b)*sin(4*d*x + 4*
c))*sin(6*d*x + 6*c))/(b^3*cos(8*d*x + 8*c)^2 + 16*b^3*cos(6*d*x + 6*c)^2 +
 16*b^3*cos(2*d*x + 2*c)^2 + b^3*sin(8*d*x + 8*c)^2 + 16*b^3*sin(6*d*x + 6*
c)^2 + 16*b^3*sin(2*d*x + 2*c)^2 - 8*b^3*cos(2*d*x + 2*c) + b^3 + 4*(64*a^2
*b - 48*a*b^2 + 9*b^3)*cos(4*d*x + 4*c)^2 + 4*(64*a^2*b - 48*a*b^2 + 9*b^3)
*sin(4*d*x + 4*c)^2 + 16*(8*a*b^2 - 3*b^3)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c
) - 2*(4*b^3*cos(6*d*x + 6*c) + 4*b^3*cos(2*d*x + 2*c) - b^3 + 2*(8*a*b^2 -
 3*b^3)*cos(4*d*x + 4*c))*cos(8*d*x + 8*c) + 8*(4*b^3*cos(2*d*x + 2*c) - b^
3 + 2*(8*a*b^2 - 3*b^3)*cos(4*d*x + 4*c))*cos(6*d*x + 6*c) - 4*(8*a*b^2 - 3
*b^3 - 4*(8*a*b^2 - 3*b^3)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 4*(2*b^3*si
n(6*d*x + 6*c) + 2*b^3*sin(2*d*x + 2*c) + (8*a*b^2 - 3*b^3)*sin(4*d*x + 4*c
))*sin(8*d*x + 8*c) + 16*(2*b^3*sin(2*d*x + 2*c) + (8*a*b^2 - 3*b^3)*sin(4*
d*x + 4*c))*sin(6*d*x + 6*c)), x) - 2*d*x + sin(2*d*x + 2*c))/(b*d)
```

Fricas [B] time = 3.63702, size = 2641, normalized size = 17.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/8*(b*d*sqrt(-((a*b^3 - b^4)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)
) + a^2)/((a*b^3 - b^4)*d^2))*log(1/4*a^2*cos(d*x + c)^2 - 1/4*a^2 - 1/4*(2
*(a^2*b^2 - a*b^3)*d^2*cos(d*x + c)^2 - (a^2*b^2 - a*b^3)*d^2)*sqrt(a^3/((a
^2*b^5 - 2*a*b^6 + b^7)*d^4)) + 1/2*((a*b^4 - b^5)*d^3*sqrt(a^3/((a^2*b^5 -
 2*a*b^6 + b^7)*d^4))*cos(d*x + c)*sin(d*x + c) - a^2*b*d*cos(d*x + c)*sin(
d*x + c))*sqrt(-((a*b^3 - b^4)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)
) + a^2)/((a*b^3 - b^4)*d^2))) - b*d*sqrt(-((a*b^3 - b^4)*d^2*sqrt(a^3/((a^
2*b^5 - 2*a*b^6 + b^7)*d^4)) + a^2)/((a*b^3 - b^4)*d^2))*log(1/4*a^2*cos(d*
x + c)^2 - 1/4*a^2 - 1/4*(2*(a^2*b^2 - a*b^3)*d^2*cos(d*x + c)^2 - (a^2*b^2
 - a*b^3)*d^2)*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - 1/2*((a*b^4 - b^
5)*d^3*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4))*cos(d*x + c)*sin(d*x + c)
- a^2*b*d*cos(d*x + c)*sin(d*x + c))*sqrt(-((a*b^3 - b^4)*d^2*sqrt(a^3/((a^
2*b^5 - 2*a*b^6 + b^7)*d^4)) + a^2)/((a*b^3 - b^4)*d^2))) + b*d*sqrt(((a*b^
3 - b^4)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - a^2)/((a*b^3 - b^4
)*d^2))*log(-1/4*a^2*cos(d*x + c)^2 + 1/4*a^2 - 1/4*(2*(a^2*b^2 - a*b^3)*d^
2*cos(d*x + c)^2 - (a^2*b^2 - a*b^3)*d^2)*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^
7)*d^4)) + 1/2*((a*b^4 - b^5)*d^3*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4))
*cos(d*x + c)*sin(d*x + c) + a^2*b*d*cos(d*x + c)*sin(d*x + c))*sqrt(((a*b^
3 - b^4)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - a^2)/((a*b^3 - b^4
)*d^2))) - b*d*sqrt(((a*b^3 - b^4)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*
d^4)) - a^2)/((a*b^3 - b^4)*d^2))*log(-1/4*a^2*cos(d*x + c)^2 + 1/4*a^2 - 1
/4*(2*(a^2*b^2 - a*b^3)*d^2*cos(d*x + c)^2 - (a^2*b^2 - a*b^3)*d^2)*sqrt(a^
3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - 1/2*((a*b^4 - b^5)*d^3*sqrt(a^3/((a^2*
b^5 - 2*a*b^6 + b^7)*d^4))*cos(d*x + c)*sin(d*x + c) + a^2*b*d*cos(d*x + c)
*sin(d*x + c))*sqrt(((a*b^3 - b^4)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*
d^4)) - a^2)/((a*b^3 - b^4)*d^2))) + 4*d*x - 4*cos(d*x + c)*sin(d*x + c))/(
b*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**6/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.205 $\int \frac{\sin^4(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=127

$$\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2bd\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2bd\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{x}{b}$$

```
[Out] -(x/b) + (a^(1/4)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(
2*Sqrt[Sqrt[a] - Sqrt[b]]*b*d) + (a^(1/4)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*T
an[c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b*d)
```

Rubi [A] time = 0.185167, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3217, 1287, 203, 1166, 205}

$$\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2bd\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2bd\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -(x/b) + (a^(1/4)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(
2*Sqrt[Sqrt[a] - Sqrt[b]]*b*d) + (a^(1/4)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*T
an[c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b*d)
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^4(c+dx)}{a-b\sin^4(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)\left(a+2ax^2+(a-b)x^4\right)}\,dx, x, \tan(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{b(1+x^2)}+\frac{a(1+x^2)}{b\left(a+2ax^2+(a-b)x^4\right)}\right) dx, x, \tan(c+dx)\right)}{d}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(c+dx)\right)}{bd}+\frac{a\,\text{Subst}\left(\int \frac{1+x^2}{a+2ax^2+(a-b)x^4}\,dx, x, \tan(c+dx)\right)}{bd}\\
&= -\frac{x}{b}+\frac{\left(a\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\right)\text{Subst}\left(\int \frac{1}{a-\sqrt{a}\sqrt{b}+(a-b)x^2}\,dx, x, \tan(c+dx)\right)}{2bd}+\frac{\left(a\left(1+\frac{\sqrt{b}}{\sqrt{a}}\right)\right)\text{Subst}\left(\int \frac{}{a+}\right.}{}\\
&= -\frac{x}{b}+\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}bd}+\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}bd}
\end{aligned}$$

Mathematica [A] time = 0.411802, size = 143, normalized size = 1.13

$$\frac{\frac{\sqrt{a}\tan^{-1}\left(\frac{(\sqrt{a}+\sqrt{b})\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}+a}}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{(\sqrt{a}-\sqrt{b})\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}-a}}-2(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (-2*(c + d*x) + (Sqrt[a]*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/Sqrt[a +
 Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]] - (Sqrt[a]*ArcTanh[((Sqrt[a]
- Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a + Sqrt[a]*Sqr
t[b]])/(2*b*d)
```

Maple [B] time = 0.095, size = 517, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a-b*sin(d*x+c)^4),x)
```

```
[Out] 1/2/d*a^2/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(
d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+1/2/d*a^2/b/(a-b)/(((a*b)^(1/2)+a)*(a
```

```
-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+1/2/d*a^2
/b/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1
/2)-a)*(a-b))^(1/2))-1/2/d*a^2/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1
/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-1/2/d*a/(a*b)^
(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(
1/2)+a)*(a-b))^(1/2))*b-1/2/d*a/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan(
(a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-1/2/d*a/(a-b)/(((a*b)^(1/2)
-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))+1
/2/d*a/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d
*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*b-1/d/b*arctan(tan(d*x+c))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a-b*sin(d*x+c)^4),x, algorithm="maxima")

[Out] Timed out
```

Fricas [B] time = 3.08797, size = 2385, normalized size = 18.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a-b*sin(d*x+c)^4),x, algorithm="fricas")

[Out] 1/8*(b*sqrt(-((a*b^2 - b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + a
)/((a*b^2 - b^3)*d^2))*log(1/4*cos(d*x + c)^2 + 1/2*((a*b^2 - b^3)*d^3*sqrt
(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4))*cos(d*x + c)*sin(d*x + c) - b*d*cos(d*x
 + c)*sin(d*x + c))*sqrt(-((a*b^2 - b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b
^5)*d^4)) + a)/((a*b^2 - b^3)*d^2)) - 1/4*(2*(a*b - b^2)*d^2*cos(d*x + c)^2
 - (a*b - b^2)*d^2)*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - 1/4) - b*sqrt
(-((a*b^2 - b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + a)/((a*b^2 -
 b^3)*d^2))*log(1/4*cos(d*x + c)^2 - 1/2*((a*b^2 - b^3)*d^3*sqrt(a/((a^2*b^
3 - 2*a*b^4 + b^5)*d^4))*cos(d*x + c)*sin(d*x + c) - b*d*cos(d*x + c)*sin(d
*x + c))*sqrt(-((a*b^2 - b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) +
 a)/((a*b^2 - b^3)*d^2)) - 1/4*(2*(a*b - b^2)*d^2*cos(d*x + c)^2 - (a*b - b
^2)*d^2)*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - 1/4) + b*sqrt(((a*b^2 -
b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - a)/((a*b^2 - b^3)*d^2))*
log(-1/4*cos(d*x + c)^2 + 1/2*((a*b^2 - b^3)*d^3*sqrt(a/((a^2*b^3 - 2*a*b^4
 + b^5)*d^4))*cos(d*x + c)*sin(d*x + c) + b*d*cos(d*x + c)*sin(d*x + c))*sq
rt(((a*b^2 - b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - a)/((a*b^2
- b^3)*d^2)) - 1/4*(2*(a*b - b^2)*d^2*cos(d*x + c)^2 - (a*b - b^2)*d^2)*sqr
t(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + 1/4) - b*sqrt(((a*b^2 - b^3)*d^2*sqr
t(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - a)/((a*b^2 - b^3)*d^2))*log(-1/4*cos
(d*x + c)^2 - 1/2*((a*b^2 - b^3)*d^3*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)
)*cos(d*x + c)*sin(d*x + c) + b*d*cos(d*x + c)*sin(d*x + c))*sqrt(((a*b^2 -
 b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - a)/((a*b^2 - b^3)*d^2))
 - 1/4*(2*(a*b - b^2)*d^2*cos(d*x + c)^2 - (a*b - b^2)*d^2)*sqrt(a/((a^2*b^
3 - 2*a*b^4 + b^5)*d^4)) + 1/4) - 8*x)/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.206 $\int \frac{\sin^2(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=125

$$\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{b}d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{b}d\sqrt{\sqrt{a}+\sqrt{b}}}$$

```
[Out] ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)]/(2*a^(1/4)*Sqrt[Sqrt
[a] - Sqrt[b]]*Sqrt[b]*d) - ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a
^(1/4)]/(2*a^(1/4)*Sqrt[Sqrt[a] + Sqrt[b]]*Sqrt[b]*d)
```

Rubi [A] time = 0.112376, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3217, 1130, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{b}d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{b}d\sqrt{\sqrt{a}+\sqrt{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)]/(2*a^(1/4)*Sqrt[Sqrt
[a] - Sqrt[b]]*Sqrt[b]*d) - ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a
^(1/4)]/(2*a^(1/4)*Sqrt[Sqrt[a] + Sqrt[b]]*Sqrt[b]*d)
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{a+2ax^2+(a-b)x^4}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\left(1-\frac{\sqrt{a}}{\sqrt{b}}\right)\text{Subst}\left(\int \frac{1}{a-\sqrt{a}\sqrt{b}+(a-b)x^2}\,dx, x, \tan(c+dx)\right)}{2d} + \frac{\left(1+\frac{\sqrt{a}}{\sqrt{b}}\right)\text{Subst}\left(\int \frac{1}{a+\sqrt{a}\sqrt{b}+(a-b)x^2}\,d\right.}{2d}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{\sqrt{a}-\sqrt{b}}\sqrt{b}d} - \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{\sqrt{a}+\sqrt{b}}\sqrt{b}d}$$

Mathematica [A] time = 0.34862, size = 137, normalized size = 1.1

$$-\frac{\tan^{-1}\left(\frac{(\sqrt{a}+\sqrt{b})\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{2\sqrt{b}d\sqrt{\sqrt{a}\sqrt{b}+a}} - \frac{\tanh^{-1}\left(\frac{(\sqrt{a}-\sqrt{b})\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{2\sqrt{b}d\sqrt{\sqrt{a}\sqrt{b}-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]]/(2*Sq
rt[a + Sqrt[a]*Sqrt[b]]*Sqrt[b]*d) - ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d
*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]]/(2*Sqrt[-a + Sqrt[a]*Sqrt[b]]*Sqrt[b]*d)
```

Maple [B] time = 0.096, size = 492, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a-b*sin(d*x+c)^4),x)
```

```
[Out] 1/2/d*a/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)
^(1/2)+a)*(a-b))^(1/2))+1/2/d*a^2/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))
^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+1/2/d*a/(a-b)
/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(
a-b))^(1/2))-1/2/d*a^2/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arct
anh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-1/2/d*b/(a-b)/(((a*b)^
(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2)
)-1/2/d*a/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(
d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*b-1/2/d*b/(a-b)/(((a*b)^(1/2)-a)*(a-b
))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))+1/2/d*a/(
a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/((
(a*b)^(1/2)-a)*(a-b))^(1/2))*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sin(dx+c)^2}{b\sin(dx+c)^4 - a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -integrate(sin(d*x + c)^2/(b*sin(d*x + c)^4 - a), x)
```

Fricas [B] time = 3.27401, size = 2390, normalized size = 19.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(-((a*b - b^2)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1)/
((a*b - b^2)*d^2))*log(1/4*cos(d*x + c)^2 + 1/2*((a^2*b - a*b^2)*d^3*sqrt(1
/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4))*cos(d*x + c)*sin(d*x + c) - a*d*cos(d*x
 + c)*sin(d*x + c))*sqrt(-((a*b - b^2)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b
^3)*d^4)) + 1)/((a*b - b^2)*d^2)) - 1/4*(2*(a^2 - a*b)*d^2*cos(d*x + c)^2 -
 (a^2 - a*b)*d^2)*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1/4) + 1/8*sq
rt(-((a*b - b^2)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1)/((a*b -
 b^2)*d^2))*log(1/4*cos(d*x + c)^2 - 1/2*((a^2*b - a*b^2)*d^3*sqrt(1/((a^3*
b - 2*a^2*b^2 + a*b^3)*d^4))*cos(d*x + c)*sin(d*x + c) - a*d*cos(d*x + c)*s
in(d*x + c))*sqrt(-((a*b - b^2)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4
)) + 1)/((a*b - b^2)*d^2)) - 1/4*(2*(a^2 - a*b)*d^2*cos(d*x + c)^2 - (a^2 -
 a*b)*d^2)*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1/4) - 1/8*sqrt(((a*
b - b^2)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1)/((a*b - b^2)*d^
2))*log(-1/4*cos(d*x + c)^2 + 1/2*((a^2*b - a*b^2)*d^3*sqrt(1/((a^3*b - 2*a
^2*b^2 + a*b^3)*d^4))*cos(d*x + c)*sin(d*x + c) + a*d*cos(d*x + c)*sin(d*x
+ c))*sqrt(((a*b - b^2)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1)/
((a*b - b^2)*d^2)) - 1/4*(2*(a^2 - a*b)*d^2*cos(d*x + c)^2 - (a^2 - a*b)*d^
2)*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1/4) + 1/8*sqrt(((a*b - b^2)
*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1)/((a*b - b^2)*d^2))*log(
-1/4*cos(d*x + c)^2 - 1/2*((a^2*b - a*b^2)*d^3*sqrt(1/((a^3*b - 2*a^2*b^2 +
 a*b^3)*d^4))*cos(d*x + c)*sin(d*x + c) + a*d*cos(d*x + c)*sin(d*x + c))*sq
rt(((a*b - b^2)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1)/((a*b -
b^2)*d^2)) - 1/4*(2*(a^2 - a*b)*d^2*cos(d*x + c)^2 - (a^2 - a*b)*d^2)*sqrt(
1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1/4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.207 $\int \frac{1}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=115

$$\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\sqrt{\sqrt{a}+\sqrt{b}}}$$

```
[Out] ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)]/(2*a^(3/4)*Sqrt[Sqrt
[a] - Sqrt[b]]*d) + ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)]/
(2*a^(3/4)*Sqrt[Sqrt[a] + Sqrt[b]]*d)
```

Rubi [A] time = 0.0904601, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3209, 1166, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\sqrt{\sqrt{a}+\sqrt{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*Sin[c + d*x]^4)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)]/(2*a^(3/4)*Sqrt[Sqrt
[a] - Sqrt[b]]*d) + ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)]/
(2*a^(3/4)*Sqrt[Sqrt[a] + Sqrt[b]]*d)
```

Rule 3209

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{a-b\sin^4(c+dx)}\,dx = \frac{\mathrm{Subst}\left(\int \frac{1+x^2}{a+2ax^2+(a-b)x^4}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\mathrm{Subst}\left(\int \frac{1}{a-\sqrt{a}\sqrt{b}+(a-b)x^2}\,dx, x, \tan(c+dx)\right)}{2d} + \frac{\left(1+\frac{\sqrt{b}}{\sqrt{a}}\right)\mathrm{Subst}\left(\int \frac{1}{a+\sqrt{a}\sqrt{b}+(a-b)x^2}\,d\right.}{2d}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{\sqrt{a}-\sqrt{b}}d} + \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{\sqrt{a}+\sqrt{b}}d}$$

Mathematica [A] time = 0.256142, size = 128, normalized size = 1.11

$$\frac{\frac{\tan^{-1}\left(\frac{(\sqrt{a}+\sqrt{b})\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}+a}} - \frac{\tanh^{-1}\left(\frac{(\sqrt{a}-\sqrt{b})\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}-a}}}{2\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*Sin[c + d*x]^4)^(-1),x]
```

```
[Out] (ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]]/Sqrt[
a + Sqrt[a]*Sqrt[b]] - ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a +
 Sqrt[a]*Sqrt[b]]]/Sqrt[-a + Sqrt[a]*Sqrt[b]])/(2*Sqrt[a]*d)
```

Maple [B] time = 0.094, size = 492, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-b*sin(d*x+c)^4),x)
```

```
[Out] 1/2/d*a/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*
x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*b+1/2/d*a/(a-b)/(((a*b)^(1/2)+a)*(a-b))
^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+1/2/d*a/(a-b)
/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(
a-b))^(1/2))-1/2/d*a/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctan
h((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*b-1/2/d/(a*b)^(1/2)/(a-b
)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a
-b))^(1/2))*b^2-1/2/d*b/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*ta
n(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-1/2/d*b/(a-b)/(((a*b)^(1/2)-a)*(a-b
))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))+1/2/d/(a*
b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a
*b)^(1/2)-a)*(a-b))^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{b\sin(dx+c)^4 - a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -integrate(1/(b*sin(d*x + c)^4 - a), x)
```

Fricas [B] time = 3.06246, size = 2388, normalized size = 20.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(-((a^2 - a*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + 1)/((a
^2 - a*b)*d^2))*log(1/4*b*cos(d*x + c)^2 + 1/2*((a^4 - a^3*b)*d^3*sqrt(b/((
a^5 - 2*a^4*b + a^3*b^2)*d^4))*cos(d*x + c)*sin(d*x + c) - a*b*d*cos(d*x +
c)*sin(d*x + c))*sqrt(-((a^2 - a*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d
^4)) + 1)/((a^2 - a*b)*d^2)) - 1/4*(2*(a^3 - a^2*b)*d^2*cos(d*x + c)^2 - (a
^3 - a^2*b)*d^2)*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - 1/4*b) - 1/8*sqr
t(-((a^2 - a*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + 1)/((a^2 - a*
b)*d^2))*log(1/4*b*cos(d*x + c)^2 - 1/2*((a^4 - a^3*b)*d^3*sqrt(b/((a^5 - 2
*a^4*b + a^3*b^2)*d^4))*cos(d*x + c)*sin(d*x + c) - a*b*d*cos(d*x + c)*sin(
d*x + c))*sqrt(-((a^2 - a*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) +
1)/((a^2 - a*b)*d^2)) - 1/4*(2*(a^3 - a^2*b)*d^2*cos(d*x + c)^2 - (a^3 - a^
2*b)*d^2)*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - 1/4*b) + 1/8*sqrt(((a^2
 - a*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - 1)/((a^2 - a*b)*d^2))
*log(-1/4*b*cos(d*x + c)^2 + 1/2*((a^4 - a^3*b)*d^3*sqrt(b/((a^5 - 2*a^4*b
+ a^3*b^2)*d^4))*cos(d*x + c)*sin(d*x + c) + a*b*d*cos(d*x + c)*sin(d*x + c
))*sqrt(((a^2 - a*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - 1)/((a^2
 - a*b)*d^2)) - 1/4*(2*(a^3 - a^2*b)*d^2*cos(d*x + c)^2 - (a^3 - a^2*b)*d^2
)*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + 1/4*b) - 1/8*sqrt(((a^2 - a*b)*
d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - 1)/((a^2 - a*b)*d^2))*log(-1/
4*b*cos(d*x + c)^2 - 1/2*((a^4 - a^3*b)*d^3*sqrt(b/((a^5 - 2*a^4*b + a^3*b^
2)*d^4))*cos(d*x + c)*sin(d*x + c) + a*b*d*cos(d*x + c)*sin(d*x + c))*sqrt(
((a^2 - a*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - 1)/((a^2 - a*b)*
d^2)) - 1/4*(2*(a^3 - a^2*b)*d^2*cos(d*x + c)^2 - (a^3 - a^2*b)*d^2)*sqrt(b
/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + 1/4*b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.208 $\int \frac{\csc^2(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=139

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{5/4}d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{5/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\cot(c+dx)}{ad}$$

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(5/4)
*Sqrt[Sqrt[a] - Sqrt[b]]*d) - (Sqrt[b]*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[
c + d*x])/a^(1/4)])/(2*a^(5/4)*Sqrt[Sqrt[a] + Sqrt[b]]*d) - Cot[c + d*x]/(a
*d)
```

Rubi [A] time = 0.178117, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3217, 1287, 1130, 205}

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{5/4}d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{5/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\cot(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(5/4)
*Sqrt[Sqrt[a] - Sqrt[b]]*d) - (Sqrt[b]*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[
c + d*x])/a^(1/4)])/(2*a^(5/4)*Sqrt[Sqrt[a] + Sqrt[b]]*d) - Cot[c + d*x]/(a
*d)
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^2(a+2ax^2+(a-b)x^4)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^2}+\frac{bx^2}{a(a+2ax^2+(a-b)x^4)}\right)dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{\cot(c+dx)}{ad}+\frac{b\,\text{Subst}\left(\int \frac{x^2}{a+2ax^2+(a-b)x^4}\,dx, x, \tan(c+dx)\right)}{ad}$$

$$= -\frac{\cot(c+dx)}{ad}+\frac{\left(\left(1-\frac{\sqrt{a}}{\sqrt{b}}\right)b\right)\text{Subst}\left(\int \frac{1}{a-\sqrt{a}\sqrt{b}+(a-b)x^2}\,dx, x, \tan(c+dx)\right)}{2ad}+\frac{\left(\left(1+\frac{\sqrt{a}}{\sqrt{b}}\right)b\right)\text{Su}}{}$$

$$= \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{5/4}\sqrt{\sqrt{a}-\sqrt{b}}d}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{5/4}\sqrt{\sqrt{a}+\sqrt{b}}d}-\frac{\cot(c+dx)}{ad}$$

Mathematica [A] time = 1.04256, size = 143, normalized size = 1.03

$$-\frac{\frac{\sqrt{b}\tan^{-1}\left(\frac{(\sqrt{a}+\sqrt{b})\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}+a}}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{(\sqrt{a}-\sqrt{b})\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}-a}}+2\cot(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -((Sqrt[b]*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[
b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]] + (Sqrt[b]*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan
[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a + Sqrt[a]*Sqrt[b]] + 2*Cot[
c + d*x])/(2*a*d)
```

Maple [B] time = 0.122, size = 518, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a-b*sin(d*x+c)^4),x)
```

```
[Out] -1/d/a/tan(d*x+c)+1/2/d*b/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*
tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+1/2/d*a/(a*b)^(1/2)/(a-b)/(((a*b)
^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2
))*b+1/2/d*b/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/
(((a*b)^(1/2)-a)*(a-b))^(1/2))-1/2/d*a/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(
```

```
a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*b-1/2/
d/a*b^2/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)
^(1/2)+a)*(a-b))^(1/2))-1/2/d/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/
2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*b^2-1/2/d/a*b^2/(
a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-
a)*(a-b))^(1/2))+1/2/d/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arct
anh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] ((a*d*cos(2*d*x + 2*c)^2 + a*d*sin(2*d*x + 2*c)^2 - 2*a*d*cos(2*d*x + 2*c)
+ a*d)*integrate(-4*(4*b^2*cos(6*d*x + 6*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 +
4*b^2*sin(6*d*x + 6*c)^2 + 4*b^2*sin(2*d*x + 2*c)^2 - 4*(8*a*b - 3*b^2)*cos
(4*d*x + 4*c)^2 - b^2*cos(2*d*x + 2*c) - 4*(8*a*b - 3*b^2)*sin(4*d*x + 4*c)
^2 + 2*(8*a*b - 7*b^2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) - (b^2*cos(6*d*x +
 6*c) - 2*b^2*cos(4*d*x + 4*c) + b^2*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) + (
8*b^2*cos(2*d*x + 2*c) - b^2 + 2*(8*a*b - 7*b^2)*cos(4*d*x + 4*c))*cos(6*d*
x + 6*c) + 2*(b^2 + (8*a*b - 7*b^2)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - (b
^2*sin(6*d*x + 6*c) - 2*b^2*sin(4*d*x + 4*c) + b^2*sin(2*d*x + 2*c))*sin(8*
d*x + 8*c) + 2*(4*b^2*sin(2*d*x + 2*c) + (8*a*b - 7*b^2)*sin(4*d*x + 4*c))*
sin(6*d*x + 6*c))/(a*b^2*cos(8*d*x + 8*c)^2 + 16*a*b^2*cos(6*d*x + 6*c)^2 +
 16*a*b^2*cos(2*d*x + 2*c)^2 + a*b^2*sin(8*d*x + 8*c)^2 + 16*a*b^2*sin(6*d*
x + 6*c)^2 + 16*a*b^2*sin(2*d*x + 2*c)^2 - 8*a*b^2*cos(2*d*x + 2*c) + a*b^2
 + 4*(64*a^3 - 48*a^2*b + 9*a*b^2)*cos(4*d*x + 4*c)^2 + 4*(64*a^3 - 48*a^2*
b + 9*a*b^2)*sin(4*d*x + 4*c)^2 + 16*(8*a^2*b - 3*a*b^2)*sin(4*d*x + 4*c)*s
in(2*d*x + 2*c) - 2*(4*a*b^2*cos(6*d*x + 6*c) + 4*a*b^2*cos(2*d*x + 2*c) -
a*b^2 + 2*(8*a^2*b - 3*a*b^2)*cos(4*d*x + 4*c))*cos(8*d*x + 8*c) + 8*(4*a*b
^2*cos(2*d*x + 2*c) - a*b^2 + 2*(8*a^2*b - 3*a*b^2)*cos(4*d*x + 4*c))*cos(6
*d*x + 6*c) - 4*(8*a^2*b - 3*a*b^2 - 4*(8*a^2*b - 3*a*b^2)*cos(2*d*x + 2*c)
)*cos(4*d*x + 4*c) - 4*(2*a*b^2*sin(6*d*x + 6*c) + 2*a*b^2*sin(2*d*x + 2*c)
 + (8*a^2*b - 3*a*b^2)*sin(4*d*x + 4*c))*sin(8*d*x + 8*c) + 16*(2*a*b^2*sin
(2*d*x + 2*c) + (8*a^2*b - 3*a*b^2)*sin(4*d*x + 4*c))*sin(6*d*x + 6*c)), x)
 - 2*sin(2*d*x + 2*c))/(a*d*cos(2*d*x + 2*c)^2 + a*d*sin(2*d*x + 2*c)^2 - 2
*a*d*cos(2*d*x + 2*c) + a*d)
```

Fricas [B] time = 3.46123, size = 2635, normalized size = 18.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/8*(a*d*sqrt(-((a^3 - a^2*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)
) + b)/((a^3 - a^2*b)*d^2))*log(1/4*b^2*cos(d*x + c)^2 - 1/4*b^2 - 1/4*(2*(
a^4 - a^3*b)*d^2*cos(d*x + c)^2 - (a^4 - a^3*b)*d^2)*sqrt(b^3/((a^7 - 2*a^6
*b + a^5*b^2)*d^4)) + 1/2*((a^5 - a^4*b)*d^3*sqrt(b^3/((a^7 - 2*a^6*b + a^5
*b^2)*d^4))*cos(d*x + c)*sin(d*x + c) - a^2*b*d*cos(d*x + c)*sin(d*x + c))*
sqrt(-((a^3 - a^2*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) + b)/((a
```

```
^3 - a^2*b)*d^2)))*sin(d*x + c) - a*d*sqrt(-((a^3 - a^2*b)*d^2*sqrt(b^3/((a
^7 - 2*a^6*b + a^5*b^2)*d^4)) + b)/((a^3 - a^2*b)*d^2))*log(1/4*b^2*cos(d*x
 + c)^2 - 1/4*b^2 - 1/4*(2*(a^4 - a^3*b)*d^2*cos(d*x + c)^2 - (a^4 - a^3*b)
*d^2)*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - 1/2*((a^5 - a^4*b)*d^3*sq
rt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4))*cos(d*x + c)*sin(d*x + c) - a^2*b*d
*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^3 - a^2*b)*d^2*sqrt(b^3/((a^7 - 2*a^6
*b + a^5*b^2)*d^4)) + b)/((a^3 - a^2*b)*d^2)))*sin(d*x + c) + a*d*sqrt(((a^
3 - a^2*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - b)/((a^3 - a^2*b
)*d^2))*log(-1/4*b^2*cos(d*x + c)^2 + 1/4*b^2 - 1/4*(2*(a^4 - a^3*b)*d^2*co
s(d*x + c)^2 - (a^4 - a^3*b)*d^2)*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4))
 + 1/2*((a^5 - a^4*b)*d^3*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4))*cos(d*x
 + c)*sin(d*x + c) + a^2*b*d*cos(d*x + c)*sin(d*x + c))*sqrt(((a^3 - a^2*b)
*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - b)/((a^3 - a^2*b)*d^2)))*s
in(d*x + c) - a*d*sqrt(((a^3 - a^2*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^
2)*d^4)) - b)/((a^3 - a^2*b)*d^2))*log(-1/4*b^2*cos(d*x + c)^2 + 1/4*b^2 -
1/4*(2*(a^4 - a^3*b)*d^2*cos(d*x + c)^2 - (a^4 - a^3*b)*d^2)*sqrt(b^3/((a^7
 - 2*a^6*b + a^5*b^2)*d^4)) - 1/2*((a^5 - a^4*b)*d^3*sqrt(b^3/((a^7 - 2*a^6
*b + a^5*b^2)*d^4))*cos(d*x + c)*sin(d*x + c) + a^2*b*d*cos(d*x + c)*sin(d*
x + c))*sqrt(((a^3 - a^2*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) -
 b)/((a^3 - a^2*b)*d^2)))*sin(d*x + c) + 8*cos(d*x + c))/(a*d*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a-b*sin(d*x+c)**4),x)

[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a-b*sin(d*x+c)^4),x, algorithm="giac")

[Out] Exception raised: NotImplementedError
```

3.209 $\int \frac{\csc^4(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=149

$$\frac{b\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{7/4}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{b\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{7/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\cot^3(c+dx)}{3ad}-\frac{\cot(c+dx)}{ad}$$

```
[Out] (b*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(7/4)*Sqrt[
Sqrt[a] - Sqrt[b]]*d) + (b*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^
(1/4)])/(2*a^(7/4)*Sqrt[Sqrt[a] + Sqrt[b]]*d) - Cot[c + d*x]/(a*d) - Cot[c
+ d*x]^3/(3*a*d)
```

Rubi [A] time = 0.193441, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3217, 1287, 1166, 205}

$$\frac{b\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{7/4}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{b\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{7/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\cot^3(c+dx)}{3ad}-\frac{\cot(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (b*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(7/4)*Sqrt[
Sqrt[a] - Sqrt[b]]*d) + (b*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^
(1/4)])/(2*a^(7/4)*Sqrt[Sqrt[a] + Sqrt[b]]*d) - Cot[c + d*x]/(a*d) - Cot[c
+ d*x]^3/(3*a*d)
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^4(c+dx)}{a-b\sin^4(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{x^4(a+2ax^2+(a-b)x^4)}\,dx, x, \tan(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^4}+\frac{1}{ax^2}+\frac{b(1+x^2)}{a(a+2ax^2+(a-b)x^4)}\right)dx, x, \tan(c+dx)\right)}{d}\\ &= -\frac{\cot(c+dx)}{ad}-\frac{\cot^3(c+dx)}{3ad}+\frac{b\,\text{Subst}\left(\int \frac{1+x^2}{a+2ax^2+(a-b)x^4}\,dx, x, \tan(c+dx)\right)}{ad}\\ &= -\frac{\cot(c+dx)}{ad}-\frac{\cot^3(c+dx)}{3ad}+\frac{\left(\left(\sqrt{a}+\sqrt{b}\right)b\right)\text{Subst}\left(\int \frac{1}{a+\sqrt{a}\sqrt{b}+(a-b)x^2}\,dx, x, \tan(c+dx)\right)}{2a^{3/2}d}\\ &= \frac{b\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{7/4}\sqrt{\sqrt{a}-\sqrt{b}}\,d}+\frac{b\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{7/4}\sqrt{\sqrt{a}+\sqrt{b}}\,d}-\frac{\cot(c+dx)}{ad}-\frac{\cot^3(c+dx)}{3ad}\end{aligned}$$

Mathematica [A] time = 1.52324, size = 165, normalized size = 1.11

$$\frac{\frac{3b\tan^{-1}\left(\frac{(\sqrt{a}+\sqrt{b})\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}+a}}-\frac{3b\tanh^{-1}\left(\frac{(\sqrt{a}-\sqrt{b})\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}-a}}-4\sqrt{a}\cot(c+dx)-2\sqrt{a}\cot(c+dx)\csc^2(c+dx)}{6a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] ((3*b*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])
/Sqrt[a + Sqrt[a]*Sqrt[b]] - (3*b*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x]
)/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a + Sqrt[a]*Sqrt[b]] - 4*Sqrt[a]*Cot[c
 + d*x] - 2*Sqrt[a]*Cot[c + d*x]*Csc[c + d*x]^2)/(6*a^(3/2)*d)
```

Maple [B] time = 0.138, size = 542, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4/(a-b*sin(d*x+c)^4),x)
```

```
[Out] 1/2/d/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+
c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*b^2+1/2/d*b/(a-b)/(((a*b)^(1/2)+a)*(a-b))
^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+1/2/d*b/(a-b)
/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(
a-b))^(1/2))-1/2/d/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh(
```

```
(-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*b^2-1/2/d/a*b^3/(a*b)^(1/2
)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)
+a)*(a-b))^(1/2))-1/2/d/a*b^2/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a
-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-1/2/d/a*b^2/(a-b)/(((a*b)^(1/
2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))
+1/2/d/a*b^3/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)
*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-1/3/d/a/tan(d*x+c)^3-1/d/a/tan(d
*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 3.33001, size = 2907, normalized size = 19.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/24*(3*(a*d*cos(d*x + c)^2 - a*d)*sqrt(-((a^4 - a^3*b)*d^2*sqrt(b^5/((a^9
 - 2*a^8*b + a^7*b^2)*d^4)) + b^2)/((a^4 - a^3*b)*d^2))*log(1/4*b^4*cos(d*x
 + c)^2 - 1/4*b^4 - 1/4*(2*(a^5*b - a^4*b^2)*d^2*cos(d*x + c)^2 - (a^5*b -
a^4*b^2)*d^2)*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) + 1/2*(a^2*b^3*d*co
s(d*x + c)*sin(d*x + c) - (a^7 - a^6*b)*d^3*sqrt(b^5/((a^9 - 2*a^8*b + a^7*
b^2)*d^4))*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^4 - a^3*b)*d^2*sqrt(b^5/((a
^9 - 2*a^8*b + a^7*b^2)*d^4)) + b^2)/((a^4 - a^3*b)*d^2)))*sin(d*x + c) - 3
*(a*d*cos(d*x + c)^2 - a*d)*sqrt(-((a^4 - a^3*b)*d^2*sqrt(b^5/((a^9 - 2*a^8
*b + a^7*b^2)*d^4)) + b^2)/((a^4 - a^3*b)*d^2))*log(1/4*b^4*cos(d*x + c)^2
- 1/4*b^4 - 1/4*(2*(a^5*b - a^4*b^2)*d^2*cos(d*x + c)^2 - (a^5*b - a^4*b^2)
*d^2)*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) - 1/2*(a^2*b^3*d*cos(d*x +
c)*sin(d*x + c) - (a^7 - a^6*b)*d^3*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4
))*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^4 - a^3*b)*d^2*sqrt(b^5/((a^9 - 2*a
^8*b + a^7*b^2)*d^4)) + b^2)/((a^4 - a^3*b)*d^2)))*sin(d*x + c) - 3*(a*d*co
s(d*x + c)^2 - a*d)*sqrt(((a^4 - a^3*b)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*
b^2)*d^4)) - b^2)/((a^4 - a^3*b)*d^2))*log(-1/4*b^4*cos(d*x + c)^2 + 1/4*b^
4 - 1/4*(2*(a^5*b - a^4*b^2)*d^2*cos(d*x + c)^2 - (a^5*b - a^4*b^2)*d^2)*sq
rt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) + 1/2*(a^2*b^3*d*cos(d*x + c)*sin(d
*x + c) + (a^7 - a^6*b)*d^3*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4))*cos(d
*x + c)*sin(d*x + c))*sqrt(((a^4 - a^3*b)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^
7*b^2)*d^4)) - b^2)/((a^4 - a^3*b)*d^2)))*sin(d*x + c) + 3*(a*d*cos(d*x + c
)^2 - a*d)*sqrt(((a^4 - a^3*b)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)
) - b^2)/((a^4 - a^3*b)*d^2))*log(-1/4*b^4*cos(d*x + c)^2 + 1/4*b^4 - 1/4*(
2*(a^5*b - a^4*b^2)*d^2*cos(d*x + c)^2 - (a^5*b - a^4*b^2)*d^2)*sqrt(b^5/((
a^9 - 2*a^8*b + a^7*b^2)*d^4)) - 1/2*(a^2*b^3*d*cos(d*x + c)*sin(d*x + c) +
 (a^7 - a^6*b)*d^3*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4))*cos(d*x + c)*s
in(d*x + c))*sqrt(((a^4 - a^3*b)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^
4)) - b^2)/((a^4 - a^3*b)*d^2)))*sin(d*x + c) + 16*cos(d*x + c)^3 - 24*cos(
```

```
d*x + c))/((a*d*cos(d*x + c)^2 - a*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.210 $\int \frac{\csc^6(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=178

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{9/4}d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{9/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{(a+b)\cot(c+dx)}{a^2d}-\frac{\cot^5(c+dx)}{5ad}-\frac{2\cot^3(c+dx)}{3ad}$$

```
[Out] (b^(3/2)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(9/4)
*Sqrt[Sqrt[a] - Sqrt[b]]*d) - (b^(3/2)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[
c + d*x])/a^(1/4)])/(2*a^(9/4)*Sqrt[Sqrt[a] + Sqrt[b]]*d) - ((a + b)*Cot[c
+ d*x])/(a^2*d) - (2*Cot[c + d*x]^3)/(3*a*d) - Cot[c + d*x]^5/(5*a*d)
```

Rubi [A] time = 0.202967, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3217, 1287, 1130, 205}

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{9/4}d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{9/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{(a+b)\cot(c+dx)}{a^2d}-\frac{\cot^5(c+dx)}{5ad}-\frac{2\cot^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (b^(3/2)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(9/4)
*Sqrt[Sqrt[a] - Sqrt[b]]*d) - (b^(3/2)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[
c + d*x])/a^(1/4)])/(2*a^(9/4)*Sqrt[Sqrt[a] + Sqrt[b]]*d) - ((a + b)*Cot[c
+ d*x])/(a^2*d) - (2*Cot[c + d*x]^3)/(3*a*d) - Cot[c + d*x]^5/(5*a*d)
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\csc^6(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^4}{x^6(a+2ax^2+(a-b)x^4)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^6}+\frac{2}{ax^4}+\frac{a+b}{a^2x^2}+\frac{b^2x^2}{a^2(a+2ax^2+(a-b)x^4)}\right)dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{(a+b)\cot(c+dx)}{a^2d} - \frac{2\cot^3(c+dx)}{3ad} - \frac{\cot^5(c+dx)}{5ad} + \frac{b^2\,\text{Subst}\left(\int \frac{x^2}{a+2ax^2+(a-b)x^4}\,dx, x, \text{ta}\right.}{a^2d}$$

$$= -\frac{(a+b)\cot(c+dx)}{a^2d} - \frac{2\cot^3(c+dx)}{3ad} - \frac{\cot^5(c+dx)}{5ad} + \frac{\left(\left(\sqrt{a}+\sqrt{b}\right)b^{3/2}\right)\text{Subst}\left(\int \frac{}{a+\sqrt{a}\sqrt{b}}\right.}{2a^2d}$$

$$= \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{9/4}\sqrt{\sqrt{a}-\sqrt{b}}\,d} - \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{9/4}\sqrt{\sqrt{a}+\sqrt{b}}\,d} - \frac{(a+b)\cot(c+dx)}{a^2d} - \frac{2\cot^3}{3}$$

Mathematica [A] time = 4.56645, size = 174, normalized size = 0.98

$$-\frac{\frac{15b^{3/2}\tan^{-1}\left(\frac{(\sqrt{a}+\sqrt{b})\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}+a}} + \frac{15b^{3/2}\tanh^{-1}\left(\frac{(\sqrt{a}-\sqrt{b})\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}-a}} + 2\cot(c+dx)\left(3a\csc^4(c+dx)+4a\csc^2(c+dx)+8a+15b\right.}{30a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -((15*b^(3/2)*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/Sqrt[a + Sqrt[a]*Sq
rt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]] + (15*b^(3/2)*ArcTanh[((Sqrt[a] - Sqrt[b
])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a + Sqrt[a]*Sqrt[b]] +
2*Cot[c + d*x]*(8*a + 15*b + 4*a*Csc[c + d*x]^2 + 3*a*Csc[c + d*x]^4))/(30*
a^2*d)
```

Maple [B] time = 0.137, size = 585, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6/(a-b*sin(d*x+c)^4),x)
```

```
[Out] 1/2/d/a*b^2/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((
a*b)^(1/2)+a)*(a-b))^(1/2))+1/2/d/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))
^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*b^2+1/2/d/a*b
^2/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1
```

```
/2)-a)*(a-b))^(1/2))-1/2/d/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*
arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*b^2-1/2/d/a^2*b^3/
(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a
)*(a-b))^(1/2))-1/2/d/a*b^3/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)
*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-1/2/d/a^2*b^3/(a-b)
/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(
a-b))^(1/2))+1/2/d/a*b^3/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*ar
ctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-1/5/d/a/tan(d*x+c)^5
-1/d/a/tan(d*x+c)-1/d/a^2/tan(d*x+c)*b-2/3/d/a/tan(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] 1/15*(300*b*cos(4*d*x + 4*c)*sin(2*d*x + 2*c) + 10*(3*b*sin(8*d*x + 8*c) -
12*b*sin(6*d*x + 6*c) + 2*(8*a + 9*b)*sin(4*d*x + 4*c) - 4*(2*a + 3*b)*sin(
2*d*x + 2*c))*cos(10*d*x + 10*c) + 50*(6*b*sin(6*d*x + 6*c) - 4*(4*a + 3*b)
*sin(4*d*x + 4*c) + (8*a + 9*b)*sin(2*d*x + 2*c))*cos(8*d*x + 8*c) + 200*((
8*a + 3*b)*sin(4*d*x + 4*c) - (4*a + 3*b)*sin(2*d*x + 2*c))*cos(6*d*x + 6*c
) + 15*(a^2*d*cos(10*d*x + 10*c)^2 + 25*a^2*d*cos(8*d*x + 8*c)^2 + 100*a^2*
d*cos(6*d*x + 6*c)^2 + 100*a^2*d*cos(4*d*x + 4*c)^2 + 25*a^2*d*cos(2*d*x +
2*c)^2 + a^2*d*sin(10*d*x + 10*c)^2 + 25*a^2*d*sin(8*d*x + 8*c)^2 + 100*a^2
*d*sin(6*d*x + 6*c)^2 + 100*a^2*d*sin(4*d*x + 4*c)^2 - 100*a^2*d*sin(4*d*x
+ 4*c)*sin(2*d*x + 2*c) + 25*a^2*d*sin(2*d*x + 2*c)^2 - 10*a^2*d*cos(2*d*x
+ 2*c) + a^2*d - 2*(5*a^2*d*cos(8*d*x + 8*c) - 10*a^2*d*cos(6*d*x + 6*c) +
10*a^2*d*cos(4*d*x + 4*c) - 5*a^2*d*cos(2*d*x + 2*c) + a^2*d)*cos(10*d*x +
10*c) - 10*(10*a^2*d*cos(6*d*x + 6*c) - 10*a^2*d*cos(4*d*x + 4*c) + 5*a^2*d
*cos(2*d*x + 2*c) - a^2*d)*cos(8*d*x + 8*c) - 20*(10*a^2*d*cos(4*d*x + 4*c)
 - 5*a^2*d*cos(2*d*x + 2*c) + a^2*d)*cos(6*d*x + 6*c) - 20*(5*a^2*d*cos(2*d
*x + 2*c) - a^2*d)*cos(4*d*x + 4*c) - 10*(a^2*d*sin(8*d*x + 8*c) - 2*a^2*d*
sin(6*d*x + 6*c) + 2*a^2*d*sin(4*d*x + 4*c) - a^2*d*sin(2*d*x + 2*c))*sin(1
0*d*x + 10*c) - 50*(2*a^2*d*sin(6*d*x + 6*c) - 2*a^2*d*sin(4*d*x + 4*c) + a
^2*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) - 100*(2*a^2*d*sin(4*d*x + 4*c) - a
^2*d*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*integrate(-4*(4*b^3*cos(6*d*x + 6*
c)^2 + 4*b^3*cos(2*d*x + 2*c)^2 + 4*b^3*sin(6*d*x + 6*c)^2 + 4*b^3*sin(2*d*
x + 2*c)^2 - b^3*cos(2*d*x + 2*c) - 4*(8*a*b^2 - 3*b^3)*cos(4*d*x + 4*c)^2
- 4*(8*a*b^2 - 3*b^3)*sin(4*d*x + 4*c)^2 + 2*(8*a*b^2 - 7*b^3)*sin(4*d*x +
4*c)*sin(2*d*x + 2*c) - (b^3*cos(6*d*x + 6*c) - 2*b^3*cos(4*d*x + 4*c) + b^
3*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) + (8*b^3*cos(2*d*x + 2*c) - b^3 + 2*(8
*a*b^2 - 7*b^3)*cos(4*d*x + 4*c))*cos(6*d*x + 6*c) + 2*(b^3 + (8*a*b^2 - 7*
b^3)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - (b^3*sin(6*d*x + 6*c) - 2*b^3*sin
(4*d*x + 4*c) + b^3*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 2*(4*b^3*sin(2*d*x
 + 2*c) + (8*a*b^2 - 7*b^3)*sin(4*d*x + 4*c))*sin(6*d*x + 6*c))/(a^2*b^2*co
s(8*d*x + 8*c)^2 + 16*a^2*b^2*cos(6*d*x + 6*c)^2 + 16*a^2*b^2*cos(2*d*x + 2
*c)^2 + a^2*b^2*sin(8*d*x + 8*c)^2 + 16*a^2*b^2*sin(6*d*x + 6*c)^2 + 16*a^2
*b^2*sin(2*d*x + 2*c)^2 - 8*a^2*b^2*cos(2*d*x + 2*c) + a^2*b^2 + 4*(64*a^4
- 48*a^3*b + 9*a^2*b^2)*cos(4*d*x + 4*c)^2 + 4*(64*a^4 - 48*a^3*b + 9*a^2*b
^2)*sin(4*d*x + 4*c)^2 + 16*(8*a^3*b - 3*a^2*b^2)*sin(4*d*x + 4*c)*sin(2*d*
x + 2*c) - 2*(4*a^2*b^2*cos(6*d*x + 6*c) + 4*a^2*b^2*cos(2*d*x + 2*c) - a^2
*b^2 + 2*(8*a^3*b - 3*a^2*b^2)*cos(4*d*x + 4*c))*cos(8*d*x + 8*c) + 8*(4*a^
2*b^2*cos(2*d*x + 2*c) - a^2*b^2 + 2*(8*a^3*b - 3*a^2*b^2)*cos(4*d*x + 4*c)
)*cos(6*d*x + 6*c) - 4*(8*a^3*b - 3*a^2*b^2 - 4*(8*a^3*b - 3*a^2*b^2)*cos(2
*d*x + 2*c))*cos(4*d*x + 4*c) - 4*(2*a^2*b^2*sin(6*d*x + 6*c) + 2*a^2*b^2*s
```

```
in(2*d*x + 2*c) + (8*a^3*b - 3*a^2*b^2)*sin(4*d*x + 4*c))*sin(8*d*x + 8*c)
+ 16*(2*a^2*b^2*sin(2*d*x + 2*c) + (8*a^3*b - 3*a^2*b^2)*sin(4*d*x + 4*c))*
sin(6*d*x + 6*c)), x) - 2*(15*b*cos(8*d*x + 8*c) - 60*b*cos(6*d*x + 6*c) +
10*(8*a + 9*b)*cos(4*d*x + 4*c) - 20*(2*a + 3*b)*cos(2*d*x + 2*c) + 8*a + 1
5*b)*sin(10*d*x + 10*c) - 10*(30*b*cos(6*d*x + 6*c) - 20*(4*a + 3*b)*cos(4*
d*x + 4*c) + 5*(8*a + 9*b)*cos(2*d*x + 2*c) - 8*a - 12*b)*sin(8*d*x + 8*c)
- 20*(10*(8*a + 3*b)*cos(4*d*x + 4*c) - 10*(4*a + 3*b)*cos(2*d*x + 2*c) + 8
*a + 9*b)*sin(6*d*x + 6*c) - 60*(5*b*cos(2*d*x + 2*c) - 2*b)*sin(4*d*x + 4*
c) - 30*b*sin(2*d*x + 2*c))/(a^2*d*cos(10*d*x + 10*c)^2 + 25*a^2*d*cos(8*d*
x + 8*c)^2 + 100*a^2*d*cos(6*d*x + 6*c)^2 + 100*a^2*d*cos(4*d*x + 4*c)^2 +
25*a^2*d*cos(2*d*x + 2*c)^2 + a^2*d*sin(10*d*x + 10*c)^2 + 25*a^2*d*sin(8*d
*x + 8*c)^2 + 100*a^2*d*sin(6*d*x + 6*c)^2 + 100*a^2*d*sin(4*d*x + 4*c)^2 -
 100*a^2*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 25*a^2*d*sin(2*d*x + 2*c)^2
- 10*a^2*d*cos(2*d*x + 2*c) + a^2*d - 2*(5*a^2*d*cos(8*d*x + 8*c) - 10*a^2*
d*cos(6*d*x + 6*c) + 10*a^2*d*cos(4*d*x + 4*c) - 5*a^2*d*cos(2*d*x + 2*c) +
 a^2*d)*cos(10*d*x + 10*c) - 10*(10*a^2*d*cos(6*d*x + 6*c) - 10*a^2*d*cos(4
*d*x + 4*c) + 5*a^2*d*cos(2*d*x + 2*c) - a^2*d)*cos(8*d*x + 8*c) - 20*(10*a
^2*d*cos(4*d*x + 4*c) - 5*a^2*d*cos(2*d*x + 2*c) + a^2*d)*cos(6*d*x + 6*c)
- 20*(5*a^2*d*cos(2*d*x + 2*c) - a^2*d)*cos(4*d*x + 4*c) - 10*(a^2*d*sin(8*
d*x + 8*c) - 2*a^2*d*sin(6*d*x + 6*c) + 2*a^2*d*sin(4*d*x + 4*c) - a^2*d*si
n(2*d*x + 2*c))*sin(10*d*x + 10*c) - 50*(2*a^2*d*sin(6*d*x + 6*c) - 2*a^2*d
*sin(4*d*x + 4*c) + a^2*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) - 100*(2*a^2*d
*sin(4*d*x + 4*c) - a^2*d*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))
```

Fricas [B] time = 3.64392, size = 3224, normalized size = 18.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/120*(8*(8*a + 15*b)*cos(d*x + c)^5 - 80*(2*a + 3*b)*cos(d*x + c)^3 - 15*
(a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2 + a^2*d)*sqrt(-((a^5 - a^4*b
)*sqrt(b^7/((a^11 - 2*a^10*b + a^9*b^2)*d^4))*d^2 + b^3)/((a^5 - a^4*b)*d^2
))*log(1/4*b^5*cos(d*x + c)^2 - 1/4*b^5 - 1/4*(2*(a^6*b - a^5*b^2)*d^2*cos(
d*x + c)^2 - (a^6*b - a^5*b^2)*d^2)*sqrt(b^7/((a^11 - 2*a^10*b + a^9*b^2)*d
^4)) + 1/2*(a^3*b^3*d*cos(d*x + c)*sin(d*x + c) - (a^8 - a^7*b)*sqrt(b^7/((
a^11 - 2*a^10*b + a^9*b^2)*d^4))*d^3*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^5
 - a^4*b)*sqrt(b^7/((a^11 - 2*a^10*b + a^9*b^2)*d^4))*d^2 + b^3)/((a^5 - a^
4*b)*d^2)))*sin(d*x + c) + 15*(a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^
2 + a^2*d)*sqrt(-((a^5 - a^4*b)*sqrt(b^7/((a^11 - 2*a^10*b + a^9*b^2)*d^4))
*d^2 + b^3)/((a^5 - a^4*b)*d^2))*log(1/4*b^5*cos(d*x + c)^2 - 1/4*b^5 - 1/4
*(2*(a^6*b - a^5*b^2)*d^2*cos(d*x + c)^2 - (a^6*b - a^5*b^2)*d^2)*sqrt(b^7/
((a^11 - 2*a^10*b + a^9*b^2)*d^4)) - 1/2*(a^3*b^3*d*cos(d*x + c)*sin(d*x +
c) - (a^8 - a^7*b)*sqrt(b^7/((a^11 - 2*a^10*b + a^9*b^2)*d^4))*d^3*cos(d*x
+ c)*sin(d*x + c))*sqrt(-((a^5 - a^4*b)*sqrt(b^7/((a^11 - 2*a^10*b + a^9*b^
2)*d^4))*d^2 + b^3)/((a^5 - a^4*b)*d^2)))*sin(d*x + c) + 15*(a^2*d*cos(d*x
+ c)^4 - 2*a^2*d*cos(d*x + c)^2 + a^2*d)*sqrt(((a^5 - a^4*b)*sqrt(b^7/((a^1
1 - 2*a^10*b + a^9*b^2)*d^4))*d^2 - b^3)/((a^5 - a^4*b)*d^2))*log(-1/4*b^5*
cos(d*x + c)^2 + 1/4*b^5 - 1/4*(2*(a^6*b - a^5*b^2)*d^2*cos(d*x + c)^2 - (a
^6*b - a^5*b^2)*d^2)*sqrt(b^7/((a^11 - 2*a^10*b + a^9*b^2)*d^4)) + 1/2*(a^3
*b^3*d*cos(d*x + c)*sin(d*x + c) + (a^8 - a^7*b)*sqrt(b^7/((a^11 - 2*a^10*b
 + a^9*b^2)*d^4))*d^3*cos(d*x + c)*sin(d*x + c))*sqrt(((a^5 - a^4*b)*sqrt(b
^7/((a^11 - 2*a^10*b + a^9*b^2)*d^4))*d^2 - b^3)/((a^5 - a^4*b)*d^2)))*sin(
d*x + c) - 15*(a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2 + a^2*d)*sqrt(
((a^5 - a^4*b)*sqrt(b^7/((a^11 - 2*a^10*b + a^9*b^2)*d^4))*d^2 - b^3)/((a^5
```

```
 - a^4*b)*d^2))*log(-1/4*b^5*cos(d*x + c)^2 + 1/4*b^5 - 1/4*(2*(a^6*b - a^5
*b^2)*d^2*cos(d*x + c)^2 - (a^6*b - a^5*b^2)*d^2)*sqrt(b^7/((a^11 - 2*a^10*
b + a^9*b^2)*d^4)) - 1/2*(a^3*b^3*d*cos(d*x + c)*sin(d*x + c) + (a^8 - a^7*
b)*sqrt(b^7/((a^11 - 2*a^10*b + a^9*b^2)*d^4))*d^3*cos(d*x + c)*sin(d*x + c
))*sqrt(((a^5 - a^4*b)*sqrt(b^7/((a^11 - 2*a^10*b + a^9*b^2)*d^4))*d^2 - b^
3)/((a^5 - a^4*b)*d^2)))*sin(d*x + c) + 120*(a + b)*cos(d*x + c))/((a^2*d*c
os(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2 + a^2*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.211 $\int \frac{\csc^8(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=197

$$\frac{b^2\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{11/4}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{b^2\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{11/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{(3a+b)\cot^3(c+dx)}{3a^2d}-\frac{(a+b)\cot(c+dx)}{a^2d}-\frac{\cot^7(c+dx)}{7ad}$$

```
[Out] (b^2*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(11/4)*Sq
rt[Sqrt[a] - Sqrt[b]]*d) + (b^2*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x
])/a^(1/4)])/(2*a^(11/4)*Sqrt[Sqrt[a] + Sqrt[b]]*d) - ((a + b)*Cot[c + d*x]
)/(a^2*d) - ((3*a + b)*Cot[c + d*x]^3)/(3*a^2*d) - (3*Cot[c + d*x]^5)/(5*a*
d) - Cot[c + d*x]^7/(7*a*d)
```

Rubi [A] time = 0.235356, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3217, 1287, 1166, 205}

$$\frac{b^2\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{11/4}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{b^2\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{11/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{(3a+b)\cot^3(c+dx)}{3a^2d}-\frac{(a+b)\cot(c+dx)}{a^2d}-\frac{\cot^7(c+dx)}{7ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^8/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (b^2*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(11/4)*Sq
rt[Sqrt[a] - Sqrt[b]]*d) + (b^2*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x
])/a^(1/4)])/(2*a^(11/4)*Sqrt[Sqrt[a] + Sqrt[b]]*d) - ((a + b)*Cot[c + d*x]
)/(a^2*d) - ((3*a + b)*Cot[c + d*x]^3)/(3*a^2*d) - (3*Cot[c + d*x]^5)/(5*a*
d) - Cot[c + d*x]^7/(7*a*d)
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\csc^8(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^5}{x^8(a+2ax^2+(a-b)x^4)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^8}+\frac{3}{ax^6}+\frac{3a+b}{a^2x^4}+\frac{a+b}{a^2x^2}+\frac{b^2(1+x^2)}{a^2(a+2ax^2+(a-b)x^4)}\right) dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{(a+b)\cot(c+dx)}{a^2d} - \frac{(3a+b)\cot^3(c+dx)}{3a^2d} - \frac{3\cot^5(c+dx)}{5ad} - \frac{\cot^7(c+dx)}{7ad} + \frac{b^2\,\text{Subst}\left(\int \right.}{}$$

$$= -\frac{(a+b)\cot(c+dx)}{a^2d} - \frac{(3a+b)\cot^3(c+dx)}{3a^2d} - \frac{3\cot^5(c+dx)}{5ad} - \frac{\cot^7(c+dx)}{7ad} + \frac{\left(\left(\sqrt{a}+\sqrt{b}\right)b\right.}{}$$

$$= \frac{b^2\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{11/4}\sqrt{\sqrt{a}-\sqrt{b}}d} + \frac{b^2\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{11/4}\sqrt{\sqrt{a}+\sqrt{b}}d} - \frac{(a+b)\cot(c+dx)}{a^2d} - \frac{(3a+b)\cot}{3a^2}$$

Mathematica [A] time = 6.32048, size = 277, normalized size = 1.41

$$\frac{b^2\tan^{-1}\left(\frac{(\sqrt{a}\sqrt{b}+b)\tan(c+dx)}{\sqrt{b}\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{2a^{5/2}d\sqrt{\sqrt{a}\sqrt{b}+a}} - \frac{b^2\tanh^{-1}\left(\frac{(\sqrt{a}\sqrt{b}-b)\tan(c+dx)}{\sqrt{b}\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{2a^{5/2}d\sqrt{\sqrt{a}\sqrt{b}-a}} + \frac{\csc^3(c+dx)(-24a\cos(c+dx)-35b\cos(c+dx))}{105a^2d} - \frac{2\csc}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^8/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (b^2*ArcTan[((Sqrt[a]*Sqrt[b] + b)*Tan[c + d*x])/(Sqrt[a + Sqrt[a]*Sqrt[b]]
*Sqrt[b])])/(2*a^(5/2)*Sqrt[a + Sqrt[a]*Sqrt[b]]*d) - (b^2*ArcTanh[((Sqrt[a
]*Sqrt[b] - b)*Tan[c + d*x])/(Sqrt[-a + Sqrt[a]*Sqrt[b]]*Sqrt[b])])/(2*a^(5
/2)*Sqrt[-a + Sqrt[a]*Sqrt[b]]*d) - (2*(24*a*Cos[c + d*x] + 35*b*Cos[c + d*
x])*Csc[c + d*x])/(105*a^2*d) + ((-24*a*Cos[c + d*x] - 35*b*Cos[c + d*x])*C
sc[c + d*x]^3)/(105*a^2*d) - (6*Cot[c + d*x]*Csc[c + d*x]^4)/(35*a*d) - (Co
t[c + d*x]*Csc[c + d*x]^6)/(7*a*d)
```

Maple [B] time = 0.141, size = 624, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^8/(a-b*sin(d*x+c)^4),x)
```

```
[Out] 1/2/d/a*b^3/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*ta
n(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+1/2/d/a*b^2/(a-b)/(((a*b)^(1/2)+a)*
(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+1/2/d/a
*b^2/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^
(1/2)-a)*(a-b))^(1/2))-1/2/d/a*b^3/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b)
)^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-1/2/d/a^2*
b^4/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)
/(((a*b)^(1/2)+a)*(a-b))^(1/2))-1/2/d/a^2*b^3/(a-b)/(((a*b)^(1/2)+a)*(a-b))
^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-1/2/d/a^2*b^3
/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2
)-a)*(a-b))^(1/2))+1/2/d/a^2*b^4/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^
(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-1/7/d/a/tan(
d*x+c)^7-1/d/a/tan(d*x+c)-1/d/a^2/tan(d*x+c)*b-1/d/a/tan(d*x+c)^3-1/3/d/a^2
/tan(d*x+c)^3*b-3/5/d/a/tan(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 3.47417, size = 3502, normalized size = 17.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/840*(16*(24*a + 35*b)*cos(d*x + c)^7 - 56*(24*a + 35*b)*cos(d*x + c)^5 +
 560*(3*a + 4*b)*cos(d*x + c)^3 + 105*(a^2*d*cos(d*x + c)^6 - 3*a^2*d*cos(d
*x + c)^4 + 3*a^2*d*cos(d*x + c)^2 - a^2*d)*sqrt(-(b^4 + (a^6 - a^5*b)*sqrt
(b^9/((a^13 - 2*a^12*b + a^11*b^2)*d^4))*d^2)/((a^6 - a^5*b)*d^2))*log(1/4*
b^7*cos(d*x + c)^2 - 1/4*b^7 - 1/4*(2*(a^7*b^2 - a^6*b^3)*d^2*cos(d*x + c)^
2 - (a^7*b^2 - a^6*b^3)*d^2)*sqrt(b^9/((a^13 - 2*a^12*b + a^11*b^2)*d^4)) +
 1/2*(a^3*b^5*d*cos(d*x + c)*sin(d*x + c) - (a^10 - a^9*b)*sqrt(b^9/((a^13
- 2*a^12*b + a^11*b^2)*d^4))*d^3*cos(d*x + c)*sin(d*x + c))*sqrt(-(b^4 + (a
^6 - a^5*b)*sqrt(b^9/((a^13 - 2*a^12*b + a^11*b^2)*d^4))*d^2)/((a^6 - a^5*b
)*d^2)))*sin(d*x + c) - 105*(a^2*d*cos(d*x + c)^6 - 3*a^2*d*cos(d*x + c)^4
+ 3*a^2*d*cos(d*x + c)^2 - a^2*d)*sqrt(-(b^4 + (a^6 - a^5*b)*sqrt(b^9/((a^1
3 - 2*a^12*b + a^11*b^2)*d^4))*d^2)/((a^6 - a^5*b)*d^2))*log(1/4*b^7*cos(d*
x + c)^2 - 1/4*b^7 - 1/4*(2*(a^7*b^2 - a^6*b^3)*d^2*cos(d*x + c)^2 - (a^7*b
^2 - a^6*b^3)*d^2)*sqrt(b^9/((a^13 - 2*a^12*b + a^11*b^2)*d^4)) - 1/2*(a^3*
b^5*d*cos(d*x + c)*sin(d*x + c) - (a^10 - a^9*b)*sqrt(b^9/((a^13 - 2*a^12*b
 + a^11*b^2)*d^4))*d^3*cos(d*x + c)*sin(d*x + c))*sqrt(-(b^4 + (a^6 - a^5*b
)*sqrt(b^9/((a^13 - 2*a^12*b + a^11*b^2)*d^4))*d^2)/((a^6 - a^5*b)*d^2)))*s
in(d*x + c) - 105*(a^2*d*cos(d*x + c)^6 - 3*a^2*d*cos(d*x + c)^4 + 3*a^2*d*
cos(d*x + c)^2 - a^2*d)*sqrt(-(b^4 - (a^6 - a^5*b)*sqrt(b^9/((a^13 - 2*a^12
*b + a^11*b^2)*d^4))*d^2)/((a^6 - a^5*b)*d^2))*log(-1/4*b^7*cos(d*x + c)^2
+ 1/4*b^7 - 1/4*(2*(a^7*b^2 - a^6*b^3)*d^2*cos(d*x + c)^2 - (a^7*b^2 - a^6*
b^3)*d^2)*sqrt(b^9/((a^13 - 2*a^12*b + a^11*b^2)*d^4)) + 1/2*(a^3*b^5*d*cos
```

```
(d*x + c)*sin(d*x + c) + (a^10 - a^9*b)*sqrt(b^9/((a^13 - 2*a^12*b + a^11*b
^2)*d^4))*d^3*cos(d*x + c)*sin(d*x + c))*sqrt(-(b^4 - (a^6 - a^5*b)*sqrt(b^
9/((a^13 - 2*a^12*b + a^11*b^2)*d^4))*d^2)/((a^6 - a^5*b)*d^2)))*sin(d*x +
c) + 105*(a^2*d*cos(d*x + c)^6 - 3*a^2*d*cos(d*x + c)^4 + 3*a^2*d*cos(d*x +
 c)^2 - a^2*d)*sqrt(-(b^4 - (a^6 - a^5*b)*sqrt(b^9/((a^13 - 2*a^12*b + a^11
*b^2)*d^4))*d^2)/((a^6 - a^5*b)*d^2))*log(-1/4*b^7*cos(d*x + c)^2 + 1/4*b^7
 - 1/4*(2*(a^7*b^2 - a^6*b^3)*d^2*cos(d*x + c)^2 - (a^7*b^2 - a^6*b^3)*d^2)
*sqrt(b^9/((a^13 - 2*a^12*b + a^11*b^2)*d^4)) - 1/2*(a^3*b^5*d*cos(d*x + c)
*sin(d*x + c) + (a^10 - a^9*b)*sqrt(b^9/((a^13 - 2*a^12*b + a^11*b^2)*d^4))
*d^3*cos(d*x + c)*sin(d*x + c))*sqrt(-(b^4 - (a^6 - a^5*b)*sqrt(b^9/((a^13
- 2*a^12*b + a^11*b^2)*d^4))*d^2)/((a^6 - a^5*b)*d^2)))*sin(d*x + c) - 840*
(a + b)*cos(d*x + c))/((a^2*d*cos(d*x + c)^6 - 3*a^2*d*cos(d*x + c)^4 + 3*a
^2*d*cos(d*x + c)^2 - a^2*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**8/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.212 $\int \frac{\sin^9(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=236

$$-\frac{a\cos(c+dx)\left(a-b\cos^2(c+dx)+b\right)}{4b^2d(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}+\frac{\sqrt{a}\left(5\sqrt{a}-6\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8b^{9/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\sqrt{a}\left(5\sqrt{a}+6\sqrt{b}\right)\text{ta}}{8b^{9/4}d\left(\sqrt{a}\right.}$$

```
[Out] (Sqrt[a]*(5*Sqrt[a] - 6*Sqrt[b])*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a]
 - Sqrt[b]]])/(8*(Sqrt[a] - Sqrt[b])^(3/2)*b^(9/4)*d) + (Sqrt[a]*(5*Sqrt[a]
 + 6*Sqrt[b])*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(8*(
Sqrt[a] + Sqrt[b])^(3/2)*b^(9/4)*d) - Cos[c + d*x]/(b^2*d) - (a*Cos[c + d*x
]*(a + b - b*Cos[c + d*x]^2))/(4*(a - b)*b^2*d*(a - b + 2*b*Cos[c + d*x]^2
 - b*Cos[c + d*x]^4))
```

Rubi [A] time = 0.479794, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3215, 1205, 1676, 1166, 205, 208}

$$-\frac{a\cos(c+dx)\left(a-b\cos^2(c+dx)+b\right)}{4b^2d(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}+\frac{\sqrt{a}\left(5\sqrt{a}-6\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8b^{9/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\sqrt{a}\left(5\sqrt{a}+6\sqrt{b}\right)\text{ta}}{8b^{9/4}d\left(\sqrt{a}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^9/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] (Sqrt[a]*(5*Sqrt[a] - 6*Sqrt[b])*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a]
 - Sqrt[b]]])/(8*(Sqrt[a] - Sqrt[b])^(3/2)*b^(9/4)*d) + (Sqrt[a]*(5*Sqrt[a]
 + 6*Sqrt[b])*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(8*(
Sqrt[a] + Sqrt[b])^(3/2)*b^(9/4)*d) - Cos[c + d*x]/(b^2*d) - (a*Cos[c + d*x
]*(a + b - b*Cos[c + d*x]^2))/(4*(a - b)*b^2*d*(a - b + 2*b*Cos[c + d*x]^2
 - b*Cos[c + d*x]^4))
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
```

```
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sin^9(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^4}{\left(a-b+2bx^2-bx^4\right)^2}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{a\cos(c+dx)\left(a+b-b\cos^2(c+dx)\right)}{4(a-b)b^2d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{2a\left(a+\frac{a^2}{b}-4b\right)-2a(7a-8b)x^2+}{a-b+2bx^2-bx^4}\right.}{8a(a-b)}$$

$$= -\frac{a\cos(c+dx)\left(a+b-b\cos^2(c+dx)\right)}{4(a-b)b^2d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)} + \frac{\text{Subst}\left(\int \left(-\frac{8a(a-b)}{b} + \frac{2\left(a^2(5a-7b)+\right.}{b\left(a-b+2bx^2-\right.}\right.\right.}{8a(a-b)}$$

$$= -\frac{\cos(c+dx)}{b^2d} - \frac{a\cos(c+dx)\left(a+b-b\cos^2(c+dx)\right)}{4(a-b)b^2d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{a^2(5a-7b)}{a-b+2bx}\right.}{4a(}$$

$$= -\frac{\cos(c+dx)}{b^2d} - \frac{a\cos(c+dx)\left(a+b-b\cos^2(c+dx)\right)}{4(a-b)b^2d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)} - \frac{\left(\sqrt{a}\left(5\sqrt{a}-6\sqrt{b}\right)\right.}{}$$

$$= \frac{\sqrt{a}\left(5\sqrt{a}-6\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{9/4}d} + \frac{\sqrt{a}\left(5\sqrt{a}+6\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\left(\sqrt{a}+\sqrt{b}\right)^{3/2}b^{9/4}d} - \frac{\cos(c+d}{b^2d}$$

Mathematica [C] time = 1.11392, size = 486, normalized size = 2.06

$$-\frac{ia\text{RootSum}\left[-16\#1^4a+\#1^8b-4\#1^6b+6\#1^4b-4\#1^2b+b\&, \frac{-20i\#1^4a\log\left(\#1^2-2\#1\cos(c+dx)+1\right)+20i\#1^2a\log\left(\#1^2-2\#1\cos(c+dx)+1\right)+40\#1^4a\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)-40\#1^2a\tan\right.}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]^9/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] -(32*Cos[c + d*x] + (32*a*Cos[c + d*x]*(2*a + b - b*Cos[2*(c + d*x)]))/((a
- b)*(8*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*x)])) + (I*a*RootSu
m[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-2*b*ArcTan[
Sin[c + d*x]/(Cos[c + d*x] - #1)] + I*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] -
 40*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 + 54*b*ArcTan[Sin[c + d
*x]/(Cos[c + d*x] - #1)]*#1^2 + (20*I)*a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*
#1^2 - (27*I)*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + 40*a*ArcTan[Sin[c
+ d*x]/(Cos[c + d*x] - #1)]*#1^4 - 54*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] -
 #1)]*#1^4 - (20*I)*a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 + (27*I)*b*Log
[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 + 2*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x]
 - #1)]*#1^6 - I*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6)/(-(b*#1) - 8*a*#
1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/(a - b))/(32*b^2*d)
```

Maple [B] time = 0.114, size = 482, normalized size = 2.

$$-\frac{\cos(dx+c)}{b^2 d} - \frac{a(\cos(dx+c))^3}{4bd\left(b(\cos(dx+c))^4 - 2b(\cos(dx+c))^2 - a + b\right)(a-b)} + \frac{a^2\cos(dx+c)}{4b^2d\left(b(\cos(dx+c))^4 - 2b(\cos(dx+c\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^9/(a-b*sin(d*x+c)^4)^2,x)
```

```
[Out] -cos(d*x+c)/b^2/d-1/4/d/b*a/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)/(a-b)*cos
(d*x+c)^3+1/4/d/b^2*a^2/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)/(a-b)*cos(d*x
+c)+1/4/d/b*a/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)/(a-b)*cos(d*x+c)-1/8/d/
b*a/(a-b)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)
^(1/2))-3/4/d*a/(a-b)/(a*b)^(1/2)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+
c)*b/(((a*b)^(1/2)-b)*b)^(1/2))+5/8/d/b*a^2/(a-b)/(a*b)^(1/2)/(((a*b)^(1/2)
-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))+1/8/d/b*a/(a-b)
/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))-
3/4/d*a/(a-b)/(a*b)^(1/2)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/((
(a*b)^(1/2)+b)*b)^(1/2))+5/8/d/b*a^2/(a-b)/(a*b)^(1/2)/(((a*b)^(1/2)+b)*b)^
(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^9/(a-b*sin(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((2*a*b^2 - 3*b^3)*cos(2*d*x + 2*c)*cos(d*x + c) - 4*(2*a*b^2 - 3*b^3)*
sin(3*d*x + 3*c)*sin(2*d*x + 2*c) + (2*a*b^2 - 3*b^3)*sin(2*d*x + 2*c)*sin(
d*x + c) - ((a*b^2 - b^3)*cos(9*d*x + 9*c) - 4*(a*b^2 - b^3)*cos(7*d*x + 7*
c) - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*cos(5*d*x + 5*c) - 4*(a*b^2 - b^3)*cos(
3*d*x + 3*c) + (a*b^2 - b^3)*cos(d*x + c))*cos(10*d*x + 10*c) - (a*b^2 - b^
3 - (2*a*b^2 - 3*b^3)*cos(8*d*x + 8*c) - (20*a^2*b - 17*a*b^2 + 2*b^3)*cos(
6*d*x + 6*c) - (20*a^2*b - 17*a*b^2 + 2*b^3)*cos(4*d*x + 4*c) - (2*a*b^2 -
```

```
3*b^3)*cos(2*d*x + 2*c))*cos(9*d*x + 9*c) - (4*(2*a*b^2 - 3*b^3)*cos(7*d*x
+ 7*c) + 2*(16*a^2*b - 30*a*b^2 + 9*b^3)*cos(5*d*x + 5*c) + 4*(2*a*b^2 - 3*
b^3)*cos(3*d*x + 3*c) - (2*a*b^2 - 3*b^3)*cos(d*x + c))*cos(8*d*x + 8*c) +
4*(a*b^2 - b^3 - (20*a^2*b - 17*a*b^2 + 2*b^3)*cos(6*d*x + 6*c) - (20*a^2*b
 - 17*a*b^2 + 2*b^3)*cos(4*d*x + 4*c) - (2*a*b^2 - 3*b^3)*cos(2*d*x + 2*c))
*cos(7*d*x + 7*c) - (2*(160*a^3 - 196*a^2*b + 67*a*b^2 - 6*b^3)*cos(5*d*x +
 5*c) + 4*(20*a^2*b - 17*a*b^2 + 2*b^3)*cos(3*d*x + 3*c) - (20*a^2*b - 17*a
*b^2 + 2*b^3)*cos(d*x + c))*cos(6*d*x + 6*c) + 2*(8*a^2*b - 11*a*b^2 + 3*b^
3 - (160*a^3 - 196*a^2*b + 67*a*b^2 - 6*b^3)*cos(4*d*x + 4*c) - (16*a^2*b -
 30*a*b^2 + 9*b^3)*cos(2*d*x + 2*c))*cos(5*d*x + 5*c) - (4*(20*a^2*b - 17*a
*b^2 + 2*b^3)*cos(3*d*x + 3*c) - (20*a^2*b - 17*a*b^2 + 2*b^3)*cos(d*x + c)
)*cos(4*d*x + 4*c) + 4*(a*b^2 - b^3 - (2*a*b^2 - 3*b^3)*cos(2*d*x + 2*c))*c
os(3*d*x + 3*c) - (a*b^2 - b^3)*cos(d*x + c) + 2*((a*b^4 - b^5)*d*cos(9*d*x
 + 9*c)^2 + 16*(a*b^4 - b^5)*d*cos(7*d*x + 7*c)^2 + 4*(64*a^3*b^2 - 112*a^2
*b^3 + 57*a*b^4 - 9*b^5)*d*cos(5*d*x + 5*c)^2 + 16*(a*b^4 - b^5)*d*cos(3*d*
x + 3*c)^2 - 8*(a*b^4 - b^5)*d*cos(3*d*x + 3*c)*cos(d*x + c) + (a*b^4 - b^5
)*d*cos(d*x + c)^2 + (a*b^4 - b^5)*d*sin(9*d*x + 9*c)^2 + 16*(a*b^4 - b^5)*
d*sin(7*d*x + 7*c)^2 + 4*(64*a^3*b^2 - 112*a^2*b^3 + 57*a*b^4 - 9*b^5)*d*si
n(5*d*x + 5*c)^2 + 16*(a*b^4 - b^5)*d*sin(3*d*x + 3*c)^2 - 8*(a*b^4 - b^5)*
d*sin(3*d*x + 3*c)*sin(d*x + c) + (a*b^4 - b^5)*d*sin(d*x + c)^2 - 2*(4*(a*
b^4 - b^5)*d*cos(7*d*x + 7*c) + 2*(8*a^2*b^3 - 11*a*b^4 + 3*b^5)*d*cos(5*d*
x + 5*c) + 4*(a*b^4 - b^5)*d*cos(3*d*x + 3*c) - (a*b^4 - b^5)*d*cos(d*x + c
))*cos(9*d*x + 9*c) + 8*(2*(8*a^2*b^3 - 11*a*b^4 + 3*b^5)*d*cos(5*d*x + 5*c
) + 4*(a*b^4 - b^5)*d*cos(3*d*x + 3*c) - (a*b^4 - b^5)*d*cos(d*x + c))*cos(
7*d*x + 7*c) + 4*(4*(8*a^2*b^3 - 11*a*b^4 + 3*b^5)*d*cos(3*d*x + 3*c) - (8*
a^2*b^3 - 11*a*b^4 + 3*b^5)*d*cos(d*x + c))*cos(5*d*x + 5*c) - 2*(4*(a*b^4
 - b^5)*d*sin(7*d*x + 7*c) + 2*(8*a^2*b^3 - 11*a*b^4 + 3*b^5)*d*sin(5*d*x +
5*c) + 4*(a*b^4 - b^5)*d*sin(3*d*x + 3*c) - (a*b^4 - b^5)*d*sin(d*x + c))*s
in(9*d*x + 9*c) + 8*(2*(8*a^2*b^3 - 11*a*b^4 + 3*b^5)*d*sin(5*d*x + 5*c) +
4*(a*b^4 - b^5)*d*sin(3*d*x + 3*c) - (a*b^4 - b^5)*d*sin(d*x + c))*sin(7*d*
x + 7*c) + 4*(4*(8*a^2*b^3 - 11*a*b^4 + 3*b^5)*d*sin(3*d*x + 3*c) - (8*a^2*
b^3 - 11*a*b^4 + 3*b^5)*d*sin(d*x + c))*sin(5*d*x + 5*c))*integrate(-1/2*(4
*a*b^2*cos(d*x + c)*sin(2*d*x + 2*c) - 4*a*b^2*cos(2*d*x + 2*c)*sin(d*x + c
) + a*b^2*sin(d*x + c) + 4*(20*a^2*b - 27*a*b^2)*cos(3*d*x + 3*c)*sin(2*d*x
 + 2*c) - (a*b^2*sin(7*d*x + 7*c) - a*b^2*sin(d*x + c) + (20*a^2*b - 27*a*b
^2)*sin(5*d*x + 5*c) - (20*a^2*b - 27*a*b^2)*sin(3*d*x + 3*c))*cos(8*d*x +
8*c) - 2*(2*a*b^2*sin(6*d*x + 6*c) + 2*a*b^2*sin(2*d*x + 2*c) + (8*a^2*b -
3*a*b^2)*sin(4*d*x + 4*c))*cos(7*d*x + 7*c) - 4*(a*b^2*sin(d*x + c) - (20*a
^2*b - 27*a*b^2)*sin(5*d*x + 5*c) + (20*a^2*b - 27*a*b^2)*sin(3*d*x + 3*c))
*cos(6*d*x + 6*c) - 2*((160*a^3 - 276*a^2*b + 81*a*b^2)*sin(4*d*x + 4*c) +
2*(20*a^2*b - 27*a*b^2)*sin(2*d*x + 2*c))*cos(5*d*x + 5*c) - 2*((160*a^3 -
276*a^2*b + 81*a*b^2)*sin(3*d*x + 3*c) + (8*a^2*b - 3*a*b^2)*sin(d*x + c))*
cos(4*d*x + 4*c) + (a*b^2*cos(7*d*x + 7*c) - a*b^2*cos(d*x + c) + (20*a^2*b
 - 27*a*b^2)*cos(5*d*x + 5*c) - (20*a^2*b - 27*a*b^2)*cos(3*d*x + 3*c))*sin
(8*d*x + 8*c) + (4*a*b^2*cos(6*d*x + 6*c) + 4*a*b^2*cos(2*d*x + 2*c) - a*b^
2 + 2*(8*a^2*b - 3*a*b^2)*cos(4*d*x + 4*c))*sin(7*d*x + 7*c) + 4*(a*b^2*cos
(d*x + c) - (20*a^2*b - 27*a*b^2)*cos(5*d*x + 5*c) + (20*a^2*b - 27*a*b^2)*
cos(3*d*x + 3*c))*sin(6*d*x + 6*c) - (20*a^2*b - 27*a*b^2 - 2*(160*a^3 - 27
6*a^2*b + 81*a*b^2)*cos(4*d*x + 4*c) - 4*(20*a^2*b - 27*a*b^2)*cos(2*d*x +
2*c))*sin(5*d*x + 5*c) + 2*((160*a^3 - 276*a^2*b + 81*a*b^2)*cos(3*d*x + 3*
c) + (8*a^2*b - 3*a*b^2)*cos(d*x + c))*sin(4*d*x + 4*c) + (20*a^2*b - 27*a*
b^2 - 4*(20*a^2*b - 27*a*b^2)*cos(2*d*x + 2*c))*sin(3*d*x + 3*c))/(a*b^4 -
b^5 + (a*b^4 - b^5)*cos(8*d*x + 8*c)^2 + 16*(a*b^4 - b^5)*cos(6*d*x + 6*c)^
2 + 4*(64*a^3*b^2 - 112*a^2*b^3 + 57*a*b^4 - 9*b^5)*cos(4*d*x + 4*c)^2 + 16
*(a*b^4 - b^5)*cos(2*d*x + 2*c)^2 + (a*b^4 - b^5)*sin(8*d*x + 8*c)^2 + 16*(
a*b^4 - b^5)*sin(6*d*x + 6*c)^2 + 4*(64*a^3*b^2 - 112*a^2*b^3 + 57*a*b^4 -
9*b^5)*sin(4*d*x + 4*c)^2 + 16*(8*a^2*b^3 - 11*a*b^4 + 3*b^5)*sin(4*d*x + 4
*c)*sin(2*d*x + 2*c) + 16*(a*b^4 - b^5)*sin(2*d*x + 2*c)^2 + 2*(a*b^4 - b^5
 - 4*(a*b^4 - b^5)*cos(6*d*x + 6*c) - 2*(8*a^2*b^3 - 11*a*b^4 + 3*b^5)*cos(
```

```
4*d*x + 4*c) - 4*(a*b^4 - b^5)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) - 8*(a*b^
4 - b^5 - 2*(8*a^2*b^3 - 11*a*b^4 + 3*b^5)*cos(4*d*x + 4*c) - 4*(a*b^4 - b^
5)*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) - 4*(8*a^2*b^3 - 11*a*b^4 + 3*b^5 - 4
*(8*a^2*b^3 - 11*a*b^4 + 3*b^5)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 8*(a*b
^4 - b^5)*cos(2*d*x + 2*c) - 4*(2*(a*b^4 - b^5)*sin(6*d*x + 6*c) + (8*a^2*b
^3 - 11*a*b^4 + 3*b^5)*sin(4*d*x + 4*c) + 2*(a*b^4 - b^5)*sin(2*d*x + 2*c))
*sin(8*d*x + 8*c) + 16*((8*a^2*b^3 - 11*a*b^4 + 3*b^5)*sin(4*d*x + 4*c) + 2
*(a*b^4 - b^5)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)), x) - ((a*b^2 - b^3)*sin
(9*d*x + 9*c) - 4*(a*b^2 - b^3)*sin(7*d*x + 7*c) - 2*(8*a^2*b - 11*a*b^2 +
3*b^3)*sin(5*d*x + 5*c) - 4*(a*b^2 - b^3)*sin(3*d*x + 3*c) + (a*b^2 - b^3)*
sin(d*x + c))*sin(10*d*x + 10*c) + ((2*a*b^2 - 3*b^3)*sin(8*d*x + 8*c) + (2
0*a^2*b - 17*a*b^2 + 2*b^3)*sin(6*d*x + 6*c) + (20*a^2*b - 17*a*b^2 + 2*b^3
)*sin(4*d*x + 4*c) + (2*a*b^2 - 3*b^3)*sin(2*d*x + 2*c))*sin(9*d*x + 9*c) -
 (4*(2*a*b^2 - 3*b^3)*sin(7*d*x + 7*c) + 2*(16*a^2*b - 30*a*b^2 + 9*b^3)*si
n(5*d*x + 5*c) + 4*(2*a*b^2 - 3*b^3)*sin(3*d*x + 3*c) - (2*a*b^2 - 3*b^3)*s
in(d*x + c))*sin(8*d*x + 8*c) - 4*((20*a^2*b - 17*a*b^2 + 2*b^3)*sin(6*d*x
+ 6*c) + (20*a^2*b - 17*a*b^2 + 2*b^3)*sin(4*d*x + 4*c) + (2*a*b^2 - 3*b^3)
*sin(2*d*x + 2*c))*sin(7*d*x + 7*c) - (2*(160*a^3 - 196*a^2*b + 67*a*b^2 -
6*b^3)*sin(5*d*x + 5*c) + 4*(20*a^2*b - 17*a*b^2 + 2*b^3)*sin(3*d*x + 3*c)
- (20*a^2*b - 17*a*b^2 + 2*b^3)*sin(d*x + c))*sin(6*d*x + 6*c) - 2*((160*a^
3 - 196*a^2*b + 67*a*b^2 - 6*b^3)*sin(4*d*x + 4*c) + (16*a^2*b - 30*a*b^2 +
 9*b^3)*sin(2*d*x + 2*c))*sin(5*d*x + 5*c) - (4*(20*a^2*b - 17*a*b^2 + 2*b^
3)*sin(3*d*x + 3*c) - (20*a^2*b - 17*a*b^2 + 2*b^3)*sin(d*x + c))*sin(4*d*x
 + 4*c))/((a*b^4 - b^5)*d*cos(9*d*x + 9*c)^2 + 16*(a*b^4 - b^5)*d*cos(7*d*x
 + 7*c)^2 + 4*(64*a^3*b^2 - 112*a^2*b^3 + 57*a*b^4 - 9*b^5)*d*cos(5*d*x + 5
*c)^2 + 16*(a*b^4 - b^5)*d*cos(3*d*x + 3*c)^2 - 8*(a*b^4 - b^5)*d*cos(3*d*x
 + 3*c)*cos(d*x + c) + (a*b^4 - b^5)*d*cos(d*x + c)^2 + (a*b^4 - b^5)*d*sin
(9*d*x + 9*c)^2 + 16*(a*b^4 - b^5)*d*sin(7*d*x + 7*c)^2 + 4*(64*a^3*b^2 - 1
12*a^2*b^3 + 57*a*b^4 - 9*b^5)*d*sin(5*d*x + 5*c)^2 + 16*(a*b^4 - b^5)*d*si
n(3*d*x + 3*c)^2 - 8*(a*b^4 - b^5)*d*sin(3*d*x + 3*c)*sin(d*x + c) + (a*b^4
 - b^5)*d*sin(d*x + c)^2 - 2*(4*(a*b^4 - b^5)*d*cos(7*d*x + 7*c) + 2*(8*a^2
*b^3 - 11*a*b^4 + 3*b^5)*d*cos(5*d*x + 5*c) + 4*(a*b^4 - b^5)*d*cos(3*d*x +
 3*c) - (a*b^4 - b^5)*d*cos(d*x + c))*cos(9*d*x + 9*c) + 8*(2*(8*a^2*b^3 -
11*a*b^4 + 3*b^5)*d*cos(5*d*x + 5*c) + 4*(a*b^4 - b^5)*d*cos(3*d*x + 3*c) -
 (a*b^4 - b^5)*d*cos(d*x + c))*cos(7*d*x + 7*c) + 4*(4*(8*a^2*b^3 - 11*a*b^
4 + 3*b^5)*d*cos(3*d*x + 3*c) - (8*a^2*b^3 - 11*a*b^4 + 3*b^5)*d*cos(d*x +
c))*cos(5*d*x + 5*c) - 2*(4*(a*b^4 - b^5)*d*sin(7*d*x + 7*c) + 2*(8*a^2*b^3
 - 11*a*b^4 + 3*b^5)*d*sin(5*d*x + 5*c) + 4*(a*b^4 - b^5)*d*sin(3*d*x + 3*c
) - (a*b^4 - b^5)*d*sin(d*x + c))*sin(9*d*x + 9*c) + 8*(2*(8*a^2*b^3 - 11*a
*b^4 + 3*b^5)*d*sin(5*d*x + 5*c) + 4*(a*b^4 - b^5)*d*sin(3*d*x + 3*c) - (a*
b^4 - b^5)*d*sin(d*x + c))*sin(7*d*x + 7*c) + 4*(4*(8*a^2*b^3 - 11*a*b^4 +
3*b^5)*d*sin(3*d*x + 3*c) - (8*a^2*b^3 - 11*a*b^4 + 3*b^5)*d*sin(d*x + c))*
sin(5*d*x + 5*c))
```

Fricas [B] time = 5.96874, size = 5901, normalized size = 25.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^9/(a-b*sin(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(16*(a*b - b^2)*cos(d*x + c)^5 - 4*(7*a*b - 8*b^2)*cos(d*x + c)^3 + (
(a*b^3 - b^4)*d*cos(d*x + c)^4 - 2*(a*b^3 - b^4)*d*cos(d*x + c)^2 - (a^2*b^
2 - 2*a*b^3 + b^4)*d)*sqrt(-((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt
((625*a^7 - 3450*a^6*b + 7161*a^5*b^2 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a^6*
b^9 - 6*a^5*b^10 + 15*a^4*b^11 - 20*a^3*b^12 + 15*a^2*b^13 - 6*a*b^14 + b^1
```

```
5)*d^4)) + 15*a^3 - 47*a^2*b + 36*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 -
b^7)*d^2))*log((625*a^5 - 2625*a^4*b + 3684*a^3*b^2 - 1728*a^2*b^3)*cos(d*x
 + c) + (2*(2*a^4*b^7 - 9*a^3*b^8 + 15*a^2*b^9 - 11*a*b^10 + 3*b^11)*d^3*sq
rt((625*a^7 - 3450*a^6*b + 7161*a^5*b^2 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a^
6*b^9 - 6*a^5*b^10 + 15*a^4*b^11 - 20*a^3*b^12 + 15*a^2*b^13 - 6*a*b^14 + b
^15)*d^4)) - (125*a^5*b^2 - 520*a^4*b^3 + 723*a^3*b^4 - 336*a^2*b^5)*d)*sqr
t(-((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((625*a^7 - 3450*a^6*b +
7161*a^5*b^2 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a^6*b^9 - 6*a^5*b^10 + 15*a^4
*b^11 - 20*a^3*b^12 + 15*a^2*b^13 - 6*a*b^14 + b^15)*d^4)) + 15*a^3 - 47*a^
2*b + 36*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))) - ((a*b^3 - b
^4)*d*cos(d*x + c)^4 - 2*(a*b^3 - b^4)*d*cos(d*x + c)^2 - (a^2*b^2 - 2*a*b^
3 + b^4)*d)*sqrt(((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((625*a^7 -
 3450*a^6*b + 7161*a^5*b^2 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a^6*b^9 - 6*a^5
*b^10 + 15*a^4*b^11 - 20*a^3*b^12 + 15*a^2*b^13 - 6*a*b^14 + b^15)*d^4)) -
15*a^3 + 47*a^2*b - 36*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))*
log((625*a^5 - 2625*a^4*b + 3684*a^3*b^2 - 1728*a^2*b^3)*cos(d*x + c) + (2*
(2*a^4*b^7 - 9*a^3*b^8 + 15*a^2*b^9 - 11*a*b^10 + 3*b^11)*d^3*sqrt((625*a^7
 - 3450*a^6*b + 7161*a^5*b^2 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a^6*b^9 - 6*a
^5*b^10 + 15*a^4*b^11 - 20*a^3*b^12 + 15*a^2*b^13 - 6*a*b^14 + b^15)*d^4))
+ (125*a^5*b^2 - 520*a^4*b^3 + 723*a^3*b^4 - 336*a^2*b^5)*d)*sqrt(((a^3*b^4
 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((625*a^7 - 3450*a^6*b + 7161*a^5*b^2
 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a^6*b^9 - 6*a^5*b^10 + 15*a^4*b^11 - 20*a
^3*b^12 + 15*a^2*b^13 - 6*a*b^14 + b^15)*d^4)) - 15*a^3 + 47*a^2*b - 36*a*b
^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))) - ((a*b^3 - b^4)*d*cos(d*
x + c)^4 - 2*(a*b^3 - b^4)*d*cos(d*x + c)^2 - (a^2*b^2 - 2*a*b^3 + b^4)*d)*
sqrt(-((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((625*a^7 - 3450*a^6*b
 + 7161*a^5*b^2 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a^6*b^9 - 6*a^5*b^10 + 15*
a^4*b^11 - 20*a^3*b^12 + 15*a^2*b^13 - 6*a*b^14 + b^15)*d^4)) + 15*a^3 - 47
*a^2*b + 36*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))*log(-(625*a
^5 - 2625*a^4*b + 3684*a^3*b^2 - 1728*a^2*b^3)*cos(d*x + c) + (2*(2*a^4*b^7
 - 9*a^3*b^8 + 15*a^2*b^9 - 11*a*b^10 + 3*b^11)*d^3*sqrt((625*a^7 - 3450*a^
6*b + 7161*a^5*b^2 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a^6*b^9 - 6*a^5*b^10 +
15*a^4*b^11 - 20*a^3*b^12 + 15*a^2*b^13 - 6*a*b^14 + b^15)*d^4)) - (125*a^5
*b^2 - 520*a^4*b^3 + 723*a^3*b^4 - 336*a^2*b^5)*d)*sqrt(-((a^3*b^4 - 3*a^2*
b^5 + 3*a*b^6 - b^7)*d^2*sqrt((625*a^7 - 3450*a^6*b + 7161*a^5*b^2 - 6624*a
^4*b^3 + 2304*a^3*b^4)/((a^6*b^9 - 6*a^5*b^10 + 15*a^4*b^11 - 20*a^3*b^12 +
 15*a^2*b^13 - 6*a*b^14 + b^15)*d^4)) + 15*a^3 - 47*a^2*b + 36*a*b^2)/((a^3
*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))) + ((a*b^3 - b^4)*d*cos(d*x + c)^4
- 2*(a*b^3 - b^4)*d*cos(d*x + c)^2 - (a^2*b^2 - 2*a*b^3 + b^4)*d)*sqrt(((a^
3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((625*a^7 - 3450*a^6*b + 7161*a^
5*b^2 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a^6*b^9 - 6*a^5*b^10 + 15*a^4*b^11 -
 20*a^3*b^12 + 15*a^2*b^13 - 6*a*b^14 + b^15)*d^4)) - 15*a^3 + 47*a^2*b - 3
6*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))*log(-(625*a^5 - 2625*
a^4*b + 3684*a^3*b^2 - 1728*a^2*b^3)*cos(d*x + c) + (2*(2*a^4*b^7 - 9*a^3*b
^8 + 15*a^2*b^9 - 11*a*b^10 + 3*b^11)*d^3*sqrt((625*a^7 - 3450*a^6*b + 7161
*a^5*b^2 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a^6*b^9 - 6*a^5*b^10 + 15*a^4*b^1
1 - 20*a^3*b^12 + 15*a^2*b^13 - 6*a*b^14 + b^15)*d^4)) + (125*a^5*b^2 - 520
*a^4*b^3 + 723*a^3*b^4 - 336*a^2*b^5)*d)*sqrt(((a^3*b^4 - 3*a^2*b^5 + 3*a*b
^6 - b^7)*d^2*sqrt((625*a^7 - 3450*a^6*b + 7161*a^5*b^2 - 6624*a^4*b^3 + 23
04*a^3*b^4)/((a^6*b^9 - 6*a^5*b^10 + 15*a^4*b^11 - 20*a^3*b^12 + 15*a^2*b^1
3 - 6*a*b^14 + b^15)*d^4)) - 15*a^3 + 47*a^2*b - 36*a*b^2)/((a^3*b^4 - 3*a^
2*b^5 + 3*a*b^6 - b^7)*d^2))) - 4*(5*a^2 - 7*a*b + 4*b^2)*cos(d*x + c))/((a
*b^3 - b^4)*d*cos(d*x + c)^4 - 2*(a*b^3 - b^4)*d*cos(d*x + c)^2 - (a^2*b^2
- 2*a*b^3 + b^4)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**9/(a-b*sin(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^9/(a-b*sin(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.213 $\int \frac{\sin^7(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=210

$$\frac{\left(3\sqrt{a}-4\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8b^{7/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\left(3\sqrt{a}+4\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8b^{7/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{a\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{4bd(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx\right.}$$

```
[Out] ((3*Sqrt[a] - 4*Sqrt[b])*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[
b]]])/(8*(Sqrt[a] - Sqrt[b])^(3/2)*b^(7/4)*d) - ((3*Sqrt[a] + 4*Sqrt[b])*Ar
cTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(8*(Sqrt[a] + Sqrt[b
])^(3/2)*b^(7/4)*d) - (a*Cos[c + d*x]*(2 - Cos[c + d*x]^2))/(4*(a - b)*b*d*
(a - b + 2*b*Cos[c + d*x]^2 - b*Cos[c + d*x]^4))
```

Rubi [A] time = 0.334937, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3215, 1205, 1166, 205, 208}

$$\frac{\left(3\sqrt{a}-4\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8b^{7/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\left(3\sqrt{a}+4\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8b^{7/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{a\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{4bd(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^7/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] ((3*Sqrt[a] - 4*Sqrt[b])*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[
b]]])/(8*(Sqrt[a] - Sqrt[b])^(3/2)*b^(7/4)*d) - ((3*Sqrt[a] + 4*Sqrt[b])*Ar
cTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(8*(Sqrt[a] + Sqrt[b
])^(3/2)*b^(7/4)*d) - (a*Cos[c + d*x]*(2 - Cos[c + d*x]^2))/(4*(a - b)*b*d*
(a - b + 2*b*Cos[c + d*x]^2 - b*Cos[c + d*x]^4))
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sin^7(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{\left(a-b+2bx^2-bx^4\right)^2}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{a\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{4(a-b)bd\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{4a(a-2b)-2a(3a-4b)x^2}{a-b+2bx^2-bx^4}\,dx\right.}{8a(a-b)bd}$$

$$= -\frac{a\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{4(a-b)bd\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)} - \frac{\left(3a-\sqrt{a}\sqrt{b}-4b\right)\text{Subst}\left(\int\right.}{8(a-}$$

$$= \frac{\left(3\sqrt{a}-4\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{7/4}d} - \frac{\left(3\sqrt{a}+4\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\left(\sqrt{a}+\sqrt{b}\right)^{3/2}b^{7/4}d} - \frac{a\,c}{4(a-b)bd\left(a\right.}$$

Mathematica [C] time = 0.568934, size = 565, normalized size = 2.69

$$\frac{\frac{16a(\cos(3(c+dx))-5\cos(c+dx))}{8a+4b\cos(2(c+dx))-b\cos(4(c+dx))-3b} - i\text{RootSum}\left[-16\#1^4a+\#1^8b-4\#1^6b+6\#1^4b-4\#1^2b+b\&, \frac{3i\#1^6a\log\left(\#1^2-2\#1\cos(c+dx\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]^7/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] ((16*a*(-5*Cos[c + d*x] + Cos[3*(c + d*x)]))/(8*a - 3*b + 4*b*Cos[2*(c + d*
x)] - b*Cos[4*(c + d*x)]) - I*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 -
 4*b*#1^6 + b*#1^8 & , (6*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - 8*b*
ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - (3*I)*a*Log[1 - 2*Cos[c + d*x]*#
1 + #1^2] + (4*I)*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] - 10*a*ArcTan[Sin[c +
 d*x]/(Cos[c + d*x] - #1)]*#1^2 + 24*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] -
#1)]*#1^2 + (5*I)*a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (12*I)*b*Log[1
 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + 10*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x]
- #1)]*#1^4 - 24*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - (5*I)*a*
Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 + (12*I)*b*Log[1 - 2*Cos[c + d*x]*#1
 + #1^2]*#1^4 - 6*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 + 8*b*Arc
```

```
Tan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 + (3*I)*a*Log[1 - 2*Cos[c + d*x]
*#1 + #1^2]*#1^6 - (4*I)*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6)/(-(b*#1)
 - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/(32*(a - b)*b*d)
```

Maple [B] time = 0.111, size = 394, normalized size = 1.9

$$-\frac{a(\cos(dx+c))^3}{4\,bd\left(b(\cos(dx+c))^4-2\,b(\cos(dx+c))^2-a+b\right)(a-b)}+\frac{\cos(dx+c)\,a}{2\,bd\left(b(\cos(dx+c))^4-2\,b(\cos(dx+c))^2-a+b\right)(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^7/(a-b*sin(d*x+c)^4)^2,x)
```

```
[Out] -1/4/d/b*a/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)/(a-b)*cos(d*x+c)^3+1/2/d/b
*a/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)/(a-b)*cos(d*x+c)+3/8/d/b*a/(a-b)/(
((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))-1/2
/d/(a-b)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^
(1/2))-1/8/d*a/(a-b)/(a*b)^(1/2)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c
)*b/(((a*b)^(1/2)-b)*b)^(1/2))-3/8/d/b*a/(a-b)/(((a*b)^(1/2)+b)*b)^(1/2)*ar
ctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))+1/2/d/(a-b)/(((a*b)^(1/2)+b)*
b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))-1/8/d*a/(a-b)/(a*b
)^(1/2)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^
(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a-b*sin(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(4*a*b*cos(2*d*x + 2*c)*cos(d*x + c) - 20*a*b*sin(3*d*x + 3*c)*sin(2*d*
x + 2*c) + 4*a*b*sin(2*d*x + 2*c)*sin(d*x + c) - a*b*cos(d*x + c) - (a*b*co
s(7*d*x + 7*c) - 5*a*b*cos(5*d*x + 5*c) - 5*a*b*cos(3*d*x + 3*c) + a*b*cos(
d*x + c))*cos(8*d*x + 8*c) + (4*a*b*cos(6*d*x + 6*c) + 4*a*b*cos(2*d*x + 2*
c) - a*b + 2*(8*a^2 - 3*a*b)*cos(4*d*x + 4*c))*cos(7*d*x + 7*c) - 4*(5*a*b*
cos(5*d*x + 5*c) + 5*a*b*cos(3*d*x + 3*c) - a*b*cos(d*x + c))*cos(6*d*x + 6
*c) - 5*(4*a*b*cos(2*d*x + 2*c) - a*b + 2*(8*a^2 - 3*a*b)*cos(4*d*x + 4*c))
*cos(5*d*x + 5*c) - 2*(5*(8*a^2 - 3*a*b)*cos(3*d*x + 3*c) - (8*a^2 - 3*a*b)
*cos(d*x + c))*cos(4*d*x + 4*c) - 5*(4*a*b*cos(2*d*x + 2*c) - a*b)*cos(3*d*
x + 3*c) + 2*((a*b^3 - b^4)*d*cos(8*d*x + 8*c)^2 + 16*(a*b^3 - b^4)*d*cos(6
*d*x + 6*c)^2 + 4*(64*a^3*b - 112*a^2*b^2 + 57*a*b^3 - 9*b^4)*d*cos(4*d*x +
 4*c)^2 + 16*(a*b^3 - b^4)*d*cos(2*d*x + 2*c)^2 + (a*b^3 - b^4)*d*sin(8*d*x
 + 8*c)^2 + 16*(a*b^3 - b^4)*d*sin(6*d*x + 6*c)^2 + 4*(64*a^3*b - 112*a^2*b
^2 + 57*a*b^3 - 9*b^4)*d*sin(4*d*x + 4*c)^2 + 16*(8*a^2*b^2 - 11*a*b^3 + 3*
b^4)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a*b^3 - b^4)*d*sin(2*d*x + 2
*c)^2 - 8*(a*b^3 - b^4)*d*cos(2*d*x + 2*c) + (a*b^3 - b^4)*d - 2*(4*(a*b^3
- b^4)*d*cos(6*d*x + 6*c) + 2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cos(4*d*x +
4*c) + 4*(a*b^3 - b^4)*d*cos(2*d*x + 2*c) - (a*b^3 - b^4)*d)*cos(8*d*x + 8*
c) + 8*(2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cos(4*d*x + 4*c) + 4*(a*b^3 - b^
4)*d*cos(2*d*x + 2*c) - (a*b^3 - b^4)*d)*cos(6*d*x + 6*c) + 4*(4*(8*a^2*b^2
 - 11*a*b^3 + 3*b^4)*d*cos(2*d*x + 2*c) - (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d)
```

*cos(4*d*x + 4*c) - 4*(2*(a*b^3 - b^4)*d*sin(6*d*x + 6*c) + (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*sin(4*d*x + 4*c) + 2*(a*b^3 - b^4)*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*sin(4*d*x + 4*c) + 2*(a*b^3 - b^4)*d*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*integrate(-1/2*(4*(5*a*b - 12*b^2)*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) - 4*(3*a*b - 4*b^2)*cos(d*x + c)*sin(2*d*x + 2*c) + 4*(3*a*b - 4*b^2)*cos(2*d*x + 2*c)*sin(d*x + c) + ((3*a*b - 4*b^2)*sin(7*d*x + 7*c) - (5*a*b - 12*b^2)*sin(5*d*x + 5*c) + (5*a*b - 12*b^2)*sin(3*d*x + 3*c) - (3*a*b - 4*b^2)*sin(d*x + c))*cos(8*d*x + 8*c) + 2*(2*(3*a*b - 4*b^2)*sin(6*d*x + 6*c) + (24*a^2 - 41*a*b + 12*b^2)*sin(4*d*x + 4*c) + 2*(3*a*b - 4*b^2)*sin(2*d*x + 2*c))*cos(7*d*x + 7*c) + 4*((5*a*b - 12*b^2)*sin(5*d*x + 5*c) - (5*a*b - 12*b^2)*sin(3*d*x + 3*c) + (3*a*b - 4*b^2)*sin(d*x + c))*cos(6*d*x + 6*c) - 2*((40*a^2 - 111*a*b + 36*b^2)*sin(4*d*x + 4*c) + 2*(5*a*b - 12*b^2)*sin(2*d*x + 2*c))*cos(5*d*x + 5*c) - 2*((40*a^2 - 111*a*b + 36*b^2)*sin(3*d*x + 3*c) - (24*a^2 - 41*a*b + 12*b^2)*sin(d*x + c))*cos(4*d*x + 4*c) - ((3*a*b - 4*b^2)*cos(7*d*x + 7*c) - (5*a*b - 12*b^2)*cos(5*d*x + 5*c) + (5*a*b - 12*b^2)*cos(3*d*x + 3*c) - (3*a*b - 4*b^2)*cos(d*x + c))*sin(8*d*x + 8*c) + (3*a*b - 4*b^2 - 4*(3*a*b - 4*b^2)*cos(6*d*x + 6*c) - 2*(24*a^2 - 41*a*b + 12*b^2)*cos(4*d*x + 4*c) - 4*(3*a*b - 4*b^2)*cos(2*d*x + 2*c))*sin(7*d*x + 7*c) - 4*((5*a*b - 12*b^2)*cos(5*d*x + 5*c) - (5*a*b - 12*b^2)*cos(3*d*x + 3*c) + (3*a*b - 4*b^2)*cos(d*x + c))*sin(6*d*x + 6*c) - (5*a*b - 12*b^2 - 2*(40*a^2 - 111*a*b + 36*b^2)*cos(4*d*x + 4*c) - 4*(5*a*b - 12*b^2)*cos(2*d*x + 2*c))*sin(5*d*x + 5*c) + 2*((40*a^2 - 111*a*b + 36*b^2)*cos(3*d*x + 3*c) - (24*a^2 - 41*a*b + 12*b^2)*cos(d*x + c))*sin(4*d*x + 4*c) + (5*a*b - 12*b^2 - 4*(5*a*b - 12*b^2)*cos(2*d*x + 2*c))*sin(3*d*x + 3*c) - (3*a*b - 4*b^2)*sin(d*x + c))/(a*b^3 - b^4 + (a*b^3 - b^4)*cos(8*d*x + 8*c)^2 + 16*(a*b^3 - b^4)*cos(6*d*x + 6*c)^2 + 4*(64*a^3*b - 112*a^2*b^2 + 57*a*b^3 - 9*b^4)*cos(4*d*x + 4*c)^2 + 16*(a*b^3 - b^4)*cos(2*d*x + 2*c)^2 + (a*b^3 - b^4)*sin(8*d*x + 8*c)^2 + 16*(a*b^3 - b^4)*sin(6*d*x + 6*c)^2 + 4*(64*a^3*b - 112*a^2*b^2 + 57*a*b^3 - 9*b^4)*sin(4*d*x + 4*c)^2 + 16*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a*b^3 - b^4)*sin(2*d*x + 2*c)^2 + 2*(a*b^3 - b^4 - 4*(a*b^3 - b^4)*cos(6*d*x + 6*c) - 2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*cos(4*d*x + 4*c) - 4*(a*b^3 - b^4)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) - 8*(a*b^3 - b^4 - 2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*cos(4*d*x + 4*c) - 4*(a*b^3 - b^4)*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) - 4*(8*a^2*b^2 - 11*a*b^3 + 3*b^4 - 4*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 8*(a*b^3 - b^4)*cos(2*d*x + 2*c) - 4*(2*(a*b^3 - b^4)*sin(6*d*x + 6*c) + (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*sin(4*d*x + 4*c) + 2*(a*b^3 - b^4)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^2*b^2 - 11*a*b^3 + 3*b^4)*sin(4*d*x + 4*c) + 2*(a*b^3 - b^4)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)), x) - (a*b*sin(7*d*x + 7*c) - 5*a*b*sin(5*d*x + 5*c) - 5*a*b*sin(3*d*x + 3*c) + a*b*sin(d*x + c))*sin(8*d*x + 8*c) + 2*(2*a*b*sin(6*d*x + 6*c) + 2*a*b*sin(2*d*x + 2*c) + (8*a^2 - 3*a*b)*sin(4*d*x + 4*c))*sin(7*d*x + 7*c) - 4*(5*a*b*sin(5*d*x + 5*c) + 5*a*b*sin(3*d*x + 3*c) - a*b*sin(d*x + c))*sin(6*d*x + 6*c) - 10*(2*a*b*sin(2*d*x + 2*c) + (8*a^2 - 3*a*b)*sin(4*d*x + 4*c))*sin(5*d*x + 5*c) - 2*(5*(8*a^2 - 3*a*b)*sin(3*d*x + 3*c) - (8*a^2 - 3*a*b)*sin(d*x + c))*sin(4*d*x + 4*c))/((a*b^3 - b^4)*d*cos(8*d*x + 8*c)^2 + 16*(a*b^3 - b^4)*d*cos(6*d*x + 6*c)^2 + 4*(64*a^3*b - 112*a^2*b^2 + 57*a*b^3 - 9*b^4)*d*cos(4*d*x + 4*c)^2 + 16*(a*b^3 - b^4)*d*cos(2*d*x + 2*c)^2 + (a*b^3 - b^4)*d*sin(8*d*x + 8*c)^2 + 16*(a*b^3 - b^4)*d*sin(6*d*x + 6*c)^2 + 4*(64*a^3*b - 112*a^2*b^2 + 57*a*b^3 - 9*b^4)*d*sin(4*d*x + 4*c)^2 + 16*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a*b^3 - b^4)*d*sin(2*d*x + 2*c)^2 - 8*(a*b^3 - b^4)*d*cos(2*d*x + 2*c) + (a*b^3 - b^4)*d - 2*(4*(a*b^3 - b^4)*d*cos(6*d*x + 6*c) + 2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cos(4*d*x + 4*c) + 4*(a*b^3 - b^4)*d*cos(2*d*x + 2*c) - (a*b^3 - b^4)*d)*cos(8*d*x + 8*c) + 8*(2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cos(4*d*x + 4*c) + 4*(a*b^3 - b^4)*d*cos(2*d*x + 2*c) - (a*b^3 - b^4)*d)*cos(6*d*x + 6*c) + 4*(4*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cos(2*d*x + 2*c) - (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d)*cos(4*d*x + 4*c) - 4*(2*(a*b^3 - b^4)*d*sin(6*d*x + 6*c) + (8*a^2*b^2 - 11*a*b^3 + 3*b^

```
4)*d*sin(4*d*x + 4*c) + 2*(a*b^3 - b^4)*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c
) + 16*((8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*sin(4*d*x + 4*c) + 2*(a*b^3 - b^4)
*d*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))
```

Fricas [B] time = 5.10174, size = 5509, normalized size = 26.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a-b*sin(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*a*cos(d*x + c)^3 - ((a*b^2 - b^3)*d*cos(d*x + c)^4 - 2*(a*b^2 - b^
3)*d*cos(d*x + c)^2 - (a^2*b - 2*a*b^2 + b^3)*d)*sqrt(-((a^3*b^3 - 3*a^2*b^
4 + 3*a*b^5 - b^6)*d^2*sqrt((81*a^5 - 522*a^4*b + 1273*a^3*b^2 - 1392*a^2*b
^3 + 576*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b
^11 - 6*a*b^12 + b^13)*d^4)) + 3*a^2 - 15*a*b + 16*b^2)/((a^3*b^3 - 3*a^2*b
^4 + 3*a*b^5 - b^6)*d^2))*log((81*a^3 - 405*a^2*b + 680*a*b^2 - 384*b^3)*co
s(d*x + c) + ((3*a^4*b^5 - 14*a^3*b^6 + 24*a^2*b^7 - 18*a*b^8 + 5*b^9)*d^3*
sqrt((81*a^5 - 522*a^4*b + 1273*a^3*b^2 - 1392*a^2*b^3 + 576*a*b^4)/((a^6*b
^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*
d^4)) - 2*(9*a^3*b^2 - 47*a^2*b^3 + 82*a*b^4 - 48*b^5)*d)*sqrt(-((a^3*b^3 -
 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((81*a^5 - 522*a^4*b + 1273*a^3*b^2 - 1
392*a^2*b^3 + 576*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 +
 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) + 3*a^2 - 15*a*b + 16*b^2)/((a^3*b^3
- 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))) + ((a*b^2 - b^3)*d*cos(d*x + c)^4 - 2*(
a*b^2 - b^3)*d*cos(d*x + c)^2 - (a^2*b - 2*a*b^2 + b^3)*d)*sqrt(((a^3*b^3 -
 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((81*a^5 - 522*a^4*b + 1273*a^3*b^2 - 1
392*a^2*b^3 + 576*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 +
 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) - 3*a^2 + 15*a*b - 16*b^2)/((a^3*b^3
- 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))*log((81*a^3 - 405*a^2*b + 680*a*b^2 - 38
4*b^3)*cos(d*x + c) + ((3*a^4*b^5 - 14*a^3*b^6 + 24*a^2*b^7 - 18*a*b^8 + 5*
b^9)*d^3*sqrt((81*a^5 - 522*a^4*b + 1273*a^3*b^2 - 1392*a^2*b^3 + 576*a*b^4
)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12
 + b^13)*d^4)) + 2*(9*a^3*b^2 - 47*a^2*b^3 + 82*a*b^4 - 48*b^5)*d)*sqrt(((a
^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((81*a^5 - 522*a^4*b + 1273*a^3
*b^2 - 1392*a^2*b^3 + 576*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^
3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) - 3*a^2 + 15*a*b - 16*b^2)/((
a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))) + ((a*b^2 - b^3)*d*cos(d*x + c)
^4 - 2*(a*b^2 - b^3)*d*cos(d*x + c)^2 - (a^2*b - 2*a*b^2 + b^3)*d)*sqrt(-((
a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((81*a^5 - 522*a^4*b + 1273*a^
3*b^2 - 1392*a^2*b^3 + 576*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a
^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) + 3*a^2 - 15*a*b + 16*b^2)/(
(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))*log(-(81*a^3 - 405*a^2*b + 680*
a*b^2 - 384*b^3)*cos(d*x + c) + ((3*a^4*b^5 - 14*a^3*b^6 + 24*a^2*b^7 - 18*
a*b^8 + 5*b^9)*d^3*sqrt((81*a^5 - 522*a^4*b + 1273*a^3*b^2 - 1392*a^2*b^3 +
 576*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11
- 6*a*b^12 + b^13)*d^4)) - 2*(9*a^3*b^2 - 47*a^2*b^3 + 82*a*b^4 - 48*b^5)*d
)*sqrt(-((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((81*a^5 - 522*a^4*b
 + 1273*a^3*b^2 - 1392*a^2*b^3 + 576*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*
b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) + 3*a^2 - 15*a*b +
 16*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))) - ((a*b^2 - b^3)*d*c
os(d*x + c)^4 - 2*(a*b^2 - b^3)*d*cos(d*x + c)^2 - (a^2*b - 2*a*b^2 + b^3)*
d)*sqrt(((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((81*a^5 - 522*a^4*b
 + 1273*a^3*b^2 - 1392*a^2*b^3 + 576*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*
b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) - 3*a^2 + 15*a*b -
 16*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))*log(-(81*a^3 - 405*a^
```

```
2*b + 680*a*b^2 - 384*b^3)*cos(d*x + c) + ((3*a^4*b^5 - 14*a^3*b^6 + 24*a^2
*b^7 - 18*a*b^8 + 5*b^9)*d^3*sqrt((81*a^5 - 522*a^4*b + 1273*a^3*b^2 - 1392
*a^2*b^3 + 576*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15
*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) + 2*(9*a^3*b^2 - 47*a^2*b^3 + 82*a*b^4 -
 48*b^5)*d)*sqrt(((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((81*a^5 -
522*a^4*b + 1273*a^3*b^2 - 1392*a^2*b^3 + 576*a*b^4)/((a^6*b^7 - 6*a^5*b^8
+ 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) - 3*a^2 +
 15*a*b - 16*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))) - 8*a*cos(d
*x + c))/((a*b^2 - b^3)*d*cos(d*x + c)^4 - 2*(a*b^2 - b^3)*d*cos(d*x + c)^2
 - (a^2*b - 2*a*b^2 + b^3)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**7/(a-b*sin(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a-b*sin(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.214 $\int \frac{\sin^5(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=217

$$\frac{\left(\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\sqrt{a}b^{5/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\left(\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\sqrt{a}b^{5/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{\cos(c+dx)\left(a-b\cos^2(c+dx)+b\right)}{4bd(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-\right.}$$

```
[Out] ((Sqrt[a] - 2*Sqrt[b])*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]
]])/(8*Sqrt[a]*(Sqrt[a] - Sqrt[b])^(3/2)*b^(5/4)*d) + ((Sqrt[a] + 2*Sqrt[b]
)*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(8*Sqrt[a]*(Sqrt
[a] + Sqrt[b])^(3/2)*b^(5/4)*d) - (Cos[c + d*x]*(a + b - b*Cos[c + d*x]^2))
/(4*(a - b)*b*d*(a - b + 2*b*Cos[c + d*x]^2 - b*Cos[c + d*x]^4))
```

Rubi [A] time = 0.264157, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3215, 1205, 1166, 205, 208}

$$\frac{\left(\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\sqrt{a}b^{5/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\left(\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\sqrt{a}b^{5/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{\cos(c+dx)\left(a-b\cos^2(c+dx)+b\right)}{4bd(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^5/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] ((Sqrt[a] - 2*Sqrt[b])*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]
]])/(8*Sqrt[a]*(Sqrt[a] - Sqrt[b])^(3/2)*b^(5/4)*d) + ((Sqrt[a] + 2*Sqrt[b]
)*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(8*Sqrt[a]*(Sqrt
[a] + Sqrt[b])^(3/2)*b^(5/4)*d) - (Cos[c + d*x]*(a + b - b*Cos[c + d*x]^2))
/(4*(a - b)*b*d*(a - b + 2*b*Cos[c + d*x]^2 - b*Cos[c + d*x]^4))
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sin^5(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\left(a-b+2bx^2-bx^4\right)^2}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\cos(c+dx)\left(a+b-b\cos^2(c+dx)\right)}{4(a-b)bd\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{2a(a-3b)+2abx^2}{a-b+2bx^2-bx^4}\,dx, x, \text{c}\right.}{8a(a-b)bd}$$

$$= -\frac{\cos(c+dx)\left(a+b-b\cos^2(c+dx)\right)}{4(a-b)bd\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)} - \frac{\left(\sqrt{a}-2\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{-\sqrt{a}\sqrt{b}+b}\right.}{8\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right.}$$

$$= \frac{\left(\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{5/4}d} + \frac{\left(\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}b^{5/4}d} - \frac{\cos(c+}{4(a-b)bd\left(a-\right.}$$

Mathematica [C] time = 0.629844, size = 469, normalized size = 2.16

$$-\frac{\frac{32\cos(c+dx)(2a-b\cos(2(c+dx))+b)}{8a+4b\cos(2(c+dx))-b\cos(4(c+dx))-3b} + i\text{RootSum}\left[-16\#1^4a+\#1^8b-4\#1^6b+6\#1^4b-4\#1^2b+b\&, \frac{-4i\#1^4a\log\left(\#1^2-2\#1\cos(c\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]^5/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] -((32*Cos[c + d*x]*(2*a + b - b*Cos[2*(c + d*x)]))/(8*a - 3*b + 4*b*Cos[2*(
c + d*x)] - b*Cos[4*(c + d*x)]) + I*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*
#1^4 - 4*b*#1^6 + b*#1^8 & , (-2*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]
 + I*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] - 8*a*ArcTan[Sin[c + d*x]/(Cos[c +
 d*x] - #1)]*#1^2 + 22*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 + (4
*I)*a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (11*I)*b*Log[1 - 2*Cos[c + d
*x]*#1 + #1^2]*#1^2 + 8*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - 2
2*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - (4*I)*a*Log[1 - 2*Cos[c
 + d*x]*#1 + #1^2]*#1^4 + (11*I)*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 +
 2*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 - I*b*Log[1 - 2*Cos[c +
```

```
d*x]*#1 + #1^2]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) &
 ])/(32*(a - b)*b*d)
```

Maple [B] time = 0.105, size = 440, normalized size = 2.

$$-\frac{(\cos(dx+c))^3}{4d\left(b(\cos(dx+c))^4-2b(\cos(dx+c))^2-a+b\right)(a-b)}+\frac{\cos(dx+c)\,a}{4bd\left(b(\cos(dx+c))^4-2b(\cos(dx+c))^2-a+b\right)(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5/(a-b*sin(d*x+c)^4)^2,x)
```

```
[Out] -1/4/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)/(a-b)*cos(d*x+c)^3+1/4/d/b*a/(
b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)/(a-b)*cos(d*x+c)+1/4/d/(b*cos(d*x+c)^4
-2*b*cos(d*x+c)^2-a+b)/(a-b)*cos(d*x+c)-1/8/d/(a-b)/(((a*b)^(1/2)-b)*b)^(1/
2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))-1/4/d/(a-b)/(a*b)^(1/2)/(
((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))*b+1
/8/d*a/(a-b)/(a*b)^(1/2)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a
*b)^(1/2)-b)*b)^(1/2))+1/8/d/(a-b)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*
x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))-1/4/d/(a-b)/(a*b)^(1/2)/(((a*b)^(1/2)+b)*
b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))*b+1/8/d*a/(a-b)/(a
*b)^(1/2)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b
)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a-b*sin(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(4*b^2*cos(2*d*x + 2*c)*cos(d*x + c) + 4*b^2*sin(2*d*x + 2*c)*sin(d*x +
 c) - b^2*cos(d*x + c) - 4*(4*a*b + b^2)*sin(3*d*x + 3*c)*sin(2*d*x + 2*c)
- (b^2*cos(7*d*x + 7*c) + b^2*cos(d*x + c) - (4*a*b + b^2)*cos(5*d*x + 5*c)
 - (4*a*b + b^2)*cos(3*d*x + 3*c))*cos(8*d*x + 8*c) + (4*b^2*cos(6*d*x + 6*
c) + 4*b^2*cos(2*d*x + 2*c) - b^2 + 2*(8*a*b - 3*b^2)*cos(4*d*x + 4*c))*cos
(7*d*x + 7*c) + 4*(b^2*cos(d*x + c) - (4*a*b + b^2)*cos(5*d*x + 5*c) - (4*a
*b + b^2)*cos(3*d*x + 3*c))*cos(6*d*x + 6*c) + (4*a*b + b^2 - 2*(32*a^2 - 4
*a*b - 3*b^2)*cos(4*d*x + 4*c) - 4*(4*a*b + b^2)*cos(2*d*x + 2*c))*cos(5*d*
x + 5*c) - 2*((32*a^2 - 4*a*b - 3*b^2)*cos(3*d*x + 3*c) - (8*a*b - 3*b^2)*c
os(d*x + c))*cos(4*d*x + 4*c) + (4*a*b + b^2 - 4*(4*a*b + b^2)*cos(2*d*x +
2*c))*cos(3*d*x + 3*c) + 2*((a*b^3 - b^4)*d*cos(8*d*x + 8*c)^2 + 16*(a*b^3
- b^4)*d*cos(6*d*x + 6*c)^2 + 4*(64*a^3*b - 112*a^2*b^2 + 57*a*b^3 - 9*b^4)
*d*cos(4*d*x + 4*c)^2 + 16*(a*b^3 - b^4)*d*cos(2*d*x + 2*c)^2 + (a*b^3 - b^
4)*d*sin(8*d*x + 8*c)^2 + 16*(a*b^3 - b^4)*d*sin(6*d*x + 6*c)^2 + 4*(64*a^3
*b - 112*a^2*b^2 + 57*a*b^3 - 9*b^4)*d*sin(4*d*x + 4*c)^2 + 16*(8*a^2*b^2 -
 11*a*b^3 + 3*b^4)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a*b^3 - b^4)*d
*sin(2*d*x + 2*c)^2 - 8*(a*b^3 - b^4)*d*cos(2*d*x + 2*c) + (a*b^3 - b^4)*d
- 2*(4*(a*b^3 - b^4)*d*cos(6*d*x + 6*c) + 2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*
d*cos(4*d*x + 4*c) + 4*(a*b^3 - b^4)*d*cos(2*d*x + 2*c) - (a*b^3 - b^4)*d)*
cos(8*d*x + 8*c) + 8*(2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cos(4*d*x + 4*c) +
 4*(a*b^3 - b^4)*d*cos(2*d*x + 2*c) - (a*b^3 - b^4)*d)*cos(6*d*x + 6*c) + 4
```

```
*(4*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cos(2*d*x + 2*c) - (8*a^2*b^2 - 11*a*b
^3 + 3*b^4)*d)*cos(4*d*x + 4*c) - 4*(2*(a*b^3 - b^4)*d*sin(6*d*x + 6*c) + (
8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*sin(4*d*x + 4*c) + 2*(a*b^3 - b^4)*d*sin(2*
d*x + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*sin(4*d
*x + 4*c) + 2*(a*b^3 - b^4)*d*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*integrate
(-1/2*(4*b^2*cos(d*x + c)*sin(2*d*x + 2*c) - 4*b^2*cos(2*d*x + 2*c)*sin(d*x
 + c) + 4*(4*a*b - 11*b^2)*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) + b^2*sin(d*x
+ c) - (b^2*sin(7*d*x + 7*c) - b^2*sin(d*x + c) + (4*a*b - 11*b^2)*sin(5*d*
x + 5*c) - (4*a*b - 11*b^2)*sin(3*d*x + 3*c))*cos(8*d*x + 8*c) - 2*(2*b^2*s
in(6*d*x + 6*c) + 2*b^2*sin(2*d*x + 2*c) + (8*a*b - 3*b^2)*sin(4*d*x + 4*c)
)*cos(7*d*x + 7*c) - 4*(b^2*sin(d*x + c) - (4*a*b - 11*b^2)*sin(5*d*x + 5*c
) + (4*a*b - 11*b^2)*sin(3*d*x + 3*c))*cos(6*d*x + 6*c) - 2*((32*a^2 - 100*
a*b + 33*b^2)*sin(4*d*x + 4*c) + 2*(4*a*b - 11*b^2)*sin(2*d*x + 2*c))*cos(5
*d*x + 5*c) - 2*((32*a^2 - 100*a*b + 33*b^2)*sin(3*d*x + 3*c) + (8*a*b - 3*
b^2)*sin(d*x + c))*cos(4*d*x + 4*c) + (b^2*cos(7*d*x + 7*c) - b^2*cos(d*x +
 c) + (4*a*b - 11*b^2)*cos(5*d*x + 5*c) - (4*a*b - 11*b^2)*cos(3*d*x + 3*c)
)*sin(8*d*x + 8*c) + (4*b^2*cos(6*d*x + 6*c) + 4*b^2*cos(2*d*x + 2*c) - b^2
 + 2*(8*a*b - 3*b^2)*cos(4*d*x + 4*c))*sin(7*d*x + 7*c) + 4*(b^2*cos(d*x +
c) - (4*a*b - 11*b^2)*cos(5*d*x + 5*c) + (4*a*b - 11*b^2)*cos(3*d*x + 3*c))
*sin(6*d*x + 6*c) - (4*a*b - 11*b^2 - 2*(32*a^2 - 100*a*b + 33*b^2)*cos(4*d
*x + 4*c) - 4*(4*a*b - 11*b^2)*cos(2*d*x + 2*c))*sin(5*d*x + 5*c) + 2*((32*
a^2 - 100*a*b + 33*b^2)*cos(3*d*x + 3*c) + (8*a*b - 3*b^2)*cos(d*x + c))*si
n(4*d*x + 4*c) + (4*a*b - 11*b^2 - 4*(4*a*b - 11*b^2)*cos(2*d*x + 2*c))*sin
(3*d*x + 3*c))/(a*b^3 - b^4 + (a*b^3 - b^4)*cos(8*d*x + 8*c)^2 + 16*(a*b^3
- b^4)*cos(6*d*x + 6*c)^2 + 4*(64*a^3*b - 112*a^2*b^2 + 57*a*b^3 - 9*b^4)*c
os(4*d*x + 4*c)^2 + 16*(a*b^3 - b^4)*cos(2*d*x + 2*c)^2 + (a*b^3 - b^4)*sin
(8*d*x + 8*c)^2 + 16*(a*b^3 - b^4)*sin(6*d*x + 6*c)^2 + 4*(64*a^3*b - 112*a
^2*b^2 + 57*a*b^3 - 9*b^4)*sin(4*d*x + 4*c)^2 + 16*(8*a^2*b^2 - 11*a*b^3 +
3*b^4)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a*b^3 - b^4)*sin(2*d*x + 2*c
)^2 + 2*(a*b^3 - b^4 - 4*(a*b^3 - b^4)*cos(6*d*x + 6*c) - 2*(8*a^2*b^2 - 11
*a*b^3 + 3*b^4)*cos(4*d*x + 4*c) - 4*(a*b^3 - b^4)*cos(2*d*x + 2*c))*cos(8*
d*x + 8*c) - 8*(a*b^3 - b^4 - 2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*cos(4*d*x +
4*c) - 4*(a*b^3 - b^4)*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) - 4*(8*a^2*b^2 -
11*a*b^3 + 3*b^4 - 4*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*cos(2*d*x + 2*c))*cos(4
*d*x + 4*c) - 8*(a*b^3 - b^4)*cos(2*d*x + 2*c) - 4*(2*(a*b^3 - b^4)*sin(6*d
*x + 6*c) + (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*sin(4*d*x + 4*c) + 2*(a*b^3 - b^
4)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^2*b^2 - 11*a*b^3 + 3*b^4)*
sin(4*d*x + 4*c) + 2*(a*b^3 - b^4)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)), x)
- (b^2*sin(7*d*x + 7*c) + b^2*sin(d*x + c) - (4*a*b + b^2)*sin(5*d*x + 5*c)
 - (4*a*b + b^2)*sin(3*d*x + 3*c))*sin(8*d*x + 8*c) + 2*(2*b^2*sin(6*d*x +
6*c) + 2*b^2*sin(2*d*x + 2*c) + (8*a*b - 3*b^2)*sin(4*d*x + 4*c))*sin(7*d*x
 + 7*c) + 4*(b^2*sin(d*x + c) - (4*a*b + b^2)*sin(5*d*x + 5*c) - (4*a*b + b
^2)*sin(3*d*x + 3*c))*sin(6*d*x + 6*c) - 2*((32*a^2 - 4*a*b - 3*b^2)*sin(4*
d*x + 4*c) + 2*(4*a*b + b^2)*sin(2*d*x + 2*c))*sin(5*d*x + 5*c) - 2*((32*a^
2 - 4*a*b - 3*b^2)*sin(3*d*x + 3*c) - (8*a*b - 3*b^2)*sin(d*x + c))*sin(4*d
*x + 4*c))/((a*b^3 - b^4)*d*cos(8*d*x + 8*c)^2 + 16*(a*b^3 - b^4)*d*cos(6*d
*x + 6*c)^2 + 4*(64*a^3*b - 112*a^2*b^2 + 57*a*b^3 - 9*b^4)*d*cos(4*d*x + 4
*c)^2 + 16*(a*b^3 - b^4)*d*cos(2*d*x + 2*c)^2 + (a*b^3 - b^4)*d*sin(8*d*x +
 8*c)^2 + 16*(a*b^3 - b^4)*d*sin(6*d*x + 6*c)^2 + 4*(64*a^3*b - 112*a^2*b^2
 + 57*a*b^3 - 9*b^4)*d*sin(4*d*x + 4*c)^2 + 16*(8*a^2*b^2 - 11*a*b^3 + 3*b^
4)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a*b^3 - b^4)*d*sin(2*d*x + 2*c
)^2 - 8*(a*b^3 - b^4)*d*cos(2*d*x + 2*c) + (a*b^3 - b^4)*d - 2*(4*(a*b^3 -
b^4)*d*cos(6*d*x + 6*c) + 2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cos(4*d*x + 4*
c) + 4*(a*b^3 - b^4)*d*cos(2*d*x + 2*c) - (a*b^3 - b^4)*d)*cos(8*d*x + 8*c)
 + 8*(2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cos(4*d*x + 4*c) + 4*(a*b^3 - b^4)
*d*cos(2*d*x + 2*c) - (a*b^3 - b^4)*d)*cos(6*d*x + 6*c) + 4*(4*(8*a^2*b^2 -
 11*a*b^3 + 3*b^4)*d*cos(2*d*x + 2*c) - (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d)*c
os(4*d*x + 4*c) - 4*(2*(a*b^3 - b^4)*d*sin(6*d*x + 6*c) + (8*a^2*b^2 - 11*a
*b^3 + 3*b^4)*d*sin(4*d*x + 4*c) + 2*(a*b^3 - b^4)*d*sin(2*d*x + 2*c))*sin(
```

```
8*d*x + 8*c) + 16*((8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*sin(4*d*x + 4*c) + 2*(a
*b^3 - b^4)*d*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))
```

Fricas [B] time = 5.03459, size = 5237, normalized size = 24.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a-b*sin(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*b*cos(d*x + c)^3 - ((a*b^2 - b^3)*d*cos(d*x + c)^4 - 2*(a*b^2 - b^
3)*d*cos(d*x + c)^2 - (a^2*b - 2*a*b^2 + b^3)*d)*sqrt(((a^4*b^2 - 3*a^3*b^3
 + 3*a^2*b^4 - a*b^5)*d^2*sqrt((a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 + 64
*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^2*
b^10 + a*b^11)*d^4)) + a^2 - a*b - 4*b^2)/((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4
 - a*b^5)*d^2))*log((a^3 - 9*a^2*b + 28*a*b^2 - 32*b^3)*cos(d*x + c) - (2*(
a^4*b^5 - 3*a^3*b^6 + 3*a^2*b^7 - a*b^8)*d^3*sqrt((a^4 - 10*a^3*b + 41*a^2*
b^2 - 80*a*b^3 + 64*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 +
15*a^3*b^9 - 6*a^2*b^10 + a*b^11)*d^4)) - (a^4*b - 8*a^3*b^2 + 23*a^2*b^3 -
 24*a*b^4)*d)*sqrt(((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2*sqrt((a^4
 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 + 64*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^
5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^2*b^10 + a*b^11)*d^4)) + a^2 - a*b -
4*b^2)/((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2))) + ((a*b^2 - b^3)*d
*cos(d*x + c)^4 - 2*(a*b^2 - b^3)*d*cos(d*x + c)^2 - (a^2*b - 2*a*b^2 + b^3
)*d)*sqrt(-((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2*sqrt((a^4 - 10*a^
3*b + 41*a^2*b^2 - 80*a*b^3 + 64*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 -
20*a^4*b^8 + 15*a^3*b^9 - 6*a^2*b^10 + a*b^11)*d^4)) - a^2 + a*b + 4*b^2)/(
(a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2))*log((a^3 - 9*a^2*b + 28*a*b
^2 - 32*b^3)*cos(d*x + c) - (2*(a^4*b^5 - 3*a^3*b^6 + 3*a^2*b^7 - a*b^8)*d^
3*sqrt((a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 + 64*b^4)/((a^7*b^5 - 6*a^6*
b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^2*b^10 + a*b^11)*d^4)) + (
a^4*b - 8*a^3*b^2 + 23*a^2*b^3 - 24*a*b^4)*d)*sqrt(-((a^4*b^2 - 3*a^3*b^3 +
 3*a^2*b^4 - a*b^5)*d^2*sqrt((a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 + 64*b
^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^2*b^
10 + a*b^11)*d^4)) - a^2 + a*b + 4*b^2)/((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 -
 a*b^5)*d^2))) + ((a*b^2 - b^3)*d*cos(d*x + c)^4 - 2*(a*b^2 - b^3)*d*cos(d*
x + c)^2 - (a^2*b - 2*a*b^2 + b^3)*d)*sqrt(((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^
4 - a*b^5)*d^2*sqrt((a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 + 64*b^4)/((a^7
*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^2*b^10 + a*b^
11)*d^4)) + a^2 - a*b - 4*b^2)/((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d
^2))*log(-(a^3 - 9*a^2*b + 28*a*b^2 - 32*b^3)*cos(d*x + c) - (2*(a^4*b^5 -
3*a^3*b^6 + 3*a^2*b^7 - a*b^8)*d^3*sqrt((a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a
*b^3 + 64*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9
 - 6*a^2*b^10 + a*b^11)*d^4)) - (a^4*b - 8*a^3*b^2 + 23*a^2*b^3 - 24*a*b^4)
*d)*sqrt(((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2*sqrt((a^4 - 10*a^3*
b + 41*a^2*b^2 - 80*a*b^3 + 64*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20
*a^4*b^8 + 15*a^3*b^9 - 6*a^2*b^10 + a*b^11)*d^4)) + a^2 - a*b - 4*b^2)/((a
^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2))) - ((a*b^2 - b^3)*d*cos(d*x +
 c)^4 - 2*(a*b^2 - b^3)*d*cos(d*x + c)^2 - (a^2*b - 2*a*b^2 + b^3)*d)*sqrt(
-((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2*sqrt((a^4 - 10*a^3*b + 41*a
^2*b^2 - 80*a*b^3 + 64*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8
 + 15*a^3*b^9 - 6*a^2*b^10 + a*b^11)*d^4)) - a^2 + a*b + 4*b^2)/((a^4*b^2 -
 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2))*log(-(a^3 - 9*a^2*b + 28*a*b^2 - 32*b
^3)*cos(d*x + c) - (2*(a^4*b^5 - 3*a^3*b^6 + 3*a^2*b^7 - a*b^8)*d^3*sqrt((a
^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 + 64*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*
a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^2*b^10 + a*b^11)*d^4)) + (a^4*b - 8
```

```
*a^3*b^2 + 23*a^2*b^3 - 24*a*b^4)*d)*sqrt(-((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^
4 - a*b^5)*d^2*sqrt((a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 + 64*b^4)/((a^7
*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^2*b^10 + a*b^
11)*d^4)) - a^2 + a*b + 4*b^2)/((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d
^2))) - 4*(a + b)*cos(d*x + c))/((a*b^2 - b^3)*d*cos(d*x + c)^4 - 2*(a*b^2
- b^3)*d*cos(d*x + c)^2 - (a^2*b - 2*a*b^2 + b^3)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5/(a-b*sin(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a-b*sin(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.215 $\int \frac{\sin^3(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=186

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\sqrt{a}b^{3/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\sqrt{a}b^{3/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{4d(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}$$

```
[Out] -ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]]/(8*Sqrt[a]*(Sqrt[a]
 - Sqrt[b])^(3/2)*b^(3/4)*d) + ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a]
+ Sqrt[b]]]/(8*Sqrt[a]*(Sqrt[a] + Sqrt[b])^(3/2)*b^(3/4)*d) - (Cos[c + d*x]
*(2 - Cos[c + d*x]^2))/(4*(a - b)*d*(a - b + 2*b*Cos[c + d*x]^2 - b*Cos[c +
 d*x]^4))
```

Rubi [A] time = 0.184763, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3215, 1178, 1166, 205, 208}

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\sqrt{a}b^{3/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\sqrt{a}b^{3/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{4d(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] -ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]]/(8*Sqrt[a]*(Sqrt[a]
 - Sqrt[b])^(3/2)*b^(3/4)*d) + ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a]
+ Sqrt[b]]]/(8*Sqrt[a]*(Sqrt[a] + Sqrt[b])^(3/2)*b^(3/4)*d) - (Cos[c + d*x]
*(2 - Cos[c + d*x]^2))/(4*(a - b)*d*(a - b + 2*b*Cos[c + d*x]^2 - b*Cos[c +
 d*x]^4))
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{\left(a-b+2bx^2-bx^4\right)^2}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{4(a-b)d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{-4ab+2abx^2}{a-b+2bx^2-bx^4}\,dx, x, \cos\right.}{8a(a-b)bd}$$

$$= -\frac{\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{4(a-b)d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{-\sqrt{a}\sqrt{b}+b-bx^2}\,dx, x, \cos\right.}{8\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)d}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{3/4}d} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}b^{3/4}d} - \frac{\cos(c+dx)\left(2-\cos^2(\right.}{4(a-b)d\left(a-b+2b\cos^2(c+dx\right.}$$

Mathematica [C] time = 0.327431, size = 345, normalized size = 1.85

$$\frac{\frac{16(\cos(3(c+dx))-5\cos(c+dx))}{8a+4b\cos(2(c+dx))-b\cos(4(c+dx))-3b} - i\text{RootSum}\left[-16\#1^4a+\#1^8b-4\#1^6b+6\#1^4b-4\#1^2b+b\&, \frac{-i\#1^6\log\left(\#1^2-2\#1\cos(c+dx)\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]^3/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] ((16*(-5*Cos[c + d*x] + Cos[3*(c + d*x)]))/(8*a - 3*b + 4*b*Cos[2*(c + d*x)
] - b*Cos[4*(c + d*x)]) - I*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4
*b*#1^6 + b*#1^8 & , (-2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + I*Log[1
 - 2*Cos[c + d*x]*#1 + #1^2] + 14*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*
#1^2 - (7*I)*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - 14*ArcTan[Sin[c + d*x
]/(Cos[c + d*x] - #1)]*#1^4 + (7*I)*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4
+ 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 - I*Log[1 - 2*Cos[c + d*x
]*#1 + #1^2]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])
/(32*(a - b)*d)
```

Maple [A] time = 0.109, size = 213, normalized size = 1.2

$$\frac{\cos(dx+c)}{8da}\sqrt{ab}\left(\sqrt{ab}+b\right)^{-1}\left(-b\left(\cos(dx+c)\right)^2+\sqrt{ab}+b\right)^{-1}+\frac{1}{8da}\sqrt{ab}\mathrm{Artanh}\left(\cos(dx+c)\,b\frac{1}{\sqrt{\left(\sqrt{ab}+b\right)b}}\right)\left(\sqrt{ab}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a-b*sin(d*x+c)^4)^2,x)
```

```
[Out] 1/8/d*(a*b)^(1/2)/a*cos(d*x+c)/((a*b)^(1/2)+b)/(-b*cos(d*x+c)^2+(a*b)^(1/2)
+b)+1/8/d*(a*b)^(1/2)/a/((a*b)^(1/2)+b)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(c
os(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))-1/8/d*(a*b)^(1/2)/a*cos(d*x+c)/((a*b
)^(1/2)-b)/(b*cos(d*x+c)^2-b+(a*b)^(1/2))-1/8/d*(a*b)^(1/2)/a/((a*b)^(1/2)-
b)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a-b*sin(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 3.83853, size = 4251, normalized size = 22.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a-b*sin(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*cos(d*x + c)^3 - ((a*b - b^2)*d*cos(d*x + c)^4 - 2*(a*b - b^2)*d*c
os(d*x + c)^2 - (a^2 - 2*a*b + b^2)*d)*sqrt(-((a^4*b - 3*a^3*b^2 + 3*a^2*b^
3 - a*b^4)*d^2*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^
5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + 3*a + b)/((a^4*b -
 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2))*log((a + 3*b)*cos(d*x + c) - ((a^5*b^
2 - 2*a^4*b^3 + 2*a^2*b^5 - a*b^6)*d^3*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3
 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^
4)) - 2*(a^2*b + 3*a*b^2)*d)*sqrt(-((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)
*d^2*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4
*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + 3*a + b)/((a^4*b - 3*a^3*b^2
 + 3*a^2*b^3 - a*b^4)*d^2))) + ((a*b - b^2)*d*cos(d*x + c)^4 - 2*(a*b - b^2
)*d*cos(d*x + c)^2 - (a^2 - 2*a*b + b^2)*d)*sqrt(((a^4*b - 3*a^3*b^2 + 3*a^
2*b^3 - a*b^4)*d^2*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^
5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) - 3*a - b)/((a^4
*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2))*log((a + 3*b)*cos(d*x + c) - ((a^
5*b^2 - 2*a^4*b^3 + 2*a^2*b^5 - a*b^6)*d^3*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7
*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9
)*d^4)) + 2*(a^2*b + 3*a*b^2)*d)*sqrt(((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b
^4)*d^2*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*
a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) - 3*a - b)/((a^4*b - 3*a^3*
```

```
b^2 + 3*a^2*b^3 - a*b^4)*d^2))) + ((a*b - b^2)*d*cos(d*x + c)^4 - 2*(a*b -
b^2)*d*cos(d*x + c)^2 - (a^2 - 2*a*b + b^2)*d)*sqrt(-((a^4*b - 3*a^3*b^2 +
3*a^2*b^3 - a*b^4)*d^2*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^4 + 1
5*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + 3*a + b)/(
(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2))*log(-(a + 3*b)*cos(d*x + c) -
 ((a^5*b^2 - 2*a^4*b^3 + 2*a^2*b^5 - a*b^6)*d^3*sqrt((a^2 + 6*a*b + 9*b^2)/
((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 +
a*b^9)*d^4)) - 2*(a^2*b + 3*a*b^2)*d)*sqrt(-((a^4*b - 3*a^3*b^2 + 3*a^2*b^3
 - a*b^4)*d^2*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5
 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + 3*a + b)/((a^4*b -
3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2))) - ((a*b - b^2)*d*cos(d*x + c)^4 - 2*(
a*b - b^2)*d*cos(d*x + c)^2 - (a^2 - 2*a*b + b^2)*d)*sqrt(((a^4*b - 3*a^3*b
^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^
4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) - 3*a -
 b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2))*log(-(a + 3*b)*cos(d*x +
 c) - ((a^5*b^2 - 2*a^4*b^3 + 2*a^2*b^5 - a*b^6)*d^3*sqrt((a^2 + 6*a*b + 9*
b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b
^8 + a*b^9)*d^4)) + 2*(a^2*b + 3*a*b^2)*d)*sqrt(((a^4*b - 3*a^3*b^2 + 3*a^2
*b^3 - a*b^4)*d^2*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5
*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) - 3*a - b)/((a^4*
b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2))) - 8*cos(d*x + c))/((a*b - b^2)*d*
cos(d*x + c)^4 - 2*(a*b - b^2)*d*cos(d*x + c)^2 - (a^2 - 2*a*b + b^2)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a-b*sin(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a-b*sin(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.216 $\int \frac{\sin(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=221

$$-\frac{\left(3\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{3/2}\sqrt[4]{b}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\left(3\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8a^{3/2}\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{\cos(c+dx)\left(a-b\cos^2(c+dx)+b\right)}{4ad(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+a\right.}$$

```
[Out] -((3*Sqrt[a] - 2*Sqrt[b])*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt
[b]]])/(8*a^(3/2)*(Sqrt[a] - Sqrt[b])^(3/2)*b^(1/4)*d) - ((3*Sqrt[a] + 2*Sq
rt[b])*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(8*a^(3/2)*
(Sqrt[a] + Sqrt[b])^(3/2)*b^(1/4)*d) - (Cos[c + d*x]*(a + b - b*Cos[c + d*x
]^2))/(4*a*(a - b)*d*(a - b + 2*b*Cos[c + d*x]^2 - b*Cos[c + d*x]^4))
```

Rubi [A] time = 0.2653, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {3215, 1092, 1166, 205, 208}

$$-\frac{\left(3\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{3/2}\sqrt[4]{b}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\left(3\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8a^{3/2}\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{\cos(c+dx)\left(a-b\cos^2(c+dx)+b\right)}{4ad(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a - b*Sin[c + d*x]^4)^2,x]
```


```
[Out] -((3*Sqrt[a] - 2*Sqrt[b])*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt
[b]]])/(8*a^(3/2)*(Sqrt[a] - Sqrt[b])^(3/2)*b^(1/4)*d) - ((3*Sqrt[a] + 2*Sq
rt[b])*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(8*a^(3/2)*
(Sqrt[a] + Sqrt[b])^(3/2)*b^(1/4)*d) - (Cos[c + d*x]*(a + b - b*Cos[c + d*x
]^2))/(4*a*(a - b)*d*(a - b + 2*b*Cos[c + d*x]^2 - b*Cos[c + d*x]^4))
```


Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
```

```
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{\left(a-b+2bx^2-bx^4\right)^2}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\cos(c+dx)\left(a+b-b\cos^2(c+dx)\right)}{4a(a-b)d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{2(a-b)b+4b^2-2\left(4(a-b)b+4\right.}{a-b+2bx^2-bx^4}\right.}{8a(a-}$$

$$= -\frac{\cos(c+dx)\left(a+b-b\cos^2(c+dx)\right)}{4a(a-b)d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)} + \frac{\left(\left(3\sqrt{a}-2\sqrt{b}\right)\sqrt{b}\right)\text{Subst}\left(\int\right.}{8a^{3/2}\left(\sqrt{a}\right.}$$

$$= -\frac{\left(3\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{3/2}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\sqrt[4]{b}d} - \frac{\left(3\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8a^{3/2}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\sqrt[4]{b}d} - \frac{\cos}{4a(a-b)d\left(\right.}$$

Mathematica [C] time = 0.444927, size = 469, normalized size = 2.12

$$-\frac{\frac{32\cos(c+dx)(2a-b\cos(2(c+dx))+b)}{8a+4b\cos(2(c+dx))-b\cos(4(c+dx))-3b} + i\text{RootSum}\left[-16\#1^4a+\#1^8b-4\#1^6b+6\#1^4b-4\#1^2b+b\&, \frac{12i\#1^4a\log\left(\#1^2-2\#1\cos(c\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] -((32*Cos[c + d*x]*(2*a + b - b*Cos[2*(c + d*x)]))/(8*a - 3*b + 4*b*Cos[2*(
c + d*x)] - b*Cos[4*(c + d*x)]) + I*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*
#1^4 - 4*b*#1^6 + b*#1^8 & , (-2*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]
 + I*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] + 24*a*ArcTan[Sin[c + d*x]/(Cos[c
+ d*x] - #1)]*#1^2 - 10*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 - (
12*I)*a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + (5*I)*b*Log[1 - 2*Cos[c +
d*x]*#1 + #1^2]*#1^2 - 24*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 +
 10*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 + (12*I)*a*Log[1 - 2*Co
s[c + d*x]*#1 + #1^2]*#1^4 - (5*I)*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4
 + 2*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 - I*b*Log[1 - 2*Cos[c
+ d*x]*#1 + #1^2]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7)
 & ])/(32*a*(a - b)*d)
```

Maple [B] time = 0.146, size = 488, normalized size = 2.2

$$-\frac{\cos(dx+c)}{8da(a-b)}\left((\cos(dx+c))^2-1-\frac{1}{b}\sqrt{ab}\right)^{-1}+\frac{\cos(dx+c)}{8d(a-b)}\frac{1}{\sqrt{ab}}\left((\cos(dx+c))^2-1-\frac{1}{b}\sqrt{ab}\right)^{-1}+\frac{b}{8da(a-b)}\mathrm{Artanh}\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a-b*sin(d*x+c)^4)^2,x)
```

```
[Out] -1/8/d/a/(a-b)*cos(d*x+c)/(cos(d*x+c)^2-1-(a*b)^(1/2)/b)+1/8/d/(a*b)^(1/2)/
(a-b)*cos(d*x+c)/(cos(d*x+c)^2-1-(a*b)^(1/2)/b)+1/8/d*b/a/(a-b)/(((a*b)^(1/
2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))-3/8/d/(a-b)/
(a*b)^(1/2)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)
*b)^(1/2))*b+1/4/d*b^2/(a*b)^(1/2)/a/(a-b)/(((a*b)^(1/2)+b)*b)^(1/2)*arctan
h(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))-1/8/d/a/(a-b)*cos(d*x+c)/(cos(d*x
+c)^2+(a*b)^(1/2)/b-1)-1/8/d/(a*b)^(1/2)/(a-b)*cos(d*x+c)/(cos(d*x+c)^2+(a*
b)^(1/2)/b-1)-1/8/d*b/a/(a-b)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b
/(((a*b)^(1/2)-b)*b)^(1/2))-3/8/d/(a-b)/(a*b)^(1/2)/(((a*b)^(1/2)-b)*b)^(1/
2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))*b+1/4/d*b^2/(a*b)^(1/2)/a
/(a-b)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1
/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-b*sin(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(4*b^2*cos(2*d*x + 2*c)*cos(d*x + c) + 4*b^2*sin(2*d*x + 2*c)*sin(d*x +
 c) - b^2*cos(d*x + c) - 4*(4*a*b + b^2)*sin(3*d*x + 3*c)*sin(2*d*x + 2*c)
- (b^2*cos(7*d*x + 7*c) + b^2*cos(d*x + c) - (4*a*b + b^2)*cos(5*d*x + 5*c)
 - (4*a*b + b^2)*cos(3*d*x + 3*c))*cos(8*d*x + 8*c) + (4*b^2*cos(6*d*x + 6*
c) + 4*b^2*cos(2*d*x + 2*c) - b^2 + 2*(8*a*b - 3*b^2)*cos(4*d*x + 4*c))*cos
(7*d*x + 7*c) + 4*(b^2*cos(d*x + c) - (4*a*b + b^2)*cos(5*d*x + 5*c) - (4*a
*b + b^2)*cos(3*d*x + 3*c))*cos(6*d*x + 6*c) + (4*a*b + b^2 - 2*(32*a^2 - 4
*a*b - 3*b^2)*cos(4*d*x + 4*c) - 4*(4*a*b + b^2)*cos(2*d*x + 2*c))*cos(5*d*
x + 5*c) - 2*((32*a^2 - 4*a*b - 3*b^2)*cos(3*d*x + 3*c) - (8*a*b - 3*b^2)*c
os(d*x + c))*cos(4*d*x + 4*c) + (4*a*b + b^2 - 4*(4*a*b + b^2)*cos(2*d*x +
2*c))*cos(3*d*x + 3*c) + 2*((a^2*b^2 - a*b^3)*d*cos(8*d*x + 8*c)^2 + 16*(a^
2*b^2 - a*b^3)*d*cos(6*d*x + 6*c)^2 + 4*(64*a^4 - 112*a^3*b + 57*a^2*b^2 -
9*a*b^3)*d*cos(4*d*x + 4*c)^2 + 16*(a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c)^2 +
 (a^2*b^2 - a*b^3)*d*sin(8*d*x + 8*c)^2 + 16*(a^2*b^2 - a*b^3)*d*sin(6*d*x
+ 6*c)^2 + 4*(64*a^4 - 112*a^3*b + 57*a^2*b^2 - 9*a*b^3)*d*sin(4*d*x + 4*c)
^2 + 16*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c
) + 16*(a^2*b^2 - a*b^3)*d*sin(2*d*x + 2*c)^2 - 8*(a^2*b^2 - a*b^3)*d*cos(2
*d*x + 2*c) + (a^2*b^2 - a*b^3)*d - 2*(4*(a^2*b^2 - a*b^3)*d*cos(6*d*x + 6*
c) + 2*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*cos(4*d*x + 4*c) + 4*(a^2*b^2 - a
*b^3)*d*cos(2*d*x + 2*c) - (a^2*b^2 - a*b^3)*d)*cos(8*d*x + 8*c) + 8*(2*(8*
a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*cos(4*d*x + 4*c) + 4*(a^2*b^2 - a*b^3)*d*co
s(2*d*x + 2*c) - (a^2*b^2 - a*b^3)*d)*cos(6*d*x + 6*c) + 4*(4*(8*a^3*b - 11
*a^2*b^2 + 3*a*b^3)*d*cos(2*d*x + 2*c) - (8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d
)*cos(4*d*x + 4*c) - 4*(2*(a^2*b^2 - a*b^3)*d*sin(6*d*x + 6*c) + (8*a^3*b -
 11*a^2*b^2 + 3*a*b^3)*d*sin(4*d*x + 4*c) + 2*(a^2*b^2 - a*b^3)*d*sin(2*d*x
```

```
 + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*sin(4*d*
x + 4*c) + 2*(a^2*b^2 - a*b^3)*d*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*integr
ate(-1/2*(4*b^2*cos(d*x + c)*sin(2*d*x + 2*c) - 4*b^2*cos(2*d*x + 2*c)*sin(
d*x + c) - 4*(12*a*b - 5*b^2)*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) + b^2*sin(d
*x + c) - (b^2*sin(7*d*x + 7*c) - b^2*sin(d*x + c) - (12*a*b - 5*b^2)*sin(5
*d*x + 5*c) + (12*a*b - 5*b^2)*sin(3*d*x + 3*c))*cos(8*d*x + 8*c) - 2*(2*b^
2*sin(6*d*x + 6*c) + 2*b^2*sin(2*d*x + 2*c) + (8*a*b - 3*b^2)*sin(4*d*x + 4
*c))*cos(7*d*x + 7*c) - 4*(b^2*sin(d*x + c) + (12*a*b - 5*b^2)*sin(5*d*x +
5*c) - (12*a*b - 5*b^2)*sin(3*d*x + 3*c))*cos(6*d*x + 6*c) + 2*((96*a^2 - 7
6*a*b + 15*b^2)*sin(4*d*x + 4*c) + 2*(12*a*b - 5*b^2)*sin(2*d*x + 2*c))*cos
(5*d*x + 5*c) + 2*((96*a^2 - 76*a*b + 15*b^2)*sin(3*d*x + 3*c) - (8*a*b - 3
*b^2)*sin(d*x + c))*cos(4*d*x + 4*c) + (b^2*cos(7*d*x + 7*c) - b^2*cos(d*x
+ c) - (12*a*b - 5*b^2)*cos(5*d*x + 5*c) + (12*a*b - 5*b^2)*cos(3*d*x + 3*c
))*sin(8*d*x + 8*c) + (4*b^2*cos(6*d*x + 6*c) + 4*b^2*cos(2*d*x + 2*c) - b^
2 + 2*(8*a*b - 3*b^2)*cos(4*d*x + 4*c))*sin(7*d*x + 7*c) + 4*(b^2*cos(d*x +
 c) + (12*a*b - 5*b^2)*cos(5*d*x + 5*c) - (12*a*b - 5*b^2)*cos(3*d*x + 3*c)
)*sin(6*d*x + 6*c) + (12*a*b - 5*b^2 - 2*(96*a^2 - 76*a*b + 15*b^2)*cos(4*d
*x + 4*c) - 4*(12*a*b - 5*b^2)*cos(2*d*x + 2*c))*sin(5*d*x + 5*c) - 2*((96*
a^2 - 76*a*b + 15*b^2)*cos(3*d*x + 3*c) - (8*a*b - 3*b^2)*cos(d*x + c))*sin
(4*d*x + 4*c) - (12*a*b - 5*b^2 - 4*(12*a*b - 5*b^2)*cos(2*d*x + 2*c))*sin(
3*d*x + 3*c))/(a^2*b^2 - a*b^3 + (a^2*b^2 - a*b^3)*cos(8*d*x + 8*c)^2 + 16*
(a^2*b^2 - a*b^3)*cos(6*d*x + 6*c)^2 + 4*(64*a^4 - 112*a^3*b + 57*a^2*b^2 -
 9*a*b^3)*cos(4*d*x + 4*c)^2 + 16*(a^2*b^2 - a*b^3)*cos(2*d*x + 2*c)^2 + (a
^2*b^2 - a*b^3)*sin(8*d*x + 8*c)^2 + 16*(a^2*b^2 - a*b^3)*sin(6*d*x + 6*c)^
2 + 4*(64*a^4 - 112*a^3*b + 57*a^2*b^2 - 9*a*b^3)*sin(4*d*x + 4*c)^2 + 16*(
8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a^2
*b^2 - a*b^3)*sin(2*d*x + 2*c)^2 + 2*(a^2*b^2 - a*b^3 - 4*(a^2*b^2 - a*b^3)
*cos(6*d*x + 6*c) - 2*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*cos(4*d*x + 4*c) - 4
*(a^2*b^2 - a*b^3)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) - 8*(a^2*b^2 - a*b^3
- 2*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*cos(4*d*x + 4*c) - 4*(a^2*b^2 - a*b^3)
*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) - 4*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3 - 4
*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 8*(a
^2*b^2 - a*b^3)*cos(2*d*x + 2*c) - 4*(2*(a^2*b^2 - a*b^3)*sin(6*d*x + 6*c)
+ (8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*sin(4*d*x + 4*c) + 2*(a^2*b^2 - a*b^3)*s
in(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*si
n(4*d*x + 4*c) + 2*(a^2*b^2 - a*b^3)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)), x
) - (b^2*sin(7*d*x + 7*c) + b^2*sin(d*x + c) - (4*a*b + b^2)*sin(5*d*x + 5*
c) - (4*a*b + b^2)*sin(3*d*x + 3*c))*sin(8*d*x + 8*c) + 2*(2*b^2*sin(6*d*x
+ 6*c) + 2*b^2*sin(2*d*x + 2*c) + (8*a*b - 3*b^2)*sin(4*d*x + 4*c))*sin(7*d
*x + 7*c) + 4*(b^2*sin(d*x + c) - (4*a*b + b^2)*sin(5*d*x + 5*c) - (4*a*b +
 b^2)*sin(3*d*x + 3*c))*sin(6*d*x + 6*c) - 2*((32*a^2 - 4*a*b - 3*b^2)*sin(
4*d*x + 4*c) + 2*(4*a*b + b^2)*sin(2*d*x + 2*c))*sin(5*d*x + 5*c) - 2*((32*
a^2 - 4*a*b - 3*b^2)*sin(3*d*x + 3*c) - (8*a*b - 3*b^2)*sin(d*x + c))*sin(4
*d*x + 4*c))/((a^2*b^2 - a*b^3)*d*cos(8*d*x + 8*c)^2 + 16*(a^2*b^2 - a*b^3)
*d*cos(6*d*x + 6*c)^2 + 4*(64*a^4 - 112*a^3*b + 57*a^2*b^2 - 9*a*b^3)*d*cos
(4*d*x + 4*c)^2 + 16*(a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c)^2 + (a^2*b^2 - a*
b^3)*d*sin(8*d*x + 8*c)^2 + 16*(a^2*b^2 - a*b^3)*d*sin(6*d*x + 6*c)^2 + 4*(
64*a^4 - 112*a^3*b + 57*a^2*b^2 - 9*a*b^3)*d*sin(4*d*x + 4*c)^2 + 16*(8*a^3
*b - 11*a^2*b^2 + 3*a*b^3)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a^2*b^
2 - a*b^3)*d*sin(2*d*x + 2*c)^2 - 8*(a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c) +
(a^2*b^2 - a*b^3)*d - 2*(4*(a^2*b^2 - a*b^3)*d*cos(6*d*x + 6*c) + 2*(8*a^3*
b - 11*a^2*b^2 + 3*a*b^3)*d*cos(4*d*x + 4*c) + 4*(a^2*b^2 - a*b^3)*d*cos(2*
d*x + 2*c) - (a^2*b^2 - a*b^3)*d)*cos(8*d*x + 8*c) + 8*(2*(8*a^3*b - 11*a^2
*b^2 + 3*a*b^3)*d*cos(4*d*x + 4*c) + 4*(a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c)
 - (a^2*b^2 - a*b^3)*d)*cos(6*d*x + 6*c) + 4*(4*(8*a^3*b - 11*a^2*b^2 + 3*a
*b^3)*d*cos(2*d*x + 2*c) - (8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d)*cos(4*d*x +
4*c) - 4*(2*(a^2*b^2 - a*b^3)*d*sin(6*d*x + 6*c) + (8*a^3*b - 11*a^2*b^2 +
3*a*b^3)*d*sin(4*d*x + 4*c) + 2*(a^2*b^2 - a*b^3)*d*sin(2*d*x + 2*c))*sin(8
*d*x + 8*c) + 16*((8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*sin(4*d*x + 4*c) + 2*(
```

```
a^2*b^2 - a*b^3)*d*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))
```

Fricas [B] time = 5.24439, size = 4799, normalized size = 21.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-b*sin(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*b*cos(d*x + c)^3 - ((a^2*b - a*b^2)*d*cos(d*x + c)^4 - 2*(a^2*b -
a*b^2)*d*cos(d*x + c)^2 - (a^3 - 2*a^2*b + a*b^2)*d)*sqrt(-((a^6 - 3*a^5*b
+ 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8*
b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + 1
5*a^2 - 15*a*b + 4*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))*log((8
1*a^2 - 81*a*b + 20*b^2)*cos(d*x + c) + (2*(2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3
 - 5*a^4*b^4 + a^3*b^5)*d^3*sqrt((81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8
*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) -
(27*a^4 - 24*a^3*b + 5*a^2*b^2)*d)*sqrt(-((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*
b^3)*d^2*sqrt((81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 -
 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + 15*a^2 - 15*a*b + 4
*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))) + ((a^2*b - a*b^2)*d*co
s(d*x + c)^4 - 2*(a^2*b - a*b^2)*d*cos(d*x + c)^2 - (a^3 - 2*a^2*b + a*b^2)
*d)*sqrt(((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((81*a^2 - 90*a*b +
 25*b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4
*b^6 + a^3*b^7)*d^4)) - 15*a^2 + 15*a*b - 4*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^
2 - a^3*b^3)*d^2))*log((81*a^2 - 81*a*b + 20*b^2)*cos(d*x + c) + (2*(2*a^7*
b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*d^3*sqrt((81*a^2 - 90*a*b
+ 25*b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^
4*b^6 + a^3*b^7)*d^4)) + (27*a^4 - 24*a^3*b + 5*a^2*b^2)*d)*sqrt(((a^6 - 3*
a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((81*a^2 - 90*a*b + 25*b^2)/((a^9*b -
6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4
)) - 15*a^2 + 15*a*b - 4*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2)))
 + ((a^2*b - a*b^2)*d*cos(d*x + c)^4 - 2*(a^2*b - a*b^2)*d*cos(d*x + c)^2 -
 (a^3 - 2*a^2*b + a*b^2)*d)*sqrt(-((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^
2*sqrt((81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6
*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + 15*a^2 - 15*a*b + 4*b^2)/(
(a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))*log(-(81*a^2 - 81*a*b + 20*b^2)
*cos(d*x + c) + (2*(2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*
d^3*sqrt((81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a
^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) - (27*a^4 - 24*a^3*b + 5*a
^2*b^2)*d)*sqrt(-((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((81*a^2 -
90*a*b + 25*b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5
 - 6*a^4*b^6 + a^3*b^7)*d^4)) + 15*a^2 - 15*a*b + 4*b^2)/((a^6 - 3*a^5*b +
3*a^4*b^2 - a^3*b^3)*d^2))) - ((a^2*b - a*b^2)*d*cos(d*x + c)^4 - 2*(a^2*b
- a*b^2)*d*cos(d*x + c)^2 - (a^3 - 2*a^2*b + a*b^2)*d)*sqrt(((a^6 - 3*a^5*b
 + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8
*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) -
15*a^2 + 15*a*b - 4*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))*log(-
(81*a^2 - 81*a*b + 20*b^2)*cos(d*x + c) + (2*(2*a^7*b - 7*a^6*b^2 + 9*a^5*b
^3 - 5*a^4*b^4 + a^3*b^5)*d^3*sqrt((81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a
^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4))
+ (27*a^4 - 24*a^3*b + 5*a^2*b^2)*d)*sqrt(((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3
*b^3)*d^2*sqrt((81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3
- 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) - 15*a^2 + 15*a*b -
4*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))) - 4*(a + b)*cos(d*x +
c))/((a^2*b - a*b^2)*d*cos(d*x + c)^4 - 2*(a^2*b - a*b^2)*d*cos(d*x + c)^2
```

```
- (a^3 - 2*a^2*b + a*b^2)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-b*sin(d*x+c)**4)**2,x)

[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-b*sin(d*x+c)^4)^2,x, algorithm="giac")

[Out] Exception raised: NotImplementedError
```

3.217 $\int \frac{\csc(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=325

$$-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^2d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{3/2}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^2d\sqrt{\sqrt{a}+\sqrt{b}}}+\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8a^{3/2}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{\tanh^{-1}(\cos}{a^2d}$$

```
[Out] -(b^(1/4)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(8*a^(3/2
)*(Sqrt[a] - Sqrt[b])^(3/2)*d) - (b^(1/4)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqr
t[Sqrt[a] - Sqrt[b]]])/(2*a^2*Sqrt[Sqrt[a] - Sqrt[b]]*d) - ArcTanh[Cos[c +
d*x]]/(a^2*d) + (b^(1/4)*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt
[b]]])/(8*a^(3/2)*(Sqrt[a] + Sqrt[b])^(3/2)*d) + (b^(1/4)*ArcTanh[(b^(1/4)*
Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*a^2*Sqrt[Sqrt[a] + Sqrt[b]]*d) -
 (b*Cos[c + d*x]*(2 - Cos[c + d*x]^2))/(4*a*(a - b)*d*(a - b + 2*b*Cos[c +
d*x]^2 - b*Cos[c + d*x]^4))
```

Rubi [A] time = 0.335504, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {3215, 1238, 207, 1178, 1166, 205, 208}

$$-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^2d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{3/2}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^2d\sqrt{\sqrt{a}+\sqrt{b}}}+\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8a^{3/2}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{\tanh^{-1}(\cos}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] -(b^(1/4)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(8*a^(3/2
)*(Sqrt[a] - Sqrt[b])^(3/2)*d) - (b^(1/4)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqr
t[Sqrt[a] - Sqrt[b]]])/(2*a^2*Sqrt[Sqrt[a] - Sqrt[b]]*d) - ArcTanh[Cos[c +
d*x]]/(a^2*d) + (b^(1/4)*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt
[b]]])/(8*a^(3/2)*(Sqrt[a] + Sqrt[b])^(3/2)*d) + (b^(1/4)*ArcTanh[(b^(1/4)*
Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*a^2*Sqrt[Sqrt[a] + Sqrt[b]]*d) -
 (b*Cos[c + d*x]*(2 - Cos[c + d*x]^2))/(4*a*(a - b)*d*(a - b + 2*b*Cos[c +
d*x]^2 - b*Cos[c + d*x]^4))
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1238

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b^2 - 4*a*c, 0] && ((IntegerQ[p]
 && IntegerQ[q]) || IGtQ[p, 0] || IGtQ[q, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\csc(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\left(a-b+2bx^2-bx^4\right)^2}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(-\frac{1}{a^2(-1+x^2)} + \frac{b-bx^2}{a\left(a-b+2bx^2-bx^4\right)^2} + \frac{b-bx^2}{a^2\left(a-b+2bx^2-bx^4\right)}\right)dx, x, \cos(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \cos(c+dx)\right)}{a^2 d} - \frac{\text{Subst}\left(\int \frac{b-bx^2}{a-b+2bx^2-bx^4}\,dx, x, \cos(c+dx)\right)}{a^2 d} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{a^2 d} - \frac{b\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{4a(a-b)d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^2\sqrt{\sqrt{a}-\sqrt{b}}\,d} - \frac{\tanh^{-1}(\cos(c+dx))}{a^2 d} + \frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^2\sqrt{\sqrt{a}+\sqrt{b}}\,d} - \frac{b}{4a(a-b)d\left(\right.}$$

$$= -\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{3/2}\left(\sqrt{a}-\sqrt{b}\right)^{3/2} d} - \frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^2\sqrt{\sqrt{a}-\sqrt{b}}\,d} - \frac{\tanh^{-1}(\cos(c+dx))}{a^2 d} + \frac{\sqrt[4]{b}\tanh^{-1}\left(\right.}{8a^{3/2}\left(\sqrt{a}+\right.}$$

Mathematica [C] time = 0.831079, size = 600, normalized size = 1.85

$$-\frac{ib\,\text{RootSum}\left[-16\#1^4a+\#1^8b-4\#1^6b+6\#1^4b-4\#1^2b+b\&, \frac{-5i\#1^6a\log\left(\#1^2-2\#1\cos(c+dx)+1\right)+19i\#1^4a\log\left(\#1^2-2\#1\cos(c+dx)+1\right)-19i\#1^2a\log\left(\#1^2-2\#1\cos(c+dx)+1\right)+5ia\log\left(\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] ((16*a*b*(-5*Cos[c + d*x] + Cos[3*(c + d*x)]))/((a - b)*(8*a - 3*b + 4*b*Co
s[2*(c + d*x)] - b*Cos[4*(c + d*x)])) - 32*Log[Cos[(c + d*x)/2]] + 32*Log[S
in[(c + d*x)/2]] - (I*b*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#
1^6 + b*#1^8 & , (-10*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + 8*b*ArcT
an[Sin[c + d*x]/(Cos[c + d*x] - #1)] + (5*I)*a*Log[1 - 2*Cos[c + d*x]*#1 +
#1^2] - (4*I)*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] + 38*a*ArcTan[Sin[c + d*x
]/(Cos[c + d*x] - #1)]*#1^2 - 24*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]
*#1^2 - (19*I)*a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + (12*I)*b*Log[1 -
2*Cos[c + d*x]*#1 + #1^2]*#1^2 - 38*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #
1)]*#1^4 + 24*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 + (19*I)*a*Lo
g[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 - (12*I)*b*Log[1 - 2*Cos[c + d*x]*#1 +
 #1^2]*#1^4 + 10*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 - 8*b*ArcT
an[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 - (5*I)*a*Log[1 - 2*Cos[c + d*x]*
#1 + #1^2]*#1^6 + (4*I)*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6)/(-(b*#1)
- 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/(a - b))/(32*a^2*d)
```

Maple [A] time = 0.147, size = 450, normalized size = 1.4

$$\frac{\ln(-1+\cos(dx+c))}{2a^2d} - \frac{\ln(1+\cos(dx+c))}{2a^2d} - \frac{b(\cos(dx+c))^3}{4da\left(b(\cos(dx+c))^4-2b(\cos(dx+c))^2-a+b\right)(a-b)} + \frac{}{2da\left(b(\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a-b*sin(d*x+c)^4)^2,x)
```

```
[Out] 1/2/d/a^2*ln(-1+cos(d*x+c))-1/2/d/a^2*ln(1+cos(d*x+c))-1/4/d*b/a/(b*cos(d*x
+c)^4-2*b*cos(d*x+c)^2-a+b)/(a-b)*cos(d*x+c)^3+1/2/d*b/a/(b*cos(d*x+c)^4-2*
b*cos(d*x+c)^2-a+b)/(a-b)*cos(d*x+c)-5/8/d*b/a/(a-b)/(((a*b)^(1/2)-b)*b)^(1
/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))+1/2/d*b^2/a^2/(a-b)/(((a
*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))-1/8/d*
b^2/(a*b)^(1/2)/a/(a-b)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*
b)^(1/2)-b)*b)^(1/2))+5/8/d*b/a/(a-b)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos
(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))-1/2/d*b^2/a^2/(a-b)/(((a*b)^(1/2)+b)*b
)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))-1/8/d*b^2/(a*b)^(1/
2)/a/(a-b)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*
b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-b*sin(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(4*a*b^2*cos(2*d*x + 2*c)*cos(d*x + c) - 20*a*b^2*sin(3*d*x + 3*c)*sin(
2*d*x + 2*c) + 4*a*b^2*sin(2*d*x + 2*c)*sin(d*x + c) - a*b^2*cos(d*x + c) -
 (a*b^2*cos(7*d*x + 7*c) - 5*a*b^2*cos(5*d*x + 5*c) - 5*a*b^2*cos(3*d*x + 3
*c) + a*b^2*cos(d*x + c))*cos(8*d*x + 8*c) + (4*a*b^2*cos(6*d*x + 6*c) + 4*
a*b^2*cos(2*d*x + 2*c) - a*b^2 + 2*(8*a^2*b - 3*a*b^2)*cos(4*d*x + 4*c))*co
s(7*d*x + 7*c) - 4*(5*a*b^2*cos(5*d*x + 5*c) + 5*a*b^2*cos(3*d*x + 3*c) - a
*b^2*cos(d*x + c))*cos(6*d*x + 6*c) - 5*(4*a*b^2*cos(2*d*x + 2*c) - a*b^2 +
 2*(8*a^2*b - 3*a*b^2)*cos(4*d*x + 4*c))*cos(5*d*x + 5*c) - 2*(5*(8*a^2*b -
 3*a*b^2)*cos(3*d*x + 3*c) - (8*a^2*b - 3*a*b^2)*cos(d*x + c))*cos(4*d*x +
4*c) - 5*(4*a*b^2*cos(2*d*x + 2*c) - a*b^2)*cos(3*d*x + 3*c) - 2*((a^3*b^2
- a^2*b^3)*d*cos(8*d*x + 8*c)^2 + 16*(a^3*b^2 - a^2*b^3)*d*cos(6*d*x + 6*c)
^2 + 4*(64*a^5 - 112*a^4*b + 57*a^3*b^2 - 9*a^2*b^3)*d*cos(4*d*x + 4*c)^2 +
 16*(a^3*b^2 - a^2*b^3)*d*cos(2*d*x + 2*c)^2 + (a^3*b^2 - a^2*b^3)*d*sin(8*
d*x + 8*c)^2 + 16*(a^3*b^2 - a^2*b^3)*d*sin(6*d*x + 6*c)^2 + 4*(64*a^5 - 11
2*a^4*b + 57*a^3*b^2 - 9*a^2*b^3)*d*sin(4*d*x + 4*c)^2 + 16*(8*a^4*b - 11*a
^3*b^2 + 3*a^2*b^3)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a^3*b^2 - a^2
*b^3)*d*sin(2*d*x + 2*c)^2 - 8*(a^3*b^2 - a^2*b^3)*d*cos(2*d*x + 2*c) + (a^
3*b^2 - a^2*b^3)*d - 2*(4*(a^3*b^2 - a^2*b^3)*d*cos(6*d*x + 6*c) + 2*(8*a^4
*b - 11*a^3*b^2 + 3*a^2*b^3)*d*cos(4*d*x + 4*c) + 4*(a^3*b^2 - a^2*b^3)*d*c
os(2*d*x + 2*c) - (a^3*b^2 - a^2*b^3)*d)*cos(8*d*x + 8*c) + 8*(2*(8*a^4*b -
 11*a^3*b^2 + 3*a^2*b^3)*d*cos(4*d*x + 4*c) + 4*(a^3*b^2 - a^2*b^3)*d*cos(2
*d*x + 2*c) - (a^3*b^2 - a^2*b^3)*d)*cos(6*d*x + 6*c) + 4*(4*(8*a^4*b - 11*
a^3*b^2 + 3*a^2*b^3)*d*cos(2*d*x + 2*c) - (8*a^4*b - 11*a^3*b^2 + 3*a^2*b^3
)*d)*cos(4*d*x + 4*c) - 4*(2*(a^3*b^2 - a^2*b^3)*d*sin(6*d*x + 6*c) + (8*a^
4*b - 11*a^3*b^2 + 3*a^2*b^3)*d*sin(4*d*x + 4*c) + 2*(a^3*b^2 - a^2*b^3)*d*
sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^4*b - 11*a^3*b^2 + 3*a^2*b^3)
*d*sin(4*d*x + 4*c) + 2*(a^3*b^2 - a^2*b^3)*d*sin(2*d*x + 2*c))*sin(6*d*x +
 6*c))*integrate(-1/2*(4*(19*a*b^2 - 12*b^3)*cos(3*d*x + 3*c)*sin(2*d*x + 2
*c) - 4*(5*a*b^2 - 4*b^3)*cos(d*x + c)*sin(2*d*x + 2*c) + 4*(5*a*b^2 - 4*b^
3)*cos(2*d*x + 2*c)*sin(d*x + c) + ((5*a*b^2 - 4*b^3)*sin(7*d*x + 7*c) - (1
9*a*b^2 - 12*b^3)*sin(5*d*x + 5*c) + (19*a*b^2 - 12*b^3)*sin(3*d*x + 3*c) -
 (5*a*b^2 - 4*b^3)*sin(d*x + c))*cos(8*d*x + 8*c) + 2*(2*(5*a*b^2 - 4*b^3)*
sin(6*d*x + 6*c) + (40*a^2*b - 47*a*b^2 + 12*b^3)*sin(4*d*x + 4*c) + 2*(5*a
```

```
*b^2 - 4*b^3)*sin(2*d*x + 2*c))*cos(7*d*x + 7*c) + 4*((19*a*b^2 - 12*b^3)*s
in(5*d*x + 5*c) - (19*a*b^2 - 12*b^3)*sin(3*d*x + 3*c) + (5*a*b^2 - 4*b^3)*
sin(d*x + c))*cos(6*d*x + 6*c) - 2*((152*a^2*b - 153*a*b^2 + 36*b^3)*sin(4*
d*x + 4*c) + 2*(19*a*b^2 - 12*b^3)*sin(2*d*x + 2*c))*cos(5*d*x + 5*c) - 2*(
(152*a^2*b - 153*a*b^2 + 36*b^3)*sin(3*d*x + 3*c) - (40*a^2*b - 47*a*b^2 +
12*b^3)*sin(d*x + c))*cos(4*d*x + 4*c) - ((5*a*b^2 - 4*b^3)*cos(7*d*x + 7*c
) - (19*a*b^2 - 12*b^3)*cos(5*d*x + 5*c) + (19*a*b^2 - 12*b^3)*cos(3*d*x +
3*c) - (5*a*b^2 - 4*b^3)*cos(d*x + c))*sin(8*d*x + 8*c) + (5*a*b^2 - 4*b^3
- 4*(5*a*b^2 - 4*b^3)*cos(6*d*x + 6*c) - 2*(40*a^2*b - 47*a*b^2 + 12*b^3)*c
os(4*d*x + 4*c) - 4*(5*a*b^2 - 4*b^3)*cos(2*d*x + 2*c))*sin(7*d*x + 7*c) -
4*((19*a*b^2 - 12*b^3)*cos(5*d*x + 5*c) - (19*a*b^2 - 12*b^3)*cos(3*d*x + 3
*c) + (5*a*b^2 - 4*b^3)*cos(d*x + c))*sin(6*d*x + 6*c) - (19*a*b^2 - 12*b^3
 - 2*(152*a^2*b - 153*a*b^2 + 36*b^3)*cos(4*d*x + 4*c) - 4*(19*a*b^2 - 12*b
^3)*cos(2*d*x + 2*c))*sin(5*d*x + 5*c) + 2*((152*a^2*b - 153*a*b^2 + 36*b^3
)*cos(3*d*x + 3*c) - (40*a^2*b - 47*a*b^2 + 12*b^3)*cos(d*x + c))*sin(4*d*x
 + 4*c) + (19*a*b^2 - 12*b^3 - 4*(19*a*b^2 - 12*b^3)*cos(2*d*x + 2*c))*sin(
3*d*x + 3*c) - (5*a*b^2 - 4*b^3)*sin(d*x + c))/(a^3*b^2 - a^2*b^3 + (a^3*b^
2 - a^2*b^3)*cos(8*d*x + 8*c)^2 + 16*(a^3*b^2 - a^2*b^3)*cos(6*d*x + 6*c)^2
 + 4*(64*a^5 - 112*a^4*b + 57*a^3*b^2 - 9*a^2*b^3)*cos(4*d*x + 4*c)^2 + 16*
(a^3*b^2 - a^2*b^3)*cos(2*d*x + 2*c)^2 + (a^3*b^2 - a^2*b^3)*sin(8*d*x + 8*
c)^2 + 16*(a^3*b^2 - a^2*b^3)*sin(6*d*x + 6*c)^2 + 4*(64*a^5 - 112*a^4*b +
57*a^3*b^2 - 9*a^2*b^3)*sin(4*d*x + 4*c)^2 + 16*(8*a^4*b - 11*a^3*b^2 + 3*a
^2*b^3)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a^3*b^2 - a^2*b^3)*sin(2*d*
x + 2*c)^2 + 2*(a^3*b^2 - a^2*b^3 - 4*(a^3*b^2 - a^2*b^3)*cos(6*d*x + 6*c)
- 2*(8*a^4*b - 11*a^3*b^2 + 3*a^2*b^3)*cos(4*d*x + 4*c) - 4*(a^3*b^2 - a^2*
b^3)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) - 8*(a^3*b^2 - a^2*b^3 - 2*(8*a^4*b
 - 11*a^3*b^2 + 3*a^2*b^3)*cos(4*d*x + 4*c) - 4*(a^3*b^2 - a^2*b^3)*cos(2*d
*x + 2*c))*cos(6*d*x + 6*c) - 4*(8*a^4*b - 11*a^3*b^2 + 3*a^2*b^3 - 4*(8*a^
4*b - 11*a^3*b^2 + 3*a^2*b^3)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 8*(a^3*b
^2 - a^2*b^3)*cos(2*d*x + 2*c) - 4*(2*(a^3*b^2 - a^2*b^3)*sin(6*d*x + 6*c)
+ (8*a^4*b - 11*a^3*b^2 + 3*a^2*b^3)*sin(4*d*x + 4*c) + 2*(a^3*b^2 - a^2*b^
3)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^4*b - 11*a^3*b^2 + 3*a^2*b
^3)*sin(4*d*x + 4*c) + 2*(a^3*b^2 - a^2*b^3)*sin(2*d*x + 2*c))*sin(6*d*x +
6*c)), x) - (a*b^2 - b^3 + (a*b^2 - b^3)*cos(8*d*x + 8*c)^2 + 16*(a*b^2 - b
^3)*cos(6*d*x + 6*c)^2 + 4*(64*a^3 - 112*a^2*b + 57*a*b^2 - 9*b^3)*cos(4*d*
x + 4*c)^2 + 16*(a*b^2 - b^3)*cos(2*d*x + 2*c)^2 + (a*b^2 - b^3)*sin(8*d*x
+ 8*c)^2 + 16*(a*b^2 - b^3)*sin(6*d*x + 6*c)^2 + 4*(64*a^3 - 112*a^2*b + 57
*a*b^2 - 9*b^3)*sin(4*d*x + 4*c)^2 + 16*(8*a^2*b - 11*a*b^2 + 3*b^3)*sin(4*
d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a*b^2 - b^3)*sin(2*d*x + 2*c)^2 + 2*(a*b^
2 - b^3 - 4*(a*b^2 - b^3)*cos(6*d*x + 6*c) - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)
*cos(4*d*x + 4*c) - 4*(a*b^2 - b^3)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) - 8*
(a*b^2 - b^3 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*cos(4*d*x + 4*c) - 4*(a*b^2 -
 b^3)*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) - 4*(8*a^2*b - 11*a*b^2 + 3*b^3 -
4*(8*a^2*b - 11*a*b^2 + 3*b^3)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 8*(a*b^
2 - b^3)*cos(2*d*x + 2*c) - 4*(2*(a*b^2 - b^3)*sin(6*d*x + 6*c) + (8*a^2*b
- 11*a*b^2 + 3*b^3)*sin(4*d*x + 4*c) + 2*(a*b^2 - b^3)*sin(2*d*x + 2*c))*si
n(8*d*x + 8*c) + 16*((8*a^2*b - 11*a*b^2 + 3*b^3)*sin(4*d*x + 4*c) + 2*(a*b
^2 - b^3)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(cos(d*x)^2 + 2*cos(d*x)*c
os(c) + cos(c)^2 + sin(d*x)^2 - 2*sin(d*x)*sin(c) + sin(c)^2) + (a*b^2 - b^
3 + (a*b^2 - b^3)*cos(8*d*x + 8*c)^2 + 16*(a*b^2 - b^3)*cos(6*d*x + 6*c)^2
+ 4*(64*a^3 - 112*a^2*b + 57*a*b^2 - 9*b^3)*cos(4*d*x + 4*c)^2 + 16*(a*b^2
- b^3)*cos(2*d*x + 2*c)^2 + (a*b^2 - b^3)*sin(8*d*x + 8*c)^2 + 16*(a*b^2 -
b^3)*sin(6*d*x + 6*c)^2 + 4*(64*a^3 - 112*a^2*b + 57*a*b^2 - 9*b^3)*sin(4*d
*x + 4*c)^2 + 16*(8*a^2*b - 11*a*b^2 + 3*b^3)*sin(4*d*x + 4*c)*sin(2*d*x +
2*c) + 16*(a*b^2 - b^3)*sin(2*d*x + 2*c)^2 + 2*(a*b^2 - b^3 - 4*(a*b^2 - b^
3)*cos(6*d*x + 6*c) - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*cos(4*d*x + 4*c) - 4*(
a*b^2 - b^3)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) - 8*(a*b^2 - b^3 - 2*(8*a^2
*b - 11*a*b^2 + 3*b^3)*cos(4*d*x + 4*c) - 4*(a*b^2 - b^3)*cos(2*d*x + 2*c))
*cos(6*d*x + 6*c) - 4*(8*a^2*b - 11*a*b^2 + 3*b^3 - 4*(8*a^2*b - 11*a*b^2 +
```

```
3*b^3)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 8*(a*b^2 - b^3)*cos(2*d*x + 2*
c) - 4*(2*(a*b^2 - b^3)*sin(6*d*x + 6*c) + (8*a^2*b - 11*a*b^2 + 3*b^3)*sin
(4*d*x + 4*c) + 2*(a*b^2 - b^3)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*((8
*a^2*b - 11*a*b^2 + 3*b^3)*sin(4*d*x + 4*c) + 2*(a*b^2 - b^3)*sin(2*d*x + 2
*c))*sin(6*d*x + 6*c))*log(cos(d*x)^2 - 2*cos(d*x)*cos(c) + cos(c)^2 + sin(
d*x)^2 + 2*sin(d*x)*sin(c) + sin(c)^2) - (a*b^2*sin(7*d*x + 7*c) - 5*a*b^2*
sin(5*d*x + 5*c) - 5*a*b^2*sin(3*d*x + 3*c) + a*b^2*sin(d*x + c))*sin(8*d*x
 + 8*c) + 2*(2*a*b^2*sin(6*d*x + 6*c) + 2*a*b^2*sin(2*d*x + 2*c) + (8*a^2*b
 - 3*a*b^2)*sin(4*d*x + 4*c))*sin(7*d*x + 7*c) - 4*(5*a*b^2*sin(5*d*x + 5*c
) + 5*a*b^2*sin(3*d*x + 3*c) - a*b^2*sin(d*x + c))*sin(6*d*x + 6*c) - 10*(2
*a*b^2*sin(2*d*x + 2*c) + (8*a^2*b - 3*a*b^2)*sin(4*d*x + 4*c))*sin(5*d*x +
 5*c) - 2*(5*(8*a^2*b - 3*a*b^2)*sin(3*d*x + 3*c) - (8*a^2*b - 3*a*b^2)*sin
(d*x + c))*sin(4*d*x + 4*c))/((a^3*b^2 - a^2*b^3)*d*cos(8*d*x + 8*c)^2 + 16
*(a^3*b^2 - a^2*b^3)*d*cos(6*d*x + 6*c)^2 + 4*(64*a^5 - 112*a^4*b + 57*a^3*
b^2 - 9*a^2*b^3)*d*cos(4*d*x + 4*c)^2 + 16*(a^3*b^2 - a^2*b^3)*d*cos(2*d*x
+ 2*c)^2 + (a^3*b^2 - a^2*b^3)*d*sin(8*d*x + 8*c)^2 + 16*(a^3*b^2 - a^2*b^3
)*d*sin(6*d*x + 6*c)^2 + 4*(64*a^5 - 112*a^4*b + 57*a^3*b^2 - 9*a^2*b^3)*d*
sin(4*d*x + 4*c)^2 + 16*(8*a^4*b - 11*a^3*b^2 + 3*a^2*b^3)*d*sin(4*d*x + 4*
c)*sin(2*d*x + 2*c) + 16*(a^3*b^2 - a^2*b^3)*d*sin(2*d*x + 2*c)^2 - 8*(a^3*
b^2 - a^2*b^3)*d*cos(2*d*x + 2*c) + (a^3*b^2 - a^2*b^3)*d - 2*(4*(a^3*b^2 -
 a^2*b^3)*d*cos(6*d*x + 6*c) + 2*(8*a^4*b - 11*a^3*b^2 + 3*a^2*b^3)*d*cos(4
*d*x + 4*c) + 4*(a^3*b^2 - a^2*b^3)*d*cos(2*d*x + 2*c) - (a^3*b^2 - a^2*b^3
)*d)*cos(8*d*x + 8*c) + 8*(2*(8*a^4*b - 11*a^3*b^2 + 3*a^2*b^3)*d*cos(4*d*x
 + 4*c) + 4*(a^3*b^2 - a^2*b^3)*d*cos(2*d*x + 2*c) - (a^3*b^2 - a^2*b^3)*d)
*cos(6*d*x + 6*c) + 4*(4*(8*a^4*b - 11*a^3*b^2 + 3*a^2*b^3)*d*cos(2*d*x + 2
*c) - (8*a^4*b - 11*a^3*b^2 + 3*a^2*b^3)*d)*cos(4*d*x + 4*c) - 4*(2*(a^3*b^
2 - a^2*b^3)*d*sin(6*d*x + 6*c) + (8*a^4*b - 11*a^3*b^2 + 3*a^2*b^3)*d*sin(
4*d*x + 4*c) + 2*(a^3*b^2 - a^2*b^3)*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) +
 16*((8*a^4*b - 11*a^3*b^2 + 3*a^2*b^3)*d*sin(4*d*x + 4*c) + 2*(a^3*b^2 - a
^2*b^3)*d*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))
```

Fricas [B] time = 9.26515, size = 6010, normalized size = 18.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-b*sin(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*a*b*cos(d*x + c)^3 - 8*a*b*cos(d*x + c) + ((a^3*b - a^2*b^2)*d*cos
(d*x + c)^4 - 2*(a^3*b - a^2*b^2)*d*cos(d*x + c)^2 - (a^4 - 2*a^3*b + a^2*b
^2)*d)*sqrt(-((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2*sqrt((625*a^4*b - 1
450*a^3*b^2 + 1241*a^2*b^3 - 464*a*b^4 + 64*b^5)/((a^13 - 6*a^12*b + 15*a^1
1*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) + 35*a^2*b -
47*a*b^2 + 16*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2))*log((625*a^
3*b - 1125*a^2*b^2 + 664*a*b^3 - 128*b^4)*cos(d*x + c) + ((5*a^10 - 18*a^9*
b + 24*a^8*b^2 - 14*a^7*b^3 + 3*a^6*b^4)*d^3*sqrt((625*a^4*b - 1450*a^3*b^2
 + 1241*a^2*b^3 - 464*a*b^4 + 64*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*
a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) - 2*(75*a^5*b - 137*a^4*
b^2 + 82*a^3*b^3 - 16*a^2*b^4)*d)*sqrt(-((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b
^3)*d^2*sqrt((625*a^4*b - 1450*a^3*b^2 + 1241*a^2*b^3 - 464*a*b^4 + 64*b^5)
/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a
^7*b^6)*d^4)) + 35*a^2*b - 47*a*b^2 + 16*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 -
 a^4*b^3)*d^2))) - ((a^3*b - a^2*b^2)*d*cos(d*x + c)^4 - 2*(a^3*b - a^2*b^2
)*d*cos(d*x + c)^2 - (a^4 - 2*a^3*b + a^2*b^2)*d)*sqrt(((a^7 - 3*a^6*b + 3*
a^5*b^2 - a^4*b^3)*d^2*sqrt((625*a^4*b - 1450*a^3*b^2 + 1241*a^2*b^3 - 464*
a*b^4 + 64*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4
```

```
- 6*a^8*b^5 + a^7*b^6)*d^4)) - 35*a^2*b + 47*a*b^2 - 16*b^3)/((a^7 - 3*a^6*
b + 3*a^5*b^2 - a^4*b^3)*d^2))*log((625*a^3*b - 1125*a^2*b^2 + 664*a*b^3 -
128*b^4)*cos(d*x + c) + ((5*a^10 - 18*a^9*b + 24*a^8*b^2 - 14*a^7*b^3 + 3*a
^6*b^4)*d^3*sqrt((625*a^4*b - 1450*a^3*b^2 + 1241*a^2*b^3 - 464*a*b^4 + 64*
b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5
 + a^7*b^6)*d^4)) + 2*(75*a^5*b - 137*a^4*b^2 + 82*a^3*b^3 - 16*a^2*b^4)*d)
*sqrt(((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2*sqrt((625*a^4*b - 1450*a^3
*b^2 + 1241*a^2*b^3 - 464*a*b^4 + 64*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 -
 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) - 35*a^2*b + 47*a*b^
2 - 16*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2))) - ((a^3*b - a^2*b
^2)*d*cos(d*x + c)^4 - 2*(a^3*b - a^2*b^2)*d*cos(d*x + c)^2 - (a^4 - 2*a^3*
b + a^2*b^2)*d)*sqrt(-((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2*sqrt((625*
a^4*b - 1450*a^3*b^2 + 1241*a^2*b^3 - 464*a*b^4 + 64*b^5)/((a^13 - 6*a^12*b
 + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) + 35
*a^2*b - 47*a*b^2 + 16*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2))*lo
g(-(625*a^3*b - 1125*a^2*b^2 + 664*a*b^3 - 128*b^4)*cos(d*x + c) + ((5*a^10
 - 18*a^9*b + 24*a^8*b^2 - 14*a^7*b^3 + 3*a^6*b^4)*d^3*sqrt((625*a^4*b - 14
50*a^3*b^2 + 1241*a^2*b^3 - 464*a*b^4 + 64*b^5)/((a^13 - 6*a^12*b + 15*a^11
*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) - 2*(75*a^5*b
- 137*a^4*b^2 + 82*a^3*b^3 - 16*a^2*b^4)*d)*sqrt(-((a^7 - 3*a^6*b + 3*a^5*b
^2 - a^4*b^3)*d^2*sqrt((625*a^4*b - 1450*a^3*b^2 + 1241*a^2*b^3 - 464*a*b^4
 + 64*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a
^8*b^5 + a^7*b^6)*d^4)) + 35*a^2*b - 47*a*b^2 + 16*b^3)/((a^7 - 3*a^6*b + 3
*a^5*b^2 - a^4*b^3)*d^2))) + ((a^3*b - a^2*b^2)*d*cos(d*x + c)^4 - 2*(a^3*b
 - a^2*b^2)*d*cos(d*x + c)^2 - (a^4 - 2*a^3*b + a^2*b^2)*d)*sqrt(((a^7 - 3*
a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2*sqrt((625*a^4*b - 1450*a^3*b^2 + 1241*a^2*
b^3 - 464*a*b^4 + 64*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 1
5*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) - 35*a^2*b + 47*a*b^2 - 16*b^3)/((a^
7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2))*log(-(625*a^3*b - 1125*a^2*b^2 + 6
64*a*b^3 - 128*b^4)*cos(d*x + c) + ((5*a^10 - 18*a^9*b + 24*a^8*b^2 - 14*a^
7*b^3 + 3*a^6*b^4)*d^3*sqrt((625*a^4*b - 1450*a^3*b^2 + 1241*a^2*b^3 - 464*
a*b^4 + 64*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4
- 6*a^8*b^5 + a^7*b^6)*d^4)) + 2*(75*a^5*b - 137*a^4*b^2 + 82*a^3*b^3 - 16*
a^2*b^4)*d)*sqrt(((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2*sqrt((625*a^4*b
 - 1450*a^3*b^2 + 1241*a^2*b^3 - 464*a*b^4 + 64*b^5)/((a^13 - 6*a^12*b + 15
*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) - 35*a^2*
b + 47*a*b^2 - 16*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2))) + 8*((
a*b - b^2)*cos(d*x + c)^4 - 2*(a*b - b^2)*cos(d*x + c)^2 - a^2 + 2*a*b - b^
2)*log(1/2*cos(d*x + c) + 1/2) - 8*((a*b - b^2)*cos(d*x + c)^4 - 2*(a*b - b
^2)*cos(d*x + c)^2 - a^2 + 2*a*b - b^2)*log(-1/2*cos(d*x + c) + 1/2))/((a^3
*b - a^2*b^2)*d*cos(d*x + c)^4 - 2*(a^3*b - a^2*b^2)*d*cos(d*x + c)^2 - (a^
4 - 2*a^3*b + a^2*b^2)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-b*sin(d*x+c)**4)**2,x)

[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-b*sin(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.218 $\int \frac{\sin^8(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=320

$$\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8b^{3/2}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2b^2d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8b^{3/2}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2b^2d\sqrt{\sqrt{a}+\sqrt{b}}}$$

```
[Out] x/b^2 - (a^(1/4)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2
*Sqrt[Sqrt[a] - Sqrt[b]]*b^2*d) + (a^(1/4)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*
Tan[c + d*x])/a^(1/4)])/(8*(Sqrt[a] - Sqrt[b])^(3/2)*b^(3/2)*d) - (a^(1/4)*
ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt[a] + S
qrt[b]]*b^2*d) - (a^(1/4)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(
1/4)])/(8*(Sqrt[a] + Sqrt[b])^(3/2)*b^(3/2)*d) - Tan[c + d*x]/(4*(a - b)*b*
d) + Tan[c + d*x]^5/(4*b*d*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan[c + d*x]^4
))
```

Rubi [A] time = 0.446449, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3217, 1313, 1275, 12, 1122, 1166, 205, 1287, 203}

$$\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8b^{3/2}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2b^2d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8b^{3/2}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2b^2d\sqrt{\sqrt{a}+\sqrt{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^8/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] x/b^2 - (a^(1/4)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2
*Sqrt[Sqrt[a] - Sqrt[b]]*b^2*d) + (a^(1/4)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*
Tan[c + d*x])/a^(1/4)])/(8*(Sqrt[a] - Sqrt[b])^(3/2)*b^(3/2)*d) - (a^(1/4)*
ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt[a] + S
qrt[b]]*b^2*d) - (a^(1/4)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(
1/4)])/(8*(Sqrt[a] + Sqrt[b])^(3/2)*b^(3/2)*d) - Tan[c + d*x]/(4*(a - b)*b*
d) + Tan[c + d*x]^5/(4*b*d*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan[c + d*x]^4
))
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 1313

```
Int[(((f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> -Dist[f^4/(c*d^2 - b*d*e + a*e^2), Int[(f*x)^
(m - 4)*(a*d + (b*d - a*e)*x^2)*(a + b*x^2 + c*x^4)^p, x], x] + Dist[(d^2*f
^4)/(c*d^2 - b*d*e + a*e^2), Int[((f*x)^(m - 4)*(a + b*x^2 + c*x^4)^(p + 1)
)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0
```

```
] && LtQ[p, -1] && GtQ[m, 2]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin^8(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^8}{(1+x^2)\left(a+2ax^2+(a-b)x^4\right)^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^4(a+ax^2)}{\left(a+2ax^2+(a-b)x^4\right)^2}\,dx, x, \tan(c+dx)\right)}{bd} - \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)\left(a+2ax^2+(a-b)x^4\right)}\,dx, x, \tan(c\right.}{bd}$$

$$= \frac{\tan^5(c+dx)}{4bd\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)} + \frac{\text{Subst}\left(\int -\frac{2abx^4}{a+2ax^2+(a-b)x^4}\,dx, x, \tan(c+\right.}{8ab^2d}$$

$$= \frac{\tan^5(c+dx)}{4bd\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(c+dx)\right)}{b^2d} - \frac{a\,\text{S}}{}$$

$$= \frac{x}{b^2} - \frac{\tan(c+dx)}{4(a-b)bd} + \frac{\tan^5(c+dx)}{4bd\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)} - \frac{\left(a\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\right)\text{Subst}}{}$$

$$= \frac{x}{b^2} - \frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}b^2d} - \frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}b^2d} - \frac{\tan(c+dx)}{4(a-b)bd} + \frac{}{4bd\left(a+\right.}$$

$$= \frac{x}{b^2} - \frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}b^2d} + \frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{3/2}d} - \frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c}{\sqrt[4]{a}}\right.}{2\sqrt{\sqrt{a}+\sqrt{b}}b^2d}$$

Mathematica [A] time = 4.95567, size = 262, normalized size = 0.82

$$\frac{-\frac{\sqrt{a}\left(4\sqrt{a}+5\sqrt{b}\right)\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{\sqrt{a}\sqrt{b}+a}} + \frac{2ab(\sin(4(c+dx))-6\sin(2(c+dx)))}{(a-b)(8a+4b\cos(2(c+dx))-b\cos(4(c+dx))-3b)} + \frac{\sqrt{a}\left(4\sqrt{a}-5\sqrt{b}\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\left(\sqrt{a}-\sqrt{b}\right)\sqrt{\sqrt{a}\sqrt{b}-a}} + 8(c+dx)}{8b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^8/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] (8*(c + d*x) - (Sqrt[a]*(4*Sqrt[a] + 5*Sqrt[b])*ArcTan[((Sqrt[a] + Sqrt[b])
*Tan[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/((Sqrt[a] + Sqrt[b])*Sqrt[a + Sq
rt[a]*Sqrt[b]]) + (Sqrt[a]*(4*Sqrt[a] - 5*Sqrt[b])*ArcTanh[((Sqrt[a] - Sqrt
[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/((Sqrt[a] - Sqrt[b])*Sqrt[-
a + Sqrt[a]*Sqrt[b]]) + (2*a*b*(-6*Sin[2*(c + d*x)] + Sin[4*(c + d*x)]))/((
a - b)*(8*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*x)])))/(8*b^2*d)
```

Maple [B] time = 0.125, size = 644, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^8/(a-b*sin(d*x+c)^4)^2,x)
```

```
[Out] -1/2/d*a/b/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)/(a-b)*tan(d*x
+c)^3-1/4/d*a/b/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)/(a-b)*ta
n(d*x+c)-1/2/d*a^2/b^2/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan
(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+3/4/d*a/b/(a-b)/(((a*b)^(1/2)+a)*(a-
b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-3/8/d*a^2/
b/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(
((a*b)^(1/2)+a)*(a-b))^(1/2))+5/8/d*a/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a
-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-1/2/d*a^2
/b^2/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^
(1/2)-a)*(a-b))^(1/2))+3/4/d*a/b/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctan
h((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))+3/8/d*a^2/b/(a*b)^(1/2)/
(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)
-a)*(a-b))^(1/2))-5/8/d*a/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*a
rctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))+1/d/b^2*arctan(tan(
d*x+c))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a-b*sin(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 9.06827, size = 8038, normalized size = 25.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a-b*sin(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/32*(32*(a*b - b^2)*d*x*cos(d*x + c)^4 - 64*(a*b - b^2)*d*x*cos(d*x + c)^2
 - 32*(a^2 - 2*a*b + b^2)*d*x + ((a*b^3 - b^4)*d*cos(d*x + c)^4 - 2*(a*b^3
- b^4)*d*cos(d*x + c)^2 - (a^2*b^2 - 2*a*b^3 + b^4)*d)*sqrt(-((a^3*b^4 - 3*
a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450
*a^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15
*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) + 16*a^3 - 47*a^2*b + 35*a*b^2)/((a^3*b^
4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))*log(32*a^3 - 166*a^2*b + 1125/4*a*b^2
- 625/4*b^3 - 1/4*(128*a^3 - 664*a^2*b + 1125*a*b^2 - 625*b^3)*cos(d*x + c)
^2 + 1/2*(2*(2*a^4*b^5 - 9*a^3*b^6 + 15*a^2*b^7 - 11*a*b^8 + 3*b^9)*d^3*sqr
t((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((a^6*b^7
- 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4
))*cos(d*x + c)*sin(d*x + c) - (24*a^3*b^2 - 127*a^2*b^3 + 220*a*b^4 - 125*
b^5)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b
^7)*d^2*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)
/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12
+ b^13)*d^4)) + 16*a^3 - 47*a^2*b + 35*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b
^6 - b^7)*d^2)) + 1/4*(2*(16*a^4*b^3 - 73*a^3*b^4 + 123*a^2*b^5 - 91*a*b^6
+ 25*b^7)*d^2*cos(d*x + c)^2 - (16*a^4*b^3 - 73*a^3*b^4 + 123*a^2*b^5 - 91*
a*b^6 + 25*b^7)*d^2)*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3
 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^1
1 - 6*a*b^12 + b^13)*d^4))) - ((a*b^3 - b^4)*d*cos(d*x + c)^4 - 2*(a*b^3 -
```

b^4)*d*cos(d*x + c)^2 - (a^2*b^2 - 2*a*b^3 + b^4)*d)*sqrt(-((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) + 16*a^3 - 47*a^2*b + 35*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))*log(32*a^3 - 166*a^2*b + 1125/4*a*b^2 - 625/4*b^3 - 1/4*(128*a^3 - 664*a^2*b + 1125*a*b^2 - 625*b^3)*cos(d*x + c)^2 - 1/2*(2*(2*a^4*b^5 - 9*a^3*b^6 + 15*a^2*b^7 - 11*a*b^8 + 3*b^9)*d^3*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4))*cos(d*x + c)*sin(d*x + c) - (24*a^3*b^2 - 127*a^2*b^3 + 220*a*b^4 - 125*b^5)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) + 16*a^3 - 47*a^2*b + 35*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2)) + 1/4*(2*(16*a^4*b^3 - 73*a^3*b^4 + 123*a^2*b^5 - 91*a*b^6 + 25*b^7)*d^2*cos(d*x + c)^2 - (16*a^4*b^3 - 73*a^3*b^4 + 123*a^2*b^5 - 91*a*b^6 + 25*b^7)*d^2)*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4))) + ((a*b^3 - b^4)*d*cos(d*x + c)^4 - 2*(a*b^3 - b^4)*d*cos(d*x + c)^2 - (a^2*b^2 - 2*a*b^3 + b^4)*d)*sqrt(((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) - 16*a^3 + 47*a^2*b - 35*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))*log(-32*a^3 + 166*a^2*b - 1125/4*a*b^2 + 625/4*b^3 + 1/4*(128*a^3 - 664*a^2*b + 1125*a*b^2 - 625*b^3)*cos(d*x + c)^2 + 1/2*(2*(2*a^4*b^5 - 9*a^3*b^6 + 15*a^2*b^7 - 11*a*b^8 + 3*b^9)*d^3*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4))*cos(d*x + c)*sin(d*x + c) + (24*a^3*b^2 - 127*a^2*b^3 + 220*a*b^4 - 125*b^5)*d*cos(d*x + c)*sin(d*x + c))*sqrt(((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) - 16*a^3 + 47*a^2*b - 35*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2)) + 1/4*(2*(16*a^4*b^3 - 73*a^3*b^4 + 123*a^2*b^5 - 91*a*b^6 + 25*b^7)*d^2*cos(d*x + c)^2 - (16*a^4*b^3 - 73*a^3*b^4 + 123*a^2*b^5 - 91*a*b^6 + 25*b^7)*d^2)*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4))) - ((a*b^3 - b^4)*d*cos(d*x + c)^4 - 2*(a*b^3 - b^4)*d*cos(d*x + c)^2 - (a^2*b^2 - 2*a*b^3 + b^4)*d)*sqrt(((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) - 16*a^3 + 47*a^2*b - 35*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))*log(-32*a^3 + 166*a^2*b - 1125/4*a*b^2 + 625/4*b^3 + 1/4*(128*a^3 - 664*a^2*b + 1125*a*b^2 - 625*b^3)*cos(d*x + c)^2 - 1/2*(2*(2*a^4*b^5 - 9*a^3*b^6 + 15*a^2*b^7 - 11*a*b^8 + 3*b^9)*d^3*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4))*cos(d*x + c)*sin(d*x + c) + (24*a^3*b^2 - 127*a^2*b^3 + 220*a*b^4 - 125*b^5)*d*cos(d*x + c)*sin(d*x + c))*sqrt(((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) - 16*a^3 + 47*a^2*b - 35*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2)) + 1/4*(2*(16*a^4*b^3 - 73*a^3*b^4 + 123*a^2*b^5 - 91*a*b^6 + 25*b^7)*d^2*cos(d*x + c)^2 - (16*a^4*b^3 - 73*a^3*b^4 + 123*a^2*b^5 - 91*a*b^6 + 25*b^7)*d^2)*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4))) - 8*(a*b*cos(d*x + c)^3 - 2*a*b*cos(d*x + c))*sin(d*x + c))/((a*b^3 - b^4)*d*cos(d*x + c)^4 - 2*(a*b^3 - b^4)*d*cos(d*x + c)^2 - (a^2*b^2 - 2*a*b^3 + b^4)*d)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**8/(a-b*sin(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a-b*sin(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.219 $\int \frac{\sin^6(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=233

$$-\frac{\left(2\sqrt{a}-3\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}b^{3/2}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\left(2\sqrt{a}+3\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}b^{3/2}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{\tan(c+dx)}{4bd(a-b)}+\frac{\tan^3(c+dx)}{4bd\left((a-b)\tan^4(c+ \ldots\right.}$$

```
[Out] -((2*Sqrt[a] - 3*Sqrt[b])*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(
1/4)])/(8*a^(1/4)*(Sqrt[a] - Sqrt[b])^(3/2)*b^(3/2)*d) + ((2*Sqrt[a] + 3*Sq
rt[b])*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(8*a^(1/4)*(
Sqrt[a] + Sqrt[b])^(3/2)*b^(3/2)*d) - Tan[c + d*x]/(4*(a - b)*b*d) + (Sec[c
 + d*x]^2*Tan[c + d*x]^3)/(4*b*d*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan[c +
d*x]^4))
```

Rubi [A] time = 0.351164, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3217, 1120, 1279, 1166, 205}

$$-\frac{\left(2\sqrt{a}-3\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}b^{3/2}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\left(2\sqrt{a}+3\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}b^{3/2}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{\tan(c+dx)}{4bd(a-b)}+\frac{\tan^3(c+dx)}{4bd\left((a-b)\tan^4(c+ \ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^6/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] -((2*Sqrt[a] - 3*Sqrt[b])*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(
1/4)])/(8*a^(1/4)*(Sqrt[a] - Sqrt[b])^(3/2)*b^(3/2)*d) + ((2*Sqrt[a] + 3*Sq
rt[b])*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(8*a^(1/4)*(
Sqrt[a] + Sqrt[b])^(3/2)*b^(3/2)*d) - Tan[c + d*x]/(4*(a - b)*b*d) + (Sec[c
 + d*x]^2*Tan[c + d*x]^3)/(4*b*d*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan[c +
d*x]^4))
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^6(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{\left(a+2ax^2+(a-b)x^4\right)^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\sec^2(c+dx)\tan^3(c+dx)}{4bd\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{x^2\left(6a+2ax^2\right)}{a+2ax^2+(a-b)x^4}\,dx, x, \tan(c\ \cdots\right.}{8abd}$$

$$= -\frac{\tan(c+dx)}{4(a-b)bd} + \frac{\sec^2(c+dx)\tan^3(c+dx)}{4bd\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{2a^2-2a(a-3\cdots}{a+2ax^2+(a-\cdots}\right.}{8a(\cdots}$$

$$= -\frac{\tan(c+dx)}{4(a-b)bd} + \frac{\sec^2(c+dx)\tan^3(c+dx)}{4bd\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)} - \frac{\left(a-\frac{2\sqrt{a}(a-2b)}{\sqrt{b}}-3b\right)\cdots}{\cdots}$$

$$= -\frac{\left(2\sqrt{a}-3\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{3/2}d} + \frac{\left(2\sqrt{a}+3\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}b^{3/2}d} - \frac{\tan\cdots}{4(\cdots}$$

Mathematica [A] time = 2.5863, size = 238, normalized size = 1.02

$$\frac{\frac{\sqrt{b}\left(\sqrt{a}\sqrt{b}+2a-3b\right)\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}+a}} + \frac{4b\sin(2(c+dx))(-2a+b\cos(2(c+dx))-b)}{8a+4b\cos(2(c+dx))-b\cos(4(c+dx))-3b} - \frac{\sqrt{b}\left(\sqrt{a}\sqrt{b}-2a+3b\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}-a}}}{8b^2d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^6/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] (((2*a + Sqrt[a]*Sqrt[b] - 3*b)*Sqrt[b]*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c +
 d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]] - (Sqrt[b]*(-2
*a + Sqrt[a]*Sqrt[b] + 3*b)*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt
```

```
[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a + Sqrt[a]*Sqrt[b]] + (4*b*(-2*a - b + b*Co
s[2*(c + d*x)])*Sin[2*(c + d*x)])/(8*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos
[4*(c + d*x)]))/(8*(a - b)*b^2*d)
```

Maple [B] time = 0.119, size = 674, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^6/(a-b*sin(d*x+c)^4)^2,x)
```

```
[Out] -1/4/d*a/b/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)/(a-b)*tan(d*x
+c)^3-1/4/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)/(a-b)*tan(d*
x+c)^3-1/4/d*a/b/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)/(a-b)*t
an(d*x+c)-1/8/d*a/b/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*
x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+3/8/d/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/
2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-1/4/d*a^2/b/(a*b)
^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^
(1/2)+a)*(a-b))^(1/2))+1/2/d*a/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1
/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-1/8/d*a/b/(a-b)/
(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a
-b))^(1/2))+3/8/d/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*
x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))+1/4/d*a^2/b/(a*b)^(1/2)/(a-b)/(((a*b)^(
1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2
))-1/2/d*a/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*t
an(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a-b*sin(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(16*a^2 + 2*a*b - 3*b^2)*cos(4*d*x + 4*c)*sin(2*d*x + 2*c) + ((2*a*b
 - b^2)*sin(6*d*x + 6*c) - (8*a*b - 3*b^2)*sin(4*d*x + 4*c) - (2*a*b + 3*b^
2)*sin(2*d*x + 2*c))*cos(8*d*x + 8*c) + 2*((16*a^2 + 2*a*b - 3*b^2)*sin(4*d
*x + 4*c) + 4*(2*a*b + b^2)*sin(2*d*x + 2*c))*cos(6*d*x + 6*c) - 2*((a*b^3
- b^4)*d*cos(8*d*x + 8*c)^2 + 16*(a*b^3 - b^4)*d*cos(6*d*x + 6*c)^2 + 4*(64
*a^3*b - 112*a^2*b^2 + 57*a*b^3 - 9*b^4)*d*cos(4*d*x + 4*c)^2 + 16*(a*b^3 -
 b^4)*d*cos(2*d*x + 2*c)^2 + (a*b^3 - b^4)*d*sin(8*d*x + 8*c)^2 + 16*(a*b^3
 - b^4)*d*sin(6*d*x + 6*c)^2 + 4*(64*a^3*b - 112*a^2*b^2 + 57*a*b^3 - 9*b^4
)*d*sin(4*d*x + 4*c)^2 + 16*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*sin(4*d*x + 4*
c)*sin(2*d*x + 2*c) + 16*(a*b^3 - b^4)*d*sin(2*d*x + 2*c)^2 - 8*(a*b^3 - b^
4)*d*cos(2*d*x + 2*c) + (a*b^3 - b^4)*d - 2*(4*(a*b^3 - b^4)*d*cos(6*d*x +
6*c) + 2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cos(4*d*x + 4*c) + 4*(a*b^3 - b^4
)*d*cos(2*d*x + 2*c) - (a*b^3 - b^4)*d)*cos(8*d*x + 8*c) + 8*(2*(8*a^2*b^2
- 11*a*b^3 + 3*b^4)*d*cos(4*d*x + 4*c) + 4*(a*b^3 - b^4)*d*cos(2*d*x + 2*c)
 - (a*b^3 - b^4)*d)*cos(6*d*x + 6*c) + 4*(4*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*
d*cos(2*d*x + 2*c) - (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d)*cos(4*d*x + 4*c) - 4
*(2*(a*b^3 - b^4)*d*sin(6*d*x + 6*c) + (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*sin
(4*d*x + 4*c) + 2*(a*b^3 - b^4)*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(
```

```
(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*sin(4*d*x + 4*c) + 2*(a*b^3 - b^4)*d*sin(2
*d*x + 2*c))*sin(6*d*x + 6*c))*integrate(-(4*(2*a*b - 3*b^2)*cos(6*d*x + 6*
c)^2 + 12*(8*a*b - 3*b^2)*cos(4*d*x + 4*c)^2 + 4*(2*a*b - 3*b^2)*cos(2*d*x
+ 2*c)^2 + 4*(2*a*b - 3*b^2)*sin(6*d*x + 6*c)^2 + 12*(8*a*b - 3*b^2)*sin(4*
d*x + 4*c)^2 + 2*(16*a^2 - 30*a*b + 21*b^2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*
c) + 4*(2*a*b - 3*b^2)*sin(2*d*x + 2*c)^2 - (6*b^2*cos(4*d*x + 4*c) + (2*a*
b - 3*b^2)*cos(6*d*x + 6*c) + (2*a*b - 3*b^2)*cos(2*d*x + 2*c))*cos(8*d*x +
 8*c) - (2*a*b - 3*b^2 - 2*(16*a^2 - 30*a*b + 21*b^2)*cos(4*d*x + 4*c) - 8*
(2*a*b - 3*b^2)*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) - 2*(3*b^2 - (16*a^2 - 3
0*a*b + 21*b^2)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - (2*a*b - 3*b^2)*cos(2*
d*x + 2*c) - (6*b^2*sin(4*d*x + 4*c) + (2*a*b - 3*b^2)*sin(6*d*x + 6*c) + (
2*a*b - 3*b^2)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 2*((16*a^2 - 30*a*b + 2
1*b^2)*sin(4*d*x + 4*c) + 4*(2*a*b - 3*b^2)*sin(2*d*x + 2*c))*sin(6*d*x + 6
*c))/(a*b^3 - b^4 + (a*b^3 - b^4)*cos(8*d*x + 8*c)^2 + 16*(a*b^3 - b^4)*cos
(6*d*x + 6*c)^2 + 4*(64*a^3*b - 112*a^2*b^2 + 57*a*b^3 - 9*b^4)*cos(4*d*x +
 4*c)^2 + 16*(a*b^3 - b^4)*cos(2*d*x + 2*c)^2 + (a*b^3 - b^4)*sin(8*d*x + 8
*c)^2 + 16*(a*b^3 - b^4)*sin(6*d*x + 6*c)^2 + 4*(64*a^3*b - 112*a^2*b^2 + 5
7*a*b^3 - 9*b^4)*sin(4*d*x + 4*c)^2 + 16*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*sin
(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a*b^3 - b^4)*sin(2*d*x + 2*c)^2 + 2*(a
*b^3 - b^4 - 4*(a*b^3 - b^4)*cos(6*d*x + 6*c) - 2*(8*a^2*b^2 - 11*a*b^3 + 3
*b^4)*cos(4*d*x + 4*c) - 4*(a*b^3 - b^4)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c)
 - 8*(a*b^3 - b^4 - 2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*cos(4*d*x + 4*c) - 4*(
a*b^3 - b^4)*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) - 4*(8*a^2*b^2 - 11*a*b^3 +
 3*b^4 - 4*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c
) - 8*(a*b^3 - b^4)*cos(2*d*x + 2*c) - 4*(2*(a*b^3 - b^4)*sin(6*d*x + 6*c)
+ (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*sin(4*d*x + 4*c) + 2*(a*b^3 - b^4)*sin(2*d
*x + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^2*b^2 - 11*a*b^3 + 3*b^4)*sin(4*d*x
+ 4*c) + 2*(a*b^3 - b^4)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)), x) - (b^2 + (
2*a*b - b^2)*cos(6*d*x + 6*c) - (8*a*b - 3*b^2)*cos(4*d*x + 4*c) - (2*a*b +
 3*b^2)*cos(2*d*x + 2*c))*sin(8*d*x + 8*c) + (2*a*b + 3*b^2 - 2*(16*a^2 + 2
*a*b - 3*b^2)*cos(4*d*x + 4*c) - 8*(2*a*b + b^2)*cos(2*d*x + 2*c))*sin(6*d*
x + 6*c) + (8*a*b - 3*b^2 - 2*(16*a^2 + 2*a*b - 3*b^2)*cos(2*d*x + 2*c))*si
n(4*d*x + 4*c) - (2*a*b - b^2)*sin(2*d*x + 2*c))/((a*b^3 - b^4)*d*cos(8*d*x
 + 8*c)^2 + 16*(a*b^3 - b^4)*d*cos(6*d*x + 6*c)^2 + 4*(64*a^3*b - 112*a^2*b
^2 + 57*a*b^3 - 9*b^4)*d*cos(4*d*x + 4*c)^2 + 16*(a*b^3 - b^4)*d*cos(2*d*x
+ 2*c)^2 + (a*b^3 - b^4)*d*sin(8*d*x + 8*c)^2 + 16*(a*b^3 - b^4)*d*sin(6*d*
x + 6*c)^2 + 4*(64*a^3*b - 112*a^2*b^2 + 57*a*b^3 - 9*b^4)*d*sin(4*d*x + 4*
c)^2 + 16*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c
) + 16*(a*b^3 - b^4)*d*sin(2*d*x + 2*c)^2 - 8*(a*b^3 - b^4)*d*cos(2*d*x + 2
*c) + (a*b^3 - b^4)*d - 2*(4*(a*b^3 - b^4)*d*cos(6*d*x + 6*c) + 2*(8*a^2*b^
2 - 11*a*b^3 + 3*b^4)*d*cos(4*d*x + 4*c) + 4*(a*b^3 - b^4)*d*cos(2*d*x + 2*
c) - (a*b^3 - b^4)*d)*cos(8*d*x + 8*c) + 8*(2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4
)*d*cos(4*d*x + 4*c) + 4*(a*b^3 - b^4)*d*cos(2*d*x + 2*c) - (a*b^3 - b^4)*d
)*cos(6*d*x + 6*c) + 4*(4*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cos(2*d*x + 2*c)
 - (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d)*cos(4*d*x + 4*c) - 4*(2*(a*b^3 - b^4)*
d*sin(6*d*x + 6*c) + (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*sin(4*d*x + 4*c) + 2*
(a*b^3 - b^4)*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^2*b^2 - 11*a*
b^3 + 3*b^4)*d*sin(4*d*x + 4*c) + 2*(a*b^3 - b^4)*d*sin(2*d*x + 2*c))*sin(6
*d*x + 6*c))
```

Fricas [B] time = 8.75668, size = 6782, normalized size = 29.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a-b*sin(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/32*(((a*b^2 - b^3)*d*cos(d*x + c)^4 - 2*(a*b^2 - b^3)*d*cos(d*x + c)^2 -
 (a^2*b - 2*a*b^2 + b^3)*d)*sqrt(-((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^
2*sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a
^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + 4*a^2 - 15*a*b + 15*b^2)/(
(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))*log(1/4*(20*a^2 - 81*a*b + 81*b
^2)*cos(d*x + c)^2 - 5*a^2 + 81/4*a*b - 81/4*b^2 + 1/2*((a^5*b^3 - 6*a^4*b^
4 + 12*a^3*b^5 - 10*a^2*b^6 + 3*a*b^7)*d^3*sqrt((25*a^2 - 90*a*b + 81*b^2)/
((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 +
a*b^9)*d^4))*cos(d*x + c)*sin(d*x + c) + 2*(5*a^3*b - 19*a^2*b^2 + 18*a*b^3
)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)
*d^2*sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 2
0*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + 4*a^2 - 15*a*b + 15*b^2
)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2)) + 1/4*(2*(4*a^5*b - 21*a^4*b
^2 + 39*a^3*b^3 - 31*a^2*b^4 + 9*a*b^5)*d^2*cos(d*x + c)^2 - (4*a^5*b - 21*
a^4*b^2 + 39*a^3*b^3 - 31*a^2*b^4 + 9*a*b^5)*d^2)*sqrt((25*a^2 - 90*a*b + 8
1*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2
*b^8 + a*b^9)*d^4))) - ((a*b^2 - b^3)*d*cos(d*x + c)^4 - 2*(a*b^2 - b^3)*d*
cos(d*x + c)^2 - (a^2*b - 2*a*b^2 + b^3)*d)*sqrt(-((a^3*b^3 - 3*a^2*b^4 + 3
*a*b^5 - b^6)*d^2*sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 1
5*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + 4*a^2 - 15
*a*b + 15*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))*log(1/4*(20*a^2
 - 81*a*b + 81*b^2)*cos(d*x + c)^2 - 5*a^2 + 81/4*a*b - 81/4*b^2 - 1/2*((a^
5*b^3 - 6*a^4*b^4 + 12*a^3*b^5 - 10*a^2*b^6 + 3*a*b^7)*d^3*sqrt((25*a^2 - 9
0*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^
7 - 6*a^2*b^8 + a*b^9)*d^4))*cos(d*x + c)*sin(d*x + c) + 2*(5*a^3*b - 19*a^
2*b^2 + 18*a*b^3)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^3*b^3 - 3*a^2*b^4
+ 3*a*b^5 - b^6)*d^2*sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4
+ 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + 4*a^2 -
 15*a*b + 15*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2)) + 1/4*(2*(4*
a^5*b - 21*a^4*b^2 + 39*a^3*b^3 - 31*a^2*b^4 + 9*a*b^5)*d^2*cos(d*x + c)^2
- (4*a^5*b - 21*a^4*b^2 + 39*a^3*b^3 - 31*a^2*b^4 + 9*a*b^5)*d^2)*sqrt((25*
a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15
*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4))) + ((a*b^2 - b^3)*d*cos(d*x + c)^4 - 2*
(a*b^2 - b^3)*d*cos(d*x + c)^2 - (a^2*b - 2*a*b^2 + b^3)*d)*sqrt(((a^3*b^3
- 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3
- 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4
)) - 4*a^2 + 15*a*b - 15*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))*
log(-1/4*(20*a^2 - 81*a*b + 81*b^2)*cos(d*x + c)^2 + 5*a^2 - 81/4*a*b + 81/
4*b^2 + 1/2*((a^5*b^3 - 6*a^4*b^4 + 12*a^3*b^5 - 10*a^2*b^6 + 3*a*b^7)*d^3*
sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4
*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4))*cos(d*x + c)*sin(d*x + c) - 2*
(5*a^3*b - 19*a^2*b^2 + 18*a*b^3)*d*cos(d*x + c)*sin(d*x + c))*sqrt(((a^3*b
^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*b
^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*
d^4)) - 4*a^2 + 15*a*b - 15*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2
)) + 1/4*(2*(4*a^5*b - 21*a^4*b^2 + 39*a^3*b^3 - 31*a^2*b^4 + 9*a*b^5)*d^2*
cos(d*x + c)^2 - (4*a^5*b - 21*a^4*b^2 + 39*a^3*b^3 - 31*a^2*b^4 + 9*a*b^5)
*d^2)*sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 -
20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4))) - ((a*b^2 - b^3)*d*cos(
d*x + c)^4 - 2*(a*b^2 - b^3)*d*cos(d*x + c)^2 - (a^2*b - 2*a*b^2 + b^3)*d)*
sqrt(((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((25*a^2 - 90*a*b + 81*
b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b
^8 + a*b^9)*d^4)) - 4*a^2 + 15*a*b - 15*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^
5 - b^6)*d^2))*log(-1/4*(20*a^2 - 81*a*b + 81*b^2)*cos(d*x + c)^2 + 5*a^2 -
 81/4*a*b + 81/4*b^2 - 1/2*((a^5*b^3 - 6*a^4*b^4 + 12*a^3*b^5 - 10*a^2*b^6
+ 3*a*b^7)*d^3*sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a
^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4))*cos(d*x + c)*si
n(d*x + c) - 2*(5*a^3*b - 19*a^2*b^2 + 18*a*b^3)*d*cos(d*x + c)*sin(d*x + c
))*sqrt(((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((25*a^2 - 90*a*b +
```

```
81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^
2*b^8 + a*b^9)*d^4)) - 4*a^2 + 15*a*b - 15*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3*a
*b^5 - b^6)*d^2)) + 1/4*(2*(4*a^5*b - 21*a^4*b^2 + 39*a^3*b^3 - 31*a^2*b^4
+ 9*a*b^5)*d^2*cos(d*x + c)^2 - (4*a^5*b - 21*a^4*b^2 + 39*a^3*b^3 - 31*a^2
*b^4 + 9*a*b^5)*d^2)*sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4
+ 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4))) + 8*(b*c
os(d*x + c)^3 - (a + b)*cos(d*x + c))*sin(d*x + c))/((a*b^2 - b^3)*d*cos(d*
x + c)^4 - 2*(a*b^2 - b^3)*d*cos(d*x + c)^2 - (a^2*b - 2*a*b^2 + b^3)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**6/(a-b*sin(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a-b*sin(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.220 $\int \frac{\sin^4(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=195

$$\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{3/4}\sqrt{b}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{3/4}\sqrt{b}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}+\frac{\tan^5(c+dx)}{4ad\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}-\frac{\tan(c+dx)}{4ad(a-b)}$$

```
[Out] ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)]/(8*a^(3/4)*(Sqrt[a]
- Sqrt[b])^(3/2)*Sqrt[b]*d) - ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])
/a^(1/4)]/(8*a^(3/4)*(Sqrt[a] + Sqrt[b])^(3/2)*Sqrt[b]*d) - Tan[c + d*x]/(4
*a*(a - b)*d) + Tan[c + d*x]^5/(4*a*d*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan
[c + d*x]^4))
```

Rubi [A] time = 0.229434, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3217, 1275, 12, 1122, 1166, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{3/4}\sqrt{b}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{3/4}\sqrt{b}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}+\frac{\tan^5(c+dx)}{4ad\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}-\frac{\tan(c+dx)}{4ad(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a - b*Sin[c + d*x]^4)^2,x]
```


```
[Out] ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)]/(8*a^(3/4)*(Sqrt[a]
- Sqrt[b])^(3/2)*Sqrt[b]*d) - ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])
/a^(1/4)]/(8*a^(3/4)*(Sqrt[a] + Sqrt[b])^(3/2)*Sqrt[b]*d) - Tan[c + d*x]/(4
*a*(a - b)*d) + Tan[c + d*x]^5/(4*a*d*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan
[c + d*x]^4))
```


Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^4(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^4(1+x^2)}{\left(a+2ax^2+(a-b)x^4\right)^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\tan^5(c+dx)}{4ad\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)} + \frac{\text{Subst}\left(\int -\frac{2bx^4}{a+2ax^2+(a-b)x^4}\,dx, x, \tan(\right.}{8abd}$$

$$= \frac{\tan^5(c+dx)}{4ad\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{x^4}{a+2ax^2+(a-b)x^4}\,dx, x, \tan(c\right.}{4ad}$$

$$= -\frac{\tan(c+dx)}{4a(a-b)d} + \frac{\tan^5(c+dx)}{4ad\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{a+2ax^2}{a+2ax^2+(a-}\right.}{4a(}$$

$$= -\frac{\tan(c+dx)}{4a(a-b)d} + \frac{\tan^5(c+dx)}{4ad\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)} + \frac{\left(2\sqrt{a}-\frac{a+b}{\sqrt{b}}\right)\text{Subst}}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\sqrt{b}d} - \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\sqrt{b}d} - \frac{\tan(c+dx)}{4a(a-b)d} + \frac{}{4ad\left(a+2a\tan^2(\right.}$$

Mathematica [A] time = 4.23528, size = 225, normalized size = 1.15

$$-\frac{\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{a}\sqrt{b}\sqrt{\sqrt{a}\sqrt{b}+a}} - \frac{2(\sin(4(c+dx))-6\sin(2(c+dx)))}{8a+4b\cos(2(c+dx))-b\cos(4(c+dx))-3b} + \frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{a}\sqrt{b}\sqrt{\sqrt{a}\sqrt{b}-a}}}{8d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] -(((Sqrt[a] - Sqrt[b])*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/Sqrt[a + S
qrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[a + Sqrt[a]*Sqrt[b]]*Sqrt[b]) + ((Sqrt[a] +
 Sqrt[b])*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt
[b]]])/(Sqrt[a]*Sqrt[-a + Sqrt[a]*Sqrt[b]]*Sqrt[b]) - (2*(-6*Sin[2*(c + d*x
)] + Sin[4*(c + d*x)]))/(8*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*
x)]))/(8*(a - b)*d)
```

Maple [B] time = 0.108, size = 478, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a-b*sin(d*x+c)^4)^2,x)
```

```
[Out] -1/2/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)/(a-b)*tan(d*x+c)^
3-1/4/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)/(a-b)*tan(d*x+c)
+1/8/d*a/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d
*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+1/8/d/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+
a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*b+1/
4/d/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/
2)+a)*(a-b))^(1/2))-1/8/d*a/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)
*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-1/8/d/(a*b)^(1/2)
/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2
)-a)*(a-b))^(1/2))*b+1/4/d/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+
b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a-b*sin(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 5.51345, size = 5956, normalized size = 30.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a-b*sin(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/32*(((a*b - b^2)*d*cos(d*x + c)^4 - 2*(a*b - b^2)*d*cos(d*x + c)^2 - (a^
2 - 2*a*b + b^2)*d)*sqrt(-((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt
((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a
^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + a + 3*b)/((a^4*b - 3*a^3*b^2 + 3*a^2*
```

```
b^3 - a*b^4)*d^2))*log(1/4*(3*a + b)*cos(d*x + c)^2 + 1/2*(2*(a^6*b - 3*a^5
*b^2 + 3*a^4*b^3 - a^3*b^4)*d^3*sqrt((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*
b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4))*cos
(d*x + c)*sin(d*x + c) - (3*a^3 + 4*a^2*b + a*b^2)*d*cos(d*x + c)*sin(d*x +
 c))*sqrt(-((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt((9*a^2 + 6*a*b
 + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*
b^6 + a^3*b^7)*d^4)) + a + 3*b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^
2)) - 1/4*(2*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d^2*cos(d*x + c)^2 - (a^
5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d^2)*sqrt((9*a^2 + 6*a*b + b^2)/((a^9*b
- 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d
^4)) - 3/4*a - 1/4*b) - ((a*b - b^2)*d*cos(d*x + c)^4 - 2*(a*b - b^2)*d*cos
(d*x + c)^2 - (a^2 - 2*a*b + b^2)*d)*sqrt(-((a^4*b - 3*a^3*b^2 + 3*a^2*b^3
- a*b^4)*d^2*sqrt((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 -
20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + a + 3*b)/((a^4*b - 3
*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2))*log(1/4*(3*a + b)*cos(d*x + c)^2 - 1/2*
(2*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*d^3*sqrt((9*a^2 + 6*a*b + b^2)
/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a
^3*b^7)*d^4))*cos(d*x + c)*sin(d*x + c) - (3*a^3 + 4*a^2*b + a*b^2)*d*cos(d
*x + c)*sin(d*x + c))*sqrt(-((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*sq
rt((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15
*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + a + 3*b)/((a^4*b - 3*a^3*b^2 + 3*a^
2*b^3 - a*b^4)*d^2)) - 1/4*(2*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d^2*cos
(d*x + c)^2 - (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d^2)*sqrt((9*a^2 + 6*a*
b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4
*b^6 + a^3*b^7)*d^4)) - 3/4*a - 1/4*b) + ((a*b - b^2)*d*cos(d*x + c)^4 - 2*
(a*b - b^2)*d*cos(d*x + c)^2 - (a^2 - 2*a*b + b^2)*d)*sqrt(((a^4*b - 3*a^3*
b^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2
 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) - a -
3*b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2))*log(-1/4*(3*a + b)*cos(
d*x + c)^2 + 1/2*(2*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*d^3*sqrt((9*a
^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^
5 - 6*a^4*b^6 + a^3*b^7)*d^4))*cos(d*x + c)*sin(d*x + c) + (3*a^3 + 4*a^2*b
 + a*b^2)*d*cos(d*x + c)*sin(d*x + c))*sqrt(((a^4*b - 3*a^3*b^2 + 3*a^2*b^3
 - a*b^4)*d^2*sqrt((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 -
 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) - a - 3*b)/((a^4*b -
3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2)) - 1/4*(2*(a^5 - 3*a^4*b + 3*a^3*b^2 -
a^2*b^3)*d^2*cos(d*x + c)^2 - (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d^2)*sq
rt((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15
*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + 3/4*a + 1/4*b) - ((a*b - b^2)*d*cos
(d*x + c)^4 - 2*(a*b - b^2)*d*cos(d*x + c)^2 - (a^2 - 2*a*b + b^2)*d)*sqrt(
((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt((9*a^2 + 6*a*b + b^2)/((a
^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b
^7)*d^4)) - a - 3*b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2))*log(-1/
4*(3*a + b)*cos(d*x + c)^2 - 1/2*(2*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^
4)*d^3*sqrt((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6
*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4))*cos(d*x + c)*sin(d*x + c) +
(3*a^3 + 4*a^2*b + a*b^2)*d*cos(d*x + c)*sin(d*x + c))*sqrt(((a^4*b - 3*a^3
*b^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^
2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) - a -
 3*b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2)) - 1/4*(2*(a^5 - 3*a^4*
b + 3*a^3*b^2 - a^2*b^3)*d^2*cos(d*x + c)^2 - (a^5 - 3*a^4*b + 3*a^3*b^2 -
a^2*b^3)*d^2)*sqrt((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 -
 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + 3/4*a + 1/4*b) + 8*
(cos(d*x + c)^3 - 2*cos(d*x + c))*sin(d*x + c))/((a*b - b^2)*d*cos(d*x + c)
^4 - 2*(a*b - b^2)*d*cos(d*x + c)^2 - (a^2 - 2*a*b + b^2)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a-b*sin(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a-b*sin(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.221 $\int \frac{\sin^2(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=219

$$\frac{\left(2\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{5/4}\sqrt{b}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\left(2\sqrt{a}+\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{5/4}\sqrt{b}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{\tan(c+dx)\left((a+b)\tan^2(c+\right.}{4ad(a-b)\left((a-b)\tan^4(c+dx)+2a\right.}$$

```
[Out] ((2*Sqrt[a] - Sqrt[b])*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4
)])/(8*a^(5/4)*(Sqrt[a] - Sqrt[b])^(3/2)*Sqrt[b]*d) - ((2*Sqrt[a] + Sqrt[b]
)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(8*a^(5/4)*(Sqrt[
a] + Sqrt[b])^(3/2)*Sqrt[b]*d) - (Tan[c + d*x]*(a + (a + b)*Tan[c + d*x]^2)
)/(4*a*(a - b)*d*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan[c + d*x]^4))
```

Rubi [A] time = 0.29745, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3217, 1333, 1166, 205}

$$\frac{\left(2\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{5/4}\sqrt{b}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\left(2\sqrt{a}+\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{5/4}\sqrt{b}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{\tan(c+dx)\left((a+b)\tan^2(c+\right.}{4ad(a-b)\left((a-b)\tan^4(c+dx)+2a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] ((2*Sqrt[a] - Sqrt[b])*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4
)])/(8*a^(5/4)*(Sqrt[a] - Sqrt[b])^(3/2)*Sqrt[b]*d) - ((2*Sqrt[a] + Sqrt[b]
)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(8*a^(5/4)*(Sqrt[
a] + Sqrt[b])^(3/2)*Sqrt[b]*d) - (Tan[c + d*x]*(a + (a + b)*Tan[c + d*x]^2)
)/(4*a*(a - b)*d*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan[c + d*x]^4))
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 1333

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*S
imp[ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d + e*x^2
)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g +
c*(4*p + 7)*(b*f - 2*a*g)*x^2, x], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1] && IGtQ[m/2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(1+x^2\right)^2}{\left(a+2ax^2+(a-b)x^4\right)^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{\tan(c+dx)\left(a+(a+b)\tan^2(c+dx)\right)}{4a(a-b)d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-\frac{2a^2b}{a-b}-\frac{2a(3a-b)bx^2}{a-b}}{a+2ax^2+(a-b)x^4}\,dx, x, \ldots\right)}{8a^2bd}$$

$$= -\frac{\tan(c+dx)\left(a+(a+b)\tan^2(c+dx)\right)}{4a(a-b)d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)} + \frac{\left(2a+\sqrt{a}\sqrt{b}-b\right)\text{Subst}\left(\int \frac{\ldots}{a+\sqrt{\ldots}}\ldots\right)}{8a\left(\sqrt{a}-\ldots\right)}$$

$$= \frac{\left(2\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{5/4}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\sqrt{b}d} - \frac{\left(2\sqrt{a}+\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{5/4}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\sqrt{b}d} - \frac{\text{ta}\ldots}{4a(a-b)d\ldots}$$

Mathematica [A] time = 2.08881, size = 255, normalized size = 1.16

$$\frac{-\frac{\sqrt{a}\left(-\sqrt{a}\sqrt{b}+2a-b\right)\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{b}\sqrt{\sqrt{a}\sqrt{b}+a}}-\frac{4\sqrt{a}\sin(2(c+dx))(2a-b\cos(2(c+dx))+b)}{8a+4b\cos(2(c+dx))-b\cos(4(c+dx))-3b}-\frac{\sqrt{a}\left(\sqrt{a}\sqrt{b}+2a-b\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{b}\sqrt{\sqrt{a}\sqrt{b}-a}}}{8a^{3/2}d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] (-((Sqrt[a]*(2*a - Sqrt[a]*Sqrt[b] - b)*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c +
 d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a + Sqrt[a]*Sqrt[b]]*Sqrt[b])) - (
Sqrt[a]*(2*a + Sqrt[a]*Sqrt[b] - b)*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*
x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/(Sqrt[-a + Sqrt[a]*Sqrt[b]]*Sqrt[b]) - (4*
Sqrt[a]*(2*a + b - b*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])/(8*a - 3*b + 4*b*C
os[2*(c + d*x)] - b*Cos[4*(c + d*x)]))/(8*a^(3/2)*(a - b)*d)
```

Maple [B] time = 0.127, size = 534, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a-b*sin(d*x+c)^4)^2,x)
```

```
[Out] -1/4/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)/(a-b)*tan(d*x+c)^
3-1/4/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)/(a-b)/a*tan(d*x+
c)^3*b-1/4/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)/(a-b)*tan(d
*x+c)+3/8/d/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((
a*b)^(1/2)+a)*(a-b))^(1/2))-1/8/d/a/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arc
tan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*b+1/4/d*a/(a*b)^(1/2)/(
a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)
*(a-b))^(1/2))+3/8/d/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan
(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-1/8/d/a/(a-b)/(((a*b)^(1/2)-a)*(a-b)
)^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*b-1/4/d*a/
(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(
((a*b)^(1/2)-a)*(a-b))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a-b*sin(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(16*a^2 + 2*a*b - 3*b^2)*cos(4*d*x + 4*c)*sin(2*d*x + 2*c) + ((2*a*b
 - b^2)*sin(6*d*x + 6*c) - (8*a*b - 3*b^2)*sin(4*d*x + 4*c) - (2*a*b + 3*b^
2)*sin(2*d*x + 2*c))*cos(8*d*x + 8*c) + 2*((16*a^2 + 2*a*b - 3*b^2)*sin(4*d
*x + 4*c) + 4*(2*a*b + b^2)*sin(2*d*x + 2*c))*cos(6*d*x + 6*c) + 2*((a^2*b^
2 - a*b^3)*d*cos(8*d*x + 8*c)^2 + 16*(a^2*b^2 - a*b^3)*d*cos(6*d*x + 6*c)^2
 + 4*(64*a^4 - 112*a^3*b + 57*a^2*b^2 - 9*a*b^3)*d*cos(4*d*x + 4*c)^2 + 16*
(a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c)^2 + (a^2*b^2 - a*b^3)*d*sin(8*d*x + 8*
c)^2 + 16*(a^2*b^2 - a*b^3)*d*sin(6*d*x + 6*c)^2 + 4*(64*a^4 - 112*a^3*b +
57*a^2*b^2 - 9*a*b^3)*d*sin(4*d*x + 4*c)^2 + 16*(8*a^3*b - 11*a^2*b^2 + 3*a
*b^3)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a^2*b^2 - a*b^3)*d*sin(2*d*
x + 2*c)^2 - 8*(a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c) + (a^2*b^2 - a*b^3)*d -
 2*(4*(a^2*b^2 - a*b^3)*d*cos(6*d*x + 6*c) + 2*(8*a^3*b - 11*a^2*b^2 + 3*a*
b^3)*d*cos(4*d*x + 4*c) + 4*(a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c) - (a^2*b^2
 - a*b^3)*d)*cos(8*d*x + 8*c) + 8*(2*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*cos
(4*d*x + 4*c) + 4*(a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c) - (a^2*b^2 - a*b^3)*
d)*cos(6*d*x + 6*c) + 4*(4*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*cos(2*d*x + 2
*c) - (8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d)*cos(4*d*x + 4*c) - 4*(2*(a^2*b^2
- a*b^3)*d*sin(6*d*x + 6*c) + (8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*sin(4*d*x
+ 4*c) + 2*(a^2*b^2 - a*b^3)*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*((8*
a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*sin(4*d*x + 4*c) + 2*(a^2*b^2 - a*b^3)*d*si
n(2*d*x + 2*c))*sin(6*d*x + 6*c))*integrate(-(4*(2*a*b - b^2)*cos(6*d*x + 6
*c)^2 - 4*(32*a^2 - 20*a*b + 3*b^2)*cos(4*d*x + 4*c)^2 + 4*(2*a*b - b^2)*co
s(2*d*x + 2*c)^2 + 4*(2*a*b - b^2)*sin(6*d*x + 6*c)^2 - 4*(32*a^2 - 20*a*b
+ 3*b^2)*sin(4*d*x + 4*c)^2 + 2*(16*a^2 - 30*a*b + 7*b^2)*sin(4*d*x + 4*c)*
sin(2*d*x + 2*c) + 4*(2*a*b - b^2)*sin(2*d*x + 2*c)^2 - ((2*a*b - b^2)*cos(
6*d*x + 6*c) - 2*(4*a*b - b^2)*cos(4*d*x + 4*c) + (2*a*b - b^2)*cos(2*d*x +
 2*c))*cos(8*d*x + 8*c) - (2*a*b - b^2 - 2*(16*a^2 - 30*a*b + 7*b^2)*cos(4*
d*x + 4*c) - 8*(2*a*b - b^2)*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 2*(4*a*b
- b^2 + (16*a^2 - 30*a*b + 7*b^2)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - (2*a
*b - b^2)*cos(2*d*x + 2*c) - ((2*a*b - b^2)*sin(6*d*x + 6*c) - 2*(4*a*b - b
^2)*sin(4*d*x + 4*c) + (2*a*b - b^2)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 2
*((16*a^2 - 30*a*b + 7*b^2)*sin(4*d*x + 4*c) + 4*(2*a*b - b^2)*sin(2*d*x +
```

```
2*c))*sin(6*d*x + 6*c))/(a^2*b^2 - a*b^3 + (a^2*b^2 - a*b^3)*cos(8*d*x + 8*
c)^2 + 16*(a^2*b^2 - a*b^3)*cos(6*d*x + 6*c)^2 + 4*(64*a^4 - 112*a^3*b + 57
*a^2*b^2 - 9*a*b^3)*cos(4*d*x + 4*c)^2 + 16*(a^2*b^2 - a*b^3)*cos(2*d*x + 2
*c)^2 + (a^2*b^2 - a*b^3)*sin(8*d*x + 8*c)^2 + 16*(a^2*b^2 - a*b^3)*sin(6*d
*x + 6*c)^2 + 4*(64*a^4 - 112*a^3*b + 57*a^2*b^2 - 9*a*b^3)*sin(4*d*x + 4*c
)^2 + 16*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
 + 16*(a^2*b^2 - a*b^3)*sin(2*d*x + 2*c)^2 + 2*(a^2*b^2 - a*b^3 - 4*(a^2*b^
2 - a*b^3)*cos(6*d*x + 6*c) - 2*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*cos(4*d*x
+ 4*c) - 4*(a^2*b^2 - a*b^3)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) - 8*(a^2*b^
2 - a*b^3 - 2*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*cos(4*d*x + 4*c) - 4*(a^2*b^
2 - a*b^3)*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) - 4*(8*a^3*b - 11*a^2*b^2 + 3
*a*b^3 - 4*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*cos(2*d*x + 2*c))*cos(4*d*x + 4
*c) - 8*(a^2*b^2 - a*b^3)*cos(2*d*x + 2*c) - 4*(2*(a^2*b^2 - a*b^3)*sin(6*d
*x + 6*c) + (8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*sin(4*d*x + 4*c) + 2*(a^2*b^2
- a*b^3)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^3*b - 11*a^2*b^2 + 3
*a*b^3)*sin(4*d*x + 4*c) + 2*(a^2*b^2 - a*b^3)*sin(2*d*x + 2*c))*sin(6*d*x
+ 6*c)), x) - (b^2 + (2*a*b - b^2)*cos(6*d*x + 6*c) - (8*a*b - 3*b^2)*cos(4
*d*x + 4*c) - (2*a*b + 3*b^2)*cos(2*d*x + 2*c))*sin(8*d*x + 8*c) + (2*a*b +
 3*b^2 - 2*(16*a^2 + 2*a*b - 3*b^2)*cos(4*d*x + 4*c) - 8*(2*a*b + b^2)*cos(
2*d*x + 2*c))*sin(6*d*x + 6*c) + (8*a*b - 3*b^2 - 2*(16*a^2 + 2*a*b - 3*b^2
)*cos(2*d*x + 2*c))*sin(4*d*x + 4*c) - (2*a*b - b^2)*sin(2*d*x + 2*c))/((a^
2*b^2 - a*b^3)*d*cos(8*d*x + 8*c)^2 + 16*(a^2*b^2 - a*b^3)*d*cos(6*d*x + 6*
c)^2 + 4*(64*a^4 - 112*a^3*b + 57*a^2*b^2 - 9*a*b^3)*d*cos(4*d*x + 4*c)^2 +
 16*(a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c)^2 + (a^2*b^2 - a*b^3)*d*sin(8*d*x
+ 8*c)^2 + 16*(a^2*b^2 - a*b^3)*d*sin(6*d*x + 6*c)^2 + 4*(64*a^4 - 112*a^3*
b + 57*a^2*b^2 - 9*a*b^3)*d*sin(4*d*x + 4*c)^2 + 16*(8*a^3*b - 11*a^2*b^2 +
 3*a*b^3)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a^2*b^2 - a*b^3)*d*sin(
2*d*x + 2*c)^2 - 8*(a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c) + (a^2*b^2 - a*b^3)
*d - 2*(4*(a^2*b^2 - a*b^3)*d*cos(6*d*x + 6*c) + 2*(8*a^3*b - 11*a^2*b^2 +
3*a*b^3)*d*cos(4*d*x + 4*c) + 4*(a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c) - (a^2
*b^2 - a*b^3)*d)*cos(8*d*x + 8*c) + 8*(2*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d
*cos(4*d*x + 4*c) + 4*(a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c) - (a^2*b^2 - a*b
^3)*d)*cos(6*d*x + 6*c) + 4*(4*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*cos(2*d*x
 + 2*c) - (8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d)*cos(4*d*x + 4*c) - 4*(2*(a^2*
b^2 - a*b^3)*d*sin(6*d*x + 6*c) + (8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*sin(4*
d*x + 4*c) + 2*(a^2*b^2 - a*b^3)*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*
((8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*sin(4*d*x + 4*c) + 2*(a^2*b^2 - a*b^3)*
d*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))
```

Fricas [B] time = 8.09228, size = 7386, normalized size = 33.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a-b*sin(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/32*(((a^2*b - a*b^2)*d*cos(d*x + c)^4 - 2*(a^2*b - a*b^2)*d*cos(d*x + c)^
2 - (a^3 - 2*a^2*b + a*b^2)*d)*sqrt(-((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*
b^4)*d^2*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b -
6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^
4)) + 4*a^2 + a*b - b^2)/((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2))*l
og(8*a^3 - 7*a^2*b + 9/4*a*b^2 - 1/4*b^3 - 1/4*(32*a^3 - 28*a^2*b + 9*a*b^2
 - b^3)*cos(d*x + c)^2 + 1/2*((3*a^8*b - 10*a^7*b^2 + 12*a^6*b^3 - 6*a^5*b^
4 + a^4*b^5)*d^3*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a
^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5
*b^7)*d^4))*cos(d*x + c)*sin(d*x + c) - 2*(8*a^5 - 5*a^4*b + a^3*b^2)*d*cos
```

(d*x + c)*sin(d*x + c))*sqrt(-((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^
2*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^10*
b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4)) + 4
*a^2 + a*b - b^2)/((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2)) + 1/4*(2
*(4*a^7 - 13*a^6*b + 15*a^5*b^2 - 7*a^4*b^3 + a^3*b^4)*d^2*cos(d*x + c)^2 -
(4*a^7 - 13*a^6*b + 15*a^5*b^2 - 7*a^4*b^3 + a^3*b^4)*d^2)*sqrt((64*a^4 -
80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3
- 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4))) - ((a^2*b - a*b^2)*
d*cos(d*x + c)^4 - 2*(a^2*b - a*b^2)*d*cos(d*x + c)^2 - (a^3 - 2*a^2*b + a*
b^2)*d)*sqrt(-((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2*sqrt((64*a^4 -
80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3
- 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4)) + 4*a^2 + a*b - b^2
)/((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2))*log(8*a^3 - 7*a^2*b + 9/
4*a*b^2 - 1/4*b^3 - 1/4*(32*a^3 - 28*a^2*b + 9*a*b^2 - b^3)*cos(d*x + c)^2
- 1/2*((3*a^8*b - 10*a^7*b^2 + 12*a^6*b^3 - 6*a^5*b^4 + a^4*b^5)*d^3*sqrt((
64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15
*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4))*cos(d*x + c
)*sin(d*x + c) - 2*(8*a^5 - 5*a^4*b + a^3*b^2)*d*cos(d*x + c)*sin(d*x + c))
*sqrt(-((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2*sqrt((64*a^4 - 80*a^3
*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a
^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4)) + 4*a^2 + a*b - b^2)/((a^5
*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2)) + 1/4*(2*(4*a^7 - 13*a^6*b + 15
*a^5*b^2 - 7*a^4*b^3 + a^3*b^4)*d^2*cos(d*x + c)^2 - (4*a^7 - 13*a^6*b + 15
*a^5*b^2 - 7*a^4*b^3 + a^3*b^4)*d^2)*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 -
10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b
^5 - 6*a^6*b^6 + a^5*b^7)*d^4))) + ((a^2*b - a*b^2)*d*cos(d*x + c)^4 - 2*(a
^2*b - a*b^2)*d*cos(d*x + c)^2 - (a^3 - 2*a^2*b + a*b^2)*d)*sqrt(((a^5*b -
3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 -
10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b
^5 - 6*a^6*b^6 + a^5*b^7)*d^4)) - 4*a^2 - a*b + b^2)/((a^5*b - 3*a^4*b^2 +
3*a^3*b^3 - a^2*b^4)*d^2))*log(-8*a^3 + 7*a^2*b - 9/4*a*b^2 + 1/4*b^3 + 1/4
*(32*a^3 - 28*a^2*b + 9*a*b^2 - b^3)*cos(d*x + c)^2 + 1/2*((3*a^8*b - 10*a^
7*b^2 + 12*a^6*b^3 - 6*a^5*b^4 + a^4*b^5)*d^3*sqrt((64*a^4 - 80*a^3*b + 41*
a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 +
15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4))*cos(d*x + c)*sin(d*x + c) + 2*(8*a
^5 - 5*a^4*b + a^3*b^2)*d*cos(d*x + c)*sin(d*x + c))*sqrt(((a^5*b - 3*a^4*b
^2 + 3*a^3*b^3 - a^2*b^4)*d^2*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b
^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*
a^6*b^6 + a^5*b^7)*d^4)) - 4*a^2 - a*b + b^2)/((a^5*b - 3*a^4*b^2 + 3*a^3*b
^3 - a^2*b^4)*d^2)) + 1/4*(2*(4*a^7 - 13*a^6*b + 15*a^5*b^2 - 7*a^4*b^3 + a
^3*b^4)*d^2*cos(d*x + c)^2 - (4*a^7 - 13*a^6*b + 15*a^5*b^2 - 7*a^4*b^3 + a
^3*b^4)*d^2)*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*
b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7
)*d^4))) - ((a^2*b - a*b^2)*d*cos(d*x + c)^4 - 2*(a^2*b - a*b^2)*d*cos(d*x
+ c)^2 - (a^3 - 2*a^2*b + a*b^2)*d)*sqrt(((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 -
a^2*b^4)*d^2*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*
b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7
)*d^4)) - 4*a^2 - a*b + b^2)/((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2
))*log(-8*a^3 + 7*a^2*b - 9/4*a*b^2 + 1/4*b^3 + 1/4*(32*a^3 - 28*a^2*b + 9*
a*b^2 - b^3)*cos(d*x + c)^2 - 1/2*((3*a^8*b - 10*a^7*b^2 + 12*a^6*b^3 - 6*a
^5*b^4 + a^4*b^5)*d^3*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4
)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6
+ a^5*b^7)*d^4))*cos(d*x + c)*sin(d*x + c) + 2*(8*a^5 - 5*a^4*b + a^3*b^2)*
d*cos(d*x + c)*sin(d*x + c))*sqrt(((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4
)*d^2*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a
^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4))
- 4*a^2 - a*b + b^2)/((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2)) + 1/
4*(2*(4*a^7 - 13*a^6*b + 15*a^5*b^2 - 7*a^4*b^3 + a^3*b^4)*d^2*cos(d*x + c)
^2 - (4*a^7 - 13*a^6*b + 15*a^5*b^2 - 7*a^4*b^3 + a^3*b^4)*d^2)*sqrt((64*a^

```
4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*
b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4))) - 8*(b*cos(d*x
+ c)^3 - (a + b)*cos(d*x + c))*sin(d*x + c))/((a^2*b - a*b^2)*d*cos(d*x + c
)^4 - 2*(a^2*b - a*b^2)*d*cos(d*x + c)^2 - (a^3 - 2*a^2*b + a*b^2)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a-b*sin(d*x+c)**4)**2,x)

[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a-b*sin(d*x+c)^4)^2,x, algorithm="giac")

[Out] Exception raised: NotImplementedError
```

3.222 $\int \frac{1}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=210

$$\frac{\left(4\sqrt{a}-3\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{7/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\left(4\sqrt{a}+3\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{7/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{b\tan(c+dx)\left(2\tan^2(c -\right.}{4ad(a-b)\left((a-b)\tan^4(c+dx)+2\right.}$$

```
[Out] ((4*Sqrt[a] - 3*Sqrt[b])*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1
/4)])/(8*a^(7/4)*(Sqrt[a] - Sqrt[b])^(3/2)*d) + ((4*Sqrt[a] + 3*Sqrt[b])*Ar
cTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(8*a^(7/4)*(Sqrt[a] +
 Sqrt[b])^(3/2)*d) - (b*Tan[c + d*x]*(1 + 2*Tan[c + d*x]^2))/(4*a*(a - b)*d
*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan[c + d*x]^4))
```

Rubi [A] time = 0.260398, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3209, 1205, 1166, 205}

$$\frac{\left(4\sqrt{a}-3\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{7/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\left(4\sqrt{a}+3\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{7/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{b\tan(c+dx)\left(2\tan^2(c -\right.}{4ad(a-b)\left((a-b)\tan^4(c+dx)+2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*Sin[c + d*x]^4)^(-2),x]
```

```
[Out] ((4*Sqrt[a] - 3*Sqrt[b])*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1
/4)])/(8*a^(7/4)*(Sqrt[a] - Sqrt[b])^(3/2)*d) + ((4*Sqrt[a] + 3*Sqrt[b])*Ar
cTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(8*a^(7/4)*(Sqrt[a] +
 Sqrt[b])^(3/2)*d) - (b*Tan[c + d*x]*(1 + 2*Tan[c + d*x]^2))/(4*a*(a - b)*d
*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan[c + d*x]^4))
```

Rule 3209

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a-b\sin^4(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{\left(a+2ax^2+(a-b)x^4\right)^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{b\tan(c+dx)\left(1+2\tan^2(c+dx)\right)}{4a(a-b)d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-\frac{2a(4a-3b)b}{a-b}-\frac{4a(2a-b)bx^2}{a-b}}{a+2ax^2+(a-b)x^4}\,dx\right.}{8a^2bd}$$

$$= -\frac{b\tan(c+dx)\left(1+2\tan^2(c+dx)\right)}{4a(a-b)d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)} + \frac{\left(4a-\sqrt{a}\sqrt{b}-3b\right)\text{Subst}\left(\int \frac{}{a-}\right.}{8a^{3/2}\left(\sqrt{a}\right.}$$

$$= \frac{\left(4\sqrt{a}-3\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{7/4}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}d} + \frac{\left(4\sqrt{a}+3\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{7/4}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}d} - \frac{}{4a(a-b}$$

Mathematica [A] time = 2.82413, size = 230, normalized size = 1.1

$$\frac{\frac{\left(-\sqrt{a}\sqrt{b}+4a-3b\right)\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}+a}} + \frac{2\sqrt{a}b(\sin(4(c+dx))-6\sin(2(c+dx)))}{8a+4b\cos(2(c+dx))-b\cos(4(c+dx))-3b} - \frac{\left(\sqrt{a}\sqrt{b}+4a-3b\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}-a}}}{8a^{3/2}d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*Sin[c + d*x]^4)^(-2),x]
```

```
[Out] (((4*a - Sqrt[a]*Sqrt[b] - 3*b)*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/S
qrt[a + Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]] - ((4*a + Sqrt[a]*Sqrt
[b] - 3*b)*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqr
t[b]]])/Sqrt[-a + Sqrt[a]*Sqrt[b]] + (2*Sqrt[a]*b*(-6*Sin[2*(c + d*x)] + Si
n[4*(c + d*x)]))/(8*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*x)]))/(
8*a^(3/2)*(a - b)*d)
```

Maple [B] time = 0.129, size = 618, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-b*sin(d*x+c)^4)^2,x)
```

```
[Out] -1/2/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)/(a-b)/a*tan(d*x+c
)^3*b-1/4/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)*b/a/(a-b)*ta
n(d*x+c)+1/2/d/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/
(((a*b)^(1/2)+a)*(a-b))^(1/2))-1/4/d/a/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*
arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*b+5/8/d/(a*b)^(1/2)/
(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a
)*(a-b))^(1/2))*b-3/8/d/a/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*a
rctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*b^2+1/2/d/(a-b)/(((a*
b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^
(1/2))-1/4/d/a/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c
)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*b-5/8/d/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)
*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*b+3/
8/d/a/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*
x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*b^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 8.20175, size = 7862, normalized size = 37.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/32*(((a^2*b - a*b^2)*d*cos(d*x + c)^4 - 2*(a^2*b - a*b^2)*d*cos(d*x + c)
^2 - (a^3 - 2*a^2*b + a*b^2)*d)*sqrt(-((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3
)*d^2*sqrt((576*a^4*b - 1392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/(
(a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7
*b^6)*d^4)) + 16*a^2 - 15*a*b + 3*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^
3)*d^2))*log(96*a^3*b - 170*a^2*b^2 + 405/4*a*b^3 - 81/4*b^4 - 1/4*(384*a^3
*b - 680*a^2*b^2 + 405*a*b^3 - 81*b^4)*cos(d*x + c)^2 + 1/2*(2*(2*a^10 - 7*
a^9*b + 9*a^8*b^2 - 5*a^7*b^3 + a^6*b^4)*d^3*sqrt((576*a^4*b - 1392*a^3*b^2
 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*
a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4))*cos(d*x + c)*sin(d*x + c
) - (120*a^5*b - 217*a^4*b^2 + 132*a^3*b^3 - 27*a^2*b^4)*d*cos(d*x + c)*sin
(d*x + c))*sqrt(-((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((576*a^4*b
 - 1392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15
*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) + 16*a^2
- 15*a*b + 3*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2)) + 1/4*(2*(16
*a^8 - 57*a^7*b + 75*a^6*b^2 - 43*a^5*b^3 + 9*a^4*b^4)*d^2*cos(d*x + c)^2 -
 (16*a^8 - 57*a^7*b + 75*a^6*b^2 - 43*a^5*b^3 + 9*a^4*b^4)*d^2)*sqrt((576*a
^4*b - 1392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b
+ 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4))) - ((
```

a^2*b - a*b^2)*d*cos(d*x + c)^4 - 2*(a^2*b - a*b^2)*d*cos(d*x + c)^2 - (a^3
 - 2*a^2*b + a*b^2)*d)*sqrt(-((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqr
t((576*a^4*b - 1392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6
*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4
)) + 16*a^2 - 15*a*b + 3*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))*
log(96*a^3*b - 170*a^2*b^2 + 405/4*a*b^3 - 81/4*b^4 - 1/4*(384*a^3*b - 680*
a^2*b^2 + 405*a*b^3 - 81*b^4)*cos(d*x + c)^2 - 1/2*(2*(2*a^10 - 7*a^9*b + 9
*a^8*b^2 - 5*a^7*b^3 + a^6*b^4)*d^3*sqrt((576*a^4*b - 1392*a^3*b^2 + 1273*a
^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3
 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4))*cos(d*x + c)*sin(d*x + c) - (120*
a^5*b - 217*a^4*b^2 + 132*a^3*b^3 - 27*a^2*b^4)*d*cos(d*x + c)*sin(d*x + c)
)*sqrt(-((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((576*a^4*b - 1392*a
^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2
 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) + 16*a^2 - 15*a*b
 + 3*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2)) + 1/4*(2*(16*a^8 - 57
*a^7*b + 75*a^6*b^2 - 43*a^5*b^3 + 9*a^4*b^4)*d^2*cos(d*x + c)^2 - (16*a^8
 - 57*a^7*b + 75*a^6*b^2 - 43*a^5*b^3 + 9*a^4*b^4)*d^2)*sqrt((576*a^4*b - 13
92*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11
*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4))) + ((a^2*b - a
*b^2)*d*cos(d*x + c)^4 - 2*(a^2*b - a*b^2)*d*cos(d*x + c)^2 - (a^3 - 2*a^2*
b + a*b^2)*d)*sqrt(((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((576*a^4
*b - 1392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b +
15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) - 16*a^
2 + 15*a*b - 3*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))*log(-96*a^
3*b + 170*a^2*b^2 - 405/4*a*b^3 + 81/4*b^4 + 1/4*(384*a^3*b - 680*a^2*b^2 +
 405*a*b^3 - 81*b^4)*cos(d*x + c)^2 + 1/2*(2*(2*a^10 - 7*a^9*b + 9*a^8*b^2
 - 5*a^7*b^3 + a^6*b^4)*d^3*sqrt((576*a^4*b - 1392*a^3*b^2 + 1273*a^2*b^3 -
522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*
b^4 - 6*a^8*b^5 + a^7*b^6)*d^4))*cos(d*x + c)*sin(d*x + c) + (120*a^5*b - 2
17*a^4*b^2 + 132*a^3*b^3 - 27*a^2*b^4)*d*cos(d*x + c)*sin(d*x + c))*sqrt(((
a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((576*a^4*b - 1392*a^3*b^2 + 1
273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10
*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) - 16*a^2 + 15*a*b - 3*b^2)/(
(a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2)) + 1/4*(2*(16*a^8 - 57*a^7*b + 7
5*a^6*b^2 - 43*a^5*b^3 + 9*a^4*b^4)*d^2*cos(d*x + c)^2 - (16*a^8 - 57*a^7*b
 + 75*a^6*b^2 - 43*a^5*b^3 + 9*a^4*b^4)*d^2)*sqrt((576*a^4*b - 1392*a^3*b^2
 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*
a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4))) - ((a^2*b - a*b^2)*d*co
s(d*x + c)^4 - 2*(a^2*b - a*b^2)*d*cos(d*x + c)^2 - (a^3 - 2*a^2*b + a*b^2)
*d)*sqrt(((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((576*a^4*b - 1392*
a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^
2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) - 16*a^2 + 15*a*b
 - 3*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))*log(-96*a^3*b + 170*
a^2*b^2 - 405/4*a*b^3 + 81/4*b^4 + 1/4*(384*a^3*b - 680*a^2*b^2 + 405*a*b^3
 - 81*b^4)*cos(d*x + c)^2 - 1/2*(2*(2*a^10 - 7*a^9*b + 9*a^8*b^2 - 5*a^7*b^
3 + a^6*b^4)*d^3*sqrt((576*a^4*b - 1392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4
 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^
8*b^5 + a^7*b^6)*d^4))*cos(d*x + c)*sin(d*x + c) + (120*a^5*b - 217*a^4*b^2
 + 132*a^3*b^3 - 27*a^2*b^4)*d*cos(d*x + c)*sin(d*x + c))*sqrt(((a^6 - 3*a^
5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((576*a^4*b - 1392*a^3*b^2 + 1273*a^2*b^
3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*
a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) - 16*a^2 + 15*a*b - 3*b^2)/((a^6 - 3*a
^5*b + 3*a^4*b^2 - a^3*b^3)*d^2)) + 1/4*(2*(16*a^8 - 57*a^7*b + 75*a^6*b^2
 - 43*a^5*b^3 + 9*a^4*b^4)*d^2*cos(d*x + c)^2 - (16*a^8 - 57*a^7*b + 75*a^6*
b^2 - 43*a^5*b^3 + 9*a^4*b^4)*d^2)*sqrt((576*a^4*b - 1392*a^3*b^2 + 1273*a^
2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 +
 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4))) + 8*(b*cos(d*x + c)^3 - 2*b*cos(d
*x + c))*sin(d*x + c))/((a^2*b - a*b^2)*d*cos(d*x + c)^4 - 2*(a^2*b - a*b^2
)*d*cos(d*x + c)^2 - (a^3 - 2*a^2*b + a*b^2)*d)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.223 $\int \frac{\csc^2(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=236

$$\frac{\sqrt{b}\left(6\sqrt{a}-5\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{9/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\sqrt{b}\left(6\sqrt{a}+5\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{9/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{b\tan(c+dx)\left((a+b)\,\text{t}\cdots\right.}{4a^2d(a-b)\left((a-b)\tan^4(c+d\cdots\right.}$$

```
[Out] ((6*Sqrt[a] - 5*Sqrt[b])*Sqrt[b]*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*
x])/a^(1/4)])/(8*a^(9/4)*(Sqrt[a] - Sqrt[b])^(3/2)*d) - ((6*Sqrt[a] + 5*Sqr
t[b])*Sqrt[b]*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(8*a^
(9/4)*(Sqrt[a] + Sqrt[b])^(3/2)*d) - Cot[c + d*x]/(a^2*d) - (b*Tan[c + d*x]
*(a + (a + b)*Tan[c + d*x]^2))/(4*a^2*(a - b)*d*(a + 2*a*Tan[c + d*x]^2 + (
a - b)*Tan[c + d*x]^4))
```

Rubi [A] time = 0.534179, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3217, 1334, 1664, 1166, 205}

$$\frac{\sqrt{b}\left(6\sqrt{a}-5\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{9/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\sqrt{b}\left(6\sqrt{a}+5\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{9/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{b\tan(c+dx)\left((a+b)\,\text{t}\cdots\right.}{4a^2d(a-b)\left((a-b)\tan^4(c+d\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] ((6*Sqrt[a] - 5*Sqrt[b])*Sqrt[b]*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*
x])/a^(1/4)])/(8*a^(9/4)*(Sqrt[a] - Sqrt[b])^(3/2)*d) - ((6*Sqrt[a] + 5*Sqr
t[b])*Sqrt[b]*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(8*a^
(9/4)*(Sqrt[a] + Sqrt[b])^(3/2)*d) - Cot[c + d*x]/(a^2*d) - (b*Tan[c + d*x]
*(a + (a + b)*Tan[c + d*x]^2))/(4*a^2*(a - b)*d*(a + 2*a*Tan[c + d*x]^2 + (
a - b)*Tan[c + d*x]^4))
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 1334

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[x^m*(a + b*x^2 + c*x^4)^(p +
1)*Simp[ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d +
e*x^2)^q, a + b*x^2 + c*x^4, x])/x^m + (b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5)
 - a*b*g)/x^m + c*(4*p + 7)*(b*f - 2*a*g)*x^(2 - m), x], x], x], x]] /; Fre
```

```
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1] &
& ILtQ[m/2, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^4}{x^2\left(a+2ax^2+(a-b)x^4\right)^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{b\tan(c+dx)\left(a+(a+b)\tan^2(c+dx)\right)}{4a^2(a-b)d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-8ab-\frac{2a(8a-7b)bx^2}{a-b}-\frac{2b}{\cdots}}{x^2\left(a+2ax^2+(a\cdots\right.}\right.}{8\cdots}$$

$$= -\frac{b\tan(c+dx)\left(a+(a+b)\tan^2(c+dx)\right)}{4a^2(a-b)d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)} - \frac{\text{Subst}\left(\int\left(-\frac{8b}{x^2}+\frac{2b^2\left(-a-(\cdots\right.}{(a-b)\left(a+2a\cdots\right.}\right.\right.}{8\cdots}$$

$$= -\frac{\cot(c+dx)}{a^2d} - \frac{b\tan(c+dx)\left(a+(a+b)\tan^2(c+dx)\right)}{4a^2(a-b)d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)} - \frac{b\,\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= -\frac{\cot(c+dx)}{a^2d} - \frac{b\tan(c+dx)\left(a+(a+b)\tan^2(c+dx)\right)}{4a^2(a-b)d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)} + \frac{\left(\left(7a+\frac{2\sqrt{a}(3\cdots}{\sqrt{\cdots}}\right.\right.}{\cdots}$$

$$= \frac{\left(6\sqrt{a}-5\sqrt{b}\right)\sqrt{b}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{9/4}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}d} - \frac{\left(6\sqrt{a}+5\sqrt{b}\right)\sqrt{b}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{9/4}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}d} \cdots$$

Mathematica [A] time = 2.15961, size = 274, normalized size = 1.16

$$\frac{-\frac{\left(6a\sqrt{b}+5\sqrt{ab}\right)\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{\sqrt{a}\sqrt{b}+a}} - \frac{4\sqrt{ab}\sin(2(c+dx))(2a-b\cos(2(c+dx))+b)}{(a-b)(8a+4b\cos(2(c+dx))-b\cos(4(c+dx))-3b)} - \frac{\left(6a\sqrt{b}-5\sqrt{ab}\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\left(\sqrt{a}-\sqrt{b}\right)\sqrt{\sqrt{a}\sqrt{b}-a}} - 8\sqrt{a}\cot(\cdots}{8a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] (-(((6*a*Sqrt[b] + 5*Sqrt[a]*b)*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/S
qrt[a + Sqrt[a]*Sqrt[b]]])/((Sqrt[a] + Sqrt[b])*Sqrt[a + Sqrt[a]*Sqrt[b]]))
 - ((6*a*Sqrt[b] - 5*Sqrt[a]*b)*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/
Sqrt[-a + Sqrt[a]*Sqrt[b]]])/((Sqrt[a] - Sqrt[b])*Sqrt[-a + Sqrt[a]*Sqrt[b]
]) - 8*Sqrt[a]*Cot[c + d*x] - (4*Sqrt[a]*b*(2*a + b - b*Cos[2*(c + d*x)])*S
in[2*(c + d*x)])/((a - b)*(8*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c +
d*x)])))/(8*a^(5/2)*d)
```

Maple [B] time = 0.154, size = 708, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a-b*sin(d*x+c)^4)^2,x)
```

```
[Out] -1/4/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)/(a-b)/a*tan(d*x+c
)^3*b-1/4/d*b^2/a^2/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)/(a-b
)*tan(d*x+c)^3-1/4/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)*b/a
/(a-b)*tan(d*x+c)+7/8/d/a/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*
tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*b-5/8/d*b^2/a^2/(a-b)/(((a*b)^(1/
2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+3
/4/d/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c
)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*b-1/2/d/a/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+
a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*b^2+
7/8/d/a/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*
b)^(1/2)-a)*(a-b))^(1/2))*b-5/8/d*b^2/a^2/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/
2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-3/4/d/(a*b)^(1/
2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1
/2)-a)*(a-b))^(1/2))*b+1/2/d/a/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1
/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*b^2-1/d/a^2/ta
n(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a-b*sin(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(48*a^2*b - 5*a*b^2 - 25*b^3)*cos(4*d*x + 4*c)*sin(2*d*x + 2*c) + ((
6*a*b^2 - 5*b^3)*sin(8*d*x + 8*c) - 2*(13*a*b^2 - 10*b^3)*sin(6*d*x + 6*c)
- 2*(32*a^2*b - 47*a*b^2 + 15*b^3)*sin(4*d*x + 4*c) - 2*(7*a*b^2 - 10*b^3)*
sin(2*d*x + 2*c))*cos(10*d*x + 10*c) + (2*(48*a^2*b - 5*a*b^2 - 25*b^3)*sin
(6*d*x + 6*c) + 2*(112*a^2*b - 165*a*b^2 + 50*b^3)*sin(4*d*x + 4*c) + 5*(8*
a*b^2 - 15*b^3)*sin(2*d*x + 2*c))*cos(8*d*x + 8*c) + 2*(2*(256*a^3 - 432*a^
2*b + 210*a*b^2 - 25*b^3)*sin(4*d*x + 4*c) + (112*a^2*b - 165*a*b^2 + 50*b^
3)*sin(2*d*x + 2*c))*cos(6*d*x + 6*c) + 2*((a^3*b^2 - a^2*b^3)*d*cos(10*d*x
 + 10*c)^2 + 25*(a^3*b^2 - a^2*b^3)*d*cos(8*d*x + 8*c)^2 + 4*(64*a^5 - 144*
a^4*b + 105*a^3*b^2 - 25*a^2*b^3)*d*cos(6*d*x + 6*c)^2 + 4*(64*a^5 - 144*a^
4*b + 105*a^3*b^2 - 25*a^2*b^3)*d*cos(4*d*x + 4*c)^2 + 25*(a^3*b^2 - a^2*b^
```

3)*d*cos(2*d*x + 2*c)^2 + (a^3*b^2 - a^2*b^3)*d*sin(10*d*x + 10*c)^2 + 25*(
a^3*b^2 - a^2*b^3)*d*sin(8*d*x + 8*c)^2 + 4*(64*a^5 - 144*a^4*b + 105*a^3*b
^2 - 25*a^2*b^3)*d*sin(6*d*x + 6*c)^2 + 4*(64*a^5 - 144*a^4*b + 105*a^3*b^2
 - 25*a^2*b^3)*d*sin(4*d*x + 4*c)^2 + 20*(8*a^4*b - 13*a^3*b^2 + 5*a^2*b^3)
*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 25*(a^3*b^2 - a^2*b^3)*d*sin(2*d*x +
 2*c)^2 - 10*(a^3*b^2 - a^2*b^3)*d*cos(2*d*x + 2*c) + (a^3*b^2 - a^2*b^3)*d
 - 2*(5*(a^3*b^2 - a^2*b^3)*d*cos(8*d*x + 8*c) + 2*(8*a^4*b - 13*a^3*b^2 +
5*a^2*b^3)*d*cos(6*d*x + 6*c) - 2*(8*a^4*b - 13*a^3*b^2 + 5*a^2*b^3)*d*cos(
4*d*x + 4*c) - 5*(a^3*b^2 - a^2*b^3)*d*cos(2*d*x + 2*c) + (a^3*b^2 - a^2*b^
3)*d)*cos(10*d*x + 10*c) + 10*(2*(8*a^4*b - 13*a^3*b^2 + 5*a^2*b^3)*d*cos(6
*d*x + 6*c) - 2*(8*a^4*b - 13*a^3*b^2 + 5*a^2*b^3)*d*cos(4*d*x + 4*c) - 5*(
a^3*b^2 - a^2*b^3)*d*cos(2*d*x + 2*c) + (a^3*b^2 - a^2*b^3)*d)*cos(8*d*x +
8*c) - 4*(2*(64*a^5 - 144*a^4*b + 105*a^3*b^2 - 25*a^2*b^3)*d*cos(4*d*x + 4
c) + 5(8*a^4*b - 13*a^3*b^2 + 5*a^2*b^3)*d*cos(2*d*x + 2*c) - (8*a^4*b -
13*a^3*b^2 + 5*a^2*b^3)*d)*cos(6*d*x + 6*c) + 4*(5*(8*a^4*b - 13*a^3*b^2 +
5*a^2*b^3)*d*cos(2*d*x + 2*c) - (8*a^4*b - 13*a^3*b^2 + 5*a^2*b^3)*d)*cos(4
*d*x + 4*c) - 2*(5*(a^3*b^2 - a^2*b^3)*d*sin(8*d*x + 8*c) + 2*(8*a^4*b - 13
*a^3*b^2 + 5*a^2*b^3)*d*sin(6*d*x + 6*c) - 2*(8*a^4*b - 13*a^3*b^2 + 5*a^2*
b^3)*d*sin(4*d*x + 4*c) - 5*(a^3*b^2 - a^2*b^3)*d*sin(2*d*x + 2*c))*sin(10*
d*x + 10*c) + 10*(2*(8*a^4*b - 13*a^3*b^2 + 5*a^2*b^3)*d*sin(6*d*x + 6*c) -
 2*(8*a^4*b - 13*a^3*b^2 + 5*a^2*b^3)*d*sin(4*d*x + 4*c) - 5*(a^3*b^2 - a^2
*b^3)*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) - 4*(2*(64*a^5 - 144*a^4*b + 105
*a^3*b^2 - 25*a^2*b^3)*d*sin(4*d*x + 4*c) + 5*(8*a^4*b - 13*a^3*b^2 + 5*a^2
*b^3)*d*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*integrate(-(4*(6*a*b^2 - 5*b^3)
*cos(6*d*x + 6*c)^2 - 4*(64*a^2*b - 64*a*b^2 + 15*b^3)*cos(4*d*x + 4*c)^2 +
 4*(6*a*b^2 - 5*b^3)*cos(2*d*x + 2*c)^2 + 4*(6*a*b^2 - 5*b^3)*sin(6*d*x + 6
c)^2 - 4(64*a^2*b - 64*a*b^2 + 15*b^3)*sin(4*d*x + 4*c)^2 + 2*(48*a^2*b -
 90*a*b^2 + 35*b^3)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*(6*a*b^2 - 5*b^3)
*sin(2*d*x + 2*c)^2 - ((6*a*b^2 - 5*b^3)*cos(6*d*x + 6*c) - 2*(8*a*b^2 - 5*
b^3)*cos(4*d*x + 4*c) + (6*a*b^2 - 5*b^3)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c
) - (6*a*b^2 - 5*b^3 - 2*(48*a^2*b - 90*a*b^2 + 35*b^3)*cos(4*d*x + 4*c) -
8*(6*a*b^2 - 5*b^3)*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 2*(8*a*b^2 - 5*b^3
 + (48*a^2*b - 90*a*b^2 + 35*b^3)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - (6*a
*b^2 - 5*b^3)*cos(2*d*x + 2*c) - ((6*a*b^2 - 5*b^3)*sin(6*d*x + 6*c) - 2*(8
*a*b^2 - 5*b^3)*sin(4*d*x + 4*c) + (6*a*b^2 - 5*b^3)*sin(2*d*x + 2*c))*sin(
8*d*x + 8*c) + 2*((48*a^2*b - 90*a*b^2 + 35*b^3)*sin(4*d*x + 4*c) + 4*(6*a*
b^2 - 5*b^3)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))/(a^3*b^2 - a^2*b^3 + (a^3*
b^2 - a^2*b^3)*cos(8*d*x + 8*c)^2 + 16*(a^3*b^2 - a^2*b^3)*cos(6*d*x + 6*c)
^2 + 4*(64*a^5 - 112*a^4*b + 57*a^3*b^2 - 9*a^2*b^3)*cos(4*d*x + 4*c)^2 + 1
6*(a^3*b^2 - a^2*b^3)*cos(2*d*x + 2*c)^2 + (a^3*b^2 - a^2*b^3)*sin(8*d*x +
8*c)^2 + 16*(a^3*b^2 - a^2*b^3)*sin(6*d*x + 6*c)^2 + 4*(64*a^5 - 112*a^4*b
+ 57*a^3*b^2 - 9*a^2*b^3)*sin(4*d*x + 4*c)^2 + 16*(8*a^4*b - 11*a^3*b^2 + 3
*a^2*b^3)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a^3*b^2 - a^2*b^3)*sin(2*
d*x + 2*c)^2 + 2*(a^3*b^2 - a^2*b^3 - 4*(a^3*b^2 - a^2*b^3)*cos(6*d*x + 6*c
) - 2*(8*a^4*b - 11*a^3*b^2 + 3*a^2*b^3)*cos(4*d*x + 4*c) - 4*(a^3*b^2 - a^
2*b^3)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) - 8*(a^3*b^2 - a^2*b^3 - 2*(8*a^4
*b - 11*a^3*b^2 + 3*a^2*b^3)*cos(4*d*x + 4*c) - 4*(a^3*b^2 - a^2*b^3)*cos(2
*d*x + 2*c))*cos(6*d*x + 6*c) - 4*(8*a^4*b - 11*a^3*b^2 + 3*a^2*b^3 - 4*(8*
a^4*b - 11*a^3*b^2 + 3*a^2*b^3)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 8*(a^3
*b^2 - a^2*b^3)*cos(2*d*x + 2*c) - 4*(2*(a^3*b^2 - a^2*b^3)*sin(6*d*x + 6*c
) + (8*a^4*b - 11*a^3*b^2 + 3*a^2*b^3)*sin(4*d*x + 4*c) + 2*(a^3*b^2 - a^2*
b^3)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^4*b - 11*a^3*b^2 + 3*a^2
*b^3)*sin(4*d*x + 4*c) + 2*(a^3*b^2 - a^2*b^3)*sin(2*d*x + 2*c))*sin(6*d*x
+ 6*c)), x) - (4*a*b^2 - 5*b^3 + (6*a*b^2 - 5*b^3)*cos(8*d*x + 8*c) - 2*(13
*a*b^2 - 10*b^3)*cos(6*d*x + 6*c) - 2*(32*a^2*b - 47*a*b^2 + 15*b^3)*cos(4*
d*x + 4*c) - 2*(7*a*b^2 - 10*b^3)*cos(2*d*x + 2*c))*sin(10*d*x + 10*c) + (1
4*a*b^2 - 20*b^3 - 2*(48*a^2*b - 5*a*b^2 - 25*b^3)*cos(6*d*x + 6*c) - 2*(11
2*a^2*b - 165*a*b^2 + 50*b^3)*cos(4*d*x + 4*c) - 5*(8*a*b^2 - 15*b^3)*cos(2
*d*x + 2*c))*sin(8*d*x + 8*c) + 2*(32*a^2*b - 47*a*b^2 + 15*b^3 - 2*(256*a^

```
3 - 432*a^2*b + 210*a*b^2 - 25*b^3)*cos(4*d*x + 4*c) - (112*a^2*b - 165*a*b
^2 + 50*b^3)*cos(2*d*x + 2*c))*sin(6*d*x + 6*c) + 2*(13*a*b^2 - 10*b^3 - (4
8*a^2*b - 5*a*b^2 - 25*b^3)*cos(2*d*x + 2*c))*sin(4*d*x + 4*c) - (6*a*b^2 -
 5*b^3)*sin(2*d*x + 2*c))/((a^3*b^2 - a^2*b^3)*d*cos(10*d*x + 10*c)^2 + 25*
(a^3*b^2 - a^2*b^3)*d*cos(8*d*x + 8*c)^2 + 4*(64*a^5 - 144*a^4*b + 105*a^3*
b^2 - 25*a^2*b^3)*d*cos(6*d*x + 6*c)^2 + 4*(64*a^5 - 144*a^4*b + 105*a^3*b^
2 - 25*a^2*b^3)*d*cos(4*d*x + 4*c)^2 + 25*(a^3*b^2 - a^2*b^3)*d*cos(2*d*x +
 2*c)^2 + (a^3*b^2 - a^2*b^3)*d*sin(10*d*x + 10*c)^2 + 25*(a^3*b^2 - a^2*b^
3)*d*sin(8*d*x + 8*c)^2 + 4*(64*a^5 - 144*a^4*b + 105*a^3*b^2 - 25*a^2*b^3)
*d*sin(6*d*x + 6*c)^2 + 4*(64*a^5 - 144*a^4*b + 105*a^3*b^2 - 25*a^2*b^3)*d
*sin(4*d*x + 4*c)^2 + 20*(8*a^4*b - 13*a^3*b^2 + 5*a^2*b^3)*d*sin(4*d*x + 4
*c)*sin(2*d*x + 2*c) + 25*(a^3*b^2 - a^2*b^3)*d*sin(2*d*x + 2*c)^2 - 10*(a^
3*b^2 - a^2*b^3)*d*cos(2*d*x + 2*c) + (a^3*b^2 - a^2*b^3)*d - 2*(5*(a^3*b^2
 - a^2*b^3)*d*cos(8*d*x + 8*c) + 2*(8*a^4*b - 13*a^3*b^2 + 5*a^2*b^3)*d*cos
(6*d*x + 6*c) - 2*(8*a^4*b - 13*a^3*b^2 + 5*a^2*b^3)*d*cos(4*d*x + 4*c) - 5
*(a^3*b^2 - a^2*b^3)*d*cos(2*d*x + 2*c) + (a^3*b^2 - a^2*b^3)*d)*cos(10*d*x
 + 10*c) + 10*(2*(8*a^4*b - 13*a^3*b^2 + 5*a^2*b^3)*d*cos(6*d*x + 6*c) - 2*
(8*a^4*b - 13*a^3*b^2 + 5*a^2*b^3)*d*cos(4*d*x + 4*c) - 5*(a^3*b^2 - a^2*b^
3)*d*cos(2*d*x + 2*c) + (a^3*b^2 - a^2*b^3)*d)*cos(8*d*x + 8*c) - 4*(2*(64*
a^5 - 144*a^4*b + 105*a^3*b^2 - 25*a^2*b^3)*d*cos(4*d*x + 4*c) + 5*(8*a^4*b
 - 13*a^3*b^2 + 5*a^2*b^3)*d*cos(2*d*x + 2*c) - (8*a^4*b - 13*a^3*b^2 + 5*a
^2*b^3)*d)*cos(6*d*x + 6*c) + 4*(5*(8*a^4*b - 13*a^3*b^2 + 5*a^2*b^3)*d*cos
(2*d*x + 2*c) - (8*a^4*b - 13*a^3*b^2 + 5*a^2*b^3)*d)*cos(4*d*x + 4*c) - 2*
(5*(a^3*b^2 - a^2*b^3)*d*sin(8*d*x + 8*c) + 2*(8*a^4*b - 13*a^3*b^2 + 5*a^2
*b^3)*d*sin(6*d*x + 6*c) - 2*(8*a^4*b - 13*a^3*b^2 + 5*a^2*b^3)*d*sin(4*d*x
 + 4*c) - 5*(a^3*b^2 - a^2*b^3)*d*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 10
*(2*(8*a^4*b - 13*a^3*b^2 + 5*a^2*b^3)*d*sin(6*d*x + 6*c) - 2*(8*a^4*b - 13
*a^3*b^2 + 5*a^2*b^3)*d*sin(4*d*x + 4*c) - 5*(a^3*b^2 - a^2*b^3)*d*sin(2*d*
x + 2*c))*sin(8*d*x + 8*c) - 4*(2*(64*a^5 - 144*a^4*b + 105*a^3*b^2 - 25*a^
2*b^3)*d*sin(4*d*x + 4*c) + 5*(8*a^4*b - 13*a^3*b^2 + 5*a^2*b^3)*d*sin(2*d*
x + 2*c))*sin(6*d*x + 6*c))
```

Fricas [B] time = 10.5434, size = 8411, normalized size = 35.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a-b*sin(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/32*(8*(4*a*b - 5*b^2)*cos(d*x + c)^5 - 8*(7*a*b - 10*b^2)*cos(d*x + c)^3
 - ((a^3*b - a^2*b^2)*d*cos(d*x + c)^4 - 2*(a^3*b - a^2*b^2)*d*cos(d*x + c)
^2 - (a^4 - 2*a^3*b + a^2*b^2)*d)*sqrt(-((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b
^3)*d^2*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b^6 + 625
*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 - 6*a^10*
b^5 + a^9*b^6)*d^4)) + 36*a^2*b - 47*a*b^2 + 15*b^3)/((a^7 - 3*a^6*b + 3*a^
5*b^2 - a^4*b^3)*d^2))*log(432*a^3*b^2 - 921*a^2*b^3 + 2625/4*a*b^4 - 625/4
*b^5 - 1/4*(1728*a^3*b^2 - 3684*a^2*b^3 + 2625*a*b^4 - 625*b^5)*cos(d*x + c
)^2 + 1/2*((7*a^11 - 26*a^10*b + 36*a^9*b^2 - 22*a^8*b^3 + 5*a^7*b^4)*d^3*s
qrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b^6 + 625*b^7)/((a
^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 - 6*a^10*b^5 + a^9
*b^6)*d^4))*cos(d*x + c)*sin(d*x + c) - 2*(144*a^6*b - 303*a^5*b^2 + 213*a^
4*b^3 - 50*a^3*b^4)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^7 - 3*a^6*b + 3*
a^5*b^2 - a^4*b^3)*d^2*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3
450*a*b^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^1
1*b^4 - 6*a^10*b^5 + a^9*b^6)*d^4)) + 36*a^2*b - 47*a*b^2 + 15*b^3)/((a^7 -
 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2)) + 1/4*(2*(36*a^9 - 133*a^8*b + 183*a^
```

7*b^2 - 111*a^6*b^3 + 25*a^5*b^4)*d^2*cos(d*x + c)^2 - (36*a^9 - 133*a^8*b + 183*a^7*b^2 - 111*a^6*b^3 + 25*a^5*b^4)*d^2)*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 - 6*a^10*b^5 + a^9*b^6)*d^4)))*sin(d*x + c) + ((a^3*b - a^2*b^2)*d*cos(d*x + c)^4 - 2*(a^3*b - a^2*b^2)*d*cos(d*x + c)^2 - (a^4 - 2*a^3*b + a^2*b^2)*d)*sqrt(-((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 - 6*a^10*b^5 + a^9*b^6)*d^4)) + 36*a^2*b - 47*a*b^2 + 15*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2))*log(432*a^3*b^2 - 921*a^2*b^3 + 2625/4*a*b^4 - 625/4*b^5 - 1/4*(1728*a^3*b^2 - 3684*a^2*b^3 + 2625*a*b^4 - 625*b^5)*cos(d*x + c)^2 - 1/2*((7*a^11 - 26*a^10*b + 36*a^9*b^2 - 22*a^8*b^3 + 5*a^7*b^4)*d^3*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 - 6*a^10*b^5 + a^9*b^6)*d^4))*cos(d*x + c)*sin(d*x + c) - 2*(144*a^6*b - 303*a^5*b^2 + 213*a^4*b^3 - 50*a^3*b^4)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 - 6*a^10*b^5 + a^9*b^6)*d^4)) + 36*a^2*b - 47*a*b^2 + 15*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2)) + 1/4*(2*(36*a^9 - 133*a^8*b + 183*a^7*b^2 - 111*a^6*b^3 + 25*a^5*b^4)*d^2*cos(d*x + c)^2 - (36*a^9 - 133*a^8*b + 183*a^7*b^2 - 111*a^6*b^3 + 25*a^5*b^4)*d^2)*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 - 6*a^10*b^5 + a^9*b^6)*d^4)))*sin(d*x + c) - ((a^3*b - a^2*b^2)*d*cos(d*x + c)^4 - 2*(a^3*b - a^2*b^2)*d*cos(d*x + c)^2 - (a^4 - 2*a^3*b + a^2*b^2)*d)*sqrt(((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 - 6*a^10*b^5 + a^9*b^6)*d^4)) - 36*a^2*b + 47*a*b^2 - 15*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2))*log(-432*a^3*b^2 + 921*a^2*b^3 - 2625/4*a*b^4 + 625/4*b^5 + 1/4*(1728*a^3*b^2 - 3684*a^2*b^3 + 2625*a*b^4 - 625*b^5)*cos(d*x + c)^2 + 1/2*((7*a^11 - 26*a^10*b + 36*a^9*b^2 - 22*a^8*b^3 + 5*a^7*b^4)*d^3*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 - 6*a^10*b^5 + a^9*b^6)*d^4))*cos(d*x + c)*sin(d*x + c) + 2*(144*a^6*b - 303*a^5*b^2 + 213*a^4*b^3 - 50*a^3*b^4)*d*cos(d*x + c)*sin(d*x + c))*sqrt(((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 - 6*a^10*b^5 + a^9*b^6)*d^4)) - 36*a^2*b + 47*a*b^2 - 15*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2)) + 1/4*(2*(36*a^9 - 133*a^8*b + 183*a^7*b^2 - 111*a^6*b^3 + 25*a^5*b^4)*d^2*cos(d*x + c)^2 - (36*a^9 - 133*a^8*b + 183*a^7*b^2 - 111*a^6*b^3 + 25*a^5*b^4)*d^2)*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 - 6*a^10*b^5 + a^9*b^6)*d^4)))*sin(d*x + c) + ((a^3*b - a^2*b^2)*d*cos(d*x + c)^4 - 2*(a^3*b - a^2*b^2)*d*cos(d*x + c)^2 - (a^4 - 2*a^3*b + a^2*b^2)*d)*sqrt(((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 - 6*a^10*b^5 + a^9*b^6)*d^4)) - 36*a^2*b + 47*a*b^2 - 15*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2))*log(-432*a^3*b^2 + 921*a^2*b^3 - 2625/4*a*b^4 + 625/4*b^5 + 1/4*(1728*a^3*b^2 - 3684*a^2*b^3 + 2625*a*b^4 - 625*b^5)*cos(d*x + c)^2 - 1/2*((7*a^11 - 26*a^10*b + 36*a^9*b^2 - 22*a^8*b^3 + 5*a^7*b^4)*d^3*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 - 6*a^10*b^5 + a^9*b^6)*d^4))*cos(d*x + c)*sin(d*x + c) + 2*(144*a^6*b - 303*a^5*b^2 + 213*a^4*b^3 - 50*a^3*b^4)*d*cos(d*x + c)*sin(d*x + c))*sqrt(((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 - 6*a^10*b^5 + a^9*b^6)*d^4)) - 36*a^2*b + 47*a*b^2 - 15*b^3)/((a^7 - 3*a^6*b

```
 + 3*a^5*b^2 - a^4*b^3)*d^2)) + 1/4*(2*(36*a^9 - 133*a^8*b + 183*a^7*b^2 -
111*a^6*b^3 + 25*a^5*b^4)*d^2*cos(d*x + c)^2 - (36*a^9 - 133*a^8*b + 183*a^
7*b^2 - 111*a^6*b^3 + 25*a^5*b^4)*d^2)*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 +
7161*a^2*b^5 - 3450*a*b^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a
^12*b^3 + 15*a^11*b^4 - 6*a^10*b^5 + a^9*b^6)*d^4)))*sin(d*x + c) - 8*(4*a^
2 - 7*a*b + 5*b^2)*cos(d*x + c))/(((a^3*b - a^2*b^2)*d*cos(d*x + c)^4 - 2*(
a^3*b - a^2*b^2)*d*cos(d*x + c)^2 - (a^4 - 2*a^3*b + a^2*b^2)*d)*sin(d*x +
c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a-b*sin(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a-b*sin(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.224 $\int \frac{\sin^9(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=315

$$\frac{\cos(c+dx)\left(9a^2-2b(2a-5b)\cos^2(c+dx)-11ab-10b^2\right)}{32b^2d(a-b)^2\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}-\frac{a\cos(c+dx)\left(a-b\cos^2(c+dx)+b\right)}{8b^2d(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)^2}$$

```
[Out] -((5*a - 14*Sqrt[a]*Sqrt[b] + 12*b)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt
[a] - Sqrt[b]]])/(64*Sqrt[a]*(Sqrt[a] - Sqrt[b])^(5/2)*b^(9/4)*d) - ((5*a +
 14*Sqrt[a]*Sqrt[b] + 12*b)*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + S
qrt[b]]])/(64*Sqrt[a]*(Sqrt[a] + Sqrt[b])^(5/2)*b^(9/4)*d) - (a*Cos[c + d*x
]*(a + b - b*Cos[c + d*x]^2))/(8*(a - b)*b^2*d*(a - b + 2*b*Cos[c + d*x]^2
- b*Cos[c + d*x]^4)^2) + (Cos[c + d*x]*(9*a^2 - 11*a*b - 10*b^2 - 2*(2*a -
5*b)*b*Cos[c + d*x]^2))/(32*(a - b)^2*b^2*d*(a - b + 2*b*Cos[c + d*x]^2 - b
*Cos[c + d*x]^4))
```

Rubi [A] time = 0.574759, antiderivative size = 315, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3215, 1205, 1678, 1166, 205, 208}

$$\frac{\cos(c+dx)\left(9a^2-2b(2a-5b)\cos^2(c+dx)-11ab-10b^2\right)}{32b^2d(a-b)^2\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}-\frac{a\cos(c+dx)\left(a-b\cos^2(c+dx)+b\right)}{8b^2d(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^9/(a - b*Sin[c + d*x]^4)^3,x]
```


```
[Out] -((5*a - 14*Sqrt[a]*Sqrt[b] + 12*b)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt
[a] - Sqrt[b]]])/(64*Sqrt[a]*(Sqrt[a] - Sqrt[b])^(5/2)*b^(9/4)*d) - ((5*a +
 14*Sqrt[a]*Sqrt[b] + 12*b)*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + S
qrt[b]]])/(64*Sqrt[a]*(Sqrt[a] + Sqrt[b])^(5/2)*b^(9/4)*d) - (a*Cos[c + d*x
]*(a + b - b*Cos[c + d*x]^2))/(8*(a - b)*b^2*d*(a - b + 2*b*Cos[c + d*x]^2
- b*Cos[c + d*x]^4)^2) + (Cos[c + d*x]*(9*a^2 - 11*a*b - 10*b^2 - 2*(2*a -
5*b)*b*Cos[c + d*x]^2))/(32*(a - b)^2*b^2*d*(a - b + 2*b*Cos[c + d*x]^2 - b
*Cos[c + d*x]^4))
```


Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
```

```
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sin^9(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^4}{\left(a-b+2bx^2-bx^4\right)^3}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{a\cos(c+dx)\left(a+b-b\cos^2(c+dx)\right)}{8(a-b)b^2d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{\frac{2a\left(a^2+ab-8b^2\right)}{b}-2a(11a-\ldots}{\left(a-b+2bx^2\ldots\right.}\ldots\right)}{16a\ldots}$$

$$= -\frac{a\cos(c+dx)\left(a+b-b\cos^2(c+dx)\right)}{8(a-b)b^2d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)^2} + \frac{\cos(c+dx)\left(9a^2-11ab-10\ldots\right.}{32(a-b)^2b^2d\left(a-b+2b\,c\ldots\right.}$$

$$= -\frac{a\cos(c+dx)\left(a+b-b\cos^2(c+dx)\right)}{8(a-b)b^2d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)^2} + \frac{\cos(c+dx)\left(9a^2-11ab-10\ldots\right.}{32(a-b)^2b^2d\left(a-b+2b\,c\ldots\right.}$$

$$= -\frac{\left(5a-14\sqrt{a}\sqrt{b}+12b\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{9/4}d} - \frac{\left(5a+14\sqrt{a}\sqrt{b}+12b\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx\ldots}{\sqrt{\sqrt{a}+\sqrt{b}}}\right.}{64\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}b^{9/4}d}$$

Mathematica [C] time = 1.49247, size = 785, normalized size = 2.49

$$i\text{RootSum}\left[-16\#1^4a+\#1^8b-4\#1^6b+6\#1^4b-4\#1^2b+b\&, \frac{-10i\#1^4a^2\log\left(\#1^2-2\#1\cos(c+dx)+1\right)+10i\#1^2a^2\log\left(\#1^2-2\#1\cos(c+dx)\ldots\right.}{\ldots}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]^9/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] ((-32*Cos[c + d*x]*(-9*a^2 + 13*a*b + 5*b^2 + (2*a - 5*b)*b*Cos[2*(c + d*x)
]))/(8*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*x)]) - (512*a*(a - b
)*Cos[c + d*x]*(2*a + b - b*Cos[2*(c + d*x)]))/(-8*a + 3*b - 4*b*Cos[2*(c +
 d*x)] + b*Cos[4*(c + d*x)])^2 + I*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#
1^4 - 4*b*#1^6 + b*#1^8 & , (-4*a*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)
] + 10*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + (2*I)*a*b*Log[1 - 2*C
os[c + d*x]*#1 + #1^2] - (5*I)*b^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] - 20*a
^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 + 56*a*b*ArcTan[Sin[c + d*
x]/(Cos[c + d*x] - #1)]*#1^2 - 78*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #
1)]*#1^2 + (10*I)*a^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (28*I)*a*b*L
og[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + (39*I)*b^2*Log[1 - 2*Cos[c + d*x]*#
1 + #1^2]*#1^2 + 20*a^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - 56*
a*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 + 78*b^2*ArcTan[Sin[c + d
*x]/(Cos[c + d*x] - #1)]*#1^4 - (10*I)*a^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2
]*#1^4 + (28*I)*a*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 - (39*I)*b^2*Log
[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 + 4*a*b*ArcTan[Sin[c + d*x]/(Cos[c + d*
x] - #1)]*#1^6 - 10*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 - (2*
I)*a*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6 + (5*I)*b^2*Log[1 - 2*Cos[c +
 d*x]*#1 + #1^2]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7)
& ])/(128*(a - b)^2*b^2*d)
```

Maple [B] time = 0.145, size = 1164, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^9/(a-b*sin(d*x+c)^4)^3,x)
```

```
[Out] 1/8/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2/(a^2-2*a*b+b^2)*cos(d*x+c)^7*
a-5/16/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2/(a^2-2*a*b+b^2)*cos(d*x+c)
^7*b-9/32/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2/b/(a^2-2*a*b+b^2)*cos(d
*x+c)^5*a^2+3/32/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2/(a^2-2*a*b+b^2)*
cos(d*x+c)^5*a+15/16/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2*b/(a^2-2*a*b
+b^2)*cos(d*x+c)^5+9/16/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2/b/(a^2-2*
a*b+b^2)*cos(d*x+c)^3*a^2-3/8/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2/(a^
2-2*a*b+b^2)*cos(d*x+c)^3*a-15/16/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2
*b/(a^2-2*a*b+b^2)*cos(d*x+c)^3+5/32/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b
)^2/b^2/(a-b)*cos(d*x+c)*a^2-15/32/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^
2/b/(a-b)*cos(d*x+c)*a-5/16/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2/(a-b)
*cos(d*x+c)+1/16/d/(a^2-2*a*b+b^2)/b/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d
*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))*a-5/32/d/(a^2-2*a*b+b^2)/(((a*b)^(1/2)-b
)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))-5/64/d/(a^2-2*a*b
+b^2)/b/(a*b)^(1/2)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(
1/2)-b)*b)^(1/2))*a^2+11/64/d/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(((a*b)^(1/2)-b)*
b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))*a-3/16/d/(a^2-2*a*b
+b^2)*b/(a*b)^(1/2)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(
1/2)-b)*b)^(1/2))-1/16/d/(a^2-2*a*b+b^2)/b/(((a*b)^(1/2)+b)*b)^(1/2)*arctan
h(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))*a+5/32/d/(a^2-2*a*b+b^2)/(((a*b)^
(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))-5/64/d/(a
^2-2*a*b+b^2)/b/(a*b)^(1/2)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/
(((a*b)^(1/2)+b)*b)^(1/2))*a^2+11/64/d/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(((a*b)^
(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))*a-3/16/d/
(a^2-2*a*b+b^2)*b/(a*b)^(1/2)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*
b/(((a*b)^(1/2)+b)*b)^(1/2))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^9/(a-b*sin(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 10.5783, size = 10985, normalized size = 34.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^9/(a-b*sin(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/128*(8*(2*a*b^2 - 5*b^3)*cos(d*x + c)^7 - 12*(3*a^2*b - a*b^2 - 10*b^3)*c
os(d*x + c)^5 + 24*(3*a^2*b - 2*a*b^2 - 5*b^3)*cos(d*x + c)^3 + ((a^2*b^4 -
 2*a*b^5 + b^6)*d*cos(d*x + c)^8 - 4*(a^2*b^4 - 2*a*b^5 + b^6)*d*cos(d*x +
c)^6 - 2*(a^3*b^3 - 5*a^2*b^4 + 7*a*b^5 - 3*b^6)*d*cos(d*x + c)^4 + 4*(a^3*
b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d*cos(d*x + c)^2 + (a^4*b^2 - 4*a^3*b^3 +
```

6*a^2*b^4 - 4*a*b^5 + b^6)*d)*sqrt((15*a^4 - 94*a^3*b + 155*a^2*b^2 - 76*a*
b^3 - 144*b^4 + (a^6*b^4 - 5*a^5*b^5 + 10*a^4*b^6 - 10*a^3*b^7 + 5*a^2*b^8
- a*b^9)*d^2*sqrt((625*a^8 - 6700*a^7*b + 35406*a^6*b^2 - 117532*a^5*b^3 +
269641*a^4*b^4 - 437952*a^3*b^5 + 498432*a^2*b^6 - 368640*a*b^7 + 147456*b^
8)/((a^11*b^9 - 10*a^10*b^10 + 45*a^9*b^11 - 120*a^8*b^12 + 210*a^7*b^13 -
252*a^6*b^14 + 210*a^5*b^15 - 120*a^4*b^16 + 45*a^3*b^17 - 10*a^2*b^18 + a*
b^19)*d^4)))/((a^6*b^4 - 5*a^5*b^5 + 10*a^4*b^6 - 10*a^3*b^7 + 5*a^2*b^8 -
a*b^9)*d^2))*log((625*a^6 - 5250*a^5*b + 22509*a^4*b^2 - 57820*a^3*b^3 + 96
336*a^2*b^4 - 98304*a*b^5 + 55296*b^6)*cos(d*x + c) - ((a^8*b^7 - 6*a^7*b^8
 + 27*a^6*b^9 - 80*a^5*b^10 + 135*a^4*b^11 - 126*a^3*b^12 + 61*a^2*b^13 - 1
2*a*b^14)*d^3*sqrt((625*a^8 - 6700*a^7*b + 35406*a^6*b^2 - 117532*a^5*b^3 +
 269641*a^4*b^4 - 437952*a^3*b^5 + 498432*a^2*b^6 - 368640*a*b^7 + 147456*b
^8)/((a^11*b^9 - 10*a^10*b^10 + 45*a^9*b^11 - 120*a^8*b^12 + 210*a^7*b^13 -
 252*a^6*b^14 + 210*a^5*b^15 - 120*a^4*b^16 + 45*a^3*b^17 - 10*a^2*b^18 + a
*b^19)*d^4)) + (125*a^7*b^2 - 1045*a^6*b^3 + 4305*a^5*b^4 - 10583*a^4*b^5 +
 16798*a^3*b^6 - 16320*a^2*b^7 + 8448*a*b^8)*d)*sqrt((15*a^4 - 94*a^3*b + 1
55*a^2*b^2 - 76*a*b^3 - 144*b^4 + (a^6*b^4 - 5*a^5*b^5 + 10*a^4*b^6 - 10*a^
3*b^7 + 5*a^2*b^8 - a*b^9)*d^2*sqrt((625*a^8 - 6700*a^7*b + 35406*a^6*b^2 -
 117532*a^5*b^3 + 269641*a^4*b^4 - 437952*a^3*b^5 + 498432*a^2*b^6 - 368640
*a*b^7 + 147456*b^8)/((a^11*b^9 - 10*a^10*b^10 + 45*a^9*b^11 - 120*a^8*b^12
 + 210*a^7*b^13 - 252*a^6*b^14 + 210*a^5*b^15 - 120*a^4*b^16 + 45*a^3*b^17
- 10*a^2*b^18 + a*b^19)*d^4)))/((a^6*b^4 - 5*a^5*b^5 + 10*a^4*b^6 - 10*a^3*
b^7 + 5*a^2*b^8 - a*b^9)*d^2))) - ((a^2*b^4 - 2*a*b^5 + b^6)*d*cos(d*x + c)
^8 - 4*(a^2*b^4 - 2*a*b^5 + b^6)*d*cos(d*x + c)^6 - 2*(a^3*b^3 - 5*a^2*b^4
+ 7*a*b^5 - 3*b^6)*d*cos(d*x + c)^4 + 4*(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^
6)*d*cos(d*x + c)^2 + (a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*d)*
sqrt((15*a^4 - 94*a^3*b + 155*a^2*b^2 - 76*a*b^3 - 144*b^4 - (a^6*b^4 - 5*a
^5*b^5 + 10*a^4*b^6 - 10*a^3*b^7 + 5*a^2*b^8 - a*b^9)*d^2*sqrt((625*a^8 - 6
700*a^7*b + 35406*a^6*b^2 - 117532*a^5*b^3 + 269641*a^4*b^4 - 437952*a^3*b^
5 + 498432*a^2*b^6 - 368640*a*b^7 + 147456*b^8)/((a^11*b^9 - 10*a^10*b^10 +
 45*a^9*b^11 - 120*a^8*b^12 + 210*a^7*b^13 - 252*a^6*b^14 + 210*a^5*b^15 -
120*a^4*b^16 + 45*a^3*b^17 - 10*a^2*b^18 + a*b^19)*d^4)))/((a^6*b^4 - 5*a^5
*b^5 + 10*a^4*b^6 - 10*a^3*b^7 + 5*a^2*b^8 - a*b^9)*d^2))*log((625*a^6 - 52
50*a^5*b + 22509*a^4*b^2 - 57820*a^3*b^3 + 96336*a^2*b^4 - 98304*a*b^5 + 55
296*b^6)*cos(d*x + c) - ((a^8*b^7 - 6*a^7*b^8 + 27*a^6*b^9 - 80*a^5*b^10 +
135*a^4*b^11 - 126*a^3*b^12 + 61*a^2*b^13 - 12*a*b^14)*d^3*sqrt((625*a^8 -
6700*a^7*b + 35406*a^6*b^2 - 117532*a^5*b^3 + 269641*a^4*b^4 - 437952*a^3*b
^5 + 498432*a^2*b^6 - 368640*a*b^7 + 147456*b^8)/((a^11*b^9 - 10*a^10*b^10
+ 45*a^9*b^11 - 120*a^8*b^12 + 210*a^7*b^13 - 252*a^6*b^14 + 210*a^5*b^15 -
 120*a^4*b^16 + 45*a^3*b^17 - 10*a^2*b^18 + a*b^19)*d^4)) - (125*a^7*b^2 -
1045*a^6*b^3 + 4305*a^5*b^4 - 10583*a^4*b^5 + 16798*a^3*b^6 - 16320*a^2*b^7
 + 8448*a*b^8)*d)*sqrt((15*a^4 - 94*a^3*b + 155*a^2*b^2 - 76*a*b^3 - 144*b^
4 - (a^6*b^4 - 5*a^5*b^5 + 10*a^4*b^6 - 10*a^3*b^7 + 5*a^2*b^8 - a*b^9)*d^2
*sqrt((625*a^8 - 6700*a^7*b + 35406*a^6*b^2 - 117532*a^5*b^3 + 269641*a^4*b
^4 - 437952*a^3*b^5 + 498432*a^2*b^6 - 368640*a*b^7 + 147456*b^8)/((a^11*b^
9 - 10*a^10*b^10 + 45*a^9*b^11 - 120*a^8*b^12 + 210*a^7*b^13 - 252*a^6*b^14
 + 210*a^5*b^15 - 120*a^4*b^16 + 45*a^3*b^17 - 10*a^2*b^18 + a*b^19)*d^4)))
/((a^6*b^4 - 5*a^5*b^5 + 10*a^4*b^6 - 10*a^3*b^7 + 5*a^2*b^8 - a*b^9)*d^2))
) - ((a^2*b^4 - 2*a*b^5 + b^6)*d*cos(d*x + c)^8 - 4*(a^2*b^4 - 2*a*b^5 + b^
6)*d*cos(d*x + c)^6 - 2*(a^3*b^3 - 5*a^2*b^4 + 7*a*b^5 - 3*b^6)*d*cos(d*x +
 c)^4 + 4*(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d*cos(d*x + c)^2 + (a^4*b^2
 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*d)*sqrt((15*a^4 - 94*a^3*b + 155*
a^2*b^2 - 76*a*b^3 - 144*b^4 + (a^6*b^4 - 5*a^5*b^5 + 10*a^4*b^6 - 10*a^3*b
^7 + 5*a^2*b^8 - a*b^9)*d^2*sqrt((625*a^8 - 6700*a^7*b + 35406*a^6*b^2 - 11
7532*a^5*b^3 + 269641*a^4*b^4 - 437952*a^3*b^5 + 498432*a^2*b^6 - 368640*a*
b^7 + 147456*b^8)/((a^11*b^9 - 10*a^10*b^10 + 45*a^9*b^11 - 120*a^8*b^12 +
210*a^7*b^13 - 252*a^6*b^14 + 210*a^5*b^15 - 120*a^4*b^16 + 45*a^3*b^17 - 1
0*a^2*b^18 + a*b^19)*d^4)))/((a^6*b^4 - 5*a^5*b^5 + 10*a^4*b^6 - 10*a^3*b^7
 + 5*a^2*b^8 - a*b^9)*d^2))*log(-(625*a^6 - 5250*a^5*b + 22509*a^4*b^2 - 57

```
820*a^3*b^3 + 96336*a^2*b^4 - 98304*a*b^5 + 55296*b^6)*cos(d*x + c) - ((a^8
*b^7 - 6*a^7*b^8 + 27*a^6*b^9 - 80*a^5*b^10 + 135*a^4*b^11 - 126*a^3*b^12 +
 61*a^2*b^13 - 12*a*b^14)*d^3*sqrt((625*a^8 - 6700*a^7*b + 35406*a^6*b^2 -
117532*a^5*b^3 + 269641*a^4*b^4 - 437952*a^3*b^5 + 498432*a^2*b^6 - 368640*
a*b^7 + 147456*b^8)/((a^11*b^9 - 10*a^10*b^10 + 45*a^9*b^11 - 120*a^8*b^12
+ 210*a^7*b^13 - 252*a^6*b^14 + 210*a^5*b^15 - 120*a^4*b^16 + 45*a^3*b^17 -
 10*a^2*b^18 + a*b^19)*d^4)) + (125*a^7*b^2 - 1045*a^6*b^3 + 4305*a^5*b^4 -
 10583*a^4*b^5 + 16798*a^3*b^6 - 16320*a^2*b^7 + 8448*a*b^8)*d)*sqrt((15*a^
4 - 94*a^3*b + 155*a^2*b^2 - 76*a*b^3 - 144*b^4 + (a^6*b^4 - 5*a^5*b^5 + 10
*a^4*b^6 - 10*a^3*b^7 + 5*a^2*b^8 - a*b^9)*d^2*sqrt((625*a^8 - 6700*a^7*b +
 35406*a^6*b^2 - 117532*a^5*b^3 + 269641*a^4*b^4 - 437952*a^3*b^5 + 498432*
a^2*b^6 - 368640*a*b^7 + 147456*b^8)/((a^11*b^9 - 10*a^10*b^10 + 45*a^9*b^1
1 - 120*a^8*b^12 + 210*a^7*b^13 - 252*a^6*b^14 + 210*a^5*b^15 - 120*a^4*b^1
6 + 45*a^3*b^17 - 10*a^2*b^18 + a*b^19)*d^4)))/((a^6*b^4 - 5*a^5*b^5 + 10*a
^4*b^6 - 10*a^3*b^7 + 5*a^2*b^8 - a*b^9)*d^2))) + ((a^2*b^4 - 2*a*b^5 + b^6
)*d*cos(d*x + c)^8 - 4*(a^2*b^4 - 2*a*b^5 + b^6)*d*cos(d*x + c)^6 - 2*(a^3*
b^3 - 5*a^2*b^4 + 7*a*b^5 - 3*b^6)*d*cos(d*x + c)^4 + 4*(a^3*b^3 - 3*a^2*b^
4 + 3*a*b^5 - b^6)*d*cos(d*x + c)^2 + (a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*
a*b^5 + b^6)*d)*sqrt((15*a^4 - 94*a^3*b + 155*a^2*b^2 - 76*a*b^3 - 144*b^4
- (a^6*b^4 - 5*a^5*b^5 + 10*a^4*b^6 - 10*a^3*b^7 + 5*a^2*b^8 - a*b^9)*d^2*s
qrt((625*a^8 - 6700*a^7*b + 35406*a^6*b^2 - 117532*a^5*b^3 + 269641*a^4*b^4
 - 437952*a^3*b^5 + 498432*a^2*b^6 - 368640*a*b^7 + 147456*b^8)/((a^11*b^9
- 10*a^10*b^10 + 45*a^9*b^11 - 120*a^8*b^12 + 210*a^7*b^13 - 252*a^6*b^14 +
 210*a^5*b^15 - 120*a^4*b^16 + 45*a^3*b^17 - 10*a^2*b^18 + a*b^19)*d^4)))/(
(a^6*b^4 - 5*a^5*b^5 + 10*a^4*b^6 - 10*a^3*b^7 + 5*a^2*b^8 - a*b^9)*d^2))*l
og(-(625*a^6 - 5250*a^5*b + 22509*a^4*b^2 - 57820*a^3*b^3 + 96336*a^2*b^4 -
 98304*a*b^5 + 55296*b^6)*cos(d*x + c) - ((a^8*b^7 - 6*a^7*b^8 + 27*a^6*b^9
 - 80*a^5*b^10 + 135*a^4*b^11 - 126*a^3*b^12 + 61*a^2*b^13 - 12*a*b^14)*d^3
*sqrt((625*a^8 - 6700*a^7*b + 35406*a^6*b^2 - 117532*a^5*b^3 + 269641*a^4*b
^4 - 437952*a^3*b^5 + 498432*a^2*b^6 - 368640*a*b^7 + 147456*b^8)/((a^11*b^
9 - 10*a^10*b^10 + 45*a^9*b^11 - 120*a^8*b^12 + 210*a^7*b^13 - 252*a^6*b^14
 + 210*a^5*b^15 - 120*a^4*b^16 + 45*a^3*b^17 - 10*a^2*b^18 + a*b^19)*d^4))
- (125*a^7*b^2 - 1045*a^6*b^3 + 4305*a^5*b^4 - 10583*a^4*b^5 + 16798*a^3*b^
6 - 16320*a^2*b^7 + 8448*a*b^8)*d)*sqrt((15*a^4 - 94*a^3*b + 155*a^2*b^2 -
76*a*b^3 - 144*b^4 - (a^6*b^4 - 5*a^5*b^5 + 10*a^4*b^6 - 10*a^3*b^7 + 5*a^2
*b^8 - a*b^9)*d^2*sqrt((625*a^8 - 6700*a^7*b + 35406*a^6*b^2 - 117532*a^5*b
^3 + 269641*a^4*b^4 - 437952*a^3*b^5 + 498432*a^2*b^6 - 368640*a*b^7 + 1474
56*b^8)/((a^11*b^9 - 10*a^10*b^10 + 45*a^9*b^11 - 120*a^8*b^12 + 210*a^7*b^
13 - 252*a^6*b^14 + 210*a^5*b^15 - 120*a^4*b^16 + 45*a^3*b^17 - 10*a^2*b^18
 + a*b^19)*d^4)))/((a^6*b^4 - 5*a^5*b^5 + 10*a^4*b^6 - 10*a^3*b^7 + 5*a^2*b
^8 - a*b^9)*d^2))) + 20*(a^3 - 4*a^2*b + a*b^2 + 2*b^3)*cos(d*x + c))/((a^2
*b^4 - 2*a*b^5 + b^6)*d*cos(d*x + c)^8 - 4*(a^2*b^4 - 2*a*b^5 + b^6)*d*cos(
d*x + c)^6 - 2*(a^3*b^3 - 5*a^2*b^4 + 7*a*b^5 - 3*b^6)*d*cos(d*x + c)^4 + 4
*(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d*cos(d*x + c)^2 + (a^4*b^2 - 4*a^3*
b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**9/(a-b*sin(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^9/(a-b*sin(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.225 $\int \frac{\sin^7(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=290

$$\frac{3\left(\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64\sqrt{a}b^{7/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}-\frac{3\left(\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64\sqrt{a}b^{7/4}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}+\frac{\cos(c+dx)\left(-3(a-3b)\cos^2(c+dx)+5a\right.}{32bd(a-b)^2\left(a-b\cos^4(c+dx)+2b\cos^2(c+\right.}$$

```
[Out] (3*(Sqrt[a] - 2*Sqrt[b])*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[
b]]])/(64*Sqrt[a]*(Sqrt[a] - Sqrt[b])^(5/2)*b^(7/4)*d) - (3*(Sqrt[a] + 2*Sq
rt[b])*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(64*Sqrt[a]
*(Sqrt[a] + Sqrt[b])^(5/2)*b^(7/4)*d) - (a*Cos[c + d*x]*(2 - Cos[c + d*x]^2
))/(8*(a - b)*b*d*(a - b + 2*b*Cos[c + d*x]^2 - b*Cos[c + d*x]^4)^2) + (Cos
[c + d*x]*(5*a - 17*b - 3*(a - 3*b)*Cos[c + d*x]^2))/(32*(a - b)^2*b*d*(a -
 b + 2*b*Cos[c + d*x]^2 - b*Cos[c + d*x]^4))
```

Rubi [A] time = 0.434751, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3215, 1205, 1178, 1166, 205, 208}

$$\frac{3\left(\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64\sqrt{a}b^{7/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}-\frac{3\left(\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64\sqrt{a}b^{7/4}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}+\frac{\cos(c+dx)\left(-3(a-3b)\cos^2(c+dx)+5a\right.}{32bd(a-b)^2\left(a-b\cos^4(c+dx)+2b\cos^2(c+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^7/(a - b*Sin[c + d*x]^4)^3,x]
```


```
[Out] (3*(Sqrt[a] - 2*Sqrt[b])*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[
b]]])/(64*Sqrt[a]*(Sqrt[a] - Sqrt[b])^(5/2)*b^(7/4)*d) - (3*(Sqrt[a] + 2*Sq
rt[b])*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(64*Sqrt[a]
*(Sqrt[a] + Sqrt[b])^(5/2)*b^(7/4)*d) - (a*Cos[c + d*x]*(2 - Cos[c + d*x]^2
))/(8*(a - b)*b*d*(a - b + 2*b*Cos[c + d*x]^2 - b*Cos[c + d*x]^4)^2) + (Cos
[c + d*x]*(5*a - 17*b - 3*(a - 3*b)*Cos[c + d*x]^2))/(32*(a - b)^2*b*d*(a -
 b + 2*b*Cos[c + d*x]^2 - b*Cos[c + d*x]^4))
```


Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
```

```
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sin^7(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{\left(a-b+2bx^2-bx^4\right)^3}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{a\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{8(a-b)bd\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{4a(a-4b)-2a(3a-8b)x^2}{\left(a-b+2bx^2-bx^4\right)^2}\,d\right.}{16a(a-b)bd}$$

$$= -\frac{a\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{8(a-b)bd\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)^2} + \frac{\cos(c+dx)\left(5a-17b-3\right.}{32(a-b)^2bd\left(a-b+2b\cos^2\right.}$$

$$= -\frac{a\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{8(a-b)bd\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)^2} + \frac{\cos(c+dx)\left(5a-17b-3\right.}{32(a-b)^2bd\left(a-b+2b\cos^2\right.}$$

$$= \frac{3\left(\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{7/4}d} - \frac{3\left(\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}b^{7/4}d} - \frac{a}{8(a-b)bd\left(a\right.}$$

Mathematica [C] time = 1.1391, size = 630, normalized size = 2.17

$$-3i\text{RootSum}\left[-16\#1^4a+\#1^8b-4\#1^6b+6\#1^4b-4\#1^2b+b\&,\frac{i\#1^6a\log\left(\#1^2-2\#1\cos(c+dx)+1\right)-3i\#1^4a\log\left(\#1^2-2\#1\cos(c+dx)+1\right)+3i\#}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]^7/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] ((-32*Cos[c + d*x]*(-7*a + 25*b + 3*(a - 3*b)*Cos[2*(c + d*x)]))/(8*a - 3*b
 + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*x)]) + (512*a*(a - b)*(-5*Cos[c +
d*x] + Cos[3*(c + d*x)]))/(-8*a + 3*b - 4*b*Cos[2*(c + d*x)] + b*Cos[4*(c +
 d*x)])^2 - (3*I)*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 +
b*#1^8 & , (2*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - 6*b*ArcTan[Sin[c
 + d*x]/(Cos[c + d*x] - #1)] - I*a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] + (3*I
)*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] - 6*a*ArcTan[Sin[c + d*x]/(Cos[c + d*
x] - #1)]*#1^2 + 34*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 + (3*I)
*a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (17*I)*b*Log[1 - 2*Cos[c + d*x]
*#1 + #1^2]*#1^2 + 6*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - 34*b
*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - (3*I)*a*Log[1 - 2*Cos[c +
d*x]*#1 + #1^2]*#1^4 + (17*I)*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 - 2*
a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 + 6*b*ArcTan[Sin[c + d*x]/(
Cos[c + d*x] - #1)]*#1^6 + I*a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6 - (3*
I)*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3
 - 3*b*#1^5 + b*#1^7) & ])/(256*(a - b)^2*b*d)
```

Maple [B] time = 0.121, size = 814, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^7/(a-b*sin(d*x+c)^4)^3,x)
```

```
[Out] 3/32/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2/(a^2-2*a*b+b^2)*cos(d*x+c)^7
*a-9/32/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2/(a^2-2*a*b+b^2)*cos(d*x+c
)^7*b-11/32/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2/(a^2-2*a*b+b^2)*cos(d
*x+c)^5*a+35/32/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2*b/(a^2-2*a*b+b^2)
*cos(d*x+c)^5+1/32/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2/b/(a^2-2*a*b+b
^2)*cos(d*x+c)^3*a^2+9/16/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2/(a^2-2*
a*b+b^2)*cos(d*x+c)^3*a-43/32/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2*b/(
a^2-2*a*b+b^2)*cos(d*x+c)^3-3/32/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2/
b/(a-b)*cos(d*x+c)*a-17/32/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2/(a-b)*
cos(d*x+c)+3/64/d/(a^2-2*a*b+b^2)/b/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*
x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))*a-9/64/d/(a^2-2*a*b+b^2)/(((a*b)^(1/2)-b)
*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))-3/32/d/(a^2-2*a*b+
b^2)*b/(a*b)^(1/2)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1
/2)-b)*b)^(1/2))-3/64/d/(a^2-2*a*b+b^2)/b/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh
(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))*a+9/64/d/(a^2-2*a*b+b^2)/(((a*b)^(
1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))-3/32/d/(a^
2-2*a*b+b^2)*b/(a*b)^(1/2)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(
((a*b)^(1/2)+b)*b)^(1/2))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a-b*sin(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 7.89863, size = 9296, normalized size = 32.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a-b*sin(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/128*(12*(a*b - 3*b^2)*cos(d*x + c)^7 - 4*(11*a*b - 35*b^2)*cos(d*x + c)^5
 + 4*(a^2 + 18*a*b - 43*b^2)*cos(d*x + c)^3 + 3*((a^2*b^3 - 2*a*b^4 + b^5)*
d*cos(d*x + c)^8 - 4*(a^2*b^3 - 2*a*b^4 + b^5)*d*cos(d*x + c)^6 - 2*(a^3*b^
2 - 5*a^2*b^3 + 7*a*b^4 - 3*b^5)*d*cos(d*x + c)^4 + 4*(a^3*b^2 - 3*a^2*b^3
+ 3*a*b^4 - b^5)*d*cos(d*x + c)^2 + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^
4 + b^5)*d)*sqrt(-((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b
^7 - a*b^8)*d^2*sqrt((a^6 - 12*a^5*b + 46*a^4*b^2 - 28*a^3*b^3 - 167*a^2*b^
4 + 160*a*b^5 + 256*b^6)/((a^11*b^7 - 10*a^10*b^8 + 45*a^9*b^9 - 120*a^8*b^
10 + 210*a^7*b^11 - 252*a^6*b^12 + 210*a^5*b^13 - 120*a^4*b^14 + 45*a^3*b^1
5 - 10*a^2*b^16 + a*b^17)*d^4)) + a^3 - 10*a^2*b + 21*a*b^2 + 4*b^3)/((a^6*
b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2))*log(27
*(a^4 - 10*a^3*b + 29*a^2*b^2 - 4*a*b^3 - 64*b^4)*cos(d*x + c) + 27*((a^8*b
^5 - 8*a^7*b^6 + 23*a^6*b^7 - 30*a^5*b^8 + 15*a^4*b^9 + 4*a^3*b^10 - 7*a^2*
b^11 + 2*a*b^12)*d^3*sqrt((a^6 - 12*a^5*b + 46*a^4*b^2 - 28*a^3*b^3 - 167*a
^2*b^4 + 160*a*b^5 + 256*b^6)/((a^11*b^7 - 10*a^10*b^8 + 45*a^9*b^9 - 120*a
^8*b^10 + 210*a^7*b^11 - 252*a^6*b^12 + 210*a^5*b^13 - 120*a^4*b^14 + 45*a^
3*b^15 - 10*a^2*b^16 + a*b^17)*d^4)) - (a^5*b^2 - 11*a^4*b^3 + 35*a^3*b^4 -
 9*a^2*b^5 - 80*a*b^6)*d)*sqrt(-((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3
*b^6 + 5*a^2*b^7 - a*b^8)*d^2*sqrt((a^6 - 12*a^5*b + 46*a^4*b^2 - 28*a^3*b^
3 - 167*a^2*b^4 + 160*a*b^5 + 256*b^6)/((a^11*b^7 - 10*a^10*b^8 + 45*a^9*b^
9 - 120*a^8*b^10 + 210*a^7*b^11 - 252*a^6*b^12 + 210*a^5*b^13 - 120*a^4*b^1
4 + 45*a^3*b^15 - 10*a^2*b^16 + a*b^17)*d^4)) + a^3 - 10*a^2*b + 21*a*b^2 +
 4*b^3)/((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a*b^8
)*d^2))) - 3*((a^2*b^3 - 2*a*b^4 + b^5)*d*cos(d*x + c)^8 - 4*(a^2*b^3 - 2*a
*b^4 + b^5)*d*cos(d*x + c)^6 - 2*(a^3*b^2 - 5*a^2*b^3 + 7*a*b^4 - 3*b^5)*d*
cos(d*x + c)^4 + 4*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*d*cos(d*x + c)^2 +
 (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*d)*sqrt(((a^6*b^3 - 5*a^5*
b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2*sqrt((a^6 - 12*a^5*b
 + 46*a^4*b^2 - 28*a^3*b^3 - 167*a^2*b^4 + 160*a*b^5 + 256*b^6)/((a^11*b^7
- 10*a^10*b^8 + 45*a^9*b^9 - 120*a^8*b^10 + 210*a^7*b^11 - 252*a^6*b^12 + 2
10*a^5*b^13 - 120*a^4*b^14 + 45*a^3*b^15 - 10*a^2*b^16 + a*b^17)*d^4)) - a^
3 + 10*a^2*b - 21*a*b^2 - 4*b^3)/((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^
3*b^6 + 5*a^2*b^7 - a*b^8)*d^2))*log(27*(a^4 - 10*a^3*b + 29*a^2*b^2 - 4*a*
b^3 - 64*b^4)*cos(d*x + c) + 27*((a^8*b^5 - 8*a^7*b^6 + 23*a^6*b^7 - 30*a^5
*b^8 + 15*a^4*b^9 + 4*a^3*b^10 - 7*a^2*b^11 + 2*a*b^12)*d^3*sqrt((a^6 - 12*
a^5*b + 46*a^4*b^2 - 28*a^3*b^3 - 167*a^2*b^4 + 160*a*b^5 + 256*b^6)/((a^11
*b^7 - 10*a^10*b^8 + 45*a^9*b^9 - 120*a^8*b^10 + 210*a^7*b^11 - 252*a^6*b^1
2 + 210*a^5*b^13 - 120*a^4*b^14 + 45*a^3*b^15 - 10*a^2*b^16 + a*b^17)*d^4))
 + (a^5*b^2 - 11*a^4*b^3 + 35*a^3*b^4 - 9*a^2*b^5 - 80*a*b^6)*d)*sqrt(((a^6
```

```
*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2*sqrt((a
^6 - 12*a^5*b + 46*a^4*b^2 - 28*a^3*b^3 - 167*a^2*b^4 + 160*a*b^5 + 256*b^6
)/((a^11*b^7 - 10*a^10*b^8 + 45*a^9*b^9 - 120*a^8*b^10 + 210*a^7*b^11 - 252
*a^6*b^12 + 210*a^5*b^13 - 120*a^4*b^14 + 45*a^3*b^15 - 10*a^2*b^16 + a*b^1
7)*d^4)) - a^3 + 10*a^2*b - 21*a*b^2 - 4*b^3)/((a^6*b^3 - 5*a^5*b^4 + 10*a^
4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2))) - 3*((a^2*b^3 - 2*a*b^4 + b^
5)*d*cos(d*x + c)^8 - 4*(a^2*b^3 - 2*a*b^4 + b^5)*d*cos(d*x + c)^6 - 2*(a^3
*b^2 - 5*a^2*b^3 + 7*a*b^4 - 3*b^5)*d*cos(d*x + c)^4 + 4*(a^3*b^2 - 3*a^2*b
^3 + 3*a*b^4 - b^5)*d*cos(d*x + c)^2 + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a
*b^4 + b^5)*d)*sqrt(-((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^
2*b^7 - a*b^8)*d^2*sqrt((a^6 - 12*a^5*b + 46*a^4*b^2 - 28*a^3*b^3 - 167*a^2
*b^4 + 160*a*b^5 + 256*b^6)/((a^11*b^7 - 10*a^10*b^8 + 45*a^9*b^9 - 120*a^8
*b^10 + 210*a^7*b^11 - 252*a^6*b^12 + 210*a^5*b^13 - 120*a^4*b^14 + 45*a^3*
b^15 - 10*a^2*b^16 + a*b^17)*d^4)) + a^3 - 10*a^2*b + 21*a*b^2 + 4*b^3)/((a
^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2))*log
(-27*(a^4 - 10*a^3*b + 29*a^2*b^2 - 4*a*b^3 - 64*b^4)*cos(d*x + c) + 27*((a
^8*b^5 - 8*a^7*b^6 + 23*a^6*b^7 - 30*a^5*b^8 + 15*a^4*b^9 + 4*a^3*b^10 - 7*
a^2*b^11 + 2*a*b^12)*d^3*sqrt((a^6 - 12*a^5*b + 46*a^4*b^2 - 28*a^3*b^3 - 1
67*a^2*b^4 + 160*a*b^5 + 256*b^6)/((a^11*b^7 - 10*a^10*b^8 + 45*a^9*b^9 - 1
20*a^8*b^10 + 210*a^7*b^11 - 252*a^6*b^12 + 210*a^5*b^13 - 120*a^4*b^14 + 4
5*a^3*b^15 - 10*a^2*b^16 + a*b^17)*d^4)) - (a^5*b^2 - 11*a^4*b^3 + 35*a^3*b
^4 - 9*a^2*b^5 - 80*a*b^6)*d)*sqrt(-((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10
*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2*sqrt((a^6 - 12*a^5*b + 46*a^4*b^2 - 28*a^
3*b^3 - 167*a^2*b^4 + 160*a*b^5 + 256*b^6)/((a^11*b^7 - 10*a^10*b^8 + 45*a^
9*b^9 - 120*a^8*b^10 + 210*a^7*b^11 - 252*a^6*b^12 + 210*a^5*b^13 - 120*a^4
*b^14 + 45*a^3*b^15 - 10*a^2*b^16 + a*b^17)*d^4)) + a^3 - 10*a^2*b + 21*a*b
^2 + 4*b^3)/((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a
*b^8)*d^2))) + 3*((a^2*b^3 - 2*a*b^4 + b^5)*d*cos(d*x + c)^8 - 4*(a^2*b^3 -
 2*a*b^4 + b^5)*d*cos(d*x + c)^6 - 2*(a^3*b^2 - 5*a^2*b^3 + 7*a*b^4 - 3*b^5
)*d*cos(d*x + c)^4 + 4*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*d*cos(d*x + c)
^2 + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*d)*sqrt(((a^6*b^3 - 5*
a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2*sqrt((a^6 - 12*a
^5*b + 46*a^4*b^2 - 28*a^3*b^3 - 167*a^2*b^4 + 160*a*b^5 + 256*b^6)/((a^11*
b^7 - 10*a^10*b^8 + 45*a^9*b^9 - 120*a^8*b^10 + 210*a^7*b^11 - 252*a^6*b^12
 + 210*a^5*b^13 - 120*a^4*b^14 + 45*a^3*b^15 - 10*a^2*b^16 + a*b^17)*d^4))
- a^3 + 10*a^2*b - 21*a*b^2 - 4*b^3)/((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 1
0*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2))*log(-27*(a^4 - 10*a^3*b + 29*a^2*b^2 -
 4*a*b^3 - 64*b^4)*cos(d*x + c) + 27*((a^8*b^5 - 8*a^7*b^6 + 23*a^6*b^7 - 3
0*a^5*b^8 + 15*a^4*b^9 + 4*a^3*b^10 - 7*a^2*b^11 + 2*a*b^12)*d^3*sqrt((a^6
- 12*a^5*b + 46*a^4*b^2 - 28*a^3*b^3 - 167*a^2*b^4 + 160*a*b^5 + 256*b^6)/(
(a^11*b^7 - 10*a^10*b^8 + 45*a^9*b^9 - 120*a^8*b^10 + 210*a^7*b^11 - 252*a^
6*b^12 + 210*a^5*b^13 - 120*a^4*b^14 + 45*a^3*b^15 - 10*a^2*b^16 + a*b^17)*
d^4)) + (a^5*b^2 - 11*a^4*b^3 + 35*a^3*b^4 - 9*a^2*b^5 - 80*a*b^6)*d)*sqrt(
((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2*sq
rt((a^6 - 12*a^5*b + 46*a^4*b^2 - 28*a^3*b^3 - 167*a^2*b^4 + 160*a*b^5 + 25
6*b^6)/((a^11*b^7 - 10*a^10*b^8 + 45*a^9*b^9 - 120*a^8*b^10 + 210*a^7*b^11
- 252*a^6*b^12 + 210*a^5*b^13 - 120*a^4*b^14 + 45*a^3*b^15 - 10*a^2*b^16 +
a*b^17)*d^4)) - a^3 + 10*a^2*b - 21*a*b^2 - 4*b^3)/((a^6*b^3 - 5*a^5*b^4 +
10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2))) - 4*(3*a^2 + 14*a*b - 1
7*b^2)*cos(d*x + c))/((a^2*b^3 - 2*a*b^4 + b^5)*d*cos(d*x + c)^8 - 4*(a^2*b
^3 - 2*a*b^4 + b^5)*d*cos(d*x + c)^6 - 2*(a^3*b^2 - 5*a^2*b^3 + 7*a*b^4 - 3
*b^5)*d*cos(d*x + c)^4 + 4*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*d*cos(d*x
+ c)^2 + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**7/(a-b*sin(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a-b*sin(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.226 $\int \frac{\sin^5(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=313

$$\frac{\cos(c+dx)\left(a^2+2b(2a+b)\cos^2(c+dx)-11ab-2b^2\right)}{32abd(a-b)^2\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}+\frac{\left(-10\sqrt{a}\sqrt{b}+3a+4b\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{3/2}b^{5/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{\left(10\sqrt{a}\sqrt{b}+3a\right.}{64a^{3/2}b}$$

```
[Out] ((3*a - 10*Sqrt[a]*Sqrt[b] + 4*b)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a
] - Sqrt[b]]])/(64*a^(3/2)*(Sqrt[a] - Sqrt[b])^(5/2)*b^(5/4)*d) + ((3*a + 1
0*Sqrt[a]*Sqrt[b] + 4*b)*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt
[b]]])/(64*a^(3/2)*(Sqrt[a] + Sqrt[b])^(5/2)*b^(5/4)*d) - (Cos[c + d*x]*(a
+ b - b*Cos[c + d*x]^2))/(8*(a - b)*b*d*(a - b + 2*b*Cos[c + d*x]^2 - b*Cos
[c + d*x]^4)^2) + (Cos[c + d*x]*(a^2 - 11*a*b - 2*b^2 + 2*b*(2*a + b)*Cos[c
 + d*x]^2))/(32*a*(a - b)^2*b*d*(a - b + 2*b*Cos[c + d*x]^2 - b*Cos[c + d*x
]^4))
```

Rubi [A] time = 0.472061, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3215, 1205, 1178, 1166, 205, 208}

$$\frac{\cos(c+dx)\left(a^2+2b(2a+b)\cos^2(c+dx)-11ab-2b^2\right)}{32abd(a-b)^2\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}+\frac{\left(-10\sqrt{a}\sqrt{b}+3a+4b\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{3/2}b^{5/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{\left(10\sqrt{a}\sqrt{b}+3a\right.}{64a^{3/2}b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^5/(a - b*Sin[c + d*x]^4)^3,x]
```


```
[Out] ((3*a - 10*Sqrt[a]*Sqrt[b] + 4*b)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a
] - Sqrt[b]]])/(64*a^(3/2)*(Sqrt[a] - Sqrt[b])^(5/2)*b^(5/4)*d) + ((3*a + 1
0*Sqrt[a]*Sqrt[b] + 4*b)*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt
[b]]])/(64*a^(3/2)*(Sqrt[a] + Sqrt[b])^(5/2)*b^(5/4)*d) - (Cos[c + d*x]*(a
+ b - b*Cos[c + d*x]^2))/(8*(a - b)*b*d*(a - b + 2*b*Cos[c + d*x]^2 - b*Cos
[c + d*x]^4)^2) + (Cos[c + d*x]*(a^2 - 11*a*b - 2*b^2 + 2*b*(2*a + b)*Cos[c
 + d*x]^2))/(32*a*(a - b)^2*b*d*(a - b + 2*b*Cos[c + d*x]^2 - b*Cos[c + d*x
]^4))
```


Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
```

```
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sin^5(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\left(a-b+2bx^2-bx^4\right)^3}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\cos(c+dx)\left(a+b-b\cos^2(c+dx)\right)}{8(a-b)bd\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{2a(a-7b)+10abx^2}{\left(a-b+2bx^2-bx^4\right)^2}\,dx, x\right.}{16a(a-b)bd}$$

$$= -\frac{\cos(c+dx)\left(a+b-b\cos^2(c+dx)\right)}{8(a-b)bd\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)^2} + \frac{\cos(c+dx)\left(a^2-11ab-2b^2\right.}{32a(a-b)^2bd\left(a-b+2b\cos\right.}$$

$$= -\frac{\cos(c+dx)\left(a+b-b\cos^2(c+dx)\right)}{8(a-b)bd\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)^2} + \frac{\cos(c+dx)\left(a^2-11ab-2b^2\right.}{32a(a-b)^2bd\left(a-b+2b\cos\right.}$$

$$= \frac{\left(3a-10\sqrt{a}\sqrt{b}+4b\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{3/2}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{5/4}d} + \frac{\left(3a+10\sqrt{a}\sqrt{b}+4b\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64a^{3/2}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}b^{5/4}d}$$

Mathematica [C] time = 1.37601, size = 786, normalized size = 2.51

$$i\text{RootSum}\left[-16\#1^4a+\#1^8b-4\#1^6b+6\#1^4b-4\#1^2b+b\&,\frac{6i\#1^4a^2\log\left(\#1^2-2\#1\cos(c+dx)+1\right)-6i\#1^2a^2\log\left(\#1^2-2\#1\cos(c+dx)+1\right)-12\#1^4a^2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)+12\#1^2a^2\tan^{-1}}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]^5/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] ((32*Cos[c + d*x]*(a^2 - 9*a*b - b^2 + b*(2*a + b)*Cos[2*(c + d*x)]))/(a*(8
*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*x)])) - (512*(a - b)*Cos[c
 + d*x]*(2*a + b - b*Cos[2*(c + d*x)]))/(-8*a + 3*b - 4*b*Cos[2*(c + d*x)]
+ b*Cos[4*(c + d*x)])^2 + (I*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 -
4*b*#1^6 + b*#1^8 & , (4*a*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + 2*b
^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - (2*I)*a*b*Log[1 - 2*Cos[c + d
*x]*#1 + #1^2] - I*b^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] + 12*a^2*ArcTan[Si
n[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 - 64*a*b*ArcTan[Sin[c + d*x]/(Cos[c +
d*x] - #1)]*#1^2 + 10*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 - (
6*I)*a^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + (32*I)*a*b*Log[1 - 2*Cos[
c + d*x]*#1 + #1^2]*#1^2 - (5*I)*b^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2
 - 12*a^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 + 64*a*b*ArcTan[Sin
[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - 10*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d
*x] - #1)]*#1^4 + (6*I)*a^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 - (32*I)
*a*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 + (5*I)*b^2*Log[1 - 2*Cos[c + d
*x]*#1 + #1^2]*#1^4 - 4*a*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 -
 2*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 + (2*I)*a*b*Log[1 - 2*
Cos[c + d*x]*#1 + #1^2]*#1^6 + I*b^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6
)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/a)/(128*(a - b)^
2*b*d)
```

Maple [B] time = 0.125, size = 1167, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5/(a-b*sin(d*x+c)^4)^3,x)
```

```
[Out] -1/8/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2/(a^2-2*a*b+b^2)*cos(d*x+c)^7
*b-1/16/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2*b^2/a/(a^2-2*a*b+b^2)*cos
(d*x+c)^7-1/32/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2/(a^2-2*a*b+b^2)*co
s(d*x+c)^5*a+19/32/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2*b/(a^2-2*a*b+b
^2)*cos(d*x+c)^5+3/16/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2/a/(a^2-2*a*
b+b^2)*cos(d*x+c)^5*b^2+5/16/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2/(a^2
-2*a*b+b^2)*cos(d*x+c)^3*a-7/8/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2*b/
(a^2-2*a*b+b^2)*cos(d*x+c)^3-3/16/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2
/a/(a^2-2*a*b+b^2)*cos(d*x+c)^3*b^2-3/32/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2
-a+b)^2/b/(a-b)*cos(d*x+c)*a-15/32/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^
2/(a-b)*cos(d*x+c)-1/16/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2/(a-b)/a*b
*cos(d*x+c)-1/16/d/(a^2-2*a*b+b^2)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x
+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))-1/32/d/a/(a^2-2*a*b+b^2)/(((a*b)^(1/2)-b)*
b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))*b+3/64/d/(a^2-2*a*b
+b^2)/(a*b)^(1/2)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/
2)-b)*b)^(1/2))*a-13/64/d/(a^2-2*a*b+b^2)*b/(a*b)^(1/2)/(((a*b)^(1/2)-b)*b)
^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))+1/16/d/a/(a^2-2*a*b+b
```

```
^2)/(a*b)^(1/2)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)
-b)*b)^(1/2))*b^2+1/16/d/(a^2-2*a*b+b^2)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(
cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))+1/32/d/a/(a^2-2*a*b+b^2)/(((a*b)^(1
/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))*b+3/64/d/(a
^2-2*a*b+b^2)/(a*b)^(1/2)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/((
(a*b)^(1/2)+b)*b)^(1/2))*a-13/64/d/(a^2-2*a*b+b^2)*b/(a*b)^(1/2)/(((a*b)^(1
/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))+1/16/d/a/(a
^2-2*a*b+b^2)/(a*b)^(1/2)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/((
(a*b)^(1/2)+b)*b)^(1/2))*b^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a-b*sin(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 10.2006, size = 10311, normalized size = 32.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a-b*sin(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(8*(2*a*b^2 + b^3)*cos(d*x + c)^7 + 4*(a^2*b - 19*a*b^2 - 6*b^3)*cos
(d*x + c)^5 - 8*(5*a^2*b - 14*a*b^2 - 3*b^3)*cos(d*x + c)^3 + ((a^3*b^3 - 2
*a^2*b^4 + a*b^5)*d*cos(d*x + c)^8 - 4*(a^3*b^3 - 2*a^2*b^4 + a*b^5)*d*cos(
d*x + c)^6 - 2*(a^4*b^2 - 5*a^3*b^3 + 7*a^2*b^4 - 3*a*b^5)*d*cos(d*x + c)^4
 + 4*(a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d*cos(d*x + c)^2 + (a^5*b -
4*a^4*b^2 + 6*a^3*b^3 - 4*a^2*b^4 + a*b^5)*d)*sqrt((15*a^4 - 30*a^3*b - 229
*a^2*b^2 + 116*a*b^3 - 16*b^4 + (a^8*b^2 - 5*a^7*b^3 + 10*a^6*b^4 - 10*a^5*
b^5 + 5*a^4*b^6 - a^3*b^7)*d^2*sqrt((81*a^6 - 1548*a^5*b + 12814*a^4*b^2 -
53212*a^3*b^3 + 104361*a^2*b^4 - 48160*a*b^5 + 6400*b^6)/((a^13*b^5 - 10*a^
12*b^6 + 45*a^11*b^7 - 120*a^10*b^8 + 210*a^9*b^9 - 252*a^8*b^10 + 210*a^7*
b^11 - 120*a^6*b^12 + 45*a^5*b^13 - 10*a^4*b^14 + a^3*b^15)*d^4)))/((a^8*b^
2 - 5*a^7*b^3 + 10*a^6*b^4 - 10*a^5*b^5 + 5*a^4*b^6 - a^3*b^7)*d^2))*log((8
1*a^5 - 1458*a^4*b + 9389*a^3*b^2 - 24972*a^2*b^3 + 10896*a*b^4 - 1280*b^5)
*cos(d*x + c) + ((a^10*b^4 + 10*a^9*b^5 - 69*a^8*b^6 + 160*a^7*b^7 - 185*a^
6*b^8 + 114*a^5*b^9 - 35*a^4*b^10 + 4*a^3*b^11)*d^3*sqrt((81*a^6 - 1548*a^5
*b + 12814*a^4*b^2 - 53212*a^3*b^3 + 104361*a^2*b^4 - 48160*a*b^5 + 6400*b^
6)/((a^13*b^5 - 10*a^12*b^6 + 45*a^11*b^7 - 120*a^10*b^8 + 210*a^9*b^9 - 25
2*a^8*b^10 + 210*a^7*b^11 - 120*a^6*b^12 + 45*a^5*b^13 - 10*a^4*b^14 + a^3*
b^15)*d^4)) - (27*a^7*b - 411*a^6*b^2 + 2383*a^5*b^3 - 5529*a^4*b^4 + 1962*
a^3*b^5 - 160*a^2*b^6)*d)*sqrt((15*a^4 - 30*a^3*b - 229*a^2*b^2 + 116*a*b^3
 - 16*b^4 + (a^8*b^2 - 5*a^7*b^3 + 10*a^6*b^4 - 10*a^5*b^5 + 5*a^4*b^6 - a^
3*b^7)*d^2*sqrt((81*a^6 - 1548*a^5*b + 12814*a^4*b^2 - 53212*a^3*b^3 + 1043
61*a^2*b^4 - 48160*a*b^5 + 6400*b^6)/((a^13*b^5 - 10*a^12*b^6 + 45*a^11*b^7
 - 120*a^10*b^8 + 210*a^9*b^9 - 252*a^8*b^10 + 210*a^7*b^11 - 120*a^6*b^12
+ 45*a^5*b^13 - 10*a^4*b^14 + a^3*b^15)*d^4)))/((a^8*b^2 - 5*a^7*b^3 + 10*a
^6*b^4 - 10*a^5*b^5 + 5*a^4*b^6 - a^3*b^7)*d^2))) - ((a^3*b^3 - 2*a^2*b^4 +
 a*b^5)*d*cos(d*x + c)^8 - 4*(a^3*b^3 - 2*a^2*b^4 + a*b^5)*d*cos(d*x + c)^6
```

```
- 2*(a^4*b^2 - 5*a^3*b^3 + 7*a^2*b^4 - 3*a*b^5)*d*cos(d*x + c)^4 + 4*(a^4*
b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d*cos(d*x + c)^2 + (a^5*b - 4*a^4*b^2
+ 6*a^3*b^3 - 4*a^2*b^4 + a*b^5)*d)*sqrt((15*a^4 - 30*a^3*b - 229*a^2*b^2 +
 116*a*b^3 - 16*b^4 - (a^8*b^2 - 5*a^7*b^3 + 10*a^6*b^4 - 10*a^5*b^5 + 5*a^
4*b^6 - a^3*b^7)*d^2*sqrt((81*a^6 - 1548*a^5*b + 12814*a^4*b^2 - 53212*a^3*
b^3 + 104361*a^2*b^4 - 48160*a*b^5 + 6400*b^6)/((a^13*b^5 - 10*a^12*b^6 + 4
5*a^11*b^7 - 120*a^10*b^8 + 210*a^9*b^9 - 252*a^8*b^10 + 210*a^7*b^11 - 120
*a^6*b^12 + 45*a^5*b^13 - 10*a^4*b^14 + a^3*b^15)*d^4)))/((a^8*b^2 - 5*a^7*
b^3 + 10*a^6*b^4 - 10*a^5*b^5 + 5*a^4*b^6 - a^3*b^7)*d^2))*log((81*a^5 - 14
58*a^4*b + 9389*a^3*b^2 - 24972*a^2*b^3 + 10896*a*b^4 - 1280*b^5)*cos(d*x +
 c) + ((a^10*b^4 + 10*a^9*b^5 - 69*a^8*b^6 + 160*a^7*b^7 - 185*a^6*b^8 + 11
4*a^5*b^9 - 35*a^4*b^10 + 4*a^3*b^11)*d^3*sqrt((81*a^6 - 1548*a^5*b + 12814
*a^4*b^2 - 53212*a^3*b^3 + 104361*a^2*b^4 - 48160*a*b^5 + 6400*b^6)/((a^13*
b^5 - 10*a^12*b^6 + 45*a^11*b^7 - 120*a^10*b^8 + 210*a^9*b^9 - 252*a^8*b^10
 + 210*a^7*b^11 - 120*a^6*b^12 + 45*a^5*b^13 - 10*a^4*b^14 + a^3*b^15)*d^4)
) + (27*a^7*b - 411*a^6*b^2 + 2383*a^5*b^3 - 5529*a^4*b^4 + 1962*a^3*b^5 -
160*a^2*b^6)*d)*sqrt((15*a^4 - 30*a^3*b - 229*a^2*b^2 + 116*a*b^3 - 16*b^4
- (a^8*b^2 - 5*a^7*b^3 + 10*a^6*b^4 - 10*a^5*b^5 + 5*a^4*b^6 - a^3*b^7)*d^2
*sqrt((81*a^6 - 1548*a^5*b + 12814*a^4*b^2 - 53212*a^3*b^3 + 104361*a^2*b^4
 - 48160*a*b^5 + 6400*b^6)/((a^13*b^5 - 10*a^12*b^6 + 45*a^11*b^7 - 120*a^1
0*b^8 + 210*a^9*b^9 - 252*a^8*b^10 + 210*a^7*b^11 - 120*a^6*b^12 + 45*a^5*b
^13 - 10*a^4*b^14 + a^3*b^15)*d^4)))/((a^8*b^2 - 5*a^7*b^3 + 10*a^6*b^4 - 1
0*a^5*b^5 + 5*a^4*b^6 - a^3*b^7)*d^2))) - ((a^3*b^3 - 2*a^2*b^4 + a*b^5)*d*
cos(d*x + c)^8 - 4*(a^3*b^3 - 2*a^2*b^4 + a*b^5)*d*cos(d*x + c)^6 - 2*(a^4*
b^2 - 5*a^3*b^3 + 7*a^2*b^4 - 3*a*b^5)*d*cos(d*x + c)^4 + 4*(a^4*b^2 - 3*a^
3*b^3 + 3*a^2*b^4 - a*b^5)*d*cos(d*x + c)^2 + (a^5*b - 4*a^4*b^2 + 6*a^3*b^
3 - 4*a^2*b^4 + a*b^5)*d)*sqrt((15*a^4 - 30*a^3*b - 229*a^2*b^2 + 116*a*b^3
 - 16*b^4 + (a^8*b^2 - 5*a^7*b^3 + 10*a^6*b^4 - 10*a^5*b^5 + 5*a^4*b^6 - a^
3*b^7)*d^2*sqrt((81*a^6 - 1548*a^5*b + 12814*a^4*b^2 - 53212*a^3*b^3 + 1043
61*a^2*b^4 - 48160*a*b^5 + 6400*b^6)/((a^13*b^5 - 10*a^12*b^6 + 45*a^11*b^7
 - 120*a^10*b^8 + 210*a^9*b^9 - 252*a^8*b^10 + 210*a^7*b^11 - 120*a^6*b^12
+ 45*a^5*b^13 - 10*a^4*b^14 + a^3*b^15)*d^4)))/((a^8*b^2 - 5*a^7*b^3 + 10*a
^6*b^4 - 10*a^5*b^5 + 5*a^4*b^6 - a^3*b^7)*d^2))*log(-(81*a^5 - 1458*a^4*b
+ 9389*a^3*b^2 - 24972*a^2*b^3 + 10896*a*b^4 - 1280*b^5)*cos(d*x + c) + ((a
^10*b^4 + 10*a^9*b^5 - 69*a^8*b^6 + 160*a^7*b^7 - 185*a^6*b^8 + 114*a^5*b^9
 - 35*a^4*b^10 + 4*a^3*b^11)*d^3*sqrt((81*a^6 - 1548*a^5*b + 12814*a^4*b^2
- 53212*a^3*b^3 + 104361*a^2*b^4 - 48160*a*b^5 + 6400*b^6)/((a^13*b^5 - 10*
a^12*b^6 + 45*a^11*b^7 - 120*a^10*b^8 + 210*a^9*b^9 - 252*a^8*b^10 + 210*a^
7*b^11 - 120*a^6*b^12 + 45*a^5*b^13 - 10*a^4*b^14 + a^3*b^15)*d^4)) - (27*a
^7*b - 411*a^6*b^2 + 2383*a^5*b^3 - 5529*a^4*b^4 + 1962*a^3*b^5 - 160*a^2*b
^6)*d)*sqrt((15*a^4 - 30*a^3*b - 229*a^2*b^2 + 116*a*b^3 - 16*b^4 + (a^8*b^
2 - 5*a^7*b^3 + 10*a^6*b^4 - 10*a^5*b^5 + 5*a^4*b^6 - a^3*b^7)*d^2*sqrt((81
*a^6 - 1548*a^5*b + 12814*a^4*b^2 - 53212*a^3*b^3 + 104361*a^2*b^4 - 48160*
a*b^5 + 6400*b^6)/((a^13*b^5 - 10*a^12*b^6 + 45*a^11*b^7 - 120*a^10*b^8 + 2
10*a^9*b^9 - 252*a^8*b^10 + 210*a^7*b^11 - 120*a^6*b^12 + 45*a^5*b^13 - 10*
a^4*b^14 + a^3*b^15)*d^4)))/((a^8*b^2 - 5*a^7*b^3 + 10*a^6*b^4 - 10*a^5*b^5
 + 5*a^4*b^6 - a^3*b^7)*d^2))) + ((a^3*b^3 - 2*a^2*b^4 + a*b^5)*d*cos(d*x +
 c)^8 - 4*(a^3*b^3 - 2*a^2*b^4 + a*b^5)*d*cos(d*x + c)^6 - 2*(a^4*b^2 - 5*a
^3*b^3 + 7*a^2*b^4 - 3*a*b^5)*d*cos(d*x + c)^4 + 4*(a^4*b^2 - 3*a^3*b^3 + 3
*a^2*b^4 - a*b^5)*d*cos(d*x + c)^2 + (a^5*b - 4*a^4*b^2 + 6*a^3*b^3 - 4*a^2
*b^4 + a*b^5)*d)*sqrt((15*a^4 - 30*a^3*b - 229*a^2*b^2 + 116*a*b^3 - 16*b^4
 - (a^8*b^2 - 5*a^7*b^3 + 10*a^6*b^4 - 10*a^5*b^5 + 5*a^4*b^6 - a^3*b^7)*d^
2*sqrt((81*a^6 - 1548*a^5*b + 12814*a^4*b^2 - 53212*a^3*b^3 + 104361*a^2*b^
4 - 48160*a*b^5 + 6400*b^6)/((a^13*b^5 - 10*a^12*b^6 + 45*a^11*b^7 - 120*a^
10*b^8 + 210*a^9*b^9 - 252*a^8*b^10 + 210*a^7*b^11 - 120*a^6*b^12 + 45*a^5*
b^13 - 10*a^4*b^14 + a^3*b^15)*d^4)))/((a^8*b^2 - 5*a^7*b^3 + 10*a^6*b^4 -
10*a^5*b^5 + 5*a^4*b^6 - a^3*b^7)*d^2))*log(-(81*a^5 - 1458*a^4*b + 9389*a^
3*b^2 - 24972*a^2*b^3 + 10896*a*b^4 - 1280*b^5)*cos(d*x + c) + ((a^10*b^4 +
 10*a^9*b^5 - 69*a^8*b^6 + 160*a^7*b^7 - 185*a^6*b^8 + 114*a^5*b^9 - 35*a^4
```

```
*b^10 + 4*a^3*b^11)*d^3*sqrt((81*a^6 - 1548*a^5*b + 12814*a^4*b^2 - 53212*a
^3*b^3 + 104361*a^2*b^4 - 48160*a*b^5 + 6400*b^6)/((a^13*b^5 - 10*a^12*b^6
+ 45*a^11*b^7 - 120*a^10*b^8 + 210*a^9*b^9 - 252*a^8*b^10 + 210*a^7*b^11 -
120*a^6*b^12 + 45*a^5*b^13 - 10*a^4*b^14 + a^3*b^15)*d^4)) + (27*a^7*b - 41
1*a^6*b^2 + 2383*a^5*b^3 - 5529*a^4*b^4 + 1962*a^3*b^5 - 160*a^2*b^6)*d)*sq
rt((15*a^4 - 30*a^3*b - 229*a^2*b^2 + 116*a*b^3 - 16*b^4 - (a^8*b^2 - 5*a^7
*b^3 + 10*a^6*b^4 - 10*a^5*b^5 + 5*a^4*b^6 - a^3*b^7)*d^2*sqrt((81*a^6 - 15
48*a^5*b + 12814*a^4*b^2 - 53212*a^3*b^3 + 104361*a^2*b^4 - 48160*a*b^5 + 6
400*b^6)/((a^13*b^5 - 10*a^12*b^6 + 45*a^11*b^7 - 120*a^10*b^8 + 210*a^9*b^
9 - 252*a^8*b^10 + 210*a^7*b^11 - 120*a^6*b^12 + 45*a^5*b^13 - 10*a^4*b^14
+ a^3*b^15)*d^4)))/((a^8*b^2 - 5*a^7*b^3 + 10*a^6*b^4 - 10*a^5*b^5 + 5*a^4*
b^6 - a^3*b^7)*d^2))) + 4*(3*a^3 + 12*a^2*b - 13*a*b^2 - 2*b^3)*cos(d*x + c
))/((a^3*b^3 - 2*a^2*b^4 + a*b^5)*d*cos(d*x + c)^8 - 4*(a^3*b^3 - 2*a^2*b^4
 + a*b^5)*d*cos(d*x + c)^6 - 2*(a^4*b^2 - 5*a^3*b^3 + 7*a^2*b^4 - 3*a*b^5)*
d*cos(d*x + c)^4 + 4*(a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d*cos(d*x +
c)^2 + (a^5*b - 4*a^4*b^2 + 6*a^3*b^3 - 4*a^2*b^4 + a*b^5)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5/(a-b*sin(d*x+c)**4)**3,x)

[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a-b*sin(d*x+c)^4)^3,x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.227 $\int \frac{\sin^3(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=288

$$-\frac{\left(5\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{3/2}b^{3/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{\left(5\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64a^{3/2}b^{3/4}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}-\frac{\cos(c+dx)\left(-(5a+b)\cos^2(c+dx)+11a\right.}{32ad(a-b)^2\left(a-b\cos^4(c+dx)+2b\cos^2(c\right.}$$

```
[Out] -((5*Sqrt[a] - 2*Sqrt[b])*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt
[b]]])/(64*a^(3/2)*(Sqrt[a] - Sqrt[b])^(5/2)*b^(3/4)*d) + ((5*Sqrt[a] + 2*S
qrt[b])*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(64*a^(3/2
)*(Sqrt[a] + Sqrt[b])^(5/2)*b^(3/4)*d) - (Cos[c + d*x]*(2 - Cos[c + d*x]^2)
)/(8*(a - b)*d*(a - b + 2*b*Cos[c + d*x]^2 - b*Cos[c + d*x]^4)^2) - (Cos[c
+ d*x]*(11*a + b - (5*a + b)*Cos[c + d*x]^2))/(32*a*(a - b)^2*d*(a - b + 2*
b*Cos[c + d*x]^2 - b*Cos[c + d*x]^4))
```

Rubi [A] time = 0.497525, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3215, 1178, 1166, 205, 208}

$$-\frac{\left(5\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{3/2}b^{3/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{\left(5\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64a^{3/2}b^{3/4}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}-\frac{\cos(c+dx)\left(-(5a+b)\cos^2(c+dx)+11a\right.}{32ad(a-b)^2\left(a-b\cos^4(c+dx)+2b\cos^2(c\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a - b*Sin[c + d*x]^4)^3,x]
```


```
[Out] -((5*Sqrt[a] - 2*Sqrt[b])*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt
[b]]])/(64*a^(3/2)*(Sqrt[a] - Sqrt[b])^(5/2)*b^(3/4)*d) + ((5*Sqrt[a] + 2*S
qrt[b])*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(64*a^(3/2
)*(Sqrt[a] + Sqrt[b])^(5/2)*b^(3/4)*d) - (Cos[c + d*x]*(2 - Cos[c + d*x]^2)
)/(8*(a - b)*d*(a - b + 2*b*Cos[c + d*x]^2 - b*Cos[c + d*x]^4)^2) - (Cos[c
+ d*x]*(11*a + b - (5*a + b)*Cos[c + d*x]^2))/(32*a*(a - b)^2*d*(a - b + 2*
b*Cos[c + d*x]^2 - b*Cos[c + d*x]^4))
```


Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{\left(a-b+2bx^2-bx^4\right)^3}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{8(a-b)d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{-12ab+10abx^2}{\left(a-b+2bx^2-bx^4\right)^2}\,dx, x, \ldots\right.}{16a(a-b)bd}$$

$$= -\frac{\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{8(a-b)d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)^2} - \frac{\cos(c+dx)\left(11a+b-(5a\ldots\right.}{32a(a-b)^2d\left(a-b+2b\cos^2(c\ldots\right.}$$

$$= -\frac{\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{8(a-b)d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)^2} - \frac{\cos(c+dx)\left(11a+b-(5a\ldots\right.}{32a(a-b)^2d\left(a-b+2b\cos^2(c\ldots\right.}$$

$$= -\frac{\left(5\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{3/2}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{3/4}d} + \frac{\left(5\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64a^{3/2}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}b^{3/4}d} - \frac{\ldots}{8(a-b)d\left(a\ldots\right.}$$

Mathematica [C] time = 1.12044, size = 631, normalized size = 2.19

$$i\text{RootSum}\left[-16\#1^4a+\#1^8b-4\#1^6b+6\#1^4b-4\#1^2b+b\&, \frac{5i\#1^6a\log\left(\#1^2-2\#1\cos(c+dx)+1\right)-47i\#1^4a\log\left(\#1^2-2\#1\cos(c+dx)+1\right)+47i\#1^2a\log\left(\#1^2-2\#1\cos(c+dx)+1\right)-5ia\log\left(\ldots\right.}{\ldots}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]^3/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] ((32*Cos[c + d*x]*(-17*a - b + (5*a + b)*Cos[2*(c + d*x)]))/(a*(8*a - 3*b +
 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*x)])) + (512*(a - b)*(-5*Cos[c + d*x
] + Cos[3*(c + d*x)]))/(-8*a + 3*b - 4*b*Cos[2*(c + d*x)] + b*Cos[4*(c + d*
x)])^2 + (I*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8
 & , (10*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + 2*b*ArcTan[Sin[c + d*
x]/(Cos[c + d*x] - #1)] - (5*I)*a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] - I*b*L
```

```
og[1 - 2*Cos[c + d*x]*#1 + #1^2] - 94*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] -
 #1)]*#1^2 + 10*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 + (47*I)*a*
Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (5*I)*b*Log[1 - 2*Cos[c + d*x]*#1
+ #1^2]*#1^2 + 94*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - 10*b*Ar
cTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - (47*I)*a*Log[1 - 2*Cos[c + d*
x]*#1 + #1^2]*#1^4 + (5*I)*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 - 10*a*
ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 - 2*b*ArcTan[Sin[c + d*x]/(Co
s[c + d*x] - #1)]*#1^6 + (5*I)*a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6 + I
*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 -
 3*b*#1^5 + b*#1^7) & ])/a)/(256*(a - b)^2*d)
```

Maple [B] time = 0.181, size = 1153, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a-b*sin(d*x+c)^4)^3,x)
```

```
[Out] -5/64/d/b/(cos(d*x+c)^2-1-(a*b)^(1/2)/b)^2/(a^2-2*a*b+b^2)*cos(d*x+c)^3-1/6
4/d/a/(cos(d*x+c)^2-1-(a*b)^(1/2)/b)^2/(a^2-2*a*b+b^2)*cos(d*x+c)^3+1/8/d/(
a*b)^(1/2)/(cos(d*x+c)^2-1-(a*b)^(1/2)/b)^2/(a^2-2*a*b+b^2)*cos(d*x+c)^3-1/
32/d*b/(a*b)^(1/2)/a/(cos(d*x+c)^2-1-(a*b)^(1/2)/b)^2/(a^2-2*a*b+b^2)*cos(d
*x+c)^3+7/64/d/b/(a*b)^(1/2)/(cos(d*x+c)^2-1-(a*b)^(1/2)/b)^2/(a-b)*cos(d*x
+c)-5/64/d/b/a/(cos(d*x+c)^2-1-(a*b)^(1/2)/b)^2/(a-b)*cos(d*x+c)-1/32/d/(a*
b)^(1/2)/a/(cos(d*x+c)^2-1-(a*b)^(1/2)/b)^2/(a-b)*cos(d*x+c)+5/64/d/(a^2-2*
a*b+b^2)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)
^(1/2))+1/64/d/a/(a^2-2*a*b+b^2)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+
c)*b/(((a*b)^(1/2)+b)*b)^(1/2))*b-1/8/d/(a^2-2*a*b+b^2)*b/(a*b)^(1/2)/(((a*
b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))+1/32/d
/a/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)
*b/(((a*b)^(1/2)+b)*b)^(1/2))*b^2-5/64/d/b/(cos(d*x+c)^2+(a*b)^(1/2)/b-1)^2
/(a^2-2*a*b+b^2)*cos(d*x+c)^3-1/64/d/a/(cos(d*x+c)^2+(a*b)^(1/2)/b-1)^2/(a^
2-2*a*b+b^2)*cos(d*x+c)^3-1/8/d/(a*b)^(1/2)/(cos(d*x+c)^2+(a*b)^(1/2)/b-1)^
2/(a^2-2*a*b+b^2)*cos(d*x+c)^3+1/32/d*b/(a*b)^(1/2)/a/(cos(d*x+c)^2+(a*b)^(
1/2)/b-1)^2/(a^2-2*a*b+b^2)*cos(d*x+c)^3-7/64/d/b/(a*b)^(1/2)/(cos(d*x+c)^2
+(a*b)^(1/2)/b-1)^2/(a-b)*cos(d*x+c)-5/64/d/b/a/(cos(d*x+c)^2+(a*b)^(1/2)/b
-1)^2/(a-b)*cos(d*x+c)+1/32/d/(a*b)^(1/2)/a/(cos(d*x+c)^2+(a*b)^(1/2)/b-1)^
2/(a-b)*cos(d*x+c)-5/64/d/(a^2-2*a*b+b^2)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(
cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))-1/64/d/a/(a^2-2*a*b+b^2)/(((a*b)^(1
/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))*b-1/8/d/(a^2
-2*a*b+b^2)*b/(a*b)^(1/2)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((
a*b)^(1/2)-b)*b)^(1/2))+1/32/d/a/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(((a*b)^(1/2)-
b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))*b^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a-b*sin(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 8.15395, size = 9106, normalized size = 31.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a-b*sin(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(4*(5*a*b + b^2)*cos(d*x + c)^7 - 12*(7*a*b + b^2)*cos(d*x + c)^5 -
12*(3*a^2 - 10*a*b - b^2)*cos(d*x + c)^3 + ((a^3*b^2 - 2*a^2*b^3 + a*b^4)*d
*cos(d*x + c)^8 - 4*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cos(d*x + c)^6 - 2*(a^4
*b - 5*a^3*b^2 + 7*a^2*b^3 - 3*a*b^4)*d*cos(d*x + c)^4 + 4*(a^4*b - 3*a^3*b
^2 + 3*a^2*b^3 - a*b^4)*d*cos(d*x + c)^2 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a
^2*b^3 + a*b^4)*d)*sqrt(-((a^8*b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*
a^4*b^5 - a^3*b^6)*d^2*sqrt((625*a^4 + 7700*a^3*b + 21966*a^2*b^2 - 10780*a
*b^3 + 1225*b^4)/((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 + 21
0*a^9*b^7 - 252*a^8*b^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*a^4
*b^12 + a^3*b^13)*d^4)) + 105*a^3 + 70*a^2*b - 35*a*b^2 + 4*b^3)/((a^8*b -
5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2))*log((625*a
^3 + 3750*a^2*b - 1491*a*b^2 + 140*b^3)*cos(d*x + c) + ((5*a^10*b^2 - 16*a^
9*b^3 + 3*a^8*b^4 + 50*a^7*b^5 - 85*a^6*b^6 + 60*a^5*b^7 - 19*a^4*b^8 + 2*a
^3*b^9)*d^3*sqrt((625*a^4 + 7700*a^3*b + 21966*a^2*b^2 - 10780*a*b^3 + 1225
*b^4)/((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 + 210*a^9*b^7 -
 252*a^8*b^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*a^4*b^12 + a^3
*b^13)*d^4)) - (325*a^5*b + 1977*a^4*b^2 - 609*a^3*b^3 + 35*a^2*b^4)*d)*sqr
t(-((a^8*b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2
*sqrt((625*a^4 + 7700*a^3*b + 21966*a^2*b^2 - 10780*a*b^3 + 1225*b^4)/((a^1
3*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 + 210*a^9*b^7 - 252*a^8*b^
8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*a^4*b^12 + a^3*b^13)*d^4)
) + 105*a^3 + 70*a^2*b - 35*a*b^2 + 4*b^3)/((a^8*b - 5*a^7*b^2 + 10*a^6*b^3
 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2))) - ((a^3*b^2 - 2*a^2*b^3 + a*b^4
)*d*cos(d*x + c)^8 - 4*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cos(d*x + c)^6 - 2*(
a^4*b - 5*a^3*b^2 + 7*a^2*b^3 - 3*a*b^4)*d*cos(d*x + c)^4 + 4*(a^4*b - 3*a^
3*b^2 + 3*a^2*b^3 - a*b^4)*d*cos(d*x + c)^2 + (a^5 - 4*a^4*b + 6*a^3*b^2 -
4*a^2*b^3 + a*b^4)*d)*sqrt(((a^8*b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 +
5*a^4*b^5 - a^3*b^6)*d^2*sqrt((625*a^4 + 7700*a^3*b + 21966*a^2*b^2 - 10780
*a*b^3 + 1225*b^4)/((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 +
210*a^9*b^7 - 252*a^8*b^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*a
^4*b^12 + a^3*b^13)*d^4)) - 105*a^3 - 70*a^2*b + 35*a*b^2 - 4*b^3)/((a^8*b
- 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2))*log((625
*a^3 + 3750*a^2*b - 1491*a*b^2 + 140*b^3)*cos(d*x + c) + ((5*a^10*b^2 - 16*
a^9*b^3 + 3*a^8*b^4 + 50*a^7*b^5 - 85*a^6*b^6 + 60*a^5*b^7 - 19*a^4*b^8 + 2
*a^3*b^9)*d^3*sqrt((625*a^4 + 7700*a^3*b + 21966*a^2*b^2 - 10780*a*b^3 + 12
25*b^4)/((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 + 210*a^9*b^7
 - 252*a^8*b^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*a^4*b^12 + a
^3*b^13)*d^4)) + (325*a^5*b + 1977*a^4*b^2 - 609*a^3*b^3 + 35*a^2*b^4)*d)*s
qrt(((a^8*b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^
2*sqrt((625*a^4 + 7700*a^3*b + 21966*a^2*b^2 - 10780*a*b^3 + 1225*b^4)/((a^
13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 + 210*a^9*b^7 - 252*a^8*b
^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*a^4*b^12 + a^3*b^13)*d^4
)) - 105*a^3 - 70*a^2*b + 35*a*b^2 - 4*b^3)/((a^8*b - 5*a^7*b^2 + 10*a^6*b^
3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2))) - ((a^3*b^2 - 2*a^2*b^3 + a*b^
4)*d*cos(d*x + c)^8 - 4*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cos(d*x + c)^6 - 2*
(a^4*b - 5*a^3*b^2 + 7*a^2*b^3 - 3*a*b^4)*d*cos(d*x + c)^4 + 4*(a^4*b - 3*a
^3*b^2 + 3*a^2*b^3 - a*b^4)*d*cos(d*x + c)^2 + (a^5 - 4*a^4*b + 6*a^3*b^2 -
 4*a^2*b^3 + a*b^4)*d)*sqrt(-((a^8*b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4
+ 5*a^4*b^5 - a^3*b^6)*d^2*sqrt((625*a^4 + 7700*a^3*b + 21966*a^2*b^2 - 107
80*a*b^3 + 1225*b^4)/((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6
```

```
+ 210*a^9*b^7 - 252*a^8*b^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10
*a^4*b^12 + a^3*b^13)*d^4)) + 105*a^3 + 70*a^2*b - 35*a*b^2 + 4*b^3)/((a^8*
b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2))*log(-(
625*a^3 + 3750*a^2*b - 1491*a*b^2 + 140*b^3)*cos(d*x + c) + ((5*a^10*b^2 -
16*a^9*b^3 + 3*a^8*b^4 + 50*a^7*b^5 - 85*a^6*b^6 + 60*a^5*b^7 - 19*a^4*b^8
+ 2*a^3*b^9)*d^3*sqrt((625*a^4 + 7700*a^3*b + 21966*a^2*b^2 - 10780*a*b^3 +
 1225*b^4)/((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 + 210*a^9*
b^7 - 252*a^8*b^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*a^4*b^12
+ a^3*b^13)*d^4)) - (325*a^5*b + 1977*a^4*b^2 - 609*a^3*b^3 + 35*a^2*b^4)*d
)*sqrt(-((a^8*b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6
)*d^2*sqrt((625*a^4 + 7700*a^3*b + 21966*a^2*b^2 - 10780*a*b^3 + 1225*b^4)/
((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 + 210*a^9*b^7 - 252*a
^8*b^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*a^4*b^12 + a^3*b^13)
*d^4)) + 105*a^3 + 70*a^2*b - 35*a*b^2 + 4*b^3)/((a^8*b - 5*a^7*b^2 + 10*a^
6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2))) + ((a^3*b^2 - 2*a^2*b^3 +
a*b^4)*d*cos(d*x + c)^8 - 4*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cos(d*x + c)^6
- 2*(a^4*b - 5*a^3*b^2 + 7*a^2*b^3 - 3*a*b^4)*d*cos(d*x + c)^4 + 4*(a^4*b -
 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d*cos(d*x + c)^2 + (a^5 - 4*a^4*b + 6*a^3*b
^2 - 4*a^2*b^3 + a*b^4)*d)*sqrt(((a^8*b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b
^4 + 5*a^4*b^5 - a^3*b^6)*d^2*sqrt((625*a^4 + 7700*a^3*b + 21966*a^2*b^2 -
10780*a*b^3 + 1225*b^4)/((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b
^6 + 210*a^9*b^7 - 252*a^8*b^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 -
 10*a^4*b^12 + a^3*b^13)*d^4)) - 105*a^3 - 70*a^2*b + 35*a*b^2 - 4*b^3)/((a
^8*b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2))*log
(-(625*a^3 + 3750*a^2*b - 1491*a*b^2 + 140*b^3)*cos(d*x + c) + ((5*a^10*b^2
 - 16*a^9*b^3 + 3*a^8*b^4 + 50*a^7*b^5 - 85*a^6*b^6 + 60*a^5*b^7 - 19*a^4*b
^8 + 2*a^3*b^9)*d^3*sqrt((625*a^4 + 7700*a^3*b + 21966*a^2*b^2 - 10780*a*b^
3 + 1225*b^4)/((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 + 210*a
^9*b^7 - 252*a^8*b^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*a^4*b^
12 + a^3*b^13)*d^4)) + (325*a^5*b + 1977*a^4*b^2 - 609*a^3*b^3 + 35*a^2*b^4
)*d)*sqrt(((a^8*b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b
^6)*d^2*sqrt((625*a^4 + 7700*a^3*b + 21966*a^2*b^2 - 10780*a*b^3 + 1225*b^4
)/((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 + 210*a^9*b^7 - 252
*a^8*b^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*a^4*b^12 + a^3*b^1
3)*d^4)) - 105*a^3 - 70*a^2*b + 35*a*b^2 - 4*b^3)/((a^8*b - 5*a^7*b^2 + 10*
a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2))) + 4*(19*a^2 - 18*a*b - b
^2)*cos(d*x + c))/((a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cos(d*x + c)^8 - 4*(a^3*
b^2 - 2*a^2*b^3 + a*b^4)*d*cos(d*x + c)^6 - 2*(a^4*b - 5*a^3*b^2 + 7*a^2*b^
3 - 3*a*b^4)*d*cos(d*x + c)^4 + 4*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d
*cos(d*x + c)^2 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a-b*sin(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a-b*sin(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.228 $\int \frac{\sin(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=313

$$-\frac{\cos(c+dx)\left((7a-3b)(a+2b)-6b(2a-b)\cos^2(c+dx)\right)}{32a^2d(a-b)^2\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}-\frac{3\left(-10\sqrt{a}\sqrt{b}+7a+4b\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{5/2}\sqrt[4]{b}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}-\frac{3\left(10\sqrt{a}\sqrt{b}\right.}{64}$$

```
[Out] (-3*(7*a - 10*Sqrt[a]*Sqrt[b] + 4*b)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqr
t[a] - Sqrt[b]]])/(64*a^(5/2)*(Sqrt[a] - Sqrt[b])^(5/2)*b^(1/4)*d) - (3*(7*
a + 10*Sqrt[a]*Sqrt[b] + 4*b)*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] +
 Sqrt[b]]])/(64*a^(5/2)*(Sqrt[a] + Sqrt[b])^(5/2)*b^(1/4)*d) - (Cos[c + d*x
]*(a + b - b*Cos[c + d*x]^2))/(8*a*(a - b)*d*(a - b + 2*b*Cos[c + d*x]^2 -
b*Cos[c + d*x]^4)^2) - (Cos[c + d*x]*((7*a - 3*b)*(a + 2*b) - 6*(2*a - b)*b
*Cos[c + d*x]^2))/(32*a^2*(a - b)^2*d*(a - b + 2*b*Cos[c + d*x]^2 - b*Cos[c
 + d*x]^4))
```

Rubi [A] time = 0.458556, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3215, 1092, 1178, 1166, 205, 208}

$$-\frac{\cos(c+dx)\left((7a-3b)(a+2b)-6b(2a-b)\cos^2(c+dx)\right)}{32a^2d(a-b)^2\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}-\frac{3\left(-10\sqrt{a}\sqrt{b}+7a+4b\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{5/2}\sqrt[4]{b}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}-\frac{3\left(10\sqrt{a}\sqrt{b}\right.}{64}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] (-3*(7*a - 10*Sqrt[a]*Sqrt[b] + 4*b)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqr
t[a] - Sqrt[b]]])/(64*a^(5/2)*(Sqrt[a] - Sqrt[b])^(5/2)*b^(1/4)*d) - (3*(7*
a + 10*Sqrt[a]*Sqrt[b] + 4*b)*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] +
 Sqrt[b]]])/(64*a^(5/2)*(Sqrt[a] + Sqrt[b])^(5/2)*b^(1/4)*d) - (Cos[c + d*x
]*(a + b - b*Cos[c + d*x]^2))/(8*a*(a - b)*d*(a - b + 2*b*Cos[c + d*x]^2 -
b*Cos[c + d*x]^4)^2) - (Cos[c + d*x]*((7*a - 3*b)*(a + 2*b) - 6*(2*a - b)*b
*Cos[c + d*x]^2))/(32*a^2*(a - b)^2*d*(a - b + 2*b*Cos[c + d*x]^2 - b*Cos[c
 + d*x]^4))
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{\left(a-b+2bx^2-bx^4\right)^3}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\cos(c+dx)\left(a+b-b\cos^2(c+dx)\right)}{8a(a-b)d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{2(a-b)b+4b^2-4(4(a-b)b+\cdots}{\left(a-b+2bx^2-bx\cdots\right.}\cdots\right.}{16a(a\cdots}$$

$$= -\frac{\cos(c+dx)\left(a+b-b\cos^2(c+dx)\right)}{8a(a-b)d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)^2} - \frac{\cos(c+dx)\left((7a-3b)(a+2b\cdots\right.}{32a^2(a-b)^2d\left(a-b+2b\cos\cdots\right.}$$

$$= -\frac{\cos(c+dx)\left(a+b-b\cos^2(c+dx)\right)}{8a(a-b)d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)^2} - \frac{\cos(c+dx)\left((7a-3b)(a+2b\cdots\right.}{32a^2(a-b)^2d\left(a-b+2b\cos\cdots\right.}$$

$$= -\frac{3\left(7a-10\sqrt{a}\sqrt{b}+4b\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{5/2}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}\sqrt[4]{b}d} - \frac{3\left(7a+10\sqrt{a}\sqrt{b}+4b\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+\cdots}{\sqrt{\sqrt{a}+\sqrt{b}\cdots}}\right.}{64a^{5/2}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}\sqrt[4]{b}d}$$

Mathematica [C] time = 1.24294, size = 784, normalized size = 2.5

$$3i\text{RootSum}\left[-16\#1^4a+\#1^8b-4\#1^6b+6\#1^4b-4\#1^2b+b\&, \frac{-14i\#1^4a^2\log\left(\#1^2-2\#1\cos(c+dx)+1\right)+14i\#1^2a^2\log\left(\#1^2-2\#1\cos(c+dx\cdots\right.}{\cdots}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] ((-32*Cos[c + d*x]*(7*a^2 + 5*a*b - 3*b^2 + 3*b*(-2*a + b)*Cos[2*(c + d*x)]
))/(8*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*x)]) - (512*a*(a - b)
*Cos[c + d*x]*(2*a + b - b*Cos[2*(c + d*x)]))/(-8*a + 3*b - 4*b*Cos[2*(c +
d*x)] + b*Cos[4*(c + d*x)])^2 + (3*I)*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*
b*#1^4 - 4*b*#1^6 + b*#1^8 & , (4*a*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #
1)] - 2*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - (2*I)*a*b*Log[1 - 2*
Cos[c + d*x]*#1 + #1^2] + I*b^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] - 28*a^2*
ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 + 24*a*b*ArcTan[Sin[c + d*x]/
(Cos[c + d*x] - #1)]*#1^2 - 10*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]
*#1^2 + (14*I)*a^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (12*I)*a*b*Log[
1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + (5*I)*b^2*Log[1 - 2*Cos[c + d*x]*#1 +
#1^2]*#1^2 + 28*a^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - 24*a*b*
ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 + 10*b^2*ArcTan[Sin[c + d*x]/
(Cos[c + d*x] - #1)]*#1^4 - (14*I)*a^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1
^4 + (12*I)*a*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 - (5*I)*b^2*Log[1 -
2*Cos[c + d*x]*#1 + #1^2]*#1^4 - 4*a*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] -
#1)]*#1^6 + 2*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 + (2*I)*a*b
*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6 - I*b^2*Log[1 - 2*Cos[c + d*x]*#1 +
 #1^2]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/(128*
a^2*(a - b)^2*d)
```

Maple [B] time = 0.188, size = 1281, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a-b*sin(d*x+c)^4)^3,x)
```

```
[Out] -3/16/d/a/(cos(d*x+c)^2-1-(a*b)^(1/2)/b)^2/(a^2-2*a*b+b^2)*cos(d*x+c)^3+3/3
2/d*b/a^2/(cos(d*x+c)^2-1-(a*b)^(1/2)/b)^2/(a^2-2*a*b+b^2)*cos(d*x+c)^3+9/6
4/d/(a*b)^(1/2)/(cos(d*x+c)^2-1-(a*b)^(1/2)/b)^2/(a^2-2*a*b+b^2)*cos(d*x+c)
^3-3/64/d*b/(a*b)^(1/2)/a/(cos(d*x+c)^2-1-(a*b)^(1/2)/b)^2/(a^2-2*a*b+b^2)*
cos(d*x+c)^3-11/64/d/b/a/(cos(d*x+c)^2-1-(a*b)^(1/2)/b)^2/(a-b)*cos(d*x+c)+
3/32/d/a^2/(cos(d*x+c)^2-1-(a*b)^(1/2)/b)^2/(a-b)*cos(d*x+c)+5/64/d/(a*b)^(
1/2)/a/(cos(d*x+c)^2-1-(a*b)^(1/2)/b)^2/(a-b)*cos(d*x+c)+3/16/d/a/(a^2-2*a*
b+b^2)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(
1/2))*b-3/32/d*b^2/a^2/(a^2-2*a*b+b^2)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(co
s(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))-21/64/d/(a^2-2*a*b+b^2)*b/(a*b)^(1/2)
/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))+
27/64/d/a/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos
(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))*b^2-3/16/d*b^3/a^2/(a*b)^(1/2)/(a^2-2*
a*b+b^2)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)
^(1/2))-3/16/d/a/(cos(d*x+c)^2+(a*b)^(1/2)/b-1)^2/(a^2-2*a*b+b^2)*cos(d*x+c
)^3+3/32/d*b/a^2/(cos(d*x+c)^2+(a*b)^(1/2)/b-1)^2/(a^2-2*a*b+b^2)*cos(d*x+c
)^3-9/64/d/(a*b)^(1/2)/(cos(d*x+c)^2+(a*b)^(1/2)/b-1)^2/(a^2-2*a*b+b^2)*cos
(d*x+c)^3+3/64/d*b/(a*b)^(1/2)/a/(cos(d*x+c)^2+(a*b)^(1/2)/b-1)^2/(a^2-2*a*
b+b^2)*cos(d*x+c)^3-11/64/d/b/a/(cos(d*x+c)^2+(a*b)^(1/2)/b-1)^2/(a-b)*cos(
d*x+c)+3/32/d/a^2/(cos(d*x+c)^2+(a*b)^(1/2)/b-1)^2/(a-b)*cos(d*x+c)-5/64/d/
(a*b)^(1/2)/a/(cos(d*x+c)^2+(a*b)^(1/2)/b-1)^2/(a-b)*cos(d*x+c)-3/16/d/a/(a
^2-2*a*b+b^2)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b
)*b)^(1/2))*b+3/32/d*b^2/a^2/(a^2-2*a*b+b^2)/(((a*b)^(1/2)-b)*b)^(1/2)*arct
an(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))-21/64/d/(a^2-2*a*b+b^2)*b/(a*b)^
(1/2)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/
2))+27/64/d/a/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(
```

```
cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))*b^2-3/16/d*b^3/a^2/(a*b)^(1/2)/(a^2
-2*a*b+b^2)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*
b)^(1/2))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-b*sin(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 10.5685, size = 9480, normalized size = 30.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-b*sin(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(24*(2*a*b^2 - b^3)*cos(d*x + c)^7 - 4*(7*a^2*b + 35*a*b^2 - 18*b^3)
*cos(d*x + c)^5 - 8*(a^2*b - 22*a*b^2 + 9*b^3)*cos(d*x + c)^3 + 3*((a^4*b^2
 - 2*a^3*b^3 + a^2*b^4)*d*cos(d*x + c)^8 - 4*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4
)*d*cos(d*x + c)^6 - 2*(a^5*b - 5*a^4*b^2 + 7*a^3*b^3 - 3*a^2*b^4)*d*cos(d*
x + c)^4 + 4*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*cos(d*x + c)^2 + (
a^6 - 4*a^5*b + 6*a^4*b^2 - 4*a^3*b^3 + a^2*b^4)*d)*sqrt(-(105*a^4 - 210*a^
3*b + 189*a^2*b^2 - 84*a*b^3 + 16*b^4 + (a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a
^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2*sqrt((2401*a^4 - 5292*a^3*b + 4974*a^2*b^
2 - 2268*a*b^3 + 441*b^4)/((a^15*b - 10*a^14*b^2 + 45*a^13*b^3 - 120*a^12*b
^4 + 210*a^11*b^5 - 252*a^10*b^6 + 210*a^9*b^7 - 120*a^8*b^8 + 45*a^7*b^9 -
 10*a^6*b^10 + a^5*b^11)*d^4)))/((a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3
+ 5*a^6*b^4 - a^5*b^5)*d^2))*log(27*(2401*a^4 - 4802*a^3*b + 4189*a^2*b^2 -
 1788*a*b^3 + 336*b^4)*cos(d*x + c) - 27*((11*a^12*b - 66*a^11*b^2 + 169*a^
10*b^3 - 240*a^9*b^4 + 205*a^8*b^5 - 106*a^7*b^6 + 31*a^6*b^7 - 4*a^5*b^8)*
d^3*sqrt((2401*a^4 - 5292*a^3*b + 4974*a^2*b^2 - 2268*a*b^3 + 441*b^4)/((a^
15*b - 10*a^14*b^2 + 45*a^13*b^3 - 120*a^12*b^4 + 210*a^11*b^5 - 252*a^10*b
^6 + 210*a^9*b^7 - 120*a^8*b^8 + 45*a^7*b^9 - 10*a^6*b^10 + a^5*b^11)*d^4))
 - (343*a^7 - 623*a^6*b + 515*a^5*b^2 - 213*a^4*b^3 + 42*a^3*b^4)*d)*sqrt(-
(105*a^4 - 210*a^3*b + 189*a^2*b^2 - 84*a*b^3 + 16*b^4 + (a^10 - 5*a^9*b +
10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2*sqrt((2401*a^4 - 5292*a^
3*b + 4974*a^2*b^2 - 2268*a*b^3 + 441*b^4)/((a^15*b - 10*a^14*b^2 + 45*a^13
*b^3 - 120*a^12*b^4 + 210*a^11*b^5 - 252*a^10*b^6 + 210*a^9*b^7 - 120*a^8*b
^8 + 45*a^7*b^9 - 10*a^6*b^10 + a^5*b^11)*d^4)))/((a^10 - 5*a^9*b + 10*a^8*
b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2))) - 3*((a^4*b^2 - 2*a^3*b^3 +
a^2*b^4)*d*cos(d*x + c)^8 - 4*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d*cos(d*x + c
)^6 - 2*(a^5*b - 5*a^4*b^2 + 7*a^3*b^3 - 3*a^2*b^4)*d*cos(d*x + c)^4 + 4*(a
^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*cos(d*x + c)^2 + (a^6 - 4*a^5*b +
 6*a^4*b^2 - 4*a^3*b^3 + a^2*b^4)*d)*sqrt(-(105*a^4 - 210*a^3*b + 189*a^2*b
^2 - 84*a*b^3 + 16*b^4 - (a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*
b^4 - a^5*b^5)*d^2*sqrt((2401*a^4 - 5292*a^3*b + 4974*a^2*b^2 - 2268*a*b^3
+ 441*b^4)/((a^15*b - 10*a^14*b^2 + 45*a^13*b^3 - 120*a^12*b^4 + 210*a^11*b
^5 - 252*a^10*b^6 + 210*a^9*b^7 - 120*a^8*b^8 + 45*a^7*b^9 - 10*a^6*b^10 +
a^5*b^11)*d^4)))/((a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a
```

```
^5*b^5)*d^2))*log(27*(2401*a^4 - 4802*a^3*b + 4189*a^2*b^2 - 1788*a*b^3 + 3
36*b^4)*cos(d*x + c) - 27*((11*a^12*b - 66*a^11*b^2 + 169*a^10*b^3 - 240*a^
9*b^4 + 205*a^8*b^5 - 106*a^7*b^6 + 31*a^6*b^7 - 4*a^5*b^8)*d^3*sqrt((2401*
a^4 - 5292*a^3*b + 4974*a^2*b^2 - 2268*a*b^3 + 441*b^4)/((a^15*b - 10*a^14*
b^2 + 45*a^13*b^3 - 120*a^12*b^4 + 210*a^11*b^5 - 252*a^10*b^6 + 210*a^9*b^
7 - 120*a^8*b^8 + 45*a^7*b^9 - 10*a^6*b^10 + a^5*b^11)*d^4)) + (343*a^7 - 6
23*a^6*b + 515*a^5*b^2 - 213*a^4*b^3 + 42*a^3*b^4)*d)*sqrt(-(105*a^4 - 210*
a^3*b + 189*a^2*b^2 - 84*a*b^3 + 16*b^4 - (a^10 - 5*a^9*b + 10*a^8*b^2 - 10
*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2*sqrt((2401*a^4 - 5292*a^3*b + 4974*a^2*
b^2 - 2268*a*b^3 + 441*b^4)/((a^15*b - 10*a^14*b^2 + 45*a^13*b^3 - 120*a^12
*b^4 + 210*a^11*b^5 - 252*a^10*b^6 + 210*a^9*b^7 - 120*a^8*b^8 + 45*a^7*b^9
 - 10*a^6*b^10 + a^5*b^11)*d^4)))/((a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^
3 + 5*a^6*b^4 - a^5*b^5)*d^2))) - 3*((a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d*cos(
d*x + c)^8 - 4*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d*cos(d*x + c)^6 - 2*(a^5*b
 - 5*a^4*b^2 + 7*a^3*b^3 - 3*a^2*b^4)*d*cos(d*x + c)^4 + 4*(a^5*b - 3*a^4*b^
2 + 3*a^3*b^3 - a^2*b^4)*d*cos(d*x + c)^2 + (a^6 - 4*a^5*b + 6*a^4*b^2 - 4*
a^3*b^3 + a^2*b^4)*d)*sqrt(-(105*a^4 - 210*a^3*b + 189*a^2*b^2 - 84*a*b^3 +
 16*b^4 + (a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*
d^2*sqrt((2401*a^4 - 5292*a^3*b + 4974*a^2*b^2 - 2268*a*b^3 + 441*b^4)/((a^
15*b - 10*a^14*b^2 + 45*a^13*b^3 - 120*a^12*b^4 + 210*a^11*b^5 - 252*a^10*b
^6 + 210*a^9*b^7 - 120*a^8*b^8 + 45*a^7*b^9 - 10*a^6*b^10 + a^5*b^11)*d^4))
)/((a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2))*l
og(-27*(2401*a^4 - 4802*a^3*b + 4189*a^2*b^2 - 1788*a*b^3 + 336*b^4)*cos(d*
x + c) - 27*((11*a^12*b - 66*a^11*b^2 + 169*a^10*b^3 - 240*a^9*b^4 + 205*a^
8*b^5 - 106*a^7*b^6 + 31*a^6*b^7 - 4*a^5*b^8)*d^3*sqrt((2401*a^4 - 5292*a^3
*b + 4974*a^2*b^2 - 2268*a*b^3 + 441*b^4)/((a^15*b - 10*a^14*b^2 + 45*a^13*
b^3 - 120*a^12*b^4 + 210*a^11*b^5 - 252*a^10*b^6 + 210*a^9*b^7 - 120*a^8*b^
8 + 45*a^7*b^9 - 10*a^6*b^10 + a^5*b^11)*d^4)) - (343*a^7 - 623*a^6*b + 515
*a^5*b^2 - 213*a^4*b^3 + 42*a^3*b^4)*d)*sqrt(-(105*a^4 - 210*a^3*b + 189*a^
2*b^2 - 84*a*b^3 + 16*b^4 + (a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a
^6*b^4 - a^5*b^5)*d^2*sqrt((2401*a^4 - 5292*a^3*b + 4974*a^2*b^2 - 2268*a*b
^3 + 441*b^4)/((a^15*b - 10*a^14*b^2 + 45*a^13*b^3 - 120*a^12*b^4 + 210*a^1
1*b^5 - 252*a^10*b^6 + 210*a^9*b^7 - 120*a^8*b^8 + 45*a^7*b^9 - 10*a^6*b^10
 + a^5*b^11)*d^4)))/((a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4
- a^5*b^5)*d^2))) + 3*((a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d*cos(d*x + c)^8 - 4
*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d*cos(d*x + c)^6 - 2*(a^5*b - 5*a^4*b^2 +
7*a^3*b^3 - 3*a^2*b^4)*d*cos(d*x + c)^4 + 4*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3
- a^2*b^4)*d*cos(d*x + c)^2 + (a^6 - 4*a^5*b + 6*a^4*b^2 - 4*a^3*b^3 + a^2*
b^4)*d)*sqrt(-(105*a^4 - 210*a^3*b + 189*a^2*b^2 - 84*a*b^3 + 16*b^4 - (a^1
0 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2*sqrt((2401
*a^4 - 5292*a^3*b + 4974*a^2*b^2 - 2268*a*b^3 + 441*b^4)/((a^15*b - 10*a^14
*b^2 + 45*a^13*b^3 - 120*a^12*b^4 + 210*a^11*b^5 - 252*a^10*b^6 + 210*a^9*b
^7 - 120*a^8*b^8 + 45*a^7*b^9 - 10*a^6*b^10 + a^5*b^11)*d^4)))/((a^10 - 5*a
^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2))*log(-27*(2401*a
^4 - 4802*a^3*b + 4189*a^2*b^2 - 1788*a*b^3 + 336*b^4)*cos(d*x + c) - 27*((
11*a^12*b - 66*a^11*b^2 + 169*a^10*b^3 - 240*a^9*b^4 + 205*a^8*b^5 - 106*a^
7*b^6 + 31*a^6*b^7 - 4*a^5*b^8)*d^3*sqrt((2401*a^4 - 5292*a^3*b + 4974*a^2*
b^2 - 2268*a*b^3 + 441*b^4)/((a^15*b - 10*a^14*b^2 + 45*a^13*b^3 - 120*a^12
*b^4 + 210*a^11*b^5 - 252*a^10*b^6 + 210*a^9*b^7 - 120*a^8*b^8 + 45*a^7*b^9
 - 10*a^6*b^10 + a^5*b^11)*d^4)) + (343*a^7 - 623*a^6*b + 515*a^5*b^2 - 213
*a^4*b^3 + 42*a^3*b^4)*d)*sqrt(-(105*a^4 - 210*a^3*b + 189*a^2*b^2 - 84*a*b
^3 + 16*b^4 - (a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b
^5)*d^2*sqrt((2401*a^4 - 5292*a^3*b + 4974*a^2*b^2 - 2268*a*b^3 + 441*b^4)/
((a^15*b - 10*a^14*b^2 + 45*a^13*b^3 - 120*a^12*b^4 + 210*a^11*b^5 - 252*a^
10*b^6 + 210*a^9*b^7 - 120*a^8*b^8 + 45*a^7*b^9 - 10*a^6*b^10 + a^5*b^11)*d
^4)))/((a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2
))) + 4*(11*a^3 + 4*a^2*b - 21*a*b^2 + 6*b^3)*cos(d*x + c))/((a^4*b^2 - 2*a
^3*b^3 + a^2*b^4)*d*cos(d*x + c)^8 - 4*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d*co
s(d*x + c)^6 - 2*(a^5*b - 5*a^4*b^2 + 7*a^3*b^3 - 3*a^2*b^4)*d*cos(d*x + c)
```

```
^4 + 4*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*cos(d*x + c)^2 + (a^6 -
4*a^5*b + 6*a^4*b^2 - 4*a^3*b^3 + a^2*b^4)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-b*sin(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-b*sin(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.229 $\int \frac{\csc(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=617

$$-\frac{b\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{4a^2d(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}-\frac{b\cos(c+dx)\left(-(5a+b)\cos^2(c+dx)+11a+b\right)}{32a^2d(a-b)^2\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}-\frac{\sqrt[4]{b}}{}$$

```
[Out] -((5*Sqrt[a] - 2*Sqrt[b])*b^(1/4)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a
] - Sqrt[b]]])/(64*a^(5/2)*(Sqrt[a] - Sqrt[b])^(5/2)*d) - (b^(1/4)*ArcTan[(
b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(8*a^(5/2)*(Sqrt[a] - Sqrt[
b])^(3/2)*d) - (b^(1/4)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b
]]])/(2*a^3*Sqrt[Sqrt[a] - Sqrt[b]]*d) - ArcTanh[Cos[c + d*x]]/(a^3*d) + (b
^(1/4)*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(8*a^(5/2)*
(Sqrt[a] + Sqrt[b])^(3/2)*d) + (b^(1/4)*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt
[Sqrt[a] + Sqrt[b]]])/(2*a^3*Sqrt[Sqrt[a] + Sqrt[b]]*d) + ((5*Sqrt[a] + 2*S
qrt[b])*b^(1/4)*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(6
4*a^(5/2)*(Sqrt[a] + Sqrt[b])^(5/2)*d) - (b*Cos[c + d*x]*(2 - Cos[c + d*x]^
2))/(8*a*(a - b)*d*(a - b + 2*b*Cos[c + d*x]^2 - b*Cos[c + d*x]^4)^2) - (b*
Cos[c + d*x]*(2 - Cos[c + d*x]^2))/(4*a^2*(a - b)*d*(a - b + 2*b*Cos[c + d*
x]^2 - b*Cos[c + d*x]^4)) - (b*Cos[c + d*x]*(11*a + b - (5*a + b)*Cos[c + d
*x]^2))/(32*a^2*(a - b)^2*d*(a - b + 2*b*Cos[c + d*x]^2 - b*Cos[c + d*x]^4)
)
```

Rubi [A] time = 0.836883, antiderivative size = 617, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {3215, 1238, 207, 1178, 1166, 205, 208}

$$-\frac{b\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{4a^2d(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}-\frac{b\cos(c+dx)\left(-(5a+b)\cos^2(c+dx)+11a+b\right)}{32a^2d(a-b)^2\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}-\frac{\sqrt[4]{b}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a - b*Sin[c + d*x]^4)^3,x]
```


```
[Out] -((5*Sqrt[a] - 2*Sqrt[b])*b^(1/4)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a
] - Sqrt[b]]])/(64*a^(5/2)*(Sqrt[a] - Sqrt[b])^(5/2)*d) - (b^(1/4)*ArcTan[(
b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(8*a^(5/2)*(Sqrt[a] - Sqrt[
b])^(3/2)*d) - (b^(1/4)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b
]]])/(2*a^3*Sqrt[Sqrt[a] - Sqrt[b]]*d) - ArcTanh[Cos[c + d*x]]/(a^3*d) + (b
^(1/4)*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(8*a^(5/2)*
(Sqrt[a] + Sqrt[b])^(3/2)*d) + (b^(1/4)*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt
[Sqrt[a] + Sqrt[b]]])/(2*a^3*Sqrt[Sqrt[a] + Sqrt[b]]*d) + ((5*Sqrt[a] + 2*S
qrt[b])*b^(1/4)*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(6
4*a^(5/2)*(Sqrt[a] + Sqrt[b])^(5/2)*d) - (b*Cos[c + d*x]*(2 - Cos[c + d*x]^
2))/(8*a*(a - b)*d*(a - b + 2*b*Cos[c + d*x]^2 - b*Cos[c + d*x]^4)^2) - (b*
Cos[c + d*x]*(2 - Cos[c + d*x]^2))/(4*a^2*(a - b)*d*(a - b + 2*b*Cos[c + d*
x]^2 - b*Cos[c + d*x]^4)) - (b*Cos[c + d*x]*(11*a + b - (5*a + b)*Cos[c + d
*x]^2))/(32*a^2*(a - b)^2*d*(a - b + 2*b*Cos[c + d*x]^2 - b*Cos[c + d*x]^4)
)
```


Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1238

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b^2 - 4*a*c, 0] && ((IntegerQ[p]
 && IntegerQ[q]) || IGtQ[p, 0] || IGtQ[q, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\csc(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\left(a-b+2bx^2-bx^4\right)^3}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(-\frac{1}{a^3(-1+x^2)} + \frac{b-bx^2}{a\left(a-b+2bx^2-bx^4\right)^3} + \frac{b-bx^2}{a^2\left(a-b+2bx^2-bx^4\right)^2} + \frac{b-bx^2}{a^3\left(a-b+2bx^2-bx^4\right)}\right)dx, x, \cos(c\right.}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \cos(c+dx)\right)}{a^3 d} - \frac{\text{Subst}\left(\int \frac{b-bx^2}{a-b+2bx^2-bx^4}\,dx, x, \cos(c+dx)\right)}{a^3 d} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{a^3 d} - \frac{b\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{8a(a-b)d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)^2} - \frac{}{4a^2(a-b}$$

$$= -\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^3\sqrt{\sqrt{a}-\sqrt{b}}\,d} - \frac{\tanh^{-1}(\cos(c+dx))}{a^3 d} + \frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^3\sqrt{\sqrt{a}+\sqrt{b}}\,d} - \frac{b}{8a(a-b)d\left(\right.}$$

$$= -\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{5/2}\left(\sqrt{a}-\sqrt{b}\right)^{3/2} d} - \frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^3\sqrt{\sqrt{a}-\sqrt{b}}\,d} - \frac{\tanh^{-1}(\cos(c+dx))}{a^3 d} + \frac{\sqrt[4]{b}\tanh^{-1}\left(\right.}{8a^{5/2}\left(\sqrt{a}+\right.}$$

$$= -\frac{\left(5\sqrt{a}-2\sqrt{b}\right)\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{5/2}\left(\sqrt{a}-\sqrt{b}\right)^{5/2} d} - \frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{5/2}\left(\sqrt{a}-\sqrt{b}\right)^{3/2} d} - \frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^3\sqrt{\sqrt{a}-\sqrt{b}}\,d}$$

Mathematica [C] time = 4.27989, size = 920, normalized size = 1.49

$$\frac{512b(\cos(3(c+dx))-5\cos(c+dx))a^2}{(a-b)(-8a+3b-4b\cos(2(c+dx))+b\cos(4(c+dx)))^2} + \frac{32b\cos(c+dx)(-41a+23b+(13a-7b)\cos(2(c+dx)))a}{(a-b)^2(8a-3b+4b\cos(2(c+dx))-b\cos(4(c+dx)))} - 256\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) + 256\log\left(\text{si}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] ((32*a*b*Cos[c + d*x]*(-41*a + 23*b + (13*a - 7*b)*Cos[2*(c + d*x)]))/((a -
 b)^2*(8*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*x)])) + (512*a^2*b
*(-5*Cos[c + d*x] + Cos[3*(c + d*x)]))/((a - b)*(-8*a + 3*b - 4*b*Cos[2*(c
+ d*x)] + b*Cos[4*(c + d*x)])^2) - 256*Log[Cos[(c + d*x)/2]] + 256*Log[Sin[
(c + d*x)/2]] - (I*b*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6
 + b*#1^8 & , (-90*a^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + 142*a*b*A
rcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - 64*b^2*ArcTan[Sin[c + d*x]/(Cos[c
 + d*x] - #1)] + (45*I)*a^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] - (71*I)*a*b*
Log[1 - 2*Cos[c + d*x]*#1 + #1^2] + (32*I)*b^2*Log[1 - 2*Cos[c + d*x]*#1 +
#1^2] + 398*a^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 - 506*a*b*Arc
Tan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 + 192*b^2*ArcTan[Sin[c + d*x]/(C
os[c + d*x] - #1)]*#1^2 - (199*I)*a^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^
2 + (253*I)*a*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (96*I)*b^2*Log[1 -
 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - 398*a^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x]
 - #1)]*#1^4 + 506*a*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - 192*
b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 + (199*I)*a^2*Log[1 - 2*C
os[c + d*x]*#1 + #1^2]*#1^4 - (253*I)*a*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]
```

```
*#1^4 + (96*I)*b^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 + 90*a^2*ArcTan[S
in[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 - 142*a*b*ArcTan[Sin[c + d*x]/(Cos[c
+ d*x] - #1)]*#1^6 + 64*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 -
 (45*I)*a^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6 + (71*I)*a*b*Log[1 - 2*C
os[c + d*x]*#1 + #1^2]*#1^6 - (32*I)*b^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*
#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/(a - b)^2)/(
256*a^3*d)
```

Maple [B] time = 0.182, size = 1139, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a-b*sin(d*x+c)^4)^3,x)
```

```
[Out] 1/2/d/a^3*ln(-1+cos(d*x+c))-1/2/d/a^3*ln(1+cos(d*x+c))-13/32/d/(b*cos(d*x+c
)^4-2*b*cos(d*x+c)^2-a+b)^2*b^2/a/(a^2-2*a*b+b^2)*cos(d*x+c)^7+7/32/d*b^3/a
^2/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2/(a^2-2*a*b+b^2)*cos(d*x+c)^7+53/
32/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2/a/(a^2-2*a*b+b^2)*cos(d*x+c)^5
*b^2-29/32/d*b^3/a^2/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2/(a^2-2*a*b+b^2
)*cos(d*x+c)^5+17/32/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2*b/(a^2-2*a*b
+b^2)*cos(d*x+c)^3-39/16/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2/a/(a^2-2
*a*b+b^2)*cos(d*x+c)^3*b^2+37/32/d*b^3/a^2/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2
-a+b)^2/(a^2-2*a*b+b^2)*cos(d*x+c)^3-35/32/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)
^2-a+b)^2/(a-b)/a*b*cos(d*x+c)+15/32/d*b^2/a^2/(b*cos(d*x+c)^4-2*b*cos(d*x+
c)^2-a+b)^2/(a-b)*cos(d*x+c)-45/64/d/a/(a^2-2*a*b+b^2)/(((a*b)^(1/2)-b)*b)^
(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))*b+71/64/d*b^2/a^2/(a^2
-2*a*b+b^2)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*
b)^(1/2))-1/2/d*b^3/a^3/(a^2-2*a*b+b^2)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(co
s(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))-1/4/d/a/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(
((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))*b^2
+5/32/d*b^3/a^2/(a*b)^(1/2)/(a^2-2*a*b+b^2)/(((a*b)^(1/2)-b)*b)^(1/2)*arcta
n(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))+45/64/d/a/(a^2-2*a*b+b^2)/(((a*b)
^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))*b-71/64/
d*b^2/a^2/(a^2-2*a*b+b^2)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/((
(a*b)^(1/2)+b)*b)^(1/2))+1/2/d*b^3/a^3/(a^2-2*a*b+b^2)/(((a*b)^(1/2)+b)*b)^
(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))-1/4/d/a/(a^2-2*a*b+b^
2)/(a*b)^(1/2)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)
+b)*b)^(1/2))*b^2+5/32/d*b^3/a^2/(a*b)^(1/2)/(a^2-2*a*b+b^2)/(((a*b)^(1/2)+
b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-b*sin(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 25.2048, size = 12519, normalized size = 20.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-b*sin(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(4*(13*a^2*b^2 - 7*a*b^3)*cos(d*x + c)^7 - 4*(53*a^2*b^2 - 29*a*b^3)
*cos(d*x + c)^5 - 4*(17*a^3*b - 78*a^2*b^2 + 37*a*b^3)*cos(d*x + c)^3 - ((a
^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*cos(d*x + c)^8 - 4*(a^5*b^2 - 2*a^4*b^3 + a
^3*b^4)*d*cos(d*x + c)^6 - 2*(a^6*b - 5*a^5*b^2 + 7*a^4*b^3 - 3*a^3*b^4)*d*
cos(d*x + c)^4 + 4*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*d*cos(d*x + c)
^2 + (a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*d)*sqrt(-(3465*a^4*b
 - 9306*a^3*b^2 + 10045*a^2*b^3 - 5084*a*b^4 + 1024*b^5 + (a^11 - 5*a^10*b
+ 10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d^2*sqrt((4100625*a^8*b -
19010700*a^7*b^2 + 39971086*a^6*b^3 - 49679452*a^5*b^4 + 39947241*a^4*b^5 -
 21320992*a^3*b^6 + 7401472*a^2*b^7 - 1536000*a*b^8 + 147456*b^9)/((a^21 -
10*a^20*b + 45*a^19*b^2 - 120*a^18*b^3 + 210*a^17*b^4 - 252*a^16*b^5 + 210*
a^15*b^6 - 120*a^14*b^7 + 45*a^13*b^8 - 10*a^12*b^9 + a^11*b^10)*d^4)))/((a
^11 - 5*a^10*b + 10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d^2))*log((
4100625*a^6*b - 14762250*a^5*b^2 + 23227949*a^4*b^3 - 20354340*a^3*b^4 + 10
504896*a^2*b^5 - 3044864*a*b^6 + 393216*b^7)*cos(d*x + c) - ((45*a^16 - 280
*a^15*b + 747*a^14*b^2 - 1110*a^13*b^3 + 995*a^12*b^4 - 540*a^11*b^5 + 165*
a^10*b^6 - 22*a^9*b^7)*d^3*sqrt((4100625*a^8*b - 19010700*a^7*b^2 + 3997108
6*a^6*b^3 - 49679452*a^5*b^4 + 39947241*a^4*b^5 - 21320992*a^3*b^6 + 740147
2*a^2*b^7 - 1536000*a*b^8 + 147456*b^9)/((a^21 - 10*a^20*b + 45*a^19*b^2 -
120*a^18*b^3 + 210*a^17*b^4 - 252*a^16*b^5 + 210*a^15*b^6 - 120*a^14*b^7 +
45*a^13*b^8 - 10*a^12*b^9 + a^11*b^10)*d^4)) - (123525*a^9*b - 450359*a^8*b
^2 + 715183*a^7*b^3 - 630957*a^6*b^4 + 327152*a^5*b^5 - 95104*a^4*b^6 + 122
88*a^3*b^7)*d)*sqrt(-(3465*a^4*b - 9306*a^3*b^2 + 10045*a^2*b^3 - 5084*a*b^
4 + 1024*b^5 + (a^11 - 5*a^10*b + 10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6
*b^5)*d^2*sqrt((4100625*a^8*b - 19010700*a^7*b^2 + 39971086*a^6*b^3 - 49679
452*a^5*b^4 + 39947241*a^4*b^5 - 21320992*a^3*b^6 + 7401472*a^2*b^7 - 15360
00*a*b^8 + 147456*b^9)/((a^21 - 10*a^20*b + 45*a^19*b^2 - 120*a^18*b^3 + 21
0*a^17*b^4 - 252*a^16*b^5 + 210*a^15*b^6 - 120*a^14*b^7 + 45*a^13*b^8 - 10*
a^12*b^9 + a^11*b^10)*d^4)))/((a^11 - 5*a^10*b + 10*a^9*b^2 - 10*a^8*b^3 +
5*a^7*b^4 - a^6*b^5)*d^2))) + ((a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*cos(d*x +
c)^8 - 4*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*cos(d*x + c)^6 - 2*(a^6*b - 5*a^
5*b^2 + 7*a^4*b^3 - 3*a^3*b^4)*d*cos(d*x + c)^4 + 4*(a^6*b - 3*a^5*b^2 + 3*
a^4*b^3 - a^3*b^4)*d*cos(d*x + c)^2 + (a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^
3 + a^3*b^4)*d)*sqrt(-(3465*a^4*b - 9306*a^3*b^2 + 10045*a^2*b^3 - 5084*a*b
^4 + 1024*b^5 - (a^11 - 5*a^10*b + 10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^
6*b^5)*d^2*sqrt((4100625*a^8*b - 19010700*a^7*b^2 + 39971086*a^6*b^3 - 4967
9452*a^5*b^4 + 39947241*a^4*b^5 - 21320992*a^3*b^6 + 7401472*a^2*b^7 - 1536
000*a*b^8 + 147456*b^9)/((a^21 - 10*a^20*b + 45*a^19*b^2 - 120*a^18*b^3 + 2
10*a^17*b^4 - 252*a^16*b^5 + 210*a^15*b^6 - 120*a^14*b^7 + 45*a^13*b^8 - 10
*a^12*b^9 + a^11*b^10)*d^4)))/((a^11 - 5*a^10*b + 10*a^9*b^2 - 10*a^8*b^3 +
 5*a^7*b^4 - a^6*b^5)*d^2))*log((4100625*a^6*b - 14762250*a^5*b^2 + 2322794
9*a^4*b^3 - 20354340*a^3*b^4 + 10504896*a^2*b^5 - 3044864*a*b^6 + 393216*b^
7)*cos(d*x + c) - ((45*a^16 - 280*a^15*b + 747*a^14*b^2 - 1110*a^13*b^3 + 9
95*a^12*b^4 - 540*a^11*b^5 + 165*a^10*b^6 - 22*a^9*b^7)*d^3*sqrt((4100625*a
^8*b - 19010700*a^7*b^2 + 39971086*a^6*b^3 - 49679452*a^5*b^4 + 39947241*a^
4*b^5 - 21320992*a^3*b^6 + 7401472*a^2*b^7 - 1536000*a*b^8 + 147456*b^9)/((
a^21 - 10*a^20*b + 45*a^19*b^2 - 120*a^18*b^3 + 210*a^17*b^4 - 252*a^16*b^5
 + 210*a^15*b^6 - 120*a^14*b^7 + 45*a^13*b^8 - 10*a^12*b^9 + a^11*b^10)*d^4
)) + (123525*a^9*b - 450359*a^8*b^2 + 715183*a^7*b^3 - 630957*a^6*b^4 + 327
152*a^5*b^5 - 95104*a^4*b^6 + 12288*a^3*b^7)*d)*sqrt(-(3465*a^4*b - 9306*a^
3*b^2 + 10045*a^2*b^3 - 5084*a*b^4 + 1024*b^5 - (a^11 - 5*a^10*b + 10*a^9*b
```

^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d^2*sqrt((4100625*a^8*b - 19010700*a
^7*b^2 + 39971086*a^6*b^3 - 49679452*a^5*b^4 + 39947241*a^4*b^5 - 21320992*
a^3*b^6 + 7401472*a^2*b^7 - 1536000*a*b^8 + 147456*b^9)/((a^21 - 10*a^20*b
+ 45*a^19*b^2 - 120*a^18*b^3 + 210*a^17*b^4 - 252*a^16*b^5 + 210*a^15*b^6 -
120*a^14*b^7 + 45*a^13*b^8 - 10*a^12*b^9 + a^11*b^10)*d^4)))/((a^11 - 5*a^
10*b + 10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d^2))) + ((a^5*b^2 -
2*a^4*b^3 + a^3*b^4)*d*cos(d*x + c)^8 - 4*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d
*cos(d*x + c)^6 - 2*(a^6*b - 5*a^5*b^2 + 7*a^4*b^3 - 3*a^3*b^4)*d*cos(d*x +
c)^4 + 4*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*d*cos(d*x + c)^2 + (a^7
- 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*d)*sqrt(-(3465*a^4*b - 9306*a
^3*b^2 + 10045*a^2*b^3 - 5084*a*b^4 + 1024*b^5 + (a^11 - 5*a^10*b + 10*a^9*
b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d^2*sqrt((4100625*a^8*b - 19010700*
a^7*b^2 + 39971086*a^6*b^3 - 49679452*a^5*b^4 + 39947241*a^4*b^5 - 21320992
*a^3*b^6 + 7401472*a^2*b^7 - 1536000*a*b^8 + 147456*b^9)/((a^21 - 10*a^20*b
+ 45*a^19*b^2 - 120*a^18*b^3 + 210*a^17*b^4 - 252*a^16*b^5 + 210*a^15*b^6
- 120*a^14*b^7 + 45*a^13*b^8 - 10*a^12*b^9 + a^11*b^10)*d^4)))/((a^11 - 5*a
^10*b + 10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d^2))*log(-(4100625*
a^6*b - 14762250*a^5*b^2 + 23227949*a^4*b^3 - 20354340*a^3*b^4 + 10504896*a
^2*b^5 - 3044864*a*b^6 + 393216*b^7)*cos(d*x + c) - ((45*a^16 - 280*a^15*b
+ 747*a^14*b^2 - 1110*a^13*b^3 + 995*a^12*b^4 - 540*a^11*b^5 + 165*a^10*b^6
- 22*a^9*b^7)*d^3*sqrt((4100625*a^8*b - 19010700*a^7*b^2 + 39971086*a^6*b^
3 - 49679452*a^5*b^4 + 39947241*a^4*b^5 - 21320992*a^3*b^6 + 7401472*a^2*b^
7 - 1536000*a*b^8 + 147456*b^9)/((a^21 - 10*a^20*b + 45*a^19*b^2 - 120*a^18
*b^3 + 210*a^17*b^4 - 252*a^16*b^5 + 210*a^15*b^6 - 120*a^14*b^7 + 45*a^13*
b^8 - 10*a^12*b^9 + a^11*b^10)*d^4)) - (123525*a^9*b - 450359*a^8*b^2 + 715
183*a^7*b^3 - 630957*a^6*b^4 + 327152*a^5*b^5 - 95104*a^4*b^6 + 12288*a^3*b
^7)*d)*sqrt(-(3465*a^4*b - 9306*a^3*b^2 + 10045*a^2*b^3 - 5084*a*b^4 + 1024
*b^5 + (a^11 - 5*a^10*b + 10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d^
2*sqrt((4100625*a^8*b - 19010700*a^7*b^2 + 39971086*a^6*b^3 - 49679452*a^5*
b^4 + 39947241*a^4*b^5 - 21320992*a^3*b^6 + 7401472*a^2*b^7 - 1536000*a*b^8
+ 147456*b^9)/((a^21 - 10*a^20*b + 45*a^19*b^2 - 120*a^18*b^3 + 210*a^17*b
^4 - 252*a^16*b^5 + 210*a^15*b^6 - 120*a^14*b^7 + 45*a^13*b^8 - 10*a^12*b^9
+ a^11*b^10)*d^4)))/((a^11 - 5*a^10*b + 10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b^
4 - a^6*b^5)*d^2))) - ((a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*cos(d*x + c)^8 - 4
*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*cos(d*x + c)^6 - 2*(a^6*b - 5*a^5*b^2 +
7*a^4*b^3 - 3*a^3*b^4)*d*cos(d*x + c)^4 + 4*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3
- a^3*b^4)*d*cos(d*x + c)^2 + (a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*
b^4)*d)*sqrt(-(3465*a^4*b - 9306*a^3*b^2 + 10045*a^2*b^3 - 5084*a*b^4 + 102
4*b^5 - (a^11 - 5*a^10*b + 10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d
^2*sqrt((4100625*a^8*b - 19010700*a^7*b^2 + 39971086*a^6*b^3 - 49679452*a^5
*b^4 + 39947241*a^4*b^5 - 21320992*a^3*b^6 + 7401472*a^2*b^7 - 1536000*a*b^
8 + 147456*b^9)/((a^21 - 10*a^20*b + 45*a^19*b^2 - 120*a^18*b^3 + 210*a^17*
b^4 - 252*a^16*b^5 + 210*a^15*b^6 - 120*a^14*b^7 + 45*a^13*b^8 - 10*a^12*b^
9 + a^11*b^10)*d^4)))/((a^11 - 5*a^10*b + 10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b
^4 - a^6*b^5)*d^2))*log(-(4100625*a^6*b - 14762250*a^5*b^2 + 23227949*a^4*b
^3 - 20354340*a^3*b^4 + 10504896*a^2*b^5 - 3044864*a*b^6 + 393216*b^7)*cos(
d*x + c) - ((45*a^16 - 280*a^15*b + 747*a^14*b^2 - 1110*a^13*b^3 + 995*a^12
*b^4 - 540*a^11*b^5 + 165*a^10*b^6 - 22*a^9*b^7)*d^3*sqrt((4100625*a^8*b -
19010700*a^7*b^2 + 39971086*a^6*b^3 - 49679452*a^5*b^4 + 39947241*a^4*b^5 -
21320992*a^3*b^6 + 7401472*a^2*b^7 - 1536000*a*b^8 + 147456*b^9)/((a^21 -
10*a^20*b + 45*a^19*b^2 - 120*a^18*b^3 + 210*a^17*b^4 - 252*a^16*b^5 + 210*
a^15*b^6 - 120*a^14*b^7 + 45*a^13*b^8 - 10*a^12*b^9 + a^11*b^10)*d^4)) + (1
23525*a^9*b - 450359*a^8*b^2 + 715183*a^7*b^3 - 630957*a^6*b^4 + 327152*a^5
*b^5 - 95104*a^4*b^6 + 12288*a^3*b^7)*d)*sqrt(-(3465*a^4*b - 9306*a^3*b^2 +
10045*a^2*b^3 - 5084*a*b^4 + 1024*b^5 - (a^11 - 5*a^10*b + 10*a^9*b^2 - 10
*a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d^2*sqrt((4100625*a^8*b - 19010700*a^7*b^2
+ 39971086*a^6*b^3 - 49679452*a^5*b^4 + 39947241*a^4*b^5 - 21320992*a^3*b^6
+ 7401472*a^2*b^7 - 1536000*a*b^8 + 147456*b^9)/((a^21 - 10*a^20*b + 45*a^
19*b^2 - 120*a^18*b^3 + 210*a^17*b^4 - 252*a^16*b^5 + 210*a^15*b^6 - 120*a^

```
14*b^7 + 45*a^13*b^8 - 10*a^12*b^9 + a^11*b^10)*d^4)))/((a^11 - 5*a^10*b +
10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d^2))) + 20*(7*a^3*b - 10*a^
2*b^2 + 3*a*b^3)*cos(d*x + c) + 64*((a^2*b^2 - 2*a*b^3 + b^4)*cos(d*x + c)^
8 - 4*(a^2*b^2 - 2*a*b^3 + b^4)*cos(d*x + c)^6 - 2*(a^3*b - 5*a^2*b^2 + 7*a
*b^3 - 3*b^4)*cos(d*x + c)^4 + a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4 +
4*(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*cos(d*x + c)^2)*log(1/2*cos(d*x + c)
+ 1/2) - 64*((a^2*b^2 - 2*a*b^3 + b^4)*cos(d*x + c)^8 - 4*(a^2*b^2 - 2*a*b^
3 + b^4)*cos(d*x + c)^6 - 2*(a^3*b - 5*a^2*b^2 + 7*a*b^3 - 3*b^4)*cos(d*x +
 c)^4 + a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4 + 4*(a^3*b - 3*a^2*b^2 +
3*a*b^3 - b^4)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2))/((a^5*b^2 - 2*
a^4*b^3 + a^3*b^4)*d*cos(d*x + c)^8 - 4*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*c
os(d*x + c)^6 - 2*(a^6*b - 5*a^5*b^2 + 7*a^4*b^3 - 3*a^3*b^4)*d*cos(d*x + c
)^4 + 4*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*d*cos(d*x + c)^2 + (a^7 -
 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-b*sin(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-b*sin(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.230 $\int \frac{\sin^8(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=319

$$-\frac{\left(2\sqrt{a}-5\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{3/4}b^{3/2}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{\left(2\sqrt{a}+5\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{3/4}b^{3/2}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}+\frac{\tan^9(c+dx)}{8ad\left((a-b)\tan^4(c+dx)+2a\tan\right.}$$

```
[Out] -((2*Sqrt[a] - 5*Sqrt[b])*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(
1/4)])/(64*a^(3/4)*(Sqrt[a] - Sqrt[b])^(5/2)*b^(3/2)*d) + ((2*Sqrt[a] + 5*S
qrt[b])*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(64*a^(3/4)
*(Sqrt[a] + Sqrt[b])^(5/2)*b^(3/2)*d) - ((a + 5*b)*Tan[c + d*x])/(32*a*(a -
 b)^2*b*d) + Tan[c + d*x]^3/(32*a*(a - b)*b*d) + Tan[c + d*x]^9/(8*a*d*(a +
 2*a*Tan[c + d*x]^2 + (a - b)*Tan[c + d*x]^4)^2) - (Sec[c + d*x]^2*Tan[c +
d*x]^5)/(32*a*b*d*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan[c + d*x]^4))
```

Rubi [A] time = 0.530171, antiderivative size = 319, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {3217, 1275, 12, 1120, 1279, 1166, 205}

$$-\frac{\left(2\sqrt{a}-5\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{3/4}b^{3/2}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{\left(2\sqrt{a}+5\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{3/4}b^{3/2}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}+\frac{\tan^9(c+dx)}{8ad\left((a-b)\tan^4(c+dx)+2a\tan\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^8/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] -((2*Sqrt[a] - 5*Sqrt[b])*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(
1/4)])/(64*a^(3/4)*(Sqrt[a] - Sqrt[b])^(5/2)*b^(3/2)*d) + ((2*Sqrt[a] + 5*S
qrt[b])*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(64*a^(3/4)
*(Sqrt[a] + Sqrt[b])^(5/2)*b^(3/2)*d) - ((a + 5*b)*Tan[c + d*x])/(32*a*(a -
 b)^2*b*d) + Tan[c + d*x]^3/(32*a*(a - b)*b*d) + Tan[c + d*x]^9/(8*a*d*(a +
 2*a*Tan[c + d*x]^2 + (a - b)*Tan[c + d*x]^4)^2) - (Sec[c + d*x]^2*Tan[c +
d*x]^5)/(32*a*b*d*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan[c + d*x]^4))
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^8(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^8(1+x^2)}{\left(a+2ax^2+(a-b)x^4\right)^3}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\tan^9(c+dx)}{8ad\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} + \frac{\text{Subst}\left(\int -\frac{2bx^8}{\left(a+2ax^2+(a-b)x^4\right)^2}\,dx, x, \text{ta}\right.}{16abd}$$

$$= \frac{\tan^9(c+dx)}{8ad\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{x^8}{\left(a+2ax^2+(a-b)x^4\right)^2}\,dx, x, \tan\right.}{8ad}$$

$$= \frac{\tan^9(c+dx)}{8ad\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{\sec^2(c+dx)\tan^5(c+}{32abd\left(a+2a\tan^2(c+dx)+(a-\right.}$$

$$= \frac{\tan^3(c+dx)}{32a(a-b)bd} + \frac{\tan^9(c+dx)}{8ad\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{\sec^2(c}{32abd\left(a+2a\tan^2\right.}$$

$$= -\frac{(a+5b)\tan(c+dx)}{32a(a-b)^2bd} + \frac{\tan^3(c+dx)}{32a(a-b)bd} + \frac{\tan^9(c+dx)}{8ad\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx\right.}$$

$$= -\frac{(a+5b)\tan(c+dx)}{32a(a-b)^2bd} + \frac{\tan^3(c+dx)}{32a(a-b)bd} + \frac{\tan^9(c+dx)}{8ad\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx\right.}$$

$$= -\frac{\left(2\sqrt{a}-5\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{3/2}d} + \frac{\left(2\sqrt{a}+5\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}b^{3/2}d} - \frac{(a-}{}$$

Mathematica [A] time = 4.01175, size = 331, normalized size = 1.04

$$\frac{\frac{\left(2a^{3/2}\sqrt{b}-8\sqrt{a}b^{3/2}+ab+5b^2\right)\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}+a}} + \frac{8b\sin(2(c+dx))((5b-2a)\cos(2(c+dx))+5a-14b)}{8a+4b\cos(2(c+dx))-b\cos(4(c+dx))-3b} + \frac{64ab(a-b)(\sin(4(c+dx))-6\sin(2(c+dx)))}{(-8a-4b\cos(2(c+dx))+b\cos(4(c+dx))+3b)^2}}{64b^2d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^8/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] (((2*a^(3/2)*Sqrt[b] + a*b - 8*Sqrt[a]*b^(3/2) + 5*b^2)*ArcTan[((Sqrt[a] +
Sqrt[b])*Tan[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[a + Sqrt[a
]*Sqrt[b]]) + ((2*Sqrt[a] - 5*Sqrt[b])*(Sqrt[a] + Sqrt[b])^2*Sqrt[b]*ArcTan
h[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*
Sqrt[-a + Sqrt[a]*Sqrt[b]]) + (8*b*(5*a - 14*b + (-2*a + 5*b)*Cos[2*(c + d*
x)])*Sin[2*(c + d*x)])/(8*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*x
)]) + (64*a*(a - b)*b*(-6*Sin[2*(c + d*x)] + Sin[4*(c + d*x)]))/(-8*a + 3*b
 - 4*b*Cos[2*(c + d*x)] + b*Cos[4*(c + d*x)])^2)/(64*(a - b)^2*b^2*d)
```

Maple [B] time = 0.135, size = 1634, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^8/(a-b*sin(d*x+c)^4)^3,x)
```

```
[Out] -1/32/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2/(a-b)/b*tan(d*
x+c)^7*a-19/32/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2/(a-b)
*tan(d*x+c)^7-3/32/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2/b
/(a^2-2*a*b+b^2)*tan(d*x+c)^5*a^2-15/16/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*
a*tan(d*x+c)^2+a)^2/(a^2-2*a*b+b^2)*tan(d*x+c)^5*a+9/32/d/(tan(d*x+c)^4*a-t
an(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2*b/(a^2-2*a*b+b^2)*tan(d*x+c)^5-3/32/d/(
tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2*a^2/b/(a^2-2*a*b+b^2)*t
an(d*x+c)^3-21/32/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2*a/
(a^2-2*a*b+b^2)*tan(d*x+c)^3-1/32/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(
d*x+c)^2+a)^2*a^2/b/(a^2-2*a*b+b^2)*tan(d*x+c)-5/32/d/(tan(d*x+c)^4*a-tan(d
*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2*a/(a^2-2*a*b+b^2)*tan(d*x+c)-1/64/d/b/(a^2-
2*a*b+b^2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a
*b)^(1/2)+a)*(a-b))^(1/2))*a^2+7/32/d/(a^2-2*a*b+b^2)*a/(a-b)/(((a*b)^(1/2)
+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-1/3
2/d/b/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arcta
n((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*a^3+11/64/d/(a^2-2*a*b+b^
2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/
(((a*b)^(1/2)+a)*(a-b))^(1/2))*a^2-1/16/d*b/(a^2-2*a*b+b^2)*a/(a*b)^(1/2)/(
a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)
*(a-b))^(1/2))-1/64/d/b/(a^2-2*a*b+b^2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)
*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*a^2+7/32/d/(a^2-2
*a*b+b^2)*a/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(
((a*b)^(1/2)-a)*(a-b))^(1/2))+1/32/d/b/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(a-b)/((
(a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b
))^(1/2))*a^3-11/64/d/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a
-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*a^2+1/1
6/d*b/(a^2-2*a*b+b^2)*a/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arc
tanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-13/64/d*b/(a^2-2*a*b+
b^2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1
/2)+a)*(a-b))^(1/2))-5/64/d*b^2/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(
1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))
-13/64/d*b/(a^2-2*a*b+b^2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+
b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))+5/64/d*b^2/(a^2-2*a*b+b^2)/(a*
b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a
*b)^(1/2)-a)*(a-b))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a-b*sin(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(4*(72*a^2*b^2 - 155*a*b^3 + 26*b^4)*cos(4*d*x + 4*c)*sin(2*d*x + 2*c)
 + ((a*b^3 - 4*b^4)*sin(14*d*x + 14*c) - (32*a^2*b^2 - 58*a*b^3 - b^4)*sin(
12*d*x + 12*c) + 3*(48*a^2*b^2 - 73*a*b^3 + 20*b^4)*sin(10*d*x + 10*c) + (2
56*a^3*b - 832*a^2*b^2 + 550*a*b^3 - 175*b^4)*sin(8*d*x + 8*c) + (112*a^2*b
^2 - 533*a*b^3 + 220*b^4)*sin(6*d*x + 6*c) - (32*a^2*b^2 - 158*a*b^3 + 141*
b^4)*sin(4*d*x + 4*c) - (17*a*b^3 - 44*b^4)*sin(2*d*x + 2*c))*cos(16*d*x +
16*c) + 2*(2*(72*a^2*b^2 - 155*a*b^3 + 26*b^4)*sin(12*d*x + 12*c) - 8*(80*a
^2*b^2 - 145*a*b^3 + 44*b^4)*sin(10*d*x + 10*c) - 3*(384*a^3*b - 1312*a^2*b
^2 + 873*a*b^3 - 280*b^4)*sin(8*d*x + 8*c) - 16*(32*a^2*b^2 - 151*a*b^3 + 6
```

2*b^4)*sin(6*d*x + 6*c) + 2*(72*a^2*b^2 - 355*a*b^3 + 310*b^4)*sin(4*d*x + 4*c) + 24*(3*a*b^3 - 8*b^4)*sin(2*d*x + 2*c))*cos(14*d*x + 14*c) - 2*(2*(128*a^3*b - 456*a^2*b^2 + 1233*a*b^3 - 434*b^4)*sin(10*d*x + 10*c) - (6400*a^3*b - 13888*a^2*b^2 + 8566*a*b^3 - 2485*b^4)*sin(8*d*x + 8*c) - 2*(128*a^3*b + 2744*a^2*b^2 - 4711*a*b^3 + 1554*b^4)*sin(6*d*x + 6*c) + 4*(400*a^2*b^2 - 918*a*b^3 + 497*b^4)*sin(4*d*x + 4*c) - 2*(72*a^2*b^2 - 355*a*b^3 + 310*b^4)*sin(2*d*x + 2*c))*cos(12*d*x + 12*c) - 2*((2048*a^4 + 18560*a^3*b - 24752*a^2*b^2 + 13175*a*b^3 - 2800*b^4)*sin(8*d*x + 8*c) + 8*(256*a^3*b + 2400*a^2*b^2 - 2379*a*b^3 + 560*b^4)*sin(6*d*x + 6*c) - 2*(128*a^3*b + 2744*a^2*b^2 - 4711*a*b^3 + 1554*b^4)*sin(4*d*x + 4*c) + 16*(32*a^2*b^2 - 151*a*b^3 + 62*b^4)*sin(2*d*x + 2*c))*cos(10*d*x + 10*c) - 2*((2048*a^4 + 18560*a^3*b - 24752*a^2*b^2 + 13175*a*b^3 - 2800*b^4)*sin(6*d*x + 6*c) - (6400*a^3*b - 13888*a^2*b^2 + 8566*a*b^3 - 2485*b^4)*sin(4*d*x + 4*c) + 3*(384*a^3*b - 1312*a^2*b^2 + 873*a*b^3 - 280*b^4)*sin(2*d*x + 2*c))*cos(8*d*x + 8*c) - 4*((128*a^3*b - 456*a^2*b^2 + 1233*a*b^3 - 434*b^4)*sin(4*d*x + 4*c) + 4*(80*a^2*b^2 - 145*a*b^3 + 44*b^4)*sin(2*d*x + 2*c))*cos(6*d*x + 6*c) - 8*((a^2*b^5 - 2*a*b^6 + b^7)*d*cos(16*d*x + 16*c)^2 + 64*(a^2*b^5 - 2*a*b^6 + b^7)*d*cos(14*d*x + 14*c)^2 + 16*(64*a^4*b^3 - 240*a^3*b^4 + 337*a^2*b^5 - 210*a*b^6 + 49*b^7)*d*cos(12*d*x + 12*c)^2 + 64*(256*a^4*b^3 - 736*a^3*b^4 + 753*a^2*b^5 - 322*a*b^6 + 49*b^7)*d*cos(10*d*x + 10*c)^2 + 4*(16384*a^6*b - 57344*a^5*b^2 + 83712*a^4*b^3 - 67648*a^3*b^4 + 32841*a^2*b^5 - 9170*a*b^6 + 1225*b^7)*d*cos(8*d*x + 8*c)^2 + 64*(256*a^4*b^3 - 736*a^3*b^4 + 753*a^2*b^5 - 322*a*b^6 + 49*b^7)*d*cos(6*d*x + 6*c)^2 + 16*(64*a^4*b^3 - 240*a^3*b^4 + 337*a^2*b^5 - 210*a*b^6 + 49*b^7)*d*cos(4*d*x + 4*c)^2 + 64*(a^2*b^5 - 2*a*b^6 + b^7)*d*cos(2*d*x + 2*c)^2 + (a^2*b^5 - 2*a*b^6 + b^7)*d*sin(16*d*x + 16*c)^2 + 64*(a^2*b^5 - 2*a*b^6 + b^7)*d*sin(14*d*x + 14*c)^2 + 16*(64*a^4*b^3 - 240*a^3*b^4 + 337*a^2*b^5 - 210*a*b^6 + 49*b^7)*d*sin(12*d*x + 12*c)^2 + 64*(256*a^4*b^3 - 736*a^3*b^4 + 753*a^2*b^5 - 322*a*b^6 + 49*b^7)*d*sin(10*d*x + 10*c)^2 + 4*(16384*a^6*b - 57344*a^5*b^2 + 83712*a^4*b^3 - 67648*a^3*b^4 + 32841*a^2*b^5 - 9170*a*b^6 + 1225*b^7)*d*sin(8*d*x + 8*c)^2 + 64*(256*a^4*b^3 - 736*a^3*b^4 + 753*a^2*b^5 - 322*a*b^6 + 49*b^7)*d*sin(6*d*x + 6*c)^2 + 16*(64*a^4*b^3 - 240*a^3*b^4 + 337*a^2*b^5 - 210*a*b^6 + 49*b^7)*d*sin(4*d*x + 4*c)^2 + 64*(8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 64*(a^2*b^5 - 2*a*b^6 + b^7)*d*sin(2*d*x + 2*c)^2 - 16*(a^2*b^5 - 2*a*b^6 + b^7)*d*cos(2*d*x + 2*c) + (a^2*b^5 - 2*a*b^6 + b^7)*d - 2*(8*(a^2*b^5 - 2*a*b^6 + b^7)*d*cos(14*d*x + 14*c) + 4*(8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d*cos(12*d*x + 12*c) - 8*(16*a^3*b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d*cos(10*d*x + 10*c) - 2*(128*a^4*b^3 - 352*a^3*b^4 + 355*a^2*b^5 - 166*a*b^6 + 35*b^7)*d*cos(8*d*x + 8*c) - 8*(16*a^3*b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d*cos(6*d*x + 6*c) + 4*(8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d*cos(4*d*x + 4*c) + 8*(a^2*b^5 - 2*a*b^6 + b^7)*d*cos(2*d*x + 2*c) - (a^2*b^5 - 2*a*b^6 + b^7)*d)*cos(16*d*x + 16*c) + 16*(4*(8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d*cos(12*d*x + 12*c) - 8*(16*a^3*b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d*cos(10*d*x + 10*c) - 2*(128*a^4*b^3 - 352*a^3*b^4 + 355*a^2*b^5 - 166*a*b^6 + 35*b^7)*d*cos(8*d*x + 8*c) - 8*(16*a^3*b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d*cos(6*d*x + 6*c) + 4*(8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d*cos(4*d*x + 4*c) + 8*(a^2*b^5 - 2*a*b^6 + b^7)*d*cos(2*d*x + 2*c) - (a^2*b^5 - 2*a*b^6 + b^7)*d)*cos(14*d*x + 14*c) - 8*(8*(128*a^4*b^3 - 424*a^3*b^4 + 513*a^2*b^5 - 266*a*b^6 + 49*b^7)*d*cos(10*d*x + 10*c) + 2*(1024*a^5*b^2 - 3712*a^4*b^3 + 5304*a^3*b^4 - 3813*a^2*b^5 + 1442*a*b^6 - 245*b^7)*d*cos(8*d*x + 8*c) + 8*(128*a^4*b^3 - 424*a^3*b^4 + 513*a^2*b^5 - 266*a*b^6 + 49*b^7)*d*cos(6*d*x + 6*c) - 4*(64*a^4*b^3 - 240*a^3*b^4 + 337*a^2*b^5 - 210*a*b^6 + 49*b^7)*d*cos(4*d*x + 4*c) - 8*(8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d*cos(2*d*x + 2*c) + (8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d)*cos(12*d*x + 12*c) + 16*(2*(2048*a^5*b^2 - 6528*a^4*b^3 + 8144*a^3*b^4 - 5141*a^2*b^5 + 1722*a*b^6 - 245*b^7)*d*cos(8*d*x + 8*c) + 8*(256*a^4*b^3 - 736*a^3*b^4 + 753*a^2*b^5 - 322*a*b^6 + 49*b^7)*d*cos(6*d*x + 6*c) - 4*(128*a^4*b^3 - 424*a^3*b^4 + 513*a^2*b^5 - 266*a*b^6 + 49*b^7)*d*cos(4*d*x + 4*c) - 8*(16*a^3*b^4 - 39* a^2*b^5 + 30*a*b^6 - 7*b^7)*d*cos(2*d*x + 2*c) + (16*a^3*b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d)*cos(10*d*x + 10*c) + 4*(8*(2048*a^5*b^2 - 6528*a^4*b^3 + 8144*a^3*b^4 - 5141*a^2*b^5 + 1722*a*b^6 - 245*b^7)*d*cos(6*d*x + 6*c) - 4*(1024*a^5*b^2 - 3712*a^4*b^3 + 5304*a^3*b^4 - 3813*a^2*b^5 + 1442*a*b^6 - 245*b^7)*d*cos(4*d*x + 4*c) - 8*(128*a^4*b^3 - 352*a^3*b^4 + 355*a^2*b^5 - 166*a*b^6 + 35*b^7)*d*cos(2*d*x + 2*c) + (128*a^4*b^3 - 352*a^3*b^4 + 355*a^2*b^5 - 166*a*b^6 + 35*b^7)*d)*cos(8*d*x + 8*c) - 16*(4*(128*a^4*b^3 - 424*a^3*b^4 + 513*a^2*b^5 - 266*a*b^6 + 49*b^7)*d*cos(4*d*x + 4*c) + 8*(16*a^3*b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d*cos(2*d*x + 2*c) - (16*a^3*b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d)*cos(6*d*x + 6*c) + 8*(8*(8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d*cos(2*d*x + 2*c) - (8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d)*cos(4*d*x + 4*c) - 4*(4*(a^2*b^5 - 2*a*b^6 + b^7)*d*sin(14*d*x + 14*c) + 2*(8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d*sin(12*d*x + 12*c) - 4*(16*a^3*b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d*sin(10*d*x + 10*c) - (128*a^4*b^3 - 352*a^3*b^4 + 355*a^2*b^5 - 166*a*b^6 + 35*b^7)*d*sin(8*d*x + 8*c) - 4*(16*a^3*b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d*sin(6*d*x + 6*c) + 2*(8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d*sin(4*d*x + 4*c) + 4*(a^2*b^5 - 2*a*b^6 + b^7)*d*sin(2*d*x + 2*c))*sin(16*d*x + 16*c) + 32*(2*(8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d*sin(12*d*x + 12*c) - 4*(16*a^3*b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d*sin(10*d*x + 10*c) - (128*a^4*b^3 - 352*a^3*b^4 + 355*a^2*b^5 - 166*a*b^6 + 35*b^7)*d*sin(8*d*x + 8*c) - 4*(16*a^3*b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d*sin(6*d*x + 6*c) + 2*(8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d*sin(4*d*x + 4*c) + 4*(a^2*b^5 - 2*a*b^6 + b^7)*d*sin(2*d*x + 2*c))*sin(14*d*x + 14*c) - 16*(4*(128*a^4*b^3 - 424*a^3*b^4 + 513*a^2*b^5 - 266*a*b^6 + 49*b^7)*d*sin(10*d*x + 10*c) + (1024*a^5*b^2 - 3712*a^4*b^3 + 5304*a^3*b^4 - 3813*a^2*b^5 + 1442*a*b^6 - 245*b^7)*d*sin(8*d*x + 8*c) + 4*(128*a^4*b^3 - 424*a^3*b^4 + 513*a^2*b^5 - 266*a*b^6 + 49*b^7)*d*sin(6*d*x + 6*c) - 2*(64*a^4*b^3 - 240*a^3*b^4 + 337*a^2*b^5 - 210*a*b^6 + 49*b^7)*d*sin(4*d*x + 4*c) - 4*(8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d*sin(2*d*x + 2*c))*sin(12*d*x + 12*c) + 32*((2048*a^5*b^2 - 6528*a^4*b^3 + 8144*a^3*b^4 - 5141*a^2*b^5 + 1722*a*b^6 - 245*b^7)*d*sin(8*d*x + 8*c) + 4*(256*a^4*b^3 - 736*a^3*b^4 + 753*a^2*b^5 - 322*a*b^6 + 49*b^7)*d*sin(6*d*x + 6*c) - 2*(128*a^4*b^3 - 424*a^3*b^4 + 513*a^2*b^5 - 266*a*b^6 + 49*b^7)*d*sin(4*d*x + 4*c) - 4*(16*a^3*b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 16*(2*(2048*a^5*b^2 - 6528*a^4*b^3 + 8144*a^3*b^4 - 5141*a^2*b^5 + 1722*a*b^6 - 245*b^7)*d*sin(6*d*x + 6*c) - (1024*a^5*b^2 - 3712*a^4*b^3 + 5304*a^3*b^4 - 3813*a^2*b^5 + 1442*a*b^6 - 245*b^7)*d*sin(4*d*x + 4*c) - 2*(128*a^4*b^3 - 352*a^3*b^4 + 355*a^2*b^5 - 166*a*b^6 + 35*b^7)*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) - 64*((128*a^4*b^3 - 424*a^3*b^4 + 513*a^2*b^5 - 266*a*b^6 + 49*b^7)*d*sin(4*d*x + 4*c) + 2*(16*a^3*b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*integrate(1/4*(4*(a*b - 4*b^2)*cos(6*d*x + 6*c)^2 + 36*(8*a*b - 3*b^2)*cos(4*d*x + 4*c)^2 + 4*(a*b - 4*b^2)*cos(2*d*x + 2*c)^2 + 4*(a*b - 4*b^2)*sin(6*d*x + 6*c)^2 + 36*(8*a*b - 3*b^2)*sin(4*d*x + 4*c)^2 + 2*(8*a^2 - 35*a*b + 48*b^2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*(a*b - 4*b^2)*sin(2*d*x + 2*c)^2 - (18*b^2*cos(4*d*x + 4*c) + (a*b - 4*b^2)*cos(6*d*x + 6*c) + (a*b - 4*b^2)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) - (a*b - 4*b^2 - 2*(8*a^2 - 35*a*b + 48*b^2)*cos(4*d*x + 4*c) - 8*(a*b - 4*b^2)*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) - 2*(9*b^2 - (8*a^2 - 35*a*b + 48*b^2)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - (a*b - 4*b^2)*cos(2*d*x + 2*c) - (18*b^2*sin(4*d*x + 4*c) + (a*b - 4*b^2)*sin(6*d*x + 6*c) + (a*b - 4*b^2)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 2*((8*a^2 - 35*a*b + 48*b^2)*sin(4*d*x + 4*c) + 4*(a*b - 4*b^2)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))/(a^2*b^3 - 2*a*b^4 + b^5 + (a^2*b^3 - 2*a*b^4 + b^5)*cos(8*d*x + 8*c)^2 + 16*(a^2*b^3 - 2*a*b^4 + b^5)*cos(6*d*x + 6*c)^2 + 4*(64*a^4*b - 176*a^3*b^2 + 169*a^2*b^3 - 66*a*b^4 + 9*b^5)*cos(4*d*x + 4*c)^2 + 16*(a^2*b^3 - 2*a*b^4 + b^5)*cos(2*d*x + 2*c)^2 + (a^2*b^3 - 2*a*b^4 + b^5)*sin(8*d*x + 8*c)^2 + 16*(a^2*b^3 - 2*a*b^4 + b^5)*sin(6*d*x + 6*c)^2 + 4*(64*a^4*b - 176*a^3*b^2 + 169*a^2*b^3 - 66*a*b^4 + 9*b^5)*sin(4*d*x + 4*c)^2 + 16*(8*a^3*b^2 - 19*a^2*b^3 + 14*a*b^4 - 3*b^5)*sin(

4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a^2*b^3 - 2*a*b^4 + b^5)*sin(2*d*x + 2*c)^2 + 2*(a^2*b^3 - 2*a*b^4 + b^5 - 4*(a^2*b^3 - 2*a*b^4 + b^5)*cos(6*d*x + 6*c) - 2*(8*a^3*b^2 - 19*a^2*b^3 + 14*a*b^4 - 3*b^5)*cos(4*d*x + 4*c) - 4*(a^2*b^3 - 2*a*b^4 + b^5)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) - 8*(a^2*b^3 - 2*a*b^4 + b^5 - 2*(8*a^3*b^2 - 19*a^2*b^3 + 14*a*b^4 - 3*b^5)*cos(4*d*x + 4*c) - 4*(a^2*b^3 - 2*a*b^4 + b^5)*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) - 4*(8*a^3*b^2 - 19*a^2*b^3 + 14*a*b^4 - 3*b^5 - 4*(8*a^3*b^2 - 19*a^2*b^3 + 14*a*b^4 - 3*b^5)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 8*(a^2*b^3 - 2*a*b^4 + b^5)*cos(2*d*x + 2*c) - 4*(2*(a^2*b^3 - 2*a*b^4 + b^5)*sin(6*d*x + 6*c) + (8*a^3*b^2 - 19*a^2*b^3 + 14*a*b^4 - 3*b^5)*sin(4*d*x + 4*c) + 2*(a^2*b^3 - 2*a*b^4 + b^5)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^3*b^2 - 19*a^2*b^3 + 14*a*b^4 - 3*b^5)*sin(4*d*x + 4*c) + 2*(a^2*b^3 - 2*a*b^4 + b^5)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)), x) - (2*a*b^3 - 5*b^4 + (a*b^3 - 4*b^4)*cos(14*d*x + 14*c) - (32*a^2*b^2 - 58*a*b^3 - b^4)*cos(12*d*x + 12*c) + 3*(48*a^2*b^2 - 73*a*b^3 + 20*b^4)*cos(10*d*x + 10*c) + (256*a^3*b - 832*a^2*b^2 + 550*a*b^3 - 175*b^4)*cos(8*d*x + 8*c) + (112*a^2*b^2 - 533*a*b^3 + 220*b^4)*cos(6*d*x + 6*c) - (32*a^2*b^2 - 158*a*b^3 + 141*b^4)*cos(4*d*x + 4*c) - (17*a*b^3 - 44*b^4)*cos(2*d*x + 2*c))*sin(16*d*x + 16*c) + (17*a*b^3 - 44*b^4 - 4*(72*a^2*b^2 - 155*a*b^3 + 26*b^4)*cos(12*d*x + 12*c) + 16*(80*a^2*b^2 - 145*a*b^3 + 44*b^4)*cos(10*d*x + 10*c) + 6*(384*a^3*b - 1312*a^2*b^2 + 873*a*b^3 - 280*b^4)*cos(8*d*x + 8*c) + 32*(32*a^2*b^2 - 151*a*b^3 + 62*b^4)*cos(6*d*x + 6*c) - 4*(72*a^2*b^2 - 355*a*b^3 + 310*b^4)*cos(4*d*x + 4*c) - 48*(3*a*b^3 - 8*b^4)*cos(2*d*x + 2*c))*sin(14*d*x + 14*c) + (32*a^2*b^2 - 158*a*b^3 + 141*b^4 + 4*(128*a^3*b - 456*a^2*b^2 + 1233*a*b^3 - 434*b^4)*cos(10*d*x + 10*c) - 2*(6400*a^3*b - 13888*a^2*b^2 + 8566*a*b^3 - 2485*b^4)*cos(8*d*x + 8*c) - 4*(128*a^3*b + 2744*a^2*b^2 - 4711*a*b^3 + 1554*b^4)*cos(6*d*x + 6*c) + 8*(400*a^2*b^2 - 918*a*b^3 + 497*b^4)*cos(4*d*x + 4*c) - 4*(72*a^2*b^2 - 355*a*b^3 + 310*b^4)*cos(2*d*x + 2*c))*sin(12*d*x + 12*c) - (112*a^2*b^2 - 533*a*b^3 + 220*b^4 - 2*(2048*a^4 + 18560*a^3*b - 24752*a^2*b^2 + 13175*a*b^3 - 2800*b^4)*cos(8*d*x + 8*c) - 16*(256*a^3*b + 2400*a^2*b^2 - 2379*a*b^3 + 560*b^4)*cos(6*d*x + 6*c) + 4*(128*a^3*b + 2744*a^2*b^2 - 4711*a*b^3 + 1554*b^4)*cos(4*d*x + 4*c) - 32*(32*a^2*b^2 - 151*a*b^3 + 62*b^4)*cos(2*d*x + 2*c))*sin(10*d*x + 10*c) - (256*a^3*b - 832*a^2*b^2 + 550*a*b^3 - 175*b^4 - 2*(2048*a^4 + 18560*a^3*b - 24752*a^2*b^2 + 13175*a*b^3 - 2800*b^4)*cos(6*d*x + 6*c) + 2*(6400*a^3*b - 13888*a^2*b^2 + 8566*a*b^3 - 2485*b^4)*cos(4*d*x + 4*c) - 6*(384*a^3*b - 1312*a^2*b^2 + 873*a*b^3 - 280*b^4)*cos(2*d*x + 2*c))*sin(8*d*x + 8*c) - (144*a^2*b^2 - 219*a*b^3 + 60*b^4 - 4*(128*a^3*b - 456*a^2*b^2 + 1233*a*b^3 - 434*b^4)*cos(4*d*x + 4*c) - 16*(80*a^2*b^2 - 145*a*b^3 + 44*b^4)*cos(2*d*x + 2*c))*sin(6*d*x + 6*c) + (32*a^2*b^2 - 58*a*b^3 - b^4 - 4*(72*a^2*b^2 - 155*a*b^3 + 26*b^4)*cos(2*d*x + 2*c))*sin(4*d*x + 4*c) - (a*b^3 - 4*b^4)*sin(2*d*x + 2*c))/((a^2*b^5 - 2*a*b^6 + b^7)*d*cos(16*d*x + 16*c)^2 + 64*(a^2*b^5 - 2*a*b^6 + b^7)*d*cos(14*d*x + 14*c)^2 + 16*(64*a^4*b^3 - 240*a^3*b^4 + 337*a^2*b^5 - 210*a*b^6 + 49*b^7)*d*cos(12*d*x + 12*c)^2 + 64*(256*a^4*b^3 - 736*a^3*b^4 + 753*a^2*b^5 - 322*a*b^6 + 49*b^7)*d*cos(10*d*x + 10*c)^2 + 4*(16384*a^6*b - 57344*a^5*b^2 + 83712*a^4*b^3 - 67648*a^3*b^4 + 32841*a^2*b^5 - 9170*a*b^6 + 1225*b^7)*d*cos(8*d*x + 8*c)^2 + 64*(256*a^4*b^3 - 736*a^3*b^4 + 753*a^2*b^5 - 322*a*b^6 + 49*b^7)*d*cos(6*d*x + 6*c)^2 + 16*(64*a^4*b^3 - 240*a^3*b^4 + 337*a^2*b^5 - 210*a*b^6 + 49*b^7)*d*cos(4*d*x + 4*c)^2 + 64*(a^2*b^5 - 2*a*b^6 + b^7)*d*cos(2*d*x + 2*c)^2 + (a^2*b^5 - 2*a*b^6 + b^7)*d*sin(16*d*x + 16*c)^2 + 64*(a^2*b^5 - 2*a*b^6 + b^7)*d*sin(14*d*x + 14*c)^2 + 16*(64*a^4*b^3 - 240*a^3*b^4 + 337*a^2*b^5 - 210*a*b^6 + 49*b^7)*d*sin(12*d*x + 12*c)^2 + 64*(256*a^4*b^3 - 736*a^3*b^4 + 753*a^2*b^5 - 322*a*b^6 + 49*b^7)*d*sin(10*d*x + 10*c)^2 + 4*(16384*a^6*b - 57344*a^5*b^2 + 83712*a^4*b^3 - 67648*a^3*b^4 + 32841*a^2*b^5 - 9170*a*b^6 + 1225*b^7)*d*sin(8*d*x + 8*c)^2 + 64*(256*a^4*b^3 - 736*a^3*b^4 + 753*a^2*b^5 - 322*a*b^6 + 49*b^7)*d*sin(6*d*x + 6*c)^2 + 16*(64*a^4*b^3 - 240*a^3*b^4 + 337*a^2*b^5 - 210*a*b^6 + 49*b^7)*d*sin(4*d*x + 4*c)^2 + 64*(8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 64*(a^2*b^5 - 2*a*b^6 + b^7)*d*sin(2*d*x + 2* c)^2 - 16*(a^2*b^5 - 2*a*b^6 + b^7)*d*cos(2*d*x + 2*c) + (a^2*b^5 - 2*a*b^6 + b^7)*d - 2*(8*(a^2*b^5 - 2*a*b^6 + b^7)*d*cos(14*d*x + 14*c) + 4*(8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d*cos(12*d*x + 12*c) - 8*(16*a^3*b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d*cos(10*d*x + 10*c) - 2*(128*a^4*b^3 - 352*a^3*b^4 + 355*a^2*b^5 - 166*a*b^6 + 35*b^7)*d*cos(8*d*x + 8*c) - 8*(16*a^3*b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d*cos(6*d*x + 6*c) + 4*(8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d*cos(4*d*x + 4*c) + 8*(a^2*b^5 - 2*a*b^6 + b^7)*d*cos(2*d*x + 2*c) - (a^2*b^5 - 2*a*b^6 + b^7)*d)*cos(16*d*x + 16*c) + 16*(4*(8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d*cos(12*d*x + 12*c) - 8*(16*a^3*b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d*cos(10*d*x + 10*c) - 2*(128*a^4*b^3 - 352*a^3*b^4 + 355*a^2*b^5 - 166*a*b^6 + 35*b^7)*d*cos(8*d*x + 8*c) - 8*(16*a^3*b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d*cos(6*d*x + 6*c) + 4*(8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d*cos(4*d*x + 4*c) + 8*(a^2*b^5 - 2*a*b^6 + b^7)*d*cos(2*d*x + 2*c) - (a^2*b^5 - 2*a*b^6 + b^7)*d)*cos(14*d*x + 14*c) - 8*(8*(128*a^4*b^3 - 424*a^3*b^4 + 513*a^2*b^5 - 266*a*b^6 + 49*b^7)*d*cos(10*d*x + 10*c) + 2*(1024*a^5*b^2 - 3712*a^4*b^3 + 5304*a^3*b^4 - 3813*a^2*b^5 + 1442*a*b^6 - 245*b^7)*d*cos(8*d*x + 8*c) + 8*(128*a^4*b^3 - 424*a^3*b^4 + 513*a^2*b^5 - 266*a*b^6 + 49*b^7)*d*cos(6*d*x + 6*c) - 4*(64*a^4*b^3 - 240*a^3*b^4 + 337*a^2*b^5 - 210*a*b^6 + 49*b^7)*d*cos(4*d*x + 4*c) - 8*(8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d*cos(2*d*x + 2*c) + (8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d)*cos(12*d*x + 12*c) + 16*(2*(2048*a^5*b^2 - 6528*a^4*b^3 + 8144*a^3*b^4 - 5141*a^2*b^5 + 1722*a*b^6 - 245*b^7)*d*cos(8*d*x + 8*c) + 8*(256*a^4*b^3 - 736*a^3*b^4 + 753*a^2*b^5 - 322*a*b^6 + 49*b^7)*d*cos(6*d*x + 6*c) - 4*(128*a^4*b^3 - 424*a^3*b^4 + 513*a^2*b^5 - 266*a*b^6 + 49*b^7)*d*cos(4*d*x + 4*c) - 8*(16*a^3*b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d*cos(2*d*x + 2*c) + (16*a^3*b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d)*cos(10*d*x + 10*c) + 4*(8*(2048*a^5*b^2 - 6528*a^4*b^3 + 8144*a^3*b^4 - 5141*a^2*b^5 + 1722*a*b^6 - 245*b^7)*d*cos(6*d*x + 6*c) - 4*(1024*a^5*b^2 - 3712*a^4*b^3 + 5304*a^3*b^4 - 3813*a^2*b^5 + 1442*a*b^6 - 245*b^7)*d*cos(4*d*x + 4*c) - 8*(128*a^4*b^3 - 352*a^3*b^4 + 355*a^2*b^5 - 166*a*b^6 + 35*b^7)*d*cos(2*d*x + 2*c) + (128*a^4*b^3 - 352*a^3*b^4 + 355*a^2*b^5 - 166*a*b^6 + 35*b^7)*d)*cos(8*d*x + 8*c) - 16*(4*(128*a^4*b^3 - 424*a^3*b^4 + 513*a^2*b^5 - 266*a*b^6 + 49*b^7)*d*cos(4*d*x + 4*c) + 8*(16*a^3*b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d*cos(2*d*x + 2*c) - (16*a^3*b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d)*cos(6*d*x + 6*c) + 8*(8*(8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d*cos(2*d*x + 2*c) - (8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d)*cos(4*d*x + 4*c) - 4*(4*(a^2*b^5 - 2*a*b^6 + b^7)*d*sin(14*d*x + 14*c) + 2*(8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d*sin(12*d*x + 12*c) - 4*(16*a^3*b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d*sin(10*d*x + 10*c) - (128*a^4*b^3 - 352*a^3*b^4 + 355*a^2*b^5 - 166*a*b^6 + 35*b^7)*d*sin(8*d*x + 8*c) - 4*(16*a^3*b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d*sin(6*d*x + 6*c) + 2*(8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d*sin(4*d*x + 4*c) + 4*(a^2*b^5 - 2*a*b^6 + b^7)*d*sin(2*d*x + 2*c))*sin(16*d*x + 16*c) + 32*(2*(8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d*sin(12*d*x + 12*c) - 4*(16*a^3*b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d*sin(10*d*x + 10*c) - (128*a^4*b^3 - 352*a^3*b^4 + 355*a^2*b^5 - 166*a*b^6 + 35*b^7)*d*sin(8*d*x + 8*c) - 4*(16*a^3*b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d*sin(6*d*x + 6*c) + 2*(8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d*sin(4*d*x + 4*c) + 4*(a^2*b^5 - 2*a*b^6 + b^7)*d*sin(2*d*x + 2*c))*sin(14*d*x + 14*c) - 16*(4*(128*a^4*b^3 - 424*a^3*b^4 + 513*a^2*b^5 - 266*a*b^6 + 49*b^7)*d*sin(10*d*x + 10*c) + (1024*a^5*b^2 - 3712*a^4*b^3 + 5304*a^3*b^4 - 3813*a^2*b^5 + 1442*a*b^6 - 245*b^7)*d*sin(8*d*x + 8*c) + 4*(128*a^4*b^3 - 424*a^3*b^4 + 513*a^2*b^5 - 266*a*b^6 + 49*b^7)*d*sin(6*d*x + 6*c) - 2*(64*a^4*b^3 - 240*a^3*b^4 + 337*a^2*b^5 - 210*a*b^6 + 49*b^7)*d*sin(4*d*x + 4*c) - 4*(8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d*sin(2*d*x + 2*c))*sin(12*d*x + 12*c) + 32*((2048*a^5*b^2 - 6528*a^4*b^3 + 8144*a^3*b^4 - 5141*a^2*b^5 + 1722*a*b^6 - 245*b^7)*d*sin(8*d*x + 8*c) + 4*(256*a^4*b^3 - 736*a^3*b^4 + 753*a^2*b^5 - 322*a*b^6 + 49*b^7)*d*sin(6*d*x + 6*c) - 2*(128*a^4*b^3 - 424*a^3*b^4 + 513*a^2*b^5 - 266*a*b^6 + 49*b^7)*d*sin(4*d*x + 4*c) - 4*(16*a^3*b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*

```
d*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 16*(2*(2048*a^5*b^2 - 6528*a^4*b^3
 + 8144*a^3*b^4 - 5141*a^2*b^5 + 1722*a*b^6 - 245*b^7)*d*sin(6*d*x + 6*c) -
 (1024*a^5*b^2 - 3712*a^4*b^3 + 5304*a^3*b^4 - 3813*a^2*b^5 + 1442*a*b^6 -
245*b^7)*d*sin(4*d*x + 4*c) - 2*(128*a^4*b^3 - 352*a^3*b^4 + 355*a^2*b^5 -
166*a*b^6 + 35*b^7)*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) - 64*((128*a^4*b^3
 - 424*a^3*b^4 + 513*a^2*b^5 - 266*a*b^6 + 49*b^7)*d*sin(4*d*x + 4*c) + 2*(
16*a^3*b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d*sin(2*d*x + 2*c))*sin(6*d*x +
 6*c))
```

Fricas [B] time = 14.0834, size = 12051, normalized size = 37.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a-b*sin(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] -1/256*(((a^2*b^3 - 2*a*b^4 + b^5)*d*cos(d*x + c)^8 - 4*(a^2*b^3 - 2*a*b^4
+ b^5)*d*cos(d*x + c)^6 - 2*(a^3*b^2 - 5*a^2*b^3 + 7*a*b^4 - 3*b^5)*d*cos(d
*x + c)^4 + 4*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*d*cos(d*x + c)^2 + (a^4
*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*d)*sqrt(((a^6*b^3 - 5*a^5*b^4 +
 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2*sqrt((1225*a^4 - 10780*a^
3*b + 21966*a^2*b^2 + 7700*a*b^3 + 625*b^4)/((a^13*b^3 - 10*a^12*b^4 + 45*a
^11*b^5 - 120*a^10*b^6 + 210*a^9*b^7 - 252*a^8*b^8 + 210*a^7*b^9 - 120*a^6*
b^10 + 45*a^5*b^11 - 10*a^4*b^12 + a^3*b^13)*d^4)) - 4*a^3 + 35*a^2*b - 70*
a*b^2 - 105*b^3)/((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^
7 - a*b^8)*d^2))*log(35*a^3 - 1491/4*a^2*b + 1875/2*a*b^2 + 625/4*b^3 - 1/4
*(140*a^3 - 1491*a^2*b + 3750*a*b^2 + 625*b^3)*cos(d*x + c)^2 + 1/2*((a^9*b
^3 - 18*a^8*b^4 + 75*a^7*b^5 - 140*a^6*b^6 + 135*a^5*b^7 - 66*a^4*b^8 + 13*
a^3*b^9)*d^3*sqrt((1225*a^4 - 10780*a^3*b + 21966*a^2*b^2 + 7700*a*b^3 + 62
5*b^4)/((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 + 210*a^9*b^7
- 252*a^8*b^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*a^4*b^12 + a^
3*b^13)*d^4))*cos(d*x + c)*sin(d*x + c) - (70*a^5*b - 623*a^4*b^2 + 1161*a^
3*b^3 + 995*a^2*b^4 + 125*a*b^5)*d*cos(d*x + c)*sin(d*x + c))*sqrt(((a^6*b^
3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2*sqrt((1225
*a^4 - 10780*a^3*b + 21966*a^2*b^2 + 7700*a*b^3 + 625*b^4)/((a^13*b^3 - 10*
a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 + 210*a^9*b^7 - 252*a^8*b^8 + 210*a^7
*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*a^4*b^12 + a^3*b^13)*d^4)) - 4*a^3 +
 35*a^2*b - 70*a*b^2 - 105*b^3)/((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3
*b^6 + 5*a^2*b^7 - a*b^8)*d^2)) + 1/4*(2*(4*a^8*b - 45*a^7*b^2 + 165*a^6*b^
3 - 290*a^5*b^4 + 270*a^4*b^5 - 129*a^3*b^6 + 25*a^2*b^7)*d^2*cos(d*x + c)^
2 - (4*a^8*b - 45*a^7*b^2 + 165*a^6*b^3 - 290*a^5*b^4 + 270*a^4*b^5 - 129*a
^3*b^6 + 25*a^2*b^7)*d^2)*sqrt((1225*a^4 - 10780*a^3*b + 21966*a^2*b^2 + 77
00*a*b^3 + 625*b^4)/((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 +
 210*a^9*b^7 - 252*a^8*b^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*
a^4*b^12 + a^3*b^13)*d^4))) - ((a^2*b^3 - 2*a*b^4 + b^5)*d*cos(d*x + c)^8 -
 4*(a^2*b^3 - 2*a*b^4 + b^5)*d*cos(d*x + c)^6 - 2*(a^3*b^2 - 5*a^2*b^3 + 7*
a*b^4 - 3*b^5)*d*cos(d*x + c)^4 + 4*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*d
*cos(d*x + c)^2 + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*d)*sqrt((
(a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2*sqr
t((1225*a^4 - 10780*a^3*b + 21966*a^2*b^2 + 7700*a*b^3 + 625*b^4)/((a^13*b^
3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 + 210*a^9*b^7 - 252*a^8*b^8 +
210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*a^4*b^12 + a^3*b^13)*d^4)) -
4*a^3 + 35*a^2*b - 70*a*b^2 - 105*b^3)/((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 -
 10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2))*log(35*a^3 - 1491/4*a^2*b + 1875/2*a
*b^2 + 625/4*b^3 - 1/4*(140*a^3 - 1491*a^2*b + 3750*a*b^2 + 625*b^3)*cos(d*
x + c)^2 - 1/2*((a^9*b^3 - 18*a^8*b^4 + 75*a^7*b^5 - 140*a^6*b^6 + 135*a^5*
```

b^7 - 66*a^4*b^8 + 13*a^3*b^9)*d^3*sqrt((1225*a^4 - 10780*a^3*b + 21966*a^2
*b^2 + 7700*a*b^3 + 625*b^4)/((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a
^10*b^6 + 210*a^9*b^7 - 252*a^8*b^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b
^11 - 10*a^4*b^12 + a^3*b^13)*d^4))*cos(d*x + c)*sin(d*x + c) - (70*a^5*b -
623*a^4*b^2 + 1161*a^3*b^3 + 995*a^2*b^4 + 125*a*b^5)*d*cos(d*x + c)*sin(d
*x + c))*sqrt(((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 -
a*b^8)*d^2*sqrt((1225*a^4 - 10780*a^3*b + 21966*a^2*b^2 + 7700*a*b^3 + 625
*b^4)/((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 + 210*a^9*b^7 -
252*a^8*b^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*a^4*b^12 + a^3
*b^13)*d^4)) - 4*a^3 + 35*a^2*b - 70*a*b^2 - 105*b^3)/((a^6*b^3 - 5*a^5*b^4
+ 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2)) + 1/4*(2*(4*a^8*b - 4
5*a^7*b^2 + 165*a^6*b^3 - 290*a^5*b^4 + 270*a^4*b^5 - 129*a^3*b^6 + 25*a^2*
b^7)*d^2*cos(d*x + c)^2 - (4*a^8*b - 45*a^7*b^2 + 165*a^6*b^3 - 290*a^5*b^4
+ 270*a^4*b^5 - 129*a^3*b^6 + 25*a^2*b^7)*d^2)*sqrt((1225*a^4 - 10780*a^3*
b + 21966*a^2*b^2 + 7700*a*b^3 + 625*b^4)/((a^13*b^3 - 10*a^12*b^4 + 45*a^1
1*b^5 - 120*a^10*b^6 + 210*a^9*b^7 - 252*a^8*b^8 + 210*a^7*b^9 - 120*a^6*b^
10 + 45*a^5*b^11 - 10*a^4*b^12 + a^3*b^13)*d^4))) + ((a^2*b^3 - 2*a*b^4 + b
^5)*d*cos(d*x + c)^8 - 4*(a^2*b^3 - 2*a*b^4 + b^5)*d*cos(d*x + c)^6 - 2*(a^
3*b^2 - 5*a^2*b^3 + 7*a*b^4 - 3*b^5)*d*cos(d*x + c)^4 + 4*(a^3*b^2 - 3*a^2*
b^3 + 3*a*b^4 - b^5)*d*cos(d*x + c)^2 + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*
a*b^4 + b^5)*d)*sqrt(-((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a
^2*b^7 - a*b^8)*d^2*sqrt((1225*a^4 - 10780*a^3*b + 21966*a^2*b^2 + 7700*a*b
^3 + 625*b^4)/((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 + 210*a
^9*b^7 - 252*a^8*b^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*a^4*b^
12 + a^3*b^13)*d^4)) + 4*a^3 - 35*a^2*b + 70*a*b^2 + 105*b^3)/((a^6*b^3 - 5
*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2))*log(-35*a^3 +
1491/4*a^2*b - 1875/2*a*b^2 - 625/4*b^3 + 1/4*(140*a^3 - 1491*a^2*b + 3750
*a*b^2 + 625*b^3)*cos(d*x + c)^2 + 1/2*((a^9*b^3 - 18*a^8*b^4 + 75*a^7*b^5
- 140*a^6*b^6 + 135*a^5*b^7 - 66*a^4*b^8 + 13*a^3*b^9)*d^3*sqrt((1225*a^4 -
10780*a^3*b + 21966*a^2*b^2 + 7700*a*b^3 + 625*b^4)/((a^13*b^3 - 10*a^12*b
^4 + 45*a^11*b^5 - 120*a^10*b^6 + 210*a^9*b^7 - 252*a^8*b^8 + 210*a^7*b^9 -
120*a^6*b^10 + 45*a^5*b^11 - 10*a^4*b^12 + a^3*b^13)*d^4))*cos(d*x + c)*si
n(d*x + c) + (70*a^5*b - 623*a^4*b^2 + 1161*a^3*b^3 + 995*a^2*b^4 + 125*a*b
^5)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 -
10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2*sqrt((1225*a^4 - 10780*a^3*b + 21966*a
^2*b^2 + 7700*a*b^3 + 625*b^4)/((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120
*a^10*b^6 + 210*a^9*b^7 - 252*a^8*b^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5
*b^11 - 10*a^4*b^12 + a^3*b^13)*d^4)) + 4*a^3 - 35*a^2*b + 70*a*b^2 + 105*b
^3)/((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^
2)) + 1/4*(2*(4*a^8*b - 45*a^7*b^2 + 165*a^6*b^3 - 290*a^5*b^4 + 270*a^4*b^
5 - 129*a^3*b^6 + 25*a^2*b^7)*d^2*cos(d*x + c)^2 - (4*a^8*b - 45*a^7*b^2 +
165*a^6*b^3 - 290*a^5*b^4 + 270*a^4*b^5 - 129*a^3*b^6 + 25*a^2*b^7)*d^2)*sq
rt((1225*a^4 - 10780*a^3*b + 21966*a^2*b^2 + 7700*a*b^3 + 625*b^4)/((a^13*b
^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 + 210*a^9*b^7 - 252*a^8*b^8 +
210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*a^4*b^12 + a^3*b^13)*d^4)))
- ((a^2*b^3 - 2*a*b^4 + b^5)*d*cos(d*x + c)^8 - 4*(a^2*b^3 - 2*a*b^4 + b^5)
*d*cos(d*x + c)^6 - 2*(a^3*b^2 - 5*a^2*b^3 + 7*a*b^4 - 3*b^5)*d*cos(d*x + c
)^4 + 4*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*d*cos(d*x + c)^2 + (a^4*b - 4
*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*d)*sqrt(-((a^6*b^3 - 5*a^5*b^4 + 10*a
^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2*sqrt((1225*a^4 - 10780*a^3*b +
21966*a^2*b^2 + 7700*a*b^3 + 625*b^4)/((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b
^5 - 120*a^10*b^6 + 210*a^9*b^7 - 252*a^8*b^8 + 210*a^7*b^9 - 120*a^6*b^10
+ 45*a^5*b^11 - 10*a^4*b^12 + a^3*b^13)*d^4)) + 4*a^3 - 35*a^2*b + 70*a*b^2
+ 105*b^3)/((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a
*b^8)*d^2))*log(-35*a^3 + 1491/4*a^2*b - 1875/2*a*b^2 - 625/4*b^3 + 1/4*(14
0*a^3 - 1491*a^2*b + 3750*a*b^2 + 625*b^3)*cos(d*x + c)^2 - 1/2*((a^9*b^3 -
18*a^8*b^4 + 75*a^7*b^5 - 140*a^6*b^6 + 135*a^5*b^7 - 66*a^4*b^8 + 13*a^3*
b^9)*d^3*sqrt((1225*a^4 - 10780*a^3*b + 21966*a^2*b^2 + 7700*a*b^3 + 625*b^
4)/((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 + 210*a^9*b^7 - 25

```
2*a^8*b^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*a^4*b^12 + a^3*b^
13)*d^4))*cos(d*x + c)*sin(d*x + c) + (70*a^5*b - 623*a^4*b^2 + 1161*a^3*b^
3 + 995*a^2*b^4 + 125*a*b^5)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^6*b^3 -
 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2*sqrt((1225*a^
4 - 10780*a^3*b + 21966*a^2*b^2 + 7700*a*b^3 + 625*b^4)/((a^13*b^3 - 10*a^1
2*b^4 + 45*a^11*b^5 - 120*a^10*b^6 + 210*a^9*b^7 - 252*a^8*b^8 + 210*a^7*b^
9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*a^4*b^12 + a^3*b^13)*d^4)) + 4*a^3 - 35
*a^2*b + 70*a*b^2 + 105*b^3)/((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^
6 + 5*a^2*b^7 - a*b^8)*d^2)) + 1/4*(2*(4*a^8*b - 45*a^7*b^2 + 165*a^6*b^3 -
 290*a^5*b^4 + 270*a^4*b^5 - 129*a^3*b^6 + 25*a^2*b^7)*d^2*cos(d*x + c)^2 -
 (4*a^8*b - 45*a^7*b^2 + 165*a^6*b^3 - 290*a^5*b^4 + 270*a^4*b^5 - 129*a^3*
b^6 + 25*a^2*b^7)*d^2)*sqrt((1225*a^4 - 10780*a^3*b + 21966*a^2*b^2 + 7700*
a*b^3 + 625*b^4)/((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 + 21
0*a^9*b^7 - 252*a^8*b^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*a^4
*b^12 + a^3*b^13)*d^4))) - 8*(2*(2*a*b - 5*b^2)*cos(d*x + c)^7 - 3*(5*a*b -
 13*b^2)*cos(d*x + c)^5 + 24*(a*b - 2*b^2)*cos(d*x + c)^3 - (a^2 + 18*a*b -
 19*b^2)*cos(d*x + c))*sin(d*x + c))/((a^2*b^3 - 2*a*b^4 + b^5)*d*cos(d*x +
 c)^8 - 4*(a^2*b^3 - 2*a*b^4 + b^5)*d*cos(d*x + c)^6 - 2*(a^3*b^2 - 5*a^2*b
^3 + 7*a*b^4 - 3*b^5)*d*cos(d*x + c)^4 + 4*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 -
 b^5)*d*cos(d*x + c)^2 + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**8/(a-b*sin(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a-b*sin(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.231 $\int \frac{\sin^6(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=343

$$-\frac{\left(-10\sqrt{a}\sqrt{b}+4a+3b\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{5/4}b^{3/2}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{\left(10\sqrt{a}\sqrt{b}+4a+3b\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{5/4}b^{3/2}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}-\frac{\tan(c+dx)\left(\frac{(2a^2+15\cdots}{\cdots}\right.}{32abd\left((a-b)\tan^4\cdots\right.}$$

```
[Out] -((4*a - 10*Sqrt[a]*Sqrt[b] + 3*b)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c +
d*x])/a^(1/4)])/(64*a^(5/4)*(Sqrt[a] - Sqrt[b])^(5/2)*b^(3/2)*d) + ((4*a +
10*Sqrt[a]*Sqrt[b] + 3*b)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(
1/4)])/(64*a^(5/4)*(Sqrt[a] + Sqrt[b])^(5/2)*b^(3/2)*d) - (Tan[c + d*x]*(a*
(a + 3*b) + (a^2 + 6*a*b + b^2)*Tan[c + d*x]^2))/(8*(a - b)^3*d*(a + 2*a*Ta
n[c + d*x]^2 + (a - b)*Tan[c + d*x]^4)^2) - (Tan[c + d*x]*((2*a*(a^2 - a*b
- 8*b^2))/(a - b)^3 + ((2*a^2 + 15*a*b + 3*b^2)*Tan[c + d*x]^2)/(a - b)^2))
/(32*a*b*d*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan[c + d*x]^4))
```

Rubi [A] time = 0.765268, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3217, 1333, 1678, 1166, 205}

$$-\frac{\left(-10\sqrt{a}\sqrt{b}+4a+3b\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{5/4}b^{3/2}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{\left(10\sqrt{a}\sqrt{b}+4a+3b\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{5/4}b^{3/2}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}-\frac{\tan(c+dx)\left(\frac{(2a^2+15\cdots}{\cdots}\right.}{32abd\left((a-b)\tan^4\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^6/(a - b*Sin[c + d*x]^4)^3,x]
```


```
[Out] -((4*a - 10*Sqrt[a]*Sqrt[b] + 3*b)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c +
d*x])/a^(1/4)])/(64*a^(5/4)*(Sqrt[a] - Sqrt[b])^(5/2)*b^(3/2)*d) + ((4*a +
10*Sqrt[a]*Sqrt[b] + 3*b)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(
1/4)])/(64*a^(5/4)*(Sqrt[a] + Sqrt[b])^(5/2)*b^(3/2)*d) - (Tan[c + d*x]*(a*
(a + 3*b) + (a^2 + 6*a*b + b^2)*Tan[c + d*x]^2))/(8*(a - b)^3*d*(a + 2*a*Ta
n[c + d*x]^2 + (a - b)*Tan[c + d*x]^4)^2) - (Tan[c + d*x]*((2*a*(a^2 - a*b
- 8*b^2))/(a - b)^3 + ((2*a^2 + 15*a*b + 3*b^2)*Tan[c + d*x]^2)/(a - b)^2))
/(32*a*b*d*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan[c + d*x]^4))
```


Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 1333

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)),
```

```
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*S
imp[ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d + e*x^2
)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g +
c*(4*p + 7)*(b*f - 2*a*g)*x^2, x], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1] && IGtQ[m/2, 0]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^6(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx = \frac{\mathrm{Subst}\left(\int \frac{x^6\left(1+x^2\right)^2}{\left(a+2ax^2+(a-b)x^4\right)^3}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{\tan(c+dx)\left(a(a+3b)+\left(a^2+6ab+b^2\right)\tan^2(c+dx)\right)}{8(a-b)^3d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{\mathrm{Subst}\left(\int \frac{-\frac{2a^3b(a+3b)}{(a-b)^3}+\frac{2a^2b(5a^2+\cdots}{(a\cdots}}{(a+2a\cdots}\right.}{\cdots}$$

$$= -\frac{\tan(c+dx)\left(a(a+3b)+\left(a^2+6ab+b^2\right)\tan^2(c+dx)\right)}{8(a-b)^3d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{\tan(c+dx)\left(\frac{2a\left(a^2-ab-8b^2\right)}{(a-b)^3}+\cdots\right.}{32abd\left(a+2a\tan^2(c+dx)\cdots\right.}$$

$$= -\frac{\tan(c+dx)\left(a(a+3b)+\left(a^2+6ab+b^2\right)\tan^2(c+dx)\right)}{8(a-b)^3d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{\tan(c+dx)\left(\frac{2a\left(a^2-ab-8b^2\right)}{(a-b)^3}+\cdots\right.}{32abd\left(a+2a\tan^2(c+dx)\cdots\right.}$$

$$= -\frac{\left(4a-10\sqrt{a}\sqrt{b}+3b\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{5/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{3/2}d} + \frac{\left(4a+10\sqrt{a}\sqrt{b}+3b\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\mathrm{t}\cdots}{\sqrt[4]{\cdots}}\right.}{64a^{5/4}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}b^{3/2}d}$$

Mathematica [A] time = 3.8556, size = 350, normalized size = 1.02

$$\frac{\frac{4b\sin(2(c+dx))\left(4a^2+3b(a+b)\cos(2(c+dx))-19ab-3b^2\right)}{a(8a+4b\cos(2(c+dx))-b\cos(4(c+dx))-3b)}+\frac{\sqrt{b}\left(10\sqrt{a}\sqrt{b}+4a+3b\right)\left(\sqrt{a}-\sqrt{b}\right)^2\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{a\sqrt{\sqrt{a}\sqrt{b}+a}}-\frac{128b(a-b)\sin(2(c+dx))(2a-b\cos(2(c+}{(-8a-4b\cos(2(c+dx))+b\cos(4(c+dx)}}{64b^2d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^6/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] (((Sqrt[a] - Sqrt[b])^2*Sqrt[b]*(4*a + 10*Sqrt[a]*Sqrt[b] + 3*b)*ArcTan[((S
qrt[a] + Sqrt[b])*Tan[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(a*Sqrt[a + Sqr
t[a]*Sqrt[b]]) + ((Sqrt[a] + Sqrt[b])^2*Sqrt[b]*(4*a - 10*Sqrt[a]*Sqrt[b] +
 3*b)*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]
])/(a*Sqrt[-a + Sqrt[a]*Sqrt[b]]) + (4*b*(4*a^2 - 19*a*b - 3*b^2 + 3*b*(a +
 b)*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])/(a*(8*a - 3*b + 4*b*Cos[2*(c + d*x)
] - b*Cos[4*(c + d*x)])) - (128*(a - b)*b*(2*a + b - b*Cos[2*(c + d*x)])*Si
n[2*(c + d*x)])/(-8*a + 3*b - 4*b*Cos[2*(c + d*x)] + b*Cos[4*(c + d*x)])^2)
/(64*(a - b)^2*b^2*d)
```

Maple [B] time = 0.138, size = 1909, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^6/(a-b*sin(d*x+c)^4)^3,x)
```

```
[Out] -1/16/d/b/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*a
rctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*a^3-7/32/d*b/(a^2-2*a
*b+b^2)*a/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(
d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+1/16/d/b/(a^2-2*a*b+b^2)/(a*b)^(1/2)/
(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)
-a)*(a-b))^(1/2))*a^3+7/32/d*b/(a^2-2*a*b+b^2)*a/(a*b)^(1/2)/(a-b)/(((a*b)^
(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/
2))+1/32/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2*b/(a^2-2*a*
b+b^2)*tan(d*x+c)^3+5/16/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+
a)^2*b/(a^2-2*a*b+b^2)*tan(d*x+c)^5-19/32/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+
2*a*tan(d*x+c)^2+a)^2*a/(a^2-2*a*b+b^2)*tan(d*x+c)^3-1/8/d/(tan(d*x+c)^4*a-
tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2*a/(a^2-2*a*b+b^2)*tan(d*x+c)-15/32/d/(
tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2/(a-b)*tan(d*x+c)^7-3/32
/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2/(a-b)/a*b*tan(d*x+c
)^7-1/16/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2/(a-b)/b*tan
(d*x+c)^7*a-3/16/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2/b/(
a^2-2*a*b+b^2)*tan(d*x+c)^5*a^2-3/16/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*t
an(d*x+c)^2+a)^2*a^2/b/(a^2-2*a*b+b^2)*tan(d*x+c)^3-1/16/d/(tan(d*x+c)^4*a-
tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2*a^2/b/(a^2-2*a*b+b^2)*tan(d*x+c)+19/64
/d/(a^2-2*a*b+b^2)*a/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d
*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-7/8/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2
*a*tan(d*x+c)^2+a)^2/(a^2-2*a*b+b^2)*tan(d*x+c)^5*a+19/64/d/(a^2-2*a*b+b^2)
*a/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1
/2)-a)*(a-b))^(1/2))+3/64/d/a*b^2/(a^2-2*a*b+b^2)/(a-b)/(((a*b)^(1/2)+a)*(a
-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+3/64/d/a*
b^2/(a^2-2*a*b+b^2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(
d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-5/16/d*b/(a^2-2*a*b+b^2)/(a-b)/(((a*b
```

```
)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/
2))-5/16/d*b/(a^2-2*a*b+b^2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-
a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-1/32/d/b/(a^2-2*a*b+b^2)/(a-
b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(
a-b))^(1/2))*a^2+19/64/d/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)
*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*a^2-1/
32/d/b/(a^2-2*a*b+b^2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*t
an(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*a^2-19/64/d/(a^2-2*a*b+b^2)/(a*b)^
(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)
^(1/2)-a)*(a-b))^(1/2))*a^2-1/64/d*b^2/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(a-b)/((
(a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))
^(1/2))+1/64/d*b^2/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b)
)^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a-b*sin(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(4*(32*a^3*b^2 - 84*a^2*b^3 - 83*a*b^4 + 21*b^5)*cos(4*d*x + 4*c)*sin
(2*d*x + 2*c) + ((4*a^2*b^3 - 13*a*b^4 + 3*b^5)*sin(14*d*x + 14*c) - 3*(8*a
^2*b^3 - 33*a*b^4 + 7*b^5)*sin(12*d*x + 12*c) + (64*a^3*b^2 + 68*a^2*b^3 -
225*a*b^4 + 63*b^5)*sin(10*d*x + 10*c) - 3*(128*a^3*b^2 + 32*a^2*b^3 - 61*a
*b^4 + 35*b^5)*sin(8*d*x + 8*c) - (64*a^3*b^2 + 452*a^2*b^3 - 9*a*b^4 - 105
*b^5)*sin(6*d*x + 6*c) + 3*(40*a^2*b^3 - 29*a*b^4 - 21*b^5)*sin(4*d*x + 4*c
) - (4*a^2*b^3 - 37*a*b^4 - 21*b^5)*sin(2*d*x + 2*c))*cos(16*d*x + 16*c) +
2*(2*(32*a^3*b^2 - 84*a^2*b^3 - 83*a*b^4 + 21*b^5)*sin(12*d*x + 12*c) - 8*(
64*a^3*b^2 - 84*a^2*b^3 - 43*a*b^4 + 21*b^5)*sin(10*d*x + 10*c) - (512*a^4*
b - 3584*a^3*b^2 + 1388*a^2*b^3 - 11*a*b^4 - 315*b^5)*sin(8*d*x + 8*c) + 16
*(172*a^2*b^3 - 37*a*b^4 - 21*b^5)*sin(6*d*x + 6*c) + 2*(32*a^3*b^2 - 372*a
^2*b^3 + 289*a*b^4 + 105*b^5)*sin(4*d*x + 4*c) + 8*(4*a^2*b^3 - 25*a*b^4 -
9*b^5)*sin(2*d*x + 2*c))*cos(14*d*x + 14*c) - 2*(2*(512*a^4*b - 672*a^3*b^2
 + 1228*a^2*b^3 + 21*a*b^4 - 147*b^5)*sin(10*d*x + 10*c) - 3*(3072*a^4*b -
6272*a^3*b^2 + 2920*a^2*b^3 - 413*a*b^4 - 245*b^5)*sin(8*d*x + 8*c) - 2*(51
2*a^4*b + 3936*a^3*b^2 - 6740*a^2*b^3 + 1281*a*b^4 + 441*b^5)*sin(6*d*x + 6
*c) + 12*(192*a^3*b^2 - 416*a^2*b^3 + 161*a*b^4 + 49*b^5)*sin(4*d*x + 4*c)
- 2*(32*a^3*b^2 - 372*a^2*b^3 + 289*a*b^4 + 105*b^5)*sin(2*d*x + 2*c))*cos(
12*d*x + 12*c) - 2*((8192*a^5 + 27136*a^4*b - 37696*a^3*b^2 + 17644*a^2*b^3
 - 2079*a*b^4 - 735*b^5)*sin(8*d*x + 8*c) + 8*(1024*a^4*b + 3712*a^3*b^2 -
3692*a^2*b^3 + 483*a*b^4 + 147*b^5)*sin(6*d*x + 6*c) - 2*(512*a^4*b + 3936*
a^3*b^2 - 6740*a^2*b^3 + 1281*a*b^4 + 441*b^5)*sin(4*d*x + 4*c) - 16*(172*a
^2*b^3 - 37*a*b^4 - 21*b^5)*sin(2*d*x + 2*c))*cos(10*d*x + 10*c) - 2*((8192
*a^5 + 27136*a^4*b - 37696*a^3*b^2 + 17644*a^2*b^3 - 2079*a*b^4 - 735*b^5)*
sin(6*d*x + 6*c) - 3*(3072*a^4*b - 6272*a^3*b^2 + 2920*a^2*b^3 - 413*a*b^4
- 245*b^5)*sin(4*d*x + 4*c) + (512*a^4*b - 3584*a^3*b^2 + 1388*a^2*b^3 - 11
*a*b^4 - 315*b^5)*sin(2*d*x + 2*c))*cos(8*d*x + 8*c) - 4*((512*a^4*b - 672*
a^3*b^2 + 1228*a^2*b^3 + 21*a*b^4 - 147*b^5)*sin(4*d*x + 4*c) + 4*(64*a^3*b
^2 - 84*a^2*b^3 - 43*a*b^4 + 21*b^5)*sin(2*d*x + 2*c))*cos(6*d*x + 6*c) + 1
6*((a^3*b^5 - 2*a^2*b^6 + a*b^7)*d*cos(16*d*x + 16*c)^2 + 64*(a^3*b^5 - 2*a
^2*b^6 + a*b^7)*d*cos(14*d*x + 14*c)^2 + 16*(64*a^5*b^3 - 240*a^4*b^4 + 337
*a^3*b^5 - 210*a^2*b^6 + 49*a*b^7)*d*cos(12*d*x + 12*c)^2 + 64*(256*a^5*b^3
 - 736*a^4*b^4 + 753*a^3*b^5 - 322*a^2*b^6 + 49*a*b^7)*d*cos(10*d*x + 10*c)
^2 + 4*(16384*a^7*b - 57344*a^6*b^2 + 83712*a^5*b^3 - 67648*a^4*b^4 + 32841
*a^3*b^5 - 9170*a^2*b^6 + 1225*a*b^7)*d*cos(8*d*x + 8*c)^2 + 64*(256*a^5*b^
```

3 - 736*a^4*b^4 + 753*a^3*b^5 - 322*a^2*b^6 + 49*a*b^7)*d*cos(6*d*x + 6*c)^
2 + 16*(64*a^5*b^3 - 240*a^4*b^4 + 337*a^3*b^5 - 210*a^2*b^6 + 49*a*b^7)*d*
cos(4*d*x + 4*c)^2 + 64*(a^3*b^5 - 2*a^2*b^6 + a*b^7)*d*cos(2*d*x + 2*c)^2
+ (a^3*b^5 - 2*a^2*b^6 + a*b^7)*d*sin(16*d*x + 16*c)^2 + 64*(a^3*b^5 - 2*a^
2*b^6 + a*b^7)*d*sin(14*d*x + 14*c)^2 + 16*(64*a^5*b^3 - 240*a^4*b^4 + 337*
a^3*b^5 - 210*a^2*b^6 + 49*a*b^7)*d*sin(12*d*x + 12*c)^2 + 64*(256*a^5*b^3
- 736*a^4*b^4 + 753*a^3*b^5 - 322*a^2*b^6 + 49*a*b^7)*d*sin(10*d*x + 10*c)^
2 + 4*(16384*a^7*b - 57344*a^6*b^2 + 83712*a^5*b^3 - 67648*a^4*b^4 + 32841*
a^3*b^5 - 9170*a^2*b^6 + 1225*a*b^7)*d*sin(8*d*x + 8*c)^2 + 64*(256*a^5*b^3
- 736*a^4*b^4 + 753*a^3*b^5 - 322*a^2*b^6 + 49*a*b^7)*d*sin(6*d*x + 6*c)^2
+ 16*(64*a^5*b^3 - 240*a^4*b^4 + 337*a^3*b^5 - 210*a^2*b^6 + 49*a*b^7)*d*s
in(4*d*x + 4*c)^2 + 64*(8*a^4*b^4 - 23*a^3*b^5 + 22*a^2*b^6 - 7*a*b^7)*d*si
n(4*d*x + 4*c)*sin(2*d*x + 2*c) + 64*(a^3*b^5 - 2*a^2*b^6 + a*b^7)*d*sin(2*
d*x + 2*c)^2 - 16*(a^3*b^5 - 2*a^2*b^6 + a*b^7)*d*cos(2*d*x + 2*c) + (a^3*b
^5 - 2*a^2*b^6 + a*b^7)*d - 2*(8*(a^3*b^5 - 2*a^2*b^6 + a*b^7)*d*cos(14*d*x
+ 14*c) + 4*(8*a^4*b^4 - 23*a^3*b^5 + 22*a^2*b^6 - 7*a*b^7)*d*cos(12*d*x +
12*c) - 8*(16*a^4*b^4 - 39*a^3*b^5 + 30*a^2*b^6 - 7*a*b^7)*d*cos(10*d*x +
10*c) - 2*(128*a^5*b^3 - 352*a^4*b^4 + 355*a^3*b^5 - 166*a^2*b^6 + 35*a*b^7
)*d*cos(8*d*x + 8*c) - 8*(16*a^4*b^4 - 39*a^3*b^5 + 30*a^2*b^6 - 7*a*b^7)*d
*cos(6*d*x + 6*c) + 4*(8*a^4*b^4 - 23*a^3*b^5 + 22*a^2*b^6 - 7*a*b^7)*d*cos
(4*d*x + 4*c) + 8*(a^3*b^5 - 2*a^2*b^6 + a*b^7)*d*cos(2*d*x + 2*c) - (a^3*b
^5 - 2*a^2*b^6 + a*b^7)*d)*cos(16*d*x + 16*c) + 16*(4*(8*a^4*b^4 - 23*a^3*b
^5 + 22*a^2*b^6 - 7*a*b^7)*d*cos(12*d*x + 12*c) - 8*(16*a^4*b^4 - 39*a^3*b^
5 + 30*a^2*b^6 - 7*a*b^7)*d*cos(10*d*x + 10*c) - 2*(128*a^5*b^3 - 352*a^4*b
^4 + 355*a^3*b^5 - 166*a^2*b^6 + 35*a*b^7)*d*cos(8*d*x + 8*c) - 8*(16*a^4*b
^4 - 39*a^3*b^5 + 30*a^2*b^6 - 7*a*b^7)*d*cos(6*d*x + 6*c) + 4*(8*a^4*b^4 -
23*a^3*b^5 + 22*a^2*b^6 - 7*a*b^7)*d*cos(4*d*x + 4*c) + 8*(a^3*b^5 - 2*a^2
*b^6 + a*b^7)*d*cos(2*d*x + 2*c) - (a^3*b^5 - 2*a^2*b^6 + a*b^7)*d)*cos(14*
d*x + 14*c) - 8*(8*(128*a^5*b^3 - 424*a^4*b^4 + 513*a^3*b^5 - 266*a^2*b^6 +
49*a*b^7)*d*cos(10*d*x + 10*c) + 2*(1024*a^6*b^2 - 3712*a^5*b^3 + 5304*a^4
*b^4 - 3813*a^3*b^5 + 1442*a^2*b^6 - 245*a*b^7)*d*cos(8*d*x + 8*c) + 8*(128
*a^5*b^3 - 424*a^4*b^4 + 513*a^3*b^5 - 266*a^2*b^6 + 49*a*b^7)*d*cos(6*d*x
+ 6*c) - 4*(64*a^5*b^3 - 240*a^4*b^4 + 337*a^3*b^5 - 210*a^2*b^6 + 49*a*b^7
)*d*cos(4*d*x + 4*c) - 8*(8*a^4*b^4 - 23*a^3*b^5 + 22*a^2*b^6 - 7*a*b^7)*d*
cos(2*d*x + 2*c) + (8*a^4*b^4 - 23*a^3*b^5 + 22*a^2*b^6 - 7*a*b^7)*d)*cos(1
2*d*x + 12*c) + 16*(2*(2048*a^6*b^2 - 6528*a^5*b^3 + 8144*a^4*b^4 - 5141*a^
3*b^5 + 1722*a^2*b^6 - 245*a*b^7)*d*cos(8*d*x + 8*c) + 8*(256*a^5*b^3 - 736
*a^4*b^4 + 753*a^3*b^5 - 322*a^2*b^6 + 49*a*b^7)*d*cos(6*d*x + 6*c) - 4*(12
8*a^5*b^3 - 424*a^4*b^4 + 513*a^3*b^5 - 266*a^2*b^6 + 49*a*b^7)*d*cos(4*d*x
+ 4*c) - 8*(16*a^4*b^4 - 39*a^3*b^5 + 30*a^2*b^6 - 7*a*b^7)*d*cos(2*d*x +
2*c) + (16*a^4*b^4 - 39*a^3*b^5 + 30*a^2*b^6 - 7*a*b^7)*d)*cos(10*d*x + 10*
c) + 4*(8*(2048*a^6*b^2 - 6528*a^5*b^3 + 8144*a^4*b^4 - 5141*a^3*b^5 + 1722
*a^2*b^6 - 245*a*b^7)*d*cos(6*d*x + 6*c) - 4*(1024*a^6*b^2 - 3712*a^5*b^3 +
5304*a^4*b^4 - 3813*a^3*b^5 + 1442*a^2*b^6 - 245*a*b^7)*d*cos(4*d*x + 4*c)
- 8*(128*a^5*b^3 - 352*a^4*b^4 + 355*a^3*b^5 - 166*a^2*b^6 + 35*a*b^7)*d*c
os(2*d*x + 2*c) + (128*a^5*b^3 - 352*a^4*b^4 + 355*a^3*b^5 - 166*a^2*b^6 +
35*a*b^7)*d)*cos(8*d*x + 8*c) - 16*(4*(128*a^5*b^3 - 424*a^4*b^4 + 513*a^3*
b^5 - 266*a^2*b^6 + 49*a*b^7)*d*cos(4*d*x + 4*c) + 8*(16*a^4*b^4 - 39*a^3*b
^5 + 30*a^2*b^6 - 7*a*b^7)*d*cos(2*d*x + 2*c) - (16*a^4*b^4 - 39*a^3*b^5 +
30*a^2*b^6 - 7*a*b^7)*d)*cos(6*d*x + 6*c) + 8*(8*(8*a^4*b^4 - 23*a^3*b^5 +
22*a^2*b^6 - 7*a*b^7)*d*cos(2*d*x + 2*c) - (8*a^4*b^4 - 23*a^3*b^5 + 22*a^2
*b^6 - 7*a*b^7)*d)*cos(4*d*x + 4*c) - 4*(4*(a^3*b^5 - 2*a^2*b^6 + a*b^7)*d*
sin(14*d*x + 14*c) + 2*(8*a^4*b^4 - 23*a^3*b^5 + 22*a^2*b^6 - 7*a*b^7)*d*si
n(12*d*x + 12*c) - 4*(16*a^4*b^4 - 39*a^3*b^5 + 30*a^2*b^6 - 7*a*b^7)*d*sin
(10*d*x + 10*c) - (128*a^5*b^3 - 352*a^4*b^4 + 355*a^3*b^5 - 166*a^2*b^6 +
35*a*b^7)*d*sin(8*d*x + 8*c) - 4*(16*a^4*b^4 - 39*a^3*b^5 + 30*a^2*b^6 - 7*
a*b^7)*d*sin(6*d*x + 6*c) + 2*(8*a^4*b^4 - 23*a^3*b^5 + 22*a^2*b^6 - 7*a*b^
7)*d*sin(4*d*x + 4*c) + 4*(a^3*b^5 - 2*a^2*b^6 + a*b^7)*d*sin(2*d*x + 2*c))
*sin(16*d*x + 16*c) + 32*(2*(8*a^4*b^4 - 23*a^3*b^5 + 22*a^2*b^6 - 7*a*b^7)

*d*sin(12*d*x + 12*c) - 4*(16*a^4*b^4 - 39*a^3*b^5 + 30*a^2*b^6 - 7*a*b^7)*
d*sin(10*d*x + 10*c) - (128*a^5*b^3 - 352*a^4*b^4 + 355*a^3*b^5 - 166*a^2*b
^6 + 35*a*b^7)*d*sin(8*d*x + 8*c) - 4*(16*a^4*b^4 - 39*a^3*b^5 + 30*a^2*b^6
 - 7*a*b^7)*d*sin(6*d*x + 6*c) + 2*(8*a^4*b^4 - 23*a^3*b^5 + 22*a^2*b^6 - 7
*a*b^7)*d*sin(4*d*x + 4*c) + 4*(a^3*b^5 - 2*a^2*b^6 + a*b^7)*d*sin(2*d*x +
2*c))*sin(14*d*x + 14*c) - 16*(4*(128*a^5*b^3 - 424*a^4*b^4 + 513*a^3*b^5 -
 266*a^2*b^6 + 49*a*b^7)*d*sin(10*d*x + 10*c) + (1024*a^6*b^2 - 3712*a^5*b^
3 + 5304*a^4*b^4 - 3813*a^3*b^5 + 1442*a^2*b^6 - 245*a*b^7)*d*sin(8*d*x + 8
c) + 4(128*a^5*b^3 - 424*a^4*b^4 + 513*a^3*b^5 - 266*a^2*b^6 + 49*a*b^7)*
d*sin(6*d*x + 6*c) - 2*(64*a^5*b^3 - 240*a^4*b^4 + 337*a^3*b^5 - 210*a^2*b^
6 + 49*a*b^7)*d*sin(4*d*x + 4*c) - 4*(8*a^4*b^4 - 23*a^3*b^5 + 22*a^2*b^6 -
 7*a*b^7)*d*sin(2*d*x + 2*c))*sin(12*d*x + 12*c) + 32*((2048*a^6*b^2 - 6528
*a^5*b^3 + 8144*a^4*b^4 - 5141*a^3*b^5 + 1722*a^2*b^6 - 245*a*b^7)*d*sin(8*
d*x + 8*c) + 4*(256*a^5*b^3 - 736*a^4*b^4 + 753*a^3*b^5 - 322*a^2*b^6 + 49*
a*b^7)*d*sin(6*d*x + 6*c) - 2*(128*a^5*b^3 - 424*a^4*b^4 + 513*a^3*b^5 - 26
6*a^2*b^6 + 49*a*b^7)*d*sin(4*d*x + 4*c) - 4*(16*a^4*b^4 - 39*a^3*b^5 + 30*
a^2*b^6 - 7*a*b^7)*d*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 16*(2*(2048*a^6
*b^2 - 6528*a^5*b^3 + 8144*a^4*b^4 - 5141*a^3*b^5 + 1722*a^2*b^6 - 245*a*b^
7)*d*sin(6*d*x + 6*c) - (1024*a^6*b^2 - 3712*a^5*b^3 + 5304*a^4*b^4 - 3813*
a^3*b^5 + 1442*a^2*b^6 - 245*a*b^7)*d*sin(4*d*x + 4*c) - 2*(128*a^5*b^3 - 3
52*a^4*b^4 + 355*a^3*b^5 - 166*a^2*b^6 + 35*a*b^7)*d*sin(2*d*x + 2*c))*sin(
8*d*x + 8*c) - 64*((128*a^5*b^3 - 424*a^4*b^4 + 513*a^3*b^5 - 266*a^2*b^6 +
 49*a*b^7)*d*sin(4*d*x + 4*c) + 2*(16*a^4*b^4 - 39*a^3*b^5 + 30*a^2*b^6 - 7
*a*b^7)*d*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*integrate(-1/8*(4*(4*a^2*b -
13*a*b^2 + 3*b^3)*cos(6*d*x + 6*c)^2 + 12*(56*a^2*b - 29*a*b^2 + 3*b^3)*cos
(4*d*x + 4*c)^2 + 4*(4*a^2*b - 13*a*b^2 + 3*b^3)*cos(2*d*x + 2*c)^2 + 4*(4*
a^2*b - 13*a*b^2 + 3*b^3)*sin(6*d*x + 6*c)^2 + 12*(56*a^2*b - 29*a*b^2 + 3*
b^3)*sin(4*d*x + 4*c)^2 + 2*(32*a^3 - 116*a^2*b + 147*a*b^2 - 21*b^3)*sin(4
*d*x + 4*c)*sin(2*d*x + 2*c) + 4*(4*a^2*b - 13*a*b^2 + 3*b^3)*sin(2*d*x + 2
*c)^2 - ((4*a^2*b - 13*a*b^2 + 3*b^3)*cos(6*d*x + 6*c) + 6*(7*a*b^2 - b^3)*
cos(4*d*x + 4*c) + (4*a^2*b - 13*a*b^2 + 3*b^3)*cos(2*d*x + 2*c))*cos(8*d*x
 + 8*c) - (4*a^2*b - 13*a*b^2 + 3*b^3 - 2*(32*a^3 - 116*a^2*b + 147*a*b^2 -
 21*b^3)*cos(4*d*x + 4*c) - 8*(4*a^2*b - 13*a*b^2 + 3*b^3)*cos(2*d*x + 2*c)
)*cos(6*d*x + 6*c) - 2*(21*a*b^2 - 3*b^3 - (32*a^3 - 116*a^2*b + 147*a*b^2
- 21*b^3)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - (4*a^2*b - 13*a*b^2 + 3*b^3)
*cos(2*d*x + 2*c) - ((4*a^2*b - 13*a*b^2 + 3*b^3)*sin(6*d*x + 6*c) + 6*(7*a
*b^2 - b^3)*sin(4*d*x + 4*c) + (4*a^2*b - 13*a*b^2 + 3*b^3)*sin(2*d*x + 2*c
))*sin(8*d*x + 8*c) + 2*((32*a^3 - 116*a^2*b + 147*a*b^2 - 21*b^3)*sin(4*d*
x + 4*c) + 4*(4*a^2*b - 13*a*b^2 + 3*b^3)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c
))/(a^3*b^3 - 2*a^2*b^4 + a*b^5 + (a^3*b^3 - 2*a^2*b^4 + a*b^5)*cos(8*d*x +
 8*c)^2 + 16*(a^3*b^3 - 2*a^2*b^4 + a*b^5)*cos(6*d*x + 6*c)^2 + 4*(64*a^5*b
 - 176*a^4*b^2 + 169*a^3*b^3 - 66*a^2*b^4 + 9*a*b^5)*cos(4*d*x + 4*c)^2 + 1
6*(a^3*b^3 - 2*a^2*b^4 + a*b^5)*cos(2*d*x + 2*c)^2 + (a^3*b^3 - 2*a^2*b^4 +
 a*b^5)*sin(8*d*x + 8*c)^2 + 16*(a^3*b^3 - 2*a^2*b^4 + a*b^5)*sin(6*d*x + 6
c)^2 + 4(64*a^5*b - 176*a^4*b^2 + 169*a^3*b^3 - 66*a^2*b^4 + 9*a*b^5)*sin
(4*d*x + 4*c)^2 + 16*(8*a^4*b^2 - 19*a^3*b^3 + 14*a^2*b^4 - 3*a*b^5)*sin(4*
d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a^3*b^3 - 2*a^2*b^4 + a*b^5)*sin(2*d*x +
2*c)^2 + 2*(a^3*b^3 - 2*a^2*b^4 + a*b^5 - 4*(a^3*b^3 - 2*a^2*b^4 + a*b^5)*c
os(6*d*x + 6*c) - 2*(8*a^4*b^2 - 19*a^3*b^3 + 14*a^2*b^4 - 3*a*b^5)*cos(4*d
*x + 4*c) - 4*(a^3*b^3 - 2*a^2*b^4 + a*b^5)*cos(2*d*x + 2*c))*cos(8*d*x + 8
c) - 8(a^3*b^3 - 2*a^2*b^4 + a*b^5 - 2*(8*a^4*b^2 - 19*a^3*b^3 + 14*a^2*b
^4 - 3*a*b^5)*cos(4*d*x + 4*c) - 4*(a^3*b^3 - 2*a^2*b^4 + a*b^5)*cos(2*d*x
+ 2*c))*cos(6*d*x + 6*c) - 4*(8*a^4*b^2 - 19*a^3*b^3 + 14*a^2*b^4 - 3*a*b^5
 - 4*(8*a^4*b^2 - 19*a^3*b^3 + 14*a^2*b^4 - 3*a*b^5)*cos(2*d*x + 2*c))*cos(
4*d*x + 4*c) - 8*(a^3*b^3 - 2*a^2*b^4 + a*b^5)*cos(2*d*x + 2*c) - 4*(2*(a^3
*b^3 - 2*a^2*b^4 + a*b^5)*sin(6*d*x + 6*c) + (8*a^4*b^2 - 19*a^3*b^3 + 14*a
^2*b^4 - 3*a*b^5)*sin(4*d*x + 4*c) + 2*(a^3*b^3 - 2*a^2*b^4 + a*b^5)*sin(2*
d*x + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^4*b^2 - 19*a^3*b^3 + 14*a^2*b^4 - 3
*a*b^5)*sin(4*d*x + 4*c) + 2*(a^3*b^3 - 2*a^2*b^4 + a*b^5)*sin(2*d*x + 2*c)

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)*sin(6*d*x + 6*c)), x) + (3*a*b^4 + 3*b^5 - (4*a^2*b^3 - 13*a*b^4 + 3*b^5)
*cos(14*d*x + 14*c) + 3*(8*a^2*b^3 - 33*a*b^4 + 7*b^5)*cos(12*d*x + 12*c) -
 (64*a^3*b^2 + 68*a^2*b^3 - 225*a*b^4 + 63*b^5)*cos(10*d*x + 10*c) + 3*(128
*a^3*b^2 + 32*a^2*b^3 - 61*a*b^4 + 35*b^5)*cos(8*d*x + 8*c) + (64*a^3*b^2 +
 452*a^2*b^3 - 9*a*b^4 - 105*b^5)*cos(6*d*x + 6*c) - 3*(40*a^2*b^3 - 29*a*b
^4 - 21*b^5)*cos(4*d*x + 4*c) + (4*a^2*b^3 - 37*a*b^4 - 21*b^5)*cos(2*d*x +
 2*c))*sin(16*d*x + 16*c) + (4*a^2*b^3 - 37*a*b^4 - 21*b^5 - 4*(32*a^3*b^2
- 84*a^2*b^3 - 83*a*b^4 + 21*b^5)*cos(12*d*x + 12*c) + 16*(64*a^3*b^2 - 84*
a^2*b^3 - 43*a*b^4 + 21*b^5)*cos(10*d*x + 10*c) + 2*(512*a^4*b - 3584*a^3*b
^2 + 1388*a^2*b^3 - 11*a*b^4 - 315*b^5)*cos(8*d*x + 8*c) - 32*(172*a^2*b^3
- 37*a*b^4 - 21*b^5)*cos(6*d*x + 6*c) - 4*(32*a^3*b^2 - 372*a^2*b^3 + 289*a
*b^4 + 105*b^5)*cos(4*d*x + 4*c) - 16*(4*a^2*b^3 - 25*a*b^4 - 9*b^5)*cos(2*
d*x + 2*c))*sin(14*d*x + 14*c) - (120*a^2*b^3 - 87*a*b^4 - 63*b^5 - 4*(512*
a^4*b - 672*a^3*b^2 + 1228*a^2*b^3 + 21*a*b^4 - 147*b^5)*cos(10*d*x + 10*c)
 + 6*(3072*a^4*b - 6272*a^3*b^2 + 2920*a^2*b^3 - 413*a*b^4 - 245*b^5)*cos(8
*d*x + 8*c) + 4*(512*a^4*b + 3936*a^3*b^2 - 6740*a^2*b^3 + 1281*a*b^4 + 441
*b^5)*cos(6*d*x + 6*c) - 24*(192*a^3*b^2 - 416*a^2*b^3 + 161*a*b^4 + 49*b^5
)*cos(4*d*x + 4*c) + 4*(32*a^3*b^2 - 372*a^2*b^3 + 289*a*b^4 + 105*b^5)*cos
(2*d*x + 2*c))*sin(12*d*x + 12*c) + (64*a^3*b^2 + 452*a^2*b^3 - 9*a*b^4 - 1
05*b^5 + 2*(8192*a^5 + 27136*a^4*b - 37696*a^3*b^2 + 17644*a^2*b^3 - 2079*a
*b^4 - 735*b^5)*cos(8*d*x + 8*c) + 16*(1024*a^4*b + 3712*a^3*b^2 - 3692*a^2
*b^3 + 483*a*b^4 + 147*b^5)*cos(6*d*x + 6*c) - 4*(512*a^4*b + 3936*a^3*b^2
- 6740*a^2*b^3 + 1281*a*b^4 + 441*b^5)*cos(4*d*x + 4*c) - 32*(172*a^2*b^3 -
 37*a*b^4 - 21*b^5)*cos(2*d*x + 2*c))*sin(10*d*x + 10*c) + (384*a^3*b^2 + 9
6*a^2*b^3 - 183*a*b^4 + 105*b^5 + 2*(8192*a^5 + 27136*a^4*b - 37696*a^3*b^2
 + 17644*a^2*b^3 - 2079*a*b^4 - 735*b^5)*cos(6*d*x + 6*c) - 6*(3072*a^4*b -
 6272*a^3*b^2 + 2920*a^2*b^3 - 413*a*b^4 - 245*b^5)*cos(4*d*x + 4*c) + 2*(5
12*a^4*b - 3584*a^3*b^2 + 1388*a^2*b^3 - 11*a*b^4 - 315*b^5)*cos(2*d*x + 2*
c))*sin(8*d*x + 8*c) - (64*a^3*b^2 + 68*a^2*b^3 - 225*a*b^4 + 63*b^5 - 4*(5
12*a^4*b - 672*a^3*b^2 + 1228*a^2*b^3 + 21*a*b^4 - 147*b^5)*cos(4*d*x + 4*c
) - 16*(64*a^3*b^2 - 84*a^2*b^3 - 43*a*b^4 + 21*b^5)*cos(2*d*x + 2*c))*sin(
6*d*x + 6*c) + (24*a^2*b^3 - 99*a*b^4 + 21*b^5 - 4*(32*a^3*b^2 - 84*a^2*b^3
 - 83*a*b^4 + 21*b^5)*cos(2*d*x + 2*c))*sin(4*d*x + 4*c) - (4*a^2*b^3 - 13*
a*b^4 + 3*b^5)*sin(2*d*x + 2*c))/((a^3*b^5 - 2*a^2*b^6 + a*b^7)*d*cos(16*d*
x + 16*c)^2 + 64*(a^3*b^5 - 2*a^2*b^6 + a*b^7)*d*cos(14*d*x + 14*c)^2 + 16*
(64*a^5*b^3 - 240*a^4*b^4 + 337*a^3*b^5 - 210*a^2*b^6 + 49*a*b^7)*d*cos(12*
d*x + 12*c)^2 + 64*(256*a^5*b^3 - 736*a^4*b^4 + 753*a^3*b^5 - 322*a^2*b^6 +
 49*a*b^7)*d*cos(10*d*x + 10*c)^2 + 4*(16384*a^7*b - 57344*a^6*b^2 + 83712*
a^5*b^3 - 67648*a^4*b^4 + 32841*a^3*b^5 - 9170*a^2*b^6 + 1225*a*b^7)*d*cos(
8*d*x + 8*c)^2 + 64*(256*a^5*b^3 - 736*a^4*b^4 + 753*a^3*b^5 - 322*a^2*b^6
+ 49*a*b^7)*d*cos(6*d*x + 6*c)^2 + 16*(64*a^5*b^3 - 240*a^4*b^4 + 337*a^3*b
^5 - 210*a^2*b^6 + 49*a*b^7)*d*cos(4*d*x + 4*c)^2 + 64*(a^3*b^5 - 2*a^2*b^6
 + a*b^7)*d*cos(2*d*x + 2*c)^2 + (a^3*b^5 - 2*a^2*b^6 + a*b^7)*d*sin(16*d*x
 + 16*c)^2 + 64*(a^3*b^5 - 2*a^2*b^6 + a*b^7)*d*sin(14*d*x + 14*c)^2 + 16*(
64*a^5*b^3 - 240*a^4*b^4 + 337*a^3*b^5 - 210*a^2*b^6 + 49*a*b^7)*d*sin(12*d
*x + 12*c)^2 + 64*(256*a^5*b^3 - 736*a^4*b^4 + 753*a^3*b^5 - 322*a^2*b^6 +
49*a*b^7)*d*sin(10*d*x + 10*c)^2 + 4*(16384*a^7*b - 57344*a^6*b^2 + 83712*a
^5*b^3 - 67648*a^4*b^4 + 32841*a^3*b^5 - 9170*a^2*b^6 + 1225*a*b^7)*d*sin(8
*d*x + 8*c)^2 + 64*(256*a^5*b^3 - 736*a^4*b^4 + 753*a^3*b^5 - 322*a^2*b^6 +
 49*a*b^7)*d*sin(6*d*x + 6*c)^2 + 16*(64*a^5*b^3 - 240*a^4*b^4 + 337*a^3*b^
5 - 210*a^2*b^6 + 49*a*b^7)*d*sin(4*d*x + 4*c)^2 + 64*(8*a^4*b^4 - 23*a^3*b
^5 + 22*a^2*b^6 - 7*a*b^7)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 64*(a^3*b^
5 - 2*a^2*b^6 + a*b^7)*d*sin(2*d*x + 2*c)^2 - 16*(a^3*b^5 - 2*a^2*b^6 + a*b
^7)*d*cos(2*d*x + 2*c) + (a^3*b^5 - 2*a^2*b^6 + a*b^7)*d - 2*(8*(a^3*b^5 -
2*a^2*b^6 + a*b^7)*d*cos(14*d*x + 14*c) + 4*(8*a^4*b^4 - 23*a^3*b^5 + 22*a^
2*b^6 - 7*a*b^7)*d*cos(12*d*x + 12*c) - 8*(16*a^4*b^4 - 39*a^3*b^5 + 30*a^2
*b^6 - 7*a*b^7)*d*cos(10*d*x + 10*c) - 2*(128*a^5*b^3 - 352*a^4*b^4 + 355*a
^3*b^5 - 166*a^2*b^6 + 35*a*b^7)*d*cos(8*d*x + 8*c) - 8*(16*a^4*b^4 - 39*a^
3*b^5 + 30*a^2*b^6 - 7*a*b^7)*d*cos(6*d*x + 6*c) + 4*(8*a^4*b^4 - 23*a^3*b^
```

5 + 22*a^2*b^6 - 7*a*b^7)*d*cos(4*d*x + 4*c) + 8*(a^3*b^5 - 2*a^2*b^6 + a*b
^7)*d*cos(2*d*x + 2*c) - (a^3*b^5 - 2*a^2*b^6 + a*b^7)*d)*cos(16*d*x + 16*c
) + 16*(4*(8*a^4*b^4 - 23*a^3*b^5 + 22*a^2*b^6 - 7*a*b^7)*d*cos(12*d*x + 12
c) - 8(16*a^4*b^4 - 39*a^3*b^5 + 30*a^2*b^6 - 7*a*b^7)*d*cos(10*d*x + 10*
c) - 2*(128*a^5*b^3 - 352*a^4*b^4 + 355*a^3*b^5 - 166*a^2*b^6 + 35*a*b^7)*d
*cos(8*d*x + 8*c) - 8*(16*a^4*b^4 - 39*a^3*b^5 + 30*a^2*b^6 - 7*a*b^7)*d*co
s(6*d*x + 6*c) + 4*(8*a^4*b^4 - 23*a^3*b^5 + 22*a^2*b^6 - 7*a*b^7)*d*cos(4*
d*x + 4*c) + 8*(a^3*b^5 - 2*a^2*b^6 + a*b^7)*d*cos(2*d*x + 2*c) - (a^3*b^5
- 2*a^2*b^6 + a*b^7)*d)*cos(14*d*x + 14*c) - 8*(8*(128*a^5*b^3 - 424*a^4*b^
4 + 513*a^3*b^5 - 266*a^2*b^6 + 49*a*b^7)*d*cos(10*d*x + 10*c) + 2*(1024*a^
6*b^2 - 3712*a^5*b^3 + 5304*a^4*b^4 - 3813*a^3*b^5 + 1442*a^2*b^6 - 245*a*b
^7)*d*cos(8*d*x + 8*c) + 8*(128*a^5*b^3 - 424*a^4*b^4 + 513*a^3*b^5 - 266*a
^2*b^6 + 49*a*b^7)*d*cos(6*d*x + 6*c) - 4*(64*a^5*b^3 - 240*a^4*b^4 + 337*a
^3*b^5 - 210*a^2*b^6 + 49*a*b^7)*d*cos(4*d*x + 4*c) - 8*(8*a^4*b^4 - 23*a^3
*b^5 + 22*a^2*b^6 - 7*a*b^7)*d*cos(2*d*x + 2*c) + (8*a^4*b^4 - 23*a^3*b^5 +
22*a^2*b^6 - 7*a*b^7)*d)*cos(12*d*x + 12*c) + 16*(2*(2048*a^6*b^2 - 6528*a
^5*b^3 + 8144*a^4*b^4 - 5141*a^3*b^5 + 1722*a^2*b^6 - 245*a*b^7)*d*cos(8*d*
x + 8*c) + 8*(256*a^5*b^3 - 736*a^4*b^4 + 753*a^3*b^5 - 322*a^2*b^6 + 49*a*
b^7)*d*cos(6*d*x + 6*c) - 4*(128*a^5*b^3 - 424*a^4*b^4 + 513*a^3*b^5 - 266*
a^2*b^6 + 49*a*b^7)*d*cos(4*d*x + 4*c) - 8*(16*a^4*b^4 - 39*a^3*b^5 + 30*a^
2*b^6 - 7*a*b^7)*d*cos(2*d*x + 2*c) + (16*a^4*b^4 - 39*a^3*b^5 + 30*a^2*b^6
- 7*a*b^7)*d)*cos(10*d*x + 10*c) + 4*(8*(2048*a^6*b^2 - 6528*a^5*b^3 + 814
4*a^4*b^4 - 5141*a^3*b^5 + 1722*a^2*b^6 - 245*a*b^7)*d*cos(6*d*x + 6*c) - 4
*(1024*a^6*b^2 - 3712*a^5*b^3 + 5304*a^4*b^4 - 3813*a^3*b^5 + 1442*a^2*b^6
- 245*a*b^7)*d*cos(4*d*x + 4*c) - 8*(128*a^5*b^3 - 352*a^4*b^4 + 355*a^3*b^
5 - 166*a^2*b^6 + 35*a*b^7)*d*cos(2*d*x + 2*c) + (128*a^5*b^3 - 352*a^4*b^4
+ 355*a^3*b^5 - 166*a^2*b^6 + 35*a*b^7)*d)*cos(8*d*x + 8*c) - 16*(4*(128*a
^5*b^3 - 424*a^4*b^4 + 513*a^3*b^5 - 266*a^2*b^6 + 49*a*b^7)*d*cos(4*d*x +
4*c) + 8*(16*a^4*b^4 - 39*a^3*b^5 + 30*a^2*b^6 - 7*a*b^7)*d*cos(2*d*x + 2*c
) - (16*a^4*b^4 - 39*a^3*b^5 + 30*a^2*b^6 - 7*a*b^7)*d)*cos(6*d*x + 6*c) +
8*(8*(8*a^4*b^4 - 23*a^3*b^5 + 22*a^2*b^6 - 7*a*b^7)*d*cos(2*d*x + 2*c) - (
8*a^4*b^4 - 23*a^3*b^5 + 22*a^2*b^6 - 7*a*b^7)*d)*cos(4*d*x + 4*c) - 4*(4*(
a^3*b^5 - 2*a^2*b^6 + a*b^7)*d*sin(14*d*x + 14*c) + 2*(8*a^4*b^4 - 23*a^3*b
^5 + 22*a^2*b^6 - 7*a*b^7)*d*sin(12*d*x + 12*c) - 4*(16*a^4*b^4 - 39*a^3*b^
5 + 30*a^2*b^6 - 7*a*b^7)*d*sin(10*d*x + 10*c) - (128*a^5*b^3 - 352*a^4*b^4
+ 355*a^3*b^5 - 166*a^2*b^6 + 35*a*b^7)*d*sin(8*d*x + 8*c) - 4*(16*a^4*b^4
- 39*a^3*b^5 + 30*a^2*b^6 - 7*a*b^7)*d*sin(6*d*x + 6*c) + 2*(8*a^4*b^4 - 2
3*a^3*b^5 + 22*a^2*b^6 - 7*a*b^7)*d*sin(4*d*x + 4*c) + 4*(a^3*b^5 - 2*a^2*b
^6 + a*b^7)*d*sin(2*d*x + 2*c))*sin(16*d*x + 16*c) + 32*(2*(8*a^4*b^4 - 23*
a^3*b^5 + 22*a^2*b^6 - 7*a*b^7)*d*sin(12*d*x + 12*c) - 4*(16*a^4*b^4 - 39*a
^3*b^5 + 30*a^2*b^6 - 7*a*b^7)*d*sin(10*d*x + 10*c) - (128*a^5*b^3 - 352*a^
4*b^4 + 355*a^3*b^5 - 166*a^2*b^6 + 35*a*b^7)*d*sin(8*d*x + 8*c) - 4*(16*a^
4*b^4 - 39*a^3*b^5 + 30*a^2*b^6 - 7*a*b^7)*d*sin(6*d*x + 6*c) + 2*(8*a^4*b^
4 - 23*a^3*b^5 + 22*a^2*b^6 - 7*a*b^7)*d*sin(4*d*x + 4*c) + 4*(a^3*b^5 - 2*
a^2*b^6 + a*b^7)*d*sin(2*d*x + 2*c))*sin(14*d*x + 14*c) - 16*(4*(128*a^5*b^
3 - 424*a^4*b^4 + 513*a^3*b^5 - 266*a^2*b^6 + 49*a*b^7)*d*sin(10*d*x + 10*c
) + (1024*a^6*b^2 - 3712*a^5*b^3 + 5304*a^4*b^4 - 3813*a^3*b^5 + 1442*a^2*b
^6 - 245*a*b^7)*d*sin(8*d*x + 8*c) + 4*(128*a^5*b^3 - 424*a^4*b^4 + 513*a^3
*b^5 - 266*a^2*b^6 + 49*a*b^7)*d*sin(6*d*x + 6*c) - 2*(64*a^5*b^3 - 240*a^4
*b^4 + 337*a^3*b^5 - 210*a^2*b^6 + 49*a*b^7)*d*sin(4*d*x + 4*c) - 4*(8*a^4*
b^4 - 23*a^3*b^5 + 22*a^2*b^6 - 7*a*b^7)*d*sin(2*d*x + 2*c))*sin(12*d*x + 1
2*c) + 32*((2048*a^6*b^2 - 6528*a^5*b^3 + 8144*a^4*b^4 - 5141*a^3*b^5 + 172
2*a^2*b^6 - 245*a*b^7)*d*sin(8*d*x + 8*c) + 4*(256*a^5*b^3 - 736*a^4*b^4 +
753*a^3*b^5 - 322*a^2*b^6 + 49*a*b^7)*d*sin(6*d*x + 6*c) - 2*(128*a^5*b^3 -
424*a^4*b^4 + 513*a^3*b^5 - 266*a^2*b^6 + 49*a*b^7)*d*sin(4*d*x + 4*c) - 4
*(16*a^4*b^4 - 39*a^3*b^5 + 30*a^2*b^6 - 7*a*b^7)*d*sin(2*d*x + 2*c))*sin(1
0*d*x + 10*c) + 16*(2*(2048*a^6*b^2 - 6528*a^5*b^3 + 8144*a^4*b^4 - 5141*a^
3*b^5 + 1722*a^2*b^6 - 245*a*b^7)*d*sin(6*d*x + 6*c) - (1024*a^6*b^2 - 3712
*a^5*b^3 + 5304*a^4*b^4 - 3813*a^3*b^5 + 1442*a^2*b^6 - 245*a*b^7)*d*sin(4*

```
d*x + 4*c) - 2*(128*a^5*b^3 - 352*a^4*b^4 + 355*a^3*b^5 - 166*a^2*b^6 + 35*
a*b^7)*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) - 64*((128*a^5*b^3 - 424*a^4*b^
4 + 513*a^3*b^5 - 266*a^2*b^6 + 49*a*b^7)*d*sin(4*d*x + 4*c) + 2*(16*a^4*b^
4 - 39*a^3*b^5 + 30*a^2*b^6 - 7*a*b^7)*d*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)
)
```

Fricas [B] time = 21.1073, size = 13869, normalized size = 40.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a-b*sin(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/256*(((a^3*b^3 - 2*a^2*b^4 + a*b^5)*d*cos(d*x + c)^8 - 4*(a^3*b^3 - 2*a^2
*b^4 + a*b^5)*d*cos(d*x + c)^6 - 2*(a^4*b^2 - 5*a^3*b^3 + 7*a^2*b^4 - 3*a*b
^5)*d*cos(d*x + c)^4 + 4*(a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d*cos(d*
x + c)^2 + (a^5*b - 4*a^4*b^2 + 6*a^3*b^3 - 4*a^2*b^4 + a*b^5)*d)*sqrt(-(16
*a^4 - 116*a^3*b + 229*a^2*b^2 + 30*a*b^3 - 15*b^4 + (a^7*b^3 - 5*a^6*b^4 +
 10*a^5*b^5 - 10*a^4*b^6 + 5*a^3*b^7 - a^2*b^8)*d^2*sqrt((6400*a^6 - 48160*
a^5*b + 104361*a^4*b^2 - 53212*a^3*b^3 + 12814*a^2*b^4 - 1548*a*b^5 + 81*b^
6)/((a^15*b^3 - 10*a^14*b^4 + 45*a^13*b^5 - 120*a^12*b^6 + 210*a^11*b^7 - 2
52*a^10*b^8 + 210*a^9*b^9 - 120*a^8*b^10 + 45*a^7*b^11 - 10*a^6*b^12 + a^5*
b^13)*d^4)))/((a^7*b^3 - 5*a^6*b^4 + 10*a^5*b^5 - 10*a^4*b^6 + 5*a^3*b^7 -
a^2*b^8)*d^2))*log(320*a^5 - 2724*a^4*b + 6243*a^3*b^2 - 9389/4*a^2*b^3 + 7
29/2*a*b^4 - 81/4*b^5 - 1/4*(1280*a^5 - 10896*a^4*b + 24972*a^3*b^2 - 9389*
a^2*b^3 + 1458*a*b^4 - 81*b^5)*cos(d*x + c)^2 + 1/2*((2*a^11*b^3 - 27*a^10*
b^4 + 108*a^9*b^5 - 205*a^8*b^6 + 210*a^7*b^7 - 117*a^6*b^8 + 32*a^5*b^9 -
3*a^4*b^10)*d^3*sqrt((6400*a^6 - 48160*a^5*b + 104361*a^4*b^2 - 53212*a^3*b
^3 + 12814*a^2*b^4 - 1548*a*b^5 + 81*b^6)/((a^15*b^3 - 10*a^14*b^4 + 45*a^1
3*b^5 - 120*a^12*b^6 + 210*a^11*b^7 - 252*a^10*b^8 + 210*a^9*b^9 - 120*a^8*
b^10 + 45*a^7*b^11 - 10*a^6*b^12 + a^5*b^13)*d^4))*cos(d*x + c)*sin(d*x + c
) + (320*a^7*b - 2404*a^6*b^2 + 4779*a^5*b^3 - 1025*a^4*b^4 + 49*a^3*b^5 +
9*a^2*b^6)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-(16*a^4 - 116*a^3*b + 229*a^2
*b^2 + 30*a*b^3 - 15*b^4 + (a^7*b^3 - 5*a^6*b^4 + 10*a^5*b^5 - 10*a^4*b^6 +
 5*a^3*b^7 - a^2*b^8)*d^2*sqrt((6400*a^6 - 48160*a^5*b + 104361*a^4*b^2 - 5
3212*a^3*b^3 + 12814*a^2*b^4 - 1548*a*b^5 + 81*b^6)/((a^15*b^3 - 10*a^14*b^
4 + 45*a^13*b^5 - 120*a^12*b^6 + 210*a^11*b^7 - 252*a^10*b^8 + 210*a^9*b^9
- 120*a^8*b^10 + 45*a^7*b^11 - 10*a^6*b^12 + a^5*b^13)*d^4)))/((a^7*b^3 - 5
*a^6*b^4 + 10*a^5*b^5 - 10*a^4*b^6 + 5*a^3*b^7 - a^2*b^8)*d^2)) - 1/4*(2*(1
6*a^10*b - 156*a^9*b^2 + 549*a^8*b^3 - 965*a^7*b^4 + 930*a^6*b^5 - 486*a^5*
b^6 + 121*a^4*b^7 - 9*a^3*b^8)*d^2*cos(d*x + c)^2 - (16*a^10*b - 156*a^9*b^
2 + 549*a^8*b^3 - 965*a^7*b^4 + 930*a^6*b^5 - 486*a^5*b^6 + 121*a^4*b^7 - 9
*a^3*b^8)*d^2)*sqrt((6400*a^6 - 48160*a^5*b + 104361*a^4*b^2 - 53212*a^3*b^
3 + 12814*a^2*b^4 - 1548*a*b^5 + 81*b^6)/((a^15*b^3 - 10*a^14*b^4 + 45*a^13
*b^5 - 120*a^12*b^6 + 210*a^11*b^7 - 252*a^10*b^8 + 210*a^9*b^9 - 120*a^8*b
^10 + 45*a^7*b^11 - 10*a^6*b^12 + a^5*b^13)*d^4))) - ((a^3*b^3 - 2*a^2*b^4
+ a*b^5)*d*cos(d*x + c)^8 - 4*(a^3*b^3 - 2*a^2*b^4 + a*b^5)*d*cos(d*x + c)^
6 - 2*(a^4*b^2 - 5*a^3*b^3 + 7*a^2*b^4 - 3*a*b^5)*d*cos(d*x + c)^4 + 4*(a^4
*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d*cos(d*x + c)^2 + (a^5*b - 4*a^4*b^2
 + 6*a^3*b^3 - 4*a^2*b^4 + a*b^5)*d)*sqrt(-(16*a^4 - 116*a^3*b + 229*a^2*b^
2 + 30*a*b^3 - 15*b^4 + (a^7*b^3 - 5*a^6*b^4 + 10*a^5*b^5 - 10*a^4*b^6 + 5*
a^3*b^7 - a^2*b^8)*d^2*sqrt((6400*a^6 - 48160*a^5*b + 104361*a^4*b^2 - 5321
2*a^3*b^3 + 12814*a^2*b^4 - 1548*a*b^5 + 81*b^6)/((a^15*b^3 - 10*a^14*b^4 +
 45*a^13*b^5 - 120*a^12*b^6 + 210*a^11*b^7 - 252*a^10*b^8 + 210*a^9*b^9 - 1
20*a^8*b^10 + 45*a^7*b^11 - 10*a^6*b^12 + a^5*b^13)*d^4)))/((a^7*b^3 - 5*a^
6*b^4 + 10*a^5*b^5 - 10*a^4*b^6 + 5*a^3*b^7 - a^2*b^8)*d^2))*log(320*a^5 -
```

```
2724*a^4*b + 6243*a^3*b^2 - 9389/4*a^2*b^3 + 729/2*a*b^4 - 81/4*b^5 - 1/4*(
1280*a^5 - 10896*a^4*b + 24972*a^3*b^2 - 9389*a^2*b^3 + 1458*a*b^4 - 81*b^5
)*cos(d*x + c)^2 - 1/2*((2*a^11*b^3 - 27*a^10*b^4 + 108*a^9*b^5 - 205*a^8*b
^6 + 210*a^7*b^7 - 117*a^6*b^8 + 32*a^5*b^9 - 3*a^4*b^10)*d^3*sqrt((6400*a^
6 - 48160*a^5*b + 104361*a^4*b^2 - 53212*a^3*b^3 + 12814*a^2*b^4 - 1548*a*b
^5 + 81*b^6)/((a^15*b^3 - 10*a^14*b^4 + 45*a^13*b^5 - 120*a^12*b^6 + 210*a^
11*b^7 - 252*a^10*b^8 + 210*a^9*b^9 - 120*a^8*b^10 + 45*a^7*b^11 - 10*a^6*b
^12 + a^5*b^13)*d^4))*cos(d*x + c)*sin(d*x + c) + (320*a^7*b - 2404*a^6*b^2
 + 4779*a^5*b^3 - 1025*a^4*b^4 + 49*a^3*b^5 + 9*a^2*b^6)*d*cos(d*x + c)*sin
(d*x + c))*sqrt(-(16*a^4 - 116*a^3*b + 229*a^2*b^2 + 30*a*b^3 - 15*b^4 + (a
^7*b^3 - 5*a^6*b^4 + 10*a^5*b^5 - 10*a^4*b^6 + 5*a^3*b^7 - a^2*b^8)*d^2*sqr
t((6400*a^6 - 48160*a^5*b + 104361*a^4*b^2 - 53212*a^3*b^3 + 12814*a^2*b^4
- 1548*a*b^5 + 81*b^6)/((a^15*b^3 - 10*a^14*b^4 + 45*a^13*b^5 - 120*a^12*b^
6 + 210*a^11*b^7 - 252*a^10*b^8 + 210*a^9*b^9 - 120*a^8*b^10 + 45*a^7*b^11
- 10*a^6*b^12 + a^5*b^13)*d^4)))/((a^7*b^3 - 5*a^6*b^4 + 10*a^5*b^5 - 10*a^
4*b^6 + 5*a^3*b^7 - a^2*b^8)*d^2)) - 1/4*(2*(16*a^10*b - 156*a^9*b^2 + 549*
a^8*b^3 - 965*a^7*b^4 + 930*a^6*b^5 - 486*a^5*b^6 + 121*a^4*b^7 - 9*a^3*b^8
)*d^2*cos(d*x + c)^2 - (16*a^10*b - 156*a^9*b^2 + 549*a^8*b^3 - 965*a^7*b^4
 + 930*a^6*b^5 - 486*a^5*b^6 + 121*a^4*b^7 - 9*a^3*b^8)*d^2)*sqrt((6400*a^6
 - 48160*a^5*b + 104361*a^4*b^2 - 53212*a^3*b^3 + 12814*a^2*b^4 - 1548*a*b^
5 + 81*b^6)/((a^15*b^3 - 10*a^14*b^4 + 45*a^13*b^5 - 120*a^12*b^6 + 210*a^1
1*b^7 - 252*a^10*b^8 + 210*a^9*b^9 - 120*a^8*b^10 + 45*a^7*b^11 - 10*a^6*b^
12 + a^5*b^13)*d^4))) + ((a^3*b^3 - 2*a^2*b^4 + a*b^5)*d*cos(d*x + c)^8 - 4
*(a^3*b^3 - 2*a^2*b^4 + a*b^5)*d*cos(d*x + c)^6 - 2*(a^4*b^2 - 5*a^3*b^3 +
7*a^2*b^4 - 3*a*b^5)*d*cos(d*x + c)^4 + 4*(a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4
- a*b^5)*d*cos(d*x + c)^2 + (a^5*b - 4*a^4*b^2 + 6*a^3*b^3 - 4*a^2*b^4 + a*
b^5)*d)*sqrt(-(16*a^4 - 116*a^3*b + 229*a^2*b^2 + 30*a*b^3 - 15*b^4 - (a^7*
b^3 - 5*a^6*b^4 + 10*a^5*b^5 - 10*a^4*b^6 + 5*a^3*b^7 - a^2*b^8)*d^2*sqrt((
6400*a^6 - 48160*a^5*b + 104361*a^4*b^2 - 53212*a^3*b^3 + 12814*a^2*b^4 - 1
548*a*b^5 + 81*b^6)/((a^15*b^3 - 10*a^14*b^4 + 45*a^13*b^5 - 120*a^12*b^6 +
 210*a^11*b^7 - 252*a^10*b^8 + 210*a^9*b^9 - 120*a^8*b^10 + 45*a^7*b^11 - 1
0*a^6*b^12 + a^5*b^13)*d^4)))/((a^7*b^3 - 5*a^6*b^4 + 10*a^5*b^5 - 10*a^4*b
^6 + 5*a^3*b^7 - a^2*b^8)*d^2))*log(-320*a^5 + 2724*a^4*b - 6243*a^3*b^2 +
9389/4*a^2*b^3 - 729/2*a*b^4 + 81/4*b^5 + 1/4*(1280*a^5 - 10896*a^4*b + 249
72*a^3*b^2 - 9389*a^2*b^3 + 1458*a*b^4 - 81*b^5)*cos(d*x + c)^2 + 1/2*((2*a
^11*b^3 - 27*a^10*b^4 + 108*a^9*b^5 - 205*a^8*b^6 + 210*a^7*b^7 - 117*a^6*b
^8 + 32*a^5*b^9 - 3*a^4*b^10)*d^3*sqrt((6400*a^6 - 48160*a^5*b + 104361*a^4
*b^2 - 53212*a^3*b^3 + 12814*a^2*b^4 - 1548*a*b^5 + 81*b^6)/((a^15*b^3 - 10
*a^14*b^4 + 45*a^13*b^5 - 120*a^12*b^6 + 210*a^11*b^7 - 252*a^10*b^8 + 210*
a^9*b^9 - 120*a^8*b^10 + 45*a^7*b^11 - 10*a^6*b^12 + a^5*b^13)*d^4))*cos(d*
x + c)*sin(d*x + c) - (320*a^7*b - 2404*a^6*b^2 + 4779*a^5*b^3 - 1025*a^4*b
^4 + 49*a^3*b^5 + 9*a^2*b^6)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-(16*a^4 - 1
16*a^3*b + 229*a^2*b^2 + 30*a*b^3 - 15*b^4 - (a^7*b^3 - 5*a^6*b^4 + 10*a^5*
b^5 - 10*a^4*b^6 + 5*a^3*b^7 - a^2*b^8)*d^2*sqrt((6400*a^6 - 48160*a^5*b +
104361*a^4*b^2 - 53212*a^3*b^3 + 12814*a^2*b^4 - 1548*a*b^5 + 81*b^6)/((a^1
5*b^3 - 10*a^14*b^4 + 45*a^13*b^5 - 120*a^12*b^6 + 210*a^11*b^7 - 252*a^10*
b^8 + 210*a^9*b^9 - 120*a^8*b^10 + 45*a^7*b^11 - 10*a^6*b^12 + a^5*b^13)*d^
4)))/((a^7*b^3 - 5*a^6*b^4 + 10*a^5*b^5 - 10*a^4*b^6 + 5*a^3*b^7 - a^2*b^8)
*d^2)) - 1/4*(2*(16*a^10*b - 156*a^9*b^2 + 549*a^8*b^3 - 965*a^7*b^4 + 930*
a^6*b^5 - 486*a^5*b^6 + 121*a^4*b^7 - 9*a^3*b^8)*d^2*cos(d*x + c)^2 - (16*a
^10*b - 156*a^9*b^2 + 549*a^8*b^3 - 965*a^7*b^4 + 930*a^6*b^5 - 486*a^5*b^6
 + 121*a^4*b^7 - 9*a^3*b^8)*d^2)*sqrt((6400*a^6 - 48160*a^5*b + 104361*a^4*
b^2 - 53212*a^3*b^3 + 12814*a^2*b^4 - 1548*a*b^5 + 81*b^6)/((a^15*b^3 - 10*
a^14*b^4 + 45*a^13*b^5 - 120*a^12*b^6 + 210*a^11*b^7 - 252*a^10*b^8 + 210*a
^9*b^9 - 120*a^8*b^10 + 45*a^7*b^11 - 10*a^6*b^12 + a^5*b^13)*d^4))) - ((a^
3*b^3 - 2*a^2*b^4 + a*b^5)*d*cos(d*x + c)^8 - 4*(a^3*b^3 - 2*a^2*b^4 + a*b^
5)*d*cos(d*x + c)^6 - 2*(a^4*b^2 - 5*a^3*b^3 + 7*a^2*b^4 - 3*a*b^5)*d*cos(d
*x + c)^4 + 4*(a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d*cos(d*x + c)^2 +
(a^5*b - 4*a^4*b^2 + 6*a^3*b^3 - 4*a^2*b^4 + a*b^5)*d)*sqrt(-(16*a^4 - 116*
```

```
a^3*b + 229*a^2*b^2 + 30*a*b^3 - 15*b^4 - (a^7*b^3 - 5*a^6*b^4 + 10*a^5*b^5
 - 10*a^4*b^6 + 5*a^3*b^7 - a^2*b^8)*d^2*sqrt((6400*a^6 - 48160*a^5*b + 104
361*a^4*b^2 - 53212*a^3*b^3 + 12814*a^2*b^4 - 1548*a*b^5 + 81*b^6)/((a^15*b
^3 - 10*a^14*b^4 + 45*a^13*b^5 - 120*a^12*b^6 + 210*a^11*b^7 - 252*a^10*b^8
 + 210*a^9*b^9 - 120*a^8*b^10 + 45*a^7*b^11 - 10*a^6*b^12 + a^5*b^13)*d^4))
)/((a^7*b^3 - 5*a^6*b^4 + 10*a^5*b^5 - 10*a^4*b^6 + 5*a^3*b^7 - a^2*b^8)*d^
2))*log(-320*a^5 + 2724*a^4*b - 6243*a^3*b^2 + 9389/4*a^2*b^3 - 729/2*a*b^4
 + 81/4*b^5 + 1/4*(1280*a^5 - 10896*a^4*b + 24972*a^3*b^2 - 9389*a^2*b^3 +
1458*a*b^4 - 81*b^5)*cos(d*x + c)^2 - 1/2*((2*a^11*b^3 - 27*a^10*b^4 + 108*
a^9*b^5 - 205*a^8*b^6 + 210*a^7*b^7 - 117*a^6*b^8 + 32*a^5*b^9 - 3*a^4*b^10
)*d^3*sqrt((6400*a^6 - 48160*a^5*b + 104361*a^4*b^2 - 53212*a^3*b^3 + 12814
*a^2*b^4 - 1548*a*b^5 + 81*b^6)/((a^15*b^3 - 10*a^14*b^4 + 45*a^13*b^5 - 12
0*a^12*b^6 + 210*a^11*b^7 - 252*a^10*b^8 + 210*a^9*b^9 - 120*a^8*b^10 + 45*
a^7*b^11 - 10*a^6*b^12 + a^5*b^13)*d^4))*cos(d*x + c)*sin(d*x + c) - (320*a
^7*b - 2404*a^6*b^2 + 4779*a^5*b^3 - 1025*a^4*b^4 + 49*a^3*b^5 + 9*a^2*b^6)
*d*cos(d*x + c)*sin(d*x + c))*sqrt(-(16*a^4 - 116*a^3*b + 229*a^2*b^2 + 30*
a*b^3 - 15*b^4 - (a^7*b^3 - 5*a^6*b^4 + 10*a^5*b^5 - 10*a^4*b^6 + 5*a^3*b^7
 - a^2*b^8)*d^2*sqrt((6400*a^6 - 48160*a^5*b + 104361*a^4*b^2 - 53212*a^3*b
^3 + 12814*a^2*b^4 - 1548*a*b^5 + 81*b^6)/((a^15*b^3 - 10*a^14*b^4 + 45*a^1
3*b^5 - 120*a^12*b^6 + 210*a^11*b^7 - 252*a^10*b^8 + 210*a^9*b^9 - 120*a^8*
b^10 + 45*a^7*b^11 - 10*a^6*b^12 + a^5*b^13)*d^4)))/((a^7*b^3 - 5*a^6*b^4 +
 10*a^5*b^5 - 10*a^4*b^6 + 5*a^3*b^7 - a^2*b^8)*d^2)) - 1/4*(2*(16*a^10*b -
 156*a^9*b^2 + 549*a^8*b^3 - 965*a^7*b^4 + 930*a^6*b^5 - 486*a^5*b^6 + 121*
a^4*b^7 - 9*a^3*b^8)*d^2*cos(d*x + c)^2 - (16*a^10*b - 156*a^9*b^2 + 549*a^
8*b^3 - 965*a^7*b^4 + 930*a^6*b^5 - 486*a^5*b^6 + 121*a^4*b^7 - 9*a^3*b^8)*
d^2)*sqrt((6400*a^6 - 48160*a^5*b + 104361*a^4*b^2 - 53212*a^3*b^3 + 12814*
a^2*b^4 - 1548*a*b^5 + 81*b^6)/((a^15*b^3 - 10*a^14*b^4 + 45*a^13*b^5 - 120
*a^12*b^6 + 210*a^11*b^7 - 252*a^10*b^8 + 210*a^9*b^9 - 120*a^8*b^10 + 45*a
^7*b^11 - 10*a^6*b^12 + a^5*b^13)*d^4))) - 8*(3*(a*b^2 + b^3)*cos(d*x + c)^
7 + (2*a^2*b - 17*a*b^2 - 9*b^3)*cos(d*x + c)^5 - (11*a^2*b - 26*a*b^2 - 9*
b^3)*cos(d*x + c)^3 + (2*a^3 + 13*a^2*b - 12*a*b^2 - 3*b^3)*cos(d*x + c))*s
in(d*x + c))/((a^3*b^3 - 2*a^2*b^4 + a*b^5)*d*cos(d*x + c)^8 - 4*(a^3*b^3 -
 2*a^2*b^4 + a*b^5)*d*cos(d*x + c)^6 - 2*(a^4*b^2 - 5*a^3*b^3 + 7*a^2*b^4 -
 3*a*b^5)*d*cos(d*x + c)^4 + 4*(a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d*
cos(d*x + c)^2 + (a^5*b - 4*a^4*b^2 + 6*a^3*b^3 - 4*a^2*b^4 + a*b^5)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**6/(a-b*sin(d*x+c)**4)**3,x)

[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a-b*sin(d*x+c)^4)^3,x, algorithm="giac")

[Out] Exception raised: NotImplementedError
```

3.232 $\int \frac{\sin^4(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=313

$$-\frac{\tan(c+dx)\left(\frac{9a^2-24ab-b^2}{(a-b)^3}+\frac{(17a+3b)\tan^2(c+dx)}{(a-b)^2}\right)}{32ad\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}+\frac{3\left(2\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{7/4}\sqrt{b}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}-\frac{3\left(2\sqrt{a}+\sqrt{b}\right)\tan^{-1}\left(\right.}{64a^{7/4}\sqrt{b}d\left(\sqrt{a}\right.}$$

```
[Out] (3*(2*Sqrt[a] - Sqrt[b])*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1
/4)])/(64*a^(7/4)*(Sqrt[a] - Sqrt[b])^(5/2)*Sqrt[b]*d) - (3*(2*Sqrt[a] + Sq
rt[b])*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(64*a^(7/4)*
(Sqrt[a] + Sqrt[b])^(5/2)*Sqrt[b]*d) - (b*Tan[c + d*x]*(3*a + b + 4*(a + b)
*Tan[c + d*x]^2))/(8*(a - b)^3*d*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan[c +
d*x]^4)^2) - (Tan[c + d*x]*((9*a^2 - 24*a*b - b^2)/(a - b)^3 + ((17*a + 3*b
)*Tan[c + d*x]^2)/(a - b)^2))/(32*a*d*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan
[c + d*x]^4))
```

Rubi [A] time = 0.695044, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3217, 1333, 1678, 1166, 205}

$$-\frac{\tan(c+dx)\left(\frac{9a^2-24ab-b^2}{(a-b)^3}+\frac{(17a+3b)\tan^2(c+dx)}{(a-b)^2}\right)}{32ad\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}+\frac{3\left(2\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{7/4}\sqrt{b}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}-\frac{3\left(2\sqrt{a}+\sqrt{b}\right)\tan^{-1}\left(\right.}{64a^{7/4}\sqrt{b}d\left(\sqrt{a}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] (3*(2*Sqrt[a] - Sqrt[b])*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1
/4)])/(64*a^(7/4)*(Sqrt[a] - Sqrt[b])^(5/2)*Sqrt[b]*d) - (3*(2*Sqrt[a] + Sq
rt[b])*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(64*a^(7/4)*
(Sqrt[a] + Sqrt[b])^(5/2)*Sqrt[b]*d) - (b*Tan[c + d*x]*(3*a + b + 4*(a + b)
*Tan[c + d*x]^2))/(8*(a - b)^3*d*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan[c +
d*x]^4)^2) - (Tan[c + d*x]*((9*a^2 - 24*a*b - b^2)/(a - b)^3 + ((17*a + 3*b
)*Tan[c + d*x]^2)/(a - b)^2))/(32*a*d*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan
[c + d*x]^4))
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 1333

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)),
```

```
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*S
imp[ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d + e*x^2
)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g +
c*(4*p + 7)*(b*f - 2*a*g)*x^2, x], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1] && IGtQ[m/2, 0]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^4(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^4\left(1+x^2\right)^3}{\left(a+2ax^2+(a-b)x^4\right)^3}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{b\tan(c+dx)\left(3a+b+4(a+b)\tan^2(c+dx)\right)}{8(a-b)^3 d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-\frac{2a^2b^2(3a+b)}{(a-b)^3}+\frac{8a^2(3a-b)b^2x^2}{(a-b)^3}\cdots}{\left(a+2ax^2+\cdots\right.}\right.}{\cdots}$$

$$= -\frac{b\tan(c+dx)\left(3a+b+4(a+b)\tan^2(c+dx)\right)}{8(a-b)^3 d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{\tan(c+dx)\left(\frac{9a^2-24ab-b^2}{(a-b)^3}+\cdots\right.}{32ad\left(a+2a\tan^2(c+dx)+(a\cdots\right.}$$

$$= -\frac{b\tan(c+dx)\left(3a+b+4(a+b)\tan^2(c+dx)\right)}{8(a-b)^3 d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{\tan(c+dx)\left(\frac{9a^2-24ab-b^2}{(a-b)^3}+\cdots\right.}{32ad\left(a+2a\tan^2(c+dx)+(a\cdots\right.}$$

$$= \frac{3\left(2\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{7/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}\sqrt{b}d} - \frac{3\left(2\sqrt{a}+\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{7/4}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}\sqrt{b}d} - \frac{b\,\mathrm{t}\cdots}{8(a-b\cdots}$$

Mathematica [A] time = 5.04678, size = 316, normalized size = 1.01

$$\frac{-\frac{3\left(2a^{3/2}-3a\sqrt{b}+b^{3/2}\right)\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{a^{3/2}\sqrt{b}\sqrt{\sqrt{a}\sqrt{b}+a}}-\frac{3\left(2a^{3/2}+3a\sqrt{b}-b^{3/2}\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{a^{3/2}\sqrt{b}\sqrt{\sqrt{a}\sqrt{b}-a}}+\frac{8\sin(2(c+dx))((2a+b)\cos(2(c+dx))-7a-2b)}{a(8a+4b\cos(2(c+dx))-b\cos(4(c+dx))-3b)}+\frac{\cdots}{(-\cdots}}{64d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] ((-3*(2*a^(3/2) - 3*a*Sqrt[b] + b^(3/2))*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c
+ d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(a^(3/2)*Sqrt[a + Sqrt[a]*Sqrt[b]]*Sqrt
[b]) - (3*(2*a^(3/2) + 3*a*Sqrt[b] - b^(3/2))*ArcTanh[((Sqrt[a] - Sqrt[b])*
Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/(a^(3/2)*Sqrt[-a + Sqrt[a]*Sqrt[
b]]*Sqrt[b]) + (8*(-7*a - 2*b + (2*a + b)*Cos[2*(c + d*x)])*Sin[2*(c + d*x)
])/(a*(8*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*x)])) + (64*(a - b
)*(-6*Sin[2*(c + d*x)] + Sin[4*(c + d*x)]))/(-8*a + 3*b - 4*b*Cos[2*(c + d*
x)] + b*Cos[4*(c + d*x)])^2)/(64*(a - b)^2*d)
```

Maple [B] time = 0.131, size = 1624, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a-b*sin(d*x+c)^4)^3,x)
```

```
[Out] -17/32/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2/(a-b)*tan(d*x
+c)^7-3/32/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2/(a-b)/a*b
*tan(d*x+c)^7-43/32/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2/
(a^2-2*a*b+b^2)*tan(d*x+c)^5*a+9/16/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*ta
n(d*x+c)^2+a)^2*b/(a^2-2*a*b+b^2)*tan(d*x+c)^5+1/32/d/(tan(d*x+c)^4*a-tan(d
*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2/a/(a^2-2*a*b+b^2)*tan(d*x+c)^5*b^2-35/32/d/
(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2*a/(a^2-2*a*b+b^2)*tan(
d*x+c)^3+11/32/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2*b/(a^
2-2*a*b+b^2)*tan(d*x+c)^3-9/32/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x
+c)^2+a)^2*a/(a^2-2*a*b+b^2)*tan(d*x+c)+3/32/d/(tan(d*x+c)^4*a-tan(d*x+c)^4
*b+2*a*tan(d*x+c)^2+a)^2/(a^2-2*a*b+b^2)*tan(d*x+c)*b+15/64/d/(a^2-2*a*b+b^
2)*a/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1
/2)+a)*(a-b))^(1/2))-9/32/d*b/(a^2-2*a*b+b^2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))
^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+3/32/d/(a^2-2
*a*b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(
d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*a^2+3/64/d*b/(a^2-2*a*b+b^2)*a/(a*b)^
(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(
1/2)+a)*(a-b))^(1/2))-3/16/d*b^2/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^
(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2)
)+15/64/d/(a^2-2*a*b+b^2)*a/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a
+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-9/32/d*b/(a^2-2*a*b+b^2)/(a-b
)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*
(a-b))^(1/2))-3/32/d/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-
b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*a^2-3/64
/d*b/(a^2-2*a*b+b^2)*a/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arct
anh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))+3/16/d*b^2/(a^2-2*a*b+
b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x
+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))+3/64/d/a*b^2/(a^2-2*a*b+b^2)/(a-b)/(((a*
```

```
b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1
/2))+3/64/d/a/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/
2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*b^3+3/64/d/a*b^2/
(a^2-2*a*b+b^2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+
c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-3/64/d/a/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(a-b
)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*
(a-b))^(1/2))*b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a-b*sin(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(3*a*b^3*sin(2*d*x + 2*c) - 12*(8*a^2*b^2 + 13*a*b^3 - 2*b^4)*cos(4*d*
x + 4*c)*sin(2*d*x + 2*c) - (3*a*b^3*sin(14*d*x + 14*c) - 3*(10*a*b^3 - b^4
)*sin(12*d*x + 12*c) - (80*a^2*b^2 - 111*a*b^3 + 16*b^4)*sin(10*d*x + 10*c)
 + (256*a^3*b - 64*a^2*b^2 - 26*a*b^3 + 35*b^4)*sin(8*d*x + 8*c) + (336*a^2
*b^2 - 95*a*b^3 - 40*b^4)*sin(6*d*x + 6*c) - (64*a^2*b^2 - 54*a*b^3 - 25*b^
4)*sin(4*d*x + 4*c) - (19*a*b^3 + 8*b^4)*sin(2*d*x + 2*c))*cos(16*d*x + 16*
c) - 2*(6*(8*a^2*b^2 + 13*a*b^3 - 2*b^4)*sin(12*d*x + 12*c) + 8*(16*a^2*b^2
 - 45*a*b^3 + 8*b^4)*sin(10*d*x + 10*c) - (1408*a^3*b - 544*a^2*b^2 + a*b^3
 + 140*b^4)*sin(8*d*x + 8*c) - 16*(96*a^2*b^2 - 29*a*b^3 - 10*b^4)*sin(6*d*
x + 6*c) + 2*(152*a^2*b^2 - 129*a*b^3 - 50*b^4)*sin(4*d*x + 4*c) + 8*(11*a*
b^3 + 4*b^4)*sin(2*d*x + 2*c))*cos(14*d*x + 14*c) - 2*(2*(640*a^3*b - 488*a
^2*b^2 + 389*a*b^3 - 70*b^4)*sin(10*d*x + 10*c) - (4096*a^4 - 8448*a^3*b +
3744*a^2*b^2 - 414*a*b^3 - 385*b^4)*sin(8*d*x + 8*c) - 2*(2688*a^3*b - 4072
*a^2*b^2 + 861*a*b^3 + 238*b^4)*sin(6*d*x + 6*c) + 4*(256*a^3*b - 560*a^2*b
^2 + 206*a*b^3 + 77*b^4)*sin(4*d*x + 4*c) + 2*(152*a^2*b^2 - 129*a*b^3 - 50
*b^4)*sin(2*d*x + 2*c))*cos(12*d*x + 12*c) - 2*((26624*a^4 - 33152*a^3*b +
15632*a^2*b^2 - 2453*a*b^3 - 420*b^4)*sin(8*d*x + 8*c) + 8*(3328*a^3*b - 31
04*a^2*b^2 + 529*a*b^3 + 84*b^4)*sin(6*d*x + 6*c) - 2*(2688*a^3*b - 4072*a^
2*b^2 + 861*a*b^3 + 238*b^4)*sin(4*d*x + 4*c) - 16*(96*a^2*b^2 - 29*a*b^3 -
 10*b^4)*sin(2*d*x + 2*c))*cos(10*d*x + 10*c) - 2*((26624*a^4 - 33152*a^3*b
 + 15632*a^2*b^2 - 2453*a*b^3 - 420*b^4)*sin(6*d*x + 6*c) - (4096*a^4 - 844
8*a^3*b + 3744*a^2*b^2 - 414*a*b^3 - 385*b^4)*sin(4*d*x + 4*c) - (1408*a^3*
b - 544*a^2*b^2 + a*b^3 + 140*b^4)*sin(2*d*x + 2*c))*cos(8*d*x + 8*c) - 4*(
(640*a^3*b - 488*a^2*b^2 + 389*a*b^3 - 70*b^4)*sin(4*d*x + 4*c) + 4*(16*a^2
*b^2 - 45*a*b^3 + 8*b^4)*sin(2*d*x + 2*c))*cos(6*d*x + 6*c) - 8*((a^3*b^4 -
 2*a^2*b^5 + a*b^6)*d*cos(16*d*x + 16*c)^2 + 64*(a^3*b^4 - 2*a^2*b^5 + a*b^
6)*d*cos(14*d*x + 14*c)^2 + 16*(64*a^5*b^2 - 240*a^4*b^3 + 337*a^3*b^4 - 21
0*a^2*b^5 + 49*a*b^6)*d*cos(12*d*x + 12*c)^2 + 64*(256*a^5*b^2 - 736*a^4*b^
3 + 753*a^3*b^4 - 322*a^2*b^5 + 49*a*b^6)*d*cos(10*d*x + 10*c)^2 + 4*(16384
*a^7 - 57344*a^6*b + 83712*a^5*b^2 - 67648*a^4*b^3 + 32841*a^3*b^4 - 9170*a
^2*b^5 + 1225*a*b^6)*d*cos(8*d*x + 8*c)^2 + 64*(256*a^5*b^2 - 736*a^4*b^3 +
 753*a^3*b^4 - 322*a^2*b^5 + 49*a*b^6)*d*cos(6*d*x + 6*c)^2 + 16*(64*a^5*b^
2 - 240*a^4*b^3 + 337*a^3*b^4 - 210*a^2*b^5 + 49*a*b^6)*d*cos(4*d*x + 4*c)^
2 + 64*(a^3*b^4 - 2*a^2*b^5 + a*b^6)*d*cos(2*d*x + 2*c)^2 + (a^3*b^4 - 2*a^
2*b^5 + a*b^6)*d*sin(16*d*x + 16*c)^2 + 64*(a^3*b^4 - 2*a^2*b^5 + a*b^6)*d*
sin(14*d*x + 14*c)^2 + 16*(64*a^5*b^2 - 240*a^4*b^3 + 337*a^3*b^4 - 210*a^2
*b^5 + 49*a*b^6)*d*sin(12*d*x + 12*c)^2 + 64*(256*a^5*b^2 - 736*a^4*b^3 + 7
53*a^3*b^4 - 322*a^2*b^5 + 49*a*b^6)*d*sin(10*d*x + 10*c)^2 + 4*(16384*a^7
- 57344*a^6*b + 83712*a^5*b^2 - 67648*a^4*b^3 + 32841*a^3*b^4 - 9170*a^2*b^
5 + 1225*a*b^6)*d*sin(8*d*x + 8*c)^2 + 64*(256*a^5*b^2 - 736*a^4*b^3 + 753*
a^3*b^4 - 322*a^2*b^5 + 49*a*b^6)*d*sin(6*d*x + 6*c)^2 + 16*(64*a^5*b^2 - 2
```

40*a^4*b^3 + 337*a^3*b^4 - 210*a^2*b^5 + 49*a*b^6)*d*sin(4*d*x + 4*c)^2 + 64*(8*a^4*b^3 - 23*a^3*b^4 + 22*a^2*b^5 - 7*a*b^6)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 64*(a^3*b^4 - 2*a^2*b^5 + a*b^6)*d*sin(2*d*x + 2*c)^2 - 16*(a^3*b^4 - 2*a^2*b^5 + a*b^6)*d*cos(2*d*x + 2*c) + (a^3*b^4 - 2*a^2*b^5 + a*b^6)*d - 2*(8*(a^3*b^4 - 2*a^2*b^5 + a*b^6)*d*cos(14*d*x + 14*c) + 4*(8*a^4*b^3 - 23*a^3*b^4 + 22*a^2*b^5 - 7*a*b^6)*d*cos(12*d*x + 12*c) - 8*(16*a^4*b^3 - 39*a^3*b^4 + 30*a^2*b^5 - 7*a*b^6)*d*cos(10*d*x + 10*c) - 2*(128*a^5*b^2 - 352*a^4*b^3 + 355*a^3*b^4 - 166*a^2*b^5 + 35*a*b^6)*d*cos(8*d*x + 8*c) - 8*(16*a^4*b^3 - 39*a^3*b^4 + 30*a^2*b^5 - 7*a*b^6)*d*cos(6*d*x + 6*c) + 4*(8*a^4*b^3 - 23*a^3*b^4 + 22*a^2*b^5 - 7*a*b^6)*d*cos(4*d*x + 4*c) + 8*(a^3*b^4 - 2*a^2*b^5 + a*b^6)*d*cos(2*d*x + 2*c) - (a^3*b^4 - 2*a^2*b^5 + a*b^6)*d)*cos(16*d*x + 16*c) + 16*(4*(8*a^4*b^3 - 23*a^3*b^4 + 22*a^2*b^5 - 7*a*b^6)*d*cos(12*d*x + 12*c) - 8*(16*a^4*b^3 - 39*a^3*b^4 + 30*a^2*b^5 - 7*a*b^6)*d*cos(10*d*x + 10*c) - 2*(128*a^5*b^2 - 352*a^4*b^3 + 355*a^3*b^4 - 166*a^2*b^5 + 35*a*b^6)*d*cos(8*d*x + 8*c) - 8*(16*a^4*b^3 - 39*a^3*b^4 + 30*a^2*b^5 - 7*a*b^6)*d*cos(6*d*x + 6*c) + 4*(8*a^4*b^3 - 23*a^3*b^4 + 22*a^2*b^5 - 7*a*b^6)*d*cos(4*d*x + 4*c) + 8*(a^3*b^4 - 2*a^2*b^5 + a*b^6)*d*cos(2*d*x + 2*c) - (a^3*b^4 - 2*a^2*b^5 + a*b^6)*d)*cos(14*d*x + 14*c) - 8*(8*(128*a^5*b^2 - 424*a^4*b^3 + 513*a^3*b^4 - 266*a^2*b^5 + 49*a*b^6)*d*cos(10*d*x + 10*c) + 2*(1024*a^6*b - 3712*a^5*b^2 + 5304*a^4*b^3 - 3813*a^3*b^4 + 1442*a^2*b^5 - 245*a*b^6)*d*cos(8*d*x + 8*c) + 8*(128*a^5*b^2 - 424*a^4*b^3 + 513*a^3*b^4 - 266*a^2*b^5 + 49*a*b^6)*d*cos(6*d*x + 6*c) - 4*(64*a^5*b^2 - 240*a^4*b^3 + 337*a^3*b^4 - 210*a^2*b^5 + 49*a*b^6)*d*cos(4*d*x + 4*c) - 8*(8*a^4*b^3 - 23*a^3*b^4 + 22*a^2*b^5 - 7*a*b^6)*d*cos(2*d*x + 2*c) + (8*a^4*b^3 - 23*a^3*b^4 + 22*a^2*b^5 - 7*a*b^6)*d)*cos(12*d*x + 12*c) + 16*(2*(2048*a^6*b - 6528*a^5*b^2 + 8144*a^4*b^3 - 5141*a^3*b^4 + 1722*a^2*b^5 - 245*a*b^6)*d*cos(8*d*x + 8*c) + 8*(256*a^5*b^2 - 736*a^4*b^3 + 753*a^3*b^4 - 322*a^2*b^5 + 49*a*b^6)*d*cos(6*d*x + 6*c) - 4*(128*a^5*b^2 - 424*a^4*b^3 + 513*a^3*b^4 - 266*a^2*b^5 + 49*a*b^6)*d*cos(4*d*x + 4*c) - 8*(16*a^4*b^3 - 39*a^3*b^4 + 30*a^2*b^5 - 7*a*b^6)*d*cos(2*d*x + 2*c) + (16*a^4*b^3 - 39*a^3*b^4 + 30*a^2*b^5 - 7*a*b^6)*d)*cos(10*d*x + 10*c) + 4*(8*(2048*a^6*b - 6528*a^5*b^2 + 8144*a^4*b^3 - 5141*a^3*b^4 + 1722*a^2*b^5 - 245*a*b^6)*d*cos(6*d*x + 6*c) - 4*(1024*a^6*b - 3712*a^5*b^2 + 5304*a^4*b^3 - 3813*a^3*b^4 + 1442*a^2*b^5 - 245*a*b^6)*d*cos(4*d*x + 4*c) - 8*(128*a^5*b^2 - 352*a^4*b^3 + 355*a^3*b^4 - 166*a^2*b^5 + 35*a*b^6)*d*cos(2*d*x + 2*c) + (128*a^5*b^2 - 352*a^4*b^3 + 355*a^3*b^4 - 166*a^2*b^5 + 35*a*b^6)*d)*cos(8*d*x + 8*c) - 16*(4*(128*a^5*b^2 - 424*a^4*b^3 + 513*a^3*b^4 - 266*a^2*b^5 + 49*a*b^6)*d*cos(4*d*x + 4*c) + 8*(16*a^4*b^3 - 39*a^3*b^4 + 30*a^2*b^5 - 7*a*b^6)*d*cos(2*d*x + 2*c) - (16*a^4*b^3 - 39*a^3*b^4 + 30*a^2*b^5 - 7*a*b^6)*d)*cos(6*d*x + 6*c) + 8*(8*(8*a^4*b^3 - 23*a^3*b^4 + 22*a^2*b^5 - 7*a*b^6)*d*cos(2*d*x + 2*c) - (8*a^4*b^3 - 23*a^3*b^4 + 22*a^2*b^5 - 7*a*b^6)*d)*cos(4*d*x + 4*c) - 4*(4*(a^3*b^4 - 2*a^2*b^5 + a*b^6)*d*sin(14*d*x + 14*c) + 2*(8*a^4*b^3 - 23*a^3*b^4 + 22*a^2*b^5 - 7*a*b^6)*d*sin(12*d*x + 12*c) - 4*(16*a^4*b^3 - 39*a^3*b^4 + 30*a^2*b^5 - 7*a*b^6)*d*sin(10*d*x + 10*c) - (128*a^5*b^2 - 352*a^4*b^3 + 355*a^3*b^4 - 166*a^2*b^5 + 35*a*b^6)*d*sin(8*d*x + 8*c) - 4*(16*a^4*b^3 - 39*a^3*b^4 + 30*a^2*b^5 - 7*a*b^6)*d*sin(6*d*x + 6*c) + 2*(8*a^4*b^3 - 23*a^3*b^4 + 22*a^2*b^5 - 7*a*b^6)*d*sin(4*d*x + 4*c) + 4*(a^3*b^4 - 2*a^2*b^5 + a*b^6)*d*sin(2*d*x + 2*c))*sin(16*d*x + 16*c) + 32*(2*(8*a^4*b^3 - 23*a^3*b^4 + 22*a^2*b^5 - 7*a*b^6)*d*sin(12*d*x + 12*c) - 4*(16*a^4*b^3 - 39*a^3*b^4 + 30*a^2*b^5 - 7*a*b^6)*d*sin(10*d*x + 10*c) - (128*a^5*b^2 - 352*a^4*b^3 + 355*a^3*b^4 - 166*a^2*b^5 + 35*a*b^6)*d*sin(8*d*x + 8*c) - 4*(16*a^4*b^3 - 39*a^3*b^4 + 30*a^2*b^5 - 7*a*b^6)*d*sin(6*d*x + 6*c) + 2*(8*a^4*b^3 - 23*a^3*b^4 + 22*a^2*b^5 - 7*a*b^6)*d*sin(4*d*x + 4*c) + 4*(a^3*b^4 - 2*a^2*b^5 + a*b^6)*d*sin(2*d*x + 2*c))*sin(14*d*x + 14*c) - 16*(4*(128*a^5*b^2 - 424*a^4*b^3 + 513*a^3*b^4 - 266*a^2*b^5 + 49*a*b^6)*d*sin(10*d*x + 10*c) + (1024*a^6*b - 3712*a^5*b^2 + 5304*a^4*b^3 - 3813*a^3*b^4 + 1442*a^2*b^5 - 245*a*b^6)*d*sin(8*d*x + 8*c) + 4*(128*a^5*b^2 - 424*a^4*b^3 + 513*a^3*b^4 - 266*a^2*b^5 + 49*a*b^6)*d*sin(6*d*x + 6*c) - 2*(64*a^5*b^2 - 240*a^4*b^3 + 337*a^3*b^4 - 210*a^2*b^5 + 49*a*b^6)*d*sin(4*d*x + 4*c

```
) - 4*(8*a^4*b^3 - 23*a^3*b^4 + 22*a^2*b^5 - 7*a*b^6)*d*sin(2*d*x + 2*c))*s
in(12*d*x + 12*c) + 32*((2048*a^6*b - 6528*a^5*b^2 + 8144*a^4*b^3 - 5141*a^
3*b^4 + 1722*a^2*b^5 - 245*a*b^6)*d*sin(8*d*x + 8*c) + 4*(256*a^5*b^2 - 736
*a^4*b^3 + 753*a^3*b^4 - 322*a^2*b^5 + 49*a*b^6)*d*sin(6*d*x + 6*c) - 2*(12
8*a^5*b^2 - 424*a^4*b^3 + 513*a^3*b^4 - 266*a^2*b^5 + 49*a*b^6)*d*sin(4*d*x
 + 4*c) - 4*(16*a^4*b^3 - 39*a^3*b^4 + 30*a^2*b^5 - 7*a*b^6)*d*sin(2*d*x +
2*c))*sin(10*d*x + 10*c) + 16*(2*(2048*a^6*b - 6528*a^5*b^2 + 8144*a^4*b^3
- 5141*a^3*b^4 + 1722*a^2*b^5 - 245*a*b^6)*d*sin(6*d*x + 6*c) - (1024*a^6*b
 - 3712*a^5*b^2 + 5304*a^4*b^3 - 3813*a^3*b^4 + 1442*a^2*b^5 - 245*a*b^6)*d
*sin(4*d*x + 4*c) - 2*(128*a^5*b^2 - 352*a^4*b^3 + 355*a^3*b^4 - 166*a^2*b^
5 + 35*a*b^6)*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) - 64*((128*a^5*b^2 - 424
*a^4*b^3 + 513*a^3*b^4 - 266*a^2*b^5 + 49*a*b^6)*d*sin(4*d*x + 4*c) + 2*(16
*a^4*b^3 - 39*a^3*b^4 + 30*a^2*b^5 - 7*a*b^6)*d*sin(2*d*x + 2*c))*sin(6*d*x
 + 6*c))*integrate(-3/4*(4*a*b*cos(6*d*x + 6*c)^2 + 4*a*b*cos(2*d*x + 2*c)^
2 + 4*a*b*sin(6*d*x + 6*c)^2 + 4*a*b*sin(2*d*x + 2*c)^2 - 4*(32*a^2 - 20*a*
b + 3*b^2)*cos(4*d*x + 4*c)^2 - a*b*cos(2*d*x + 2*c) - 4*(32*a^2 - 20*a*b +
 3*b^2)*sin(4*d*x + 4*c)^2 + 2*(8*a^2 - 19*a*b + 4*b^2)*sin(4*d*x + 4*c)*si
n(2*d*x + 2*c) - (a*b*cos(6*d*x + 6*c) + a*b*cos(2*d*x + 2*c) - 2*(4*a*b -
b^2)*cos(4*d*x + 4*c))*cos(8*d*x + 8*c) + (8*a*b*cos(2*d*x + 2*c) - a*b + 2
*(8*a^2 - 19*a*b + 4*b^2)*cos(4*d*x + 4*c))*cos(6*d*x + 6*c) + 2*(4*a*b - b
^2 + (8*a^2 - 19*a*b + 4*b^2)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - (a*b*sin
(6*d*x + 6*c) + a*b*sin(2*d*x + 2*c) - 2*(4*a*b - b^2)*sin(4*d*x + 4*c))*si
n(8*d*x + 8*c) + 2*(4*a*b*sin(2*d*x + 2*c) + (8*a^2 - 19*a*b + 4*b^2)*sin(4
*d*x + 4*c))*sin(6*d*x + 6*c))/(a^3*b^2 - 2*a^2*b^3 + a*b^4 + (a^3*b^2 - 2*
a^2*b^3 + a*b^4)*cos(8*d*x + 8*c)^2 + 16*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*cos(
6*d*x + 6*c)^2 + 4*(64*a^5 - 176*a^4*b + 169*a^3*b^2 - 66*a^2*b^3 + 9*a*b^4
)*cos(4*d*x + 4*c)^2 + 16*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*cos(2*d*x + 2*c)^2
+ (a^3*b^2 - 2*a^2*b^3 + a*b^4)*sin(8*d*x + 8*c)^2 + 16*(a^3*b^2 - 2*a^2*b^
3 + a*b^4)*sin(6*d*x + 6*c)^2 + 4*(64*a^5 - 176*a^4*b + 169*a^3*b^2 - 66*a^
2*b^3 + 9*a*b^4)*sin(4*d*x + 4*c)^2 + 16*(8*a^4*b - 19*a^3*b^2 + 14*a^2*b^3
 - 3*a*b^4)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a^3*b^2 - 2*a^2*b^3 + a
*b^4)*sin(2*d*x + 2*c)^2 + 2*(a^3*b^2 - 2*a^2*b^3 + a*b^4 - 4*(a^3*b^2 - 2*
a^2*b^3 + a*b^4)*cos(6*d*x + 6*c) - 2*(8*a^4*b - 19*a^3*b^2 + 14*a^2*b^3 -
3*a*b^4)*cos(4*d*x + 4*c) - 4*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*cos(2*d*x + 2*c
))*cos(8*d*x + 8*c) - 8*(a^3*b^2 - 2*a^2*b^3 + a*b^4 - 2*(8*a^4*b - 19*a^3*
b^2 + 14*a^2*b^3 - 3*a*b^4)*cos(4*d*x + 4*c) - 4*(a^3*b^2 - 2*a^2*b^3 + a*b
^4)*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) - 4*(8*a^4*b - 19*a^3*b^2 + 14*a^2*b
^3 - 3*a*b^4 - 4*(8*a^4*b - 19*a^3*b^2 + 14*a^2*b^3 - 3*a*b^4)*cos(2*d*x +
2*c))*cos(4*d*x + 4*c) - 8*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*cos(2*d*x + 2*c) -
 4*(2*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*sin(6*d*x + 6*c) + (8*a^4*b - 19*a^3*b^
2 + 14*a^2*b^3 - 3*a*b^4)*sin(4*d*x + 4*c) + 2*(a^3*b^2 - 2*a^2*b^3 + a*b^4
)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^4*b - 19*a^3*b^2 + 14*a^2*b
^3 - 3*a*b^4)*sin(4*d*x + 4*c) + 2*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*sin(2*d*x
+ 2*c))*sin(6*d*x + 6*c)), x) + (3*a*b^3*cos(14*d*x + 14*c) + 2*a*b^3 + b^4
 - 3*(10*a*b^3 - b^4)*cos(12*d*x + 12*c) - (80*a^2*b^2 - 111*a*b^3 + 16*b^4
)*cos(10*d*x + 10*c) + (256*a^3*b - 64*a^2*b^2 - 26*a*b^3 + 35*b^4)*cos(8*d
*x + 8*c) + (336*a^2*b^2 - 95*a*b^3 - 40*b^4)*cos(6*d*x + 6*c) - (64*a^2*b^
2 - 54*a*b^3 - 25*b^4)*cos(4*d*x + 4*c) - (19*a*b^3 + 8*b^4)*cos(2*d*x + 2*
c))*sin(16*d*x + 16*c) - (19*a*b^3 + 8*b^4 - 12*(8*a^2*b^2 + 13*a*b^3 - 2*b
^4)*cos(12*d*x + 12*c) - 16*(16*a^2*b^2 - 45*a*b^3 + 8*b^4)*cos(10*d*x + 10
*c) + 2*(1408*a^3*b - 544*a^2*b^2 + a*b^3 + 140*b^4)*cos(8*d*x + 8*c) + 32*
(96*a^2*b^2 - 29*a*b^3 - 10*b^4)*cos(6*d*x + 6*c) - 4*(152*a^2*b^2 - 129*a*
b^3 - 50*b^4)*cos(4*d*x + 4*c) - 16*(11*a*b^3 + 4*b^4)*cos(2*d*x + 2*c))*si
n(14*d*x + 14*c) - (64*a^2*b^2 - 54*a*b^3 - 25*b^4 - 4*(640*a^3*b - 488*a^2
*b^2 + 389*a*b^3 - 70*b^4)*cos(10*d*x + 10*c) + 2*(4096*a^4 - 8448*a^3*b +
3744*a^2*b^2 - 414*a*b^3 - 385*b^4)*cos(8*d*x + 8*c) + 4*(2688*a^3*b - 4072
*a^2*b^2 + 861*a*b^3 + 238*b^4)*cos(6*d*x + 6*c) - 8*(256*a^3*b - 560*a^2*b
^2 + 206*a*b^3 + 77*b^4)*cos(4*d*x + 4*c) - 4*(152*a^2*b^2 - 129*a*b^3 - 50
*b^4)*cos(2*d*x + 2*c))*sin(12*d*x + 12*c) + (336*a^2*b^2 - 95*a*b^3 - 40*b
```

^4 + 2*(26624*a^4 - 33152*a^3*b + 15632*a^2*b^2 - 2453*a*b^3 - 420*b^4)*cos
(8*d*x + 8*c) + 16*(3328*a^3*b - 3104*a^2*b^2 + 529*a*b^3 + 84*b^4)*cos(6*d
*x + 6*c) - 4*(2688*a^3*b - 4072*a^2*b^2 + 861*a*b^3 + 238*b^4)*cos(4*d*x +
 4*c) - 32*(96*a^2*b^2 - 29*a*b^3 - 10*b^4)*cos(2*d*x + 2*c))*sin(10*d*x +
10*c) + (256*a^3*b - 64*a^2*b^2 - 26*a*b^3 + 35*b^4 + 2*(26624*a^4 - 33152*
a^3*b + 15632*a^2*b^2 - 2453*a*b^3 - 420*b^4)*cos(6*d*x + 6*c) - 2*(4096*a^
4 - 8448*a^3*b + 3744*a^2*b^2 - 414*a*b^3 - 385*b^4)*cos(4*d*x + 4*c) - 2*(
1408*a^3*b - 544*a^2*b^2 + a*b^3 + 140*b^4)*cos(2*d*x + 2*c))*sin(8*d*x + 8
*c) - (80*a^2*b^2 - 111*a*b^3 + 16*b^4 - 4*(640*a^3*b - 488*a^2*b^2 + 389*a
*b^3 - 70*b^4)*cos(4*d*x + 4*c) - 16*(16*a^2*b^2 - 45*a*b^3 + 8*b^4)*cos(2*
d*x + 2*c))*sin(6*d*x + 6*c) - 3*(10*a*b^3 - b^4 - 4*(8*a^2*b^2 + 13*a*b^3
- 2*b^4)*cos(2*d*x + 2*c))*sin(4*d*x + 4*c))/((a^3*b^4 - 2*a^2*b^5 + a*b^6)
*d*cos(16*d*x + 16*c)^2 + 64*(a^3*b^4 - 2*a^2*b^5 + a*b^6)*d*cos(14*d*x + 1
4*c)^2 + 16*(64*a^5*b^2 - 240*a^4*b^3 + 337*a^3*b^4 - 210*a^2*b^5 + 49*a*b^
6)*d*cos(12*d*x + 12*c)^2 + 64*(256*a^5*b^2 - 736*a^4*b^3 + 753*a^3*b^4 - 3
22*a^2*b^5 + 49*a*b^6)*d*cos(10*d*x + 10*c)^2 + 4*(16384*a^7 - 57344*a^6*b
+ 83712*a^5*b^2 - 67648*a^4*b^3 + 32841*a^3*b^4 - 9170*a^2*b^5 + 1225*a*b^6
)*d*cos(8*d*x + 8*c)^2 + 64*(256*a^5*b^2 - 736*a^4*b^3 + 753*a^3*b^4 - 322*
a^2*b^5 + 49*a*b^6)*d*cos(6*d*x + 6*c)^2 + 16*(64*a^5*b^2 - 240*a^4*b^3 + 3
37*a^3*b^4 - 210*a^2*b^5 + 49*a*b^6)*d*cos(4*d*x + 4*c)^2 + 64*(a^3*b^4 - 2
*a^2*b^5 + a*b^6)*d*cos(2*d*x + 2*c)^2 + (a^3*b^4 - 2*a^2*b^5 + a*b^6)*d*si
n(16*d*x + 16*c)^2 + 64*(a^3*b^4 - 2*a^2*b^5 + a*b^6)*d*sin(14*d*x + 14*c)^
2 + 16*(64*a^5*b^2 - 240*a^4*b^3 + 337*a^3*b^4 - 210*a^2*b^5 + 49*a*b^6)*d*
sin(12*d*x + 12*c)^2 + 64*(256*a^5*b^2 - 736*a^4*b^3 + 753*a^3*b^4 - 322*a^
2*b^5 + 49*a*b^6)*d*sin(10*d*x + 10*c)^2 + 4*(16384*a^7 - 57344*a^6*b + 837
12*a^5*b^2 - 67648*a^4*b^3 + 32841*a^3*b^4 - 9170*a^2*b^5 + 1225*a*b^6)*d*s
in(8*d*x + 8*c)^2 + 64*(256*a^5*b^2 - 736*a^4*b^3 + 753*a^3*b^4 - 322*a^2*b
^5 + 49*a*b^6)*d*sin(6*d*x + 6*c)^2 + 16*(64*a^5*b^2 - 240*a^4*b^3 + 337*a^
3*b^4 - 210*a^2*b^5 + 49*a*b^6)*d*sin(4*d*x + 4*c)^2 + 64*(8*a^4*b^3 - 23*a
^3*b^4 + 22*a^2*b^5 - 7*a*b^6)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 64*(a^
3*b^4 - 2*a^2*b^5 + a*b^6)*d*sin(2*d*x + 2*c)^2 - 16*(a^3*b^4 - 2*a^2*b^5 +
 a*b^6)*d*cos(2*d*x + 2*c) + (a^3*b^4 - 2*a^2*b^5 + a*b^6)*d - 2*(8*(a^3*b^
4 - 2*a^2*b^5 + a*b^6)*d*cos(14*d*x + 14*c) + 4*(8*a^4*b^3 - 23*a^3*b^4 + 2
2*a^2*b^5 - 7*a*b^6)*d*cos(12*d*x + 12*c) - 8*(16*a^4*b^3 - 39*a^3*b^4 + 30
*a^2*b^5 - 7*a*b^6)*d*cos(10*d*x + 10*c) - 2*(128*a^5*b^2 - 352*a^4*b^3 + 3
55*a^3*b^4 - 166*a^2*b^5 + 35*a*b^6)*d*cos(8*d*x + 8*c) - 8*(16*a^4*b^3 - 3
9*a^3*b^4 + 30*a^2*b^5 - 7*a*b^6)*d*cos(6*d*x + 6*c) + 4*(8*a^4*b^3 - 23*a^
3*b^4 + 22*a^2*b^5 - 7*a*b^6)*d*cos(4*d*x + 4*c) + 8*(a^3*b^4 - 2*a^2*b^5 +
 a*b^6)*d*cos(2*d*x + 2*c) - (a^3*b^4 - 2*a^2*b^5 + a*b^6)*d)*cos(16*d*x +
16*c) + 16*(4*(8*a^4*b^3 - 23*a^3*b^4 + 22*a^2*b^5 - 7*a*b^6)*d*cos(12*d*x
+ 12*c) - 8*(16*a^4*b^3 - 39*a^3*b^4 + 30*a^2*b^5 - 7*a*b^6)*d*cos(10*d*x +
 10*c) - 2*(128*a^5*b^2 - 352*a^4*b^3 + 355*a^3*b^4 - 166*a^2*b^5 + 35*a*b^
6)*d*cos(8*d*x + 8*c) - 8*(16*a^4*b^3 - 39*a^3*b^4 + 30*a^2*b^5 - 7*a*b^6)*
d*cos(6*d*x + 6*c) + 4*(8*a^4*b^3 - 23*a^3*b^4 + 22*a^2*b^5 - 7*a*b^6)*d*co
s(4*d*x + 4*c) + 8*(a^3*b^4 - 2*a^2*b^5 + a*b^6)*d*cos(2*d*x + 2*c) - (a^3*
b^4 - 2*a^2*b^5 + a*b^6)*d)*cos(14*d*x + 14*c) - 8*(8*(128*a^5*b^2 - 424*a^
4*b^3 + 513*a^3*b^4 - 266*a^2*b^5 + 49*a*b^6)*d*cos(10*d*x + 10*c) + 2*(102
4*a^6*b - 3712*a^5*b^2 + 5304*a^4*b^3 - 3813*a^3*b^4 + 1442*a^2*b^5 - 245*a
*b^6)*d*cos(8*d*x + 8*c) + 8*(128*a^5*b^2 - 424*a^4*b^3 + 513*a^3*b^4 - 266
*a^2*b^5 + 49*a*b^6)*d*cos(6*d*x + 6*c) - 4*(64*a^5*b^2 - 240*a^4*b^3 + 337
*a^3*b^4 - 210*a^2*b^5 + 49*a*b^6)*d*cos(4*d*x + 4*c) - 8*(8*a^4*b^3 - 23*a
^3*b^4 + 22*a^2*b^5 - 7*a*b^6)*d*cos(2*d*x + 2*c) + (8*a^4*b^3 - 23*a^3*b^4
 + 22*a^2*b^5 - 7*a*b^6)*d)*cos(12*d*x + 12*c) + 16*(2*(2048*a^6*b - 6528*a
^5*b^2 + 8144*a^4*b^3 - 5141*a^3*b^4 + 1722*a^2*b^5 - 245*a*b^6)*d*cos(8*d*
x + 8*c) + 8*(256*a^5*b^2 - 736*a^4*b^3 + 753*a^3*b^4 - 322*a^2*b^5 + 49*a*
b^6)*d*cos(6*d*x + 6*c) - 4*(128*a^5*b^2 - 424*a^4*b^3 + 513*a^3*b^4 - 266*
a^2*b^5 + 49*a*b^6)*d*cos(4*d*x + 4*c) - 8*(16*a^4*b^3 - 39*a^3*b^4 + 30*a^
2*b^5 - 7*a*b^6)*d*cos(2*d*x + 2*c) + (16*a^4*b^3 - 39*a^3*b^4 + 30*a^2*b^5
 - 7*a*b^6)*d)*cos(10*d*x + 10*c) + 4*(8*(2048*a^6*b - 6528*a^5*b^2 + 8144*

```
a^4*b^3 - 5141*a^3*b^4 + 1722*a^2*b^5 - 245*a*b^6)*d*cos(6*d*x + 6*c) - 4*(
1024*a^6*b - 3712*a^5*b^2 + 5304*a^4*b^3 - 3813*a^3*b^4 + 1442*a^2*b^5 - 24
5*a*b^6)*d*cos(4*d*x + 4*c) - 8*(128*a^5*b^2 - 352*a^4*b^3 + 355*a^3*b^4 -
166*a^2*b^5 + 35*a*b^6)*d*cos(2*d*x + 2*c) + (128*a^5*b^2 - 352*a^4*b^3 + 3
55*a^3*b^4 - 166*a^2*b^5 + 35*a*b^6)*d)*cos(8*d*x + 8*c) - 16*(4*(128*a^5*b
^2 - 424*a^4*b^3 + 513*a^3*b^4 - 266*a^2*b^5 + 49*a*b^6)*d*cos(4*d*x + 4*c)
 + 8*(16*a^4*b^3 - 39*a^3*b^4 + 30*a^2*b^5 - 7*a*b^6)*d*cos(2*d*x + 2*c) -
(16*a^4*b^3 - 39*a^3*b^4 + 30*a^2*b^5 - 7*a*b^6)*d)*cos(6*d*x + 6*c) + 8*(8
*(8*a^4*b^3 - 23*a^3*b^4 + 22*a^2*b^5 - 7*a*b^6)*d*cos(2*d*x + 2*c) - (8*a^
4*b^3 - 23*a^3*b^4 + 22*a^2*b^5 - 7*a*b^6)*d)*cos(4*d*x + 4*c) - 4*(4*(a^3*
b^4 - 2*a^2*b^5 + a*b^6)*d*sin(14*d*x + 14*c) + 2*(8*a^4*b^3 - 23*a^3*b^4 +
 22*a^2*b^5 - 7*a*b^6)*d*sin(12*d*x + 12*c) - 4*(16*a^4*b^3 - 39*a^3*b^4 +
30*a^2*b^5 - 7*a*b^6)*d*sin(10*d*x + 10*c) - (128*a^5*b^2 - 352*a^4*b^3 + 3
55*a^3*b^4 - 166*a^2*b^5 + 35*a*b^6)*d*sin(8*d*x + 8*c) - 4*(16*a^4*b^3 - 3
9*a^3*b^4 + 30*a^2*b^5 - 7*a*b^6)*d*sin(6*d*x + 6*c) + 2*(8*a^4*b^3 - 23*a^
3*b^4 + 22*a^2*b^5 - 7*a*b^6)*d*sin(4*d*x + 4*c) + 4*(a^3*b^4 - 2*a^2*b^5 +
 a*b^6)*d*sin(2*d*x + 2*c))*sin(16*d*x + 16*c) + 32*(2*(8*a^4*b^3 - 23*a^3*
b^4 + 22*a^2*b^5 - 7*a*b^6)*d*sin(12*d*x + 12*c) - 4*(16*a^4*b^3 - 39*a^3*b
^4 + 30*a^2*b^5 - 7*a*b^6)*d*sin(10*d*x + 10*c) - (128*a^5*b^2 - 352*a^4*b^
3 + 355*a^3*b^4 - 166*a^2*b^5 + 35*a*b^6)*d*sin(8*d*x + 8*c) - 4*(16*a^4*b^
3 - 39*a^3*b^4 + 30*a^2*b^5 - 7*a*b^6)*d*sin(6*d*x + 6*c) + 2*(8*a^4*b^3 -
23*a^3*b^4 + 22*a^2*b^5 - 7*a*b^6)*d*sin(4*d*x + 4*c) + 4*(a^3*b^4 - 2*a^2*
b^5 + a*b^6)*d*sin(2*d*x + 2*c))*sin(14*d*x + 14*c) - 16*(4*(128*a^5*b^2 -
424*a^4*b^3 + 513*a^3*b^4 - 266*a^2*b^5 + 49*a*b^6)*d*sin(10*d*x + 10*c) +
(1024*a^6*b - 3712*a^5*b^2 + 5304*a^4*b^3 - 3813*a^3*b^4 + 1442*a^2*b^5 - 2
45*a*b^6)*d*sin(8*d*x + 8*c) + 4*(128*a^5*b^2 - 424*a^4*b^3 + 513*a^3*b^4 -
 266*a^2*b^5 + 49*a*b^6)*d*sin(6*d*x + 6*c) - 2*(64*a^5*b^2 - 240*a^4*b^3 +
 337*a^3*b^4 - 210*a^2*b^5 + 49*a*b^6)*d*sin(4*d*x + 4*c) - 4*(8*a^4*b^3 -
23*a^3*b^4 + 22*a^2*b^5 - 7*a*b^6)*d*sin(2*d*x + 2*c))*sin(12*d*x + 12*c) +
 32*((2048*a^6*b - 6528*a^5*b^2 + 8144*a^4*b^3 - 5141*a^3*b^4 + 1722*a^2*b^
5 - 245*a*b^6)*d*sin(8*d*x + 8*c) + 4*(256*a^5*b^2 - 736*a^4*b^3 + 753*a^3*
b^4 - 322*a^2*b^5 + 49*a*b^6)*d*sin(6*d*x + 6*c) - 2*(128*a^5*b^2 - 424*a^4
*b^3 + 513*a^3*b^4 - 266*a^2*b^5 + 49*a*b^6)*d*sin(4*d*x + 4*c) - 4*(16*a^4
*b^3 - 39*a^3*b^4 + 30*a^2*b^5 - 7*a*b^6)*d*sin(2*d*x + 2*c))*sin(10*d*x +
10*c) + 16*(2*(2048*a^6*b - 6528*a^5*b^2 + 8144*a^4*b^3 - 5141*a^3*b^4 + 17
22*a^2*b^5 - 245*a*b^6)*d*sin(6*d*x + 6*c) - (1024*a^6*b - 3712*a^5*b^2 + 5
304*a^4*b^3 - 3813*a^3*b^4 + 1442*a^2*b^5 - 245*a*b^6)*d*sin(4*d*x + 4*c) -
 2*(128*a^5*b^2 - 352*a^4*b^3 + 355*a^3*b^4 - 166*a^2*b^5 + 35*a*b^6)*d*sin
(2*d*x + 2*c))*sin(8*d*x + 8*c) - 64*((128*a^5*b^2 - 424*a^4*b^3 + 513*a^3*
b^4 - 266*a^2*b^5 + 49*a*b^6)*d*sin(4*d*x + 4*c) + 2*(16*a^4*b^3 - 39*a^3*b
^4 + 30*a^2*b^5 - 7*a*b^6)*d*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))
```

Fricas [B] time = 13.6166, size = 12400, normalized size = 39.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a-b*sin(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/256*(3*((a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cos(d*x + c)^8 - 4*(a^3*b^2 - 2*a
^2*b^3 + a*b^4)*d*cos(d*x + c)^6 - 2*(a^4*b - 5*a^3*b^2 + 7*a^2*b^3 - 3*a*b
^4)*d*cos(d*x + c)^4 + 4*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d*cos(d*x
+ c)^2 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d)*sqrt(-((a^8*b -
 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2*sqrt((256*a
^6 + 160*a^5*b - 167*a^4*b^2 - 28*a^3*b^3 + 46*a^2*b^4 - 12*a*b^5 + b^6)/((
a^17*b - 10*a^16*b^2 + 45*a^15*b^3 - 120*a^14*b^4 + 210*a^13*b^5 - 252*a^12
```

```
*b^6 + 210*a^11*b^7 - 120*a^10*b^8 + 45*a^9*b^9 - 10*a^8*b^10 + a^7*b^11)*d
^4)) + 4*a^3 + 21*a^2*b - 10*a*b^2 + b^3)/((a^8*b - 5*a^7*b^2 + 10*a^6*b^3
 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2))*log(432*a^4 + 27*a^3*b - 783/4*a^
2*b^2 + 135/2*a*b^3 - 27/4*b^4 - 27/4*(64*a^4 + 4*a^3*b - 29*a^2*b^2 + 10*a
*b^3 - b^4)*cos(d*x + c)^2 + 27/2*((5*a^12*b - 26*a^11*b^2 + 55*a^10*b^3 -
60*a^9*b^4 + 35*a^8*b^5 - 10*a^7*b^6 + a^6*b^7)*d^3*sqrt((256*a^6 + 160*a^5
*b - 167*a^4*b^2 - 28*a^3*b^3 + 46*a^2*b^4 - 12*a*b^5 + b^6)/((a^17*b - 10*
a^16*b^2 + 45*a^15*b^3 - 120*a^14*b^4 + 210*a^13*b^5 - 252*a^12*b^6 + 210*a
^11*b^7 - 120*a^10*b^8 + 45*a^9*b^9 - 10*a^8*b^10 + a^7*b^11)*d^4))*cos(d*x
 + c)*sin(d*x + c) - (32*a^7 + 58*a^6*b - 13*a^5*b^2 - 21*a^4*b^3 + 9*a^3*b
^4 - a^2*b^5)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^8*b - 5*a^7*b^2 + 10*a
^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2*sqrt((256*a^6 + 160*a^5*b -
167*a^4*b^2 - 28*a^3*b^3 + 46*a^2*b^4 - 12*a*b^5 + b^6)/((a^17*b - 10*a^16*
b^2 + 45*a^15*b^3 - 120*a^14*b^4 + 210*a^13*b^5 - 252*a^12*b^6 + 210*a^11*b
^7 - 120*a^10*b^8 + 45*a^9*b^9 - 10*a^8*b^10 + a^7*b^11)*d^4)) + 4*a^3 + 21
*a^2*b - 10*a*b^2 + b^3)/((a^8*b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*
a^4*b^5 - a^3*b^6)*d^2)) + 27/4*(2*(4*a^10 - 21*a^9*b + 45*a^8*b^2 - 50*a^7
*b^3 + 30*a^6*b^4 - 9*a^5*b^5 + a^4*b^6)*d^2*cos(d*x + c)^2 - (4*a^10 - 21*
a^9*b + 45*a^8*b^2 - 50*a^7*b^3 + 30*a^6*b^4 - 9*a^5*b^5 + a^4*b^6)*d^2)*sq
rt((256*a^6 + 160*a^5*b - 167*a^4*b^2 - 28*a^3*b^3 + 46*a^2*b^4 - 12*a*b^5
+ b^6)/((a^17*b - 10*a^16*b^2 + 45*a^15*b^3 - 120*a^14*b^4 + 210*a^13*b^5 -
 252*a^12*b^6 + 210*a^11*b^7 - 120*a^10*b^8 + 45*a^9*b^9 - 10*a^8*b^10 + a^
7*b^11)*d^4))) - 3*((a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cos(d*x + c)^8 - 4*(a^3
*b^2 - 2*a^2*b^3 + a*b^4)*d*cos(d*x + c)^6 - 2*(a^4*b - 5*a^3*b^2 + 7*a^2*b
^3 - 3*a*b^4)*d*cos(d*x + c)^4 + 4*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*
d*cos(d*x + c)^2 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d)*sqrt(
-((a^8*b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2*s
qrt((256*a^6 + 160*a^5*b - 167*a^4*b^2 - 28*a^3*b^3 + 46*a^2*b^4 - 12*a*b^5
 + b^6)/((a^17*b - 10*a^16*b^2 + 45*a^15*b^3 - 120*a^14*b^4 + 210*a^13*b^5
- 252*a^12*b^6 + 210*a^11*b^7 - 120*a^10*b^8 + 45*a^9*b^9 - 10*a^8*b^10 + a
^7*b^11)*d^4)) + 4*a^3 + 21*a^2*b - 10*a*b^2 + b^3)/((a^8*b - 5*a^7*b^2 + 1
0*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2))*log(432*a^4 + 27*a^3*b
- 783/4*a^2*b^2 + 135/2*a*b^3 - 27/4*b^4 - 27/4*(64*a^4 + 4*a^3*b - 29*a^2*
b^2 + 10*a*b^3 - b^4)*cos(d*x + c)^2 - 27/2*((5*a^12*b - 26*a^11*b^2 + 55*a
^10*b^3 - 60*a^9*b^4 + 35*a^8*b^5 - 10*a^7*b^6 + a^6*b^7)*d^3*sqrt((256*a^6
 + 160*a^5*b - 167*a^4*b^2 - 28*a^3*b^3 + 46*a^2*b^4 - 12*a*b^5 + b^6)/((a^
17*b - 10*a^16*b^2 + 45*a^15*b^3 - 120*a^14*b^4 + 210*a^13*b^5 - 252*a^12*b
^6 + 210*a^11*b^7 - 120*a^10*b^8 + 45*a^9*b^9 - 10*a^8*b^10 + a^7*b^11)*d^4
))*cos(d*x + c)*sin(d*x + c) - (32*a^7 + 58*a^6*b - 13*a^5*b^2 - 21*a^4*b^3
 + 9*a^3*b^4 - a^2*b^5)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^8*b - 5*a^7*
b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2*sqrt((256*a^6 + 16
0*a^5*b - 167*a^4*b^2 - 28*a^3*b^3 + 46*a^2*b^4 - 12*a*b^5 + b^6)/((a^17*b
- 10*a^16*b^2 + 45*a^15*b^3 - 120*a^14*b^4 + 210*a^13*b^5 - 252*a^12*b^6 +
210*a^11*b^7 - 120*a^10*b^8 + 45*a^9*b^9 - 10*a^8*b^10 + a^7*b^11)*d^4)) +
4*a^3 + 21*a^2*b - 10*a*b^2 + b^3)/((a^8*b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^
5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2)) + 27/4*(2*(4*a^10 - 21*a^9*b + 45*a^8*b^
2 - 50*a^7*b^3 + 30*a^6*b^4 - 9*a^5*b^5 + a^4*b^6)*d^2*cos(d*x + c)^2 - (4*
a^10 - 21*a^9*b + 45*a^8*b^2 - 50*a^7*b^3 + 30*a^6*b^4 - 9*a^5*b^5 + a^4*b^
6)*d^2)*sqrt((256*a^6 + 160*a^5*b - 167*a^4*b^2 - 28*a^3*b^3 + 46*a^2*b^4 -
 12*a*b^5 + b^6)/((a^17*b - 10*a^16*b^2 + 45*a^15*b^3 - 120*a^14*b^4 + 210*
a^13*b^5 - 252*a^12*b^6 + 210*a^11*b^7 - 120*a^10*b^8 + 45*a^9*b^9 - 10*a^8
*b^10 + a^7*b^11)*d^4))) + 3*((a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cos(d*x + c)^
8 - 4*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cos(d*x + c)^6 - 2*(a^4*b - 5*a^3*b^2
 + 7*a^2*b^3 - 3*a*b^4)*d*cos(d*x + c)^4 + 4*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3
 - a*b^4)*d*cos(d*x + c)^2 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4
)*d)*sqrt(((a^8*b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b
^6)*d^2*sqrt((256*a^6 + 160*a^5*b - 167*a^4*b^2 - 28*a^3*b^3 + 46*a^2*b^4 -
 12*a*b^5 + b^6)/((a^17*b - 10*a^16*b^2 + 45*a^15*b^3 - 120*a^14*b^4 + 210*
a^13*b^5 - 252*a^12*b^6 + 210*a^11*b^7 - 120*a^10*b^8 + 45*a^9*b^9 - 10*a^8
```

*b^10 + a^7*b^11)*d^4)) - 4*a^3 - 21*a^2*b + 10*a*b^2 - b^3)/((a^8*b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2))*log(-432*a^4 - 27*a^3*b + 783/4*a^2*b^2 - 135/2*a*b^3 + 27/4*b^4 + 27/4*(64*a^4 + 4*a^3*b - 29*a^2*b^2 + 10*a*b^3 - b^4)*cos(d*x + c)^2 + 27/2*((5*a^12*b - 26*a^11*b^2 + 55*a^10*b^3 - 60*a^9*b^4 + 35*a^8*b^5 - 10*a^7*b^6 + a^6*b^7)*d^3*sqrt((256*a^6 + 160*a^5*b - 167*a^4*b^2 - 28*a^3*b^3 + 46*a^2*b^4 - 12*a*b^5 + b^6)/((a^17*b - 10*a^16*b^2 + 45*a^15*b^3 - 120*a^14*b^4 + 210*a^13*b^5 - 252*a^12*b^6 + 210*a^11*b^7 - 120*a^10*b^8 + 45*a^9*b^9 - 10*a^8*b^10 + a^7*b^11)*d^4))*cos(d*x + c)*sin(d*x + c) + (32*a^7 + 58*a^6*b - 13*a^5*b^2 - 21*a^4*b^3 + 9*a^3*b^4 - a^2*b^5)*d*cos(d*x + c)*sin(d*x + c))*sqrt(((a^8*b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2*sqrt((256*a^6 + 160*a^5*b - 167*a^4*b^2 - 28*a^3*b^3 + 46*a^2*b^4 - 12*a*b^5 + b^6)/((a^17*b - 10*a^16*b^2 + 45*a^15*b^3 - 120*a^14*b^4 + 210*a^13*b^5 - 252*a^12*b^6 + 210*a^11*b^7 - 120*a^10*b^8 + 45*a^9*b^9 - 10*a^8*b^10 + a^7*b^11)*d^4)) - 4*a^3 - 21*a^2*b + 10*a*b^2 - b^3)/((a^8*b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2)) + 27/4*(2*(4*a^10 - 21*a^9*b + 45*a^8*b^2 - 50*a^7*b^3 + 30*a^6*b^4 - 9*a^5*b^5 + a^4*b^6)*d^2*cos(d*x + c)^2 - (4*a^10 - 21*a^9*b + 45*a^8*b^2 - 50*a^7*b^3 + 30*a^6*b^4 - 9*a^5*b^5 + a^4*b^6)*d^2)*sqrt((256*a^6 + 160*a^5*b - 167*a^4*b^2 - 28*a^3*b^3 + 46*a^2*b^4 - 12*a*b^5 + b^6)/((a^17*b - 10*a^16*b^2 + 45*a^15*b^3 - 120*a^14*b^4 + 210*a^13*b^5 - 252*a^12*b^6 + 210*a^11*b^7 - 120*a^10*b^8 + 45*a^9*b^9 - 10*a^8*b^10 + a^7*b^11)*d^4))) - 3*((a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cos(d*x + c)^8 - 4*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cos(d*x + c)^6 - 2*(a^4*b - 5*a^3*b^2 + 7*a^2*b^3 - 3*a*b^4)*d*cos(d*x + c)^4 + 4*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d*cos(d*x + c)^2 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d)*sqrt(((a^8*b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2*sqrt((256*a^6 + 160*a^5*b - 167*a^4*b^2 - 28*a^3*b^3 + 46*a^2*b^4 - 12*a*b^5 + b^6)/((a^17*b - 10*a^16*b^2 + 45*a^15*b^3 - 120*a^14*b^4 + 210*a^13*b^5 - 252*a^12*b^6 + 210*a^11*b^7 - 120*a^10*b^8 + 45*a^9*b^9 - 10*a^8*b^10 + a^7*b^11)*d^4)) - 4*a^3 - 21*a^2*b + 10*a*b^2 - b^3)/((a^8*b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2))*log(-432*a^4 - 27*a^3*b + 783/4*a^2*b^2 - 135/2*a*b^3 + 27/4*b^4 + 27/4*(64*a^4 + 4*a^3*b - 29*a^2*b^2 + 10*a*b^3 - b^4)*cos(d*x + c)^2 - 27/2*((5*a^12*b - 26*a^11*b^2 + 55*a^10*b^3 - 60*a^9*b^4 + 35*a^8*b^5 - 10*a^7*b^6 + a^6*b^7)*d^3*sqrt((256*a^6 + 160*a^5*b - 167*a^4*b^2 - 28*a^3*b^3 + 46*a^2*b^4 - 12*a*b^5 + b^6)/((a^17*b - 10*a^16*b^2 + 45*a^15*b^3 - 120*a^14*b^4 + 210*a^13*b^5 - 252*a^12*b^6 + 210*a^11*b^7 - 120*a^10*b^8 + 45*a^9*b^9 - 10*a^8*b^10 + a^7*b^11)*d^4))*cos(d*x + c)*sin(d*x + c) + (32*a^7 + 58*a^6*b - 13*a^5*b^2 - 21*a^4*b^3 + 9*a^3*b^4 - a^2*b^5)*d*cos(d*x + c)*sin(d*x + c))*sqrt(((a^8*b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2*sqrt((256*a^6 + 160*a^5*b - 167*a^4*b^2 - 28*a^3*b^3 + 46*a^2*b^4 - 12*a*b^5 + b^6)/((a^17*b - 10*a^16*b^2 + 45*a^15*b^3 - 120*a^14*b^4 + 210*a^13*b^5 - 252*a^12*b^6 + 210*a^11*b^7 - 120*a^10*b^8 + 45*a^9*b^9 - 10*a^8*b^10 + a^7*b^11)*d^4)) - 4*a^3 - 21*a^2*b + 10*a*b^2 - b^3)/((a^8*b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2)) + 27/4*(2*(4*a^10 - 21*a^9*b + 45*a^8*b^2 - 50*a^7*b^3 + 30*a^6*b^4 - 9*a^5*b^5 + a^4*b^6)*d^2*cos(d*x + c)^2 - (4*a^10 - 21*a^9*b + 45*a^8*b^2 - 50*a^7*b^3 + 30*a^6*b^4 - 9*a^5*b^5 + a^4*b^6)*d^2)*sqrt((256*a^6 + 160*a^5*b - 167*a^4*b^2 - 28*a^3*b^3 + 46*a^2*b^4 - 12*a*b^5 + b^6)/((a^17*b - 10*a^16*b^2 + 45*a^15*b^3 - 120*a^14*b^4 + 210*a^13*b^5 - 252*a^12*b^6 + 210*a^11*b^7 - 120*a^10*b^8 + 45*a^9*b^9 - 10*a^8*b^10 + a^7*b^11)*d^4))) - 8*(2*(2*a*b + b^2)*cos(d*x + c)^7 - (17*a*b + 7*b^2)*cos(d*x + c)^5 - 8*(a^2 - 3*a*b - b^2)*cos(d*x + c)^3 + (17*a^2 - 14*a*b - 3*b^2)*cos(d*x + c))*sin(d*x + c))/((a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cos(d*x + c)^8 - 4*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cos(d*x + c)^6 - 2*(a^4*b - 5*a^3*b^2 + 7*a^2*b^3 - 3*a*b^4)*d*cos(d*x + c)^4 + 4*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d*cos(d*x + c)^2 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a-b*sin(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a-b*sin(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.233 $\int \frac{\sin^2(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=347

$$-\frac{\tan(c+dx)\left(\frac{5(2a^2+3ab-b^2)\tan^2(c+dx)}{(a-b)^2}+\frac{2a(5a^2-9ab-4b^2)}{(a-b)^3}\right)}{32a^2d\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}-\frac{b\tan(c+dx)\left(\left(a^2+6ab+b^2\right)\tan^2(c+dx)+a(a+3b)\right)}{8ad(a-b)^3\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)^2}+\frac{(-1\cdots}{\cdots}$$

```
[Out] ((12*a - 14*Sqrt[a]*Sqrt[b] + 5*b)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c +
d*x])/a^(1/4)])/(64*a^(9/4)*(Sqrt[a] - Sqrt[b])^(5/2)*Sqrt[b]*d) - ((12*a +
 14*Sqrt[a]*Sqrt[b] + 5*b)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^
(1/4)])/(64*a^(9/4)*(Sqrt[a] + Sqrt[b])^(5/2)*Sqrt[b]*d) - (b*Tan[c + d*x]*
(a*(a + 3*b) + (a^2 + 6*a*b + b^2)*Tan[c + d*x]^2))/(8*a*(a - b)^3*d*(a + 2
*a*Tan[c + d*x]^2 + (a - b)*Tan[c + d*x]^4)^2) - (Tan[c + d*x]*((2*a*(5*a^2
 - 9*a*b - 4*b^2))/(a - b)^3 + (5*(2*a^2 + 3*a*b - b^2)*Tan[c + d*x]^2)/(a
- b)^2))/(32*a^2*d*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan[c + d*x]^4))
```

Rubi [A] time = 0.724284, antiderivative size = 347, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3217, 1333, 1678, 1166, 205}

$$-\frac{\tan(c+dx)\left(\frac{5(2a^2+3ab-b^2)\tan^2(c+dx)}{(a-b)^2}+\frac{2a(5a^2-9ab-4b^2)}{(a-b)^3}\right)}{32a^2d\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}-\frac{b\tan(c+dx)\left(\left(a^2+6ab+b^2\right)\tan^2(c+dx)+a(a+3b)\right)}{8ad(a-b)^3\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)^2}+\frac{(-1\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a - b*Sin[c + d*x]^4)^3,x]
```


```
[Out] ((12*a - 14*Sqrt[a]*Sqrt[b] + 5*b)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c +
d*x])/a^(1/4)])/(64*a^(9/4)*(Sqrt[a] - Sqrt[b])^(5/2)*Sqrt[b]*d) - ((12*a +
 14*Sqrt[a]*Sqrt[b] + 5*b)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^
(1/4)])/(64*a^(9/4)*(Sqrt[a] + Sqrt[b])^(5/2)*Sqrt[b]*d) - (b*Tan[c + d*x]*
(a*(a + 3*b) + (a^2 + 6*a*b + b^2)*Tan[c + d*x]^2))/(8*a*(a - b)^3*d*(a + 2
*a*Tan[c + d*x]^2 + (a - b)*Tan[c + d*x]^4)^2) - (Tan[c + d*x]*((2*a*(5*a^2
 - 9*a*b - 4*b^2))/(a - b)^3 + (5*(2*a^2 + 3*a*b - b^2)*Tan[c + d*x]^2)/(a
- b)^2))/(32*a^2*d*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan[c + d*x]^4))
```


Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 1333

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)),
```

```
x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*S
imp[ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d + e*x^2
)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g +
c*(4*p + 7)*(b*f - 2*a*g)*x^2, x], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1] && IGtQ[m/2, 0]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(1+x^2\right)^4}{\left(a+2ax^2+(a-b)x^4\right)^3}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{b\tan(c+dx)\left(a(a+3b)+\left(a^2+6ab+b^2\right)\tan^2(c+dx)\right)}{8a(a-b)^3d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-\frac{2a^2b^2(a+3b)}{(a-b)^3}-\frac{2ab(8a^3}{}}{}\right.}{}$$

$$= -\frac{b\tan(c+dx)\left(a(a+3b)+\left(a^2+6ab+b^2\right)\tan^2(c+dx)\right)}{8a(a-b)^3d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{\tan(c+dx)\left(\frac{2a\left(5a^2-9ab-4b^2\right.}{(a-b)^3}\right.}{32a^2d\left(a+2a\tan^2(c+d\right.}$$

$$= -\frac{b\tan(c+dx)\left(a(a+3b)+\left(a^2+6ab+b^2\right)\tan^2(c+dx)\right)}{8a(a-b)^3d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{\tan(c+dx)\left(\frac{2a\left(5a^2-9ab-4b^2\right.}{(a-b)^3}\right.}{32a^2d\left(a+2a\tan^2(c+d\right.}$$

$$= \frac{\left(12a-14\sqrt{a}\sqrt{b}+5b\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{9/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}\sqrt{b}d} - \frac{\left(12a+14\sqrt{a}\sqrt{b}+5b\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}}{\sqrt[4]{}}\right.}{64a^{9/4}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}\sqrt{b}d}$$

Mathematica [A] time = 6.43048, size = 457, normalized size = 1.32

$$\frac{\left(11a^{3/2}b^{3/2}-12a^{5/2}\sqrt{b}+10a^2b-4ab^2-5\sqrt{a}b^{5/2}\right)\tan^{-1}\left(\frac{\left(\sqrt{a}\sqrt{b}+b\right)\tan(c+dx)}{\sqrt{b}\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{64a^{5/2}bd\sqrt{\sqrt{a}\sqrt{b}+a}(a-b)^2}+\frac{24a^2\sin(2(c+dx))+22ab\sin(2(c+dx))-}{32a^2d(a-b)^2(-8a-}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] ((-12*a^(5/2)*Sqrt[b] + 10*a^2*b + 11*a^(3/2)*b^(3/2) - 4*a*b^2 - 5*Sqrt[a]
*b^(5/2))*ArcTan[((Sqrt[a]*Sqrt[b] + b)*Tan[c + d*x])/(Sqrt[a + Sqrt[a]*Sqr
t[b]]*Sqrt[b])])/(64*a^(5/2)*Sqrt[a + Sqrt[a]*Sqrt[b]]*(a - b)^2*b*d) - ((1
2*a^(5/2)*Sqrt[b] + 10*a^2*b - 11*a^(3/2)*b^(3/2) - 4*a*b^2 + 5*Sqrt[a]*b^(
5/2))*ArcTanh[((Sqrt[a]*Sqrt[b] - b)*Tan[c + d*x])/(Sqrt[-a + Sqrt[a]*Sqrt[
b]]*Sqrt[b])])/(64*a^(5/2)*Sqrt[-a + Sqrt[a]*Sqrt[b]]*(a - b)^2*b*d) + (-4*
a*Sin[2*(c + d*x)] - 2*b*Sin[2*(c + d*x)] + b*Sin[4*(c + d*x)])/(a*(a - b)*
d*(-8*a + 3*b - 4*b*Cos[2*(c + d*x)] + b*Cos[4*(c + d*x)])^2) + (24*a^2*Sin
[2*(c + d*x)] + 22*a*b*Sin[2*(c + d*x)] - 10*b^2*Sin[2*(c + d*x)] - 11*a*b*
Sin[4*(c + d*x)] + 5*b^2*Sin[4*(c + d*x)])/(32*a^2*(a - b)^2*d*(-8*a + 3*b
- 4*b*Cos[2*(c + d*x)] + b*Cos[4*(c + d*x)]))
```

Maple [B] time = 0.145, size = 1906, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a-b*sin(d*x+c)^4)^3,x)
```

```
[Out] -13/64/d*b/(a^2-2*a*b+b^2)*a/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2
)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+13/64/d*b/(a^2-2*a
*b+b^2)*a/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*ta
n(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-3/32/d/(tan(d*x+c)^4*a-tan(d*x+c)^4
*b+2*a*tan(d*x+c)^2+a)^2*b/(a^2-2*a*b+b^2)*tan(d*x+c)^3-3/8/d/(tan(d*x+c)^4
*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2*b/(a^2-2*a*b+b^2)*tan(d*x+c)^5-15/1
6/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2*a/(a^2-2*a*b+b^2)*
tan(d*x+c)^3-5/16/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2*a/
(a^2-2*a*b+b^2)*tan(d*x+c)+5/32/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*
x+c)^2+a)^2/a^2/(a-b)*tan(d*x+c)^7*b^2+9/32/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*
b+2*a*tan(d*x+c)^2+a)^2/a/(a^2-2*a*b+b^2)*tan(d*x+c)^3*b^2-1/64/d/a/(a^2-2*
a*b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d
*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*b^3+1/64/d/a/(a^2-2*a*b+b^2)/(a*b)^(1/
2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1
/2)-a)*(a-b))^(1/2))*b^3-5/16/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+
c)^2+a)^2/(a-b)*tan(d*x+c)^7-15/32/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan
(d*x+c)^2+a)^2/(a-b)/a*b*tan(d*x+c)^7+11/32/d/(a^2-2*a*b+b^2)*a/(a-b)/(((a*
b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1
/2))-15/16/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2/(a^2-2*a*
b+b^2)*tan(d*x+c)^5*a+11/32/d/(a^2-2*a*b+b^2)*a/(a-b)/(((a*b)^(1/2)-a)*(a-b
))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))+5/16/d/a*
b^2/(a^2-2*a*b+b^2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*
x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+5/16/d/a*b^2/(a^2-2*a*b+b^2)/(a-b)/(((a
*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))
^(1/2))-37/64/d*b/(a^2-2*a*b+b^2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arcta
```

```
n((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-37/64/d*b/(a^2-2*a*b+b^2)
/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2
)-a)*(a-b))^(1/2))+3/16/d/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a
)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*a^2-3
/16/d/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arcta
nh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*a^2+1/32/d*b^2/(a^2-2*a
*b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*
x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-1/32/d*b^2/(a^2-2*a*b+b^2)/(a*b)^(1/2)/
(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)
-a)*(a-b))^(1/2))-5/64/d/a^2/(a^2-2*a*b+b^2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^
(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*b^3-5/64/d/a^2
/(a^2-2*a*b+b^2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x
+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*b^3+9/16/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*
b+2*a*tan(d*x+c)^2+a)^2/a/(a^2-2*a*b+b^2)*tan(d*x+c)^5*b^2+1/8/d/(tan(d*x+c
)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2/(a^2-2*a*b+b^2)*tan(d*x+c)*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a-b*sin(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(4*(96*a^3*b^2 + 36*a^2*b^3 - 53*a*b^4 + 35*b^5)*cos(4*d*x + 4*c)*sin(
2*d*x + 2*c) + ((12*a^2*b^3 - 11*a*b^4 + 5*b^5)*sin(14*d*x + 14*c) - (104*a
^2*b^3 - 85*a*b^4 + 35*b^5)*sin(12*d*x + 12*c) - (320*a^3*b^2 - 652*a^2*b^3
 + 407*a*b^4 - 105*b^5)*sin(10*d*x + 10*c) + (1408*a^3*b^2 - 1696*a^2*b^3 +
 865*a*b^4 - 175*b^5)*sin(8*d*x + 8*c) + (320*a^3*b^2 + 756*a^2*b^3 - 849*a
*b^4 + 175*b^5)*sin(6*d*x + 6*c) - (248*a^2*b^3 - 383*a*b^4 + 105*b^5)*sin(
4*d*x + 4*c) - (12*a^2*b^3 + 77*a*b^4 - 35*b^5)*sin(2*d*x + 2*c))*cos(16*d*
x + 16*c) + 2*(2*(96*a^3*b^2 + 36*a^2*b^3 - 53*a*b^4 + 35*b^5)*sin(12*d*x +
 12*c) + 8*(64*a^3*b^2 - 196*a^2*b^3 + 125*a*b^4 - 35*b^5)*sin(10*d*x + 10*
c) - 3*(512*a^4*b + 1024*a^3*b^2 - 1556*a^2*b^3 + 865*a*b^4 - 175*b^5)*sin(
8*d*x + 8*c) - 16*(128*a^3*b^2 + 124*a^2*b^3 - 173*a*b^4 + 35*b^5)*sin(6*d*
x + 6*c) + 2*(96*a^3*b^2 + 324*a^2*b^3 - 649*a*b^4 + 175*b^5)*sin(4*d*x + 4
*c) + 24*(4*a^2*b^3 + 11*a*b^4 - 5*b^5)*sin(2*d*x + 2*c))*cos(14*d*x + 14*c
) + 2*(2*(2560*a^4*b - 4128*a^3*b^2 + 3644*a^2*b^3 - 1379*a*b^4 + 245*b^5)*
sin(10*d*x + 10*c) - (9216*a^4*b - 25984*a^3*b^2 + 21304*a^2*b^3 - 8575*a*b
^4 + 1225*b^5)*sin(8*d*x + 8*c) - 2*(2560*a^4*b + 480*a^3*b^2 - 7908*a^2*b^
3 + 5033*a*b^4 - 735*b^5)*sin(6*d*x + 6*c) + 4*(576*a^3*b^2 - 1696*a^2*b^3
+ 1323*a*b^4 - 245*b^5)*sin(4*d*x + 4*c) + 2*(96*a^3*b^2 + 324*a^2*b^3 - 64
9*a*b^4 + 175*b^5)*sin(2*d*x + 2*c))*cos(12*d*x + 12*c) + 2*((40960*a^5 - 2
4064*a^4*b - 22080*a^3*b^2 + 27516*a^2*b^3 - 11095*a*b^4 + 1225*b^5)*sin(8*
d*x + 8*c) + 8*(5120*a^4*b - 1408*a^3*b^2 - 3900*a^2*b^3 + 2107*a*b^4 - 245
*b^5)*sin(6*d*x + 6*c) - 2*(2560*a^4*b + 480*a^3*b^2 - 7908*a^2*b^3 + 5033*
a*b^4 - 735*b^5)*sin(4*d*x + 4*c) - 16*(128*a^3*b^2 + 124*a^2*b^3 - 173*a*b
^4 + 35*b^5)*sin(2*d*x + 2*c))*cos(10*d*x + 10*c) + 2*((40960*a^5 - 24064*a
^4*b - 22080*a^3*b^2 + 27516*a^2*b^3 - 11095*a*b^4 + 1225*b^5)*sin(6*d*x +
6*c) - (9216*a^4*b - 25984*a^3*b^2 + 21304*a^2*b^3 - 8575*a*b^4 + 1225*b^5)
*sin(4*d*x + 4*c) - 3*(512*a^4*b + 1024*a^3*b^2 - 1556*a^2*b^3 + 865*a*b^4
- 175*b^5)*sin(2*d*x + 2*c))*cos(8*d*x + 8*c) + 4*((2560*a^4*b - 4128*a^3*b
^2 + 3644*a^2*b^3 - 1379*a*b^4 + 245*b^5)*sin(4*d*x + 4*c) + 4*(64*a^3*b^2
- 196*a^2*b^3 + 125*a*b^4 - 35*b^5)*sin(2*d*x + 2*c))*cos(6*d*x + 6*c) + 16
*((a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*cos(16*d*x + 16*c)^2 + 64*(a^4*b^4 - 2*
a^3*b^5 + a^2*b^6)*d*cos(14*d*x + 14*c)^2 + 16*(64*a^6*b^2 - 240*a^5*b^3 +
337*a^4*b^4 - 210*a^3*b^5 + 49*a^2*b^6)*d*cos(12*d*x + 12*c)^2 + 64*(256*a^
```

6*b^2 - 736*a^5*b^3 + 753*a^4*b^4 - 322*a^3*b^5 + 49*a^2*b^6)*d*cos(10*d*x + 10*c)^2 + 4*(16384*a^8 - 57344*a^7*b + 83712*a^6*b^2 - 67648*a^5*b^3 + 32841*a^4*b^4 - 9170*a^3*b^5 + 1225*a^2*b^6)*d*cos(8*d*x + 8*c)^2 + 64*(256*a^6*b^2 - 736*a^5*b^3 + 753*a^4*b^4 - 322*a^3*b^5 + 49*a^2*b^6)*d*cos(6*d*x + 6*c)^2 + 16*(64*a^6*b^2 - 240*a^5*b^3 + 337*a^4*b^4 - 210*a^3*b^5 + 49*a^2*b^6)*d*cos(4*d*x + 4*c)^2 + 64*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*cos(2*d*x + 2*c)^2 + (a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*sin(16*d*x + 16*c)^2 + 64*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*sin(14*d*x + 14*c)^2 + 16*(64*a^6*b^2 - 240*a^5*b^3 + 337*a^4*b^4 - 210*a^3*b^5 + 49*a^2*b^6)*d*sin(12*d*x + 12*c)^2 + 64*(256*a^6*b^2 - 736*a^5*b^3 + 753*a^4*b^4 - 322*a^3*b^5 + 49*a^2*b^6)*d*sin(10*d*x + 10*c)^2 + 4*(16384*a^8 - 57344*a^7*b + 83712*a^6*b^2 - 67648*a^5*b^3 + 32841*a^4*b^4 - 9170*a^3*b^5 + 1225*a^2*b^6)*d*sin(8*d*x + 8*c)^2 + 64*(256*a^6*b^2 - 736*a^5*b^3 + 753*a^4*b^4 - 322*a^3*b^5 + 49*a^2*b^6)*d*sin(6*d*x + 6*c)^2 + 16*(64*a^6*b^2 - 240*a^5*b^3 + 337*a^4*b^4 - 210*a^3*b^5 + 49*a^2*b^6)*d*sin(4*d*x + 4*c)^2 + 64*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 64*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*sin(2*d*x + 2*c)^2 - 16*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*cos(2*d*x + 2*c) + (a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d - 2*(8*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*cos(14*d*x + 14*c) + 4*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*cos(12*d*x + 12*c) - 8*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*cos(10*d*x + 10*c) - 2*(128*a^6*b^2 - 352*a^5*b^3 + 355*a^4*b^4 - 166*a^3*b^5 + 35*a^2*b^6)*d*cos(8*d*x + 8*c) - 8*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*cos(6*d*x + 6*c) + 4*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*cos(4*d*x + 4*c) + 8*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*cos(2*d*x + 2*c) - (a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d)*cos(16*d*x + 16*c) + 16*(4*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*cos(12*d*x + 12*c) - 8*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*cos(10*d*x + 10*c) - 2*(128*a^6*b^2 - 352*a^5*b^3 + 355*a^4*b^4 - 166*a^3*b^5 + 35*a^2*b^6)*d*cos(8*d*x + 8*c) - 8*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*cos(6*d*x + 6*c) + 4*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*cos(4*d*x + 4*c) + 8*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*cos(2*d*x + 2*c) - (a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d)*cos(14*d*x + 14*c) - 8*(8*(128*a^6*b^2 - 424*a^5*b^3 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*cos(10*d*x + 10*c) + 2*(1024*a^7*b - 3712*a^6*b^2 + 5304*a^5*b^3 - 3813*a^4*b^4 + 1442*a^3*b^5 - 245*a^2*b^6)*d*cos(8*d*x + 8*c) + 8*(128*a^6*b^2 - 424*a^5*b^3 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*cos(6*d*x + 6*c) - 4*(64*a^6*b^2 - 240*a^5*b^3 + 337*a^4*b^4 - 210*a^3*b^5 + 49*a^2*b^6)*d*cos(4*d*x + 4*c) - 8*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*cos(2*d*x + 2*c) + (8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d)*cos(12*d*x + 12*c) + 16*(2*(2048*a^7*b - 6528*a^6*b^2 + 8144*a^5*b^3 - 5141*a^4*b^4 + 1722*a^3*b^5 - 245*a^2*b^6)*d*cos(8*d*x + 8*c) + 8*(256*a^6*b^2 - 736*a^5*b^3 + 753*a^4*b^4 - 322*a^3*b^5 + 49*a^2*b^6)*d*cos(6*d*x + 6*c) - 4*(128*a^6*b^2 - 424*a^5*b^3 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*cos(4*d*x + 4*c) - 8*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*cos(2*d*x + 2*c) + (16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d)*cos(10*d*x + 10*c) + 4*(8*(2048*a^7*b - 6528*a^6*b^2 + 8144*a^5*b^3 - 5141*a^4*b^4 + 1722*a^3*b^5 - 245*a^2*b^6)*d*cos(6*d*x + 6*c) - 4*(1024*a^7*b - 3712*a^6*b^2 + 5304*a^5*b^3 - 3813*a^4*b^4 + 1442*a^3*b^5 - 245*a^2*b^6)*d*cos(4*d*x + 4*c) - 8*(128*a^6*b^2 - 352*a^5*b^3 + 355*a^4*b^4 - 166*a^3*b^5 + 35*a^2*b^6)*d*cos(2*d*x + 2*c) + (128*a^6*b^2 - 352*a^5*b^3 + 355*a^4*b^4 - 166*a^3*b^5 + 35*a^2*b^6)*d)*cos(8*d*x + 8*c) - 16*(4*(128*a^6*b^2 - 424*a^5*b^3 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*cos(4*d*x + 4*c) + 8*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*cos(2*d*x + 2*c) - (16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d)*cos(6*d*x + 6*c) + 8*(8*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*cos(2*d*x + 2*c) - (8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d)*cos(4*d*x + 4*c) - 4*(4*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*sin(14*d*x + 14*c) + 2*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*sin(12*d*x + 12*c) - 4*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*sin(10*d*x + 10*c) - (128*a^6*b^2 - 352*a^5*b^3 + 355*a^

4*b^4 - 166*a^3*b^5 + 35*a^2*b^6)*d*sin(8*d*x + 8*c) - 4*(16*a^5*b^3 - 39*a
^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*sin(6*d*x + 6*c) + 2*(8*a^5*b^3 - 23*a^4
*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*sin(4*d*x + 4*c) + 4*(a^4*b^4 - 2*a^3*b^5
+ a^2*b^6)*d*sin(2*d*x + 2*c))*sin(16*d*x + 16*c) + 32*(2*(8*a^5*b^3 - 23*a
^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*sin(12*d*x + 12*c) - 4*(16*a^5*b^3 - 39*
a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*sin(10*d*x + 10*c) - (128*a^6*b^2 - 352
*a^5*b^3 + 355*a^4*b^4 - 166*a^3*b^5 + 35*a^2*b^6)*d*sin(8*d*x + 8*c) - 4*(
16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*sin(6*d*x + 6*c) + 2*(8
*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*sin(4*d*x + 4*c) + 4*(a^4
*b^4 - 2*a^3*b^5 + a^2*b^6)*d*sin(2*d*x + 2*c))*sin(14*d*x + 14*c) - 16*(4*
(128*a^6*b^2 - 424*a^5*b^3 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*sin(
10*d*x + 10*c) + (1024*a^7*b - 3712*a^6*b^2 + 5304*a^5*b^3 - 3813*a^4*b^4 +
1442*a^3*b^5 - 245*a^2*b^6)*d*sin(8*d*x + 8*c) + 4*(128*a^6*b^2 - 424*a^5*
b^3 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*sin(6*d*x + 6*c) - 2*(64*a^
6*b^2 - 240*a^5*b^3 + 337*a^4*b^4 - 210*a^3*b^5 + 49*a^2*b^6)*d*sin(4*d*x +
4*c) - 4*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*sin(2*d*x + 2
*c))*sin(12*d*x + 12*c) + 32*((2048*a^7*b - 6528*a^6*b^2 + 8144*a^5*b^3 - 5
141*a^4*b^4 + 1722*a^3*b^5 - 245*a^2*b^6)*d*sin(8*d*x + 8*c) + 4*(256*a^6*b
^2 - 736*a^5*b^3 + 753*a^4*b^4 - 322*a^3*b^5 + 49*a^2*b^6)*d*sin(6*d*x + 6*
c) - 2*(128*a^6*b^2 - 424*a^5*b^3 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)
*d*sin(4*d*x + 4*c) - 4*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*
d*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 16*(2*(2048*a^7*b - 6528*a^6*b^2 +
8144*a^5*b^3 - 5141*a^4*b^4 + 1722*a^3*b^5 - 245*a^2*b^6)*d*sin(6*d*x + 6*
c) - (1024*a^7*b - 3712*a^6*b^2 + 5304*a^5*b^3 - 3813*a^4*b^4 + 1442*a^3*b^
5 - 245*a^2*b^6)*d*sin(4*d*x + 4*c) - 2*(128*a^6*b^2 - 352*a^5*b^3 + 355*a^
4*b^4 - 166*a^3*b^5 + 35*a^2*b^6)*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) - 64
*((128*a^6*b^2 - 424*a^5*b^3 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*si
n(4*d*x + 4*c) + 2*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*sin
(2*d*x + 2*c))*sin(6*d*x + 6*c))*integrate(-1/8*(4*(12*a^2*b - 11*a*b^2 + 5
*b^3)*cos(6*d*x + 6*c)^2 - 4*(256*a^3 - 248*a^2*b + 97*a*b^2 - 15*b^3)*cos(
4*d*x + 4*c)^2 + 4*(12*a^2*b - 11*a*b^2 + 5*b^3)*cos(2*d*x + 2*c)^2 + 4*(12
*a^2*b - 11*a*b^2 + 5*b^3)*sin(6*d*x + 6*c)^2 - 4*(256*a^3 - 248*a^2*b + 97
*a*b^2 - 15*b^3)*sin(4*d*x + 4*c)^2 + 2*(96*a^3 - 252*a^2*b + 149*a*b^2 - 3
5*b^3)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*(12*a^2*b - 11*a*b^2 + 5*b^3)*
sin(2*d*x + 2*c)^2 - ((12*a^2*b - 11*a*b^2 + 5*b^3)*cos(6*d*x + 6*c) - 2*(3
2*a^2*b - 19*a*b^2 + 5*b^3)*cos(4*d*x + 4*c) + (12*a^2*b - 11*a*b^2 + 5*b^3
)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) - (12*a^2*b - 11*a*b^2 + 5*b^3 - 2*(96
*a^3 - 252*a^2*b + 149*a*b^2 - 35*b^3)*cos(4*d*x + 4*c) - 8*(12*a^2*b - 11*
a*b^2 + 5*b^3)*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 2*(32*a^2*b - 19*a*b^2
+ 5*b^3 + (96*a^3 - 252*a^2*b + 149*a*b^2 - 35*b^3)*cos(2*d*x + 2*c))*cos(4
*d*x + 4*c) - (12*a^2*b - 11*a*b^2 + 5*b^3)*cos(2*d*x + 2*c) - ((12*a^2*b -
11*a*b^2 + 5*b^3)*sin(6*d*x + 6*c) - 2*(32*a^2*b - 19*a*b^2 + 5*b^3)*sin(4
*d*x + 4*c) + (12*a^2*b - 11*a*b^2 + 5*b^3)*sin(2*d*x + 2*c))*sin(8*d*x + 8
c) + 2((96*a^3 - 252*a^2*b + 149*a*b^2 - 35*b^3)*sin(4*d*x + 4*c) + 4*(12
*a^2*b - 11*a*b^2 + 5*b^3)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))/(a^4*b^2 - 2
*a^3*b^3 + a^2*b^4 + (a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*cos(8*d*x + 8*c)^2 + 1
6*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*cos(6*d*x + 6*c)^2 + 4*(64*a^6 - 176*a^5*
b + 169*a^4*b^2 - 66*a^3*b^3 + 9*a^2*b^4)*cos(4*d*x + 4*c)^2 + 16*(a^4*b^2
- 2*a^3*b^3 + a^2*b^4)*cos(2*d*x + 2*c)^2 + (a^4*b^2 - 2*a^3*b^3 + a^2*b^4)
*sin(8*d*x + 8*c)^2 + 16*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*sin(6*d*x + 6*c)^2
+ 4*(64*a^6 - 176*a^5*b + 169*a^4*b^2 - 66*a^3*b^3 + 9*a^2*b^4)*sin(4*d*x
+ 4*c)^2 + 16*(8*a^5*b - 19*a^4*b^2 + 14*a^3*b^3 - 3*a^2*b^4)*sin(4*d*x + 4
*c)*sin(2*d*x + 2*c) + 16*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*sin(2*d*x + 2*c)^
2 + 2*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4 - 4*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*co
s(6*d*x + 6*c) - 2*(8*a^5*b - 19*a^4*b^2 + 14*a^3*b^3 - 3*a^2*b^4)*cos(4*d*
x + 4*c) - 4*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*cos(2*d*x + 2*c))*cos(8*d*x +
8*c) - 8*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4 - 2*(8*a^5*b - 19*a^4*b^2 + 14*a^3*
b^3 - 3*a^2*b^4)*cos(4*d*x + 4*c) - 4*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*cos(2
*d*x + 2*c))*cos(6*d*x + 6*c) - 4*(8*a^5*b - 19*a^4*b^2 + 14*a^3*b^3 - 3*a^

2*b^4 - 4*(8*a^5*b - 19*a^4*b^2 + 14*a^3*b^3 - 3*a^2*b^4)*cos(2*d*x + 2*c))
*cos(4*d*x + 4*c) - 8*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*cos(2*d*x + 2*c) - 4*
(2*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*sin(6*d*x + 6*c) + (8*a^5*b - 19*a^4*b^2
 + 14*a^3*b^3 - 3*a^2*b^4)*sin(4*d*x + 4*c) + 2*(a^4*b^2 - 2*a^3*b^3 + a^2*
b^4)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^5*b - 19*a^4*b^2 + 14*a^
3*b^3 - 3*a^2*b^4)*sin(4*d*x + 4*c) + 2*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*sin
(2*d*x + 2*c))*sin(6*d*x + 6*c)), x) - (11*a*b^4 - 5*b^5 + (12*a^2*b^3 - 11
*a*b^4 + 5*b^5)*cos(14*d*x + 14*c) - (104*a^2*b^3 - 85*a*b^4 + 35*b^5)*cos(
12*d*x + 12*c) - (320*a^3*b^2 - 652*a^2*b^3 + 407*a*b^4 - 105*b^5)*cos(10*d
*x + 10*c) + (1408*a^3*b^2 - 1696*a^2*b^3 + 865*a*b^4 - 175*b^5)*cos(8*d*x
+ 8*c) + (320*a^3*b^2 + 756*a^2*b^3 - 849*a*b^4 + 175*b^5)*cos(6*d*x + 6*c)
 - (248*a^2*b^3 - 383*a*b^4 + 105*b^5)*cos(4*d*x + 4*c) - (12*a^2*b^3 + 77*
a*b^4 - 35*b^5)*cos(2*d*x + 2*c))*sin(16*d*x + 16*c) + (12*a^2*b^3 + 77*a*b
^4 - 35*b^5 - 4*(96*a^3*b^2 + 36*a^2*b^3 - 53*a*b^4 + 35*b^5)*cos(12*d*x +
12*c) - 16*(64*a^3*b^2 - 196*a^2*b^3 + 125*a*b^4 - 35*b^5)*cos(10*d*x + 10*
c) + 6*(512*a^4*b + 1024*a^3*b^2 - 1556*a^2*b^3 + 865*a*b^4 - 175*b^5)*cos(
8*d*x + 8*c) + 32*(128*a^3*b^2 + 124*a^2*b^3 - 173*a*b^4 + 35*b^5)*cos(6*d*
x + 6*c) - 4*(96*a^3*b^2 + 324*a^2*b^3 - 649*a*b^4 + 175*b^5)*cos(4*d*x + 4
c) - 48(4*a^2*b^3 + 11*a*b^4 - 5*b^5)*cos(2*d*x + 2*c))*sin(14*d*x + 14*c
) + (248*a^2*b^3 - 383*a*b^4 + 105*b^5 - 4*(2560*a^4*b - 4128*a^3*b^2 + 364
4*a^2*b^3 - 1379*a*b^4 + 245*b^5)*cos(10*d*x + 10*c) + 2*(9216*a^4*b - 2598
4*a^3*b^2 + 21304*a^2*b^3 - 8575*a*b^4 + 1225*b^5)*cos(8*d*x + 8*c) + 4*(25
60*a^4*b + 480*a^3*b^2 - 7908*a^2*b^3 + 5033*a*b^4 - 735*b^5)*cos(6*d*x + 6
c) - 8(576*a^3*b^2 - 1696*a^2*b^3 + 1323*a*b^4 - 245*b^5)*cos(4*d*x + 4*c
) - 4*(96*a^3*b^2 + 324*a^2*b^3 - 649*a*b^4 + 175*b^5)*cos(2*d*x + 2*c))*si
n(12*d*x + 12*c) - (320*a^3*b^2 + 756*a^2*b^3 - 849*a*b^4 + 175*b^5 + 2*(40
960*a^5 - 24064*a^4*b - 22080*a^3*b^2 + 27516*a^2*b^3 - 11095*a*b^4 + 1225*
b^5)*cos(8*d*x + 8*c) + 16*(5120*a^4*b - 1408*a^3*b^2 - 3900*a^2*b^3 + 2107
*a*b^4 - 245*b^5)*cos(6*d*x + 6*c) - 4*(2560*a^4*b + 480*a^3*b^2 - 7908*a^2
*b^3 + 5033*a*b^4 - 735*b^5)*cos(4*d*x + 4*c) - 32*(128*a^3*b^2 + 124*a^2*b
^3 - 173*a*b^4 + 35*b^5)*cos(2*d*x + 2*c))*sin(10*d*x + 10*c) - (1408*a^3*b
^2 - 1696*a^2*b^3 + 865*a*b^4 - 175*b^5 + 2*(40960*a^5 - 24064*a^4*b - 2208
0*a^3*b^2 + 27516*a^2*b^3 - 11095*a*b^4 + 1225*b^5)*cos(6*d*x + 6*c) - 2*(9
216*a^4*b - 25984*a^3*b^2 + 21304*a^2*b^3 - 8575*a*b^4 + 1225*b^5)*cos(4*d*
x + 4*c) - 6*(512*a^4*b + 1024*a^3*b^2 - 1556*a^2*b^3 + 865*a*b^4 - 175*b^5
)*cos(2*d*x + 2*c))*sin(8*d*x + 8*c) + (320*a^3*b^2 - 652*a^2*b^3 + 407*a*b
^4 - 105*b^5 - 4*(2560*a^4*b - 4128*a^3*b^2 + 3644*a^2*b^3 - 1379*a*b^4 + 2
45*b^5)*cos(4*d*x + 4*c) - 16*(64*a^3*b^2 - 196*a^2*b^3 + 125*a*b^4 - 35*b^
5)*cos(2*d*x + 2*c))*sin(6*d*x + 6*c) + (104*a^2*b^3 - 85*a*b^4 + 35*b^5 -
4*(96*a^3*b^2 + 36*a^2*b^3 - 53*a*b^4 + 35*b^5)*cos(2*d*x + 2*c))*sin(4*d*x
 + 4*c) - (12*a^2*b^3 - 11*a*b^4 + 5*b^5)*sin(2*d*x + 2*c))/((a^4*b^4 - 2*a
^3*b^5 + a^2*b^6)*d*cos(16*d*x + 16*c)^2 + 64*(a^4*b^4 - 2*a^3*b^5 + a^2*b^
6)*d*cos(14*d*x + 14*c)^2 + 16*(64*a^6*b^2 - 240*a^5*b^3 + 337*a^4*b^4 - 21
0*a^3*b^5 + 49*a^2*b^6)*d*cos(12*d*x + 12*c)^2 + 64*(256*a^6*b^2 - 736*a^5*
b^3 + 753*a^4*b^4 - 322*a^3*b^5 + 49*a^2*b^6)*d*cos(10*d*x + 10*c)^2 + 4*(1
6384*a^8 - 57344*a^7*b + 83712*a^6*b^2 - 67648*a^5*b^3 + 32841*a^4*b^4 - 91
70*a^3*b^5 + 1225*a^2*b^6)*d*cos(8*d*x + 8*c)^2 + 64*(256*a^6*b^2 - 736*a^5
*b^3 + 753*a^4*b^4 - 322*a^3*b^5 + 49*a^2*b^6)*d*cos(6*d*x + 6*c)^2 + 16*(6
4*a^6*b^2 - 240*a^5*b^3 + 337*a^4*b^4 - 210*a^3*b^5 + 49*a^2*b^6)*d*cos(4*d
*x + 4*c)^2 + 64*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*cos(2*d*x + 2*c)^2 + (a^
4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*sin(16*d*x + 16*c)^2 + 64*(a^4*b^4 - 2*a^3*b
^5 + a^2*b^6)*d*sin(14*d*x + 14*c)^2 + 16*(64*a^6*b^2 - 240*a^5*b^3 + 337*a
^4*b^4 - 210*a^3*b^5 + 49*a^2*b^6)*d*sin(12*d*x + 12*c)^2 + 64*(256*a^6*b^2
 - 736*a^5*b^3 + 753*a^4*b^4 - 322*a^3*b^5 + 49*a^2*b^6)*d*sin(10*d*x + 10*
c)^2 + 4*(16384*a^8 - 57344*a^7*b + 83712*a^6*b^2 - 67648*a^5*b^3 + 32841*a
^4*b^4 - 9170*a^3*b^5 + 1225*a^2*b^6)*d*sin(8*d*x + 8*c)^2 + 64*(256*a^6*b^
2 - 736*a^5*b^3 + 753*a^4*b^4 - 322*a^3*b^5 + 49*a^2*b^6)*d*sin(6*d*x + 6*c
)^2 + 16*(64*a^6*b^2 - 240*a^5*b^3 + 337*a^4*b^4 - 210*a^3*b^5 + 49*a^2*b^6
)*d*sin(4*d*x + 4*c)^2 + 64*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^

6)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 64*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)
*d*sin(2*d*x + 2*c)^2 - 16*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*cos(2*d*x + 2*
c) + (a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d - 2*(8*(a^4*b^4 - 2*a^3*b^5 + a^2*b^
6)*d*cos(14*d*x + 14*c) + 4*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^
6)*d*cos(12*d*x + 12*c) - 8*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b
^6)*d*cos(10*d*x + 10*c) - 2*(128*a^6*b^2 - 352*a^5*b^3 + 355*a^4*b^4 - 166
*a^3*b^5 + 35*a^2*b^6)*d*cos(8*d*x + 8*c) - 8*(16*a^5*b^3 - 39*a^4*b^4 + 30
*a^3*b^5 - 7*a^2*b^6)*d*cos(6*d*x + 6*c) + 4*(8*a^5*b^3 - 23*a^4*b^4 + 22*a
^3*b^5 - 7*a^2*b^6)*d*cos(4*d*x + 4*c) + 8*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*
d*cos(2*d*x + 2*c) - (a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d)*cos(16*d*x + 16*c)
+ 16*(4*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*cos(12*d*x + 12
c) - 8(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*cos(10*d*x + 1
0*c) - 2*(128*a^6*b^2 - 352*a^5*b^3 + 355*a^4*b^4 - 166*a^3*b^5 + 35*a^2*b^
6)*d*cos(8*d*x + 8*c) - 8*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6
)*d*cos(6*d*x + 6*c) + 4*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*
d*cos(4*d*x + 4*c) + 8*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*cos(2*d*x + 2*c) -
 (a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d)*cos(14*d*x + 14*c) - 8*(8*(128*a^6*b^2
- 424*a^5*b^3 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*cos(10*d*x + 10*c
) + 2*(1024*a^7*b - 3712*a^6*b^2 + 5304*a^5*b^3 - 3813*a^4*b^4 + 1442*a^3*b
^5 - 245*a^2*b^6)*d*cos(8*d*x + 8*c) + 8*(128*a^6*b^2 - 424*a^5*b^3 + 513*a
^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*cos(6*d*x + 6*c) - 4*(64*a^6*b^2 - 240
*a^5*b^3 + 337*a^4*b^4 - 210*a^3*b^5 + 49*a^2*b^6)*d*cos(4*d*x + 4*c) - 8*(
8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*cos(2*d*x + 2*c) + (8*a^
5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d)*cos(12*d*x + 12*c) + 16*(2*
(2048*a^7*b - 6528*a^6*b^2 + 8144*a^5*b^3 - 5141*a^4*b^4 + 1722*a^3*b^5 - 2
45*a^2*b^6)*d*cos(8*d*x + 8*c) + 8*(256*a^6*b^2 - 736*a^5*b^3 + 753*a^4*b^4
 - 322*a^3*b^5 + 49*a^2*b^6)*d*cos(6*d*x + 6*c) - 4*(128*a^6*b^2 - 424*a^5*
b^3 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*cos(4*d*x + 4*c) - 8*(16*a^
5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*cos(2*d*x + 2*c) + (16*a^5*b
^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d)*cos(10*d*x + 10*c) + 4*(8*(204
8*a^7*b - 6528*a^6*b^2 + 8144*a^5*b^3 - 5141*a^4*b^4 + 1722*a^3*b^5 - 245*a
^2*b^6)*d*cos(6*d*x + 6*c) - 4*(1024*a^7*b - 3712*a^6*b^2 + 5304*a^5*b^3 -
3813*a^4*b^4 + 1442*a^3*b^5 - 245*a^2*b^6)*d*cos(4*d*x + 4*c) - 8*(128*a^6*
b^2 - 352*a^5*b^3 + 355*a^4*b^4 - 166*a^3*b^5 + 35*a^2*b^6)*d*cos(2*d*x + 2
*c) + (128*a^6*b^2 - 352*a^5*b^3 + 355*a^4*b^4 - 166*a^3*b^5 + 35*a^2*b^6)*
d)*cos(8*d*x + 8*c) - 16*(4*(128*a^6*b^2 - 424*a^5*b^3 + 513*a^4*b^4 - 266*
a^3*b^5 + 49*a^2*b^6)*d*cos(4*d*x + 4*c) + 8*(16*a^5*b^3 - 39*a^4*b^4 + 30*
a^3*b^5 - 7*a^2*b^6)*d*cos(2*d*x + 2*c) - (16*a^5*b^3 - 39*a^4*b^4 + 30*a^3
*b^5 - 7*a^2*b^6)*d)*cos(6*d*x + 6*c) + 8*(8*(8*a^5*b^3 - 23*a^4*b^4 + 22*a
^3*b^5 - 7*a^2*b^6)*d*cos(2*d*x + 2*c) - (8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b
^5 - 7*a^2*b^6)*d)*cos(4*d*x + 4*c) - 4*(4*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*
d*sin(14*d*x + 14*c) + 2*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*
d*sin(12*d*x + 12*c) - 4*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)
*d*sin(10*d*x + 10*c) - (128*a^6*b^2 - 352*a^5*b^3 + 355*a^4*b^4 - 166*a^3*
b^5 + 35*a^2*b^6)*d*sin(8*d*x + 8*c) - 4*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*
b^5 - 7*a^2*b^6)*d*sin(6*d*x + 6*c) + 2*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^
5 - 7*a^2*b^6)*d*sin(4*d*x + 4*c) + 4*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*sin
(2*d*x + 2*c))*sin(16*d*x + 16*c) + 32*(2*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*
b^5 - 7*a^2*b^6)*d*sin(12*d*x + 12*c) - 4*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3
*b^5 - 7*a^2*b^6)*d*sin(10*d*x + 10*c) - (128*a^6*b^2 - 352*a^5*b^3 + 355*a
^4*b^4 - 166*a^3*b^5 + 35*a^2*b^6)*d*sin(8*d*x + 8*c) - 4*(16*a^5*b^3 - 39*
a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*sin(6*d*x + 6*c) + 2*(8*a^5*b^3 - 23*a^
4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*sin(4*d*x + 4*c) + 4*(a^4*b^4 - 2*a^3*b^5
 + a^2*b^6)*d*sin(2*d*x + 2*c))*sin(14*d*x + 14*c) - 16*(4*(128*a^6*b^2 - 4
24*a^5*b^3 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*sin(10*d*x + 10*c) +
 (1024*a^7*b - 3712*a^6*b^2 + 5304*a^5*b^3 - 3813*a^4*b^4 + 1442*a^3*b^5 -
245*a^2*b^6)*d*sin(8*d*x + 8*c) + 4*(128*a^6*b^2 - 424*a^5*b^3 + 513*a^4*b^
4 - 266*a^3*b^5 + 49*a^2*b^6)*d*sin(6*d*x + 6*c) - 2*(64*a^6*b^2 - 240*a^5*
b^3 + 337*a^4*b^4 - 210*a^3*b^5 + 49*a^2*b^6)*d*sin(4*d*x + 4*c) - 4*(8*a^5

```
*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*sin(2*d*x + 2*c))*sin(12*d*x
+ 12*c) + 32*((2048*a^7*b - 6528*a^6*b^2 + 8144*a^5*b^3 - 5141*a^4*b^4 + 17
22*a^3*b^5 - 245*a^2*b^6)*d*sin(8*d*x + 8*c) + 4*(256*a^6*b^2 - 736*a^5*b^3
 + 753*a^4*b^4 - 322*a^3*b^5 + 49*a^2*b^6)*d*sin(6*d*x + 6*c) - 2*(128*a^6*
b^2 - 424*a^5*b^3 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*sin(4*d*x + 4
*c) - 4*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*sin(2*d*x + 2*
c))*sin(10*d*x + 10*c) + 16*(2*(2048*a^7*b - 6528*a^6*b^2 + 8144*a^5*b^3 -
5141*a^4*b^4 + 1722*a^3*b^5 - 245*a^2*b^6)*d*sin(6*d*x + 6*c) - (1024*a^7*b
 - 3712*a^6*b^2 + 5304*a^5*b^3 - 3813*a^4*b^4 + 1442*a^3*b^5 - 245*a^2*b^6)
*d*sin(4*d*x + 4*c) - 2*(128*a^6*b^2 - 352*a^5*b^3 + 355*a^4*b^4 - 166*a^3*
b^5 + 35*a^2*b^6)*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) - 64*((128*a^6*b^2 -
 424*a^5*b^3 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*sin(4*d*x + 4*c) +
 2*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*sin(2*d*x + 2*c))*s
in(6*d*x + 6*c))
```

Fricas [B] time = 24.3843, size = 15023, normalized size = 43.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a-b*sin(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] -1/256*(((a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d*cos(d*x + c)^8 - 4*(a^4*b^2 - 2*
a^3*b^3 + a^2*b^4)*d*cos(d*x + c)^6 - 2*(a^5*b - 5*a^4*b^2 + 7*a^3*b^3 - 3*
a^2*b^4)*d*cos(d*x + c)^4 + 4*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*c
os(d*x + c)^2 + (a^6 - 4*a^5*b + 6*a^4*b^2 - 4*a^3*b^3 + a^2*b^4)*d)*sqrt(-
(144*a^4 + 76*a^3*b - 155*a^2*b^2 + 94*a*b^3 - 15*b^4 - (a^9*b - 5*a^8*b^2
+ 10*a^7*b^3 - 10*a^6*b^4 + 5*a^5*b^5 - a^4*b^6)*d^2*sqrt((147456*a^8 - 368
640*a^7*b + 498432*a^6*b^2 - 437952*a^5*b^3 + 269641*a^4*b^4 - 117532*a^3*b
^5 + 35406*a^2*b^6 - 6700*a*b^7 + 625*b^8)/((a^19*b - 10*a^18*b^2 + 45*a^17
*b^3 - 120*a^16*b^4 + 210*a^15*b^5 - 252*a^14*b^6 + 210*a^13*b^7 - 120*a^12
*b^8 + 45*a^11*b^9 - 10*a^10*b^10 + a^9*b^11)*d^4)))/((a^9*b - 5*a^8*b^2 +
10*a^7*b^3 - 10*a^6*b^4 + 5*a^5*b^5 - a^4*b^6)*d^2))*log(13824*a^6 - 24576*
a^5*b + 24084*a^4*b^2 - 14455*a^3*b^3 + 22509/4*a^2*b^4 - 2625/2*a*b^5 + 62
5/4*b^6 - 1/4*(55296*a^6 - 98304*a^5*b + 96336*a^4*b^2 - 57820*a^3*b^3 + 22
509*a^2*b^4 - 5250*a*b^5 + 625*b^6)*cos(d*x + c)^2 + 1/2*((22*a^14*b - 125*
a^13*b^2 + 300*a^12*b^3 - 395*a^11*b^4 + 310*a^10*b^5 - 147*a^9*b^6 + 40*a^
8*b^7 - 5*a^7*b^8)*d^3*sqrt((147456*a^8 - 368640*a^7*b + 498432*a^6*b^2 - 4
37952*a^5*b^3 + 269641*a^4*b^4 - 117532*a^3*b^5 + 35406*a^2*b^6 - 6700*a*b^
7 + 625*b^8)/((a^19*b - 10*a^18*b^2 + 45*a^17*b^3 - 120*a^16*b^4 + 210*a^15
*b^5 - 252*a^14*b^6 + 210*a^13*b^7 - 120*a^12*b^8 + 45*a^11*b^9 - 10*a^10*b
^10 + a^9*b^11)*d^4))*cos(d*x + c)*sin(d*x + c) + (4608*a^9 - 6144*a^8*b +
5052*a^7*b^2 - 2437*a^6*b^3 + 783*a^5*b^4 - 159*a^4*b^5 + 25*a^3*b^6)*d*cos
(d*x + c)*sin(d*x + c))*sqrt(-(144*a^4 + 76*a^3*b - 155*a^2*b^2 + 94*a*b^3
- 15*b^4 - (a^9*b - 5*a^8*b^2 + 10*a^7*b^3 - 10*a^6*b^4 + 5*a^5*b^5 - a^4*b
^6)*d^2*sqrt((147456*a^8 - 368640*a^7*b + 498432*a^6*b^2 - 437952*a^5*b^3 +
 269641*a^4*b^4 - 117532*a^3*b^5 + 35406*a^2*b^6 - 6700*a*b^7 + 625*b^8)/((
a^19*b - 10*a^18*b^2 + 45*a^17*b^3 - 120*a^16*b^4 + 210*a^15*b^5 - 252*a^14
*b^6 + 210*a^13*b^7 - 120*a^12*b^8 + 45*a^11*b^9 - 10*a^10*b^10 + a^9*b^11)
*d^4)))/((a^9*b - 5*a^8*b^2 + 10*a^7*b^3 - 10*a^6*b^4 + 5*a^5*b^5 - a^4*b^6
)*d^2)) - 1/4*(2*(144*a^12 - 796*a^11*b + 1845*a^10*b^2 - 2325*a^9*b^3 + 17
30*a^8*b^4 - 774*a^7*b^5 + 201*a^6*b^6 - 25*a^5*b^7)*d^2*cos(d*x + c)^2 - (
144*a^12 - 796*a^11*b + 1845*a^10*b^2 - 2325*a^9*b^3 + 1730*a^8*b^4 - 774*a
^7*b^5 + 201*a^6*b^6 - 25*a^5*b^7)*d^2)*sqrt((147456*a^8 - 368640*a^7*b + 4
98432*a^6*b^2 - 437952*a^5*b^3 + 269641*a^4*b^4 - 117532*a^3*b^5 + 35406*a^
2*b^6 - 6700*a*b^7 + 625*b^8)/((a^19*b - 10*a^18*b^2 + 45*a^17*b^3 - 120*a^
```

16*b^4 + 210*a^15*b^5 - 252*a^14*b^6 + 210*a^13*b^7 - 120*a^12*b^8 + 45*a^1
1*b^9 - 10*a^10*b^10 + a^9*b^11)*d^4))) - ((a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*
d*cos(d*x + c)^8 - 4*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d*cos(d*x + c)^6 - 2*(
a^5*b - 5*a^4*b^2 + 7*a^3*b^3 - 3*a^2*b^4)*d*cos(d*x + c)^4 + 4*(a^5*b - 3*
a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*cos(d*x + c)^2 + (a^6 - 4*a^5*b + 6*a^4*b^
2 - 4*a^3*b^3 + a^2*b^4)*d)*sqrt(-(144*a^4 + 76*a^3*b - 155*a^2*b^2 + 94*a*
b^3 - 15*b^4 - (a^9*b - 5*a^8*b^2 + 10*a^7*b^3 - 10*a^6*b^4 + 5*a^5*b^5 - a
^4*b^6)*d^2*sqrt((147456*a^8 - 368640*a^7*b + 498432*a^6*b^2 - 437952*a^5*b
^3 + 269641*a^4*b^4 - 117532*a^3*b^5 + 35406*a^2*b^6 - 6700*a*b^7 + 625*b^8
)/((a^19*b - 10*a^18*b^2 + 45*a^17*b^3 - 120*a^16*b^4 + 210*a^15*b^5 - 252*
a^14*b^6 + 210*a^13*b^7 - 120*a^12*b^8 + 45*a^11*b^9 - 10*a^10*b^10 + a^9*b
^11)*d^4)))/((a^9*b - 5*a^8*b^2 + 10*a^7*b^3 - 10*a^6*b^4 + 5*a^5*b^5 - a^4
*b^6)*d^2))*log(13824*a^6 - 24576*a^5*b + 24084*a^4*b^2 - 14455*a^3*b^3 + 2
2509/4*a^2*b^4 - 2625/2*a*b^5 + 625/4*b^6 - 1/4*(55296*a^6 - 98304*a^5*b +
96336*a^4*b^2 - 57820*a^3*b^3 + 22509*a^2*b^4 - 5250*a*b^5 + 625*b^6)*cos(d
x + c)^2 - 1/2((22*a^14*b - 125*a^13*b^2 + 300*a^12*b^3 - 395*a^11*b^4 +
310*a^10*b^5 - 147*a^9*b^6 + 40*a^8*b^7 - 5*a^7*b^8)*d^3*sqrt((147456*a^8 -
368640*a^7*b + 498432*a^6*b^2 - 437952*a^5*b^3 + 269641*a^4*b^4 - 117532*a
^3*b^5 + 35406*a^2*b^6 - 6700*a*b^7 + 625*b^8)/((a^19*b - 10*a^18*b^2 + 45*
a^17*b^3 - 120*a^16*b^4 + 210*a^15*b^5 - 252*a^14*b^6 + 210*a^13*b^7 - 120*
a^12*b^8 + 45*a^11*b^9 - 10*a^10*b^10 + a^9*b^11)*d^4))*cos(d*x + c)*sin(d*
x + c) + (4608*a^9 - 6144*a^8*b + 5052*a^7*b^2 - 2437*a^6*b^3 + 783*a^5*b^4
- 159*a^4*b^5 + 25*a^3*b^6)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-(144*a^4 +
76*a^3*b - 155*a^2*b^2 + 94*a*b^3 - 15*b^4 - (a^9*b - 5*a^8*b^2 + 10*a^7*b^
3 - 10*a^6*b^4 + 5*a^5*b^5 - a^4*b^6)*d^2*sqrt((147456*a^8 - 368640*a^7*b +
498432*a^6*b^2 - 437952*a^5*b^3 + 269641*a^4*b^4 - 117532*a^3*b^5 + 35406*
a^2*b^6 - 6700*a*b^7 + 625*b^8)/((a^19*b - 10*a^18*b^2 + 45*a^17*b^3 - 120*
a^16*b^4 + 210*a^15*b^5 - 252*a^14*b^6 + 210*a^13*b^7 - 120*a^12*b^8 + 45*a
^11*b^9 - 10*a^10*b^10 + a^9*b^11)*d^4)))/((a^9*b - 5*a^8*b^2 + 10*a^7*b^3
- 10*a^6*b^4 + 5*a^5*b^5 - a^4*b^6)*d^2)) - 1/4*(2*(144*a^12 - 796*a^11*b +
1845*a^10*b^2 - 2325*a^9*b^3 + 1730*a^8*b^4 - 774*a^7*b^5 + 201*a^6*b^6 -
25*a^5*b^7)*d^2*cos(d*x + c)^2 - (144*a^12 - 796*a^11*b + 1845*a^10*b^2 - 2
325*a^9*b^3 + 1730*a^8*b^4 - 774*a^7*b^5 + 201*a^6*b^6 - 25*a^5*b^7)*d^2)*s
qrt((147456*a^8 - 368640*a^7*b + 498432*a^6*b^2 - 437952*a^5*b^3 + 269641*a
^4*b^4 - 117532*a^3*b^5 + 35406*a^2*b^6 - 6700*a*b^7 + 625*b^8)/((a^19*b -
10*a^18*b^2 + 45*a^17*b^3 - 120*a^16*b^4 + 210*a^15*b^5 - 252*a^14*b^6 + 21
0*a^13*b^7 - 120*a^12*b^8 + 45*a^11*b^9 - 10*a^10*b^10 + a^9*b^11)*d^4))) +
((a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d*cos(d*x + c)^8 - 4*(a^4*b^2 - 2*a^3*b^3
+ a^2*b^4)*d*cos(d*x + c)^6 - 2*(a^5*b - 5*a^4*b^2 + 7*a^3*b^3 - 3*a^2*b^4
)*d*cos(d*x + c)^4 + 4*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*cos(d*x
+ c)^2 + (a^6 - 4*a^5*b + 6*a^4*b^2 - 4*a^3*b^3 + a^2*b^4)*d)*sqrt(-(144*a^
4 + 76*a^3*b - 155*a^2*b^2 + 94*a*b^3 - 15*b^4 + (a^9*b - 5*a^8*b^2 + 10*a^
7*b^3 - 10*a^6*b^4 + 5*a^5*b^5 - a^4*b^6)*d^2*sqrt((147456*a^8 - 368640*a^7
*b + 498432*a^6*b^2 - 437952*a^5*b^3 + 269641*a^4*b^4 - 117532*a^3*b^5 + 35
406*a^2*b^6 - 6700*a*b^7 + 625*b^8)/((a^19*b - 10*a^18*b^2 + 45*a^17*b^3 -
120*a^16*b^4 + 210*a^15*b^5 - 252*a^14*b^6 + 210*a^13*b^7 - 120*a^12*b^8 +
45*a^11*b^9 - 10*a^10*b^10 + a^9*b^11)*d^4)))/((a^9*b - 5*a^8*b^2 + 10*a^7*
b^3 - 10*a^6*b^4 + 5*a^5*b^5 - a^4*b^6)*d^2))*log(-13824*a^6 + 24576*a^5*b
- 24084*a^4*b^2 + 14455*a^3*b^3 - 22509/4*a^2*b^4 + 2625/2*a*b^5 - 625/4*b^
6 + 1/4*(55296*a^6 - 98304*a^5*b + 96336*a^4*b^2 - 57820*a^3*b^3 + 22509*a^
2*b^4 - 5250*a*b^5 + 625*b^6)*cos(d*x + c)^2 + 1/2*((22*a^14*b - 125*a^13*b
^2 + 300*a^12*b^3 - 395*a^11*b^4 + 310*a^10*b^5 - 147*a^9*b^6 + 40*a^8*b^7
- 5*a^7*b^8)*d^3*sqrt((147456*a^8 - 368640*a^7*b + 498432*a^6*b^2 - 437952*
a^5*b^3 + 269641*a^4*b^4 - 117532*a^3*b^5 + 35406*a^2*b^6 - 6700*a*b^7 + 62
5*b^8)/((a^19*b - 10*a^18*b^2 + 45*a^17*b^3 - 120*a^16*b^4 + 210*a^15*b^5 -
252*a^14*b^6 + 210*a^13*b^7 - 120*a^12*b^8 + 45*a^11*b^9 - 10*a^10*b^10 +
a^9*b^11)*d^4))*cos(d*x + c)*sin(d*x + c) - (4608*a^9 - 6144*a^8*b + 5052*a
^7*b^2 - 2437*a^6*b^3 + 783*a^5*b^4 - 159*a^4*b^5 + 25*a^3*b^6)*d*cos(d*x +
c)*sin(d*x + c))*sqrt(-(144*a^4 + 76*a^3*b - 155*a^2*b^2 + 94*a*b^3 - 15*b

```
^4 + (a^9*b - 5*a^8*b^2 + 10*a^7*b^3 - 10*a^6*b^4 + 5*a^5*b^5 - a^4*b^6)*d^
2*sqrt((147456*a^8 - 368640*a^7*b + 498432*a^6*b^2 - 437952*a^5*b^3 + 26964
1*a^4*b^4 - 117532*a^3*b^5 + 35406*a^2*b^6 - 6700*a*b^7 + 625*b^8)/((a^19*b
 - 10*a^18*b^2 + 45*a^17*b^3 - 120*a^16*b^4 + 210*a^15*b^5 - 252*a^14*b^6 +
 210*a^13*b^7 - 120*a^12*b^8 + 45*a^11*b^9 - 10*a^10*b^10 + a^9*b^11)*d^4))
)/((a^9*b - 5*a^8*b^2 + 10*a^7*b^3 - 10*a^6*b^4 + 5*a^5*b^5 - a^4*b^6)*d^2)
) - 1/4*(2*(144*a^12 - 796*a^11*b + 1845*a^10*b^2 - 2325*a^9*b^3 + 1730*a^8
*b^4 - 774*a^7*b^5 + 201*a^6*b^6 - 25*a^5*b^7)*d^2*cos(d*x + c)^2 - (144*a^
12 - 796*a^11*b + 1845*a^10*b^2 - 2325*a^9*b^3 + 1730*a^8*b^4 - 774*a^7*b^5
 + 201*a^6*b^6 - 25*a^5*b^7)*d^2)*sqrt((147456*a^8 - 368640*a^7*b + 498432*
a^6*b^2 - 437952*a^5*b^3 + 269641*a^4*b^4 - 117532*a^3*b^5 + 35406*a^2*b^6
- 6700*a*b^7 + 625*b^8)/((a^19*b - 10*a^18*b^2 + 45*a^17*b^3 - 120*a^16*b^4
 + 210*a^15*b^5 - 252*a^14*b^6 + 210*a^13*b^7 - 120*a^12*b^8 + 45*a^11*b^9
- 10*a^10*b^10 + a^9*b^11)*d^4))) - ((a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d*cos(
d*x + c)^8 - 4*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d*cos(d*x + c)^6 - 2*(a^5*b
- 5*a^4*b^2 + 7*a^3*b^3 - 3*a^2*b^4)*d*cos(d*x + c)^4 + 4*(a^5*b - 3*a^4*b^
2 + 3*a^3*b^3 - a^2*b^4)*d*cos(d*x + c)^2 + (a^6 - 4*a^5*b + 6*a^4*b^2 - 4*
a^3*b^3 + a^2*b^4)*d)*sqrt(-(144*a^4 + 76*a^3*b - 155*a^2*b^2 + 94*a*b^3 -
15*b^4 + (a^9*b - 5*a^8*b^2 + 10*a^7*b^3 - 10*a^6*b^4 + 5*a^5*b^5 - a^4*b^6
)*d^2*sqrt((147456*a^8 - 368640*a^7*b + 498432*a^6*b^2 - 437952*a^5*b^3 + 2
69641*a^4*b^4 - 117532*a^3*b^5 + 35406*a^2*b^6 - 6700*a*b^7 + 625*b^8)/((a^
19*b - 10*a^18*b^2 + 45*a^17*b^3 - 120*a^16*b^4 + 210*a^15*b^5 - 252*a^14*b
^6 + 210*a^13*b^7 - 120*a^12*b^8 + 45*a^11*b^9 - 10*a^10*b^10 + a^9*b^11)*d
^4)))/((a^9*b - 5*a^8*b^2 + 10*a^7*b^3 - 10*a^6*b^4 + 5*a^5*b^5 - a^4*b^6)*
d^2))*log(-13824*a^6 + 24576*a^5*b - 24084*a^4*b^2 + 14455*a^3*b^3 - 22509/
4*a^2*b^4 + 2625/2*a*b^5 - 625/4*b^6 + 1/4*(55296*a^6 - 98304*a^5*b + 96336
*a^4*b^2 - 57820*a^3*b^3 + 22509*a^2*b^4 - 5250*a*b^5 + 625*b^6)*cos(d*x +
c)^2 - 1/2*((22*a^14*b - 125*a^13*b^2 + 300*a^12*b^3 - 395*a^11*b^4 + 310*a
^10*b^5 - 147*a^9*b^6 + 40*a^8*b^7 - 5*a^7*b^8)*d^3*sqrt((147456*a^8 - 3686
40*a^7*b + 498432*a^6*b^2 - 437952*a^5*b^3 + 269641*a^4*b^4 - 117532*a^3*b^
5 + 35406*a^2*b^6 - 6700*a*b^7 + 625*b^8)/((a^19*b - 10*a^18*b^2 + 45*a^17*
b^3 - 120*a^16*b^4 + 210*a^15*b^5 - 252*a^14*b^6 + 210*a^13*b^7 - 120*a^12*
b^8 + 45*a^11*b^9 - 10*a^10*b^10 + a^9*b^11)*d^4))*cos(d*x + c)*sin(d*x + c
) - (4608*a^9 - 6144*a^8*b + 5052*a^7*b^2 - 2437*a^6*b^3 + 783*a^5*b^4 - 15
9*a^4*b^5 + 25*a^3*b^6)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-(144*a^4 + 76*a^
3*b - 155*a^2*b^2 + 94*a*b^3 - 15*b^4 + (a^9*b - 5*a^8*b^2 + 10*a^7*b^3 - 1
0*a^6*b^4 + 5*a^5*b^5 - a^4*b^6)*d^2*sqrt((147456*a^8 - 368640*a^7*b + 4984
32*a^6*b^2 - 437952*a^5*b^3 + 269641*a^4*b^4 - 117532*a^3*b^5 + 35406*a^2*b
^6 - 6700*a*b^7 + 625*b^8)/((a^19*b - 10*a^18*b^2 + 45*a^17*b^3 - 120*a^16*
b^4 + 210*a^15*b^5 - 252*a^14*b^6 + 210*a^13*b^7 - 120*a^12*b^8 + 45*a^11*b
^9 - 10*a^10*b^10 + a^9*b^11)*d^4)))/((a^9*b - 5*a^8*b^2 + 10*a^7*b^3 - 10*
a^6*b^4 + 5*a^5*b^5 - a^4*b^6)*d^2)) - 1/4*(2*(144*a^12 - 796*a^11*b + 1845
*a^10*b^2 - 2325*a^9*b^3 + 1730*a^8*b^4 - 774*a^7*b^5 + 201*a^6*b^6 - 25*a^
5*b^7)*d^2*cos(d*x + c)^2 - (144*a^12 - 796*a^11*b + 1845*a^10*b^2 - 2325*a
^9*b^3 + 1730*a^8*b^4 - 774*a^7*b^5 + 201*a^6*b^6 - 25*a^5*b^7)*d^2)*sqrt((
147456*a^8 - 368640*a^7*b + 498432*a^6*b^2 - 437952*a^5*b^3 + 269641*a^4*b^
4 - 117532*a^3*b^5 + 35406*a^2*b^6 - 6700*a*b^7 + 625*b^8)/((a^19*b - 10*a^
18*b^2 + 45*a^17*b^3 - 120*a^16*b^4 + 210*a^15*b^5 - 252*a^14*b^6 + 210*a^1
3*b^7 - 120*a^12*b^8 + 45*a^11*b^9 - 10*a^10*b^10 + a^9*b^11)*d^4))) + 8*((
11*a*b^2 - 5*b^3)*cos(d*x + c)^7 - 3*(2*a^2*b + 11*a*b^2 - 5*b^3)*cos(d*x +
 c)^5 - 3*(a^2*b - 14*a*b^2 + 5*b^3)*cos(d*x + c)^3 + 5*(2*a^3 + a^2*b - 4*
a*b^2 + b^3)*cos(d*x + c))*sin(d*x + c))/((a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d
*cos(d*x + c)^8 - 4*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d*cos(d*x + c)^6 - 2*(a
^5*b - 5*a^4*b^2 + 7*a^3*b^3 - 3*a^2*b^4)*d*cos(d*x + c)^4 + 4*(a^5*b - 3*a
^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*cos(d*x + c)^2 + (a^6 - 4*a^5*b + 6*a^4*b^2
 - 4*a^3*b^3 + a^2*b^4)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a-b*sin(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a-b*sin(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.234 $\int \frac{1}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=319

$$-\frac{b\tan(c+dx)\left(\frac{17a^2-40ab+7b^2}{(a-b)^3}+\frac{(33a-13b)\tan^2(c+dx)}{(a-b)^2}\right)}{32a^2d\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}+\frac{\left(-50\sqrt{a}\sqrt{b}+32a+21b\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{11/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{\left(50\sqrt{a}\sqrt{b}+\right.}{}$$

```
[Out] ((32*a - 50*Sqrt[a]*Sqrt[b] + 21*b)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c +
 d*x])/a^(1/4)])/(64*a^(11/4)*(Sqrt[a] - Sqrt[b])^(5/2)*d) + ((32*a + 50*Sq
rt[a]*Sqrt[b] + 21*b)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)
])/(64*a^(11/4)*(Sqrt[a] + Sqrt[b])^(5/2)*d) - (b^2*Tan[c + d*x]*(3*a + b +
 4*(a + b)*Tan[c + d*x]^2))/(8*a*(a - b)^3*d*(a + 2*a*Tan[c + d*x]^2 + (a -
 b)*Tan[c + d*x]^4)^2) - (b*Tan[c + d*x]*((17*a^2 - 40*a*b + 7*b^2)/(a - b)
^3 + ((33*a - 13*b)*Tan[c + d*x]^2)/(a - b)^2))/(32*a^2*d*(a + 2*a*Tan[c +
d*x]^2 + (a - b)*Tan[c + d*x]^4))
```

Rubi [A] time = 0.651839, antiderivative size = 319, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3209, 1205, 1678, 1166, 205}

$$-\frac{b\tan(c+dx)\left(\frac{17a^2-40ab+7b^2}{(a-b)^3}+\frac{(33a-13b)\tan^2(c+dx)}{(a-b)^2}\right)}{32a^2d\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}+\frac{\left(-50\sqrt{a}\sqrt{b}+32a+21b\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{11/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{\left(50\sqrt{a}\sqrt{b}+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*Sin[c + d*x]^4)^(-3),x]
```


```
[Out] ((32*a - 50*Sqrt[a]*Sqrt[b] + 21*b)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c +
 d*x])/a^(1/4)])/(64*a^(11/4)*(Sqrt[a] - Sqrt[b])^(5/2)*d) + ((32*a + 50*Sq
rt[a]*Sqrt[b] + 21*b)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)
])/(64*a^(11/4)*(Sqrt[a] + Sqrt[b])^(5/2)*d) - (b^2*Tan[c + d*x]*(3*a + b +
 4*(a + b)*Tan[c + d*x]^2))/(8*a*(a - b)^3*d*(a + 2*a*Tan[c + d*x]^2 + (a -
 b)*Tan[c + d*x]^4)^2) - (b*Tan[c + d*x]*((17*a^2 - 40*a*b + 7*b^2)/(a - b)
^3 + ((33*a - 13*b)*Tan[c + d*x]^2)/(a - b)^2))/(32*a^2*d*(a + 2*a*Tan[c +
d*x]^2 + (a - b)*Tan[c + d*x]^4))
```


Rule 3209

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
```

```
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a-b\sin^4(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^5}{\left(a+2ax^2+(a-b)x^4\right)^3}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{b^2\tan(c+dx)\left(3a+b+4(a+b)\tan^2(c+dx)\right)}{8a(a-b)^3d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-\frac{2ab\left(8a^3-24a^2b+27ab^2 \ldots}{(a-b)^3} \ldots}{\ldots}\right)}{\ldots}$$

$$= -\frac{b^2\tan(c+dx)\left(3a+b+4(a+b)\tan^2(c+dx)\right)}{8a(a-b)^3d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{b\tan(c+dx)\left(\frac{17a^2-40ab+7 \ldots}{(a-b)^3} \ldots\right)}{32a^2d\left(a+2a\tan^2(c+dx \ldots\right)}$$

$$= -\frac{b^2\tan(c+dx)\left(3a+b+4(a+b)\tan^2(c+dx)\right)}{8a(a-b)^3d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{b\tan(c+dx)\left(\frac{17a^2-40ab+7 \ldots}{(a-b)^3} \ldots\right)}{32a^2d\left(a+2a\tan^2(c+dx \ldots\right)}$$

$$= \frac{\left(32a-50\sqrt{a}\sqrt{b}+21b\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{11/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}d} + \frac{\left(32a+50\sqrt{a}\sqrt{b}+21b\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+ \ldots}}{\ldots}\right)}{64a^{11/4}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}d}$$

Mathematica [A] time = 3.12397, size = 333, normalized size = 1.04

$$\frac{\frac{64a^{3/2}b(a-b)(\sin(4(c+dx))-6\sin(2(c+dx)))}{(-8a-4b\cos(2(c+dx))+b\cos(4(c+dx))+3b)^2}+\frac{\left(50\sqrt{a}\sqrt{b}+32a+21b\right)\left(\sqrt{a}-\sqrt{b}\right)^2\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}+a}}+\frac{8\sqrt{a}b\sin(2(c+dx))((6a-3b)\cos(2(c+dx))-19a+10}{8a+4b\cos(2(c+dx))-b\cos(4(c+dx))-3b}}{64a^{5/2}d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*Sin[c + d*x]^4)^(-3),x]
```

```
[Out] (((Sqrt[a] - Sqrt[b])^2*(32*a + 50*Sqrt[a]*Sqrt[b] + 21*b)*ArcTan[((Sqrt[a]
 + Sqrt[b])*Tan[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt
[b]] - ((Sqrt[a] + Sqrt[b])^2*(32*a - 50*Sqrt[a]*Sqrt[b] + 21*b)*ArcTanh[((
Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a + Sqr
t[a]*Sqrt[b]] + (8*Sqrt[a]*b*(-19*a + 10*b + (6*a - 3*b)*Cos[2*(c + d*x)])*
Sin[2*(c + d*x)])/(8*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*x)]) +
 (64*a^(3/2)*(a - b)*b*(-6*Sin[2*(c + d*x)] + Sin[4*(c + d*x)]))/(-8*a + 3*
b - 4*b*Cos[2*(c + d*x)] + b*Cos[4*(c + d*x)])^2)/(64*a^(5/2)*(a - b)^2*d)
```

Maple [B] time = 0.148, size = 1803, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-b*sin(d*x+c)^4)^3,x)
```

```
[Out] 23/32/d*b/(a^2-2*a*b+b^2)*a/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)
*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-23/32/d*b/(a^2-2*a*
b+b^2)*a/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan
(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-67/32/d/(tan(d*x+c)^4*a-tan(d*x+c)^4
*b+2*a*tan(d*x+c)^2+a)^2*b/(a^2-2*a*b+b^2)*tan(d*x+c)^3-83/32/d/(tan(d*x+c)
^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2*b/(a^2-2*a*b+b^2)*tan(d*x+c)^5+13
/32/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2/a^2/(a-b)*tan(d*
x+c)^7*b^2+43/32/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2/a/(
a^2-2*a*b+b^2)*tan(d*x+c)^3*b^2+19/16/d/a/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(a-b)
/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-
b))^(1/2))*b^3-19/16/d/a/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)
*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*b^3-
7/32/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2/a^2*b^3/(a^2-2*
a*b+b^2)*tan(d*x+c)^5-33/32/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)
^2+a)^2/(a-b)/a*b*tan(d*x+c)^7+1/2/d/(a^2-2*a*b+b^2)*a/(a-b)/(((a*b)^(1/2)+
a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-21/6
4/d/a^2/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arc
tan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*b^4+21/64/d/a^2/(a^2-2*
a*b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan
(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*b^4+1/2/d/(a^2-2*a*b+b^2)*a/(a-b)/((
(a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b
))^(1/2))+23/32/d/a*b^2/(a^2-2*a*b+b^2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)
*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+23/32/d/a*b^2/(a^2-
2*a*b+b^2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/((
(a*b)^(1/2)-a)*(a-b))^(1/2))-65/64/d*b/(a^2-2*a*b+b^2)/(a-b)/(((a*b)^(1/2)+
a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-65/6
4/d*b/(a^2-2*a*b+b^2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*ta
n(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-101/64/d*b^2/(a^2-2*a*b+b^2)/(a*b)^
```

```
(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(
1/2)+a)*(a-b))^(1/2))+101/64/d*b^2/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b
)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(
1/2))-13/64/d/a^2/(a^2-2*a*b+b^2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arcta
n((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*b^3-13/64/d/a^2/(a^2-2*a*
b+b^2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b
)^(1/2)-a)*(a-b))^(1/2))*b^3+33/16/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan
(d*x+c)^2+a)^2/a/(a^2-2*a*b+b^2)*tan(d*x+c)^5*b^2-17/32/d/(tan(d*x+c)^4*a-t
an(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2/(a^2-2*a*b+b^2)*tan(d*x+c)*b+11/32/d/(t
an(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2*b^2/a/(a^2-2*a*b+b^2)*ta
n(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(4*(120*a^2*b^3 - 77*a*b^4 + 14*b^5)*cos(4*d*x + 4*c)*sin(2*d*x + 2*c)
+ ((7*a*b^4 - 4*b^5)*sin(14*d*x + 14*c) - (32*a^2*b^3 + 2*a*b^4 - 7*b^5)*si
n(12*d*x + 12*c) - (16*a^2*b^3 - 3*a*b^4 - 28*b^5)*sin(10*d*x + 10*c) + 3*(
256*a^3*b^2 - 320*a^2*b^3 + 166*a*b^4 - 35*b^5)*sin(8*d*x + 8*c) + (784*a^2
*b^3 - 723*a*b^4 + 140*b^5)*sin(6*d*x + 6*c) - (160*a^2*b^3 - 266*a*b^4 + 9
1*b^5)*sin(4*d*x + 4*c) - (55*a*b^4 - 28*b^5)*sin(2*d*x + 2*c))*cos(16*d*x
+ 16*c) + 2*(2*(120*a^2*b^3 - 77*a*b^4 + 14*b^5)*sin(12*d*x + 12*c) - 8*(48
*a^2*b^3 - 55*a*b^4 + 28*b^5)*sin(10*d*x + 10*c) - (3968*a^3*b^2 - 5024*a^2
*b^3 + 2621*a*b^4 - 560*b^5)*sin(8*d*x + 8*c) - 16*(224*a^2*b^3 - 209*a*b^4
 + 42*b^5)*sin(6*d*x + 6*c) + 2*(376*a^2*b^3 - 613*a*b^4 + 210*b^5)*sin(4*d
*x + 4*c) + 8*(31*a*b^4 - 16*b^5)*sin(2*d*x + 2*c))*cos(14*d*x + 14*c) + 2*
(2*(1152*a^3*b^2 - 520*a^2*b^3 - 455*a*b^4 + 294*b^5)*sin(10*d*x + 10*c) -
(8192*a^4*b - 23296*a^3*b^2 + 21376*a^2*b^3 - 9394*a*b^4 + 1715*b^5)*sin(8*
d*x + 8*c) - 2*(5248*a^3*b^2 - 10888*a^2*b^3 + 6433*a*b^4 - 1078*b^5)*sin(6
*d*x + 6*c) + 4*(512*a^3*b^2 - 1520*a^2*b^3 + 1330*a*b^4 - 343*b^5)*sin(4*d
*x + 4*c) + 2*(376*a^2*b^3 - 613*a*b^4 + 210*b^5)*sin(2*d*x + 2*c))*cos(12*
d*x + 12*c) + 2*((51200*a^4*b - 84864*a^3*b^2 + 56016*a^2*b^3 - 18081*a*b^4
 + 1960*b^5)*sin(8*d*x + 8*c) + 8*(6400*a^3*b^2 - 8608*a^2*b^3 + 3437*a*b^4
 - 392*b^5)*sin(6*d*x + 6*c) - 2*(5248*a^3*b^2 - 10888*a^2*b^3 + 6433*a*b^4
 - 1078*b^5)*sin(4*d*x + 4*c) - 16*(224*a^2*b^3 - 209*a*b^4 + 42*b^5)*sin(2
*d*x + 2*c))*cos(10*d*x + 10*c) + 2*((51200*a^4*b - 84864*a^3*b^2 + 56016*a
^2*b^3 - 18081*a*b^4 + 1960*b^5)*sin(6*d*x + 6*c) - (8192*a^4*b - 23296*a^3
*b^2 + 21376*a^2*b^3 - 9394*a*b^4 + 1715*b^5)*sin(4*d*x + 4*c) - (3968*a^3*
b^2 - 5024*a^2*b^3 + 2621*a*b^4 - 560*b^5)*sin(2*d*x + 2*c))*cos(8*d*x + 8*
c) + 4*((1152*a^3*b^2 - 520*a^2*b^3 - 455*a*b^4 + 294*b^5)*sin(4*d*x + 4*c)
 - 4*(48*a^2*b^3 - 55*a*b^4 + 28*b^5)*sin(2*d*x + 2*c))*cos(6*d*x + 6*c) -
8*((a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*cos(16*d*x + 16*c)^2 + 64*(a^4*b^4 - 2
*a^3*b^5 + a^2*b^6)*d*cos(14*d*x + 14*c)^2 + 16*(64*a^6*b^2 - 240*a^5*b^3 +
 337*a^4*b^4 - 210*a^3*b^5 + 49*a^2*b^6)*d*cos(12*d*x + 12*c)^2 + 64*(256*a
^6*b^2 - 736*a^5*b^3 + 753*a^4*b^4 - 322*a^3*b^5 + 49*a^2*b^6)*d*cos(10*d*x
 + 10*c)^2 + 4*(16384*a^8 - 57344*a^7*b + 83712*a^6*b^2 - 67648*a^5*b^3 + 3
2841*a^4*b^4 - 9170*a^3*b^5 + 1225*a^2*b^6)*d*cos(8*d*x + 8*c)^2 + 64*(256*
a^6*b^2 - 736*a^5*b^3 + 753*a^4*b^4 - 322*a^3*b^5 + 49*a^2*b^6)*d*cos(6*d*x
 + 6*c)^2 + 16*(64*a^6*b^2 - 240*a^5*b^3 + 337*a^4*b^4 - 210*a^3*b^5 + 49*a
^2*b^6)*d*cos(4*d*x + 4*c)^2 + 64*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*cos(2*d
*x + 2*c)^2 + (a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*sin(16*d*x + 16*c)^2 + 64*(
a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*sin(14*d*x + 14*c)^2 + 16*(64*a^6*b^2 - 24
```

0*a^5*b^3 + 337*a^4*b^4 - 210*a^3*b^5 + 49*a^2*b^6)*d*sin(12*d*x + 12*c)^2
+ 64*(256*a^6*b^2 - 736*a^5*b^3 + 753*a^4*b^4 - 322*a^3*b^5 + 49*a^2*b^6)*d
*sin(10*d*x + 10*c)^2 + 4*(16384*a^8 - 57344*a^7*b + 83712*a^6*b^2 - 67648*
a^5*b^3 + 32841*a^4*b^4 - 9170*a^3*b^5 + 1225*a^2*b^6)*d*sin(8*d*x + 8*c)^2
 + 64*(256*a^6*b^2 - 736*a^5*b^3 + 753*a^4*b^4 - 322*a^3*b^5 + 49*a^2*b^6)*
d*sin(6*d*x + 6*c)^2 + 16*(64*a^6*b^2 - 240*a^5*b^3 + 337*a^4*b^4 - 210*a^3
*b^5 + 49*a^2*b^6)*d*sin(4*d*x + 4*c)^2 + 64*(8*a^5*b^3 - 23*a^4*b^4 + 22*a
^3*b^5 - 7*a^2*b^6)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 64*(a^4*b^4 - 2*a
^3*b^5 + a^2*b^6)*d*sin(2*d*x + 2*c)^2 - 16*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)
*d*cos(2*d*x + 2*c) + (a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d - 2*(8*(a^4*b^4 - 2
*a^3*b^5 + a^2*b^6)*d*cos(14*d*x + 14*c) + 4*(8*a^5*b^3 - 23*a^4*b^4 + 22*a
^3*b^5 - 7*a^2*b^6)*d*cos(12*d*x + 12*c) - 8*(16*a^5*b^3 - 39*a^4*b^4 + 30*
a^3*b^5 - 7*a^2*b^6)*d*cos(10*d*x + 10*c) - 2*(128*a^6*b^2 - 352*a^5*b^3 +
355*a^4*b^4 - 166*a^3*b^5 + 35*a^2*b^6)*d*cos(8*d*x + 8*c) - 8*(16*a^5*b^3
 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*cos(6*d*x + 6*c) + 4*(8*a^5*b^3 -
23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*cos(4*d*x + 4*c) + 8*(a^4*b^4 - 2*a^
3*b^5 + a^2*b^6)*d*cos(2*d*x + 2*c) - (a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d)*co
s(16*d*x + 16*c) + 16*(4*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*
d*cos(12*d*x + 12*c) - 8*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)
*d*cos(10*d*x + 10*c) - 2*(128*a^6*b^2 - 352*a^5*b^3 + 355*a^4*b^4 - 166*a^
3*b^5 + 35*a^2*b^6)*d*cos(8*d*x + 8*c) - 8*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^
3*b^5 - 7*a^2*b^6)*d*cos(6*d*x + 6*c) + 4*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*
b^5 - 7*a^2*b^6)*d*cos(4*d*x + 4*c) + 8*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*c
os(2*d*x + 2*c) - (a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d)*cos(14*d*x + 14*c) - 8
(8(128*a^6*b^2 - 424*a^5*b^3 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*
cos(10*d*x + 10*c) + 2*(1024*a^7*b - 3712*a^6*b^2 + 5304*a^5*b^3 - 3813*a^4
*b^4 + 1442*a^3*b^5 - 245*a^2*b^6)*d*cos(8*d*x + 8*c) + 8*(128*a^6*b^2 - 42
4*a^5*b^3 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*cos(6*d*x + 6*c) - 4*
(64*a^6*b^2 - 240*a^5*b^3 + 337*a^4*b^4 - 210*a^3*b^5 + 49*a^2*b^6)*d*cos(4
*d*x + 4*c) - 8*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*cos(2*d
*x + 2*c) + (8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d)*cos(12*d*x
 + 12*c) + 16*(2*(2048*a^7*b - 6528*a^6*b^2 + 8144*a^5*b^3 - 5141*a^4*b^4 +
 1722*a^3*b^5 - 245*a^2*b^6)*d*cos(8*d*x + 8*c) + 8*(256*a^6*b^2 - 736*a^5*
b^3 + 753*a^4*b^4 - 322*a^3*b^5 + 49*a^2*b^6)*d*cos(6*d*x + 6*c) - 4*(128*a
^6*b^2 - 424*a^5*b^3 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*cos(4*d*x
+ 4*c) - 8*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*cos(2*d*x +
 2*c) + (16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d)*cos(10*d*x +
10*c) + 4*(8*(2048*a^7*b - 6528*a^6*b^2 + 8144*a^5*b^3 - 5141*a^4*b^4 + 172
2*a^3*b^5 - 245*a^2*b^6)*d*cos(6*d*x + 6*c) - 4*(1024*a^7*b - 3712*a^6*b^2
+ 5304*a^5*b^3 - 3813*a^4*b^4 + 1442*a^3*b^5 - 245*a^2*b^6)*d*cos(4*d*x + 4
c) - 8(128*a^6*b^2 - 352*a^5*b^3 + 355*a^4*b^4 - 166*a^3*b^5 + 35*a^2*b^6
)*d*cos(2*d*x + 2*c) + (128*a^6*b^2 - 352*a^5*b^3 + 355*a^4*b^4 - 166*a^3*b
^5 + 35*a^2*b^6)*d)*cos(8*d*x + 8*c) - 16*(4*(128*a^6*b^2 - 424*a^5*b^3 + 5
13*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*cos(4*d*x + 4*c) + 8*(16*a^5*b^3 -
 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*cos(2*d*x + 2*c) - (16*a^5*b^3 - 39
*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d)*cos(6*d*x + 6*c) + 8*(8*(8*a^5*b^3 -
23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*cos(2*d*x + 2*c) - (8*a^5*b^3 - 23*a
^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d)*cos(4*d*x + 4*c) - 4*(4*(a^4*b^4 - 2*a^
3*b^5 + a^2*b^6)*d*sin(14*d*x + 14*c) + 2*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*
b^5 - 7*a^2*b^6)*d*sin(12*d*x + 12*c) - 4*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3
*b^5 - 7*a^2*b^6)*d*sin(10*d*x + 10*c) - (128*a^6*b^2 - 352*a^5*b^3 + 355*a
^4*b^4 - 166*a^3*b^5 + 35*a^2*b^6)*d*sin(8*d*x + 8*c) - 4*(16*a^5*b^3 - 39*
a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*sin(6*d*x + 6*c) + 2*(8*a^5*b^3 - 23*a^
4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*sin(4*d*x + 4*c) + 4*(a^4*b^4 - 2*a^3*b^5
 + a^2*b^6)*d*sin(2*d*x + 2*c))*sin(16*d*x + 16*c) + 32*(2*(8*a^5*b^3 - 23*
a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*sin(12*d*x + 12*c) - 4*(16*a^5*b^3 - 39
*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*sin(10*d*x + 10*c) - (128*a^6*b^2 - 35
2*a^5*b^3 + 355*a^4*b^4 - 166*a^3*b^5 + 35*a^2*b^6)*d*sin(8*d*x + 8*c) - 4*
(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*sin(6*d*x + 6*c) + 2*(

8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*sin(4*d*x + 4*c) + 4*(a^
4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*sin(2*d*x + 2*c))*sin(14*d*x + 14*c) - 16*(4
*(128*a^6*b^2 - 424*a^5*b^3 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*sin
(10*d*x + 10*c) + (1024*a^7*b - 3712*a^6*b^2 + 5304*a^5*b^3 - 3813*a^4*b^4
+ 1442*a^3*b^5 - 245*a^2*b^6)*d*sin(8*d*x + 8*c) + 4*(128*a^6*b^2 - 424*a^5
*b^3 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*sin(6*d*x + 6*c) - 2*(64*a
^6*b^2 - 240*a^5*b^3 + 337*a^4*b^4 - 210*a^3*b^5 + 49*a^2*b^6)*d*sin(4*d*x
+ 4*c) - 4*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*sin(2*d*x +
2*c))*sin(12*d*x + 12*c) + 32*((2048*a^7*b - 6528*a^6*b^2 + 8144*a^5*b^3 -
5141*a^4*b^4 + 1722*a^3*b^5 - 245*a^2*b^6)*d*sin(8*d*x + 8*c) + 4*(256*a^6*
b^2 - 736*a^5*b^3 + 753*a^4*b^4 - 322*a^3*b^5 + 49*a^2*b^6)*d*sin(6*d*x + 6
c) - 2(128*a^6*b^2 - 424*a^5*b^3 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6
)*d*sin(4*d*x + 4*c) - 4*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)
*d*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 16*(2*(2048*a^7*b - 6528*a^6*b^2
+ 8144*a^5*b^3 - 5141*a^4*b^4 + 1722*a^3*b^5 - 245*a^2*b^6)*d*sin(6*d*x + 6
*c) - (1024*a^7*b - 3712*a^6*b^2 + 5304*a^5*b^3 - 3813*a^4*b^4 + 1442*a^3*b
^5 - 245*a^2*b^6)*d*sin(4*d*x + 4*c) - 2*(128*a^6*b^2 - 352*a^5*b^3 + 355*a
^4*b^4 - 166*a^3*b^5 + 35*a^2*b^6)*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) - 6
4*((128*a^6*b^2 - 424*a^5*b^3 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*s
in(4*d*x + 4*c) + 2*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*si
n(2*d*x + 2*c))*sin(6*d*x + 6*c))*integrate(1/4*(4*(7*a*b^2 - 4*b^3)*cos(6*
d*x + 6*c)^2 - 4*(256*a^3 - 416*a^2*b + 256*a*b^2 - 51*b^3)*cos(4*d*x + 4*c
)^2 + 4*(7*a*b^2 - 4*b^3)*cos(2*d*x + 2*c)^2 + 4*(7*a*b^2 - 4*b^3)*sin(6*d*
x + 6*c)^2 - 4*(256*a^3 - 416*a^2*b + 256*a*b^2 - 51*b^3)*sin(4*d*x + 4*c)^
2 - 2*(72*a^2*b - 107*a*b^2 + 56*b^3)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4
*(7*a*b^2 - 4*b^3)*sin(2*d*x + 2*c)^2 - ((7*a*b^2 - 4*b^3)*cos(6*d*x + 6*c)
 - 2*(32*a^2*b - 40*a*b^2 + 17*b^3)*cos(4*d*x + 4*c) + (7*a*b^2 - 4*b^3)*co
s(2*d*x + 2*c))*cos(8*d*x + 8*c) - (7*a*b^2 - 4*b^3 + 2*(72*a^2*b - 107*a*b
^2 + 56*b^3)*cos(4*d*x + 4*c) - 8*(7*a*b^2 - 4*b^3)*cos(2*d*x + 2*c))*cos(6
*d*x + 6*c) + 2*(32*a^2*b - 40*a*b^2 + 17*b^3 - (72*a^2*b - 107*a*b^2 + 56*
b^3)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - (7*a*b^2 - 4*b^3)*cos(2*d*x + 2*c
) - ((7*a*b^2 - 4*b^3)*sin(6*d*x + 6*c) - 2*(32*a^2*b - 40*a*b^2 + 17*b^3)*
sin(4*d*x + 4*c) + (7*a*b^2 - 4*b^3)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) - 2
*((72*a^2*b - 107*a*b^2 + 56*b^3)*sin(4*d*x + 4*c) - 4*(7*a*b^2 - 4*b^3)*si
n(2*d*x + 2*c))*sin(6*d*x + 6*c))/(a^4*b^2 - 2*a^3*b^3 + a^2*b^4 + (a^4*b^2
 - 2*a^3*b^3 + a^2*b^4)*cos(8*d*x + 8*c)^2 + 16*(a^4*b^2 - 2*a^3*b^3 + a^2*
b^4)*cos(6*d*x + 6*c)^2 + 4*(64*a^6 - 176*a^5*b + 169*a^4*b^2 - 66*a^3*b^3
+ 9*a^2*b^4)*cos(4*d*x + 4*c)^2 + 16*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*cos(2*
d*x + 2*c)^2 + (a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*sin(8*d*x + 8*c)^2 + 16*(a^4
*b^2 - 2*a^3*b^3 + a^2*b^4)*sin(6*d*x + 6*c)^2 + 4*(64*a^6 - 176*a^5*b + 16
9*a^4*b^2 - 66*a^3*b^3 + 9*a^2*b^4)*sin(4*d*x + 4*c)^2 + 16*(8*a^5*b - 19*a
^4*b^2 + 14*a^3*b^3 - 3*a^2*b^4)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a^
4*b^2 - 2*a^3*b^3 + a^2*b^4)*sin(2*d*x + 2*c)^2 + 2*(a^4*b^2 - 2*a^3*b^3 +
a^2*b^4 - 4*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*cos(6*d*x + 6*c) - 2*(8*a^5*b -
 19*a^4*b^2 + 14*a^3*b^3 - 3*a^2*b^4)*cos(4*d*x + 4*c) - 4*(a^4*b^2 - 2*a^3
*b^3 + a^2*b^4)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) - 8*(a^4*b^2 - 2*a^3*b^3
 + a^2*b^4 - 2*(8*a^5*b - 19*a^4*b^2 + 14*a^3*b^3 - 3*a^2*b^4)*cos(4*d*x +
4*c) - 4*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*cos(2*d*x + 2*c))*cos(6*d*x + 6*c)
 - 4*(8*a^5*b - 19*a^4*b^2 + 14*a^3*b^3 - 3*a^2*b^4 - 4*(8*a^5*b - 19*a^4*b
^2 + 14*a^3*b^3 - 3*a^2*b^4)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 8*(a^4*b^
2 - 2*a^3*b^3 + a^2*b^4)*cos(2*d*x + 2*c) - 4*(2*(a^4*b^2 - 2*a^3*b^3 + a^2
*b^4)*sin(6*d*x + 6*c) + (8*a^5*b - 19*a^4*b^2 + 14*a^3*b^3 - 3*a^2*b^4)*si
n(4*d*x + 4*c) + 2*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*sin(2*d*x + 2*c))*sin(8*
d*x + 8*c) + 16*((8*a^5*b - 19*a^4*b^2 + 14*a^3*b^3 - 3*a^2*b^4)*sin(4*d*x
+ 4*c) + 2*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*sin(2*d*x + 2*c))*sin(6*d*x + 6*
c)), x) - (6*a*b^4 - 3*b^5 + (7*a*b^4 - 4*b^5)*cos(14*d*x + 14*c) - (32*a^2
*b^3 + 2*a*b^4 - 7*b^5)*cos(12*d*x + 12*c) - (16*a^2*b^3 - 3*a*b^4 - 28*b^5
)*cos(10*d*x + 10*c) + 3*(256*a^3*b^2 - 320*a^2*b^3 + 166*a*b^4 - 35*b^5)*c
os(8*d*x + 8*c) + (784*a^2*b^3 - 723*a*b^4 + 140*b^5)*cos(6*d*x + 6*c) - (1

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60*a^2*b^3 - 266*a*b^4 + 91*b^5)*cos(4*d*x + 4*c) - (55*a*b^4 - 28*b^5)*cos
(2*d*x + 2*c))*sin(16*d*x + 16*c) + (55*a*b^4 - 28*b^5 - 4*(120*a^2*b^3 - 7
7*a*b^4 + 14*b^5)*cos(12*d*x + 12*c) + 16*(48*a^2*b^3 - 55*a*b^4 + 28*b^5)*
cos(10*d*x + 10*c) + 2*(3968*a^3*b^2 - 5024*a^2*b^3 + 2621*a*b^4 - 560*b^5)
*cos(8*d*x + 8*c) + 32*(224*a^2*b^3 - 209*a*b^4 + 42*b^5)*cos(6*d*x + 6*c)
- 4*(376*a^2*b^3 - 613*a*b^4 + 210*b^5)*cos(4*d*x + 4*c) - 16*(31*a*b^4 - 1
6*b^5)*cos(2*d*x + 2*c))*sin(14*d*x + 14*c) + (160*a^2*b^3 - 266*a*b^4 + 91
*b^5 - 4*(1152*a^3*b^2 - 520*a^2*b^3 - 455*a*b^4 + 294*b^5)*cos(10*d*x + 10
*c) + 2*(8192*a^4*b - 23296*a^3*b^2 + 21376*a^2*b^3 - 9394*a*b^4 + 1715*b^5
)*cos(8*d*x + 8*c) + 4*(5248*a^3*b^2 - 10888*a^2*b^3 + 6433*a*b^4 - 1078*b^
5)*cos(6*d*x + 6*c) - 8*(512*a^3*b^2 - 1520*a^2*b^3 + 1330*a*b^4 - 343*b^5)
*cos(4*d*x + 4*c) - 4*(376*a^2*b^3 - 613*a*b^4 + 210*b^5)*cos(2*d*x + 2*c))
*sin(12*d*x + 12*c) - (784*a^2*b^3 - 723*a*b^4 + 140*b^5 + 2*(51200*a^4*b -
 84864*a^3*b^2 + 56016*a^2*b^3 - 18081*a*b^4 + 1960*b^5)*cos(8*d*x + 8*c) +
 16*(6400*a^3*b^2 - 8608*a^2*b^3 + 3437*a*b^4 - 392*b^5)*cos(6*d*x + 6*c) -
 4*(5248*a^3*b^2 - 10888*a^2*b^3 + 6433*a*b^4 - 1078*b^5)*cos(4*d*x + 4*c)
- 32*(224*a^2*b^3 - 209*a*b^4 + 42*b^5)*cos(2*d*x + 2*c))*sin(10*d*x + 10*c
) - (768*a^3*b^2 - 960*a^2*b^3 + 498*a*b^4 - 105*b^5 + 2*(51200*a^4*b - 848
64*a^3*b^2 + 56016*a^2*b^3 - 18081*a*b^4 + 1960*b^5)*cos(6*d*x + 6*c) - 2*(
8192*a^4*b - 23296*a^3*b^2 + 21376*a^2*b^3 - 9394*a*b^4 + 1715*b^5)*cos(4*d
*x + 4*c) - 2*(3968*a^3*b^2 - 5024*a^2*b^3 + 2621*a*b^4 - 560*b^5)*cos(2*d*
x + 2*c))*sin(8*d*x + 8*c) + (16*a^2*b^3 - 3*a*b^4 - 28*b^5 - 4*(1152*a^3*b
^2 - 520*a^2*b^3 - 455*a*b^4 + 294*b^5)*cos(4*d*x + 4*c) + 16*(48*a^2*b^3 -
 55*a*b^4 + 28*b^5)*cos(2*d*x + 2*c))*sin(6*d*x + 6*c) + (32*a^2*b^3 + 2*a*
b^4 - 7*b^5 - 4*(120*a^2*b^3 - 77*a*b^4 + 14*b^5)*cos(2*d*x + 2*c))*sin(4*d
*x + 4*c) - (7*a*b^4 - 4*b^5)*sin(2*d*x + 2*c))/((a^4*b^4 - 2*a^3*b^5 + a^2
*b^6)*d*cos(16*d*x + 16*c)^2 + 64*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*cos(14*
d*x + 14*c)^2 + 16*(64*a^6*b^2 - 240*a^5*b^3 + 337*a^4*b^4 - 210*a^3*b^5 +
49*a^2*b^6)*d*cos(12*d*x + 12*c)^2 + 64*(256*a^6*b^2 - 736*a^5*b^3 + 753*a^
4*b^4 - 322*a^3*b^5 + 49*a^2*b^6)*d*cos(10*d*x + 10*c)^2 + 4*(16384*a^8 - 5
7344*a^7*b + 83712*a^6*b^2 - 67648*a^5*b^3 + 32841*a^4*b^4 - 9170*a^3*b^5 +
 1225*a^2*b^6)*d*cos(8*d*x + 8*c)^2 + 64*(256*a^6*b^2 - 736*a^5*b^3 + 753*a
^4*b^4 - 322*a^3*b^5 + 49*a^2*b^6)*d*cos(6*d*x + 6*c)^2 + 16*(64*a^6*b^2 -
240*a^5*b^3 + 337*a^4*b^4 - 210*a^3*b^5 + 49*a^2*b^6)*d*cos(4*d*x + 4*c)^2
+ 64*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*cos(2*d*x + 2*c)^2 + (a^4*b^4 - 2*a^
3*b^5 + a^2*b^6)*d*sin(16*d*x + 16*c)^2 + 64*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6
)*d*sin(14*d*x + 14*c)^2 + 16*(64*a^6*b^2 - 240*a^5*b^3 + 337*a^4*b^4 - 210
*a^3*b^5 + 49*a^2*b^6)*d*sin(12*d*x + 12*c)^2 + 64*(256*a^6*b^2 - 736*a^5*b
^3 + 753*a^4*b^4 - 322*a^3*b^5 + 49*a^2*b^6)*d*sin(10*d*x + 10*c)^2 + 4*(16
384*a^8 - 57344*a^7*b + 83712*a^6*b^2 - 67648*a^5*b^3 + 32841*a^4*b^4 - 917
0*a^3*b^5 + 1225*a^2*b^6)*d*sin(8*d*x + 8*c)^2 + 64*(256*a^6*b^2 - 736*a^5*
b^3 + 753*a^4*b^4 - 322*a^3*b^5 + 49*a^2*b^6)*d*sin(6*d*x + 6*c)^2 + 16*(64
*a^6*b^2 - 240*a^5*b^3 + 337*a^4*b^4 - 210*a^3*b^5 + 49*a^2*b^6)*d*sin(4*d*
x + 4*c)^2 + 64*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*sin(4*d
*x + 4*c)*sin(2*d*x + 2*c) + 64*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*sin(2*d*x
 + 2*c)^2 - 16*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*cos(2*d*x + 2*c) + (a^4*b^
4 - 2*a^3*b^5 + a^2*b^6)*d - 2*(8*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*cos(14*
d*x + 14*c) + 4*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*cos(12*
d*x + 12*c) - 8*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*cos(10
*d*x + 10*c) - 2*(128*a^6*b^2 - 352*a^5*b^3 + 355*a^4*b^4 - 166*a^3*b^5 + 3
5*a^2*b^6)*d*cos(8*d*x + 8*c) - 8*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7
*a^2*b^6)*d*cos(6*d*x + 6*c) + 4*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a
^2*b^6)*d*cos(4*d*x + 4*c) + 8*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*cos(2*d*x
+ 2*c) - (a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d)*cos(16*d*x + 16*c) + 16*(4*(8*a
^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*cos(12*d*x + 12*c) - 8*(16*
a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*cos(10*d*x + 10*c) - 2*(12
8*a^6*b^2 - 352*a^5*b^3 + 355*a^4*b^4 - 166*a^3*b^5 + 35*a^2*b^6)*d*cos(8*d
*x + 8*c) - 8*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*cos(6*d*
x + 6*c) + 4*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*cos(4*d*x
```

+ 4*c) + 8*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*cos(2*d*x + 2*c) - (a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d)*cos(14*d*x + 14*c) - 8*(8*(128*a^6*b^2 - 424*a^5*b^3 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*cos(10*d*x + 10*c) + 2*(1024*a^7*b - 3712*a^6*b^2 + 5304*a^5*b^3 - 3813*a^4*b^4 + 1442*a^3*b^5 - 245*a^2*b^6)*d*cos(8*d*x + 8*c) + 8*(128*a^6*b^2 - 424*a^5*b^3 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*cos(6*d*x + 6*c) - 4*(64*a^6*b^2 - 240*a^5*b^3 + 337*a^4*b^4 - 210*a^3*b^5 + 49*a^2*b^6)*d*cos(4*d*x + 4*c) - 8*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*cos(2*d*x + 2*c) + (8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d)*cos(12*d*x + 12*c) + 16*(2*(2048*a^7*b - 6528*a^6*b^2 + 8144*a^5*b^3 - 5141*a^4*b^4 + 1722*a^3*b^5 - 245*a^2*b^6)*d*cos(8*d*x + 8*c) + 8*(256*a^6*b^2 - 736*a^5*b^3 + 753*a^4*b^4 - 322*a^3*b^5 + 49*a^2*b^6)*d*cos(6*d*x + 6*c) - 4*(128*a^6*b^2 - 424*a^5*b^3 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*cos(4*d*x + 4*c) - 8*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*cos(2*d*x + 2*c) + (16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d)*cos(10*d*x + 10*c) + 4*(8*(2048*a^7*b - 6528*a^6*b^2 + 8144*a^5*b^3 - 5141*a^4*b^4 + 1722*a^3*b^5 - 245*a^2*b^6)*d*cos(6*d*x + 6*c) - 4*(1024*a^7*b - 3712*a^6*b^2 + 5304*a^5*b^3 - 3813*a^4*b^4 + 1442*a^3*b^5 - 245*a^2*b^6)*d*cos(4*d*x + 4*c) - 8*(128*a^6*b^2 - 352*a^5*b^3 + 355*a^4*b^4 - 166*a^3*b^5 + 35*a^2*b^6)*d*cos(2*d*x + 2*c) + (128*a^6*b^2 - 352*a^5*b^3 + 355*a^4*b^4 - 166*a^3*b^5 + 35*a^2*b^6)*d)*cos(8*d*x + 8*c) - 16*(4*(128*a^6*b^2 - 424*a^5*b^3 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*cos(4*d*x + 4*c) + 8*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*cos(2*d*x + 2*c) - (16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d)*cos(6*d*x + 6*c) + 8*(8*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*cos(2*d*x + 2*c) - (8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d)*cos(4*d*x + 4*c) - 4*(4*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*sin(14*d*x + 14*c) + 2*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*sin(12*d*x + 12*c) - 4*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*sin(10*d*x + 10*c) - (128*a^6*b^2 - 352*a^5*b^3 + 355*a^4*b^4 - 166*a^3*b^5 + 35*a^2*b^6)*d*sin(8*d*x + 8*c) - 4*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*sin(6*d*x + 6*c) + 2*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*sin(4*d*x + 4*c) + 4*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*sin(2*d*x + 2*c))*sin(16*d*x + 16*c) + 32*(2*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*sin(12*d*x + 12*c) - 4*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*sin(10*d*x + 10*c) - (128*a^6*b^2 - 352*a^5*b^3 + 355*a^4*b^4 - 166*a^3*b^5 + 35*a^2*b^6)*d*sin(8*d*x + 8*c) - 4*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*sin(6*d*x + 6*c) + 2*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*sin(4*d*x + 4*c) + 4*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*sin(2*d*x + 2*c))*sin(14*d*x + 14*c) - 16*(4*(128*a^6*b^2 - 424*a^5*b^3 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*sin(10*d*x + 10*c) + (1024*a^7*b - 3712*a^6*b^2 + 5304*a^5*b^3 - 3813*a^4*b^4 + 1442*a^3*b^5 - 245*a^2*b^6)*d*sin(8*d*x + 8*c) + 4*(128*a^6*b^2 - 424*a^5*b^3 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*sin(6*d*x + 6*c) - 2*(64*a^6*b^2 - 240*a^5*b^3 + 337*a^4*b^4 - 210*a^3*b^5 + 49*a^2*b^6)*d*sin(4*d*x + 4*c) - 4*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*sin(2*d*x + 2*c))*sin(12*d*x + 12*c) + 32*((2048*a^7*b - 6528*a^6*b^2 + 8144*a^5*b^3 - 5141*a^4*b^4 + 1722*a^3*b^5 - 245*a^2*b^6)*d*sin(8*d*x + 8*c) + 4*(256*a^6*b^2 - 736*a^5*b^3 + 753*a^4*b^4 - 322*a^3*b^5 + 49*a^2*b^6)*d*sin(6*d*x + 6*c) - 2*(128*a^6*b^2 - 424*a^5*b^3 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*sin(4*d*x + 4*c) - 4*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 16*(2*(2048*a^7*b - 6528*a^6*b^2 + 8144*a^5*b^3 - 5141*a^4*b^4 + 1722*a^3*b^5 - 245*a^2*b^6)*d*sin(6*d*x + 6*c) - (1024*a^7*b - 3712*a^6*b^2 + 5304*a^5*b^3 - 3813*a^4*b^4 + 1442*a^3*b^5 - 245*a^2*b^6)*d*sin(4*d*x + 4*c) - 2*(128*a^6*b^2 - 352*a^5*b^3 + 355*a^4*b^4 - 166*a^3*b^5 + 35*a^2*b^6)*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) - 64*((128*a^6*b^2 - 424*a^5*b^3 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*sin(4*d*x + 4*c) + 2*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))

Fricas [B] time = 24.4428, size = 15823, normalized size = 49.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/256*(((a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d*cos(d*x + c)^8 - 4*(a^4*b^2 - 2*a
^3*b^3 + a^2*b^4)*d*cos(d*x + c)^6 - 2*(a^5*b - 5*a^4*b^2 + 7*a^3*b^3 - 3*a
^2*b^4)*d*cos(d*x + c)^4 + 4*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*co
s(d*x + c)^2 + (a^6 - 4*a^5*b + 6*a^4*b^2 - 4*a^3*b^3 + a^2*b^4)*d)*sqrt(-(
1024*a^4 - 1916*a^3*b + 1501*a^2*b^2 - 570*a*b^3 + 105*b^4 - (a^10 - 5*a^9*
b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2*sqrt((3686400*a^8*b
- 17817600*a^7*b^2 + 39458560*a^6*b^3 - 51952960*a^5*b^4 + 44335881*a^4*b^5
 - 25065628*a^3*b^6 + 9162574*a^2*b^7 - 1980972*a*b^8 + 194481*b^9)/((a^21
- 10*a^20*b + 45*a^19*b^2 - 120*a^18*b^3 + 210*a^17*b^4 - 252*a^16*b^5 + 21
0*a^15*b^6 - 120*a^14*b^7 + 45*a^13*b^8 - 10*a^12*b^9 + a^11*b^10)*d^4)))/(
(a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2))*log(
491520*a^6*b - 1742720*a^5*b^2 + 2747904*a^4*b^3 - 2435877*a^3*b^4 + 510698
9/4*a^2*b^5 - 750141/2*a*b^6 + 194481/4*b^7 - 1/4*(1966080*a^6*b - 6970880*
a^5*b^2 + 10991616*a^4*b^3 - 9743508*a^3*b^4 + 5106989*a^2*b^5 - 1500282*a*
b^6 + 194481*b^7)*cos(d*x + c)^2 + 1/2*((32*a^16 - 193*a^15*b + 498*a^14*b^
2 - 715*a^13*b^3 + 620*a^12*b^4 - 327*a^11*b^5 + 98*a^10*b^6 - 13*a^9*b^7)*
d^3*sqrt((3686400*a^8*b - 17817600*a^7*b^2 + 39458560*a^6*b^3 - 51952960*a^
5*b^4 + 44335881*a^4*b^5 - 25065628*a^3*b^6 + 9162574*a^2*b^7 - 1980972*a*b
^8 + 194481*b^9)/((a^21 - 10*a^20*b + 45*a^19*b^2 - 120*a^18*b^3 + 210*a^17
*b^4 - 252*a^16*b^5 + 210*a^15*b^6 - 120*a^14*b^7 + 45*a^13*b^8 - 10*a^12*b
^9 + a^11*b^10)*d^4))*cos(d*x + c)*sin(d*x + c) + (88320*a^9*b - 319040*a^8
*b^2 + 510294*a^7*b^3 - 457551*a^6*b^4 + 241865*a^5*b^5 - 71421*a^4*b^6 + 9
261*a^3*b^7)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-(1024*a^4 - 1916*a^3*b + 15
01*a^2*b^2 - 570*a*b^3 + 105*b^4 - (a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^
3 + 5*a^6*b^4 - a^5*b^5)*d^2*sqrt((3686400*a^8*b - 17817600*a^7*b^2 + 39458
560*a^6*b^3 - 51952960*a^5*b^4 + 44335881*a^4*b^5 - 25065628*a^3*b^6 + 9162
574*a^2*b^7 - 1980972*a*b^8 + 194481*b^9)/((a^21 - 10*a^20*b + 45*a^19*b^2
- 120*a^18*b^3 + 210*a^17*b^4 - 252*a^16*b^5 + 210*a^15*b^6 - 120*a^14*b^7
+ 45*a^13*b^8 - 10*a^12*b^9 + a^11*b^10)*d^4)))/((a^10 - 5*a^9*b + 10*a^8*b
^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2)) - 1/4*(2*(1024*a^13 - 6276*a^1
2*b + 16461*a^11*b^2 - 24005*a^10*b^3 + 21090*a^9*b^4 - 11214*a^8*b^5 + 336
1*a^7*b^6 - 441*a^6*b^7)*d^2*cos(d*x + c)^2 - (1024*a^13 - 6276*a^12*b + 16
461*a^11*b^2 - 24005*a^10*b^3 + 21090*a^9*b^4 - 11214*a^8*b^5 + 3361*a^7*b^
6 - 441*a^6*b^7)*d^2)*sqrt((3686400*a^8*b - 17817600*a^7*b^2 + 39458560*a^6
*b^3 - 51952960*a^5*b^4 + 44335881*a^4*b^5 - 25065628*a^3*b^6 + 9162574*a^2
*b^7 - 1980972*a*b^8 + 194481*b^9)/((a^21 - 10*a^20*b + 45*a^19*b^2 - 120*a
^18*b^3 + 210*a^17*b^4 - 252*a^16*b^5 + 210*a^15*b^6 - 120*a^14*b^7 + 45*a^
13*b^8 - 10*a^12*b^9 + a^11*b^10)*d^4))) - ((a^4*b^2 - 2*a^3*b^3 + a^2*b^4)
*d*cos(d*x + c)^8 - 4*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d*cos(d*x + c)^6 - 2*
(a^5*b - 5*a^4*b^2 + 7*a^3*b^3 - 3*a^2*b^4)*d*cos(d*x + c)^4 + 4*(a^5*b - 3
*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*cos(d*x + c)^2 + (a^6 - 4*a^5*b + 6*a^4*b
^2 - 4*a^3*b^3 + a^2*b^4)*d)*sqrt(-(1024*a^4 - 1916*a^3*b + 1501*a^2*b^2 -
570*a*b^3 + 105*b^4 - (a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4
 - a^5*b^5)*d^2*sqrt((3686400*a^8*b - 17817600*a^7*b^2 + 39458560*a^6*b^3 -
 51952960*a^5*b^4 + 44335881*a^4*b^5 - 25065628*a^3*b^6 + 9162574*a^2*b^7 -
 1980972*a*b^8 + 194481*b^9)/((a^21 - 10*a^20*b + 45*a^19*b^2 - 120*a^18*b^
3 + 210*a^17*b^4 - 252*a^16*b^5 + 210*a^15*b^6 - 120*a^14*b^7 + 45*a^13*b^8
 - 10*a^12*b^9 + a^11*b^10)*d^4)))/((a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b
^3 + 5*a^6*b^4 - a^5*b^5)*d^2))*log(491520*a^6*b - 1742720*a^5*b^2 + 274790
4*a^4*b^3 - 2435877*a^3*b^4 + 5106989/4*a^2*b^5 - 750141/2*a*b^6 + 194481/4
```

b^7 - 1/4(1966080*a^6*b - 6970880*a^5*b^2 + 10991616*a^4*b^3 - 9743508*a^
3*b^4 + 5106989*a^2*b^5 - 1500282*a*b^6 + 194481*b^7)*cos(d*x + c)^2 - 1/2*
((32*a^16 - 193*a^15*b + 498*a^14*b^2 - 715*a^13*b^3 + 620*a^12*b^4 - 327*a
^11*b^5 + 98*a^10*b^6 - 13*a^9*b^7)*d^3*sqrt((3686400*a^8*b - 17817600*a^7*
b^2 + 39458560*a^6*b^3 - 51952960*a^5*b^4 + 44335881*a^4*b^5 - 25065628*a^3
*b^6 + 9162574*a^2*b^7 - 1980972*a*b^8 + 194481*b^9)/((a^21 - 10*a^20*b + 4
5*a^19*b^2 - 120*a^18*b^3 + 210*a^17*b^4 - 252*a^16*b^5 + 210*a^15*b^6 - 12
0*a^14*b^7 + 45*a^13*b^8 - 10*a^12*b^9 + a^11*b^10)*d^4))*cos(d*x + c)*sin(
d*x + c) + (88320*a^9*b - 319040*a^8*b^2 + 510294*a^7*b^3 - 457551*a^6*b^4
+ 241865*a^5*b^5 - 71421*a^4*b^6 + 9261*a^3*b^7)*d*cos(d*x + c)*sin(d*x + c
))*sqrt(-(1024*a^4 - 1916*a^3*b + 1501*a^2*b^2 - 570*a*b^3 + 105*b^4 - (a^1
0 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2*sqrt((3686
400*a^8*b - 17817600*a^7*b^2 + 39458560*a^6*b^3 - 51952960*a^5*b^4 + 443358
81*a^4*b^5 - 25065628*a^3*b^6 + 9162574*a^2*b^7 - 1980972*a*b^8 + 194481*b^
9)/((a^21 - 10*a^20*b + 45*a^19*b^2 - 120*a^18*b^3 + 210*a^17*b^4 - 252*a^1
6*b^5 + 210*a^15*b^6 - 120*a^14*b^7 + 45*a^13*b^8 - 10*a^12*b^9 + a^11*b^10
)*d^4)))/((a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*
d^2)) - 1/4*(2*(1024*a^13 - 6276*a^12*b + 16461*a^11*b^2 - 24005*a^10*b^3 +
 21090*a^9*b^4 - 11214*a^8*b^5 + 3361*a^7*b^6 - 441*a^6*b^7)*d^2*cos(d*x +
c)^2 - (1024*a^13 - 6276*a^12*b + 16461*a^11*b^2 - 24005*a^10*b^3 + 21090*a
^9*b^4 - 11214*a^8*b^5 + 3361*a^7*b^6 - 441*a^6*b^7)*d^2)*sqrt((3686400*a^8
*b - 17817600*a^7*b^2 + 39458560*a^6*b^3 - 51952960*a^5*b^4 + 44335881*a^4*
b^5 - 25065628*a^3*b^6 + 9162574*a^2*b^7 - 1980972*a*b^8 + 194481*b^9)/((a^
21 - 10*a^20*b + 45*a^19*b^2 - 120*a^18*b^3 + 210*a^17*b^4 - 252*a^16*b^5 +
 210*a^15*b^6 - 120*a^14*b^7 + 45*a^13*b^8 - 10*a^12*b^9 + a^11*b^10)*d^4))
) + ((a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d*cos(d*x + c)^8 - 4*(a^4*b^2 - 2*a^3*
b^3 + a^2*b^4)*d*cos(d*x + c)^6 - 2*(a^5*b - 5*a^4*b^2 + 7*a^3*b^3 - 3*a^2*
b^4)*d*cos(d*x + c)^4 + 4*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*cos(d
*x + c)^2 + (a^6 - 4*a^5*b + 6*a^4*b^2 - 4*a^3*b^3 + a^2*b^4)*d)*sqrt(-(102
4*a^4 - 1916*a^3*b + 1501*a^2*b^2 - 570*a*b^3 + 105*b^4 + (a^10 - 5*a^9*b +
 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2*sqrt((3686400*a^8*b - 1
7817600*a^7*b^2 + 39458560*a^6*b^3 - 51952960*a^5*b^4 + 44335881*a^4*b^5 -
25065628*a^3*b^6 + 9162574*a^2*b^7 - 1980972*a*b^8 + 194481*b^9)/((a^21 - 1
0*a^20*b + 45*a^19*b^2 - 120*a^18*b^3 + 210*a^17*b^4 - 252*a^16*b^5 + 210*a
^15*b^6 - 120*a^14*b^7 + 45*a^13*b^8 - 10*a^12*b^9 + a^11*b^10)*d^4)))/((a^
10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2))*log(-49
1520*a^6*b + 1742720*a^5*b^2 - 2747904*a^4*b^3 + 2435877*a^3*b^4 - 5106989/
4*a^2*b^5 + 750141/2*a*b^6 - 194481/4*b^7 + 1/4*(1966080*a^6*b - 6970880*a^
5*b^2 + 10991616*a^4*b^3 - 9743508*a^3*b^4 + 5106989*a^2*b^5 - 1500282*a*b^
6 + 194481*b^7)*cos(d*x + c)^2 + 1/2*((32*a^16 - 193*a^15*b + 498*a^14*b^2
- 715*a^13*b^3 + 620*a^12*b^4 - 327*a^11*b^5 + 98*a^10*b^6 - 13*a^9*b^7)*d^
3*sqrt((3686400*a^8*b - 17817600*a^7*b^2 + 39458560*a^6*b^3 - 51952960*a^5*
b^4 + 44335881*a^4*b^5 - 25065628*a^3*b^6 + 9162574*a^2*b^7 - 1980972*a*b^8
 + 194481*b^9)/((a^21 - 10*a^20*b + 45*a^19*b^2 - 120*a^18*b^3 + 210*a^17*b
^4 - 252*a^16*b^5 + 210*a^15*b^6 - 120*a^14*b^7 + 45*a^13*b^8 - 10*a^12*b^9
 + a^11*b^10)*d^4))*cos(d*x + c)*sin(d*x + c) - (88320*a^9*b - 319040*a^8*b
^2 + 510294*a^7*b^3 - 457551*a^6*b^4 + 241865*a^5*b^5 - 71421*a^4*b^6 + 926
1*a^3*b^7)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-(1024*a^4 - 1916*a^3*b + 1501
*a^2*b^2 - 570*a*b^3 + 105*b^4 + (a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3
+ 5*a^6*b^4 - a^5*b^5)*d^2*sqrt((3686400*a^8*b - 17817600*a^7*b^2 + 3945856
0*a^6*b^3 - 51952960*a^5*b^4 + 44335881*a^4*b^5 - 25065628*a^3*b^6 + 916257
4*a^2*b^7 - 1980972*a*b^8 + 194481*b^9)/((a^21 - 10*a^20*b + 45*a^19*b^2 -
120*a^18*b^3 + 210*a^17*b^4 - 252*a^16*b^5 + 210*a^15*b^6 - 120*a^14*b^7 +
45*a^13*b^8 - 10*a^12*b^9 + a^11*b^10)*d^4)))/((a^10 - 5*a^9*b + 10*a^8*b^2
 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2)) - 1/4*(2*(1024*a^13 - 6276*a^12*
b + 16461*a^11*b^2 - 24005*a^10*b^3 + 21090*a^9*b^4 - 11214*a^8*b^5 + 3361*
a^7*b^6 - 441*a^6*b^7)*d^2*cos(d*x + c)^2 - (1024*a^13 - 6276*a^12*b + 1646
1*a^11*b^2 - 24005*a^10*b^3 + 21090*a^9*b^4 - 11214*a^8*b^5 + 3361*a^7*b^6
 - 441*a^6*b^7)*d^2)*sqrt((3686400*a^8*b - 17817600*a^7*b^2 + 39458560*a^6*b

```
^3 - 51952960*a^5*b^4 + 44335881*a^4*b^5 - 25065628*a^3*b^6 + 9162574*a^2*b
^7 - 1980972*a*b^8 + 194481*b^9)/((a^21 - 10*a^20*b + 45*a^19*b^2 - 120*a^1
8*b^3 + 210*a^17*b^4 - 252*a^16*b^5 + 210*a^15*b^6 - 120*a^14*b^7 + 45*a^13
*b^8 - 10*a^12*b^9 + a^11*b^10)*d^4))) - ((a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d
*cos(d*x + c)^8 - 4*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d*cos(d*x + c)^6 - 2*(a
^5*b - 5*a^4*b^2 + 7*a^3*b^3 - 3*a^2*b^4)*d*cos(d*x + c)^4 + 4*(a^5*b - 3*a
^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*cos(d*x + c)^2 + (a^6 - 4*a^5*b + 6*a^4*b^2
 - 4*a^3*b^3 + a^2*b^4)*d)*sqrt(-(1024*a^4 - 1916*a^3*b + 1501*a^2*b^2 - 57
0*a*b^3 + 105*b^4 + (a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 -
 a^5*b^5)*d^2*sqrt((3686400*a^8*b - 17817600*a^7*b^2 + 39458560*a^6*b^3 - 5
1952960*a^5*b^4 + 44335881*a^4*b^5 - 25065628*a^3*b^6 + 9162574*a^2*b^7 - 1
980972*a*b^8 + 194481*b^9)/((a^21 - 10*a^20*b + 45*a^19*b^2 - 120*a^18*b^3
+ 210*a^17*b^4 - 252*a^16*b^5 + 210*a^15*b^6 - 120*a^14*b^7 + 45*a^13*b^8 -
 10*a^12*b^9 + a^11*b^10)*d^4)))/((a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3
 + 5*a^6*b^4 - a^5*b^5)*d^2))*log(-491520*a^6*b + 1742720*a^5*b^2 - 2747904
*a^4*b^3 + 2435877*a^3*b^4 - 5106989/4*a^2*b^5 + 750141/2*a*b^6 - 194481/4*
b^7 + 1/4*(1966080*a^6*b - 6970880*a^5*b^2 + 10991616*a^4*b^3 - 9743508*a^3
*b^4 + 5106989*a^2*b^5 - 1500282*a*b^6 + 194481*b^7)*cos(d*x + c)^2 - 1/2*(
(32*a^16 - 193*a^15*b + 498*a^14*b^2 - 715*a^13*b^3 + 620*a^12*b^4 - 327*a^
11*b^5 + 98*a^10*b^6 - 13*a^9*b^7)*d^3*sqrt((3686400*a^8*b - 17817600*a^7*b
^2 + 39458560*a^6*b^3 - 51952960*a^5*b^4 + 44335881*a^4*b^5 - 25065628*a^3*
b^6 + 9162574*a^2*b^7 - 1980972*a*b^8 + 194481*b^9)/((a^21 - 10*a^20*b + 45
*a^19*b^2 - 120*a^18*b^3 + 210*a^17*b^4 - 252*a^16*b^5 + 210*a^15*b^6 - 120
*a^14*b^7 + 45*a^13*b^8 - 10*a^12*b^9 + a^11*b^10)*d^4))*cos(d*x + c)*sin(d
*x + c) - (88320*a^9*b - 319040*a^8*b^2 + 510294*a^7*b^3 - 457551*a^6*b^4 +
 241865*a^5*b^5 - 71421*a^4*b^6 + 9261*a^3*b^7)*d*cos(d*x + c)*sin(d*x + c)
)*sqrt(-(1024*a^4 - 1916*a^3*b + 1501*a^2*b^2 - 570*a*b^3 + 105*b^4 + (a^10
 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2*sqrt((36864
00*a^8*b - 17817600*a^7*b^2 + 39458560*a^6*b^3 - 51952960*a^5*b^4 + 4433588
1*a^4*b^5 - 25065628*a^3*b^6 + 9162574*a^2*b^7 - 1980972*a*b^8 + 194481*b^9
)/((a^21 - 10*a^20*b + 45*a^19*b^2 - 120*a^18*b^3 + 210*a^17*b^4 - 252*a^16
*b^5 + 210*a^15*b^6 - 120*a^14*b^7 + 45*a^13*b^8 - 10*a^12*b^9 + a^11*b^10)
*d^4)))/((a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d
^2)) - 1/4*(2*(1024*a^13 - 6276*a^12*b + 16461*a^11*b^2 - 24005*a^10*b^3 +
21090*a^9*b^4 - 11214*a^8*b^5 + 3361*a^7*b^6 - 441*a^6*b^7)*d^2*cos(d*x + c
)^2 - (1024*a^13 - 6276*a^12*b + 16461*a^11*b^2 - 24005*a^10*b^3 + 21090*a^
9*b^4 - 11214*a^8*b^5 + 3361*a^7*b^6 - 441*a^6*b^7)*d^2)*sqrt((3686400*a^8*
b - 17817600*a^7*b^2 + 39458560*a^6*b^3 - 51952960*a^5*b^4 + 44335881*a^4*b
^5 - 25065628*a^3*b^6 + 9162574*a^2*b^7 - 1980972*a*b^8 + 194481*b^9)/((a^2
1 - 10*a^20*b + 45*a^19*b^2 - 120*a^18*b^3 + 210*a^17*b^4 - 252*a^16*b^5 +
210*a^15*b^6 - 120*a^14*b^7 + 45*a^13*b^8 - 10*a^12*b^9 + a^11*b^10)*d^4)))
 - 8*(6*(2*a*b^2 - b^3)*cos(d*x + c)^7 - (49*a*b^2 - 25*b^3)*cos(d*x + c)^5
 - 8*(2*a^2*b - 9*a*b^2 + 4*b^3)*cos(d*x + c)^3 + (33*a^2*b - 46*a*b^2 + 13
*b^3)*cos(d*x + c))*sin(d*x + c))/((a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d*cos(d*
x + c)^8 - 4*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d*cos(d*x + c)^6 - 2*(a^5*b -
5*a^4*b^2 + 7*a^3*b^3 - 3*a^2*b^4)*d*cos(d*x + c)^4 + 4*(a^5*b - 3*a^4*b^2
+ 3*a^3*b^3 - a^2*b^4)*d*cos(d*x + c)^2 + (a^6 - 4*a^5*b + 6*a^4*b^2 - 4*a^
3*b^3 + a^2*b^4)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.235 $\int \frac{\csc^2(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=357

$$-\frac{b^2\tan(c+dx)\left(\left(a^2+6ab+b^2\right)\tan^2(c+dx)+a(a+3b)\right)}{8a^2d(a-b)^3\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)^2}-\frac{b\tan(c+dx)\left(\frac{\left(18a^2+15ab-13b^2\right)\tan^2(c+dx)}{(a-b)^2}+\frac{2a^2(9a-17b)}{(a-b)^3}\right)}{32a^3d\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}+\frac{3\sqrt{}}{}$$

```
[Out] (3*Sqrt[b]*(20*a - 34*Sqrt[a]*Sqrt[b] + 15*b)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b
]]*Tan[c + d*x])/a^(1/4)])/(64*a^(13/4)*(Sqrt[a] - Sqrt[b])^(5/2)*d) - (3*S
qrt[b]*(20*a + 34*Sqrt[a]*Sqrt[b] + 15*b)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*T
an[c + d*x])/a^(1/4)])/(64*a^(13/4)*(Sqrt[a] + Sqrt[b])^(5/2)*d) - Cot[c +
d*x]/(a^3*d) - (b^2*Tan[c + d*x]*(a*(a + 3*b) + (a^2 + 6*a*b + b^2)*Tan[c +
 d*x]^2))/(8*a^2*(a - b)^3*d*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan[c + d*x]
^4)^2) - (b*Tan[c + d*x]*((2*a^2*(9*a - 17*b))/(a - b)^3 + ((18*a^2 + 15*a*
b - 13*b^2)*Tan[c + d*x]^2)/(a - b)^2))/(32*a^3*d*(a + 2*a*Tan[c + d*x]^2 +
 (a - b)*Tan[c + d*x]^4))
```

Rubi [A] time = 1.29425, antiderivative size = 357, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3217, 1334, 1669, 1664, 1166, 205}

$$-\frac{b^2\tan(c+dx)\left(\left(a^2+6ab+b^2\right)\tan^2(c+dx)+a(a+3b)\right)}{8a^2d(a-b)^3\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)^2}-\frac{b\tan(c+dx)\left(\frac{\left(18a^2+15ab-13b^2\right)\tan^2(c+dx)}{(a-b)^2}+\frac{2a^2(9a-17b)}{(a-b)^3}\right)}{32a^3d\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}+\frac{3\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a - b*Sin[c + d*x]^4)^3,x]
```


```
[Out] (3*Sqrt[b]*(20*a - 34*Sqrt[a]*Sqrt[b] + 15*b)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b
]]*Tan[c + d*x])/a^(1/4)])/(64*a^(13/4)*(Sqrt[a] - Sqrt[b])^(5/2)*d) - (3*S
qrt[b]*(20*a + 34*Sqrt[a]*Sqrt[b] + 15*b)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*T
an[c + d*x])/a^(1/4)])/(64*a^(13/4)*(Sqrt[a] + Sqrt[b])^(5/2)*d) - Cot[c +
d*x]/(a^3*d) - (b^2*Tan[c + d*x]*(a*(a + 3*b) + (a^2 + 6*a*b + b^2)*Tan[c +
 d*x]^2))/(8*a^2*(a - b)^3*d*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan[c + d*x]
^4)^2) - (b*Tan[c + d*x]*((2*a^2*(9*a - 17*b))/(a - b)^3 + ((18*a^2 + 15*a*
b - 13*b^2)*Tan[c + d*x]^2)/(a - b)^2))/(32*a^3*d*(a + 2*a*Tan[c + d*x]^2 +
 (a - b)*Tan[c + d*x]^4))
```


Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 1334

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
```

```
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[x^m*(a + b*x^2 + c*x^4)^(p +
1)*Simp[ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d +
e*x^2)^q, a + b*x^2 + c*x^4, x])/x^m + (b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5)
 - a*b*g)/x^m + c*(4*p + 7)*(b*f - 2*a*g)*x^(2 - m), x], x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1] &
& ILtQ[m/2, 0]
```

Rule 1669

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^6}{x^2\left(a+2ax^2+(a-b)x^4\right)^3}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{b^2\tan(c+dx)\left(a(a+3b)+\left(a^2+6ab+b^2\right)\tan^2(c+dx)\right)}{8a^2(a-b)^3d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-16ab-\frac{2ab(32a^3-96a^2b+9}{(a-b)}}{}\right.}{}$$

$$= -\frac{b^2\tan(c+dx)\left(a(a+3b)+\left(a^2+6ab+b^2\right)\tan^2(c+dx)\right)}{8a^2(a-b)^3d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{b\tan(c+dx)\left(\frac{2a^2(9a-17b)}{(a-b)^3}+\right.}{32a^3d\left(a+2a\tan^2(c+dx)\right.}$$

$$= -\frac{b^2\tan(c+dx)\left(a(a+3b)+\left(a^2+6ab+b^2\right)\tan^2(c+dx)\right)}{8a^2(a-b)^3d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{b\tan(c+dx)\left(\frac{2a^2(9a-17b)}{(a-b)^3}+\right.}{32a^3d\left(a+2a\tan^2(c+dx)\right.}$$

$$= -\frac{\cot(c+dx)}{a^3d} - \frac{b^2\tan(c+dx)\left(a(a+3b)+\left(a^2+6ab+b^2\right)\tan^2(c+dx)\right)}{8a^2(a-b)^3d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{b\tan(c+dx)}{32a^3d\left(a+2\right.}$$

$$= -\frac{\cot(c+dx)}{a^3d} - \frac{b^2\tan(c+dx)\left(a(a+3b)+\left(a^2+6ab+b^2\right)\tan^2(c+dx)\right)}{8a^2(a-b)^3d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{b\tan(c+dx)}{32a^3d\left(a+2\right.}$$

$$= \frac{3\sqrt{b}\left(20a-34\sqrt{a}\sqrt{b}+15b\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{13/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}d} - \frac{3\sqrt{b}\left(20a+34\sqrt{a}\sqrt{b}+15b\right)\tan^{-1}}{64a^{13/4}\left(\sqrt{a}+\sqrt{b}\right)^{5/}}$$

Mathematica [A] time = 5.27675, size = 357, normalized size = 1.

$$-\frac{\frac{4b\sin(2(c+dx))\left(28a^2+b(13b-19a)\cos(2(c+dx))+3ab-13b^2\right)}{(a-b)^2(8a+4b\cos(2(c+dx))-b\cos(4(c+dx))-3b)} + \frac{3\sqrt{b}\left(34\sqrt{a}\sqrt{b}+20a+15b\right)\tan^{-1}\left(\frac{(\sqrt{a}+\sqrt{b})\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\left(\sqrt{a}+\sqrt{b}\right)^2\sqrt{\sqrt{a}\sqrt{b}+a}} + \frac{128ab\sin(2(c+dx))(2a-b\cos(2(c+}{(a-b)(-8a-4b\cos(2(c+dx))+b\cos(4(c-}}{64a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] -((3*Sqrt[b]*(20*a + 34*Sqrt[a]*Sqrt[b] + 15*b)*ArcTan[((Sqrt[a] + Sqrt[b])
*Tan[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/((Sqrt[a] + Sqrt[b])^2*Sqrt[a +
Sqrt[a]*Sqrt[b]]) + (3*Sqrt[b]*(20*a - 34*Sqrt[a]*Sqrt[b] + 15*b)*ArcTanh[(
(Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/((Sqrt[a] -
Sqrt[b])^2*Sqrt[-a + Sqrt[a]*Sqrt[b]]) + 64*Cot[c + d*x] + (4*b*(28*a^2 + 3
*a*b - 13*b^2 + b*(-19*a + 13*b)*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])/((a -
b)^2*(8*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*x)])) + (128*a*b*(2
*a + b - b*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])/((a - b)*(-8*a + 3*b - 4*b*C
os[2*(c + d*x)] + b*Cos[4*(c + d*x)])^2))/(64*a^3*d)
```

Maple [B] time = 0.206, size = 1959, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a-b*sin(d*x+c)^4)^3,x)
```

```
[Out] 15/16/d*b/(a^2-2*a*b+b^2)*a/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)
*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-15/16/d*b/(a^2-2*a*
b+b^2)*a/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan
(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-27/16/d/(tan(d*x+c)^4*a-tan(d*x+c)^4
*b+2*a*tan(d*x+c)^2+a)^2*b/(a^2-2*a*b+b^2)*tan(d*x+c)^3-1/d/a^3/tan(d*x+c)-
27/16/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2*b/(a^2-2*a*b+b
^2)*tan(d*x+c)^5-15/32/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)
^2/a^2/(a-b)*tan(d*x+c)^7*b^2+13/32/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*ta
n(d*x+c)^2+a)^2/a/(a^2-2*a*b+b^2)*tan(d*x+c)^3*b^2+57/32/d/a/(a^2-2*a*b+b^2
)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(
((a*b)^(1/2)+a)*(a-b))^(1/2))*b^3-57/32/d/a/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(a-
b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)
*(a-b))^(1/2))*b^3+13/16/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+
a)^2/a^2*b^3/(a^2-2*a*b+b^2)*tan(d*x+c)^5-9/16/d/(tan(d*x+c)^4*a-tan(d*x+c)
^4*b+2*a*tan(d*x+c)^2+a)^2/(a-b)/a*b*tan(d*x+c)^7-33/64/d/a^2/(a^2-2*a*b+b^
2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/
(((a*b)^(1/2)+a)*(a-b))^(1/2))*b^4+33/64/d/a^2/(a^2-2*a*b+b^2)/(a*b)^(1/2)/
(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)
-a)*(a-b))^(1/2))*b^4-189/64/d/a*b^2/(a^2-2*a*b+b^2)/(a-b)/(((a*b)^(1/2)+a)
*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-189/64
/d/a*b^2/(a^2-2*a*b+b^2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)
*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))+39/32/d*b/(a^2-2*a*b+b^2)/(a-b)/
(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b
))^(1/2))+39/32/d*b/(a^2-2*a*b+b^2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arc
tanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-141/64/d*b^2/(a^2-2*a
*b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*
x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+141/64/d*b^2/(a^2-2*a*b+b^2)/(a*b)^(1/2
)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/
2)-a)*(a-b))^(1/2))+39/16/d/a^2/(a^2-2*a*b+b^2)/(a-b)/(((a*b)^(1/2)+a)*(a-b
))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*b^3+39/16/d
/a^2/(a^2-2*a*b+b^2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan
(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*b^3-45/64/d*b^4/a^3/(a^2-2*a*b+b^2)/
(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)
-a)*(a-b))^(1/2))+1/8/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^
2/a/(a^2-2*a*b+b^2)*tan(d*x+c)^5*b^2-9/16/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+
2*a*tan(d*x+c)^2+a)^2/(a^2-2*a*b+b^2)*tan(d*x+c)*b+3/8/d/(tan(d*x+c)^4*a-ta
n(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2*b^2/a/(a^2-2*a*b+b^2)*tan(d*x+c)-45/64/d
*b^4/a^3/(a^2-2*a*b+b^2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*t
an(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+13/32/d*b^3/a^3/(tan(d*x+c)^4*a-ta
n(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2/(a-b)*tan(d*x+c)^7+17/32/d*b^3/a^2/(tan(
d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2/(a^2-2*a*b+b^2)*tan(d*x+c)^
3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a-b*sin(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 26.8953, size = 16077, normalized size = 45.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a-b*sin(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] -1/256*(8*(32*a^2*b^2 - 83*a*b^3 + 45*b^4)*cos(d*x + c)^9 - 48*(19*a^2*b^2
- 54*a*b^3 + 30*b^4)*cos(d*x + c)^7 - 8*(64*a^3*b - 301*a^2*b^2 + 555*a*b^3
 - 270*b^4)*cos(d*x + c)^5 + 16*(55*a^3*b - 188*a^2*b^2 + 235*a*b^3 - 90*b^
4)*cos(d*x + c)^3 + 3*((a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*cos(d*x + c)^8 - 4
*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*cos(d*x + c)^6 - 2*(a^6*b - 5*a^5*b^2 +
7*a^4*b^3 - 3*a^3*b^4)*d*cos(d*x + c)^4 + 4*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3
- a^3*b^4)*d*cos(d*x + c)^2 + (a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*
b^4)*d)*sqrt(-(400*a^4*b - 1044*a^3*b^2 + 1085*a^2*b^3 - 530*a*b^4 + 105*b^
5 - (a^11 - 5*a^10*b + 10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d^2*s
qrt((409600*a^8*b^3 - 2355200*a^7*b^4 + 6054400*a^6*b^5 - 9073120*a^5*b^6 +
 8661145*a^4*b^7 - 5389980*a^3*b^8 + 2135086*a^2*b^9 - 492300*a*b^10 + 5062
5*b^11)/((a^23 - 10*a^22*b + 45*a^21*b^2 - 120*a^20*b^3 + 210*a^19*b^4 - 25
2*a^18*b^5 + 210*a^17*b^6 - 120*a^16*b^7 + 45*a^15*b^8 - 10*a^14*b^9 + a^13
*b^10)*d^4)))/((a^11 - 5*a^10*b + 10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6
*b^5)*d^2))*log(1728000*a^6*b^2 - 7369920*a^5*b^3 + 13507020*a^4*b^4 - 1357
3305*a^3*b^5 + 31519503/4*a^2*b^6 - 5011875/2*a*b^7 + 1366875/4*b^8 - 27/4*
(256000*a^6*b^2 - 1091840*a^5*b^3 + 2001040*a^4*b^4 - 2010860*a^3*b^5 + 116
7389*a^2*b^6 - 371250*a*b^7 + 50625*b^8)*cos(d*x + c)^2 + 27/2*((26*a^17 -
167*a^16*b + 460*a^15*b^2 - 705*a^14*b^3 + 650*a^13*b^4 - 361*a^12*b^5 + 11
2*a^11*b^6 - 15*a^10*b^7)*d^3*sqrt((409600*a^8*b^3 - 2355200*a^7*b^4 + 6054
400*a^6*b^5 - 9073120*a^5*b^6 + 8661145*a^4*b^7 - 5389980*a^3*b^8 + 2135086
*a^2*b^9 - 492300*a*b^10 + 50625*b^11)/((a^23 - 10*a^22*b + 45*a^21*b^2 - 1
20*a^20*b^3 + 210*a^19*b^4 - 252*a^18*b^5 + 210*a^17*b^6 - 120*a^16*b^7 + 4
5*a^15*b^8 - 10*a^14*b^9 + a^13*b^10)*d^4))*cos(d*x + c)*sin(d*x + c) + (12
800*a^10*b - 54080*a^9*b^2 + 98420*a^8*b^3 - 98415*a^7*b^4 + 56973*a^6*b^5
- 18109*a^5*b^6 + 2475*a^4*b^7)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-(400*a^4
*b - 1044*a^3*b^2 + 1085*a^2*b^3 - 530*a*b^4 + 105*b^5 - (a^11 - 5*a^10*b +
 10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d^2*sqrt((409600*a^8*b^3 -
2355200*a^7*b^4 + 6054400*a^6*b^5 - 9073120*a^5*b^6 + 8661145*a^4*b^7 - 538
9980*a^3*b^8 + 2135086*a^2*b^9 - 492300*a*b^10 + 50625*b^11)/((a^23 - 10*a^
22*b + 45*a^21*b^2 - 120*a^20*b^3 + 210*a^19*b^4 - 252*a^18*b^5 + 210*a^17*
b^6 - 120*a^16*b^7 + 45*a^15*b^8 - 10*a^14*b^9 + a^13*b^10)*d^4)))/((a^11 -
 5*a^10*b + 10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d^2)) - 27/4*(2*
(400*a^14 - 2556*a^13*b + 7005*a^12*b^2 - 10685*a^11*b^3 + 9810*a^10*b^4 -
5430*a^9*b^5 + 1681*a^8*b^6 - 225*a^7*b^7)*d^2*cos(d*x + c)^2 - (400*a^14 -
 2556*a^13*b + 7005*a^12*b^2 - 10685*a^11*b^3 + 9810*a^10*b^4 - 5430*a^9*b^
5 + 1681*a^8*b^6 - 225*a^7*b^7)*d^2)*sqrt((409600*a^8*b^3 - 2355200*a^7*b^4
 + 6054400*a^6*b^5 - 9073120*a^5*b^6 + 8661145*a^4*b^7 - 5389980*a^3*b^8 +
2135086*a^2*b^9 - 492300*a*b^10 + 50625*b^11)/((a^23 - 10*a^22*b + 45*a^21*
b^2 - 120*a^20*b^3 + 210*a^19*b^4 - 252*a^18*b^5 + 210*a^17*b^6 - 120*a^16*
b^7 + 45*a^15*b^8 - 10*a^14*b^9 + a^13*b^10)*d^4)))*sin(d*x + c) - 3*((a^5*
b^2 - 2*a^4*b^3 + a^3*b^4)*d*cos(d*x + c)^8 - 4*(a^5*b^2 - 2*a^4*b^3 + a^3*
b^4)*d*cos(d*x + c)^6 - 2*(a^6*b - 5*a^5*b^2 + 7*a^4*b^3 - 3*a^3*b^4)*d*cos
(d*x + c)^4 + 4*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*d*cos(d*x + c)^2
+ (a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*d)*sqrt(-(400*a^4*b - 1
044*a^3*b^2 + 1085*a^2*b^3 - 530*a*b^4 + 105*b^5 - (a^11 - 5*a^10*b + 10*a^
9*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d^2*sqrt((409600*a^8*b^3 - 235520
0*a^7*b^4 + 6054400*a^6*b^5 - 9073120*a^5*b^6 + 8661145*a^4*b^7 - 5389980*a
^3*b^8 + 2135086*a^2*b^9 - 492300*a*b^10 + 50625*b^11)/((a^23 - 10*a^22*b +
 45*a^21*b^2 - 120*a^20*b^3 + 210*a^19*b^4 - 252*a^18*b^5 + 210*a^17*b^6 -
```

120*a^16*b^7 + 45*a^15*b^8 - 10*a^14*b^9 + a^13*b^10)*d^4)))/((a^11 - 5*a^1
0*b + 10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d^2))*log(1728000*a^6*
b^2 - 7369920*a^5*b^3 + 13507020*a^4*b^4 - 13573305*a^3*b^5 + 31519503/4*a^
2*b^6 - 5011875/2*a*b^7 + 1366875/4*b^8 - 27/4*(256000*a^6*b^2 - 1091840*a^
5*b^3 + 2001040*a^4*b^4 - 2010860*a^3*b^5 + 1167389*a^2*b^6 - 371250*a*b^7
+ 50625*b^8)*cos(d*x + c)^2 - 27/2*((26*a^17 - 167*a^16*b + 460*a^15*b^2 -
705*a^14*b^3 + 650*a^13*b^4 - 361*a^12*b^5 + 112*a^11*b^6 - 15*a^10*b^7)*d^
3*sqrt((409600*a^8*b^3 - 2355200*a^7*b^4 + 6054400*a^6*b^5 - 9073120*a^5*b^
6 + 8661145*a^4*b^7 - 5389980*a^3*b^8 + 2135086*a^2*b^9 - 492300*a*b^10 + 5
0625*b^11)/((a^23 - 10*a^22*b + 45*a^21*b^2 - 120*a^20*b^3 + 210*a^19*b^4 -
252*a^18*b^5 + 210*a^17*b^6 - 120*a^16*b^7 + 45*a^15*b^8 - 10*a^14*b^9 + a
^13*b^10)*d^4))*cos(d*x + c)*sin(d*x + c) + (12800*a^10*b - 54080*a^9*b^2 +
98420*a^8*b^3 - 98415*a^7*b^4 + 56973*a^6*b^5 - 18109*a^5*b^6 + 2475*a^4*b
^7)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-(400*a^4*b - 1044*a^3*b^2 + 1085*a^2
*b^3 - 530*a*b^4 + 105*b^5 - (a^11 - 5*a^10*b + 10*a^9*b^2 - 10*a^8*b^3 + 5
*a^7*b^4 - a^6*b^5)*d^2*sqrt((409600*a^8*b^3 - 2355200*a^7*b^4 + 6054400*a^
6*b^5 - 9073120*a^5*b^6 + 8661145*a^4*b^7 - 5389980*a^3*b^8 + 2135086*a^2*b
^9 - 492300*a*b^10 + 50625*b^11)/((a^23 - 10*a^22*b + 45*a^21*b^2 - 120*a^2
0*b^3 + 210*a^19*b^4 - 252*a^18*b^5 + 210*a^17*b^6 - 120*a^16*b^7 + 45*a^15
*b^8 - 10*a^14*b^9 + a^13*b^10)*d^4)))/((a^11 - 5*a^10*b + 10*a^9*b^2 - 10*
a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d^2)) - 27/4*(2*(400*a^14 - 2556*a^13*b + 70
05*a^12*b^2 - 10685*a^11*b^3 + 9810*a^10*b^4 - 5430*a^9*b^5 + 1681*a^8*b^6
- 225*a^7*b^7)*d^2*cos(d*x + c)^2 - (400*a^14 - 2556*a^13*b + 7005*a^12*b^2
- 10685*a^11*b^3 + 9810*a^10*b^4 - 5430*a^9*b^5 + 1681*a^8*b^6 - 225*a^7*b
^7)*d^2)*sqrt((409600*a^8*b^3 - 2355200*a^7*b^4 + 6054400*a^6*b^5 - 9073120
*a^5*b^6 + 8661145*a^4*b^7 - 5389980*a^3*b^8 + 2135086*a^2*b^9 - 492300*a*b
^10 + 50625*b^11)/((a^23 - 10*a^22*b + 45*a^21*b^2 - 120*a^20*b^3 + 210*a^1
9*b^4 - 252*a^18*b^5 + 210*a^17*b^6 - 120*a^16*b^7 + 45*a^15*b^8 - 10*a^14*
b^9 + a^13*b^10)*d^4)))*sin(d*x + c) + 3*((a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d
*cos(d*x + c)^8 - 4*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*cos(d*x + c)^6 - 2*(a
^6*b - 5*a^5*b^2 + 7*a^4*b^3 - 3*a^3*b^4)*d*cos(d*x + c)^4 + 4*(a^6*b - 3*a
^5*b^2 + 3*a^4*b^3 - a^3*b^4)*d*cos(d*x + c)^2 + (a^7 - 4*a^6*b + 6*a^5*b^2
- 4*a^4*b^3 + a^3*b^4)*d)*sqrt(-(400*a^4*b - 1044*a^3*b^2 + 1085*a^2*b^3 -
530*a*b^4 + 105*b^5 + (a^11 - 5*a^10*b + 10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b
^4 - a^6*b^5)*d^2*sqrt((409600*a^8*b^3 - 2355200*a^7*b^4 + 6054400*a^6*b^5
- 9073120*a^5*b^6 + 8661145*a^4*b^7 - 5389980*a^3*b^8 + 2135086*a^2*b^9 - 4
92300*a*b^10 + 50625*b^11)/((a^23 - 10*a^22*b + 45*a^21*b^2 - 120*a^20*b^3
+ 210*a^19*b^4 - 252*a^18*b^5 + 210*a^17*b^6 - 120*a^16*b^7 + 45*a^15*b^8 -
10*a^14*b^9 + a^13*b^10)*d^4)))/((a^11 - 5*a^10*b + 10*a^9*b^2 - 10*a^8*b^
3 + 5*a^7*b^4 - a^6*b^5)*d^2))*log(-1728000*a^6*b^2 + 7369920*a^5*b^3 - 135
07020*a^4*b^4 + 13573305*a^3*b^5 - 31519503/4*a^2*b^6 + 5011875/2*a*b^7 - 1
366875/4*b^8 + 27/4*(256000*a^6*b^2 - 1091840*a^5*b^3 + 2001040*a^4*b^4 - 2
010860*a^3*b^5 + 1167389*a^2*b^6 - 371250*a*b^7 + 50625*b^8)*cos(d*x + c)^2
+ 27/2*((26*a^17 - 167*a^16*b + 460*a^15*b^2 - 705*a^14*b^3 + 650*a^13*b^4
- 361*a^12*b^5 + 112*a^11*b^6 - 15*a^10*b^7)*d^3*sqrt((409600*a^8*b^3 - 23
55200*a^7*b^4 + 6054400*a^6*b^5 - 9073120*a^5*b^6 + 8661145*a^4*b^7 - 53899
80*a^3*b^8 + 2135086*a^2*b^9 - 492300*a*b^10 + 50625*b^11)/((a^23 - 10*a^22
*b + 45*a^21*b^2 - 120*a^20*b^3 + 210*a^19*b^4 - 252*a^18*b^5 + 210*a^17*b^
6 - 120*a^16*b^7 + 45*a^15*b^8 - 10*a^14*b^9 + a^13*b^10)*d^4))*cos(d*x + c
)*sin(d*x + c) - (12800*a^10*b - 54080*a^9*b^2 + 98420*a^8*b^3 - 98415*a^7*
b^4 + 56973*a^6*b^5 - 18109*a^5*b^6 + 2475*a^4*b^7)*d*cos(d*x + c)*sin(d*x
+ c))*sqrt(-(400*a^4*b - 1044*a^3*b^2 + 1085*a^2*b^3 - 530*a*b^4 + 105*b^5
+ (a^11 - 5*a^10*b + 10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d^2*sqr
t((409600*a^8*b^3 - 2355200*a^7*b^4 + 6054400*a^6*b^5 - 9073120*a^5*b^6 + 8
661145*a^4*b^7 - 5389980*a^3*b^8 + 2135086*a^2*b^9 - 492300*a*b^10 + 50625*
b^11)/((a^23 - 10*a^22*b + 45*a^21*b^2 - 120*a^20*b^3 + 210*a^19*b^4 - 252*
a^18*b^5 + 210*a^17*b^6 - 120*a^16*b^7 + 45*a^15*b^8 - 10*a^14*b^9 + a^13*b
^10)*d^4)))/((a^11 - 5*a^10*b + 10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6*b
^5)*d^2)) - 27/4*(2*(400*a^14 - 2556*a^13*b + 7005*a^12*b^2 - 10685*a^11*b^

```
3 + 9810*a^10*b^4 - 5430*a^9*b^5 + 1681*a^8*b^6 - 225*a^7*b^7)*d^2*cos(d*x
+ c)^2 - (400*a^14 - 2556*a^13*b + 7005*a^12*b^2 - 10685*a^11*b^3 + 9810*a^
10*b^4 - 5430*a^9*b^5 + 1681*a^8*b^6 - 225*a^7*b^7)*d^2)*sqrt((409600*a^8*b
^3 - 2355200*a^7*b^4 + 6054400*a^6*b^5 - 9073120*a^5*b^6 + 8661145*a^4*b^7
- 5389980*a^3*b^8 + 2135086*a^2*b^9 - 492300*a*b^10 + 50625*b^11)/((a^23 -
10*a^22*b + 45*a^21*b^2 - 120*a^20*b^3 + 210*a^19*b^4 - 252*a^18*b^5 + 210*
a^17*b^6 - 120*a^16*b^7 + 45*a^15*b^8 - 10*a^14*b^9 + a^13*b^10)*d^4)))*sin
(d*x + c) - 3*((a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*cos(d*x + c)^8 - 4*(a^5*b^
2 - 2*a^4*b^3 + a^3*b^4)*d*cos(d*x + c)^6 - 2*(a^6*b - 5*a^5*b^2 + 7*a^4*b^
3 - 3*a^3*b^4)*d*cos(d*x + c)^4 + 4*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^
4)*d*cos(d*x + c)^2 + (a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*d)*
sqrt(-(400*a^4*b - 1044*a^3*b^2 + 1085*a^2*b^3 - 530*a*b^4 + 105*b^5 + (a^1
1 - 5*a^10*b + 10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d^2*sqrt((409
600*a^8*b^3 - 2355200*a^7*b^4 + 6054400*a^6*b^5 - 9073120*a^5*b^6 + 8661145
*a^4*b^7 - 5389980*a^3*b^8 + 2135086*a^2*b^9 - 492300*a*b^10 + 50625*b^11)/
((a^23 - 10*a^22*b + 45*a^21*b^2 - 120*a^20*b^3 + 210*a^19*b^4 - 252*a^18*b
^5 + 210*a^17*b^6 - 120*a^16*b^7 + 45*a^15*b^8 - 10*a^14*b^9 + a^13*b^10)*d
^4)))/((a^11 - 5*a^10*b + 10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d^
2))*log(-1728000*a^6*b^2 + 7369920*a^5*b^3 - 13507020*a^4*b^4 + 13573305*a^
3*b^5 - 31519503/4*a^2*b^6 + 5011875/2*a*b^7 - 1366875/4*b^8 + 27/4*(256000
*a^6*b^2 - 1091840*a^5*b^3 + 2001040*a^4*b^4 - 2010860*a^3*b^5 + 1167389*a^
2*b^6 - 371250*a*b^7 + 50625*b^8)*cos(d*x + c)^2 - 27/2*((26*a^17 - 167*a^1
6*b + 460*a^15*b^2 - 705*a^14*b^3 + 650*a^13*b^4 - 361*a^12*b^5 + 112*a^11*
b^6 - 15*a^10*b^7)*d^3*sqrt((409600*a^8*b^3 - 2355200*a^7*b^4 + 6054400*a^6
*b^5 - 9073120*a^5*b^6 + 8661145*a^4*b^7 - 5389980*a^3*b^8 + 2135086*a^2*b^
9 - 492300*a*b^10 + 50625*b^11)/((a^23 - 10*a^22*b + 45*a^21*b^2 - 120*a^20
*b^3 + 210*a^19*b^4 - 252*a^18*b^5 + 210*a^17*b^6 - 120*a^16*b^7 + 45*a^15*
b^8 - 10*a^14*b^9 + a^13*b^10)*d^4))*cos(d*x + c)*sin(d*x + c) - (12800*a^1
0*b - 54080*a^9*b^2 + 98420*a^8*b^3 - 98415*a^7*b^4 + 56973*a^6*b^5 - 18109
*a^5*b^6 + 2475*a^4*b^7)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-(400*a^4*b - 10
44*a^3*b^2 + 1085*a^2*b^3 - 530*a*b^4 + 105*b^5 + (a^11 - 5*a^10*b + 10*a^9
*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d^2*sqrt((409600*a^8*b^3 - 2355200
*a^7*b^4 + 6054400*a^6*b^5 - 9073120*a^5*b^6 + 8661145*a^4*b^7 - 5389980*a^
3*b^8 + 2135086*a^2*b^9 - 492300*a*b^10 + 50625*b^11)/((a^23 - 10*a^22*b +
45*a^21*b^2 - 120*a^20*b^3 + 210*a^19*b^4 - 252*a^18*b^5 + 210*a^17*b^6 - 1
20*a^16*b^7 + 45*a^15*b^8 - 10*a^14*b^9 + a^13*b^10)*d^4)))/((a^11 - 5*a^10
*b + 10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d^2)) - 27/4*(2*(400*a^
14 - 2556*a^13*b + 7005*a^12*b^2 - 10685*a^11*b^3 + 9810*a^10*b^4 - 5430*a^
9*b^5 + 1681*a^8*b^6 - 225*a^7*b^7)*d^2*cos(d*x + c)^2 - (400*a^14 - 2556*a
^13*b + 7005*a^12*b^2 - 10685*a^11*b^3 + 9810*a^10*b^4 - 5430*a^9*b^5 + 168
1*a^8*b^6 - 225*a^7*b^7)*d^2)*sqrt((409600*a^8*b^3 - 2355200*a^7*b^4 + 6054
400*a^6*b^5 - 9073120*a^5*b^6 + 8661145*a^4*b^7 - 5389980*a^3*b^8 + 2135086
*a^2*b^9 - 492300*a*b^10 + 50625*b^11)/((a^23 - 10*a^22*b + 45*a^21*b^2 - 1
20*a^20*b^3 + 210*a^19*b^4 - 252*a^18*b^5 + 210*a^17*b^6 - 120*a^16*b^7 + 4
5*a^15*b^8 - 10*a^14*b^9 + a^13*b^10)*d^4)))*sin(d*x + c) + 8*(32*a^4 - 110
*a^3*b + 189*a^2*b^2 - 156*a*b^3 + 45*b^4)*cos(d*x + c))/(((a^5*b^2 - 2*a^4
*b^3 + a^3*b^4)*d*cos(d*x + c)^8 - 4*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*cos(
d*x + c)^6 - 2*(a^6*b - 5*a^5*b^2 + 7*a^4*b^3 - 3*a^3*b^4)*d*cos(d*x + c)^4
 + 4*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*d*cos(d*x + c)^2 + (a^7 - 4*
a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a-b*sin(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a-b*sin(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.236 $\int \frac{1}{1-\sin^4(x)}\,dx$

Optimal. Leaf size=25

$$\frac{\tan^{-1}\left(\sqrt{2}\tan(x)\right)}{2\sqrt{2}}+\frac{\tan(x)}{2}$$

```
[Out] ArcTan[Sqrt[2]*Tan[x]]/(2*Sqrt[2]) + Tan[x]/2
```

Rubi [A] time = 0.0192821, antiderivative size = 45, normalized size of antiderivative = 1.8, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3209, 388, 203}

$$\frac{x}{2\sqrt{2}}+\frac{\tan(x)}{2}+\frac{\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\sin^2(x)+\sqrt{2}+1}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sin[x]^4)^(-1),x]
```

```
[Out] x/(2*Sqrt[2]) + ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Sin[x]^2)]/(2*Sqrt[2]
) + Tan[x]/2
```

Rule 3209

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1-\sin^4(x)}\,dx &= \text{Subst}\left(\int \frac{1+x^2}{1+2x^2}\,dx,x,\tan(x)\right)\\ &= \frac{\tan(x)}{2}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx,x,\tan(x)\right)\\ &= \frac{x}{2\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)}{2\sqrt{2}}+\frac{\tan(x)}{2}\end{aligned}$$

Mathematica [A] time = 0.0525903, size = 24, normalized size = 0.96

$$\frac{1}{4}\left(\sqrt{2}\tan^{-1}\left(\sqrt{2}\tan(x)\right)+2\tan(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sin[x]^4)^(-1),x]
```

```
[Out] (Sqrt[2]*ArcTan[Sqrt[2]*Tan[x]] + 2*Tan[x])/4
```

Maple [A] time = 0.04, size = 18, normalized size = 0.7

$$\frac{\arctan\left(\sqrt{2}\tan(x)\right)\sqrt{2}}{4}+\frac{\tan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-sin(x)^4),x)
```

```
[Out] 1/4*arctan(2^(1/2)*tan(x))*2^(1/2)+1/2*tan(x)
```

Maxima [A] time = 1.41544, size = 23, normalized size = 0.92

$$\frac{1}{4}\sqrt{2}\arctan\left(\sqrt{2}\tan(x)\right)+\frac{1}{2}\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)^4),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(sqrt(2)*tan(x)) + 1/2*tan(x)
```

Fricas [B] time = 1.9956, size = 138, normalized size = 5.52

$$-\frac{\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(x)^2-2\sqrt{2}}{4\cos(x)\sin(x)}\right)\cos(x)-4\sin(x)}{8\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)^4),x, algorithm="fricas")
```

```
[Out] -1/8*(sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - 2*sqrt(2))/(cos(x)*sin(x)))*
cos(x) - 4*sin(x))/cos(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{1}{\sin^4(x)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)**4),x)
```

```
[Out] -Integral(1/(sin(x)**4 - 1), x)
```

Giac [B] time = 1.10003, size = 69, normalized size = 2.76

$$\frac{1}{4}\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin\left(2x\right)-2\sin\left(2x\right)}{\sqrt{2}\cos\left(2x\right)+\sqrt{2}-2\cos\left(2x\right)+2}\right)\right)+\frac{1}{2}\tan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)^4),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - 2*sin(2*x))/(sqrt(2)*cos(2*x)
+ sqrt(2) - 2*cos(2*x) + 2))) + 1/2*tan(x)
```

3.237 $\int \frac{1}{a+b\sin^4(x)}\,dx$

Optimal. Leaf size=487

$$-\frac{\left(\sqrt{a+b}+\sqrt{a}\right)\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}-\sqrt{2}(a+b)^{3/4}\tan(x)}{\sqrt[4]{a}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{a+b}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}+\frac{\left(\sqrt{a+b}+\sqrt{a}\right)\tan^{-1}\left(\frac{\sqrt{2}(a+b)^{3/4}\tan(x)+\sqrt[4]{a}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}{\sqrt[4]{a}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{a+b}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}+$$

```
[Out] -((Sqrt[a] + Sqrt[a + b])*ArcTan[(a^(1/4)*Sqrt[a + b - Sqrt[a]*Sqrt[a + b]]
 - Sqrt[2]*(a + b)^(3/4)*Tan[x])/(a^(1/4)*Sqrt[a + b + Sqrt[a]*Sqrt[a + b]]
)])/(2*Sqrt[2]*a^(3/4)*(a + b)^(1/4)*Sqrt[a + b + Sqrt[a]*Sqrt[a + b]]) + (
(Sqrt[a] + Sqrt[a + b])*ArcTan[(a^(1/4)*Sqrt[a + b - Sqrt[a]*Sqrt[a + b]] +
 Sqrt[2]*(a + b)^(3/4)*Tan[x])/(a^(1/4)*Sqrt[a + b + Sqrt[a]*Sqrt[a + b]])]
)/(2*Sqrt[2]*a^(3/4)*(a + b)^(1/4)*Sqrt[a + b + Sqrt[a]*Sqrt[a + b]]) + ((S
qrt[a] - Sqrt[a + b])*Log[Sqrt[a]*(a + b)^(1/4) - Sqrt[2]*a^(1/4)*Sqrt[a +
b - Sqrt[a]*Sqrt[a + b]]*Tan[x] + (a + b)^(3/4)*Tan[x]^2])/(4*Sqrt[2]*a^(3/
4)*(a + b)^(1/4)*Sqrt[a + b - Sqrt[a]*Sqrt[a + b]]) - ((Sqrt[a] - Sqrt[a +
b])*Log[Sqrt[a]*(a + b)^(1/4) + Sqrt[2]*a^(1/4)*Sqrt[a + b - Sqrt[a]*Sqrt[a
 + b]]*Tan[x] + (a + b)^(3/4)*Tan[x]^2])/(4*Sqrt[2]*a^(3/4)*(a + b)^(1/4)*S
qrt[a + b - Sqrt[a]*Sqrt[a + b]])
```

Rubi [A] time = 1.1374, antiderivative size = 487, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {3209, 1169, 634, 618, 204, 628}

$$-\frac{\left(\sqrt{a+b}+\sqrt{a}\right)\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}-\sqrt{2}(a+b)^{3/4}\tan(x)}{\sqrt[4]{a}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{a+b}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}+\frac{\left(\sqrt{a+b}+\sqrt{a}\right)\tan^{-1}\left(\frac{\sqrt{2}(a+b)^{3/4}\tan(x)+\sqrt[4]{a}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}}{\sqrt[4]{a}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{a+b}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[x]^4)^(-1),x]
```


```
[Out] -((Sqrt[a] + Sqrt[a + b])*ArcTan[(a^(1/4)*Sqrt[a + b - Sqrt[a]*Sqrt[a + b]]
 - Sqrt[2]*(a + b)^(3/4)*Tan[x])/(a^(1/4)*Sqrt[a + b + Sqrt[a]*Sqrt[a + b]]
)])/(2*Sqrt[2]*a^(3/4)*(a + b)^(1/4)*Sqrt[a + b + Sqrt[a]*Sqrt[a + b]]) + (
(Sqrt[a] + Sqrt[a + b])*ArcTan[(a^(1/4)*Sqrt[a + b - Sqrt[a]*Sqrt[a + b]] +
 Sqrt[2]*(a + b)^(3/4)*Tan[x])/(a^(1/4)*Sqrt[a + b + Sqrt[a]*Sqrt[a + b]])]
)/(2*Sqrt[2]*a^(3/4)*(a + b)^(1/4)*Sqrt[a + b + Sqrt[a]*Sqrt[a + b]]) + ((S
qrt[a] - Sqrt[a + b])*Log[Sqrt[a]*(a + b)^(1/4) - Sqrt[2]*a^(1/4)*Sqrt[a +
b - Sqrt[a]*Sqrt[a + b]]*Tan[x] + (a + b)^(3/4)*Tan[x]^2])/(4*Sqrt[2]*a^(3/
4)*(a + b)^(1/4)*Sqrt[a + b - Sqrt[a]*Sqrt[a + b]]) - ((Sqrt[a] - Sqrt[a +
b])*Log[Sqrt[a]*(a + b)^(1/4) + Sqrt[2]*a^(1/4)*Sqrt[a + b - Sqrt[a]*Sqrt[a
 + b]]*Tan[x] + (a + b)^(3/4)*Tan[x]^2])/(4*Sqrt[2]*a^(3/4)*(a + b)^(1/4)*S
qrt[a + b - Sqrt[a]*Sqrt[a + b]])
```


Rule 3209

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{a+b\sin^4(x)}\,dx = \text{Subst}\left(\int \frac{1+x^2}{a+2ax^2+(a+b)x^4}\,dx, x, \tan(x)\right)$$

$$= \frac{\sqrt[4]{a+b}\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}}{(a+b)^{3/4}}-\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)x}{\frac{\sqrt{a}}{\sqrt{a+b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}\,x}{(a+b)^{3/4}}+x^2}\,dx, x, \tan(x)\right)}{2\sqrt{2}a^{3/4}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}} + \frac{\sqrt[4]{a+b}\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}}{(a+b)^{3/4}}+\left(1\cdots\right.}{\frac{\sqrt{a}}{\sqrt{a+b}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a+b-\sqrt{a}\cdots}}{(a+b)^{3/4}}}\cdots\right.}{2\sqrt{2}a^{3/4}\sqrt{a+b-\sqrt{a}\cdots}}$$

$$= \frac{\left(1+\frac{\sqrt{a+b}}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{a+b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}\,x}{(a+b)^{3/4}}+x^2}\,dx, x, \tan(x)\right)}{4(a+b)} + \frac{\left(1+\frac{\sqrt{a+b}}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{\cdots}{\frac{\sqrt{a}}{\sqrt{a+b}}+\frac{\sqrt{2}\sqrt[4]{a}\cdots}{\cdots}}\cdots\right.}{4(a+b\cdots}$$

$$= -\frac{\sqrt[4]{a+b}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\log\left(\sqrt{a}\sqrt[4]{a+b}-\sqrt{2}\sqrt[4]{a}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}\tan(x)+(a+b)^{3/4}\tan^2(x)\right)}{4\sqrt{2}a^{3/4}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}} - \cdots$$

$$= -\frac{\left(\sqrt{a}+\sqrt{a+b}\right)\tan^{-1}\left(\frac{(a+b)^{3/4}\left(\frac{\sqrt[4]{a}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}}{(a+b)^{3/4}}-\sqrt{2}\tan(x)\right)}{\sqrt[4]{a}\sqrt{a+b+\sqrt{a}\sqrt{a+b}}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{a+b}\sqrt{a+b+\sqrt{a}\sqrt{a+b}}} + \frac{\left(\sqrt{a}+\sqrt{a+b}\right)\tan^{-1}\left(\frac{(a+b)^{3/4}\left(\frac{\sqrt[4]{a}\sqrt{a+b-\cdots}}{(a+b)\cdots}\cdots\right.}{\sqrt[4]{a}\sqrt{a+b\cdots}}\cdots\right.}{2\sqrt{2}a^{3/4}\sqrt[4]{a+b}\sqrt{a+b+\sqrt{a}\cdots}}$$

Mathematica [C] time = 0.299524, size = 148, normalized size = 0.3

$$\frac{\left(\sqrt{a}-i\sqrt{b}\right)\sqrt{a+i\sqrt{a}\sqrt{b}}\tan^{-1}\left(\frac{\sqrt{a+i\sqrt{a}\sqrt{b}}\tan(x)}{\sqrt{a}}\right)-\left(\sqrt{a}+i\sqrt{b}\right)\sqrt{-a+i\sqrt{a}\sqrt{b}}\tanh^{-1}\left(\frac{\sqrt{-a+i\sqrt{a}\sqrt{b}}\tan(x)}{\sqrt{a}}\right)}{2a(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[x]^4)^(-1),x]
```

```
[Out] ((Sqrt[a] - I*Sqrt[b])*Sqrt[a + I*Sqrt[a]*Sqrt[b]]*ArcTan[(Sqrt[a + I*Sqrt[
a]*Sqrt[b]]*Tan[x])/Sqrt[a]] - (Sqrt[a] + I*Sqrt[b])*Sqrt[-a + I*Sqrt[a]*Sq
rt[b]]*ArcTanh[(Sqrt[-a + I*Sqrt[a]*Sqrt[b]]*Tan[x])/Sqrt[a]])/(2*a*(a + b)
)
```

Maple [B] time = 0.169, size = 1677, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(x)^4),x)
```

```
[Out] -1/8/b/(a+b)^(1/2)*ln((a+b)^(1/2)*tan(x)^2+tan(x)*(2*(a*(a+b))^(1/2)-2*a)^(
1/2)+a^(1/2))*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-1/8/a/b/(a+b)^(1/2)*ln((a+b)^(1
/2)*tan(x)^2+tan(x)*(2*(a*(a+b))^(1/2)-2*a)^(1/2)+a^(1/2))*(a^2+a*b)^(1/2)*
(2*(a^2+a*b)^(1/2)-2*a)^(1/2)+1/8/a^(3/2)/b*ln((a+b)^(1/2)*tan(x)^2+tan(x)*
(2*(a*(a+b))^(1/2)-2*a)^(1/2)+a^(1/2))*(a^2+a*b)^(1/2)*(2*(a^2+a*b)^(1/2)-2
```

```
*a)^(1/2)+1/8/a^(1/2)/b*ln((a+b)^(1/2)*tan(x)^2+tan(x)*(2*(a*(a+b))^(1/2)-2
*a)^(1/2)+a^(1/2))*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)+1/a^(1/2)/(4*a^(1/2)*(a+b)
^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((2*(a+b)^(1/2)*tan(x)+(2*(a*(a+b
))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))+1
/4/b/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((2*(a+b)^(1
/2)*tan(x)+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b)
)^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)/(a+b)^(1/2)*(2*(a^2+a*b)^
(1/2)-2*a)^(1/2)+1/4/a/b/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2
)*arctan((2*(a+b)^(1/2)*tan(x)+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a
+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)/(a+b)
^(1/2)*(a^2+a*b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-1/4/a^(3/2)/b/(4*a^(1/
2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((2*(a+b)^(1/2)*tan(x)+(2
*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^
(1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a
)^(1/2)-1/4/a^(1/2)/b/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*a
rctan((2*(a+b)^(1/2)*tan(x)+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)
^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)*(2*(a^2+
a*b)^(1/2)-2*a)^(1/2)+1/8/b/(a+b)^(1/2)*ln(-(a+b)^(1/2)*tan(x)^2+tan(x)*(2*
(a*(a+b))^(1/2)-2*a)^(1/2)-a^(1/2))*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)+1/8/a/b/(
a+b)^(1/2)*ln(-(a+b)^(1/2)*tan(x)^2+tan(x)*(2*(a*(a+b))^(1/2)-2*a)^(1/2)-a^
(1/2))*(a^2+a*b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-1/8/a^(3/2)/b*ln(-(a+b
)^(1/2)*tan(x)^2+tan(x)*(2*(a*(a+b))^(1/2)-2*a)^(1/2)-a^(1/2))*(a^2+a*b)^(1
/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-1/8/a^(1/2)/b*ln(-(a+b)^(1/2)*tan(x)^2+ta
n(x)*(2*(a*(a+b))^(1/2)-2*a)^(1/2)-a^(1/2))*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-1
/a^(1/2)/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((-2*(a+
b)^(1/2)*tan(x)+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*
(a+b))^(1/2)+2*a)^(1/2))-1/4/b/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a
)^(1/2)*arctan((-2*(a+b)^(1/2)*tan(x)+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(
1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2
)/(a+b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-1/4/a/b/(4*a^(1/2)*(a+b)^(1/2)-
2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((-2*(a+b)^(1/2)*tan(x)+(2*(a*(a+b))^(1/
2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(
a+b))^(1/2)-2*a)^(1/2)/(a+b)^(1/2)*(a^2+a*b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^
(1/2)+1/4/a^(3/2)/b/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arc
tan((-2*(a+b)^(1/2)*tan(x)+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^
(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)*(a^2+a*b)
^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)+1/4/a^(1/2)/b/(4*a^(1/2)*(a+b)^(1/2)-2
*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((-2*(a+b)^(1/2)*tan(x)+(2*(a*(a+b))^(1/2
)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a
+b))^(1/2)-2*a)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b\sin(x)^4 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)^4),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*sin(x)^4 + a), x)
```

Fricas [B] time = 3.2451, size = 1893, normalized size = 3.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)^4),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(-((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) + 1)/(a^2 + a*b)
)*log(1/4*b*cos(x)^2 + 1/2*(a*b*cos(x)*sin(x) + (a^4 + a^3*b)*sqrt(-b/(a^5
+ 2*a^4*b + a^3*b^2))*cos(x)*sin(x))*sqrt(-((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^
4*b + a^3*b^2)) + 1)/(a^2 + a*b)) - 1/4*(a^3 + a^2*b - 2*(a^3 + a^2*b)*cos(
x)^2)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) - 1/4*b) + 1/8*sqrt(-((a^2 + a*b)*
sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) + 1)/(a^2 + a*b))*log(1/4*b*cos(x)^2 - 1
/2*(a*b*cos(x)*sin(x) + (a^4 + a^3*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2))*co
s(x)*sin(x))*sqrt(-((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) + 1)/(a^
2 + a*b)) - 1/4*(a^3 + a^2*b - 2*(a^3 + a^2*b)*cos(x)^2)*sqrt(-b/(a^5 + 2*a
^4*b + a^3*b^2)) - 1/4*b) + 1/8*sqrt(((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^4*b +
a^3*b^2)) - 1)/(a^2 + a*b))*log(-1/4*b*cos(x)^2 + 1/2*(a*b*cos(x)*sin(x) -
(a^4 + a^3*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2))*cos(x)*sin(x))*sqrt(((a^2
+ a*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) - 1)/(a^2 + a*b)) - 1/4*(a^3 + a^
2*b - 2*(a^3 + a^2*b)*cos(x)^2)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) + 1/4*b)
 - 1/8*sqrt(((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) - 1)/(a^2 + a*b
))*log(-1/4*b*cos(x)^2 - 1/2*(a*b*cos(x)*sin(x) - (a^4 + a^3*b)*sqrt(-b/(a^
5 + 2*a^4*b + a^3*b^2))*cos(x)*sin(x))*sqrt(((a^2 + a*b)*sqrt(-b/(a^5 + 2*a
^4*b + a^3*b^2)) - 1)/(a^2 + a*b)) - 1/4*(a^3 + a^2*b - 2*(a^3 + a^2*b)*cos
(x)^2)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) + 1/4*b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)**4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)^4),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.238 $\int \frac{1}{1+\sin^4(x)}\,dx$

Optimal. Leaf size=309

$$\frac{x}{2\sqrt{\sqrt{2}-1}}-\frac{1}{8}\sqrt{\sqrt{2}-1}\log\left(2\tan^2(x)-2\sqrt{\sqrt{2}-1}\tan(x)+\sqrt{2}\right)+\frac{1}{8}\sqrt{\sqrt{2}-1}\log\left(\sqrt{2}\tan^2(x)+\sqrt{2\left(\sqrt{2}-1\right)}\tan(x)+\right.$$

```
[Out] x/(2*Sqrt[-1 + Sqrt[2]]) + ArcTan[(Sqrt[-1 + Sqrt[2]] - 2*Sqrt[-1 + Sqrt[2]
]*Cos[x]^2 - (-2 + Sqrt[2])*Cos[x]*Sin[x])/(2 + Sqrt[1 + Sqrt[2]] + (-2 + S
qrt[2])*Cos[x]^2 - 2*Sqrt[-1 + Sqrt[2]]*Cos[x]*Sin[x])]/(4*Sqrt[-1 + Sqrt[2
]]) - ArcTan[(Sqrt[-1 + Sqrt[2]] - 2*Sqrt[-1 + Sqrt[2]]*Cos[x]^2 + (-2 + Sq
rt[2])*Cos[x]*Sin[x])/(2 + Sqrt[1 + Sqrt[2]] + (-2 + Sqrt[2])*Cos[x]^2 + 2*
Sqrt[-1 + Sqrt[2]]*Cos[x]*Sin[x])]/(4*Sqrt[-1 + Sqrt[2]]) - (Sqrt[-1 + Sqrt
[2]]*Log[Sqrt[2] - 2*Sqrt[-1 + Sqrt[2]]*Tan[x] + 2*Tan[x]^2])/8 + (Sqrt[-1
+ Sqrt[2]]*Log[1 + Sqrt[2*(-1 + Sqrt[2])]*Tan[x] + Sqrt[2]*Tan[x]^2])/8
```

Rubi [A] time = 0.202052, antiderivative size = 309, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {3209, 1169, 634, 618, 204, 628}

$$\frac{x}{2\sqrt{\sqrt{2}-1}}-\frac{1}{8}\sqrt{\sqrt{2}-1}\log\left(2\tan^2(x)-2\sqrt{\sqrt{2}-1}\tan(x)+\sqrt{2}\right)+\frac{1}{8}\sqrt{\sqrt{2}-1}\log\left(\sqrt{2}\tan^2(x)+\sqrt{2\left(\sqrt{2}-1\right)}\tan(x)+\right.$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sin[x]^4)^(-1),x]
```

```
[Out] x/(2*Sqrt[-1 + Sqrt[2]]) + ArcTan[(Sqrt[-1 + Sqrt[2]] - 2*Sqrt[-1 + Sqrt[2]
]*Cos[x]^2 - (-2 + Sqrt[2])*Cos[x]*Sin[x])/(2 + Sqrt[1 + Sqrt[2]] + (-2 + S
qrt[2])*Cos[x]^2 - 2*Sqrt[-1 + Sqrt[2]]*Cos[x]*Sin[x])]/(4*Sqrt[-1 + Sqrt[2
]]) - ArcTan[(Sqrt[-1 + Sqrt[2]] - 2*Sqrt[-1 + Sqrt[2]]*Cos[x]^2 + (-2 + Sq
rt[2])*Cos[x]*Sin[x])/(2 + Sqrt[1 + Sqrt[2]] + (-2 + Sqrt[2])*Cos[x]^2 + 2*
Sqrt[-1 + Sqrt[2]]*Cos[x]*Sin[x])]/(4*Sqrt[-1 + Sqrt[2]]) - (Sqrt[-1 + Sqrt
[2]]*Log[Sqrt[2] - 2*Sqrt[-1 + Sqrt[2]]*Tan[x] + 2*Tan[x]^2])/8 + (Sqrt[-1
+ Sqrt[2]]*Log[1 + Sqrt[2*(-1 + Sqrt[2])]*Tan[x] + Sqrt[2]*Tan[x]^2])/8
```

Rule 3209

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{1+\sin^4(x)}\,dx = \text{Subst}\left(\int \frac{1+x^2}{1+2x^2+2x^4}\,dx, x, \tan(x)\right)$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{-1+\sqrt{2}}-\left(1-\frac{1}{\sqrt{2}}\right)x}{\frac{1}{\sqrt{2}}-\sqrt{-1+\sqrt{2}}x+x^2}\,dx, x, \tan(x)\right)}{2\sqrt{2\left(-1+\sqrt{2}\right)}} + \frac{\text{Subst}\left(\int \frac{\sqrt{-1+\sqrt{2}}+\left(1-\frac{1}{\sqrt{2}}\right)x}{\frac{1}{\sqrt{2}}+\sqrt{-1+\sqrt{2}}x+x^2}\,dx, x, \tan(x)\right)}{2\sqrt{2\left(-1+\sqrt{2}\right)}}$$

$$= -\left(\frac{1}{8}\sqrt{-1+\sqrt{2}}\,\text{Subst}\left(\int \frac{-\sqrt{-1+\sqrt{2}}+2x}{\frac{1}{\sqrt{2}}-\sqrt{-1+\sqrt{2}}x+x^2}\,dx, x, \tan(x)\right)\right) + \frac{1}{8}\sqrt{-1+\sqrt{2}}\,\text{Subst}\left(\int \frac{\sqrt{}}{\frac{1}{\sqrt{2}}+}\right.$$

$$= -\frac{1}{8}\sqrt{-1+\sqrt{2}}\log\left(\sqrt{2}-2\sqrt{-1+\sqrt{2}}\tan(x)+2\tan^2(x)\right) + \frac{1}{8}\sqrt{-1+\sqrt{2}}\log\left(1+\sqrt{2\left(-1+\sqrt{2}\right)}\right.$$

$$= \frac{1}{2}\sqrt{1+\sqrt{2}}x + \frac{1}{4}\sqrt{1+\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{-1+\sqrt{2}}-2\sqrt{-1+\sqrt{2}}\cos^2(x)+\left(2-\sqrt{2}\right)\cos(x)\sin(x)}{2+\sqrt{1+\sqrt{2}}-\left(2-\sqrt{2}\right)\cos^2(x)-2\sqrt{-1+\sqrt{2}}\cos(x)\sin(x)}\right)$$

Mathematica [C] time = 0.0734058, size = 45, normalized size = 0.15

$$\frac{\tan^{-1}\left(\sqrt{1-i}\tan(x)\right)}{2\sqrt{1-i}} + \frac{\tan^{-1}\left(\sqrt{1+i}\tan(x)\right)}{2\sqrt{1+i}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sin[x]^4)^(-1),x]
```

```
[Out] ArcTan[Sqrt[1 - I]*Tan[x]]/(2*Sqrt[1 - I]) + ArcTan[Sqrt[1 + I]*Tan[x]]/(2*
Sqrt[1 + I])
```

Maple [A] time = 0.151, size = 239, normalized size = 0.8

$$\frac{\sqrt{2}\sqrt{-2+2\sqrt{2}}\ln\left(\sqrt{2}+\sqrt{-2+2\sqrt{2}}\sqrt{2}\tan(x)+2(\tan(x))^2\right)}{16}+\frac{\sqrt{2}}{4\sqrt{1+\sqrt{2}}}\arctan\left(\frac{\sqrt{2}\sqrt{-2+2\sqrt{2}}+4\tan(x)}{2\sqrt{1+\sqrt{2}}}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+sin(x)^4),x)
```

```
[Out] 1/16*2^(1/2)*(-2+2*2^(1/2))^(1/2)*ln(2^(1/2)+(-2+2*2^(1/2))^(1/2)*2^(1/2)*t
an(x)+2*tan(x)^2)+1/4/(1+2^(1/2))^(1/2)*arctan(1/2*(2^(1/2)*(-2+2*2^(1/2))^
(1/2)+4*tan(x))/(1+2^(1/2))^(1/2))*2^(1/2)+1/4/(1+2^(1/2))^(1/2)*arctan(1/2
*(2^(1/2)*(-2+2*2^(1/2))^(1/2)+4*tan(x))/(1+2^(1/2))^(1/2))-1/16*2^(1/2)*(-
2+2*2^(1/2))^(1/2)*ln(-(-2+2*2^(1/2))^(1/2)*2^(1/2)*tan(x)+2*tan(x)^2+2^(1/
2))+1/4/(1+2^(1/2))^(1/2)*arctan(1/2*(-2^(1/2)*(-2+2*2^(1/2))^(1/2)+4*tan(x
))/(1+2^(1/2))^(1/2))*2^(1/2)+1/4/(1+2^(1/2))^(1/2)*arctan(1/2*(-2^(1/2)*(-
2+2*2^(1/2))^(1/2)+4*tan(x))/(1+2^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sin(x)^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)^4),x, algorithm="maxima")
```

```
[Out] integrate(1/(sin(x)^4 + 1), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)**4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sin(x)^4 + 1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)^4),x, algorithm="giac")
```

```
[Out] integrate(1/(sin(x)^4 + 1), x)
```

3.239 $\int \sin(c+dx)\sqrt{a+b\sin^4(c+dx)}\,dx$

<u>**Optimal**</u>. Leaf size=477

$$-\frac{\cos(c+dx)\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}{3d}+\frac{2\sqrt{b}\cos(c+dx)\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}{3d\sqrt{a+b}\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)}+-$$

```
[Out] -(Cos[c + d*x]*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4])/(3*d) +
 (2*Sqrt[b]*Cos[c + d*x]*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4
])/(3*Sqrt[a + b]*d*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])) - (2*b^(1/4
)*(a + b)^(3/4)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])*Sqrt[(a + b - 2*
b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4)/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x]^2)
/Sqrt[a + b])^2)]*EllipticE[2*ArcTan[(b^(1/4)*Cos[c + d*x])/(a + b)^(1/4)],
 (1 + Sqrt[b]/Sqrt[a + b])/2])/(3*d*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos
[c + d*x]^4]) + ((a + b)^(3/4)*(Sqrt[b] - Sqrt[a + b])*(1 + (Sqrt[b]*Cos[c
+ d*x]^2)/Sqrt[a + b])*Sqrt[(a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4)
/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])^2)]*EllipticF[2*ArcTan
[(b^(1/4)*Cos[c + d*x])/(a + b)^(1/4)], (1 + Sqrt[b]/Sqrt[a + b])/2])/(3*b^
(1/4)*d*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4])
```

<u>**Rubi**</u> **[A]** time = 0.388257, antiderivative size = 477, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3215, 1091, 1197, 1103, 1195}

$$-\frac{\cos(c+dx)\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}{3d}+\frac{2\sqrt{b}\cos(c+dx)\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}{3d\sqrt{a+b}\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)}+-$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] -(Cos[c + d*x]*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4])/(3*d) +
 (2*Sqrt[b]*Cos[c + d*x]*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4
])/(3*Sqrt[a + b]*d*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])) - (2*b^(1/4
)*(a + b)^(3/4)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])*Sqrt[(a + b - 2*
b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4)/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x]^2)
/Sqrt[a + b])^2)]*EllipticE[2*ArcTan[(b^(1/4)*Cos[c + d*x])/(a + b)^(1/4)],
 (1 + Sqrt[b]/Sqrt[a + b])/2])/(3*d*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos
[c + d*x]^4]) + ((a + b)^(3/4)*(Sqrt[b] - Sqrt[a + b])*(1 + (Sqrt[b]*Cos[c
+ d*x]^2)/Sqrt[a + b])*Sqrt[(a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4)
/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])^2)]*EllipticF[2*ArcTan
[(b^(1/4)*Cos[c + d*x])/(a + b)^(1/4)], (1 + Sqrt[b]/Sqrt[a + b])/2])/(3*b^
(1/4)*d*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4])
```

<u>Rule 3215</u>

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
```

```
1)/2]
```

Rule 1091

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*
x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \sin(c+dx)\sqrt{a+b\sin^4(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \sqrt{a+b-2bx^2+bx^4}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\cos(c+dx)\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}{3d} - \frac{\text{Subst}\left(\int \frac{2(a+b)-2b}{\sqrt{a+b-2bx^2}}\right.}{}$$

$$= -\frac{\cos(c+dx)\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}{3d} - \frac{\left(2\sqrt{b}\sqrt{a+b}\right)\text{Subst}}{}$$

$$= -\frac{\cos(c+dx)\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}{3d} + \frac{2\sqrt{b}\cos(c+dx)\sqrt{a}}{3\sqrt{a}}$$

Mathematica [C] time = 31.5928, size = 47242, normalized size = 99.04

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]*Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.686, size = 439, normalized size = 0.9

$$-\frac{1}{4d}\left(\frac{4\cos(dx+c)}{3}\sqrt{a+b-2b(\cos(dx+c))^2+b(\cos(dx+c))^4}+4\frac{2/3a+2/3b}{\sqrt{a+b-2b(\cos(dx+c))^2+b(\cos(dx+c))^4}}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x)
```

```
[Out] -1/4/d*(4/3*cos(d*x+c)*(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)+4*(2/3*a
+2/3*b)/((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2)*(1-(I*a^(1/2)*b^(1/2)+b)/(a+b)*
cos(d*x+c)^2)^(1/2)*(1+(I*a^(1/2)*b^(1/2)-b)/(a+b)*cos(d*x+c)^2)^(1/2)/(a+b
-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)*EllipticF(cos(d*x+c)*((I*a^(1/2)*b^
(1/2)+b)/(a+b))^(1/2),(-1-2*(I*a^(1/2)*b^(1/2)-b)/(a+b))^(1/2))+16/3*b*(a+b
)/((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2)*(1-(I*a^(1/2)*b^(1/2)+b)/(a+b)*cos(d*
x+c)^2)^(1/2)*(1+(I*a^(1/2)*b^(1/2)-b)/(a+b)*cos(d*x+c)^2)^(1/2)/(a+b-2*b*c
os(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)/(-2*b+2*I*a^(1/2)*b^(1/2))*(EllipticF(cos
(d*x+c)*((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2),(-1-2*(I*a^(1/2)*b^(1/2)-b)/(a+
b))^(1/2))-EllipticE(cos(d*x+c)*((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2),(-1-2*(
I*a^(1/2)*b^(1/2)-b)/(a+b))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin(dx+c)^4+a}\sin(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(d*x + c)^4 + a)*sin(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\cos(dx+c)^4-2b\cos(dx+c)^2+a+b}\sin(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)*sin(d*x + c),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*sin(d*x+c)**4)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin{(dx+c)}^4 + a}\sin{(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(d*x + c)^4 + a)*sin(d*x + c), x)
```

3.240 $\int \csc(c+dx)\sqrt{a+b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=521

$$\frac{\sqrt{b}\cos(c+dx)\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}{d\sqrt{a+b}\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)}+\frac{\sqrt{-a}\tan^{-1}\left(\frac{\sqrt{-a}\cos(c+dx)}{\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}\right)}{2d}-\frac{\sqrt[4]{b}(a+b)^{3/4}\left(\frac{\sqrt{b}}{}\right.}{}$$

```
[Out] (Sqrt[-a]*ArcTan[(Sqrt[-a]*Cos[c + d*x])/Sqrt[a + b - 2*b*Cos[c + d*x]^2 +
b*Cos[c + d*x]^4]])/(2*d) + (Sqrt[b]*Cos[c + d*x]*Sqrt[a + b - 2*b*Cos[c +
d*x]^2 + b*Cos[c + d*x]^4])/(Sqrt[a + b]*d*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sq
rt[a + b])) - (b^(1/4)*(a + b)^(3/4)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a +
 b])*Sqrt[(a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4)/((a + b)*(1 + (Sq
rt[b]*Cos[c + d*x]^2)/Sqrt[a + b])^2)]*EllipticE[2*ArcTan[(b^(1/4)*Cos[c +
d*x])/(a + b)^(1/4)], (1 + Sqrt[b]/Sqrt[a + b])/2])/(d*Sqrt[a + b - 2*b*Cos
[c + d*x]^2 + b*Cos[c + d*x]^4]) - ((a + b)^(1/4)*(Sqrt[b] - Sqrt[a + b])^2
*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])*Sqrt[(a + b - 2*b*Cos[c + d*x]^
2 + b*Cos[c + d*x]^4)/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])^2
)]*EllipticPi[(Sqrt[b] + Sqrt[a + b])^2/(4*Sqrt[b]*Sqrt[a + b]), 2*ArcTan[(
b^(1/4)*Cos[c + d*x])/(a + b)^(1/4)], (1 + Sqrt[b]/Sqrt[a + b])/2])/(4*b^(1
/4)*d*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4])
```

Rubi [A] time = 0.678119, antiderivative size = 521, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3215, 1208, 1197, 1103, 1195, 1216, 1706}

$$\frac{\sqrt{b}\cos(c+dx)\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}{d\sqrt{a+b}\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)}+\frac{\sqrt{-a}\tan^{-1}\left(\frac{\sqrt{-a}\cos(c+dx)}{\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}\right)}{2d}-\frac{\sqrt[4]{b}(a+b)^{3/4}\left(\frac{\sqrt{b}}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] (Sqrt[-a]*ArcTan[(Sqrt[-a]*Cos[c + d*x])/Sqrt[a + b - 2*b*Cos[c + d*x]^2 +
b*Cos[c + d*x]^4]])/(2*d) + (Sqrt[b]*Cos[c + d*x]*Sqrt[a + b - 2*b*Cos[c +
d*x]^2 + b*Cos[c + d*x]^4])/(Sqrt[a + b]*d*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sq
rt[a + b])) - (b^(1/4)*(a + b)^(3/4)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a +
 b])*Sqrt[(a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4)/((a + b)*(1 + (Sq
rt[b]*Cos[c + d*x]^2)/Sqrt[a + b])^2)]*EllipticE[2*ArcTan[(b^(1/4)*Cos[c +
d*x])/(a + b)^(1/4)], (1 + Sqrt[b]/Sqrt[a + b])/2])/(d*Sqrt[a + b - 2*b*Cos
[c + d*x]^2 + b*Cos[c + d*x]^4]) - ((a + b)^(1/4)*(Sqrt[b] - Sqrt[a + b])^2
*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])*Sqrt[(a + b - 2*b*Cos[c + d*x]^
2 + b*Cos[c + d*x]^4)/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])^2
)]*EllipticPi[(Sqrt[b] + Sqrt[a + b])^2/(4*Sqrt[b]*Sqrt[a + b]), 2*ArcTan[(
b^(1/4)*Cos[c + d*x])/(a + b)^(1/4)], (1 + Sqrt[b]/Sqrt[a + b])/2])/(4*b^(1
/4)*d*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4])
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
```

```
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1208

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \csc(c+dx)\sqrt{a+b\sin^4(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{\sqrt{a+b-2bx^2+bx^4}}{1-x^2}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{-b+bx^2}{\sqrt{a+b-2bx^2+bx^4}}\,dx, x, \cos(c+dx)\right)}{d} - \frac{a\,\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+b-2bx^2+bx^4}}\,dx, x \right.}{d}$$

$$= -\frac{\left(\sqrt{b}\sqrt{a+b}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a+b}}}{\sqrt{a+b-2bx^2+bx^4}}\,dx, x, \cos(c+dx)\right)}{d} + \frac{\left((a+b)\left(-1+\frac{\sqrt{b}}{\sqrt{a+b}}\right)\right)}{}$$

$$= \frac{\sqrt{-a}\tan^{-1}\left(\frac{\sqrt{-a}\cos(c+dx)}{\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}\right)}{2d} + \frac{\sqrt{b}\cos(c+dx)\sqrt{a+b-2b\cos^2(c+}}{\sqrt{a+b}d\left(1+\frac{\sqrt{b}\cos^2(c}{\sqrt{a+b}}\right.}$$

Mathematica [C] time = 31.9618, size = 118912, normalized size = 228.24

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]*Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.81, size = 0, normalized size = 0.

$$\int \csc(dx+c)\sqrt{a+b(\sin(dx+c))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x)
```

```
[Out] int(csc(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin(dx+c)^4+a}\,\csc(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(d*x + c)^4 + a)*csc(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\cos\left(dx+c\right)^{4}-2\,b\cos\left(dx+c\right)^{2}+a+b}\csc\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)*csc(d*x + c),
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a+b\sin^{4}\left(c+dx\right)}\csc\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sin(d*x+c)**4)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(c + d*x)**4)*csc(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sin\left(dx+c\right)^{4}+a}\csc\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(d*x + c)^4 + a)*csc(d*x + c), x)
```

3.241 $\int \frac{\sin^5(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

Optimal. Leaf size=484

$$\frac{\sqrt[4]{a+b}\left(2\sqrt{b}\sqrt{a+b}+a-2b\right)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)\sqrt{\frac{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}{(a+b)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt[4]{a+b}}\right)|\frac{1}{2}\left(\frac{\sqrt{b}}{\sqrt{a+b}}+1\right)\right)}{6b^{5/4}d\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}} - \cdots 2($$

```
[Out] -(Cos[c + d*x]*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4])/(3*b*d)
 + (2*Cos[c + d*x]*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4])/(3*
Sqrt[b]*Sqrt[a + b]*d*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])) - (2*(a +
 b)^(3/4)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])*Sqrt[(a + b - 2*b*Cos[
c + d*x]^2 + b*Cos[c + d*x]^4)/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[
a + b])^2)]*EllipticE[2*ArcTan[(b^(1/4)*Cos[c + d*x])/(a + b)^(1/4)], (1 +
Sqrt[b]/Sqrt[a + b])/2])/(3*b^(3/4)*d*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*C
os[c + d*x]^4]) + ((a + b)^(1/4)*(a - 2*b + 2*Sqrt[b]*Sqrt[a + b])*(1 + (Sq
rt[b]*Cos[c + d*x]^2)/Sqrt[a + b])*Sqrt[(a + b - 2*b*Cos[c + d*x]^2 + b*Cos
[c + d*x]^4)/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])^2)]*Ellipt
icF[2*ArcTan[(b^(1/4)*Cos[c + d*x])/(a + b)^(1/4)], (1 + Sqrt[b]/Sqrt[a + b
])/2])/(6*b^(5/4)*d*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4])
```

Rubi [A] time = 0.425803, antiderivative size = 484, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3215, 1206, 1197, 1103, 1195}

$$\frac{\sqrt[4]{a+b}\left(2\sqrt{b}\sqrt{a+b}+a-2b\right)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)\sqrt{\frac{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}{(a+b)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt[4]{a+b}}\right)|\frac{1}{2}\left(\frac{\sqrt{b}}{\sqrt{a+b}}+1\right)\right)}{6b^{5/4}d\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}} - \cdots 2($$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^5/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] -(Cos[c + d*x]*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4])/(3*b*d)
 + (2*Cos[c + d*x]*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4])/(3*
Sqrt[b]*Sqrt[a + b]*d*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])) - (2*(a +
 b)^(3/4)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])*Sqrt[(a + b - 2*b*Cos[
c + d*x]^2 + b*Cos[c + d*x]^4)/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[
a + b])^2)]*EllipticE[2*ArcTan[(b^(1/4)*Cos[c + d*x])/(a + b)^(1/4)], (1 +
Sqrt[b]/Sqrt[a + b])/2])/(3*b^(3/4)*d*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*C
os[c + d*x]^4]) + ((a + b)^(1/4)*(a - 2*b + 2*Sqrt[b]*Sqrt[a + b])*(1 + (Sq
rt[b]*Cos[c + d*x]^2)/Sqrt[a + b])*Sqrt[(a + b - 2*b*Cos[c + d*x]^2 + b*Cos
[c + d*x]^4)/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])^2)]*Ellipt
icF[2*ArcTan[(b^(1/4)*Cos[c + d*x])/(a + b)^(1/4)], (1 + Sqrt[b]/Sqrt[a + b
])/2])/(6*b^(5/4)*d*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4])
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
```

```
1)/2]
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\sin^5(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\sqrt{a+b-2bx^2+bx^4}}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\cos(c+dx)\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}{3bd} - \frac{\text{Subst}\left(\int \frac{-a+2b-2bx^2}{\sqrt{a+b-2bx^2+bx^4}}\,dx, x, \text{c}\right.}{3bd}$$

$$= -\frac{\cos(c+dx)\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}{3bd} - \frac{\left(2\sqrt{a+b}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a+b}}}{\sqrt{a+b-2bx^2\ldots}}\right.}{3\sqrt{b}d}$$

$$= -\frac{\cos(c+dx)\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}{3bd} + \frac{2\cos(c+dx)\sqrt{a+b-2b\cos^2}}{3\sqrt{b}\sqrt{a+b}d\left(1+\right.}$$

Mathematica [C] time = 31.7204, size = 47246, normalized size = 97.62

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]^5/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.468, size = 837, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5/(a+b*sin(d*x+c)^4)^(1/2),x)
```

```
[Out] -1/d/((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2)*(1-(I*a^(1/2)*b^(1/2)+b)/(a+b)*cos
(d*x+c)^2)^(1/2)*(1+(I*a^(1/2)*b^(1/2)-b)/(a+b)*cos(d*x+c)^2)^(1/2)/(a+b-2*
b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)*EllipticF(cos(d*x+c)*((I*a^(1/2)*b^(1/
2)+b)/(a+b))^(1/2),(-1-2*(I*a^(1/2)*b^(1/2)-b)/(a+b))^(1/2))-4/d*(a+b)/((I*
a^(1/2)*b^(1/2)+b)/(a+b))^(1/2)*(1-(I*a^(1/2)*b^(1/2)+b)/(a+b)*cos(d*x+c)^2
)^(1/2)*(1+(I*a^(1/2)*b^(1/2)-b)/(a+b)*cos(d*x+c)^2)^(1/2)/(a+b-2*b*cos(d*x
+c)^2+b*cos(d*x+c)^4)^(1/2)/(-2*b+2*I*a^(1/2)*b^(1/2))*(EllipticF(cos(d*x+c
)*((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2),(-1-2*(I*a^(1/2)*b^(1/2)-b)/(a+b))^(1
/2))-EllipticE(cos(d*x+c)*((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2),(-1-2*(I*a^(1
/2)*b^(1/2)-b)/(a+b))^(1/2)))-4/d*(1/12/b*cos(d*x+c)*(a+b-2*b*cos(d*x+c)^2+
b*cos(d*x+c)^4)^(1/2)-1/12*(a+b)/b/((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2)*(1-(
I*a^(1/2)*b^(1/2)+b)/(a+b)*cos(d*x+c)^2)^(1/2)*(1+(I*a^(1/2)*b^(1/2)-b)/(a+
b)*cos(d*x+c)^2)^(1/2)/(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)*Elliptic
F(cos(d*x+c)*((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2),(-1-2*(I*a^(1/2)*b^(1/2)-b
)/(a+b))^(1/2))-2/3*(a+b)/((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2)*(1-(I*a^(1/2)
*b^(1/2)+b)/(a+b)*cos(d*x+c)^2)^(1/2)*(1+(I*a^(1/2)*b^(1/2)-b)/(a+b)*cos(d*
x+c)^2)^(1/2)/(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)/(-2*b+2*I*a^(1/2)
*b^(1/2))*(EllipticF(cos(d*x+c)*((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2),(-1-2*(
I*a^(1/2)*b^(1/2)-b)/(a+b))^(1/2))-EllipticE(cos(d*x+c)*((I*a^(1/2)*b^(1/2)
+b)/(a+b))^(1/2),(-1-2*(I*a^(1/2)*b^(1/2)-b)/(a+b))^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)^5}{\sqrt{b\sin(dx+c)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^5/sqrt(b*sin(d*x + c)^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\cos(dx+c)^4-2\cos(dx+c)^2+1\right)\sin(dx+c)}{\sqrt{b\cos(dx+c)^4-2\,b\cos(dx+c)^2+a+b}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*sin(d*x + c)/sqrt(b*cos(d*
x + c)^4 - 2*b*cos(d*x + c)^2 + a + b), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5/(a+b*sin(d*x+c)**4)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)^5}{\sqrt{b\sin(dx+c)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^5/sqrt(b*sin(d*x + c)^4 + a), x)
```

3.242 $\int \frac{\sin^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

Optimal. Leaf size=431

$$-\frac{\sqrt[4]{a+b}\left(\sqrt{b}-\sqrt{a+b}\right)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)\sqrt{\frac{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}{(a+b)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt[4]{a+b}}\right)|\frac{1}{2}\left(\frac{\sqrt{b}}{\sqrt{a+b}}+1\right)\right)}{2b^{3/4}d\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}-\frac{(a+b)^{3/}}{}$$

```
[Out] (Cos[c + d*x]*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4])/(Sqrt[b]
*Sqrt[a + b]*d*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])) - ((a + b)^(3/4)
*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])*Sqrt[(a + b - 2*b*Cos[c + d*x]^
2 + b*Cos[c + d*x]^4)/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])^2
)]*EllipticE[2*ArcTan[(b^(1/4)*Cos[c + d*x])/(a + b)^(1/4)], (1 + Sqrt[b]/S
qrt[a + b])/2])/(b^(3/4)*d*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]
^4]) - ((a + b)^(1/4)*(Sqrt[b] - Sqrt[a + b])*(1 + (Sqrt[b]*Cos[c + d*x]^2)
/Sqrt[a + b])*Sqrt[(a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4)/((a + b)
*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])^2)]*EllipticF[2*ArcTan[(b^(1/4)
*Cos[c + d*x])/(a + b)^(1/4)], (1 + Sqrt[b]/Sqrt[a + b])/2])/(2*b^(3/4)*d*S
qrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4])
```

Rubi [A] time = 0.302493, antiderivative size = 431, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3215, 1197, 1103, 1195}

$$-\frac{\sqrt[4]{a+b}\left(\sqrt{b}-\sqrt{a+b}\right)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)\sqrt{\frac{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}{(a+b)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt[4]{a+b}}\right)|\frac{1}{2}\left(\frac{\sqrt{b}}{\sqrt{a+b}}+1\right)\right)}{2b^{3/4}d\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}-\frac{(a+b)^{3/}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/Sqrt[a + b*Sin[c + d*x]^4],x]
```


```
[Out] (Cos[c + d*x]*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4])/(Sqrt[b]
*Sqrt[a + b]*d*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])) - ((a + b)^(3/4)
*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])*Sqrt[(a + b - 2*b*Cos[c + d*x]^
2 + b*Cos[c + d*x]^4)/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])^2
)]*EllipticE[2*ArcTan[(b^(1/4)*Cos[c + d*x])/(a + b)^(1/4)], (1 + Sqrt[b]/S
qrt[a + b])/2])/(b^(3/4)*d*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]
^4]) - ((a + b)^(1/4)*(Sqrt[b] - Sqrt[a + b])*(1 + (Sqrt[b]*Cos[c + d*x]^2)
/Sqrt[a + b])*Sqrt[(a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4)/((a + b)
*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])^2)]*EllipticF[2*ArcTan[(b^(1/4)
*Cos[c + d*x])/(a + b)^(1/4)], (1 + Sqrt[b]/Sqrt[a + b])/2])/(2*b^(3/4)*d*S
qrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4])
```


Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{\sqrt{a+b-2bx^2+bx^4}}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\sqrt{a+b}\,\text{Subst}\left(\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a+b}}}{\sqrt{a+b-2bx^2+bx^4}}\,dx, x, \cos(c+dx)\right)}{\sqrt{b}d} - \frac{\left(1-\frac{\sqrt{a+b}}{\sqrt{b}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+b-2bx^2+bx^4}}\right.}{d}$$

$$= \frac{\cos(c+dx)\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}{\sqrt{b}\sqrt{a+b}d\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)} - \frac{(a+b)^{3/4}\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)\sqrt{\frac{a-}{}}}{b^{3/4}d\sqrt{a}}$$

Mathematica [C] time = 31.8742, size = 89374, normalized size = 207.36

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]^3/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.525, size = 398, normalized size = 0.9

$$-\frac{1}{d}\sqrt{1-\frac{(\cos(dx+c))^2}{a+b}\left(i\sqrt{a}\sqrt{b}+b\right)}\sqrt{1+\frac{(\cos(dx+c))^2}{a+b}\left(i\sqrt{a}\sqrt{b}-b\right)}\,\text{EllipticF}\left(\cos(dx+c)\sqrt{\frac{1}{a+b}\left(i\sqrt{a}\sqrt{b}+b\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x)
```

```
[Out] -1/d/((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2)*(1-(I*a^(1/2)*b^(1/2)+b)/(a+b)*cos
(d*x+c)^2)^(1/2)*(1+(I*a^(1/2)*b^(1/2)-b)/(a+b)*cos(d*x+c)^2)^(1/2)/(a+b-2*
b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)*EllipticF(cos(d*x+c)*((I*a^(1/2)*b^(1/
2)+b)/(a+b))^(1/2),(-1-2*(I*a^(1/2)*b^(1/2)-b)/(a+b))^(1/2))-2/d*(a+b)/((I*
a^(1/2)*b^(1/2)+b)/(a+b))^(1/2)*(1-(I*a^(1/2)*b^(1/2)+b)/(a+b)*cos(d*x+c)^2
)^(1/2)*(1+(I*a^(1/2)*b^(1/2)-b)/(a+b)*cos(d*x+c)^2)^(1/2)/(a+b-2*b*cos(d*x
+c)^2+b*cos(d*x+c)^4)^(1/2)/(-2*b+2*I*a^(1/2)*b^(1/2))*(EllipticF(cos(d*x+c
)*((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2),(-1-2*(I*a^(1/2)*b^(1/2)-b)/(a+b))^(1
/2))-EllipticE(cos(d*x+c)*((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2),(-1-2*(I*a^(1
/2)*b^(1/2)-b)/(a+b))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(dx+c\right)^3}{\sqrt{b\sin\left(dx+c\right)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^3/sqrt(b*sin(d*x + c)^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(\cos\left(dx+c\right)^2-1\right)\sin\left(dx+c\right)}{\sqrt{b\cos\left(dx+c\right)^4-2\,b\cos\left(dx+c\right)^2+a+b}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(d*x + c)^2 - 1)*sin(d*x + c)/sqrt(b*cos(d*x + c)^4 - 2*b*cos
(d*x + c)^2 + a + b), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+b*sin(d*x+c)**4)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.243 $\int \frac{\sin(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

Optimal. Leaf size=171

$$-\frac{\sqrt[4]{a+b}\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)\sqrt{\frac{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}{(a+b)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt[4]{a+b}}\right)|\frac{1}{2}\left(\frac{\sqrt{b}}{\sqrt{a+b}}+1\right)\right)}{2\sqrt[4]{b}d\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}$$

```
[Out] -((a + b)^(1/4)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])*Sqrt[(a + b - 2*
b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4)/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x]^2)
/Sqrt[a + b])^2)]*EllipticF[2*ArcTan[(b^(1/4)*Cos[c + d*x])/(a + b)^(1/4)],
 (1 + Sqrt[b]/Sqrt[a + b])/2])/(2*b^(1/4)*d*Sqrt[a + b - 2*b*Cos[c + d*x]^2
 + b*Cos[c + d*x]^4])
```

Rubi [A] time = 0.104504, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3215, 1103}

$$-\frac{\sqrt[4]{a+b}\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)\sqrt{\frac{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}{(a+b)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt[4]{a+b}}\right)|\frac{1}{2}\left(\frac{\sqrt{b}}{\sqrt{a+b}}+1\right)\right)}{2\sqrt[4]{b}d\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] -((a + b)^(1/4)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])*Sqrt[(a + b - 2*
b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4)/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x]^2)
/Sqrt[a + b])^2)]*EllipticF[2*ArcTan[(b^(1/4)*Cos[c + d*x])/(a + b)^(1/4)],
 (1 + Sqrt[b]/Sqrt[a + b])/2])/(2*b^(1/4)*d*Sqrt[a + b - 2*b*Cos[c + d*x]^2
 + b*Cos[c + d*x]^4])
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b-2bx^2+bx^4}}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\sqrt[4]{a+b}\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)\sqrt{\frac{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}{(a+b)\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt[4]{a+b}}\right)|\frac{1}{2}\left(1+\right.\right.}{2\sqrt[4]{b}d\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}$$

Mathematica [C] time = 25.2087, size = 13300, normalized size = 77.78

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.349, size = 163, normalized size = 1.

$$-\frac{1}{d}\sqrt{1-\frac{(\cos(dx+c))^2}{a+b}\left(i\sqrt{a}\sqrt{b}+b\right)}\sqrt{1+\frac{(\cos(dx+c))^2}{a+b}\left(i\sqrt{a}\sqrt{b}-b\right)}\text{EllipticF}\left(\cos(dx+c)\sqrt{\frac{1}{a+b}\left(i\sqrt{a}\sqrt{b}+b\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x)
```

```
[Out] -1/d/((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2)*(1-(I*a^(1/2)*b^(1/2)+b)/(a+b)*cos
(d*x+c)^2)^(1/2)*(1+(I*a^(1/2)*b^(1/2)-b)/(a+b)*cos(d*x+c)^2)^(1/2)/(a+b-2*
b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)*EllipticF(cos(d*x+c)*((I*a^(1/2)*b^(1/
2)+b)/(a+b))^(1/2),(-1-2*(I*a^(1/2)*b^(1/2)-b)/(a+b))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{\sqrt{b\sin(dx+c)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/sqrt(b*sin(d*x + c)^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sin(dx+c)}{\sqrt{b\cos(dx+c)^4-2b\cos(dx+c)^2+a+b}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sin(d*x + c)/sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)**4)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{\sqrt{b\sin(dx+c)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)/sqrt(b*sin(d*x + c)^4 + a), x)
```

3.244 $\int \frac{\csc(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

Optimal. Leaf size=469

$$-\frac{\tan^{-1}\left(\frac{\sqrt{-a}\cos(c+dx)}{\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}\right)}{2\sqrt{-a}d}+\frac{\sqrt[4]{b}\sqrt[4]{a+b}\left(\sqrt{b}-\sqrt{a+b}\right)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)\sqrt{\frac{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}{(a+b)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)^2}}F\left(2\right.}{2ad\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+}}$$

```
[Out] -ArcTan[(Sqrt[-a]*Cos[c + d*x])/Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c +
 d*x]^4]]/(2*Sqrt[-a]*d) + (b^(1/4)*(a + b)^(1/4)*(Sqrt[b] - Sqrt[a + b])*(
1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])*Sqrt[(a + b - 2*b*Cos[c + d*x]^2
+ b*Cos[c + d*x]^4)/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])^2)]
*EllipticF[2*ArcTan[(b^(1/4)*Cos[c + d*x])/(a + b)^(1/4)], (1 + Sqrt[b]/Sqr
t[a + b])/2])/(2*a*d*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4]) -
 ((a + b)^(1/4)*(Sqrt[b] - Sqrt[a + b])^2*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqr
t[a + b])*Sqrt[(a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4)/((a + b)*(1
+ (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])^2)]*EllipticPi[(Sqrt[b] + Sqrt[a +
b])^2/(4*Sqrt[b]*Sqrt[a + b]), 2*ArcTan[(b^(1/4)*Cos[c + d*x])/(a + b)^(1/4
)], (1 + Sqrt[b]/Sqrt[a + b])/2])/(4*a*b^(1/4)*d*Sqrt[a + b - 2*b*Cos[c + d
*x]^2 + b*Cos[c + d*x]^4])
```

Rubi [A] time = 0.434311, antiderivative size = 469, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3215, 1216, 1103, 1706}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{-a}\cos(c+dx)}{\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}\right)}{2\sqrt{-a}d}+\frac{\sqrt[4]{b}\sqrt[4]{a+b}\left(\sqrt{b}-\sqrt{a+b}\right)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)\sqrt{\frac{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}{(a+b)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)^2}}F\left(2\right.}{2ad\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] -ArcTan[(Sqrt[-a]*Cos[c + d*x])/Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c +
 d*x]^4]]/(2*Sqrt[-a]*d) + (b^(1/4)*(a + b)^(1/4)*(Sqrt[b] - Sqrt[a + b])*(
1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])*Sqrt[(a + b - 2*b*Cos[c + d*x]^2
+ b*Cos[c + d*x]^4)/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])^2)]
*EllipticF[2*ArcTan[(b^(1/4)*Cos[c + d*x])/(a + b)^(1/4)], (1 + Sqrt[b]/Sqr
t[a + b])/2])/(2*a*d*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4]) -
 ((a + b)^(1/4)*(Sqrt[b] - Sqrt[a + b])^2*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqr
t[a + b])*Sqrt[(a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4)/((a + b)*(1
+ (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])^2)]*EllipticPi[(Sqrt[b] + Sqrt[a +
b])^2/(4*Sqrt[b]*Sqrt[a + b]), 2*ArcTan[(b^(1/4)*Cos[c + d*x])/(a + b)^(1/4
)], (1 + Sqrt[b]/Sqrt[a + b])/2])/(4*a*b^(1/4)*d*Sqrt[a + b - 2*b*Cos[c + d
*x]^2 + b*Cos[c + d*x]^4])
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{\csc(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+b-2bx^2+bx^4}}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= \frac{\left((a+b)\left(-1+\frac{\sqrt{b}}{\sqrt{a+b}}\right)\right)\text{Subst}\left(\int \frac{1+\frac{\sqrt{b}x^2}{\sqrt{a+b}}}{(1-x^2)\sqrt{a+b-2bx^2+bx^4}}\,dx, x, \cos(c+dx)\right)}{ad} + \frac{\left(\sqrt{b}\left(\sqrt{b}-\sqrt{a+b}\right)\cdots\right.}{\cdots}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{-a}\cos(c+dx)}{\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}\right)}{2\sqrt{-a}d} + \frac{\sqrt[4]{b}\sqrt[4]{a+b}\left(\sqrt{b}-\sqrt{a+b}\right)\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)\sqrt{\frac{a+b-\cdots}{\cdots}}}{2ad\sqrt{a+b-2b\cos\cdots}}$$

Mathematica [C] time = 31.4792, size = 63281, normalized size = 134.93

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.75, size = 0, normalized size = 0.

$$\int \csc(dx+c)\frac{1}{\sqrt{a+b(\sin(dx+c))^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x)
```

```
[Out] int(csc(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(dx+c)}{\sqrt{b\sin(dx+c)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(d*x + c)/sqrt(b*sin(d*x + c)^4 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)**4)**(1/2),x)
```

```
[Out] Integral(csc(c + d*x)/sqrt(a + b*sin(c + d*x)**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(dx+c)}{\sqrt{b\sin(dx+c)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(d*x + c)/sqrt(b*sin(d*x + c)^4 + a), x)
```

3.245 $\int \frac{\csc^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

Optimal. Leaf size=776

$$-\frac{\sqrt{b}\cos(c+dx)\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}{2ad\sqrt{a+b}\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{-a}\cos(c+dx)}{\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}\right)}{4\sqrt{-a}d}-\frac{\cot(c+dx)\csc(c$$

```
[Out] -ArcTan[(Sqrt[-a]*Cos[c + d*x])/Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c +
 d*x]^4]]/(4*Sqrt[-a]*d) - (Sqrt[b]*Cos[c + d*x]*Sqrt[a + b - 2*b*Cos[c + d
*x]^2 + b*Cos[c + d*x]^4])/(2*a*Sqrt[a + b]*d*(1 + (Sqrt[b]*Cos[c + d*x]^2)
/Sqrt[a + b])) - (Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4]*Cot[c
 + d*x]*Csc[c + d*x])/(2*a*d) + (b^(1/4)*(a + b)^(3/4)*(1 + (Sqrt[b]*Cos[c
+ d*x]^2)/Sqrt[a + b])*Sqrt[(a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4)
/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])^2)]*EllipticE[2*ArcTan
[(b^(1/4)*Cos[c + d*x])/(a + b)^(1/4)], (1 + Sqrt[b]/Sqrt[a + b])/2])/(2*a*
d*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4]) - (b^(1/4)*(a + b -
Sqrt[b]*Sqrt[a + b])*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])*Sqrt[(a + b
 - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4)/((a + b)*(1 + (Sqrt[b]*Cos[c + d*
x]^2)/Sqrt[a + b])^2)]*EllipticF[2*ArcTan[(b^(1/4)*Cos[c + d*x])/(a + b)^(1
/4)], (1 + Sqrt[b]/Sqrt[a + b])/2])/(2*a*(a + b)^(1/4)*d*Sqrt[a + b - 2*b*C
os[c + d*x]^2 + b*Cos[c + d*x]^4]) - ((a + b)^(1/4)*(Sqrt[b] - Sqrt[a + b])
^2*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])*Sqrt[(a + b - 2*b*Cos[c + d*x
]^2 + b*Cos[c + d*x]^4)/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])
^2)]*EllipticPi[(Sqrt[b] + Sqrt[a + b])^2/(4*Sqrt[b]*Sqrt[a + b]), 2*ArcTan
[(b^(1/4)*Cos[c + d*x])/(a + b)^(1/4)], (1 + Sqrt[b]/Sqrt[a + b])/2])/(8*a*
b^(1/4)*d*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4])
```

Rubi [A] time = 1.04339, antiderivative size = 776, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3215, 1223, 1714, 1195, 1708, 1103, 1706}

$$-\frac{\sqrt{b}\cos(c+dx)\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}{2ad\sqrt{a+b}\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{-a}\cos(c+dx)}{\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}\right)}{4\sqrt{-a}d}-\frac{\cot(c+dx)\csc(c$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/Sqrt[a + b*Sin[c + d*x]^4],x]
```


```
[Out] -ArcTan[(Sqrt[-a]*Cos[c + d*x])/Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c +
 d*x]^4]]/(4*Sqrt[-a]*d) - (Sqrt[b]*Cos[c + d*x]*Sqrt[a + b - 2*b*Cos[c + d
*x]^2 + b*Cos[c + d*x]^4])/(2*a*Sqrt[a + b]*d*(1 + (Sqrt[b]*Cos[c + d*x]^2)
/Sqrt[a + b])) - (Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4]*Cot[c
 + d*x]*Csc[c + d*x])/(2*a*d) + (b^(1/4)*(a + b)^(3/4)*(1 + (Sqrt[b]*Cos[c
+ d*x]^2)/Sqrt[a + b])*Sqrt[(a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4)
/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])^2)]*EllipticE[2*ArcTan
[(b^(1/4)*Cos[c + d*x])/(a + b)^(1/4)], (1 + Sqrt[b]/Sqrt[a + b])/2])/(2*a*
d*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4]) - (b^(1/4)*(a + b -
Sqrt[b]*Sqrt[a + b])*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])*Sqrt[(a + b
 - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4)/((a + b)*(1 + (Sqrt[b]*Cos[c + d*
x]^2)/Sqrt[a + b])^2)]*EllipticF[2*ArcTan[(b^(1/4)*Cos[c + d*x])/(a + b)^(1
```

```
/4)], (1 + Sqrt[b]/Sqrt[a + b])/2])/(2*a*(a + b)^(1/4)*d*Sqrt[a + b - 2*b*C
os[c + d*x]^2 + b*Cos[c + d*x]^4]) - ((a + b)^(1/4)*(Sqrt[b] - Sqrt[a + b])
^2*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])*Sqrt[(a + b - 2*b*Cos[c + d*x
]^2 + b*Cos[c + d*x]^4)/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])
^2)]*EllipticPi[(Sqrt[b] + Sqrt[a + b])^2/(4*Sqrt[b]*Sqrt[a + b]), 2*ArcTan
[(b^(1/4)*Cos[c + d*x])/(a + b)^(1/4)], (1 + Sqrt[b]/Sqrt[a + b])/2])/(8*a*
b^(1/4)*d*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4])
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1714

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, -Dist[C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1708

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[(a*(B*d - A*e)*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
```

```
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2\sqrt{a+b-2bx^2+bx^4}}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}\cot(c+dx)\csc(c+dx)}{2ad} + \frac{\text{Subst}\left(\int \frac{-a+b-}{\left(1-x^2\right)\sqrt{a}}\right.}{}$$

$$= -\frac{\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}\cot(c+dx)\csc(c+dx)}{2ad} - \frac{\text{Subst}\left(\int \frac{-b(-a+b)+}{}\right.}{}$$

$$= -\frac{\sqrt{b}\cos(c+dx)\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}{2a\sqrt{a+b}d\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)} - \frac{\sqrt{a+b-2b\cos^2(c+dx)+}}{}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{-a}\cos(c+dx)}{\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}\right)}{4\sqrt{-a}d} - \frac{\sqrt{b}\cos(c+dx)\sqrt{a+b-2b\cos^2(c+dx)+b\cos}}{2a\sqrt{a+b}d\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)}$$

Mathematica [C] time = 32.7293, size = 119171, normalized size = 153.57

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^3/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.727, size = 0, normalized size = 0.

$$\int (\csc(dx+c))^3 \frac{1}{\sqrt{a+b(\sin(dx+c))^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x)
```

```
[Out] int(csc(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(dx+c)^3}{\sqrt{b\sin(dx+c)^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(d*x + c)^3/sqrt(b*sin(d*x + c)^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc(dx+c)^3}{\sqrt{b\cos(dx+c)^4 - 2\,b\cos(dx+c)^2 + a + b}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(csc(d*x + c)^3/sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^3(c+dx)}{\sqrt{a + b\sin^4(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a+b*sin(d*x+c)**4)**(1/2),x)
```

```
[Out] Integral(csc(c + d*x)**3/sqrt(a + b*sin(c + d*x)**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(dx+c)^3}{\sqrt{b\sin(dx+c)^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(d*x + c)^3/sqrt(b*sin(d*x + c)^4 + a), x)
```

3.246 $\int \frac{\sin^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

Optimal. Leaf size=499

$$-\frac{\cos^2(c+dx)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}}\right)\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}}{2\sqrt{b}d\sqrt{a+b\sin^4(c+dx)}}-\frac{\sqrt[4]{a}\left(\sqrt{a+b}+\sqrt{a}\right)\mathrm{co}}{}$$

```
[Out] -(ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c
 + d*x]^4]]*Cos[c + d*x]^2*Sqrt[a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*
x]^4])/(2*Sqrt[b]*d*Sqrt[a + b*Sin[c + d*x]^4]) - (a^(1/4)*(Sqrt[a] + Sqrt[
a + b])*Cos[c + d*x]^2*EllipticF[2*ArcTan[((a + b)^(1/4)*Tan[c + d*x])/a^(1
/4)], (1 - Sqrt[a]/Sqrt[a + b])/2]*(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2)*S
qrt[(a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4)/(Sqrt[a] + Sqrt[a + b
]*Tan[c + d*x]^2)^2])/(2*b*(a + b)^(1/4)*d*Sqrt[a + b*Sin[c + d*x]^4]) + ((
Sqrt[a] + Sqrt[a + b])^2*Cos[c + d*x]^2*EllipticPi[-(Sqrt[a] - Sqrt[a + b])
^2/(4*Sqrt[a]*Sqrt[a + b]), 2*ArcTan[((a + b)^(1/4)*Tan[c + d*x])/a^(1/4)],
 (1 - Sqrt[a]/Sqrt[a + b])/2]*(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2)*Sqrt[(
a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4)/(Sqrt[a] + Sqrt[a + b]*Tan
[c + d*x]^2)^2])/(4*a^(1/4)*b*(a + b)^(1/4)*d*Sqrt[a + b*Sin[c + d*x]^4])
```

Rubi [A] time = 0.664941, antiderivative size = 499, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3219, 1319, 1103, 1706}

$$-\frac{\cos^2(c+dx)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}}\right)\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}}{2\sqrt{b}d\sqrt{a+b\sin^4(c+dx)}}-\frac{\sqrt[4]{a}\left(\sqrt{a+b}+\sqrt{a}\right)\mathrm{co}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] -(ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c
 + d*x]^4]]*Cos[c + d*x]^2*Sqrt[a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*
x]^4])/(2*Sqrt[b]*d*Sqrt[a + b*Sin[c + d*x]^4]) - (a^(1/4)*(Sqrt[a] + Sqrt[
a + b])*Cos[c + d*x]^2*EllipticF[2*ArcTan[((a + b)^(1/4)*Tan[c + d*x])/a^(1
/4)], (1 - Sqrt[a]/Sqrt[a + b])/2]*(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2)*S
qrt[(a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4)/(Sqrt[a] + Sqrt[a + b
]*Tan[c + d*x]^2)^2])/(2*b*(a + b)^(1/4)*d*Sqrt[a + b*Sin[c + d*x]^4]) + ((
Sqrt[a] + Sqrt[a + b])^2*Cos[c + d*x]^2*EllipticPi[-(Sqrt[a] - Sqrt[a + b])
^2/(4*Sqrt[a]*Sqrt[a + b]), 2*ArcTan[((a + b)^(1/4)*Tan[c + d*x])/a^(1/4)],
 (1 - Sqrt[a]/Sqrt[a + b])/2]*(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2)*Sqrt[(
a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4)/(Sqrt[a] + Sqrt[a + b]*Tan
[c + d*x]^2)^2])/(4*a^(1/4)*b*(a + b)^(1/4)*d*Sqrt[a + b*Sin[c + d*x]^4])
```

Rule 3219

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(ff^(m + 1
)*(a + b*Sin[e + f*x]^4)^p*(Sec[e + f*x]^2)^(2*p))/(f*Apart[a*(1 + Tan[e +
f*x]^2)^2 + b*Tan[e + f*x]^4]^p), Subst[Int[(x^m*ExpandToSum[a*(1 + ff^2*x^
2)^2 + b*ff^4*x^4, x]^p)/(1 + ff^2*x^2)^(m/2 + 2*p + 1), x], x, Tan[e + f*x
```

```
]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && IntegerQ[p - 1/
2]
```

Rule 1319

```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2]}, -Dist[(a*(e + d*q))/(c*d^2 - a*e^2),
Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[(a*d*(e + d*q))/(c*d^2 - a*e^2
), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[
{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
 && PosQ[c/a] && NeQ[c*d^2 - a*e^2, 0]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \frac{\left(\cos^2(c+dx)\sqrt{a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)}\right)\text{Subst}\left(\int \frac{x^2}{(1+x^2)\sqrt{a+2ax^2+(a+b)x \cdots}}\right)}{d\sqrt{a+b\sin^4(c+dx)}}$$

$$= -\frac{\left(a\left(1+\frac{\sqrt{a+b}}{\sqrt{a}}\right)\cos^2(c+dx)\sqrt{a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)}\right)\text{Subst}\left(\int \frac{\cdots}{\sqrt{a+2\cdots}}\right)}{bd\sqrt{a+b\sin^4(c+dx)}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)}}\right)\cos^2(c+dx)\sqrt{a+2a\tan^2(c+dx)+(a+b)\tan^4 \cdots}}{2\sqrt{b}d\sqrt{a+b\sin^4(c+dx)}}$$

Mathematica [C] time = 2.77753, size = 287, normalized size = 0.58

$$-\frac{2i\cos^2(c+dx)\sqrt{1+\left(1+\frac{i\sqrt{b}}{\sqrt{a}}\right)\tan^2(c+dx)}\sqrt{2+\left(2-\frac{2i\sqrt{b}}{\sqrt{a}}\right)\tan^2(c+dx)}\left(F\left(i\sinh^{-1}\left(\sqrt{1-\frac{i\sqrt{b}}{\sqrt{a}}}\tan(c+dx)\right)|\frac{\sqrt{a}+i\cdots}{\sqrt{a}-i\cdots}\right)\cdots\right)}{d\sqrt{1-\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{8a-4b\cos(2(c+dx))+b\cos(4(c+dx))+3b \cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] ((-2*I)*Cos[c + d*x]^2*(EllipticF[I*ArcSinh[Sqrt[1 - (I*Sqrt[b])/Sqrt[a]]*T
an[c + d*x]], (Sqrt[a] + I*Sqrt[b])/(Sqrt[a] - I*Sqrt[b])] - EllipticPi[Sqr
t[a]/(Sqrt[a] - I*Sqrt[b]), I*ArcSinh[Sqrt[1 - (I*Sqrt[b])/Sqrt[a]]*Tan[c +
 d*x]], (Sqrt[a] + I*Sqrt[b])/(Sqrt[a] - I*Sqrt[b])])*Sqrt[1 + (1 + (I*Sqrt
[b])/Sqrt[a])*Tan[c + d*x]^2]*Sqrt[2 + (2 - ((2*I)*Sqrt[b])/Sqrt[a])*Tan[c
+ d*x]^2])/(Sqrt[1 - (I*Sqrt[b])/Sqrt[a]]*d*Sqrt[8*a + 3*b - 4*b*Cos[2*(c +
 d*x)] + b*Cos[4*(c + d*x)]])
```

Maple [A] time = 4.555, size = 881, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x)
```

```
[Out] -1/2*((4*a+cos(2*d*x+2*c)^2*b+b-2*b*cos(2*d*x+2*c))*sin(2*d*x+2*c)^2)^(1/2)
*(-a*b)^(1/2)*((-b+(-a*b)^(1/2))*(-1+cos(2*d*x+2*c))/(-a*b)^(1/2)/(cos(2*d*
x+2*c)+1))^(1/2)*(cos(2*d*x+2*c)+1)^2*((-b*cos(2*d*x+2*c)+2*(-a*b)^(1/2)+b)
/(-a*b)^(1/2)/(cos(2*d*x+2*c)+1))^(1/2)*((b*cos(2*d*x+2*c)+2*(-a*b)^(1/2)-b
)/(-a*b)^(1/2)/(cos(2*d*x+2*c)+1))^(1/2)*(EllipticF(((-b+(-a*b)^(1/2))*(-1+
cos(2*d*x+2*c))/(-a*b)^(1/2)/(cos(2*d*x+2*c)+1))^(1/2),((b+(-a*b)^(1/2))/(-
b+(-a*b)^(1/2)))^(1/2))-2*EllipticPi(((-b+(-a*b)^(1/2))*(-1+cos(2*d*x+2*c))
/(-a*b)^(1/2)/(cos(2*d*x+2*c)+1))^(1/2),(-a*b)^(1/2)/(-b+(-a*b)^(1/2)),((b+
(-a*b)^(1/2))/(-b+(-a*b)^(1/2)))^(1/2)))/(-b+(-a*b)^(1/2))/(1/b*(-1+cos(2*d
*x+2*c))*(cos(2*d*x+2*c)+1)*(-b*cos(2*d*x+2*c)+2*(-a*b)^(1/2)+b)*(b*cos(2*d
*x+2*c)+2*(-a*b)^(1/2)-b))^(1/2)/sin(2*d*x+2*c)/(4*a+cos(2*d*x+2*c)^2*b+b-2
*b*cos(2*d*x+2*c))^(1/2)/d-1/2*((4*a+cos(2*d*x+2*c)^2*b+b-2*b*cos(2*d*x+2*c
))*sin(2*d*x+2*c)^2)^(1/2)*(-a*b)^(1/2)*((-b+(-a*b)^(1/2))*(-1+cos(2*d*x+2*
c))/(-a*b)^(1/2)/(cos(2*d*x+2*c)+1))^(1/2)*(cos(2*d*x+2*c)+1)^2*((-b*cos(2*
d*x+2*c)+2*(-a*b)^(1/2)+b)/(-a*b)^(1/2)/(cos(2*d*x+2*c)+1))^(1/2)*((b*cos(2
*d*x+2*c)+2*(-a*b)^(1/2)-b)/(-a*b)^(1/2)/(cos(2*d*x+2*c)+1))^(1/2)*Elliptic
F(((-b+(-a*b)^(1/2))*(-1+cos(2*d*x+2*c))/(-a*b)^(1/2)/(cos(2*d*x+2*c)+1))^(
1/2),((b+(-a*b)^(1/2))/(-b+(-a*b)^(1/2)))^(1/2))/(-b+(-a*b)^(1/2))/(1/b*(-1
+cos(2*d*x+2*c))*(cos(2*d*x+2*c)+1)*(-b*cos(2*d*x+2*c)+2*(-a*b)^(1/2)+b)*(b
*cos(2*d*x+2*c)+2*(-a*b)^(1/2)-b))^(1/2)/sin(2*d*x+2*c)/(4*a+cos(2*d*x+2*c)
^2*b+b-2*b*cos(2*d*x+2*c))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)^2}{\sqrt{b\sin(dx+c)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^2/sqrt(b*sin(d*x + c)^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\cos(dx+c)^2-1}{\sqrt{b\cos(dx+c)^4-2\,b\cos(dx+c)^2+a+b}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(d*x + c)^2 - 1)/sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 +
 a + b), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+b*sin(d*x+c)**4)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)^2}{\sqrt{b\sin(dx+c)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^2/sqrt(b*sin(d*x + c)^4 + a), x)
```

3.247 $\int \frac{1}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

Optimal. Leaf size=162

$$\frac{\cos^2(c+dx)\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)\sqrt{\frac{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}{\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a+b}\tan(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\right)}{2\sqrt[4]{a}d\sqrt[4]{a+b}\sqrt{a+b\sin^4(c+dx)}}$$

```
[Out] (Cos[c + d*x]^2*EllipticF[2*ArcTan[((a + b)^(1/4)*Tan[c + d*x])/a^(1/4)], (
1 - Sqrt[a]/Sqrt[a + b])/2]*(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2)*Sqrt[(a
+ 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4)/(Sqrt[a] + Sqrt[a + b]*Tan[c
 + d*x]^2)^2])/(2*a^(1/4)*(a + b)^(1/4)*d*Sqrt[a + b*Sin[c + d*x]^4])
```

Rubi [A] time = 0.0793673, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3210, 1103}

$$\frac{\cos^2(c+dx)\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)\sqrt{\frac{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}{\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a+b}\tan(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\right)}{2\sqrt[4]{a}d\sqrt[4]{a+b}\sqrt{a+b\sin^4(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sin[c + d*x]^4],x]
```


```
[Out] (Cos[c + d*x]^2*EllipticF[2*ArcTan[((a + b)^(1/4)*Tan[c + d*x])/a^(1/4)], (
1 - Sqrt[a]/Sqrt[a + b])/2]*(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2)*Sqrt[(a
+ 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4)/(Sqrt[a] + Sqrt[a + b]*Tan[c
 + d*x]^2)^2])/(2*a^(1/4)*(a + b)^(1/4)*d*Sqrt[a + b*Sin[c + d*x]^4])
```


Rule 3210

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[(ff*(a + b*Sin[e + f*x]^4)^p*(Sec[e + f
*x]^2)^(2*p))/(f*(a + 2*a*Tan[e + f*x]^2 + (a + b)*Tan[e + f*x]^4)^p), Subs
t[Int[(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x],
 x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[p - 1/2]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \frac{\left(\cos^2(c+dx)\sqrt{a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+2ax^2+(a+b)x^4}}\,dx,\right.}{d\sqrt{a+b\sin^4(c+dx)}}$$

$$= \frac{\cos^2(c+dx)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a+b}\tan(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\right)\left(\sqrt{a}+\sqrt{a+b}\tan^2(c+dx)\right)\sqrt{\frac{a+2}{}}}{2\sqrt[4]{a}\sqrt[4]{a+b}d\sqrt{a+b\sin^4(c+dx)}}$$

Mathematica [C] time = 8.40371, size = 304, normalized size = 1.88

$$\frac{2\sqrt{2}\left(\sqrt{b}+i\sqrt{a}\right)\sin^2(c+dx)\tan(c+dx)\left(2\sqrt{a}+i\sqrt{b}\cos(2(c+dx))-i\sqrt{b}\right)\left(2i\sqrt{a}+\sqrt{b}\cos(2(c+dx))-\sqrt{b}\right)\sqrt{\csc^2(}}{\sqrt{a}d(8a-4b\cos(2(c+dx))+b\cos}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] (2*Sqrt[2]*(I*Sqrt[a] + Sqrt[b])*(2*Sqrt[a] - I*Sqrt[b] + I*Sqrt[b]*Cos[2*(
c + d*x)])*((2*I)*Sqrt[a] - Sqrt[b] + Sqrt[b]*Cos[2*(c + d*x)])*Sqrt[(1 - (
(2*I)*Sqrt[a])/Sqrt[b] - Cos[2*(c + d*x)])*Csc[c + d*x]^2]*Sqrt[(Cot[c + d*
x]^2*(I*Sqrt[a]*Sqrt[b] - a*Csc[c + d*x]^2))/(Sqrt[a] - I*Sqrt[b])^2]*Ellip
ticF[ArcSin[Sqrt[((-I)*Sqrt[b] + Sqrt[a]*Csc[c + d*x]^2)/(Sqrt[a] - I*Sqrt[
b])]], 1/2 + ((I/2)*Sqrt[a])/Sqrt[b]]*Sin[c + d*x]^2*Tan[c + d*x])/(Sqrt[a]
*d*(8*a + 3*b - 4*b*Cos[2*(c + d*x)] + b*Cos[4*(c + d*x)])^(3/2))
```

Maple [B] time = 2.356, size = 396, normalized size = 2.4

$$-\frac{(\cos(2dx+2c)+1)^2}{\sin(2dx+2c)d}\sqrt{\left(4a+(\cos(2dx+2c))^2b+b-2b\cos(2dx+2c)\right)(\sin(2dx+2c))^2}\sqrt{-ab}\sqrt{\frac{-1+\cos(2d}{\cos(2dx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(d*x+c)^4)^(1/2),x)
```

```
[Out] -((4*a+cos(2*d*x+2*c)^2*b+b-2*b*cos(2*d*x+2*c))*sin(2*d*x+2*c)^2)^(1/2)*(-a
*b)^(1/2)*((-b+(-a*b)^(1/2))*(-1+cos(2*d*x+2*c))/(-a*b)^(1/2)/(cos(2*d*x+2*
c)+1))^(1/2)*(cos(2*d*x+2*c)+1)^2*((-b*cos(2*d*x+2*c)+2*(-a*b)^(1/2)+b)/(-a
*b)^(1/2)/(cos(2*d*x+2*c)+1))^(1/2)*((b*cos(2*d*x+2*c)+2*(-a*b)^(1/2)-b)/(-
a*b)^(1/2)/(cos(2*d*x+2*c)+1))^(1/2)*EllipticF(((-b+(-a*b)^(1/2))*(-1+cos(2
*d*x+2*c))/(-a*b)^(1/2)/(cos(2*d*x+2*c)+1))^(1/2),((b+(-a*b)^(1/2))/(-b+(-a
*b)^(1/2)))^(1/2))/(-b+(-a*b)^(1/2))/(1/b*(-1+cos(2*d*x+2*c))*(cos(2*d*x+2*
c)+1)*(-b*cos(2*d*x+2*c)+2*(-a*b)^(1/2)+b)*(b*cos(2*d*x+2*c)+2*(-a*b)^(1/2)
-b))^(1/2)/sin(2*d*x+2*c)/(4*a+cos(2*d*x+2*c)^2*b+b-2*b*cos(2*d*x+2*c))^(1/
2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sin(dx+c)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sin(d*x + c)^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{b\cos\left(dx+c\right)^{4}-2\,b\cos\left(dx+c\right)^{2}+a+b}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)**4)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sin\left(dx+c\right)^{4}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sin(d*x + c)^4 + a), x)
```

3.248 $\int \frac{\csc^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

Optimal. Leaf size=493

$$\frac{\left(\sqrt{a}\sqrt{a+b}+a+b\right)\cos^2(c+dx)\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)\sqrt{\frac{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}{\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a+b}\tan(c+dx)}{\sqrt[4]{a}}\right.\right.}{2a^{3/4}d(a+b)^{3/4}\sqrt{a+b\sin^4(c+dx)}}$$

```
[Out] -((Cos[c + d*x]^2*Cot[c + d*x]*(a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*
x]^4))/(a*d*Sqrt[a + b*Sin[c + d*x]^4])) + (Sqrt[a + b]*Cos[c + d*x]*Sin[c
+ d*x]*(a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4))/(a*d*Sqrt[a + b*S
in[c + d*x]^4]*(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2)) - ((a + b)^(1/4)*Cos
[c + d*x]^2*EllipticE[2*ArcTan[((a + b)^(1/4)*Tan[c + d*x])/a^(1/4)], (1 -
Sqrt[a]/Sqrt[a + b])/2]*(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2)*Sqrt[(a + 2*
a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4)/(Sqrt[a] + Sqrt[a + b]*Tan[c + d
*x]^2)^2])/(a^(3/4)*d*Sqrt[a + b*Sin[c + d*x]^4]) + ((a + b + Sqrt[a]*Sqrt[
a + b])*Cos[c + d*x]^2*EllipticF[2*ArcTan[((a + b)^(1/4)*Tan[c + d*x])/a^(1
/4)], (1 - Sqrt[a]/Sqrt[a + b])/2]*(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2)*S
qrt[(a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4)/(Sqrt[a] + Sqrt[a + b
]*Tan[c + d*x]^2)^2])/(2*a^(3/4)*(a + b)^(3/4)*d*Sqrt[a + b*Sin[c + d*x]^4]
)
```

Rubi [A] time = 0.419335, antiderivative size = 493, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3219, 1281, 1197, 1103, 1195}

$$\frac{\left(\sqrt{a}\sqrt{a+b}+a+b\right)\cos^2(c+dx)\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)\sqrt{\frac{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}{\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a+b}\tan(c+dx)}{\sqrt[4]{a}}\right.\right.}{2a^{3/4}d(a+b)^{3/4}\sqrt{a+b\sin^4(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] -((Cos[c + d*x]^2*Cot[c + d*x]*(a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*
x]^4))/(a*d*Sqrt[a + b*Sin[c + d*x]^4])) + (Sqrt[a + b]*Cos[c + d*x]*Sin[c
+ d*x]*(a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4))/(a*d*Sqrt[a + b*S
in[c + d*x]^4]*(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2)) - ((a + b)^(1/4)*Cos
[c + d*x]^2*EllipticE[2*ArcTan[((a + b)^(1/4)*Tan[c + d*x])/a^(1/4)], (1 -
Sqrt[a]/Sqrt[a + b])/2]*(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2)*Sqrt[(a + 2*
a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4)/(Sqrt[a] + Sqrt[a + b]*Tan[c + d
*x]^2)^2])/(a^(3/4)*d*Sqrt[a + b*Sin[c + d*x]^4]) + ((a + b + Sqrt[a]*Sqrt[
a + b])*Cos[c + d*x]^2*EllipticF[2*ArcTan[((a + b)^(1/4)*Tan[c + d*x])/a^(1
/4)], (1 - Sqrt[a]/Sqrt[a + b])/2]*(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2)*S
qrt[(a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4)/(Sqrt[a] + Sqrt[a + b
]*Tan[c + d*x]^2)^2])/(2*a^(3/4)*(a + b)^(3/4)*d*Sqrt[a + b*Sin[c + d*x]^4]
)
```

Rule 3219

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(ff^(m + 1
)*(a + b*Sin[e + f*x]^4)^p*(Sec[e + f*x]^2)^(2*p))/(f*Apart[a*(1 + Tan[e +
```

```
f*x]^2)^2 + b*Tan[e + f*x]^4]^p), Subst[Int[(x^m*ExpandToSum[a*(1 + ff^2*x^
2)^2 + b*ff^4*x^4, x]^p)/(1 + ff^2*x^2)^(m/2 + 2*p + 1), x], x, Tan[e + f*x
]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && IntegerQ[p - 1/
2]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \frac{\left(\cos^2(c+dx)\sqrt{a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)}\right)\text{Subst}\left(\int \frac{1+x^2}{x^2\sqrt{a+2ax^2+(a+b)x^4}}\,dx, x\right.}{d\sqrt{a+b\sin^4(c+dx)}}$$

$$= -\frac{\cos^2(c+dx)\cot(c+dx)\left(a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)\right)}{ad\sqrt{a+b\sin^4(c+dx)}} - \frac{\left(\cos^2(c+dx)\sqrt{a\right.}{}$$

$$= -\frac{\cos^2(c+dx)\cot(c+dx)\left(a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)\right)}{ad\sqrt{a+b\sin^4(c+dx)}} - \frac{\left(\sqrt{a+b}\cos^2(c+\right.}{}$$

$$= -\frac{\cos^2(c+dx)\cot(c+dx)\left(a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)\right)}{ad\sqrt{a+b\sin^4(c+dx)}} + \frac{\sqrt{a+b}\cos(c+dx}{ad\sqrt{a}}$$

Mathematica [C] time = 16.2194, size = 498, normalized size = 1.01

$$-\frac{\cot(c+dx)\sqrt{8a-4b\cos(2(c+dx))+b\cos(4(c+dx))+3b}}{2\sqrt{2}ad}-\frac{\sqrt{1-\frac{i\sqrt{b}}{\sqrt{a}}}\tan(c+dx)\left(a\left(\tan^2(c+dx)+1\right)^2+b\tan^4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] -(Sqrt[8*a + 3*b - 4*b*Cos[2*(c + d*x)] + b*Cos[4*(c + d*x)]]*Cot[c + d*x])
/(2*Sqrt[2]*a*d) - (Sqrt[a]*(I*Sqrt[a] + Sqrt[b])*EllipticE[I*ArcSinh[Sqrt[
1 - (I*Sqrt[b])/Sqrt[a]]*Tan[c + d*x]], (Sqrt[a] + I*Sqrt[b])/(Sqrt[a] - I*
Sqrt[b])]*(1 + Tan[c + d*x]^2)*Sqrt[1 + (1 - (I*Sqrt[b])/Sqrt[a])*Tan[c + d
*x]^2]*Sqrt[1 + (1 + (I*Sqrt[b])/Sqrt[a])*Tan[c + d*x]^2] - Sqrt[a]*Sqrt[b]
*EllipticF[I*ArcSinh[Sqrt[1 - (I*Sqrt[b])/Sqrt[a]]*Tan[c + d*x]], (Sqrt[a]
+ I*Sqrt[b])/(Sqrt[a] - I*Sqrt[b])]*(1 + Tan[c + d*x]^2)*Sqrt[1 + (1 - (I*S
qrt[b])/Sqrt[a])*Tan[c + d*x]^2]*Sqrt[1 + (1 + (I*Sqrt[b])/Sqrt[a])*Tan[c +
 d*x]^2] + Sqrt[1 - (I*Sqrt[b])/Sqrt[a]]*Tan[c + d*x]*(b*Tan[c + d*x]^4 + a
*(1 + Tan[c + d*x]^2)^2))/(a*Sqrt[1 - (I*Sqrt[b])/Sqrt[a]]*d*(1 + Tan[c + d
*x]^2)^2*Sqrt[(b*Tan[c + d*x]^4 + a*(1 + Tan[c + d*x]^2)^2)/(1 + Tan[c + d*
x]^2)^2])
```

Maple [F] time = 0.707, size = 0, normalized size = 0.

$$\int \left(\csc\left(dx+c\right)\right)^{2}\frac{1}{\sqrt{a+b\left(\sin\left(dx+c\right)\right)^{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x)
```

```
[Out] int(csc(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(dx+c\right)^{2}}{\sqrt{b\sin\left(dx+c\right)^{4}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(d*x + c)^2/sqrt(b*sin(d*x + c)^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\csc\left(dx+c\right)^{2}}{\sqrt{b\cos\left(dx+c\right)^{4}-2\,b\cos\left(dx+c\right)^{2}+a+b}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(csc(d*x + c)^2/sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+b*sin(d*x+c)**4)**(1/2),x)
```

```
[Out] Integral(csc(c + d*x)**2/sqrt(a + b*sin(c + d*x)**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(dx+c)^2}{\sqrt{b\sin(dx+c)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(d*x + c)^2/sqrt(b*sin(d*x + c)^4 + a), x)
```

3.249 $\int \frac{1}{a+b\sin^5(x)}\,dx$

Optimal. Leaf size=384

$$\frac{2\tan^{-1}\left(\frac{\sqrt[5]{a}\tan\left(\frac{x}{2}\right)+\sqrt[5]{b}}{\sqrt{a^{2/5}-b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-b^{2/5}}}+\frac{2\tan^{-1}\left(\frac{\sqrt[5]{a}\tan\left(\frac{x}{2}\right)+(-1)^{2/5}\sqrt[5]{b}}{\sqrt{a^{2/5}-(-1)^{4/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-(-1)^{4/5}b^{2/5}}}+\frac{2\tan^{-1}\left(\frac{\sqrt[5]{a}\tan\left(\frac{x}{2}\right)+(-1)^{4/5}\sqrt[5]{b}}{\sqrt{a^{2/5}+(-1)^{3/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}+(-1)^{3/5}b^{2/5}}}-\frac{2\tan^{-1}\left(\frac{(-1)^{3/5}\left((-1)^{2/5}\sqrt[5]{a}\tan\left(\frac{x}{2}\right)\right.}{\sqrt{a^{2/5}+\sqrt[5]{-1}b^{2/5}}}\right.}{5a^{4/5}\sqrt{a^{2/5}+\sqrt[5]{-1}b^{2/5}}}$$

```
[Out] (2*ArcTan[(b^(1/5) + a^(1/5)*Tan[x/2])/Sqrt[a^(2/5) - b^(2/5)]])/(5*a^(4/5)
*Sqrt[a^(2/5) - b^(2/5)]) + (2*ArcTan[((-1)^(2/5)*b^(1/5) + a^(1/5)*Tan[x/2
])/Sqrt[a^(2/5) - (-1)^(4/5)*b^(2/5)]])/(5*a^(4/5)*Sqrt[a^(2/5) - (-1)^(4/5
)*b^(2/5)]) + (2*ArcTan[((-1)^(4/5)*b^(1/5) + a^(1/5)*Tan[x/2])/Sqrt[a^(2/5
) + (-1)^(3/5)*b^(2/5)]])/(5*a^(4/5)*Sqrt[a^(2/5) + (-1)^(3/5)*b^(2/5)]) -
(2*ArcTan[((-1)^(3/5)*(b^(1/5) + (-1)^(2/5)*a^(1/5)*Tan[x/2]))/Sqrt[a^(2/5)
 + (-1)^(1/5)*b^(2/5)]])/(5*a^(4/5)*Sqrt[a^(2/5) + (-1)^(1/5)*b^(2/5)]) - (
2*ArcTan[((-1)^(1/5)*(b^(1/5) + (-1)^(4/5)*a^(1/5)*Tan[x/2]))/Sqrt[a^(2/5)
- (-1)^(2/5)*b^(2/5)]])/(5*a^(4/5)*Sqrt[a^(2/5) - (-1)^(2/5)*b^(2/5)])
```

Rubi [A] time = 0.714324, antiderivative size = 384, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {3213, 2660, 618, 204}

$$\frac{2\tan^{-1}\left(\frac{\sqrt[5]{a}\tan\left(\frac{x}{2}\right)+\sqrt[5]{b}}{\sqrt{a^{2/5}-b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-b^{2/5}}}+\frac{2\tan^{-1}\left(\frac{\sqrt[5]{a}\tan\left(\frac{x}{2}\right)+(-1)^{2/5}\sqrt[5]{b}}{\sqrt{a^{2/5}-(-1)^{4/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-(-1)^{4/5}b^{2/5}}}+\frac{2\tan^{-1}\left(\frac{\sqrt[5]{a}\tan\left(\frac{x}{2}\right)+(-1)^{4/5}\sqrt[5]{b}}{\sqrt{a^{2/5}+(-1)^{3/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}+(-1)^{3/5}b^{2/5}}}-\frac{2\tan^{-1}\left(\frac{(-1)^{3/5}\left((-1)^{2/5}\sqrt[5]{a}\tan\left(\frac{x}{2}\right)\right.}{\sqrt{a^{2/5}+\sqrt[5]{-1}b^{2/5}}}\right.}{5a^{4/5}\sqrt{a^{2/5}+\sqrt[5]{-1}b^{2/5}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[x]^5)^(-1),x]
```


```
[Out] (2*ArcTan[(b^(1/5) + a^(1/5)*Tan[x/2])/Sqrt[a^(2/5) - b^(2/5)]])/(5*a^(4/5)
*Sqrt[a^(2/5) - b^(2/5)]) + (2*ArcTan[((-1)^(2/5)*b^(1/5) + a^(1/5)*Tan[x/2
])/Sqrt[a^(2/5) - (-1)^(4/5)*b^(2/5)]])/(5*a^(4/5)*Sqrt[a^(2/5) - (-1)^(4/5
)*b^(2/5)]) + (2*ArcTan[((-1)^(4/5)*b^(1/5) + a^(1/5)*Tan[x/2])/Sqrt[a^(2/5
) + (-1)^(3/5)*b^(2/5)]])/(5*a^(4/5)*Sqrt[a^(2/5) + (-1)^(3/5)*b^(2/5)]) -
(2*ArcTan[((-1)^(3/5)*(b^(1/5) + (-1)^(2/5)*a^(1/5)*Tan[x/2]))/Sqrt[a^(2/5)
 + (-1)^(1/5)*b^(2/5)]])/(5*a^(4/5)*Sqrt[a^(2/5) + (-1)^(1/5)*b^(2/5)]) - (
2*ArcTan[((-1)^(1/5)*(b^(1/5) + (-1)^(4/5)*a^(1/5)*Tan[x/2]))/Sqrt[a^(2/5)
- (-1)^(2/5)*b^(2/5)]])/(5*a^(4/5)*Sqrt[a^(2/5) - (-1)^(2/5)*b^(2/5)])
```


Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{a+b\sin^5(x)}\,dx = \int\left(-\frac{1}{5a^{4/5}\left(-\sqrt[5]{a}-\sqrt[5]{b}\sin(x)\right)}-\frac{1}{5a^{4/5}\left(-\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}\sin(x)\right)}-\frac{1}{5a^{4/5}\left(-\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}\sin(x)\right)}-\cdots\right.$$

$$=-\frac{\int\frac{1}{-\sqrt[5]{a}-\sqrt[5]{b}\sin(x)}\,dx}{5a^{4/5}}-\frac{\int\frac{1}{-\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}\sin(x)}\,dx}{5a^{4/5}}-\frac{\int\frac{1}{-\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}\sin(x)}\,dx}{5a^{4/5}}-\frac{\int\frac{1}{-\sqrt[5]{a}+(-1)^{3/5}\sqrt[5]{b}\sin(x)}\,dx}{5a^{4/5}}-\cdots$$

$$=-\frac{2\,\text{Subst}\left(\int\frac{1}{-\sqrt[5]{a}-2\sqrt[5]{b}x-\sqrt[5]{a}x^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)}{5a^{4/5}}-\frac{2\,\text{Subst}\left(\int\frac{1}{-\sqrt[5]{a}+2\sqrt[5]{-1}\sqrt[5]{b}x-\sqrt[5]{a}x^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)}{5a^{4/5}}-\frac{2\,\text{Su}\cdots}{\cdots}$$

$$=\frac{4\,\text{Subst}\left(\int\frac{1}{-4\left(a^{2/5}-b^{2/5}\right)-x^2}\,dx,x,-2\sqrt[5]{b}-2\sqrt[5]{a}\tan\left(\frac{x}{2}\right)\right)}{5a^{4/5}}+\frac{4\,\text{Subst}\left(\int\frac{1}{-4\left(a^{2/5}+\sqrt[5]{-1}b^{2/5}\right)-x^2}\,dx,x,2(-1)^{3}\cdots\right.}{5a^{4/5}}$$

$$=-\frac{2\tan^{-1}\left(\frac{\sqrt[5]{-1}\sqrt[5]{b}-\sqrt[5]{a}\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}-(-1)^{2/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-(-1)^{2/5}b^{2/5}}}-\frac{2\tan^{-1}\left(\frac{(-1)^{3/5}\sqrt[5]{b}-\sqrt[5]{a}\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}+\sqrt[5]{-1}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}+\sqrt[5]{-1}b^{2/5}}}+\frac{2\tan^{-1}\left(\frac{\sqrt[5]{b}+\sqrt[5]{a}\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}-b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-b^{2/5}}}+\frac{2\tan^{-1}\left(\cdots\right)}{5a^{4/5}\sqrt{a\cdots}}$$

Mathematica [C] time = 0.211827, size = 149, normalized size = 0.39

$$\frac{8}{5}i\text{RootSum}\left[32\#1^5a-i\#1^{10}b+5i\#1^8b-10i\#1^6b+10i\#1^4b-5i\#1^2b+ib\&,\frac{2\#1^3\tan^{-1}\left(\frac{\sin(x)}{\cos(x)-\#1}\right)-i\#1^3\log\left(\#1^2-2\#\cdots\right)}{16i\#1^3a+\#1^8b-4\#1^6b+6\#1^4b-4\#\cdots}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[x]^5)^(-1),x]
```

```
[Out] ((8*I)/5)*RootSum[I*b - (5*I)*b*#1^2 + (10*I)*b*#1^4 + 32*a*#1^5 - (10*I)*b
*#1^6 + (5*I)*b*#1^8 - I*b*#1^10 & , (2*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^3 -
 I*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^3)/(b - 4*b*#1^2 + (16*I)*a*#1^3 + 6*b*#1
^4 - 4*b*#1^6 + b*#1^8) & ]
```

Maple [C] time = 0.123, size = 109, normalized size = 0.3

$$\frac{1}{5}\sum_{\_R=\text{RootOf}\left(a\_Z^{10}+5a\_Z^8+10a\_Z^6+32b\_Z^5+10a\_Z^4+5a\_Z^2+a\right)}\frac{\_R^8+4\_R^6+6\_R^4+4\_R^2+1}{\_R^9a+4\_R^7a+6\_R^5a+16\_R^4b+4\_R^3a+\_Ra}\ln\left(\tan\left(\frac{x}{2}\right)\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(x)^5),x)
```

```
[Out] 1/5*sum((_R^8+4*_R^6+6*_R^4+4*_R^2+1)/(_R^9*a+4*_R^7*a+6*_R^5*a+16*_R^4*b+4
*_R^3*a+_R*a)*ln(tan(1/2*x)-_R),_R=RootOf(_Z^10*a+5*_Z^8*a+10*_Z^6*a+32*_Z^
5*b+10*_Z^4*a+5*_Z^2*a+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b\sin(x)^5 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)^5),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*sin(x)^5 + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)^5),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)**5),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b\sin(x)^5 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)^5),x, algorithm="giac")
```

```
[Out] integrate(1/(b*sin(x)^5 + a), x)
```

3.250 $\int \frac{1}{a+b\sin^6(x)}\,dx$

Optimal. Leaf size=171

$$\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}}$$

```
[Out] ArcTan[(Sqrt[a^(1/3) + b^(1/3)]*Tan[x])/a^(1/6)]/(3*a^(5/6)*Sqrt[a^(1/3) +
b^(1/3)]) + ArcTan[(Sqrt[a^(1/3) - (-1)^(1/3)*b^(1/3)]*Tan[x])/a^(1/6)]/(3*
a^(5/6)*Sqrt[a^(1/3) - (-1)^(1/3)*b^(1/3)]) + ArcTan[(Sqrt[a^(1/3) + (-1)^(
2/3)*b^(1/3)]*Tan[x])/a^(1/6)]/(3*a^(5/6)*Sqrt[a^(1/3) + (-1)^(2/3)*b^(1/3)
])
```

Rubi [A] time = 0.25652, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3211, 3181, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[x]^6)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[a^(1/3) + b^(1/3)]*Tan[x])/a^(1/6)]/(3*a^(5/6)*Sqrt[a^(1/3) +
b^(1/3)]) + ArcTan[(Sqrt[a^(1/3) - (-1)^(1/3)*b^(1/3)]*Tan[x])/a^(1/6)]/(3*
a^(5/6)*Sqrt[a^(1/3) - (-1)^(1/3)*b^(1/3)]) + ArcTan[(Sqrt[a^(1/3) + (-1)^(
2/3)*b^(1/3)]*Tan[x])/a^(1/6)]/(3*a^(5/6)*Sqrt[a^(1/3) + (-1)^(2/3)*b^(1/3)
])
```

Rule 3211

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^((4*k)/n)*Rt[-(a/b),
n/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{a+b\sin^6(x)}\,dx = \frac{\int \frac{1}{1+\frac{\sqrt[3]{b}\sin^2(x)}{\sqrt[3]{a}}}\,dx}{3a}+\frac{\int \frac{1}{1-\frac{\sqrt[3]{-1}\sqrt[3]{b}\sin^2(x)}{\sqrt[3]{a}}}\,dx}{3a}+\frac{\int \frac{1}{1+\frac{(-1)^{2/3}\sqrt[3]{b}\sin^2(x)}{\sqrt[3]{a}}}\,dx}{3a}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1+\left(1+\frac{\sqrt[3]{b}}{\sqrt[3]{a}}\right)x^2}\,dx,x,\tan(x)\right)}{3a}+\frac{\text{Subst}\left(\int \frac{1}{1+\left(1-\frac{\sqrt[3]{-1}\sqrt[3]{b}}{\sqrt[3]{a}}\right)x^2}\,dx,x,\tan(x)\right)}{3a}+\frac{\text{Subst}\left(\int \frac{1}{1+\left(1+\right.}\right.}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}}$$

Mathematica [C] time = 0.22116, size = 148, normalized size = 0.87

$$-\frac{8}{3}\text{RootSum}\left[-64\#1^3a+\#1^6b-6\#1^5b+15\#1^4b-20\#1^3b+15\#1^2b-6\#1b+b\&,\frac{2\#1^2\tan^{-1}\left(\frac{\sin(2x)}{\cos(2x)-\#1}\right)-i\#1^2\log}{-32\#1^2a+\#1^5b-5\#1^4b+10\#1}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[x]^6)^(-1),x]
```

```
[Out] (-8*RootSum[b - 6*b*#1 + 15*b*#1^2 - 64*a*#1^3 - 20*b*#1^3 + 15*b*#1^4 - 6*
b*#1^5 + b*#1^6 & , (2*ArcTan[Sin[2*x]/(Cos[2*x] - #1)]*#1^2 - I*Log[1 - 2*
Cos[2*x]*#1 + #1^2]*#1^2)/(-b + 5*b*#1 - 32*a*#1^2 - 10*b*#1^2 + 10*b*#1^3
- 5*b*#1^4 + b*#1^5) & ])/3
```

Maple [C] time = 0.376, size = 68, normalized size = 0.4

$$\frac{1}{6}\sum_{\_R=\text{RootOf}\left((a+b)\_Z^6+3a\_Z^4+3a\_Z^2+a\right)}\frac{\left(\_R^4+2\_R^2+1\right)\ln\left(\tan(x)-\_R\right)}{\_R^5a+\_R^5b+2\_R^3a+\_R a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(x)^6),x)
```

```
[Out] 1/6*sum((_R^4+2*_R^2+1)/(_R^5*a+_R^5*b+2*_R^3*a+_R*a)*ln(tan(x)-_R),_R=Root
Of((a+b)*_Z^6+3*a*_Z^4+3*a*_Z^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b\sin(x)^6+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)^6),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*sin(x)^6 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)^6),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{a + b\sin^{6}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)**6),x)
```

```
[Out] Integral(1/(a + b*sin(x)**6), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b\sin(x)^{6} + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)^6),x, algorithm="giac")
```

```
[Out] integrate(1/(b*sin(x)^6 + a), x)
```

3.251 $\int \frac{1}{a+b\sin^8(x)}\,dx$

Optimal. Leaf size=245

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}\tan(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}\tan(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}\tan(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}\tan(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}$$

```
[Out] -ArcTan[(Sqrt[(-a)^(1/4) - b^(1/4)]*Tan[x])/(-a)^(1/8)]/(4*(-a)^(7/8)*Sqrt[
(-a)^(1/4) - b^(1/4)]) - ArcTan[(Sqrt[(-a)^(1/4) - I*b^(1/4)]*Tan[x])/(-a)^
(1/8)]/(4*(-a)^(7/8)*Sqrt[(-a)^(1/4) - I*b^(1/4)]) - ArcTan[(Sqrt[(-a)^(1/4
) + I*b^(1/4)]*Tan[x])/(-a)^(1/8)]/(4*(-a)^(7/8)*Sqrt[(-a)^(1/4) + I*b^(1/4
)]) - ArcTan[(Sqrt[(-a)^(1/4) + b^(1/4)]*Tan[x])/(-a)^(1/8)]/(4*(-a)^(7/8)*
Sqrt[(-a)^(1/4) + b^(1/4)])
```

Rubi [A] time = 0.527758, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3211, 3181, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}\tan(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}\tan(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}\tan(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt{a\sqrt[4]{b}+(-a)^{5/4}}\tan(x)}{(-a)^{5/8}}\right)}{4(-a)^{3/8}\sqrt{a\sqrt[4]{b}+(-a)^{5/4}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[x]^8)^(-1),x]
```

```
[Out] -ArcTan[(Sqrt[(-a)^(1/4) - I*b^(1/4)]*Tan[x])/(-a)^(1/8)]/(4*(-a)^(7/8)*Sqr
t[(-a)^(1/4) - I*b^(1/4)]) - ArcTan[(Sqrt[(-a)^(1/4) + I*b^(1/4)]*Tan[x])/(
-a)^(1/8)]/(4*(-a)^(7/8)*Sqrt[(-a)^(1/4) + I*b^(1/4)]) - ArcTan[(Sqrt[(-a)^
(1/4) + b^(1/4)]*Tan[x])/(-a)^(1/8)]/(4*(-a)^(7/8)*Sqrt[(-a)^(1/4) + b^(1/4
)]) - ArcTan[(Sqrt[(-a)^(5/4) + a*b^(1/4)]*Tan[x])/(-a)^(5/8)]/(4*(-a)^(3/8
)*Sqrt[(-a)^(5/4) + a*b^(1/4)])
```

Rule 3211

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^((4*k)/n)*Rt[-(a/b),
n/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{a+b\sin^8(x)}\,dx = \frac{\int \frac{1}{1-\frac{\sqrt[4]{b}\sin^2(x)}{\sqrt[4]{-a}}}dx}{4a} + \frac{\int \frac{1}{1-\frac{i\sqrt[4]{b}\sin^2(x)}{\sqrt[4]{-a}}}dx}{4a} + \frac{\int \frac{1}{1+\frac{i\sqrt[4]{b}\sin^2(x)}{\sqrt[4]{-a}}}dx}{4a} + \frac{\int \frac{1}{1+\frac{\sqrt[4]{b}\sin^2(x)}{\sqrt[4]{-a}}}dx}{4a}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1+\left(1-\frac{\sqrt[4]{b}}{\sqrt[4]{-a}}\right)x^2}dx, x, \tan(x)\right)}{4a} + \frac{\text{Subst}\left(\int \frac{1}{1+\left(1-\frac{i\sqrt[4]{b}}{\sqrt[4]{-a}}\right)x^2}dx, x, \tan(x)\right)}{4a} + \frac{\text{Subst}\left(\int \frac{1}{1+\left(1+\frac{i\sqrt[4]{b}}{\sqrt[4]{-a}}\right)x}\right.}{4a}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}\tan(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}} - \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}\tan(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}} - \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}\tan(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}} - \frac{\tan^{-1}\left(\frac{\sqrt{(-a)^{5/4}+a}}{(-a)^{5}}\right.}{4(-a)^{3/8}\sqrt{(-a)^{5/4}}}$$

Mathematica [C] time = 0.265741, size = 174, normalized size = 0.71

$$8\text{RootSum}\left[256\#1^4a+\#1^8b-8\#1^7b+28\#1^6b-56\#1^5b+70\#1^4b-56\#1^3b+28\#1^2b-8\#1b+b\&,\frac{2\#1^3\tan}{128\#1^3a+\#1^7b-}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[x]^8)^(-1),x]
```

```
[Out] 8*RootSum[b - 8*b*#1 + 28*b*#1^2 - 56*b*#1^3 + 256*a*#1^4 + 70*b*#1^4 - 56*
b*#1^5 + 28*b*#1^6 - 8*b*#1^7 + b*#1^8 & , (2*ArcTan[Sin[2*x]/(Cos[2*x] - #
1)]*#1^3 - I*Log[1 - 2*Cos[2*x]*#1 + #1^2]*#1^3)/(-b + 7*b*#1 - 21*b*#1^2 +
 128*a*#1^3 + 35*b*#1^3 - 35*b*#1^4 + 21*b*#1^5 - 7*b*#1^6 + b*#1^7) & ]
```

Maple [C] time = 0.118, size = 85, normalized size = 0.4

$$\frac{1}{8}\sum_{\_R=\text{RootOf}\left((a+b)\_Z^8+4a\_Z^6+6a\_Z^4+4a\_Z^2+a\right)}\frac{\left(\_R^6+3\_R^4+3\_R^2+1\right)\ln(\tan(x)-\_R)}{\_R^7a+\_R^7b+3\_R^5a+3\_R^3a+\_Ra}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(x)^8),x)
```

```
[Out] 1/8*sum((_R^6+3*_R^4+3*_R^2+1)/(_R^7*a+_R^7*b+3*_R^5*a+3*_R^3*a+_R*a)*ln(ta
n(x)-_R),_R=RootOf((a+b)*_Z^8+4*a*_Z^6+6*a*_Z^4+4*a*_Z^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b\sin(x)^8+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)^8),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*sin(x)^8 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)^8),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)**8),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b\sin(x)^8 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)^8),x, algorithm="giac")
```

```
[Out] integrate(1/(b*sin(x)^8 + a), x)
```

3.252 $\int \frac{1}{a-b\sin^5(x)}\,dx$

Optimal. Leaf size=379

$$-\frac{2\tan^{-1}\left(\frac{\sqrt[5]{b}-\sqrt[5]{a}\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}-b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-b^{2/5}}}-\frac{2\tan^{-1}\left(\frac{(-1)^{2/5}\sqrt[5]{b}-\sqrt[5]{a}\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}-(-1)^{4/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-(-1)^{4/5}b^{2/5}}}-\frac{2\tan^{-1}\left(\frac{(-1)^{4/5}\sqrt[5]{b}-\sqrt[5]{a}\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}+(-1)^{3/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}+(-1)^{3/5}b^{2/5}}}+\frac{2\tan^{-1}\left(\frac{\sqrt[5]{a}\tan\left(\frac{x}{2}\right)+\sqrt[5]{-1}\sqrt[5]{b}}{\sqrt{a^{2/5}-(-1)^{2/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-(-1)^{2/5}b^{2/5}}}+\cdots$$

```
[Out] (-2*ArcTan[(b^(1/5) - a^(1/5)*Tan[x/2])/Sqrt[a^(2/5) - b^(2/5)]])/(5*a^(4/5
)*Sqrt[a^(2/5) - b^(2/5)]) - (2*ArcTan[((-1)^(2/5)*b^(1/5) - a^(1/5)*Tan[x/
2])/Sqrt[a^(2/5) - (-1)^(4/5)*b^(2/5)]])/(5*a^(4/5)*Sqrt[a^(2/5) - (-1)^(4/
5)*b^(2/5)]) - (2*ArcTan[((-1)^(4/5)*b^(1/5) - a^(1/5)*Tan[x/2])/Sqrt[a^(2/
5) + (-1)^(3/5)*b^(2/5)]])/(5*a^(4/5)*Sqrt[a^(2/5) + (-1)^(3/5)*b^(2/5)]) +
 (2*ArcTan[((-1)^(1/5)*b^(1/5) + a^(1/5)*Tan[x/2])/Sqrt[a^(2/5) - (-1)^(2/5
)*b^(2/5)]])/(5*a^(4/5)*Sqrt[a^(2/5) - (-1)^(2/5)*b^(2/5)]) + (2*ArcTan[((-
1)^(3/5)*b^(1/5) + a^(1/5)*Tan[x/2])/Sqrt[a^(2/5) + (-1)^(1/5)*b^(2/5)]])/(
5*a^(4/5)*Sqrt[a^(2/5) + (-1)^(1/5)*b^(2/5)])
```

Rubi [A] time = 0.47672, antiderivative size = 379, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3213, 2660, 618, 204}

$$-\frac{2\tan^{-1}\left(\frac{\sqrt[5]{b}-\sqrt[5]{a}\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}-b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-b^{2/5}}}-\frac{2\tan^{-1}\left(\frac{(-1)^{2/5}\sqrt[5]{b}-\sqrt[5]{a}\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}-(-1)^{4/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-(-1)^{4/5}b^{2/5}}}-\frac{2\tan^{-1}\left(\frac{(-1)^{4/5}\sqrt[5]{b}-\sqrt[5]{a}\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}+(-1)^{3/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}+(-1)^{3/5}b^{2/5}}}+\frac{2\tan^{-1}\left(\frac{\sqrt[5]{a}\tan\left(\frac{x}{2}\right)+\sqrt[5]{-1}\sqrt[5]{b}}{\sqrt{a^{2/5}-(-1)^{2/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-(-1)^{2/5}b^{2/5}}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*Sin[x]^5)^(-1),x]
```

```
[Out] (-2*ArcTan[(b^(1/5) - a^(1/5)*Tan[x/2])/Sqrt[a^(2/5) - b^(2/5)]])/(5*a^(4/5
)*Sqrt[a^(2/5) - b^(2/5)]) - (2*ArcTan[((-1)^(2/5)*b^(1/5) - a^(1/5)*Tan[x/
2])/Sqrt[a^(2/5) - (-1)^(4/5)*b^(2/5)]])/(5*a^(4/5)*Sqrt[a^(2/5) - (-1)^(4/
5)*b^(2/5)]) - (2*ArcTan[((-1)^(4/5)*b^(1/5) - a^(1/5)*Tan[x/2])/Sqrt[a^(2/
5) + (-1)^(3/5)*b^(2/5)]])/(5*a^(4/5)*Sqrt[a^(2/5) + (-1)^(3/5)*b^(2/5)]) +
 (2*ArcTan[((-1)^(1/5)*b^(1/5) + a^(1/5)*Tan[x/2])/Sqrt[a^(2/5) - (-1)^(2/5
)*b^(2/5)]])/(5*a^(4/5)*Sqrt[a^(2/5) - (-1)^(2/5)*b^(2/5)]) + (2*ArcTan[((-
1)^(3/5)*b^(1/5) + a^(1/5)*Tan[x/2])/Sqrt[a^(2/5) + (-1)^(1/5)*b^(2/5)]])/(
5*a^(4/5)*Sqrt[a^(2/5) + (-1)^(1/5)*b^(2/5)])
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{a-b\sin^5(x)}dx = \int \left(\frac{1}{5a^{4/5}\left(\sqrt[5]{a}-\sqrt[5]{b}\sin(x)\right)}+\frac{1}{5a^{4/5}\left(\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}\sin(x)\right)}+\frac{1}{5a^{4/5}\left(\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}\sin(x)\right)}+\frac{}{5a}\right.$$

$$=\frac{\int \frac{1}{\sqrt[5]{a}-\sqrt[5]{b}\sin(x)}dx}{5a^{4/5}}+\frac{\int \frac{1}{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}\sin(x)}dx}{5a^{4/5}}+\frac{\int \frac{1}{\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}\sin(x)}dx}{5a^{4/5}}+\frac{\int \frac{1}{\sqrt[5]{a}+(-1)^{3/5}\sqrt[5]{b}\sin(x)}dx}{5a^{4/5}}+\frac{\int}{}$$

$$=\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt[5]{a}-2\sqrt[5]{b}x+\sqrt[5]{a}x^2}dx,x,\tan\left(\frac{x}{2}\right)\right)}{5a^{4/5}}+\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt[5]{a}+2\sqrt[5]{-1}\sqrt[5]{b}x+\sqrt[5]{a}x^2}dx,x,\tan\left(\frac{x}{2}\right)\right)}{5a^{4/5}}+\frac{2\,\text{Su}}{}$$

$$=-\frac{4\,\text{Subst}\left(\int \frac{1}{-4\left(a^{2/5}-b^{2/5}\right)-x^2}dx,x,-2\sqrt[5]{b}+2\sqrt[5]{a}\tan\left(\frac{x}{2}\right)\right)}{5a^{4/5}}-\frac{4\,\text{Subst}\left(\int \frac{1}{-4\left(a^{2/5}+\sqrt[5]{-1}b^{2/5}\right)-x^2}dx,x,2\right.}{5a^{4/5}}$$

$$=-\frac{2\tan^{-1}\left(\frac{\sqrt[5]{b}-\sqrt[5]{a}\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}-b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-b^{2/5}}}-\frac{2\tan^{-1}\left(\frac{(-1)^{2/5}\sqrt[5]{b}-\sqrt[5]{a}\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}-(-1)^{4/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-(-1)^{4/5}b^{2/5}}}-\frac{2\tan^{-1}\left(\frac{(-1)^{4/5}\sqrt[5]{b}-\sqrt[5]{a}\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}+(-1)^{3/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}+(-1)^{3/5}b^{2/5}}}+\frac{2\tan}{5a^{4/5}}$$

Mathematica [C] time = 0.19115, size = 149, normalized size = 0.39

$$-\frac{8}{5}i\text{RootSum}\left[32\#1^5a+i\#1^{10}b-5i\#1^8b+10i\#1^6b-10i\#1^4b+5i\#1^2b-ib\&,\frac{2\#1^3\tan^{-1}\left(\frac{\sin(x)}{\cos(x)-\#1}\right)-i\#1^3\log\left(\#1^2\right.}{-16i\#1^3a+\#1^8b-4\#1^6b+6\#1^4}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a - b*Sin[x]^5)^(-1),x]
```

```
[Out] ((-8*I)/5)*RootSum[(-I)*b + (5*I)*b*#1^2 - (10*I)*b*#1^4 + 32*a*#1^5 + (10*
I)*b*#1^6 - (5*I)*b*#1^8 + I*b*#1^10 & , (2*ArcTan[Sin[x]/(Cos[x] - #1)]*#1
^3 - I*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^3)/(b - 4*b*#1^2 - (16*I)*a*#1^3 + 6*
b*#1^4 - 4*b*#1^6 + b*#1^8) & ]
```

Maple [C] time = 0.125, size = 109, normalized size = 0.3

$$\frac{1}{5}\sum_{\_R=\text{RootOf}\left(a\_Z^{10}+5a\_Z^8+10a\_Z^6-32b\_Z^5+10a\_Z^4+5a\_Z^2+a\right)}\frac{\_R^8+4\_R^6+6\_R^4+4\_R^2+1}{\_R^9a+4\_R^7a+6\_R^5a-16\_R^4b+4\_R^3a+\_Ra}\ln\left(\tan\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-b*sin(x)^5),x)
```

```
[Out] 1/5*sum((_R^8+4*_R^6+6*_R^4+4*_R^2+1)/(_R^9*a+4*_R^7*a+6*_R^5*a-16*_R^4*b+4
*_R^3*a+_R*a)*ln(tan(1/2*x)-_R),_R=RootOf(_Z^10*a+5*_Z^8*a+10*_Z^6*a-32*_Z^
5*b+10*_Z^4*a+5*_Z^2*a+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{b\sin{(x)}^5 - a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(x)^5),x, algorithm="maxima")
```

```
[Out] -integrate(1/(b*sin(x)^5 - a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(x)^5),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(x)**5),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{b\sin{(x)}^5 - a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(x)^5),x, algorithm="giac")
```

```
[Out] integrate(-1/(b*sin(x)^5 - a), x)
```

3.253 $\int \frac{1}{a-b\sin^6(x)}\,dx$

Optimal. Leaf size=175

$$\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}}$$

```
[Out] ArcTan[(Sqrt[a^(1/3) - b^(1/3)]*Tan[x])/a^(1/6)]/(3*a^(5/6)*Sqrt[a^(1/3) -
b^(1/3)]) + ArcTan[(Sqrt[a^(1/3) + (-1)^(1/3)*b^(1/3)]*Tan[x])/a^(1/6)]/(3*
a^(5/6)*Sqrt[a^(1/3) + (-1)^(1/3)*b^(1/3)]) + ArcTan[(Sqrt[a^(1/3) - (-1)^(
2/3)*b^(1/3)]*Tan[x])/a^(1/6)]/(3*a^(5/6)*Sqrt[a^(1/3) - (-1)^(2/3)*b^(1/3)
])
```

Rubi [A] time = 0.260425, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3211, 3181, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*Sin[x]^6)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[a^(1/3) - b^(1/3)]*Tan[x])/a^(1/6)]/(3*a^(5/6)*Sqrt[a^(1/3) -
b^(1/3)]) + ArcTan[(Sqrt[a^(1/3) + (-1)^(1/3)*b^(1/3)]*Tan[x])/a^(1/6)]/(3*
a^(5/6)*Sqrt[a^(1/3) + (-1)^(1/3)*b^(1/3)]) + ArcTan[(Sqrt[a^(1/3) - (-1)^(
2/3)*b^(1/3)]*Tan[x])/a^(1/6)]/(3*a^(5/6)*Sqrt[a^(1/3) - (-1)^(2/3)*b^(1/3)
])
```

Rule 3211

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^((4*k)/n)*Rt[-(a/b),
n/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{a-b\sin^6(x)}\,dx = \frac{\int \frac{1}{1-\frac{\sqrt[3]{b}\sin^2(x)}{\sqrt[3]{a}}}\,dx}{3a} + \frac{\int \frac{1}{1+\frac{\sqrt[3]{-1}\sqrt[3]{b}\sin^2(x)}{\sqrt[3]{a}}}\,dx}{3a} + \frac{\int \frac{1}{1-\frac{(-1)^{2/3}\sqrt[3]{b}\sin^2(x)}{\sqrt[3]{a}}}\,dx}{3a}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1+\left(1-\frac{\sqrt[3]{b}}{\sqrt[3]{a}}\right)x^2}\,dx, x, \tan(x)\right)}{3a} + \frac{\text{Subst}\left(\int \frac{1}{1+\left(1+\frac{\sqrt[3]{-1}\sqrt[3]{b}}{\sqrt[3]{a}}\right)x^2}\,dx, x, \tan(x)\right)}{3a} + \frac{\text{Subst}\left(\int \frac{1}{1+\left(1-\frac{(-1)^{2/}}{\sqrt[3]{}}\right.}\right.}{3}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}} + \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}} + \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}}$$

Mathematica [C] time = 0.173231, size = 148, normalized size = 0.85

$$\frac{8}{3}\text{RootSum}\left[64\#1^3a+\#1^6b-6\#1^5b+15\#1^4b-20\#1^3b+15\#1^2b-6\#1b+b\&, \frac{2\#1^2\tan^{-1}\left(\frac{\sin(2x)}{\cos(2x)-\#1}\right)-i\#1^2\log\left(\#1^2\right.}{32\#1^2a+\#1^5b-5\#1^4b+10\#1^3b-1\text{(}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*Sin[x]^6)^(-1),x]
```

```
[Out] (8*RootSum[b - 6*b*#1 + 15*b*#1^2 + 64*a*#1^3 - 20*b*#1^3 + 15*b*#1^4 - 6*b
*#1^5 + b*#1^6 & , (2*ArcTan[Sin[2*x]/(Cos[2*x] - #1)]*#1^2 - I*Log[1 - 2*C
os[2*x]*#1 + #1^2]*#1^2)/(-b + 5*b*#1 + 32*a*#1^2 - 10*b*#1^2 + 10*b*#1^3 -
 5*b*#1^4 + b*#1^5) & ])/3
```

Maple [C] time = 0.358, size = 71, normalized size = 0.4

$$\frac{1}{6}\sum_{\_R=\text{RootOf}\left((a-b)\_Z^6+3a\_Z^4+3a\_Z^2+a\right)}\frac{\left(\_R^4+2\_R^2+1\right)\ln\left(\tan(x)-\_R\right)}{\_R^5a-\_R^5b+2\_R^3a+\_R\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-b*sin(x)^6),x)
```

```
[Out] 1/6*sum((_R^4+2*_R^2+1)/(_R^5*a-_R^5*b+2*_R^3*a+_R*a)*ln(tan(x)-_R),_R=Root
Of((a-b)*_Z^6+3*a*_Z^4+3*a*_Z^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{b\sin(x)^6-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(x)^6),x, algorithm="maxima")
```

```
[Out] -integrate(1/(b*sin(x)^6 - a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(x)^6),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{a - b\sin^6(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(x)**6),x)
```

```
[Out] Integral(1/(a - b*sin(x)**6), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{b\sin(x)^6 - a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(x)^6),x, algorithm="giac")
```

```
[Out] integrate(-1/(b*sin(x)^6 - a), x)
```

3.254 $\int \frac{1}{a-b\sin^8(x)}\,dx$

Optimal. Leaf size=213

$$\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}\tan(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}\tan(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{a}+i\sqrt[4]{b}}\tan(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}+i\sqrt[4]{b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{a}+\sqrt[4]{b}}\tan(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}+\sqrt[4]{b}}}$$

```
[Out] ArcTan[(Sqrt[a^(1/4) - b^(1/4)]*Tan[x])/a^(1/8)]/(4*a^(7/8)*Sqrt[a^(1/4) -
b^(1/4)]) + ArcTan[(Sqrt[a^(1/4) - I*b^(1/4)]*Tan[x])/a^(1/8)]/(4*a^(7/8)*S
qrt[a^(1/4) - I*b^(1/4)]) + ArcTan[(Sqrt[a^(1/4) + I*b^(1/4)]*Tan[x])/a^(1/
8)]/(4*a^(7/8)*Sqrt[a^(1/4) + I*b^(1/4)]) + ArcTan[(Sqrt[a^(1/4) + b^(1/4)]
*Tan[x])/a^(1/8)]/(4*a^(7/8)*Sqrt[a^(1/4) + b^(1/4)])
```

Rubi [A] time = 0.217818, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3211, 3181, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}\tan(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}\tan(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{a}+i\sqrt[4]{b}}\tan(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}+i\sqrt[4]{b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{a}+\sqrt[4]{b}}\tan(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}+\sqrt[4]{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*Sin[x]^8)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[a^(1/4) - b^(1/4)]*Tan[x])/a^(1/8)]/(4*a^(7/8)*Sqrt[a^(1/4) -
b^(1/4)]) + ArcTan[(Sqrt[a^(1/4) - I*b^(1/4)]*Tan[x])/a^(1/8)]/(4*a^(7/8)*S
qrt[a^(1/4) - I*b^(1/4)]) + ArcTan[(Sqrt[a^(1/4) + I*b^(1/4)]*Tan[x])/a^(1/
8)]/(4*a^(7/8)*Sqrt[a^(1/4) + I*b^(1/4)]) + ArcTan[(Sqrt[a^(1/4) + b^(1/4)]
*Tan[x])/a^(1/8)]/(4*a^(7/8)*Sqrt[a^(1/4) + b^(1/4)])
```

Rule 3211

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^((4*k)/n)*Rt[-(a/b),
n/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{a-b\sin^8(x)}\,dx = \frac{\int \frac{1}{1-\frac{\sqrt[4]{b}\sin^2(x)}{\sqrt[4]{a}}}\,dx}{4a} + \frac{\int \frac{1}{1-\frac{i\sqrt[4]{b}\sin^2(x)}{\sqrt[4]{a}}}\,dx}{4a} + \frac{\int \frac{1}{1+\frac{i\sqrt[4]{b}\sin^2(x)}{\sqrt[4]{a}}}\,dx}{4a} + \frac{\int \frac{1}{1+\frac{\sqrt[4]{b}\sin^2(x)}{\sqrt[4]{a}}}\,dx}{4a}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1+\left(1-\frac{\sqrt[4]{b}}{\sqrt[4]{a}}\right)x^2}\,dx, x, \tan(x)\right)}{4a} + \frac{\text{Subst}\left(\int \frac{1}{1+\left(1-\frac{i\sqrt[4]{b}}{\sqrt[4]{a}}\right)x^2}\,dx, x, \tan(x)\right)}{4a} + \frac{\text{Subst}\left(\int \frac{1}{1+\left(1+\frac{i\sqrt[4]{b}}{\sqrt[4]{a}}\right.}\right.}{4a}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}\tan(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}} + \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}\tan(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}} + \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{a}+i\sqrt[4]{b}}\tan(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}+i\sqrt[4]{b}}} + \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{a}+\sqrt[4]{b}}\tan}{\sqrt[8]{a}}\right.}{4a^{7/8}\sqrt{\sqrt[4]{a}+\sqrt[4]{b}}}$$

Mathematica [C] time = 0.213668, size = 174, normalized size = 0.82

$$-8\text{RootSum}\left[-256\#1^4a+\#1^8b-8\#1^7b+28\#1^6b-56\#1^5b+70\#1^4b-56\#1^3b+28\#1^2b-8\#1b+b\&, \frac{2\#}{-128\#1^3a+}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a - b*Sin[x]^8)^(-1),x]
```

```
[Out] -8*RootSum[b - 8*b*#1 + 28*b*#1^2 - 56*b*#1^3 - 256*a*#1^4 + 70*b*#1^4 - 56
*b*#1^5 + 28*b*#1^6 - 8*b*#1^7 + b*#1^8 & , (2*ArcTan[Sin[2*x]/(Cos[2*x] -
#1)]*#1^3 - I*Log[1 - 2*Cos[2*x]*#1 + #1^2]*#1^3)/(-b + 7*b*#1 - 21*b*#1^2
- 128*a*#1^3 + 35*b*#1^3 - 35*b*#1^4 + 21*b*#1^5 - 7*b*#1^6 + b*#1^7) & ]
```

Maple [C] time = 0.114, size = 88, normalized size = 0.4

$$\frac{1}{8}\sum_{\_R=\text{RootOf}\left((a-b)\_Z^8+4a\_Z^6+6a\_Z^4+4a\_Z^2+a\right)}\frac{\left(\_R^6+3\_R^4+3\_R^2+1\right)\ln\left(\tan(x)-\_R\right)}{\_R^7a-\_R^7b+3\_R^5a+3\_R^3a+\_Ra}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-b*sin(x)^8),x)
```

```
[Out] 1/8*sum((_R^6+3*_R^4+3*_R^2+1)/(_R^7*a-_R^7*b+3*_R^5*a+3*_R^3*a+_R*a)*ln(ta
n(x)-_R),_R=RootOf((a-b)*_Z^8+4*a*_Z^6+6*a*_Z^4+4*a*_Z^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{b\sin(x)^8-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(x)^8),x, algorithm="maxima")
```

```
[Out] -integrate(1/(b*sin(x)^8 - a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(x)^8),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(x)**8),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{b \sin(x)^8 - a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(x)^8),x, algorithm="giac")
```

```
[Out] integrate(-1/(b*sin(x)^8 - a), x)
```

3.255 $\int \frac{1}{1+\sin^5(x)}\,dx$

Optimal. Leaf size=195

$$\frac{2\tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)+(-1)^{2/5}}{\sqrt{1-(-1)^{4/5}}}\right)}{5\sqrt{1-(-1)^{4/5}}}+\frac{2\tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)+(-1)^{4/5}}{\sqrt{1+(-1)^{3/5}}}\right)}{5\sqrt{1+(-1)^{3/5}}}-\frac{2\tan^{-1}\left(\frac{(-1)^{3/5}\left((-1)^{2/5}\tan\left(\frac{x}{2}\right)+1\right)}{\sqrt{1+\sqrt[5]{-1}}}\right)}{5\sqrt{1+\sqrt[5]{-1}}}-\frac{2\tan^{-1}\left(\frac{\sqrt[5]{-1}\left((-1)^{4/5}\tan\left(\frac{x}{2}\right)+1\right)}{\sqrt{1-(-1)^{2/5}}}\right)}{5\sqrt{1-(-1)^{2/5}}}-$$

```
[Out] (2*ArcTan[((-1)^(2/5) + Tan[x/2])/Sqrt[1 - (-1)^(4/5)]])/(5*Sqrt[1 - (-1)^(
4/5)]) + (2*ArcTan[((-1)^(4/5) + Tan[x/2])/Sqrt[1 + (-1)^(3/5)]])/(5*Sqrt[1
 + (-1)^(3/5)]) - (2*ArcTan[((-1)^(3/5)*(1 + (-1)^(2/5)*Tan[x/2]))/Sqrt[1 +
 (-1)^(1/5)]])/(5*Sqrt[1 + (-1)^(1/5)]) - (2*ArcTan[((-1)^(1/5)*(1 + (-1)^(
4/5)*Tan[x/2]))/Sqrt[1 - (-1)^(2/5)]])/(5*Sqrt[1 - (-1)^(2/5)]) - Cos[x]/(5
*(1 + Sin[x]))
```

Rubi [A] time = 0.382604, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3213, 2648, 2660, 618, 204}

$$\frac{2\tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)+(-1)^{2/5}}{\sqrt{1-(-1)^{4/5}}}\right)}{5\sqrt{1-(-1)^{4/5}}}+\frac{2\tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)+(-1)^{4/5}}{\sqrt{1+(-1)^{3/5}}}\right)}{5\sqrt{1+(-1)^{3/5}}}-\frac{2\tan^{-1}\left(\frac{(-1)^{3/5}\left((-1)^{2/5}\tan\left(\frac{x}{2}\right)+1\right)}{\sqrt{1+\sqrt[5]{-1}}}\right)}{5\sqrt{1+\sqrt[5]{-1}}}-\frac{2\tan^{-1}\left(\frac{\sqrt[5]{-1}\left((-1)^{4/5}\tan\left(\frac{x}{2}\right)+1\right)}{\sqrt{1-(-1)^{2/5}}}\right)}{5\sqrt{1-(-1)^{2/5}}}-$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sin[x]^5)^(-1),x]
```

```
[Out] (2*ArcTan[((-1)^(2/5) + Tan[x/2])/Sqrt[1 - (-1)^(4/5)]])/(5*Sqrt[1 - (-1)^(
4/5)]) + (2*ArcTan[((-1)^(4/5) + Tan[x/2])/Sqrt[1 + (-1)^(3/5)]])/(5*Sqrt[1
 + (-1)^(3/5)]) - (2*ArcTan[((-1)^(3/5)*(1 + (-1)^(2/5)*Tan[x/2]))/Sqrt[1 +
 (-1)^(1/5)]])/(5*Sqrt[1 + (-1)^(1/5)]) - (2*ArcTan[((-1)^(1/5)*(1 + (-1)^(
4/5)*Tan[x/2]))/Sqrt[1 - (-1)^(2/5)]])/(5*Sqrt[1 - (-1)^(2/5)]) - Cos[x]/(5
*(1 + Sin[x]))
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{1+\sin^5(x)}\,dx = \int \left(-\frac{1}{5(-1-\sin(x))} - \frac{1}{5\left(-1+\sqrt[5]{-1}\sin(x)\right)} - \frac{1}{5\left(-1-(-1)^{2/5}\sin(x)\right)} - \frac{1}{5\left(-1+(-1)^{3/5}\sin(x)\right)} - \frac{1}{5\left(\right.}\right.$$

$$= -\left(\frac{1}{5}\int \frac{1}{-1-\sin(x)}\,dx\right) - \frac{1}{5}\int \frac{1}{-1+\sqrt[5]{-1}\sin(x)}\,dx - \frac{1}{5}\int \frac{1}{-1-(-1)^{2/5}\sin(x)}\,dx - \frac{1}{5}\int \frac{1}{-1+(-1)^{3/}}$$

$$= -\frac{\cos(x)}{5(1+\sin(x))} - \frac{2}{5}\text{Subst}\left(\int \frac{1}{-1+2\sqrt[5]{-1}x-x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right) - \frac{2}{5}\text{Subst}\left(\int \frac{1}{-1-2(-1)^{2/5}x-x^2}\right.$$

$$= -\frac{\cos(x)}{5(1+\sin(x))} + \frac{4}{5}\text{Subst}\left(\int \frac{1}{-4\left(1+\sqrt[5]{-1}\right)-x^2}\,dx, x, 2(-1)^{3/5}-2\tan\left(\frac{x}{2}\right)\right) + \frac{4}{5}\text{Subst}\left(\int \frac{}{-4\left(1-\right.}\right.$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt[5]{-1}-\tan\left(\frac{x}{2}\right)}{\sqrt{1-(-1)^{2/5}}}\right)}{5\sqrt{1-(-1)^{2/5}}} - \frac{2\tan^{-1}\left(\frac{(-1)^{3/5}-\tan\left(\frac{x}{2}\right)}{\sqrt{1+\sqrt[5]{-1}}}\right)}{5\sqrt{1+\sqrt[5]{-1}}} + \frac{2\tan^{-1}\left(\frac{(-1)^{2/5}+\tan\left(\frac{x}{2}\right)}{\sqrt{1-(-1)^{4/5}}}\right)}{5\sqrt{1-(-1)^{4/5}}} + \frac{2\tan^{-1}\left(\frac{(-1)^{4/5}+\tan\left(\frac{x}{2}\right)}{\sqrt{1+(-1)^{3/5}}}\right)}{5\sqrt{1+(-1)^{3/5}}}$$

Mathematica [C] time = 0.159132, size = 411, normalized size = 2.11

$$\frac{2\sin\left(\frac{x}{2}\right)}{5\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)} - \frac{1}{10}i\text{RootSum}\left[\#1^8 - 2i\#1^7 - 8\#1^6 + 14i\#1^5 + 30\#1^4 - 14i\#1^3 - 8\#1^2 + 2i\#1 + 1\&, \frac{-i\#1^6\log\left(\#\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sin[x]^5)^(-1),x]
```

```
[Out] (-I/10)*RootSum[1 + (2*I)*#1 - 8*#1^2 - (14*I)*#1^3 + 30*#1^4 + (14*I)*#1^5
 - 8*#1^6 - (2*I)*#1^7 + #1^8 & , (-2*ArcTan[Sin[x]/(Cos[x] - #1)] + I*Log[
1 - 2*Cos[x]*#1 + #1^2] - (8*I)*ArcTan[Sin[x]/(Cos[x] - #1)]*#1 - 4*Log[1 -
 2*Cos[x]*#1 + #1^2]*#1 + 30*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^2 - (15*I)*Log
[1 - 2*Cos[x]*#1 + #1^2]*#1^2 + (80*I)*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^3 +
40*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^3 - 30*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^4
+ (15*I)*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^4 - (8*I)*ArcTan[Sin[x]/(Cos[x] - #
1)]*#1^5 - 4*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^5 + 2*ArcTan[Sin[x]/(Cos[x] - #
1)]*#1^6 - I*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^6)/(I - 8*#1 - (21*I)*#1^2 + 60
*#1^3 + (35*I)*#1^4 - 24*#1^5 - (7*I)*#1^6 + 4*#1^7) & ] + (2*Sin[x/2])/(5*
(Cos[x/2] + Sin[x/2]))
```

Maple [C] time = 0.079, size = 133, normalized size = 0.7

$$-\frac{2}{5}\left(\tan\left(\frac{x}{2}\right)+1\right)^{-1} + \frac{2}{5}\sum_{\_R=\text{RootOf}\left(\_Z^8-2\_Z^7+8\_Z^6-14\_Z^5+30\_Z^4-14\_Z^3+8\_Z^2-2\_Z+1\right)} \frac{2\_R^6 - 3\_R^5 + 10\_R^4 - 10\_R^3}{4\_R^7 - 7\_R^6 + 24\_R^5 - 35\_R^4 + 60}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+sin(x)^5),x)
```

```
[Out] -2/5/(tan(1/2*x)+1)+2/5*sum((2*_R^6-3*_R^5+10*_R^4-10*_R^3+10*_R^2-3*_R+2)/
(4*_R^7-7*_R^6+24*_R^5-35*_R^4+60*_R^3-21*_R^2+8*_R-1)*ln(tan(1/2*x)-_R),_R
=RootOf(_Z^8-2*_Z^7+8*_Z^6-14*_Z^5+30*_Z^4-14*_Z^3+8*_Z^2-2*_Z+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)^5),x, algorithm="maxima")
```

```
[Out] -1/5*(5*(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1)*integrate(-2/5*((4*cos(6*x) -
40*cos(4*x) + 4*cos(2*x) - sin(7*x) + 15*sin(5*x) - 15*sin(3*x) + sin(x))*c
os(8*x) + 2*(22*cos(5*x) - 22*cos(3*x) + 2*cos(x) - 8*sin(6*x) + 55*sin(4*x
) - 8*sin(2*x))*cos(7*x) - 2*cos(7*x)^2 + 4*(110*cos(4*x) - 16*cos(2*x) - 4
4*sin(5*x) + 44*sin(3*x) - 4*sin(x) + 1)*cos(6*x) - 32*cos(6*x)^2 + 2*(210*
cos(3*x) - 22*cos(x) - 505*sin(4*x) + 88*sin(2*x))*cos(5*x) - 210*cos(5*x)^
2 + 10*(44*cos(2*x) - 101*sin(3*x) + 11*sin(x) - 4)*cos(4*x) - 1200*cos(4*x
)^2 + 44*(cos(x) - 4*sin(2*x))*cos(3*x) - 210*cos(3*x)^2 - 4*(4*sin(x) - 1)
*cos(2*x) - 32*cos(2*x)^2 - 2*cos(x)^2 + (cos(7*x) - 15*cos(5*x) + 15*cos(3
*x) - cos(x) + 4*sin(6*x) - 40*sin(4*x) + 4*sin(2*x))*sin(8*x) + (16*cos(6*
x) - 110*cos(4*x) + 16*cos(2*x) + 44*sin(5*x) - 44*sin(3*x) + 4*sin(x) - 1)
*sin(7*x) - 2*sin(7*x)^2 + 8*(22*cos(5*x) - 22*cos(3*x) + 2*cos(x) + 55*sin
(4*x) - 8*sin(2*x))*sin(6*x) - 32*sin(6*x)^2 + (1010*cos(4*x) - 176*cos(2*x
) + 420*sin(3*x) - 44*sin(x) + 15)*sin(5*x) - 210*sin(5*x)^2 + 10*(101*cos(
3*x) - 11*cos(x) + 44*sin(2*x))*sin(4*x) - 1200*sin(4*x)^2 + (176*cos(2*x)
+ 44*sin(x) - 15)*sin(3*x) - 210*sin(3*x)^2 + 16*cos(x)*sin(2*x) - 32*sin(2
*x)^2 - 2*sin(x)^2 + sin(x))/(2*(8*cos(6*x) - 30*cos(4*x) + 8*cos(2*x) - 2*
sin(7*x) + 14*sin(5*x) - 14*sin(3*x) + 2*sin(x) - 1)*cos(8*x) - cos(8*x)^2
+ 8*(7*cos(5*x) - 7*cos(3*x) + cos(x) - 4*sin(6*x) + 15*sin(4*x) - 4*sin(2*
x))*cos(7*x) - 4*cos(7*x)^2 + 16*(30*cos(4*x) - 8*cos(2*x) - 14*sin(5*x) +
14*sin(3*x) - 2*sin(x) + 1)*cos(6*x) - 64*cos(6*x)^2 + 56*(7*cos(3*x) - cos
(x) - 15*sin(4*x) + 4*sin(2*x))*cos(5*x) - 196*cos(5*x)^2 + 60*(8*cos(2*x)
- 14*sin(3*x) + 2*sin(x) - 1)*cos(4*x) - 900*cos(4*x)^2 + 56*(cos(x) - 4*si
n(2*x))*cos(3*x) - 196*cos(3*x)^2 - 16*(2*sin(x) - 1)*cos(2*x) - 64*cos(2*x
)^2 - 4*cos(x)^2 + 4*(cos(7*x) - 7*cos(5*x) + 7*cos(3*x) - cos(x) + 4*sin(6
*x) - 15*sin(4*x) + 4*sin(2*x))*sin(8*x) - sin(8*x)^2 + 4*(8*cos(6*x) - 30*
cos(4*x) + 8*cos(2*x) + 14*sin(5*x) - 14*sin(3*x) + 2*sin(x) - 1)*sin(7*x)
- 4*sin(7*x)^2 + 32*(7*cos(5*x) - 7*cos(3*x) + cos(x) + 15*sin(4*x) - 4*sin
(2*x))*sin(6*x) - 64*sin(6*x)^2 + 28*(30*cos(4*x) - 8*cos(2*x) + 14*sin(3*x
) - 2*sin(x) + 1)*sin(5*x) - 196*sin(5*x)^2 + 120*(7*cos(3*x) - cos(x) + 4*
sin(2*x))*sin(4*x) - 900*sin(4*x)^2 + 28*(8*cos(2*x) + 2*sin(x) - 1)*sin(3*
x) - 196*sin(3*x)^2 + 32*cos(x)*sin(2*x) - 64*sin(2*x)^2 - 4*sin(x)^2 + 4*s
in(x) - 1), x) + 2*cos(x))/(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)^5),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(\sin(x)+1)\left(\sin^4(x)-\sin^3(x)+\sin^2(x)-\sin(x)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)**5),x)
```

```
[Out] Integral(1/((sin(x) + 1)*(sin(x)**4 - sin(x)**3 + sin(x)**2 - sin(x) + 1)),
 x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sin(x)^5+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)^5),x, algorithm="giac")
```

```
[Out] integrate(1/(sin(x)^5 + 1), x)
```

3.256 $\int \frac{1}{1+\sin^6(x)} dx$

Optimal. Leaf size=103

$$\frac{x}{3\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{1-\sqrt[3]{-1}}\tan(x)\right)}{3\sqrt{1-\sqrt[3]{-1}}}+\frac{\tan^{-1}\left(\sqrt{1+(-1)^{2/3}}\tan(x)\right)}{3\sqrt{1+(-1)^{2/3}}}+\frac{\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\sin^2(x)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

```
[Out] x/(3*Sqrt[2]) + ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Sin[x]^2)]/(3*Sqrt[2]
) + ArcTan[Sqrt[1 - (-1)^(1/3)]*Tan[x]]/(3*Sqrt[1 - (-1)^(1/3)]) + ArcTan[S
qrt[1 + (-1)^(2/3)]*Tan[x]]/(3*Sqrt[1 + (-1)^(2/3)])
```

Rubi [A] time = 0.103638, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3211, 3181, 203}

$$\frac{x}{3\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{1-\sqrt[3]{-1}}\tan(x)\right)}{3\sqrt{1-\sqrt[3]{-1}}}+\frac{\tan^{-1}\left(\sqrt{1+(-1)^{2/3}}\tan(x)\right)}{3\sqrt{1+(-1)^{2/3}}}+\frac{\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\sin^2(x)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sin[x]^6)^(-1),x]
```

```
[Out] x/(3*Sqrt[2]) + ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Sin[x]^2)]/(3*Sqrt[2]
) + ArcTan[Sqrt[1 - (-1)^(1/3)]*Tan[x]]/(3*Sqrt[1 - (-1)^(1/3)]) + ArcTan[S
qrt[1 + (-1)^(2/3)]*Tan[x]]/(3*Sqrt[1 + (-1)^(2/3)])
```

Rule 3211

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^((4*k)/n)*Rt[-(a/b),
n/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{1+\sin^6(x)}\,dx = \frac{1}{3}\int \frac{1}{1+\sin^2(x)}\,dx + \frac{1}{3}\int \frac{1}{1-\sqrt[3]{-1}\sin^2(x)}\,dx + \frac{1}{3}\int \frac{1}{1+(-1)^{2/3}\sin^2(x)}\,dx$$

$$= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \tan(x)\right) + \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{1+\left(1-\sqrt[3]{-1}\right)x^2}\,dx, x, \tan(x)\right) + \frac{1}{3}\,\text{Subst}\left(\int \frac{}{1-}\right.$$

$$= \frac{x}{3\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)}{3\sqrt{2}} + \frac{\tan^{-1}\left(\sqrt{1-\sqrt[3]{-1}}\tan(x)\right)}{3\sqrt{1-\sqrt[3]{-1}}} + \frac{\tan^{-1}\left(\sqrt{1+(-1)^{2/3}}\tan(x)\right)}{3\sqrt{1+(-1)^{2/3}}}$$

Mathematica [A] time = 0.164577, size = 79, normalized size = 0.77

$$\frac{1}{12}\left(-2\sqrt{3}\tan^{-1}\left(\frac{1-2\tan(x)}{\sqrt{3}}\right) + 2\sqrt{2}\tan^{-1}\left(\sqrt{2}\tan(x)\right) + 2\sqrt{3}\tan^{-1}\left(\frac{2\tan(x)+1}{\sqrt{3}}\right) - \log(2-\sin(2x)) + \log(\sin(2x)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sin[x]^6)^(-1),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(1 - 2*Tan[x])/Sqrt[3]] + 2*Sqrt[2]*ArcTan[Sqrt[2]*Tan[x
]] + 2*Sqrt[3]*ArcTan[(1 + 2*Tan[x])/Sqrt[3]] - Log[2 - Sin[2*x]] + Log[2 +
 Sin[2*x]])/12
```

Maple [A] time = 0.044, size = 72, normalized size = 0.7

$$\frac{\arctan\left(\sqrt{2}\tan(x)\right)\sqrt{2}}{6} - \frac{\ln\left((\tan(x))^2 - \tan(x) + 1\right)}{12} + \frac{\sqrt{3}}{6}\arctan\left(\frac{(2\tan(x)-1)\sqrt{3}}{3}\right) + \frac{\ln\left((\tan(x))^2 + \tan(x) + 1\right.}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+sin(x)^6),x)
```

```
[Out] 1/6*arctan(2^(1/2)*tan(x))*2^(1/2)-1/12*ln(tan(x)^2-tan(x)+1)+1/6*3^(1/2)*a
rctan(1/3*(2*tan(x)-1)*3^(1/2))+1/12*ln(tan(x)^2+tan(x)+1)+1/6*3^(1/2)*arct
an(1/3*(1+2*tan(x))*3^(1/2))
```

Maxima [A] time = 1.46809, size = 96, normalized size = 0.93

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\tan(x)+1)\right) + \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\tan(x)-1)\right) + \frac{1}{6}\sqrt{2}\arctan\left(\sqrt{2}\tan(x)\right) + \frac{1}{12}\log\left(\tan(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)^6),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*tan(x) + 1)) + 1/6*sqrt(3)*arctan(1/3*sqr
t(3)*(2*tan(x) - 1)) + 1/6*sqrt(2)*arctan(sqrt(2)*tan(x)) + 1/12*log(tan(x)
^2 + tan(x) + 1) - 1/12*log(tan(x)^2 - tan(x) + 1)
```

Fricas [A] time = 1.8856, size = 466, normalized size = 4.52

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{4\sqrt{3}\cos\left(x\right)\sin\left(x\right)+\sqrt{3}}{3\left(2\cos\left(x\right)^{2}-1\right)}\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{4\sqrt{3}\cos\left(x\right)\sin\left(x\right)-\sqrt{3}}{3\left(2\cos\left(x\right)^{2}-1\right)}\right)-\frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos\left(x\right)^{2}\cdots}{4\cos\left(x\right)\sin\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)^6),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*(4*sqrt(3)*cos(x)*sin(x) + sqrt(3))/(2*cos(x)^2 - 1
)) + 1/12*sqrt(3)*arctan(1/3*(4*sqrt(3)*cos(x)*sin(x) - sqrt(3))/(2*cos(x)^
2 - 1)) - 1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - 2*sqrt(2))/(cos(x)*
sin(x))) + 1/24*log(-cos(x)^4 + cos(x)^2 + 2*cos(x)*sin(x) + 1) - 1/24*log(
-cos(x)^4 + cos(x)^2 - 2*cos(x)*sin(x) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)**6),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.14276, size = 250, normalized size = 2.43

$$\frac{1}{6}\sqrt{3}\left(x+\arctan\left(-\frac{\sqrt{3}\sin\left(2x\right)+\cos\left(2x\right)-2\sin\left(2x\right)+1}{\sqrt{3}\cos\left(2x\right)+\sqrt{3}-2\cos\left(2x\right)-\sin\left(2x\right)+2}\right)\right)+\frac{1}{6}\sqrt{3}\left(x+\arctan\left(-\frac{\sqrt{3}\sin\left(2x\right)-\cos\left(2\cdots\right.}{\sqrt{3}\cos\left(2x\right)+\sqrt{3}-2\cos\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)^6),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*(x + arctan(-(sqrt(3)*sin(2*x) + cos(2*x) - 2*sin(2*x) + 1)/(sq
rt(3)*cos(2*x) + sqrt(3) - 2*cos(2*x) - sin(2*x) + 2))) + 1/6*sqrt(3)*(x +
arctan(-(sqrt(3)*sin(2*x) - cos(2*x) - 2*sin(2*x) - 1)/(sqrt(3)*cos(2*x) +
sqrt(3) - 2*cos(2*x) + sin(2*x) + 2))) + 1/6*sqrt(2)*(x + arctan(-(sqrt(2)*
sin(2*x) - 2*sin(2*x))/(sqrt(2)*cos(2*x) + sqrt(2) - 2*cos(2*x) + 2))) + 1/
12*log(tan(x)^2 + tan(x) + 1) - 1/12*log(tan(x)^2 - tan(x) + 1)
```

3.257 $\int \frac{1}{1+\sin^8(x)}\,dx$

Optimal. Leaf size=218

$$\frac{1}{8}\left(\sqrt{1+\sqrt{4-2\sqrt{2}}}+\sqrt{2+2\sqrt[4]{2}+2\sqrt{1+\sqrt{2}}+2\sqrt{2+\sqrt{2}}}+\sqrt{1+\sqrt{4+2\sqrt{2}}}\right)\left(x-\tan^{-1}(\tan(x))\right)+\frac{\tan^{-1}\left(\sqrt{1-\sqrt[4]{-}}\right.}{4\sqrt{1-\sqrt[4]{}}}$$

```
[Out] ((Sqrt[1 + Sqrt[4 - 2*Sqrt[2]]] + Sqrt[2 + 2*2^(1/4) + 2*Sqrt[1 + Sqrt[2]]
+ 2*Sqrt[2 + Sqrt[2]]] + Sqrt[1 + Sqrt[4 + 2*Sqrt[2]]])*(x - ArcTan[Tan[x]]
))/8 + ArcTan[Sqrt[1 - (-1)^(1/4)]*Tan[x]]/(4*Sqrt[1 - (-1)^(1/4)]) + ArcTa
n[Sqrt[1 + (-1)^(1/4)]*Tan[x]]/(4*Sqrt[1 + (-1)^(1/4)]) + ArcTan[Sqrt[1 - (
-1)^(3/4)]*Tan[x]]/(4*Sqrt[1 - (-1)^(3/4)]) + ArcTan[Sqrt[1 + (-1)^(3/4)]*T
an[x]]/(4*Sqrt[1 + (-1)^(3/4)])
```

Rubi [A] time = 0.200197, antiderivative size = 129, normalized size of antiderivative = 0.59, number of steps used = 9, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3211, 3181, 203}

$$\frac{\tan^{-1}\left(\sqrt{1-\sqrt[4]{-1}}\tan(x)\right)}{4\sqrt{1-\sqrt[4]{-1}}}+\frac{\tan^{-1}\left(\sqrt{1+\sqrt[4]{-1}}\tan(x)\right)}{4\sqrt{1+\sqrt[4]{-1}}}+\frac{\tan^{-1}\left(\sqrt{1-(-1)^{3/4}}\tan(x)\right)}{4\sqrt{1-(-1)^{3/4}}}+\frac{\tan^{-1}\left(\sqrt{1+(-1)^{3/4}}\tan(x)\right)}{4\sqrt{1+(-1)^{3/4}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sin[x]^8)^(-1),x]
```

```
[Out] ArcTan[Sqrt[1 - (-1)^(1/4)]*Tan[x]]/(4*Sqrt[1 - (-1)^(1/4)]) + ArcTan[Sqrt[
1 + (-1)^(1/4)]*Tan[x]]/(4*Sqrt[1 + (-1)^(1/4)]) + ArcTan[Sqrt[1 - (-1)^(3/
4)]*Tan[x]]/(4*Sqrt[1 - (-1)^(3/4)]) + ArcTan[Sqrt[1 + (-1)^(3/4)]*Tan[x]]/
(4*Sqrt[1 + (-1)^(3/4)])
```

Rule 3211

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^((4*k)/n)*Rt[-(a/b),
n/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{1+\sin^8(x)}\,dx = \frac{1}{4}\int \frac{1}{1-\sqrt[4]{-1}\sin^2(x)}\,dx + \frac{1}{4}\int \frac{1}{1+\sqrt[4]{-1}\sin^2(x)}\,dx + \frac{1}{4}\int \frac{1}{1-(-1)^{3/4}\sin^2(x)}\,dx + \frac{1}{4}\int \frac{1}{1+(-1)}$$

$$= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1+\left(1-\sqrt[4]{-1}\right)x^2}\,dx, x, \tan(x)\right) + \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1+\left(1+\sqrt[4]{-1}\right)x^2}\,dx, x, \tan(x)\right) + \frac{1}{4}\,\text{S}$$

$$= \frac{\tan^{-1}\left(\sqrt{1-\sqrt[4]{-1}}\tan(x)\right)}{4\sqrt{1-\sqrt[4]{-1}}} + \frac{\tan^{-1}\left(\sqrt{1+\sqrt[4]{-1}}\tan(x)\right)}{4\sqrt{1+\sqrt[4]{-1}}} + \frac{\tan^{-1}\left(\sqrt{1-(-1)^{3/4}}\tan(x)\right)}{4\sqrt{1-(-1)^{3/4}}} + \frac{\tan^{-1}(}{}$$

Mathematica [C] time = 0.147735, size = 141, normalized size = 0.65

$$8\text{RootSum}\left[\#1^8-8\#1^7+28\#1^6-56\#1^5+326\#1^4-56\#1^3+28\#1^2-8\#1+1\&, \frac{2\#1^3\tan^{-1}\left(\frac{\sin(2x)}{\cos(2x)-\#1}\right)-i\#1^3\log(}{\#1^7-7\#1^6+21\#1^5-35\#1^4+163}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sin[x]^8)^(-1),x]
```

```
[Out] 8*RootSum[1 - 8*#1 + 28*#1^2 - 56*#1^3 + 326*#1^4 - 56*#1^5 + 28*#1^6 - 8*#
1^7 + #1^8 & , (2*ArcTan[Sin[2*x]/(Cos[2*x] - #1)]*#1^3 - I*Log[1 - 2*Cos[2
*x]*#1 + #1^2]*#1^3)/(-1 + 7*#1 - 21*#1^2 + 163*#1^3 - 35*#1^4 + 21*#1^5 -
7*#1^6 + #1^7) & ]
```

Maple [C] time = 0.066, size = 71, normalized size = 0.3

$$\frac{1}{8}\sum_{\_R=\text{RootOf}\left(2\_Z^8+4\_Z^6+6\_Z^4+4\_Z^2+1\right)}\frac{\left(\_R^6+3\_R^4+3\_R^2+1\right)\ln\left(\tan(x)-\_R\right)}{2\_R^7+3\_R^5+3\_R^3+\_R}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+sin(x)^8),x)
```

```
[Out] 1/8*sum((_R^6+3*_R^4+3*_R^2+1)/(2*_R^7+3*_R^5+3*_R^3+_R)*ln(tan(x)-_R),_R=R
ootOf(2*_Z^8+4*_Z^6+6*_Z^4+4*_Z^2+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sin(x)^8+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)^8),x, algorithm="maxima")
```

```
[Out] integrate(1/(sin(x)^8 + 1), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)^8),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)**8),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)^8),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.258 $\int \frac{1}{1-\sin^5(x)}\,dx$

Optimal. Leaf size=187

$$-\frac{2\tan^{-1}\left(\frac{(-1)^{2/5}-\tan\left(\frac{x}{2}\right)}{\sqrt{1-(-1)^{4/5}}}\right)}{5\sqrt{1-(-1)^{4/5}}}-\frac{2\tan^{-1}\left(\frac{(-1)^{4/5}-\tan\left(\frac{x}{2}\right)}{\sqrt{1+(-1)^{3/5}}}\right)}{5\sqrt{1+(-1)^{3/5}}}+\frac{2\tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)+\sqrt[5]{-1}}{\sqrt{1-(-1)^{2/5}}}\right)}{5\sqrt{1-(-1)^{2/5}}}+\frac{2\tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)+(-1)^{3/5}}{\sqrt{1+\sqrt[5]{-1}}}\right)}{5\sqrt{1+\sqrt[5]{-1}}}+\frac{\cos(x)}{5(1-\sin(x))}$$

```
[Out] (-2*ArcTan[((-1)^(2/5) - Tan[x/2])/Sqrt[1 - (-1)^(4/5)]])/(5*Sqrt[1 - (-1)^
(4/5)]) - (2*ArcTan[((-1)^(4/5) - Tan[x/2])/Sqrt[1 + (-1)^(3/5)]])/(5*Sqrt[
1 + (-1)^(3/5)]) + (2*ArcTan[((-1)^(1/5) + Tan[x/2])/Sqrt[1 - (-1)^(2/5)]])
/(5*Sqrt[1 - (-1)^(2/5)]) + (2*ArcTan[((-1)^(3/5) + Tan[x/2])/Sqrt[1 + (-1)
^(1/5)]])/(5*Sqrt[1 + (-1)^(1/5)]) + Cos[x]/(5*(1 - Sin[x]))
```

Rubi [A] time = 0.268514, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3213, 2648, 2660, 618, 204}

$$-\frac{2\tan^{-1}\left(\frac{(-1)^{2/5}-\tan\left(\frac{x}{2}\right)}{\sqrt{1-(-1)^{4/5}}}\right)}{5\sqrt{1-(-1)^{4/5}}}-\frac{2\tan^{-1}\left(\frac{(-1)^{4/5}-\tan\left(\frac{x}{2}\right)}{\sqrt{1+(-1)^{3/5}}}\right)}{5\sqrt{1+(-1)^{3/5}}}+\frac{2\tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)+\sqrt[5]{-1}}{\sqrt{1-(-1)^{2/5}}}\right)}{5\sqrt{1-(-1)^{2/5}}}+\frac{2\tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)+(-1)^{3/5}}{\sqrt{1+\sqrt[5]{-1}}}\right)}{5\sqrt{1+\sqrt[5]{-1}}}+\frac{\cos(x)}{5(1-\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sin[x]^5)^(-1),x]
```

```
[Out] (-2*ArcTan[((-1)^(2/5) - Tan[x/2])/Sqrt[1 - (-1)^(4/5)]])/(5*Sqrt[1 - (-1)^
(4/5)]) - (2*ArcTan[((-1)^(4/5) - Tan[x/2])/Sqrt[1 + (-1)^(3/5)]])/(5*Sqrt[
1 + (-1)^(3/5)]) + (2*ArcTan[((-1)^(1/5) + Tan[x/2])/Sqrt[1 - (-1)^(2/5)]])
/(5*Sqrt[1 - (-1)^(2/5)]) + (2*ArcTan[((-1)^(3/5) + Tan[x/2])/Sqrt[1 + (-1)
^(1/5)]])/(5*Sqrt[1 + (-1)^(1/5)]) + Cos[x]/(5*(1 - Sin[x]))
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{1-\sin^5(x)}\,dx = \int \left(\frac{1}{5(1-\sin(x))} + \frac{1}{5\left(1+\sqrt[5]{-1}\sin(x)\right)} + \frac{1}{5\left(1-(-1)^{2/5}\sin(x)\right)} + \frac{1}{5\left(1+(-1)^{3/5}\sin(x)\right)} + \frac{1}{5\left(1-(-1\right.}\right.$$

$$= \frac{1}{5}\int \frac{1}{1-\sin(x)}\,dx + \frac{1}{5}\int \frac{1}{1+\sqrt[5]{-1}\sin(x)}\,dx + \frac{1}{5}\int \frac{1}{1-(-1)^{2/5}\sin(x)}\,dx + \frac{1}{5}\int \frac{1}{1+(-1)^{3/5}\sin(x)}\,dx$$

$$= \frac{\cos(x)}{5(1-\sin(x))} + \frac{2}{5}\text{Subst}\left(\int \frac{1}{1+2\sqrt[5]{-1}x+x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right) + \frac{2}{5}\text{Subst}\left(\int \frac{1}{1-2(-1)^{2/5}x+x^2}\,dx, x\right.$$

$$= \frac{\cos(x)}{5(1-\sin(x))} - \frac{4}{5}\text{Subst}\left(\int \frac{1}{-4\left(1+\sqrt[5]{-1}\right)-x^2}\,dx, x, 2(-1)^{3/5}+2\tan\left(\frac{x}{2}\right)\right) - \frac{4}{5}\text{Subst}\left(\int \frac{}{-4\left(1-(\right.}\right.$$

$$= -\frac{2\tan^{-1}\left(\frac{(-1)^{2/5}-\tan\left(\frac{x}{2}\right)}{\sqrt{1-(-1)^{4/5}}}\right)}{5\sqrt{1-(-1)^{4/5}}} - \frac{2\tan^{-1}\left(\frac{(-1)^{4/5}-\tan\left(\frac{x}{2}\right)}{\sqrt{1+(-1)^{3/5}}}\right)}{5\sqrt{1+(-1)^{3/5}}} + \frac{2\tan^{-1}\left(\frac{\sqrt[5]{-1}+\tan\left(\frac{x}{2}\right)}{\sqrt{1-(-1)^{2/5}}}\right)}{5\sqrt{1-(-1)^{2/5}}} + \frac{2\tan^{-1}\left(\frac{(-1)^{3/5}+\tan\left(\frac{x}{2}\right)}{\sqrt{1+\sqrt[5]{-1}}}\right)}{5\sqrt{1+\sqrt[5]{-1}}}$$

Mathematica [C] time = 0.134343, size = 413, normalized size = 2.21

$$\frac{2\sin\left(\frac{x}{2}\right)}{5\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)} + \frac{1}{10}i\text{RootSum}\left[\#1^8+2i\#1^7-8\#1^6-14i\#1^5+30\#1^4+14i\#1^3-8\#1^2-2i\#1+1\&, \frac{-i\#1^6\log\left(\#\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sin[x]^5)^(-1),x]
```

```
[Out] (I/10)*RootSum[1 - (2*I)*#1 - 8*#1^2 + (14*I)*#1^3 + 30*#1^4 - (14*I)*#1^5
- 8*#1^6 + (2*I)*#1^7 + #1^8 & , (-2*ArcTan[Sin[x]/(Cos[x] - #1)] + I*Log[1
 - 2*Cos[x]*#1 + #1^2] + (8*I)*ArcTan[Sin[x]/(Cos[x] - #1)]*#1 + 4*Log[1 -
2*Cos[x]*#1 + #1^2]*#1 + 30*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^2 - (15*I)*Log[
1 - 2*Cos[x]*#1 + #1^2]*#1^2 - (80*I)*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^3 - 4
0*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^3 - 30*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^4 +
 (15*I)*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^4 + (8*I)*ArcTan[Sin[x]/(Cos[x] - #1
)]*#1^5 + 4*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^5 + 2*ArcTan[Sin[x]/(Cos[x] - #1
)]*#1^6 - I*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^6)/(-I - 8*#1 + (21*I)*#1^2 + 60
*#1^3 - (35*I)*#1^4 - 24*#1^5 + (7*I)*#1^6 + 4*#1^7) & ] + (2*Sin[x/2])/(5*
(Cos[x/2] - Sin[x/2]))
```

Maple [C] time = 0.082, size = 133, normalized size = 0.7

$$\frac{2}{5}\sum_{\_R=\text{RootOf}\left(\_Z^8+2\_Z^7+8\_Z^6+14\_Z^5+30\_Z^4+14\_Z^3+8\_Z^2+2\_Z+1\right)} \frac{2\_R^6+3\_R^5+10\_R^4+10\_R^3+10\_R^2+3\_R+2}{4\_R^7+7\_R^6+24\_R^5+35\_R^4+60\_R^3+21\_R^2+8\_R}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-sin(x)^5),x)
```

```
[Out] 2/5*sum((2*_R^6+3*_R^5+10*_R^4+10*_R^3+10*_R^2+3*_R+2)/(4*_R^7+7*_R^6+24*_R
^5+35*_R^4+60*_R^3+21*_R^2+8*_R+1)*ln(tan(1/2*x)-_R),_R=RootOf(_Z^8+2*_Z^7+
8*_Z^6+14*_Z^5+30*_Z^4+14*_Z^3+8*_Z^2+2*_Z+1))-2/5/(tan(1/2*x)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)^5),x, algorithm="maxima")
```

```
[Out] 1/5*(5*(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)*integrate(2/5*((4*cos(6*x) - 40
*cos(4*x) + 4*cos(2*x) + sin(7*x) - 15*sin(5*x) + 15*sin(3*x) - sin(x))*cos
(8*x) + 2*(22*cos(5*x) - 22*cos(3*x) + 2*cos(x) + 8*sin(6*x) - 55*sin(4*x)
+ 8*sin(2*x))*cos(7*x) - 2*cos(7*x)^2 + 4*(110*cos(4*x) - 16*cos(2*x) + 44*
sin(5*x) - 44*sin(3*x) + 4*sin(x) + 1)*cos(6*x) - 32*cos(6*x)^2 + 2*(210*co
s(3*x) - 22*cos(x) + 505*sin(4*x) - 88*sin(2*x))*cos(5*x) - 210*cos(5*x)^2
+ 10*(44*cos(2*x) + 101*sin(3*x) - 11*sin(x) - 4)*cos(4*x) - 1200*cos(4*x)^
2 + 44*(cos(x) + 4*sin(2*x))*cos(3*x) - 210*cos(3*x)^2 + 4*(4*sin(x) + 1)*c
os(2*x) - 32*cos(2*x)^2 - 2*cos(x)^2 - (cos(7*x) - 15*cos(5*x) + 15*cos(3*x
) - cos(x) - 4*sin(6*x) + 40*sin(4*x) - 4*sin(2*x))*sin(8*x) - (16*cos(6*x)
 - 110*cos(4*x) + 16*cos(2*x) - 44*sin(5*x) + 44*sin(3*x) - 4*sin(x) - 1)*s
in(7*x) - 2*sin(7*x)^2 - 8*(22*cos(5*x) - 22*cos(3*x) + 2*cos(x) - 55*sin(4
*x) + 8*sin(2*x))*sin(6*x) - 32*sin(6*x)^2 - (1010*cos(4*x) - 176*cos(2*x)
- 420*sin(3*x) + 44*sin(x) + 15)*sin(5*x) - 210*sin(5*x)^2 - 10*(101*cos(3*
x) - 11*cos(x) - 44*sin(2*x))*sin(4*x) - 1200*sin(4*x)^2 - (176*cos(2*x) -
44*sin(x) - 15)*sin(3*x) - 210*sin(3*x)^2 - 16*cos(x)*sin(2*x) - 32*sin(2*x
)^2 - 2*sin(x)^2 - sin(x))/(2*(8*cos(6*x) - 30*cos(4*x) + 8*cos(2*x) + 2*si
n(7*x) - 14*sin(5*x) + 14*sin(3*x) - 2*sin(x) - 1)*cos(8*x) - cos(8*x)^2 +
8*(7*cos(5*x) - 7*cos(3*x) + cos(x) + 4*sin(6*x) - 15*sin(4*x) + 4*sin(2*x)
)*cos(7*x) - 4*cos(7*x)^2 + 16*(30*cos(4*x) - 8*cos(2*x) + 14*sin(5*x) - 14
*sin(3*x) + 2*sin(x) + 1)*cos(6*x) - 64*cos(6*x)^2 + 56*(7*cos(3*x) - cos(x
) + 15*sin(4*x) - 4*sin(2*x))*cos(5*x) - 196*cos(5*x)^2 + 60*(8*cos(2*x) +
14*sin(3*x) - 2*sin(x) - 1)*cos(4*x) - 900*cos(4*x)^2 + 56*(cos(x) + 4*sin(
2*x))*cos(3*x) - 196*cos(3*x)^2 + 16*(2*sin(x) + 1)*cos(2*x) - 64*cos(2*x)^
2 - 4*cos(x)^2 - 4*(cos(7*x) - 7*cos(5*x) + 7*cos(3*x) - cos(x) - 4*sin(6*x
) + 15*sin(4*x) - 4*sin(2*x))*sin(8*x) - sin(8*x)^2 - 4*(8*cos(6*x) - 30*co
s(4*x) + 8*cos(2*x) - 14*sin(5*x) + 14*sin(3*x) - 2*sin(x) - 1)*sin(7*x) -
4*sin(7*x)^2 - 32*(7*cos(5*x) - 7*cos(3*x) + cos(x) - 15*sin(4*x) + 4*sin(2
*x))*sin(6*x) - 64*sin(6*x)^2 - 28*(30*cos(4*x) - 8*cos(2*x) - 14*sin(3*x)
+ 2*sin(x) + 1)*sin(5*x) - 196*sin(5*x)^2 - 120*(7*cos(3*x) - cos(x) - 4*si
n(2*x))*sin(4*x) - 900*sin(4*x)^2 - 28*(8*cos(2*x) - 2*sin(x) - 1)*sin(3*x)
 - 196*sin(3*x)^2 - 32*cos(x)*sin(2*x) - 64*sin(2*x)^2 - 4*sin(x)^2 - 4*sin
(x) - 1), x) + 2*cos(x))/(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)^5),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)**5),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\sin(x)^5 - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)^5),x, algorithm="giac")
```

```
[Out] integrate(-1/(sin(x)^5 - 1), x)
```

3.259 $\int \frac{1}{1-\sin^6(x)}\,dx$

Optimal. Leaf size=71

$$\frac{\tan^{-1}\left(\sqrt{1+\sqrt[3]{-1}}\tan(x)\right)}{3\sqrt{1+\sqrt[3]{-1}}}+\frac{\tan^{-1}\left(\sqrt{1-(-1)^{2/3}}\tan(x)\right)}{3\sqrt{1-(-1)^{2/3}}}+\frac{\tan(x)}{3}$$

```
[Out] ArcTan[Sqrt[1 + (-1)^(1/3)]*Tan[x]]/(3*Sqrt[1 + (-1)^(1/3)]) + ArcTan[Sqrt[
1 - (-1)^(2/3)]*Tan[x]]/(3*Sqrt[1 - (-1)^(2/3)]) + Tan[x]/3
```

Rubi [A] time = 0.135382, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {3211, 3181, 203, 3175, 3767, 8}

$$\frac{\tan^{-1}\left(\sqrt{1+\sqrt[3]{-1}}\tan(x)\right)}{3\sqrt{1+\sqrt[3]{-1}}}+\frac{\tan^{-1}\left(\sqrt{1-(-1)^{2/3}}\tan(x)\right)}{3\sqrt{1-(-1)^{2/3}}}+\frac{\tan(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sin[x]^6)^(-1),x]
```

```
[Out] ArcTan[Sqrt[1 + (-1)^(1/3)]*Tan[x]]/(3*Sqrt[1 + (-1)^(1/3)]) + ArcTan[Sqrt[
1 - (-1)^(2/3)]*Tan[x]]/(3*Sqrt[1 - (-1)^(2/3)]) + Tan[x]/3
```

Rule 3211

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^((4*k)/n)*Rt[-(a/b),
n/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1-\sin^6(x)}\,dx &= \frac{1}{3}\int \frac{1}{1-\sin^2(x)}\,dx+\frac{1}{3}\int \frac{1}{1+\sqrt[3]{-1}\sin^2(x)}\,dx+\frac{1}{3}\int \frac{1}{1-(-1)^{2/3}\sin^2(x)}\,dx\\
&= \frac{1}{3}\int \sec^2(x)\,dx+\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{1+\left(1+\sqrt[3]{-1}\right)x^2}\,dx,x,\tan(x)\right)+\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{1+\left(1-(-1)^{2/3}\right)x^2}\,dx\right.\\
&= \frac{\tan^{-1}\left(\sqrt{1+\sqrt[3]{-1}}\tan(x)\right)}{3\sqrt{1+\sqrt[3]{-1}}}+\frac{\tan^{-1}\left(\sqrt{1-(-1)^{2/3}}\tan(x)\right)}{3\sqrt{1-(-1)^{2/3}}}-\frac{1}{3}\operatorname{Subst}\left(\int 1\,dx,x,-\tan(x)\right)\\
&= \frac{\tan^{-1}\left(\sqrt{1+\sqrt[3]{-1}}\tan(x)\right)}{3\sqrt{1+\sqrt[3]{-1}}}+\frac{\tan^{-1}\left(\sqrt{1-(-1)^{2/3}}\tan(x)\right)}{3\sqrt{1-(-1)^{2/3}}}+\frac{\tan(x)}{3}\end{aligned}$$

Mathematica [C] time = 0.276495, size = 117, normalized size = 1.65

$$\frac{\cos(x)(-8\cos(2x)+\cos(4x)+15)\left(-6\sin(x)+i\sqrt[4]{-3}\left(\sqrt{3}+3i\right)\cos(x)\tan^{-1}\left(\frac{1}{2}\sqrt[4]{-\frac{1}{3}}\left(\sqrt{3}-3i\right)\tan(x)\right)+\sqrt[4]{-3}\left(\sqrt{3}-3i\right.\right.}{144\left(\sin^6(x)-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sin[x]^6)^(-1),x]
```

```
[Out] (Cos[x]*(15 - 8*Cos[2*x] + Cos[4*x])*(I*(-3)^(1/4)*(3*I + Sqrt[3])*ArcTan[(
(-1/3)^(1/4)*(-3*I + Sqrt[3])*Tan[x])/2]*Cos[x] + (-3)^(1/4)*(-3*I + Sqrt[3
])*ArcTan[((-1)^(3/4)*(3*I + Sqrt[3])*Tan[x])/(2*3^(1/4))]*Cos[x] - 6*Sin[x
]))/(144*(-1 + Sin[x]^6))
```

Maple [B] time = 0.139, size = 255, normalized size = 3.6

$$\frac{\tan(x)}{3}+\frac{\sqrt{3}\sqrt{2\sqrt{3}-3}\ln\left(\sqrt{3}+\sqrt{2\sqrt{3}-3}\sqrt{3}\tan(x)+3\,(\tan(x))^2\right)}{36}+\frac{\sqrt{3}}{3\sqrt{6\sqrt{3}+9}}\arctan\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{3}+6\tan}{\sqrt{6\sqrt{3}+9}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-sin(x)^6),x)
```

```
[Out] 1/3*tan(x)+1/36*3^(1/2)*(2*3^(1/2)-3)^(1/2)*ln(3^(1/2)+(2*3^(1/2)-3)^(1/2)*
3^(1/2)*tan(x)+3*tan(x)^2)+1/3/(6*3^(1/2)+9)^(1/2)*arctan(((2*3^(1/2)-3)^(1
/2)*3^(1/2)+6*tan(x))/(6*3^(1/2)+9)^(1/2))*3^(1/2)+1/2/(6*3^(1/2)+9)^(1/2)*
arctan(((2*3^(1/2)-3)^(1/2)*3^(1/2)+6*tan(x))/(6*3^(1/2)+9)^(1/2))-1/36*3^(
1/2)*(2*3^(1/2)-3)^(1/2)*ln(-(2*3^(1/2)-3)^(1/2)*3^(1/2)*tan(x)+3*tan(x)^2+
3^(1/2))+1/3/(6*3^(1/2)+9)^(1/2)*arctan((-(2*3^(1/2)-3)^(1/2)*3^(1/2)+6*tan
```

```
(x))/(6*3^(1/2)+9)^(1/2))*3^(1/2)+1/2/(6*3^(1/2)+9)^(1/2)*arctan((-(2*3^(1/
2)-3)^(1/2)*3^(1/2)+6*tan(x))/(6*3^(1/2)+9)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)^6),x, algorithm="maxima")
```

```
[Out] -1/3*(3*(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)*integrate(-4/3*((cos(6*x
) - 10*cos(4*x) + cos(2*x))*cos(8*x) + (110*cos(4*x) - 16*cos(2*x) + 1)*cos
(6*x) - 8*cos(6*x)^2 + 10*(11*cos(2*x) - 1)*cos(4*x) - 300*cos(4*x)^2 - 8*c
os(2*x)^2 + (sin(6*x) - 10*sin(4*x) + sin(2*x))*sin(8*x) + 2*(55*sin(4*x) -
 8*sin(2*x))*sin(6*x) - 8*sin(6*x)^2 - 300*sin(4*x)^2 + 110*sin(4*x)*sin(2*
x) - 8*sin(2*x)^2 + cos(2*x))/(2*(8*cos(6*x) - 30*cos(4*x) + 8*cos(2*x) - 1
)*cos(8*x) - cos(8*x)^2 + 16*(30*cos(4*x) - 8*cos(2*x) + 1)*cos(6*x) - 64*c
os(6*x)^2 + 60*(8*cos(2*x) - 1)*cos(4*x) - 900*cos(4*x)^2 - 64*cos(2*x)^2 +
 4*(4*sin(6*x) - 15*sin(4*x) + 4*sin(2*x))*sin(8*x) - sin(8*x)^2 + 32*(15*s
in(4*x) - 4*sin(2*x))*sin(6*x) - 64*sin(6*x)^2 - 900*sin(4*x)^2 + 480*sin(4
*x)*sin(2*x) - 64*sin(2*x)^2 + 16*cos(2*x) - 1), x) - 2*sin(2*x))/(cos(2*x)
^2 + sin(2*x)^2 + 2*cos(2*x) + 1)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)^6),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)**6),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\sin(x)^6 - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)^6),x, algorithm="giac")
```

```
[Out] integrate(-1/(sin(x)^6 - 1), x)
```

3.260 $\int \frac{1}{1-\sin^8(x)}\,dx$

Optimal. Leaf size=89

$$\frac{x}{4\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{1-i}\tan(x)\right)}{4\sqrt{1-i}}+\frac{\tan^{-1}\left(\sqrt{1+i}\tan(x)\right)}{4\sqrt{1+i}}+\frac{\tan(x)}{4}+\frac{\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\sin^2(x)+\sqrt{2}+1}\right)}{4\sqrt{2}}$$

```
[Out] x/(4*Sqrt[2]) + ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Sin[x]^2)]/(4*Sqrt[2]
) + ArcTan[Sqrt[1 - I]*Tan[x]]/(4*Sqrt[1 - I]) + ArcTan[Sqrt[1 + I]*Tan[x]]
/(4*Sqrt[1 + I]) + Tan[x]/4
```

Rubi [A] time = 0.0770086, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {3211, 3181, 203, 3175, 3767, 8}

$$\frac{x}{4\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{1-i}\tan(x)\right)}{4\sqrt{1-i}}+\frac{\tan^{-1}\left(\sqrt{1+i}\tan(x)\right)}{4\sqrt{1+i}}+\frac{\tan(x)}{4}+\frac{\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\sin^2(x)+\sqrt{2}+1}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sin[x]^8)^(-1),x]
```

```
[Out] x/(4*Sqrt[2]) + ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Sin[x]^2)]/(4*Sqrt[2]
) + ArcTan[Sqrt[1 - I]*Tan[x]]/(4*Sqrt[1 - I]) + ArcTan[Sqrt[1 + I]*Tan[x]]
/(4*Sqrt[1 + I]) + Tan[x]/4
```

Rule 3211

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^((4*k)/n)*Rt[-(a/b),
n/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1-\sin^8(x)}\,dx &= \frac{1}{4}\int \frac{1}{1-\sin^2(x)}\,dx+\frac{1}{4}\int \frac{1}{1-i\sin^2(x)}\,dx+\frac{1}{4}\int \frac{1}{1+i\sin^2(x)}\,dx+\frac{1}{4}\int \frac{1}{1+\sin^2(x)}\,dx\\ &= \frac{1}{4}\int \sec^2(x)\,dx+\frac{1}{4}\,\mathrm{Subst}\left(\int \frac{1}{1+(1-i)x^2}\,dx,x,\tan(x)\right)+\frac{1}{4}\,\mathrm{Subst}\left(\int \frac{1}{1+(1+i)x^2}\,dx,x,\tan(x)\right)\\ &= \frac{x}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{1-i}\tan(x)\right)}{4\sqrt{1-i}}+\frac{\tan^{-1}\left(\sqrt{1+i}\tan(x)\right)}{4\sqrt{1+i}}-\frac{1}{4}\,\mathrm{Subst}(\int 1\,dx,x,\\ &= \frac{x}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{1-i}\tan(x)\right)}{4\sqrt{1-i}}+\frac{\tan^{-1}\left(\sqrt{1+i}\tan(x)\right)}{4\sqrt{1+i}}+\frac{\tan(x)}{4}\end{aligned}$$

Mathematica [A] time = 0.166119, size = 64, normalized size = 0.72

$$\frac{1}{8}\left(\frac{2\tan^{-1}\left(\sqrt{1-i}\tan(x)\right)}{\sqrt{1-i}}+\frac{2\tan^{-1}\left(\sqrt{1+i}\tan(x)\right)}{\sqrt{1+i}}+\sqrt{2}\tan^{-1}\left(\sqrt{2}\tan(x)\right)+2\tan(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sin[x]^8)^(-1),x]
```

```
[Out] ((2*ArcTan[Sqrt[1 - I]*Tan[x]])/Sqrt[1 - I] + (2*ArcTan[Sqrt[1 + I]*Tan[x]]
)/Sqrt[1 + I] + Sqrt[2]*ArcTan[Sqrt[2]*Tan[x]] + 2*Tan[x])/8
```

Maple [B] time = 0.078, size = 255, normalized size = 2.9

$$\frac{\tan(x)}{4}+\frac{\arctan\left(\sqrt{2}\tan(x)\right)\sqrt{2}}{8}+\frac{\sqrt{2}\sqrt{-2+2\sqrt{2}}\ln\left(\sqrt{2}+\sqrt{-2+2\sqrt{2}}\sqrt{2}\tan(x)+2\left(\tan(x)\right)^2\right)}{32}+\frac{\sqrt{2}}{8\sqrt{1+\sqrt{2}}}\mathrm{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-sin(x)^8),x)
```

```
[Out] 1/4*tan(x)+1/8*arctan(2^(1/2)*tan(x))*2^(1/2)+1/32*2^(1/2)*(-2+2*2^(1/2))^(
1/2)*ln(2^(1/2)+(-2+2*2^(1/2))^(1/2)*2^(1/2)*tan(x)+2*tan(x)^2)+1/8/(1+2^(1
/2))^(1/2)*arctan(1/2*(2^(1/2)*(-2+2*2^(1/2))^(1/2)+4*tan(x))/(1+2^(1/2))^(
1/2))*2^(1/2)+1/8/(1+2^(1/2))^(1/2)*arctan(1/2*(2^(1/2)*(-2+2*2^(1/2))^(1/2
)+4*tan(x))/(1+2^(1/2))^(1/2))-1/32*2^(1/2)*(-2+2*2^(1/2))^(1/2)*ln(-(-2+2*
2^(1/2))^(1/2)*2^(1/2)*tan(x)+2*tan(x)^2+2^(1/2))+1/8/(1+2^(1/2))^(1/2)*arc
tan(1/2*(-2^(1/2)*(-2+2*2^(1/2))^(1/2)+4*tan(x))/(1+2^(1/2))^(1/2))*2^(1/2)
+1/8/(1+2^(1/2))^(1/2)*arctan(1/2*(-2^(1/2)*(-2+2*2^(1/2))^(1/2)+4*tan(x))/
(1+2^(1/2))^(1/2))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)^8),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)^8),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)**8),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\sin(x)^8 - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)^8),x, algorithm="giac")
```

```
[Out] integrate(-1/(sin(x)^8 - 1), x)
```

3.261 $\int \frac{\cos^9(x)}{a-a\sin^2(x)}\,dx$

Optimal. Leaf size=38

$$-\frac{\sin^7(x)}{7a}+\frac{3\sin^5(x)}{5a}-\frac{\sin^3(x)}{a}+\frac{\sin(x)}{a}$$

```
[Out] Sin[x]/a - Sin[x]^3/a + (3*Sin[x]^5)/(5*a) - Sin[x]^7/(7*a)
```

Rubi [A] time = 0.0538224, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 2633}

$$-\frac{\sin^7(x)}{7a}+\frac{3\sin^5(x)}{5a}-\frac{\sin^3(x)}{a}+\frac{\sin(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^9/(a - a*Sin[x]^2),x]

[Out] Sin[x]/a - Sin[x]^3/a + (3*Sin[x]^5)/(5*a) - Sin[x]^7/(7*a)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^9(x)}{a-a\sin^2(x)}\,dx &= \frac{\int \cos^7(x)\,dx}{a}\\ &= -\frac{\mathrm{Subst}\left(\int \left(1-3x^2+3x^4-x^6\right)\,dx,x,-\sin(x)\right)}{a}\\ &= \frac{\sin(x)}{a}-\frac{\sin^3(x)}{a}+\frac{3\sin^5(x)}{5a}-\frac{\sin^7(x)}{7a}\end{aligned}$$

Mathematica [A] time = 0.0044287, size = 35, normalized size = 0.92

$$\frac{\frac{35\sin(x)}{64}+\frac{7}{64}\sin(3x)+\frac{7}{320}\sin(5x)+\frac{1}{448}\sin(7x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^9/(a - a*Sin[x]^2),x]
```

```
[Out] ((35*Sin[x])/64 + (7*Sin[3*x])/64 + (7*Sin[5*x])/320 + Sin[7*x]/448)/a
```

Maple [A] time = 0.034, size = 26, normalized size = 0.7

$$\frac{1}{a}\left(-\frac{(\sin(x))^7}{7}+\frac{3(\sin(x))^5}{5}-(\sin(x))^3+\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^9/(a-a*sin(x)^2),x)
```

```
[Out] 1/a*(-1/7*sin(x)^7+3/5*sin(x)^5-sin(x)^3+sin(x))
```

Maxima [A] time = 0.990582, size = 38, normalized size = 1.

$$-\frac{5\sin(x)^7-21\sin(x)^5+35\sin(x)^3-35\sin(x)}{35a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^9/(a-a*sin(x)^2),x, algorithm="maxima")
```

```
[Out] -1/35*(5*sin(x)^7 - 21*sin(x)^5 + 35*sin(x)^3 - 35*sin(x))/a
```

Fricas [A] time = 2.45918, size = 80, normalized size = 2.11

$$\frac{\left(5\cos(x)^6+6\cos(x)^4+8\cos(x)^2+16\right)\sin(x)}{35a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^9/(a-a*sin(x)^2),x, algorithm="fricas")
```

```
[Out] 1/35*(5*cos(x)^6 + 6*cos(x)^4 + 8*cos(x)^2 + 16)*sin(x)/a
```

Sympy [B] time = 83.0451, size = 580, normalized size = 15.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**9/(a-a*sin(x)**2),x)
```

```
[Out] 70*tan(x/2)**13/(35*a*tan(x/2)**14 + 245*a*tan(x/2)**12 + 735*a*tan(x/2)**1
0 + 1225*a*tan(x/2)**8 + 1225*a*tan(x/2)**6 + 735*a*tan(x/2)**4 + 245*a*tan
(x/2)**2 + 35*a) + 140*tan(x/2)**11/(35*a*tan(x/2)**14 + 245*a*tan(x/2)**12
 + 735*a*tan(x/2)**10 + 1225*a*tan(x/2)**8 + 1225*a*tan(x/2)**6 + 735*a*tan
(x/2)**4 + 245*a*tan(x/2)**2 + 35*a) + 602*tan(x/2)**9/(35*a*tan(x/2)**14 +
 245*a*tan(x/2)**12 + 735*a*tan(x/2)**10 + 1225*a*tan(x/2)**8 + 1225*a*tan(
```

```
x/2)**6 + 735*a*tan(x/2)**4 + 245*a*tan(x/2)**2 + 35*a) + 424*tan(x/2)**7/(
35*a*tan(x/2)**14 + 245*a*tan(x/2)**12 + 735*a*tan(x/2)**10 + 1225*a*tan(x/
2)**8 + 1225*a*tan(x/2)**6 + 735*a*tan(x/2)**4 + 245*a*tan(x/2)**2 + 35*a)
+ 602*tan(x/2)**5/(35*a*tan(x/2)**14 + 245*a*tan(x/2)**12 + 735*a*tan(x/2)*
*10 + 1225*a*tan(x/2)**8 + 1225*a*tan(x/2)**6 + 735*a*tan(x/2)**4 + 245*a*t
an(x/2)**2 + 35*a) + 140*tan(x/2)**3/(35*a*tan(x/2)**14 + 245*a*tan(x/2)**1
2 + 735*a*tan(x/2)**10 + 1225*a*tan(x/2)**8 + 1225*a*tan(x/2)**6 + 735*a*ta
n(x/2)**4 + 245*a*tan(x/2)**2 + 35*a) + 70*tan(x/2)/(35*a*tan(x/2)**14 + 24
5*a*tan(x/2)**12 + 735*a*tan(x/2)**10 + 1225*a*tan(x/2)**8 + 1225*a*tan(x/2
)**6 + 735*a*tan(x/2)**4 + 245*a*tan(x/2)**2 + 35*a)
```

Giac [A] time = 1.10332, size = 38, normalized size = 1.

$$-\frac{5\,\sin\left(x\right)^{7}-21\,\sin\left(x\right)^{5}+35\,\sin\left(x\right)^{3}-35\,\sin\left(x\right)}{35\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^9/(a-a*sin(x)^2),x, algorithm="giac")
```

```
[Out] -1/35*(5*sin(x)^7 - 21*sin(x)^5 + 35*sin(x)^3 - 35*sin(x))/a
```

3.262 $\int \frac{\cos^7(x)}{a-a\sin^2(x)}\,dx$

Optimal. Leaf size=29

$$\frac{\sin^5(x)}{5a}-\frac{2\sin^3(x)}{3a}+\frac{\sin(x)}{a}$$

```
[Out] Sin[x]/a - (2*Sin[x]^3)/(3*a) + Sin[x]^5/(5*a)
```

Rubi [A] time = 0.0523097, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 2633}

$$\frac{\sin^5(x)}{5a}-\frac{2\sin^3(x)}{3a}+\frac{\sin(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^7/(a - a*Sin[x]^2),x]
```

```
[Out] Sin[x]/a - (2*Sin[x]^3)/(3*a) + Sin[x]^5/(5*a)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^7(x)}{a-a\sin^2(x)}\,dx &= \frac{\int \cos^5(x)\,dx}{a}\\ &= -\frac{\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx,x,-\sin(x)\right)}{a}\\ &= \frac{\sin(x)}{a}-\frac{2\sin^3(x)}{3a}+\frac{\sin^5(x)}{5a}\end{aligned}$$

Mathematica [A] time = 0.0030325, size = 27, normalized size = 0.93

$$\frac{\frac{5\sin(x)}{8}+\frac{5}{48}\sin(3x)+\frac{1}{80}\sin(5x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^7/(a - a*Sin[x]^2),x]
```

```
[Out] ((5*Sin[x])/8 + (5*Sin[3*x])/48 + Sin[5*x]/80)/a
```

Maple [A] time = 0.035, size = 20, normalized size = 0.7

$$\frac{1}{a}\left(\frac{(\sin(x))^5}{5}-\frac{2(\sin(x))^3}{3}+\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^7/(a-a*sin(x)^2),x)
```

```
[Out] 1/a*(1/5*sin(x)^5-2/3*sin(x)^3+sin(x))
```

Maxima [A] time = 1.0433, size = 30, normalized size = 1.03

$$\frac{3\sin(x)^5-10\sin(x)^3+15\sin(x)}{15a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^7/(a-a*sin(x)^2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*sin(x)^5 - 10*sin(x)^3 + 15*sin(x))/a
```

Fricas [A] time = 2.53558, size = 61, normalized size = 2.1

$$\frac{\left(3\cos(x)^4+4\cos(x)^2+8\right)\sin(x)}{15a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^7/(a-a*sin(x)^2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*cos(x)^4 + 4*cos(x)^2 + 8)*sin(x)/a
```

Sympy [B] time = 35.1572, size = 311, normalized size = 10.72

$$\frac{30\tan^9\left(\frac{x}{2}\right)}{15a\tan^{10}\left(\frac{x}{2}\right)+75a\tan^8\left(\frac{x}{2}\right)+150a\tan^6\left(\frac{x}{2}\right)+150a\tan^4\left(\frac{x}{2}\right)+75a\tan^2\left(\frac{x}{2}\right)+15a}+\frac{}{15a\tan^{10}\left(\frac{x}{2}\right)+75a\tan^8\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**7/(a-a*sin(x)**2),x)
```

```
[Out] 30*tan(x/2)**9/(15*a*tan(x/2)**10 + 75*a*tan(x/2)**8 + 150*a*tan(x/2)**6 +
150*a*tan(x/2)**4 + 75*a*tan(x/2)**2 + 15*a) + 40*tan(x/2)**7/(15*a*tan(x/2
)**10 + 75*a*tan(x/2)**8 + 150*a*tan(x/2)**6 + 150*a*tan(x/2)**4 + 75*a*tan
(x/2)**2 + 15*a) + 116*tan(x/2)**5/(15*a*tan(x/2)**10 + 75*a*tan(x/2)**8 +
150*a*tan(x/2)**6 + 150*a*tan(x/2)**4 + 75*a*tan(x/2)**2 + 15*a) + 40*tan(x
```

```
/2)**3/(15*a*tan(x/2)**10 + 75*a*tan(x/2)**8 + 150*a*tan(x/2)**6 + 150*a*ta
n(x/2)**4 + 75*a*tan(x/2)**2 + 15*a) + 30*tan(x/2)/(15*a*tan(x/2)**10 + 75*
a*tan(x/2)**8 + 150*a*tan(x/2)**6 + 150*a*tan(x/2)**4 + 75*a*tan(x/2)**2 +
15*a)
```

Giac [A] time = 1.14075, size = 30, normalized size = 1.03

$$\frac{3\,\sin\left(x\right)^{5}-10\,\sin\left(x\right)^{3}+15\,\sin\left(x\right)}{15\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^7/(a-a*sin(x)^2),x, algorithm="giac")
```

```
[Out] 1/15*(3*sin(x)^5 - 10*sin(x)^3 + 15*sin(x))/a
```

3.263 $\int \frac{\cos^5(x)}{a-a\sin^2(x)}\,dx$

Optimal. Leaf size=18

$$\frac{\sin(x)}{a}-\frac{\sin^3(x)}{3a}$$

```
[Out] Sin[x]/a - Sin[x]^3/(3*a)
```

Rubi [A] time = 0.0496366, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 2633}

$$\frac{\sin(x)}{a}-\frac{\sin^3(x)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^5/(a - a*Sin[x]^2),x]

[Out] Sin[x]/a - Sin[x]^3/(3*a)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^5(x)}{a-a\sin^2(x)}\,dx &= \frac{\int \cos^3(x)\,dx}{a}\\ &= -\frac{\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, -\sin(x)\right)}{a}\\ &= \frac{\sin(x)}{a}-\frac{\sin^3(x)}{3a}\end{aligned}$$

Mathematica [A] time = 0.003115, size = 19, normalized size = 1.06

$$\frac{\frac{3\sin(x)}{4}+\frac{1}{12}\sin(3x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^5/(a - a*Sin[x]^2),x]
```

[Out] ((3*Sin[x])/4 + Sin[3*x]/12)/a

Maple [A] time = 0.033, size = 14, normalized size = 0.8

$$\frac{1}{a}\left(-\frac{(\sin(x))^3}{3}+\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)^5/(a-a*sin(x)^2),x)

[Out] 1/a*(-1/3*sin(x)^3+sin(x))

Maxima [A] time = 1.01401, size = 19, normalized size = 1.06

$$-\frac{\sin(x)^3-3\sin(x)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^5/(a-a*sin(x)^2),x, algorithm="maxima")

[Out] -1/3*(sin(x)^3 - 3*sin(x))/a

Fricas [A] time = 2.34708, size = 39, normalized size = 2.17

$$\frac{\left(\cos(x)^2+2\right)\sin(x)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^5/(a-a*sin(x)^2),x, algorithm="fricas")

[Out] 1/3*(cos(x)^2 + 2)*sin(x)/a

Sympy [B] time = 16.3355, size = 124, normalized size = 6.89

$$\frac{6\tan^5\left(\frac{x}{2}\right)}{3a\tan^6\left(\frac{x}{2}\right)+9a\tan^4\left(\frac{x}{2}\right)+9a\tan^2\left(\frac{x}{2}\right)+3a}+\frac{4\tan^3\left(\frac{x}{2}\right)}{3a\tan^6\left(\frac{x}{2}\right)+9a\tan^4\left(\frac{x}{2}\right)+9a\tan^2\left(\frac{x}{2}\right)+3a}+\frac{6\tan\left(\frac{x}{2}\right)}{3a\tan^6\left(\frac{x}{2}\right)+9a\tan^4\left(\frac{x}{2}\right)+9a\tan^2\left(\frac{x}{2}\right)+3a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)**5/(a-a*sin(x)**2),x)

[Out] 6*tan(x/2)**5/(3*a*tan(x/2)**6 + 9*a*tan(x/2)**4 + 9*a*tan(x/2)**2 + 3*a) + 4*tan(x/2)**3/(3*a*tan(x/2)**6 + 9*a*tan(x/2)**4 + 9*a*tan(x/2)**2 + 3*a) + 6*tan(x/2)/(3*a*tan(x/2)**6 + 9*a*tan(x/2)**4 + 9*a*tan(x/2)**2 + 3*a)

Giac [A] time = 1.13555, size = 19, normalized size = 1.06

$$-\frac{\sin(x)^3 - 3\sin(x)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5/(a-a*sin(x)^2),x, algorithm="giac")
```

```
[Out] -1/3*(sin(x)^3 - 3*sin(x))/a
```

3.264 $\int \frac{\cos^3(x)}{a-a\sin^2(x)}\,dx$

Optimal. Leaf size=6

$$\frac{\sin(x)}{a}$$

```
[Out] Sin[x]/a
```

Rubi [A] time = 0.0430568, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 2637}

$$\frac{\sin(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^3/(a - a*Sin[x]^2),x]
```

```
[Out] Sin[x]/a
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(x)}{a-a\sin^2(x)}\,dx &= \frac{\int \cos(x)\,dx}{a}\\ &= \frac{\sin(x)}{a}\end{aligned}$$

Mathematica [A] time = 0.0021942, size = 6, normalized size = 1.

$$\frac{\sin(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^3/(a - a*Sin[x]^2),x]
```

```
[Out] Sin[x]/a
```

Maple [A] time = 0.031, size = 7, normalized size = 1.2

$$\frac{\sin(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3/(a-a*sin(x)^2),x)
```

```
[Out] sin(x)/a
```

Maxima [A] time = 0.9828, size = 8, normalized size = 1.33

$$\frac{\sin(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(a-a*sin(x)^2),x, algorithm="maxima")
```

```
[Out] sin(x)/a
```

Fricas [A] time = 2.15429, size = 14, normalized size = 2.33

$$\frac{\sin(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(a-a*sin(x)^2),x, algorithm="fricas")
```

```
[Out] sin(x)/a
```

Sympy [B] time = 6.9355, size = 15, normalized size = 2.5

$$\frac{2\tan\left(\frac{x}{2}\right)}{a\tan^2\left(\frac{x}{2}\right)+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3/(a-a*sin(x)**2),x)
```

```
[Out] 2*tan(x/2)/(a*tan(x/2)**2 + a)
```

Giac [A] time = 1.13852, size = 8, normalized size = 1.33

$$\frac{\sin(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(a-a*sin(x)^2),x, algorithm="giac")
```

```
[Out] sin(x)/a
```

3.265 $\int \frac{\cos(x)}{a-a\sin^2(x)}\,dx$

Optimal. Leaf size=7

$$\frac{\tanh^{-1}(\sin(x))}{a}$$

```
[Out] ArcTanh[Sin[x]]/a
```

Rubi [A] time = 0.0260759, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3175, 3770}

$$\frac{\tanh^{-1}(\sin(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/(a - a*Sin[x]^2),x]
```

```
[Out] ArcTanh[Sin[x]]/a
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)}{a-a\sin^2(x)}\,dx &= \frac{\int \sec(x)\,dx}{a}\\ &= \frac{\tanh^{-1}(\sin(x))}{a}\end{aligned}$$

Mathematica [B] time = 0.004368, size = 37, normalized size = 5.29

$$\frac{\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/(a - a*Sin[x]^2),x]
```

```
[Out] (-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]])/a
```

Maple [A] time = 0.028, size = 8, normalized size = 1.1

$$\frac{\operatorname{Artanh}(\sin(x))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(a-a*sin(x)^2),x)
```

```
[Out] arctanh(sin(x))/a
```

Maxima [B] time = 1.01844, size = 28, normalized size = 4.

$$\frac{\log(\sin(x)+1)}{2a} - \frac{\log(\sin(x)-1)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a-a*sin(x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*log(sin(x) + 1)/a - 1/2*log(sin(x) - 1)/a
```

Fricas [B] time = 1.92961, size = 59, normalized size = 8.43

$$\frac{\log(\sin(x)+1) - \log(-\sin(x)+1)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a-a*sin(x)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(log(sin(x) + 1) - log(-sin(x) + 1))/a
```

Sympy [B] time = 0.430734, size = 19, normalized size = 2.71

$$-\frac{\log(\sin(x)-1)}{2a} + \frac{\log(\sin(x)+1)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a-a*sin(x)**2),x)
```

```
[Out] -log(sin(x) - 1)/(2*a) + log(sin(x) + 1)/(2*a)
```

Giac [B] time = 1.13484, size = 31, normalized size = 4.43

$$\frac{\log(\sin(x)+1)}{2a} - \frac{\log(-\sin(x)+1)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a-a*sin(x)^2),x, algorithm="giac")
```

```
[Out] 1/2*log(sin(x) + 1)/a - 1/2*log(-sin(x) + 1)/a
```

3.266 $\int \frac{\sec^3(x)}{a-a\sin^2(x)}\,dx$

Optimal. Leaf size=35

$$\frac{3\tanh^{-1}(\sin(x))}{8a}+\frac{\tan(x)\sec^3(x)}{4a}+\frac{3\tan(x)\sec(x)}{8a}$$

```
[Out] (3*ArcTanh[Sin[x]])/(8*a) + (3*Sec[x]*Tan[x])/(8*a) + (Sec[x]^3*Tan[x])/(4*
a)
```

Rubi [A] time = 0.0583, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3175, 3768, 3770}

$$\frac{3\tanh^{-1}(\sin(x))}{8a}+\frac{\tan(x)\sec^3(x)}{4a}+\frac{3\tan(x)\sec(x)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^3/(a - a*Sin[x]^2),x]
```

```
[Out] (3*ArcTanh[Sin[x]])/(8*a) + (3*Sec[x]*Tan[x])/(8*a) + (Sec[x]^3*Tan[x])/(4*
a)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^3(x)}{a-a\sin^2(x)}\,dx &= \frac{\int \sec^5(x)\,dx}{a}\\
&= \frac{\sec^3(x)\tan(x)}{4a}+\frac{3\int \sec^3(x)\,dx}{4a}\\
&= \frac{3\sec(x)\tan(x)}{8a}+\frac{\sec^3(x)\tan(x)}{4a}+\frac{3\int \sec(x)\,dx}{8a}\\
&= \frac{3\tanh^{-1}(\sin(x))}{8a}+\frac{3\sec(x)\tan(x)}{8a}+\frac{\sec^3(x)\tan(x)}{4a}
\end{aligned}$$

Mathematica [A] time = 0.120584, size = 61, normalized size = 1.74

$$\frac{\frac{1}{2}(11\sin(x)+3\sin(3x))\sec^4(x)-6\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+6\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)}{16a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^3/(a - a*Sin[x]^2),x]
```

```
[Out] (-6*Log[Cos[x/2] - Sin[x/2]] + 6*Log[Cos[x/2] + Sin[x/2]] + (Sec[x]^4*(11*S
in[x] + 3*Sin[3*x]))/2)/(16*a)
```

Maple [B] time = 0.052, size = 66, normalized size = 1.9

$$\frac{1}{16\,a\,(-1+\sin(x))^2}-\frac{3}{16\,a\,(-1+\sin(x))}-\frac{3\,\ln(-1+\sin(x))}{16\,a}-\frac{1}{16\,a\,(1+\sin(x))^2}-\frac{3}{16\,a\,(1+\sin(x))}+\frac{3\,\ln(1+\sin(x))}{16\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^3/(a-a*sin(x)^2),x)
```

```
[Out] 1/16/a/(-1+sin(x))^2-3/16/a/(-1+sin(x))-3/16/a*ln(-1+sin(x))-1/16/a/(1+sin(
x))^2-3/16/a/(1+sin(x))+3/16/a*ln(1+sin(x))
```

Maxima [A] time = 0.967217, size = 69, normalized size = 1.97

$$-\frac{3\,\sin(x)^3-5\,\sin(x)}{8\left(a\sin(x)^4-2\,a\sin(x)^2+a\right)}+\frac{3\,\log(\sin(x)+1)}{16\,a}-\frac{3\,\log(\sin(x)-1)}{16\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3/(a-a*sin(x)^2),x, algorithm="maxima")
```

```
[Out] -1/8*(3*sin(x)^3 - 5*sin(x))/(a*sin(x)^4 - 2*a*sin(x)^2 + a) + 3/16*log(sin
(x) + 1)/a - 3/16*log(sin(x) - 1)/a
```

Fricas [A] time = 1.94446, size = 143, normalized size = 4.09

$$\frac{3\,\cos(x)^4\log(\sin(x)+1)-3\,\cos(x)^4\log(-\sin(x)+1)+2\left(3\,\cos(x)^2+2\right)\sin(x)}{16\,a\cos(x)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3/(a-a*sin(x)^2),x, algorithm="fricas")
```

```
[Out] 1/16*(3*cos(x)^4*log(sin(x) + 1) - 3*cos(x)^4*log(-sin(x) + 1) + 2*(3*cos(x
)^2 + 2)*sin(x))/(a*cos(x)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{\sec^3(x)}{\sin^2(x)-1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**3/(a-a*sin(x)**2),x)
```

```
[Out] -Integral(sec(x)**3/(sin(x)**2 - 1), x)/a
```

Giac [A] time = 1.10022, size = 63, normalized size = 1.8

$$\frac{3\,\log(\sin(x)+1)}{16\,a}-\frac{3\,\log(-\sin(x)+1)}{16\,a}-\frac{3\,\sin(x)^3-5\,\sin(x)}{8\left(\sin(x)^2-1\right)^2 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3/(a-a*sin(x)^2),x, algorithm="giac")
```

```
[Out] 3/16*log(sin(x) + 1)/a - 3/16*log(-sin(x) + 1)/a - 1/8*(3*sin(x)^3 - 5*sin(
x))/((sin(x)^2 - 1)^2*a)
```

3.267 $\int \frac{\cos^6(x)}{a-a\sin^2(x)}\,dx$

Optimal. Leaf size=33

$$\frac{3x}{8a}+\frac{\sin(x)\cos^3(x)}{4a}+\frac{3\sin(x)\cos(x)}{8a}$$

```
[Out] (3*x)/(8*a) + (3*Cos[x]*Sin[x])/(8*a) + (Cos[x]^3*Sin[x])/(4*a)
```

Rubi [A] time = 0.055304, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3175, 2635, 8}

$$\frac{3x}{8a}+\frac{\sin(x)\cos^3(x)}{4a}+\frac{3\sin(x)\cos(x)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^6/(a - a*Sin[x]^2),x]
```

```
[Out] (3*x)/(8*a) + (3*Cos[x]*Sin[x])/(8*a) + (Cos[x]^3*Sin[x])/(4*a)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^6(x)}{a-a\sin^2(x)}\,dx &= \frac{\int \cos^4(x)\,dx}{a}\\
&= \frac{\cos^3(x)\sin(x)}{4a}+\frac{3\int \cos^2(x)\,dx}{4a}\\
&= \frac{3\cos(x)\sin(x)}{8a}+\frac{\cos^3(x)\sin(x)}{4a}+\frac{3\int 1\,dx}{8a}\\
&= \frac{3x}{8a}+\frac{3\cos(x)\sin(x)}{8a}+\frac{\cos^3(x)\sin(x)}{4a}
\end{aligned}$$

Mathematica [A] time = 0.0030953, size = 26, normalized size = 0.79

$$\frac{\frac{3x}{8}+\frac{1}{4}\sin(2x)+\frac{1}{32}\sin(4x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^6/(a - a*Sin[x]^2),x]
```

```
[Out] ((3*x)/8 + Sin[2*x]/4 + Sin[4*x]/32)/a
```

Maple [A] time = 0.04, size = 40, normalized size = 1.2

$$\frac{\tan(x)}{4a\left((\tan(x))^2+1\right)^2}+\frac{3\tan(x)}{8a\left((\tan(x))^2+1\right)}+\frac{3\arctan(\tan(x))}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^6/(a-a*sin(x)^2),x)
```

```
[Out] 1/4/a*tan(x)/(tan(x)^2+1)^2+3/8/a*tan(x)/(tan(x)^2+1)+3/8/a*arctan(tan(x))
```

Maxima [A] time = 1.50521, size = 50, normalized size = 1.52

$$\frac{3\tan(x)^3+5\tan(x)}{8\left(a\tan(x)^4+2a\tan(x)^2+a\right)}+\frac{3x}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6/(a-a*sin(x)^2),x, algorithm="maxima")
```

```
[Out] 1/8*(3*tan(x)^3 + 5*tan(x))/(a*tan(x)^4 + 2*a*tan(x)^2 + a) + 3/8*x/a
```

Fricas [A] time = 1.92784, size = 62, normalized size = 1.88

$$\frac{\left(2\cos(x)^3+3\cos(x)\right)\sin(x)+3x}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6/(a-a*sin(x)^2),x, algorithm="fricas")
```

```
[Out] 1/8*((2*cos(x)^3 + 3*cos(x))*sin(x) + 3*x)/a
```

Sympy [B] time = 30.2134, size = 473, normalized size = 14.33

$$\frac{3x\tan^8\left(\frac{x}{2}\right)}{8a\tan^8\left(\frac{x}{2}\right)+32a\tan^6\left(\frac{x}{2}\right)+48a\tan^4\left(\frac{x}{2}\right)+32a\tan^2\left(\frac{x}{2}\right)+8a}+\frac{12x\tan^6\left(\frac{x}{2}\right)}{8a\tan^8\left(\frac{x}{2}\right)+32a\tan^6\left(\frac{x}{2}\right)+48a\tan^4\left(\frac{x}{2}\right)+32a\tan^2\left(\frac{x}{2}\right)+8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**6/(a-a*sin(x)**2),x)
```

```
[Out] 3*x*tan(x/2)**8/(8*a*tan(x/2)**8 + 32*a*tan(x/2)**6 + 48*a*tan(x/2)**4 + 32
*a*tan(x/2)**2 + 8*a) + 12*x*tan(x/2)**6/(8*a*tan(x/2)**8 + 32*a*tan(x/2)**
6 + 48*a*tan(x/2)**4 + 32*a*tan(x/2)**2 + 8*a) + 18*x*tan(x/2)**4/(8*a*tan(
x/2)**8 + 32*a*tan(x/2)**6 + 48*a*tan(x/2)**4 + 32*a*tan(x/2)**2 + 8*a) + 1
2*x*tan(x/2)**2/(8*a*tan(x/2)**8 + 32*a*tan(x/2)**6 + 48*a*tan(x/2)**4 + 32
*a*tan(x/2)**2 + 8*a) + 3*x/(8*a*tan(x/2)**8 + 32*a*tan(x/2)**6 + 48*a*tan(
x/2)**4 + 32*a*tan(x/2)**2 + 8*a) - 10*tan(x/2)**7/(8*a*tan(x/2)**8 + 32*a*
tan(x/2)**6 + 48*a*tan(x/2)**4 + 32*a*tan(x/2)**2 + 8*a) + 6*tan(x/2)**5/(8
*a*tan(x/2)**8 + 32*a*tan(x/2)**6 + 48*a*tan(x/2)**4 + 32*a*tan(x/2)**2 + 8
*a) - 6*tan(x/2)**3/(8*a*tan(x/2)**8 + 32*a*tan(x/2)**6 + 48*a*tan(x/2)**4
+ 32*a*tan(x/2)**2 + 8*a) + 10*tan(x/2)/(8*a*tan(x/2)**8 + 32*a*tan(x/2)**6
 + 48*a*tan(x/2)**4 + 32*a*tan(x/2)**2 + 8*a)
```

Giac [A] time = 1.0963, size = 49, normalized size = 1.48

$$\frac{3\arctan\left(\tan\left(x\right)\right)}{8\,a}+\frac{\frac{3\,\tan\left(x\right)^{3}}{a}+\frac{5\,\tan\left(x\right)}{a}}{8\left(\tan\left(x\right)^{2}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6/(a-a*sin(x)^2),x, algorithm="giac")
```

```
[Out] 3/8*arctan(tan(x))/a + 1/8*(3*tan(x)^3/a + 5*tan(x)/a)/(tan(x)^2 + 1)^2
```

3.268 $\int \frac{\cos^4(x)}{a-a\sin^2(x)}\,dx$

Optimal. Leaf size=20

$$\frac{x}{2a}+\frac{\sin(x)\cos(x)}{2a}$$

```
[Out] x/(2*a) + (Cos[x]*Sin[x])/(2*a)
```

Rubi [A] time = 0.0479074, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3175, 2635, 8}

$$\frac{x}{2a}+\frac{\sin(x)\cos(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^4/(a - a*Sin[x]^2),x]
```

```
[Out] x/(2*a) + (Cos[x]*Sin[x])/(2*a)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^4(x)}{a-a\sin^2(x)}\,dx &= \frac{\int \cos^2(x)\,dx}{a}\\
&= \frac{\cos(x)\sin(x)}{2a}+\frac{\int 1\,dx}{2a}\\
&= \frac{x}{2a}+\frac{\cos(x)\sin(x)}{2a}
\end{aligned}$$

Mathematica [A] time = 0.0028952, size = 18, normalized size = 0.9

$$\frac{\frac{x}{2}+\frac{1}{4}\sin(2x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^4/(a - a*Sin[x]^2),x]
```

```
[Out] (x/2 + Sin[2*x]/4)/a
```

Maple [A] time = 0.037, size = 25, normalized size = 1.3

$$\frac{\tan(x)}{2a\left((\tan(x))^2+1\right)}+\frac{\arctan(\tan(x))}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4/(a-a*sin(x)^2),x)
```

```
[Out] 1/2/a*tan(x)/(tan(x)^2+1)+1/2/a*arctan(tan(x))
```

Maxima [A] time = 1.48137, size = 28, normalized size = 1.4

$$\frac{x}{2a}+\frac{\tan(x)}{2\left(a\tan(x)^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4/(a-a*sin(x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*x/a + 1/2*tan(x)/(a*tan(x)^2 + a)
```

Fricas [A] time = 1.82887, size = 36, normalized size = 1.8

$$\frac{\cos(x)\sin(x)+x}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4/(a-a*sin(x)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(cos(x)*sin(x) + x)/a
```

Sympy [B] time = 8.19727, size = 153, normalized size = 7.65

$$\frac{x\tan^4\left(\frac{x}{2}\right)}{2a\tan^4\left(\frac{x}{2}\right)+4a\tan^2\left(\frac{x}{2}\right)+2a}+\frac{2x\tan^2\left(\frac{x}{2}\right)}{2a\tan^4\left(\frac{x}{2}\right)+4a\tan^2\left(\frac{x}{2}\right)+2a}+\frac{x}{2a\tan^4\left(\frac{x}{2}\right)+4a\tan^2\left(\frac{x}{2}\right)+2a}-\frac{2\tan^3\left(\right.}{2a\tan^4\left(\frac{x}{2}\right)+4a\,\mathrm{t}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**4/(a-a*sin(x)**2),x)
```

```
[Out] x*tan(x/2)**4/(2*a*tan(x/2)**4 + 4*a*tan(x/2)**2 + 2*a) + 2*x*tan(x/2)**2/(
2*a*tan(x/2)**4 + 4*a*tan(x/2)**2 + 2*a) + x/(2*a*tan(x/2)**4 + 4*a*tan(x/2
)**2 + 2*a) - 2*tan(x/2)**3/(2*a*tan(x/2)**4 + 4*a*tan(x/2)**2 + 2*a) + 2*t
an(x/2)/(2*a*tan(x/2)**4 + 4*a*tan(x/2)**2 + 2*a)
```

Giac [A] time = 1.12452, size = 32, normalized size = 1.6

$$\frac{\arctan\left(\tan\left(x\right)\right)}{2\,a}+\frac{\tan\left(x\right)}{2\left(\tan\left(x\right)^{2}+1\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4/(a-a*sin(x)^2),x, algorithm="giac")
```

```
[Out] 1/2*arctan(tan(x))/a + 1/2*tan(x)/((tan(x)^2 + 1)*a)
```

3.269 $\int \frac{\cos^2(x)}{a-a\sin^2(x)}\,dx$

Optimal. Leaf size=5

$$\frac{x}{a}$$

[Out] x/a

Rubi [A] time = 0.040586, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 8}

$$\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2/(a - a*Sin[x]^2),x]
```

```
[Out] x/a
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(x)}{a-a\sin^2(x)}\,dx &= \frac{\int 1\,dx}{a}\\ &= \frac{x}{a}\end{aligned}$$

Mathematica [A] time = 0.0007805, size = 5, normalized size = 1.

$$\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2/(a - a*Sin[x]^2),x]
```

```
[Out] x/a
```

Maple [C] time = 0.033, size = 8, normalized size = 1.6

$$\frac{\arctan\left(\tan\left(x\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/(a-a*sin(x)^2),x)
```

```
[Out] 1/a*arctan(tan(x))
```

Maxima [A] time = 1.47286, size = 7, normalized size = 1.4

$$\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a-a*sin(x)^2),x, algorithm="maxima")
```

```
[Out] x/a
```

Fricas [A] time = 1.58364, size = 7, normalized size = 1.4

$$\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a-a*sin(x)^2),x, algorithm="fricas")
```

```
[Out] x/a
```

Sympy [A] time = 2.11331, size = 2, normalized size = 0.4

$$\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2/(a-a*sin(x)**2),x)
```

```
[Out] x/a
```

Giac [B] time = 1.10916, size = 19, normalized size = 3.8

$$\frac{\arctan\left(\frac{|a|\tan\left(x\right)}{a}\right)}{|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a-a*sin(x)^2),x, algorithm="giac")
```

```
[Out] arctan(abs(a)*tan(x)/a)/abs(a)
```

3.270 $\int \frac{\sec(x)}{a-a\sin^2(x)}\,dx$

Optimal. Leaf size=22

$$\frac{\tanh^{-1}(\sin(x))}{2a}+\frac{\tan(x)\sec(x)}{2a}$$

```
[Out] ArcTanh[Sin[x]]/(2*a) + (Sec[x]*Tan[x])/(2*a)
```

Rubi [A] time = 0.0434152, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3175, 3768, 3770}

$$\frac{\tanh^{-1}(\sin(x))}{2a}+\frac{\tan(x)\sec(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]/(a - a*Sin[x]^2),x]
```

```
[Out] ArcTanh[Sin[x]]/(2*a) + (Sec[x]*Tan[x])/(2*a)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(x)}{a-a\sin^2(x)}\,dx &= \frac{\int \sec^3(x)\,dx}{a}\\ &= \frac{\sec(x)\tan(x)}{2a}+\frac{\int \sec(x)\,dx}{2a}\\ &= \frac{\tanh^{-1}(\sin(x))}{2a}+\frac{\sec(x)\tan(x)}{2a}\end{aligned}$$

Mathematica [B] time = 0.0410465, size = 45, normalized size = 2.05

$$\frac{\tan(x)\sec(x)-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]/(a - a*Sin[x]^2),x]
```

```
[Out] (-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]] + Sec[x]*Tan[x])/(2*a
)
```

Maple [B] time = 0.046, size = 44, normalized size = 2.

$$-\frac{1}{4a(-1+\sin(x))}-\frac{\ln(-1+\sin(x))}{4a}-\frac{1}{4a(1+\sin(x))}+\frac{\ln(1+\sin(x))}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)/(a-a*sin(x)^2),x)
```

```
[Out] -1/4/a/(-1+sin(x))-1/4/a*ln(-1+sin(x))-1/4/a/(1+sin(x))+1/4/a*ln(1+sin(x))
```

Maxima [B] time = 0.995265, size = 50, normalized size = 2.27

$$\frac{\log(\sin(x)+1)}{4a}-\frac{\log(\sin(x)-1)}{4a}-\frac{\sin(x)}{2\left(a\sin(x)^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a-a*sin(x)^2),x, algorithm="maxima")
```

```
[Out] 1/4*log(sin(x) + 1)/a - 1/4*log(sin(x) - 1)/a - 1/2*sin(x)/(a*sin(x)^2 - a)
```

Fricas [B] time = 1.97521, size = 113, normalized size = 5.14

$$\frac{\cos(x)^2\log(\sin(x)+1)-\cos(x)^2\log(-\sin(x)+1)+2\sin(x)}{4a\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a-a*sin(x)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(cos(x)^2*log(sin(x) + 1) - cos(x)^2*log(-sin(x) + 1) + 2*sin(x))/(a*co
s(x)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int\frac{\sec(x)}{\sin^2(x)-1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a-a*sin(x)**2),x)
```

```
[Out] -Integral(sec(x)/(sin(x)**2 - 1), x)/a
```

Giac [B] time = 1.12035, size = 51, normalized size = 2.32

$$\frac{\log\left(\sin\left(x\right)+1\right)}{4\,a}-\frac{\log\left(-\sin\left(x\right)+1\right)}{4\,a}-\frac{\sin\left(x\right)}{2\left(\sin\left(x\right)^{2}-1\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a-a*sin(x)^2),x, algorithm="giac")
```

```
[Out] 1/4*log(sin(x) + 1)/a - 1/4*log(-sin(x) + 1)/a - 1/2*sin(x)/((sin(x)^2 - 1)
*a)
```

3.271 $\int \frac{\sec^2(x)}{a-a\sin^2(x)}\,dx$

Optimal. Leaf size=18

$$\frac{\tan^3(x)}{3a}+\frac{\tan(x)}{a}$$

```
[Out] Tan[x]/a + Tan[x]^3/(3*a)
```

Rubi [A] time = 0.0494037, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 3767}

$$\frac{\tan^3(x)}{3a}+\frac{\tan(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2/(a - a*Sin[x]^2),x]
```

```
[Out] Tan[x]/a + Tan[x]^3/(3*a)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(x)}{a-a\sin^2(x)}\,dx &= \frac{\int \sec^4(x)\,dx}{a}\\
&= -\frac{\operatorname{Subst}\left(\int \left(1+x^2\right)\,dx, x, -\tan(x)\right)}{a}\\
&= \frac{\tan(x)}{a}+\frac{\tan^3(x)}{3a}
\end{aligned}$$

Mathematica [A] time = 0.0037023, size = 21, normalized size = 1.17

$$\frac{\frac{2\tan(x)}{3}+\frac{1}{3}\tan(x)\sec^2(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/(a - a*Sin[x]^2),x]
```

```
[Out] ((2*Tan[x])/3 + (Sec[x]^2*Tan[x])/3)/a
```

Maple [A] time = 0.042, size = 14, normalized size = 0.8

$$\frac{1}{a}\left(\frac{(\tan(x))^3}{3}+\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(a-a*sin(x)^2),x)
```

```
[Out] 1/a*(1/3*tan(x)^3+tan(x))
```

Maxima [A] time = 0.967517, size = 19, normalized size = 1.06

$$\frac{\tan(x)^3+3\tan(x)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a-a*sin(x)^2),x, algorithm="maxima")
```

```
[Out] 1/3*(tan(x)^3 + 3*tan(x))/a
```

Fricas [A] time = 1.80905, size = 57, normalized size = 3.17

$$\frac{\left(2\cos(x)^2+1\right)\sin(x)}{3a\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a-a*sin(x)^2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*cos(x)^2 + 1)*sin(x)/(a*cos(x)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int\frac{\sec^2(x)}{\sin^2(x)-1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(a-a*sin(x)**2),x)
```

```
[Out] -Integral(sec(x)**2/(sin(x)**2 - 1), x)/a
```

Giac [A] time = 1.16791, size = 19, normalized size = 1.06

$$\frac{\tan(x)^3 + 3\tan(x)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a-a*sin(x)^2),x, algorithm="giac")
```

```
[Out] 1/3*(tan(x)^3 + 3*tan(x))/a
```

3.272 $\int \frac{\sec^4(x)}{a-a\sin^2(x)}\,dx$

Optimal. Leaf size=29

$$\frac{\tan^5(x)}{5a}+\frac{2\tan^3(x)}{3a}+\frac{\tan(x)}{a}$$

```
[Out] Tan[x]/a + (2*Tan[x]^3)/(3*a) + Tan[x]^5/(5*a)
```

Rubi [A] time = 0.0516302, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 3767}

$$\frac{\tan^5(x)}{5a}+\frac{2\tan^3(x)}{3a}+\frac{\tan(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^4/(a - a*Sin[x]^2),x]
```

```
[Out] Tan[x]/a + (2*Tan[x]^3)/(3*a) + Tan[x]^5/(5*a)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^4(x)}{a-a\sin^2(x)}\,dx &= \frac{\int \sec^6(x)\,dx}{a}\\
&= -\frac{\text{Subst}\left(\int \left(1+2x^2+x^4\right)dx,x,-\tan(x)\right)}{a}\\
&= \frac{\tan(x)}{a}+\frac{2\tan^3(x)}{3a}+\frac{\tan^5(x)}{5a}
\end{aligned}$$

Mathematica [A] time = 0.0042628, size = 31, normalized size = 1.07

$$\frac{\frac{8\tan(x)}{15}+\frac{1}{5}\tan(x)\sec^4(x)+\frac{4}{15}\tan(x)\sec^2(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^4/(a - a*Sin[x]^2),x]
```

```
[Out] ((8*Tan[x])/15 + (4*Sec[x]^2*Tan[x])/15 + (Sec[x]^4*Tan[x])/5)/a
```

Maple [A] time = 0.046, size = 20, normalized size = 0.7

$$\frac{1}{a}\left(\frac{(\tan(x))^5}{5}+\frac{2\,(\tan(x))^3}{3}+\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^4/(a-a*sin(x)^2),x)
```

```
[Out] 1/a*(1/5*tan(x)^5+2/3*tan(x)^3+tan(x))
```

Maxima [A] time = 0.979178, size = 30, normalized size = 1.03

$$\frac{3\,\tan(x)^5+10\,\tan(x)^3+15\,\tan(x)}{15\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4/(a-a*sin(x)^2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*tan(x)^5 + 10*tan(x)^3 + 15*tan(x))/a
```

Fricas [A] time = 1.83189, size = 76, normalized size = 2.62

$$\frac{\left(8\,\cos(x)^4+4\,\cos(x)^2+3\right)\sin(x)}{15\,a\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4/(a-a*sin(x)^2),x, algorithm="fricas")
```

```
[Out] 1/15*(8*cos(x)^4 + 4*cos(x)^2 + 3)*sin(x)/(a*cos(x)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int\frac{\sec^4(x)}{\sin^2(x)-1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**4/(a-a*sin(x)**2),x)
```

```
[Out] -Integral(sec(x)**4/(sin(x)**2 - 1), x)/a
```

Giac [A] time = 1.16536, size = 30, normalized size = 1.03

$$\frac{3 \tan(x)^5 + 10 \tan(x)^3 + 15 \tan(x)}{15 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4/(a-a*sin(x)^2),x, algorithm="giac")
```

```
[Out] 1/15*(3*tan(x)^5 + 10*tan(x)^3 + 15*tan(x))/a
```

3.273 $\int \frac{\cos^9(x)}{(a-a\sin^2(x))^2}\,dx$

Optimal. Leaf size=29

$$\frac{\sin^5(x)}{5a^2}-\frac{2\sin^3(x)}{3a^2}+\frac{\sin(x)}{a^2}$$

```
[Out] Sin[x]/a^2 - (2*Sin[x]^3)/(3*a^2) + Sin[x]^5/(5*a^2)
```

Rubi [A] time = 0.0461488, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 2633}

$$\frac{\sin^5(x)}{5a^2}-\frac{2\sin^3(x)}{3a^2}+\frac{\sin(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^9/(a - a*Sin[x]^2)^2,x]
```

```
[Out] Sin[x]/a^2 - (2*Sin[x]^3)/(3*a^2) + Sin[x]^5/(5*a^2)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^9(x)}{\left(a-a\sin^2(x)\right)^2}\,dx &= \frac{\int \cos^5(x)\,dx}{a^2}\\
&= -\frac{\text{Subst}\left(\int \left(1-2x^2+x^4\right)dx, x, -\sin(x)\right)}{a^2}\\
&= \frac{\sin(x)}{a^2}-\frac{2\sin^3(x)}{3a^2}+\frac{\sin^5(x)}{5a^2}
\end{aligned}$$

Mathematica [A] time = 0.0037578, size = 27, normalized size = 0.93

$$\frac{\frac{5\sin(x)}{8}+\frac{5}{48}\sin(3x)+\frac{1}{80}\sin(5x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^9/(a - a*Sin[x]^2)^2,x]
```

```
[Out] ((5*Sin[x])/8 + (5*Sin[3*x])/48 + Sin[5*x]/80)/a^2
```

Maple [A] time = 0.035, size = 20, normalized size = 0.7

$$\frac{1}{a^2}\left(\frac{(\sin(x))^5}{5}-\frac{2(\sin(x))^3}{3}+\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^9/(a-a*sin(x)^2)^2,x)
```

```
[Out] 1/a^2*(1/5*sin(x)^5-2/3*sin(x)^3+sin(x))
```

Maxima [A] time = 0.961846, size = 30, normalized size = 1.03

$$\frac{3\sin(x)^5-10\sin(x)^3+15\sin(x)}{15a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^9/(a-a*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/15*(3*sin(x)^5 - 10*sin(x)^3 + 15*sin(x))/a^2
```

Fricas [A] time = 1.83329, size = 63, normalized size = 2.17

$$\frac{\left(3\cos(x)^4+4\cos(x)^2+8\right)\sin(x)}{15a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^9/(a-a*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/15*(3*cos(x)^4 + 4*cos(x)^2 + 8)*sin(x)/a^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**9/(a-a*sin(x)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11176, size = 30, normalized size = 1.03

$$\frac{3\sin(x)^5-10\sin(x)^3+15\sin(x)}{15a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^9/(a-a*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/15*(3*sin(x)^5 - 10*sin(x)^3 + 15*sin(x))/a^2
```

3.274 $\int \frac{\cos^7(x)}{\left(a-a\sin^2(x)\right)^2} dx$

Optimal. Leaf size=18

$$\frac{\sin(x)}{a^2} - \frac{\sin^3(x)}{3a^2}$$

```
[Out] Sin[x]/a^2 - Sin[x]^3/(3*a^2)
```

Rubi [A] time = 0.044215, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 2633}

$$\frac{\sin(x)}{a^2} - \frac{\sin^3(x)}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^7/(a - a*Sin[x]^2)^2,x]
```

```
[Out] Sin[x]/a^2 - Sin[x]^3/(3*a^2)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^7(x)}{\left(a-a\sin^2(x)\right)^2} dx &= \frac{\int \cos^3(x)\, dx}{a^2}\\
&= -\frac{\text{Subst}\left(\int \left(1-x^2\right) dx, x, -\sin(x)\right)}{a^2}\\
&= \frac{\sin(x)}{a^2} - \frac{\sin^3(x)}{3a^2}
\end{aligned}$$

Mathematica [A] time = 0.0027456, size = 19, normalized size = 1.06

$$\frac{\frac{3\sin(x)}{4} + \frac{1}{12}\sin(3x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^7/(a - a*Sin[x]^2)^2,x]
```

```
[Out] ((3*Sin[x])/4 + Sin[3*x]/12)/a^2
```

Maple [A] time = 0.033, size = 14, normalized size = 0.8

$$\frac{1}{a^2}\left(-\frac{(\sin(x))^3}{3}+\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^7/(a-a*sin(x)^2)^2,x)
```

```
[Out] 1/a^2*(-1/3*sin(x)^3+sin(x))
```

Maxima [A] time = 0.985575, size = 19, normalized size = 1.06

$$-\frac{\sin(x)^3-3\sin(x)}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^7/(a-a*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(sin(x)^3 - 3*sin(x))/a^2
```

Fricas [A] time = 1.82188, size = 42, normalized size = 2.33

$$\frac{\left(\cos(x)^2+2\right)\sin(x)}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^7/(a-a*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(cos(x)^2 + 2)*sin(x)/a^2
```

Sympy [B] time = 74.1649, size = 144, normalized size = 8.

$$\frac{6\tan^5\left(\frac{x}{2}\right)}{3a^2\tan^6\left(\frac{x}{2}\right)+9a^2\tan^4\left(\frac{x}{2}\right)+9a^2\tan^2\left(\frac{x}{2}\right)+3a^2}+\frac{4\tan^3\left(\frac{x}{2}\right)}{3a^2\tan^6\left(\frac{x}{2}\right)+9a^2\tan^4\left(\frac{x}{2}\right)+9a^2\tan^2\left(\frac{x}{2}\right)+3a^2}+\frac{}{3a^2\tan^6\left(\frac{x}{2}\right)+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**7/(a-a*sin(x)**2)**2,x)
```

```
[Out] 6*tan(x/2)**5/(3*a**2*tan(x/2)**6 + 9*a**2*tan(x/2)**4 + 9*a**2*tan(x/2)**2
 + 3*a**2) + 4*tan(x/2)**3/(3*a**2*tan(x/2)**6 + 9*a**2*tan(x/2)**4 + 9*a**
2*tan(x/2)**2 + 3*a**2) + 6*tan(x/2)/(3*a**2*tan(x/2)**6 + 9*a**2*tan(x/2)*
*4 + 9*a**2*tan(x/2)**2 + 3*a**2)
```

Giac [A] time = 1.09902, size = 19, normalized size = 1.06

$$-\frac{\sin(x)^3 - 3\sin(x)}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^7/(a-a*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] -1/3*(sin(x)^3 - 3*sin(x))/a^2
```

3.275 $\int \frac{\cos^5(x)}{(a-a\sin^2(x))^2}\,dx$

Optimal. Leaf size=6

$$\frac{\sin(x)}{a^2}$$

[Out] Sin[x]/a^2

Rubi [A] time = 0.0385285, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 2637}

$$\frac{\sin(x)}{a^2}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^5/(a - a*Sin[x]^2)^2,x]

[Out] Sin[x]/a^2

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^5(x)}{(a-a\sin^2(x))^2}\,dx &= \frac{\int \cos(x)\,dx}{a^2}\\ &= \frac{\sin(x)}{a^2}\end{aligned}$$

Mathematica [A] time = 0.0017739, size = 6, normalized size = 1.

$$\frac{\sin(x)}{a^2}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]^5/(a - a*Sin[x]^2)^2,x]

[Out] Sin[x]/a^2

Maple [A] time = 0.03, size = 7, normalized size = 1.2

$$\frac{\sin\left(x\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^5/(a-a*sin(x)^2)^2,x)
```

```
[Out] sin(x)/a^2
```

Maxima [A] time = 0.995567, size = 8, normalized size = 1.33

$$\frac{\sin\left(x\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5/(a-a*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] sin(x)/a^2
```

Fricas [A] time = 1.78371, size = 16, normalized size = 2.67

$$\frac{\sin\left(x\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5/(a-a*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] sin(x)/a^2
```

Sympy [B] time = 30.1794, size = 19, normalized size = 3.17

$$\frac{2\tan\left(\frac{x}{2}\right)}{a^{2}\tan^{2}\left(\frac{x}{2}\right)+a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**5/(a-a*sin(x)**2)**2,x)
```

```
[Out] 2*tan(x/2)/(a**2*tan(x/2)**2 + a**2)
```

Giac [A] time = 1.09659, size = 8, normalized size = 1.33

$$\frac{\sin\left(x\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5/(a-a*sin(x)^2)^2,x, algorithm="giac")

[Out] sin(x)/a^2
```

3.276 $\int \frac{\cos^3(x)}{\left(a-a\sin^2(x)\right)^2}\,dx$

Optimal. Leaf size=7

$$\frac{\tanh^{-1}(\sin(x))}{a^2}$$

```
[Out] ArcTanh[Sin[x]]/a^2
```

Rubi [A] time = 0.0393184, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 3770}

$$\frac{\tanh^{-1}(\sin(x))}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^3/(a - a*Sin[x]^2)^2,x]
```

```
[Out] ArcTanh[Sin[x]]/a^2
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(x)}{\left(a-a\sin^2(x)\right)^2}\,dx &= \frac{\int \sec(x)\,dx}{a^2}\\ &= \frac{\tanh^{-1}(\sin(x))}{a^2}\end{aligned}$$

Mathematica [B] time = 0.0035549, size = 37, normalized size = 5.29

$$\frac{\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^3/(a - a*Sin[x]^2)^2,x]
```

```
[Out] (-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]])/a^2
```

Maple [A] time = 0.033, size = 8, normalized size = 1.1

$$\frac{\operatorname{Artanh}(\sin(x))}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3/(a-a*sin(x)^2)^2,x)
```

```
[Out] arctanh(sin(x))/a^2
```

Maxima [B] time = 0.975028, size = 28, normalized size = 4.

$$\frac{\log(\sin(x)+1)}{2a^2}-\frac{\log(\sin(x)-1)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(a-a*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*log(sin(x) + 1)/a^2 - 1/2*log(sin(x) - 1)/a^2
```

Fricas [B] time = 1.92245, size = 62, normalized size = 8.86

$$\frac{\log(\sin(x)+1)-\log(-\sin(x)+1)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(a-a*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(log(sin(x) + 1) - log(-sin(x) + 1))/a^2
```

Sympy [B] time = 11.15, size = 22, normalized size = 3.14

$$-\frac{\log\left(\tan\left(\frac{x}{2}\right)-1\right)}{a^2}+\frac{\log\left(\tan\left(\frac{x}{2}\right)+1\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3/(a-a*sin(x)**2)**2,x)
```

```
[Out] -log(tan(x/2) - 1)/a**2 + log(tan(x/2) + 1)/a**2
```

Giac [B] time = 1.14169, size = 31, normalized size = 4.43

$$\frac{\log(\sin(x)+1)}{2a^2}-\frac{\log(-\sin(x)+1)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(a-a*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*log(sin(x) + 1)/a^2 - 1/2*log(-sin(x) + 1)/a^2
```

3.277 $\int \frac{\cos(x)}{\left(a-a\sin^2(x)\right)^2}\,dx$

Optimal. Leaf size=22

$$\frac{\tanh^{-1}(\sin(x))}{2a^2}+\frac{\tan(x)\sec(x)}{2a^2}$$

```
[Out] ArcTanh[Sin[x]]/(2*a^2) + (Sec[x]*Tan[x])/(2*a^2)
```

Rubi [A] time = 0.0316159, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3175, 3768, 3770}

$$\frac{\tanh^{-1}(\sin(x))}{2a^2}+\frac{\tan(x)\sec(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/(a - a*Sin[x]^2)^2,x]
```

```
[Out] ArcTanh[Sin[x]]/(2*a^2) + (Sec[x]*Tan[x])/(2*a^2)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(x)}{\left(a-a\sin^2(x)\right)^2}\,dx &= \frac{\int \sec^3(x)\,dx}{a^2}\\
&= \frac{\sec(x)\tan(x)}{2a^2}+\frac{\int \sec(x)\,dx}{2a^2}\\
&= \frac{\tanh^{-1}(\sin(x))}{2a^2}+\frac{\sec(x)\tan(x)}{2a^2}
\end{aligned}$$

Mathematica [B] time = 0.0056328, size = 45, normalized size = 2.05

$$\frac{\tan(x)\sec(x)-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/(a - a*Sin[x]^2)^2,x]
```

```
[Out] (-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]] + Sec[x]*Tan[x])/(2*a
^2)
```

Maple [B] time = 0.036, size = 44, normalized size = 2.

$$-\frac{1}{4\,a^2\,(-1+\sin(x))}-\frac{\ln(-1+\sin(x))}{4\,a^2}-\frac{1}{4\,a^2\,(1+\sin(x))}+\frac{\ln(1+\sin(x))}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(a-a*sin(x)^2)^2,x)
```

```
[Out] -1/4/a^2/(-1+sin(x))-1/4/a^2*ln(-1+sin(x))-1/4/a^2/(1+sin(x))+1/4/a^2*ln(1+
sin(x))
```

Maxima [B] time = 1.00151, size = 55, normalized size = 2.5

$$-\frac{\sin(x)}{2\left(a^2\sin(x)^2-a^2\right)}+\frac{\log(\sin(x)+1)}{4\,a^2}-\frac{\log(\sin(x)-1)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a-a*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*sin(x)/(a^2*sin(x)^2 - a^2) + 1/4*log(sin(x) + 1)/a^2 - 1/4*log(sin(x)
 - 1)/a^2
```

Fricas [B] time = 1.79149, size = 116, normalized size = 5.27

$$\frac{\cos(x)^2\log(\sin(x)+1)-\cos(x)^2\log(-\sin(x)+1)+2\sin(x)}{4\,a^2\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a-a*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(cos(x)^2*log(sin(x) + 1) - cos(x)^2*log(-sin(x) + 1) + 2*sin(x))/(a^2*
cos(x)^2)
```

Sympy [B] time = 1.25807, size = 117, normalized size = 5.32

$$-\frac{\log(\sin(x)-1)\sin^2(x)}{4a^2\sin^2(x)-4a^2}+\frac{\log(\sin(x)-1)}{4a^2\sin^2(x)-4a^2}+\frac{\log(\sin(x)+1)\sin^2(x)}{4a^2\sin^2(x)-4a^2}-\frac{\log(\sin(x)+1)}{4a^2\sin^2(x)-4a^2}-\frac{2\sin(x)}{4a^2\sin^2(x)-4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a-a*sin(x)**2)**2,x)
```

```
[Out] -log(sin(x) - 1)*sin(x)**2/(4*a**2*sin(x)**2 - 4*a**2) + log(sin(x) - 1)/(4
*a**2*sin(x)**2 - 4*a**2) + log(sin(x) + 1)*sin(x)**2/(4*a**2*sin(x)**2 - 4
*a**2) - log(sin(x) + 1)/(4*a**2*sin(x)**2 - 4*a**2) - 2*sin(x)/(4*a**2*sin
(x)**2 - 4*a**2)
```

Giac [B] time = 1.10988, size = 51, normalized size = 2.32

$$\frac{\log\left(\sin\left(x\right)+1\right)}{4\,a^{2}}-\frac{\log\left(-\sin\left(x\right)+1\right)}{4\,a^{2}}-\frac{\sin\left(x\right)}{2\left(\sin\left(x\right)^{2}-1\right)a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a-a*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/4*log(sin(x) + 1)/a^2 - 1/4*log(-sin(x) + 1)/a^2 - 1/2*sin(x)/((sin(x)^2
- 1)*a^2)
```

3.278 $\int \frac{\sec(x)}{\left(a-a\sin^2(x)\right)^2}\,dx$

Optimal. Leaf size=35

$$\frac{3\tanh^{-1}(\sin(x))}{8a^2}+\frac{\tan(x)\sec^3(x)}{4a^2}+\frac{3\tan(x)\sec(x)}{8a^2}$$

```
[Out] (3*ArcTanh[Sin[x]])/(8*a^2) + (3*Sec[x]*Tan[x])/(8*a^2) + (Sec[x]^3*Tan[x])
/(4*a^2)
```

Rubi [A] time = 0.0480322, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3175, 3768, 3770}

$$\frac{3\tanh^{-1}(\sin(x))}{8a^2}+\frac{\tan(x)\sec^3(x)}{4a^2}+\frac{3\tan(x)\sec(x)}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]/(a - a*Sin[x]^2)^2,x]
```

```
[Out] (3*ArcTanh[Sin[x]])/(8*a^2) + (3*Sec[x]*Tan[x])/(8*a^2) + (Sec[x]^3*Tan[x])
/(4*a^2)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(x)}{\left(a-a\sin^2(x)\right)^2}\,dx &= \frac{\int \sec^5(x)\,dx}{a^2}\\
&= \frac{\sec^3(x)\tan(x)}{4a^2}+\frac{3\int \sec^3(x)\,dx}{4a^2}\\
&= \frac{3\sec(x)\tan(x)}{8a^2}+\frac{\sec^3(x)\tan(x)}{4a^2}+\frac{3\int \sec(x)\,dx}{8a^2}\\
&= \frac{3\tanh^{-1}(\sin(x))}{8a^2}+\frac{3\sec(x)\tan(x)}{8a^2}+\frac{\sec^3(x)\tan(x)}{4a^2}
\end{aligned}$$

Mathematica [A] time = 0.0070534, size = 61, normalized size = 1.74

$$\frac{\frac{1}{2}(11\sin(x)+3\sin(3x))\sec^4(x)-6\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+6\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)}{16a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]/(a - a*Sin[x]^2)^2,x]
```

```
[Out] (-6*Log[Cos[x/2] - Sin[x/2]] + 6*Log[Cos[x/2] + Sin[x/2]] + (Sec[x]^4*(11*S
in[x] + 3*Sin[3*x]))/2)/(16*a^2)
```

Maple [B] time = 0.049, size = 66, normalized size = 1.9

$$\frac{1}{16\,a^2\,(-1+\sin(x))^2}-\frac{3}{16\,a^2\,(-1+\sin(x))}-\frac{3\,\ln(-1+\sin(x))}{16\,a^2}-\frac{1}{16\,a^2\,(1+\sin(x))^2}-\frac{3}{16\,a^2\,(1+\sin(x))}+\frac{3\,\ln(1+\sin(x))}{16\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)/(a-a*sin(x)^2)^2,x)
```

```
[Out] 1/16/a^2/(-1+sin(x))^2-3/16/a^2/(-1+sin(x))-3/16/a^2*ln(-1+sin(x))-1/16/a^2
/(1+sin(x))^2-3/16/a^2/(1+sin(x))+3/16/a^2*ln(1+sin(x))
```

Maxima [A] time = 0.984979, size = 77, normalized size = 2.2

$$-\frac{3\,\sin(x)^3-5\,\sin(x)}{8\left(a^2\sin(x)^4-2\,a^2\sin(x)^2+a^2\right)}+\frac{3\,\log(\sin(x)+1)}{16\,a^2}-\frac{3\,\log(\sin(x)-1)}{16\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a-a*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/8*(3*sin(x)^3 - 5*sin(x))/(a^2*sin(x)^4 - 2*a^2*sin(x)^2 + a^2) + 3/16*l
og(sin(x) + 1)/a^2 - 3/16*log(sin(x) - 1)/a^2
```

Fricas [A] time = 1.92176, size = 146, normalized size = 4.17

$$\frac{3\,\cos(x)^4\log(\sin(x)+1)-3\,\cos(x)^4\log(-\sin(x)+1)+2\left(3\,\cos(x)^2+2\right)\sin(x)}{16\,a^2\cos(x)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a-a*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(3*cos(x)^4*log(sin(x) + 1) - 3*cos(x)^4*log(-sin(x) + 1) + 2*(3*cos(x
)^2 + 2)*sin(x))/(a^2*cos(x)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec(x)}{\sin^4(x)-2\sin^2(x)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a-a*sin(x)**2)**2,x)
```

```
[Out] Integral(sec(x)/(sin(x)**4 - 2*sin(x)**2 + 1), x)/a**2
```

Giac [A] time = 1.11668, size = 63, normalized size = 1.8

$$\frac{3\log(\sin(x)+1)}{16a^2} - \frac{3\log(-\sin(x)+1)}{16a^2} - \frac{3\sin(x)^3 - 5\sin(x)}{8\left(\sin(x)^2-1\right)^2 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a-a*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] 3/16*log(sin(x) + 1)/a^2 - 3/16*log(-sin(x) + 1)/a^2 - 1/8*(3*sin(x)^3 - 5*
sin(x))/((sin(x)^2 - 1)^2*a^2)
```

3.279 $\int \frac{\cos^8(x)}{(a-a\sin^2(x))^2}\,dx$

Optimal. Leaf size=33

$$\frac{3x}{8a^2}+\frac{\sin(x)\cos^3(x)}{4a^2}+\frac{3\sin(x)\cos(x)}{8a^2}$$

```
[Out] (3*x)/(8*a^2) + (3*Cos[x]*Sin[x])/(8*a^2) + (Cos[x]^3*Sin[x])/(4*a^2)
```

Rubi [A] time = 0.0501656, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3175, 2635, 8}

$$\frac{3x}{8a^2}+\frac{\sin(x)\cos^3(x)}{4a^2}+\frac{3\sin(x)\cos(x)}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^8/(a - a*Sin[x]^2)^2,x]
```

```
[Out] (3*x)/(8*a^2) + (3*Cos[x]*Sin[x])/(8*a^2) + (Cos[x]^3*Sin[x])/(4*a^2)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^8(x)}{\left(a-a\sin^2(x)\right)^2}\,dx &= \frac{\int \cos^4(x)\,dx}{a^2}\\
&= \frac{\cos^3(x)\sin(x)}{4a^2}+\frac{3\int \cos^2(x)\,dx}{4a^2}\\
&= \frac{3\cos(x)\sin(x)}{8a^2}+\frac{\cos^3(x)\sin(x)}{4a^2}+\frac{3\int 1\,dx}{8a^2}\\
&= \frac{3x}{8a^2}+\frac{3\cos(x)\sin(x)}{8a^2}+\frac{\cos^3(x)\sin(x)}{4a^2}
\end{aligned}$$

Mathematica [A] time = 0.0027006, size = 26, normalized size = 0.79

$$\frac{\frac{3x}{8}+\frac{1}{4}\sin(2x)+\frac{1}{32}\sin(4x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^8/(a - a*Sin[x]^2)^2,x]
```

```
[Out] ((3*x)/8 + Sin[2*x]/4 + Sin[4*x]/32)/a^2
```

Maple [A] time = 0.039, size = 40, normalized size = 1.2

$$\frac{\tan(x)}{4a^2\left((\tan(x))^2+1\right)^2}+\frac{3\tan(x)}{8a^2\left((\tan(x))^2+1\right)}+\frac{3\arctan(\tan(x))}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^8/(a-a*sin(x)^2)^2,x)
```

```
[Out] 1/4/a^2*tan(x)/(tan(x)^2+1)^2+3/8/a^2*tan(x)/(tan(x)^2+1)+3/8/a^2*arctan(ta
n(x))
```

Maxima [A] time = 1.51192, size = 58, normalized size = 1.76

$$\frac{3\tan(x)^3+5\tan(x)}{8\left(a^2\tan(x)^4+2a^2\tan(x)^2+a^2\right)}+\frac{3x}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^8/(a-a*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/8*(3*tan(x)^3 + 5*tan(x))/(a^2*tan(x)^4 + 2*a^2*tan(x)^2 + a^2) + 3/8*x/a
^2
```

Fricas [A] time = 1.94281, size = 65, normalized size = 1.97

$$\frac{\left(2\cos(x)^3+3\cos(x)\right)\sin(x)+3x}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^8/(a-a*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*((2*cos(x)^3 + 3*cos(x))*sin(x) + 3*x)/a^2
```

Sympy [B] time = 123.222, size = 549, normalized size = 16.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**8/(a-a*sin(x)**2)**2,x)
```

```
[Out] 3*x*tan(x/2)**8/(8*a**2*tan(x/2)**8 + 32*a**2*tan(x/2)**6 + 48*a**2*tan(x/2
)**4 + 32*a**2*tan(x/2)**2 + 8*a**2) + 12*x*tan(x/2)**6/(8*a**2*tan(x/2)**8
 + 32*a**2*tan(x/2)**6 + 48*a**2*tan(x/2)**4 + 32*a**2*tan(x/2)**2 + 8*a**2
) + 18*x*tan(x/2)**4/(8*a**2*tan(x/2)**8 + 32*a**2*tan(x/2)**6 + 48*a**2*ta
n(x/2)**4 + 32*a**2*tan(x/2)**2 + 8*a**2) + 12*x*tan(x/2)**2/(8*a**2*tan(x/
2)**8 + 32*a**2*tan(x/2)**6 + 48*a**2*tan(x/2)**4 + 32*a**2*tan(x/2)**2 + 8
*a**2) + 3*x/(8*a**2*tan(x/2)**8 + 32*a**2*tan(x/2)**6 + 48*a**2*tan(x/2)**
4 + 32*a**2*tan(x/2)**2 + 8*a**2) - 10*tan(x/2)**7/(8*a**2*tan(x/2)**8 + 32
*a**2*tan(x/2)**6 + 48*a**2*tan(x/2)**4 + 32*a**2*tan(x/2)**2 + 8*a**2) + 6
*tan(x/2)**5/(8*a**2*tan(x/2)**8 + 32*a**2*tan(x/2)**6 + 48*a**2*tan(x/2)**
4 + 32*a**2*tan(x/2)**2 + 8*a**2) - 6*tan(x/2)**3/(8*a**2*tan(x/2)**8 + 32*
a**2*tan(x/2)**6 + 48*a**2*tan(x/2)**4 + 32*a**2*tan(x/2)**2 + 8*a**2) + 10
*tan(x/2)/(8*a**2*tan(x/2)**8 + 32*a**2*tan(x/2)**6 + 48*a**2*tan(x/2)**4 +
 32*a**2*tan(x/2)**2 + 8*a**2)
```

Giac [A] time = 1.10106, size = 42, normalized size = 1.27

$$\frac{3x}{8a^2} + \frac{3\tan(x)^3 + 5\tan(x)}{8\left(\tan(x)^2 + 1\right)^2 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^8/(a-a*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] 3/8*x/a^2 + 1/8*(3*tan(x)^3 + 5*tan(x))/((tan(x)^2 + 1)^2*a^2)
```

3.280 $\int \frac{\cos^6(x)}{\left(a-a\sin^2(x)\right)^2}\,dx$

Optimal. Leaf size=20

$$\frac{x}{2a^2}+\frac{\sin(x)\cos(x)}{2a^2}$$

```
[Out] x/(2*a^2) + (Cos[x]*Sin[x])/(2*a^2)
```

Rubi [A] time = 0.0434516, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3175, 2635, 8}

$$\frac{x}{2a^2}+\frac{\sin(x)\cos(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^6/(a - a*Sin[x]^2)^2,x]
```

```
[Out] x/(2*a^2) + (Cos[x]*Sin[x])/(2*a^2)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^6(x)}{\left(a-a\sin^2(x)\right)^2}\,dx &= \frac{\int \cos^2(x)\,dx}{a^2}\\
&= \frac{\cos(x)\sin(x)}{2a^2}+\frac{\int 1\,dx}{2a^2}\\
&= \frac{x}{2a^2}+\frac{\cos(x)\sin(x)}{2a^2}
\end{aligned}$$

Mathematica [A] time = 0.0026244, size = 18, normalized size = 0.9

$$\frac{\frac{x}{2}+\frac{1}{4}\sin(2x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^6/(a - a*Sin[x]^2)^2,x]
```

```
[Out] (x/2 + Sin[2*x]/4)/a^2
```

Maple [A] time = 0.037, size = 25, normalized size = 1.3

$$\frac{\tan(x)}{2a^2\left((\tan(x))^2+1\right)}+\frac{\arctan(\tan(x))}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^6/(a-a*sin(x)^2)^2,x)
```

```
[Out] 1/2/a^2*tan(x)/(tan(x)^2+1)+1/2/a^2*arctan(tan(x))
```

Maxima [A] time = 1.48092, size = 34, normalized size = 1.7

$$\frac{\tan(x)}{2\left(a^2\tan(x)^2+a^2\right)}+\frac{x}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6/(a-a*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*tan(x)/(a^2*tan(x)^2 + a^2) + 1/2*x/a^2
```

Fricas [A] time = 1.82461, size = 39, normalized size = 1.95

$$\frac{\cos(x)\sin(x)+x}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6/(a-a*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(cos(x)*sin(x) + x)/a^2
```

Sympy [B] time = 47.708, size = 178, normalized size = 8.9

$$\frac{x\tan^4\left(\frac{x}{2}\right)}{2a^2\tan^4\left(\frac{x}{2}\right)+4a^2\tan^2\left(\frac{x}{2}\right)+2a^2}+\frac{2x\tan^2\left(\frac{x}{2}\right)}{2a^2\tan^4\left(\frac{x}{2}\right)+4a^2\tan^2\left(\frac{x}{2}\right)+2a^2}+\frac{x}{2a^2\tan^4\left(\frac{x}{2}\right)+4a^2\tan^2\left(\frac{x}{2}\right)+2a^2}-\frac{}{2a^2\tan^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**6/(a-a*sin(x)**2)**2,x)
```

```
[Out] x*tan(x/2)**4/(2*a**2*tan(x/2)**4 + 4*a**2*tan(x/2)**2 + 2*a**2) + 2*x*tan(
x/2)**2/(2*a**2*tan(x/2)**4 + 4*a**2*tan(x/2)**2 + 2*a**2) + x/(2*a**2*tan(
x/2)**4 + 4*a**2*tan(x/2)**2 + 2*a**2) - 2*tan(x/2)**3/(2*a**2*tan(x/2)**4
+ 4*a**2*tan(x/2)**2 + 2*a**2) + 2*tan(x/2)/(2*a**2*tan(x/2)**4 + 4*a**2*ta
n(x/2)**2 + 2*a**2)
```

Giac [A] time = 1.16203, size = 30, normalized size = 1.5

$$\frac{x}{2\,a^2} + \frac{\tan\left(x\right)}{2\left(\tan\left(x\right)^2 + 1\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6/(a-a*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*x/a^2 + 1/2*tan(x)/((tan(x)^2 + 1)*a^2)
```

3.281 $\int \frac{\cos^4(x)}{\left(a-a\sin^2(x)\right)^2}\,dx$

Optimal. Leaf size=5

$$\frac{x}{a^2}$$

```
[Out] x/a^2
```

Rubi [A] time = 0.037163, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 8}

$$\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^4/(a - a*Sin[x]^2)^2,x]
```

```
[Out] x/a^2
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^4(x)}{\left(a-a\sin^2(x)\right)^2}\,dx &= \frac{\int 1\,dx}{a^2}\\ &= \frac{x}{a^2}\end{aligned}$$

Mathematica [A] time = 0.0003858, size = 5, normalized size = 1.

$$\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^4/(a - a*Sin[x]^2)^2,x]
```

```
[Out] x/a^2
```

Maple [C] time = 0.035, size = 8, normalized size = 1.6

$$\frac{\arctan(\tan(x))}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4/(a-a*sin(x)^2)^2,x)
```

```
[Out] 1/a^2*arctan(tan(x))
```

Maxima [A] time = 1.44907, size = 7, normalized size = 1.4

$$\frac{x}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4/(a-a*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] x/a^2
```

Fricas [A] time = 1.6706, size = 9, normalized size = 1.8

$$\frac{x}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4/(a-a*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] x/a^2
```

Sympy [A] time = 19.1012, size = 3, normalized size = 0.6

$$\frac{x}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**4/(a-a*sin(x)**2)**2,x)
```

```
[Out] x/a**2
```

Giac [A] time = 1.09676, size = 7, normalized size = 1.4

$$\frac{x}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4/(a-a*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] x/a^2
```

3.282 $\int \frac{\cos^2(x)}{\left(a-a\sin^2(x)\right)^2}\,dx$

Optimal. Leaf size=6

$$\frac{\tan(x)}{a^2}$$

[Out] Tan[x]/a^2

Rubi [A] time = 0.0430509, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3175, 3767, 8}

$$\frac{\tan(x)}{a^2}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^2/(a - a*Sin[x]^2)^2,x]

[Out] Tan[x]/a^2

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(x)}{\left(a-a\sin^2(x)\right)^2}\,dx &= \frac{\int \sec^2(x)\,dx}{a^2}\\ &= -\frac{\text{Subst}\left(\int 1\,dx, x, -\tan(x)\right)}{a^2}\\ &= \frac{\tan(x)}{a^2}\end{aligned}$$

Mathematica [A] time = 0.0022568, size = 6, normalized size = 1.

$$\frac{\tan(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2/(a - a*Sin[x]^2)^2,x]
```

```
[Out] Tan[x]/a^2
```

Maple [A] time = 0.033, size = 7, normalized size = 1.2

$$\frac{\tan(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/(a-a*sin(x)^2)^2,x)
```

```
[Out] tan(x)/a^2
```

Maxima [A] time = 0.989701, size = 8, normalized size = 1.33

$$\frac{\tan(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a-a*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] tan(x)/a^2
```

Fricas [A] time = 1.85202, size = 28, normalized size = 4.67

$$\frac{\sin(x)}{a^2\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a-a*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] sin(x)/(a^2*cos(x))
```

Sympy [B] time = 7.59203, size = 20, normalized size = 3.33

$$-\frac{2\tan\left(\frac{x}{2}\right)}{a^2\tan^2\left(\frac{x}{2}\right) - a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2/(a-a*sin(x)**2)**2,x)
```

```
[Out] -2*tan(x/2)/(a**2*tan(x/2)**2 - a**2)
```

Giac [A] time = 1.0964, size = 8, normalized size = 1.33

$$\frac{\tan(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a-a*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] tan(x)/a^2
```

3.283 $\int \frac{\sec^2(x)}{\left(a-a\sin^2(x)\right)^2}\,dx$

Optimal. Leaf size=29

$$\frac{\tan^5(x)}{5a^2}+\frac{2\tan^3(x)}{3a^2}+\frac{\tan(x)}{a^2}$$

```
[Out] Tan[x]/a^2 + (2*Tan[x]^3)/(3*a^2) + Tan[x]^5/(5*a^2)
```

Rubi [A] time = 0.046838, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 3767}

$$\frac{\tan^5(x)}{5a^2}+\frac{2\tan^3(x)}{3a^2}+\frac{\tan(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2/(a - a*Sin[x]^2)^2,x]
```

```
[Out] Tan[x]/a^2 + (2*Tan[x]^3)/(3*a^2) + Tan[x]^5/(5*a^2)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(x)}{\left(a-a\sin^2(x)\right)^2}\,dx &= \frac{\int \sec^6(x)\,dx}{a^2}\\
&= -\frac{\text{Subst}\left(\int \left(1+2x^2+x^4\right)dx, x, -\tan(x)\right)}{a^2}\\
&= \frac{\tan(x)}{a^2}+\frac{2\tan^3(x)}{3a^2}+\frac{\tan^5(x)}{5a^2}
\end{aligned}$$

Mathematica [A] time = 0.0034332, size = 31, normalized size = 1.07

$$\frac{\frac{8\tan(x)}{15}+\frac{1}{5}\tan(x)\sec^4(x)+\frac{4}{15}\tan(x)\sec^2(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/(a - a*Sin[x]^2)^2,x]
```

```
[Out] ((8*Tan[x])/15 + (4*Sec[x]^2*Tan[x])/15 + (Sec[x]^4*Tan[x])/5)/a^2
```

Maple [A] time = 0.043, size = 20, normalized size = 0.7

$$\frac{1}{a^2}\left(\frac{(\tan(x))^5}{5}+\frac{2\,(\tan(x))^3}{3}+\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(a-a*sin(x)^2)^2,x)
```

```
[Out] 1/a^2*(1/5*tan(x)^5+2/3*tan(x)^3+tan(x))
```

Maxima [A] time = 0.978235, size = 30, normalized size = 1.03

$$\frac{3\,\tan(x)^5+10\,\tan(x)^3+15\,\tan(x)}{15\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a-a*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/15*(3*tan(x)^5 + 10*tan(x)^3 + 15*tan(x))/a^2
```

Fricas [A] time = 1.78191, size = 78, normalized size = 2.69

$$\frac{\left(8\,\cos(x)^4+4\,\cos(x)^2+3\right)\sin(x)}{15\,a^2\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a-a*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/15*(8*cos(x)^4 + 4*cos(x)^2 + 3)*sin(x)/(a^2*cos(x)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec^2(x)}{\sin^4(x)-2\sin^2(x)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(a-a*sin(x)**2)**2,x)
```

```
[Out] Integral(sec(x)**2/(sin(x)**4 - 2*sin(x)**2 + 1), x)/a**2
```

Giac [A] time = 1.11368, size = 30, normalized size = 1.03

$$\frac{3\,\tan\left(x\right)^{5}+10\,\tan\left(x\right)^{3}+15\,\tan\left(x\right)}{15\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a-a*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/15*(3*tan(x)^5 + 10*tan(x)^3 + 15*tan(x))/a^2
```

3.284 $\int \frac{\sec^4(x)}{(a-a\sin^2(x))^2}\,dx$

Optimal. Leaf size=37

$$\frac{\tan^7(x)}{7a^2}+\frac{3\tan^5(x)}{5a^2}+\frac{\tan^3(x)}{a^2}+\frac{\tan(x)}{a^2}$$

```
[Out] Tan[x]/a^2 + Tan[x]^3/a^2 + (3*Tan[x]^5)/(5*a^2) + Tan[x]^7/(7*a^2)
```

Rubi [A] time = 0.0481523, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 3767}

$$\frac{\tan^7(x)}{7a^2}+\frac{3\tan^5(x)}{5a^2}+\frac{\tan^3(x)}{a^2}+\frac{\tan(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^4/(a - a*Sin[x]^2)^2,x]
```

```
[Out] Tan[x]/a^2 + Tan[x]^3/a^2 + (3*Tan[x]^5)/(5*a^2) + Tan[x]^7/(7*a^2)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^4(x)}{(a-a\sin^2(x))^2}\,dx &= \frac{\int \sec^8(x)\,dx}{a^2}\\ &= -\frac{\text{Subst}\left(\int \left(1+3x^2+3x^4+x^6\right)dx, x, -\tan(x)\right)}{a^2}\\ &= \frac{\tan(x)}{a^2}+\frac{\tan^3(x)}{a^2}+\frac{3\tan^5(x)}{5a^2}+\frac{\tan^7(x)}{7a^2}\end{aligned}$$

Mathematica [A] time = 0.0044555, size = 41, normalized size = 1.11

$$\frac{\frac{16\tan(x)}{35}+\frac{1}{7}\tan(x)\sec^6(x)+\frac{6}{35}\tan(x)\sec^4(x)+\frac{8}{35}\tan(x)\sec^2(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^4/(a - a*Sin[x]^2)^2,x]
```

```
[Out] ((16*Tan[x])/35 + (8*Sec[x]^2*Tan[x])/35 + (6*Sec[x]^4*Tan[x])/35 + (Sec[x]
^6*Tan[x])/7)/a^2
```

Maple [A] time = 0.047, size = 24, normalized size = 0.7

$$\frac{1}{a^2}\left(\frac{(\tan(x))^7}{7}+\frac{3\,(\tan(x))^5}{5}+(\tan(x))^3+\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^4/(a-a*sin(x)^2)^2,x)
```

```
[Out] 1/a^2*(1/7*tan(x)^7+3/5*tan(x)^5+tan(x)^3+tan(x))
```

Maxima [A] time = 0.980879, size = 38, normalized size = 1.03

$$\frac{5\,\tan(x)^7+21\,\tan(x)^5+35\,\tan(x)^3+35\,\tan(x)}{35\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4/(a-a*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/35*(5*tan(x)^7 + 21*tan(x)^5 + 35*tan(x)^3 + 35*tan(x))/a^2
```

Fricas [A] time = 1.87242, size = 97, normalized size = 2.62

$$\frac{\left(16\,\cos(x)^6+8\,\cos(x)^4+6\,\cos(x)^2+5\right)\sin(x)}{35\,a^2\cos(x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4/(a-a*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/35*(16*cos(x)^6 + 8*cos(x)^4 + 6*cos(x)^2 + 5)*sin(x)/(a^2*cos(x)^7)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec^4(x)}{\sin^4(x)-2\sin^2(x)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**4/(a-a*sin(x)**2)**2,x)
```

```
[Out] Integral(sec(x)**4/(sin(x)**4 - 2*sin(x)**2 + 1), x)/a**2
```

Giac [A] time = 1.1291, size = 38, normalized size = 1.03

$$\frac{5\,\tan\left(x\right)^{7}+21\,\tan\left(x\right)^{5}+35\,\tan\left(x\right)^{3}+35\,\tan\left(x\right)}{35\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4/(a-a*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/35*(5*tan(x)^7 + 21*tan(x)^5 + 35*tan(x)^3 + 35*tan(x))/a^2
```

3.285 $\int \cos^6(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx$

Optimal. Leaf size=109

$$\frac{(8a+b)\sin(e+fx)\cos^5(e+fx)}{48f}+\frac{5(8a+b)\sin(e+fx)\cos^3(e+fx)}{192f}+\frac{5(8a+b)\sin(e+fx)\cos(e+fx)}{128f}+\frac{5}{128}x(8a$$

```
[Out] (5*(8*a + b)*x)/128 + (5*(8*a + b)*Cos[e + f*x]*Sin[e + f*x])/(128*f) + (5*
(8*a + b)*Cos[e + f*x]^3*Sin[e + f*x])/(192*f) + ((8*a + b)*Cos[e + f*x]^5*
Sin[e + f*x])/(48*f) - (b*Cos[e + f*x]^7*Sin[e + f*x])/(8*f)
```

Rubi [A] time = 0.0645058, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3191, 385, 199, 203}

$$\frac{(8a+b)\sin(e+fx)\cos^5(e+fx)}{48f}+\frac{5(8a+b)\sin(e+fx)\cos^3(e+fx)}{192f}+\frac{5(8a+b)\sin(e+fx)\cos(e+fx)}{128f}+\frac{5}{128}x(8a$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^6*(a + b*Sin[e + f*x]^2),x]
```

```
[Out] (5*(8*a + b)*x)/128 + (5*(8*a + b)*Cos[e + f*x]*Sin[e + f*x])/(128*f) + (5*
(8*a + b)*Cos[e + f*x]^3*Sin[e + f*x])/(192*f) + ((8*a + b)*Cos[e + f*x]^5*
Sin[e + f*x])/(48*f) - (b*Cos[e + f*x]^7*Sin[e + f*x])/(8*f)
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^6(e+fx)\left(a+b\sin^2(e+fx)\right)dx = \frac{\text{Subst}\left(\int \frac{a+(a+b)x^2}{(1+x^2)^5}dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cos^7(e+fx)\sin(e+fx)}{8f} + \frac{(8a+b)\,\text{Subst}\left(\int \frac{1}{(1+x^2)^4}dx, x, \tan(e+fx)\right)}{8f}$$

$$= \frac{(8a+b)\cos^5(e+fx)\sin(e+fx)}{48f} - \frac{b\cos^7(e+fx)\sin(e+fx)}{8f} + \frac{(5(8a+b))\,\text{Su}}{}$$

$$= \frac{5(8a+b)\cos^3(e+fx)\sin(e+fx)}{192f} + \frac{(8a+b)\cos^5(e+fx)\sin(e+fx)}{48f} - \frac{b\cos}{}$$

$$= \frac{5(8a+b)\cos(e+fx)\sin(e+fx)}{128f} + \frac{5(8a+b)\cos^3(e+fx)\sin(e+fx)}{192f} + \frac{(8a+}{}$$

$$= \frac{5}{128}(8a+b)x + \frac{5(8a+b)\cos(e+fx)\sin(e+fx)}{128f} + \frac{5(8a+b)\cos^3(e+fx)\sin}{192f}$$

Mathematica [A] time = 0.311012, size = 87, normalized size = 0.8

$$\frac{48(15a+b)\sin(2(e+fx)) + 24(6a-b)\sin(4(e+fx)) + 16a\sin(6(e+fx)) + 960ae + 960afx - 16b\sin(6(e+fx)) - 3b\,\text{s}}{3072f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^6*(a + b*Sin[e + f*x]^2),x]
```

```
[Out] (960*a*e + 960*a*f*x + 120*b*f*x + 48*(15*a + b)*Sin[2*(e + f*x)] + 24*(6*a
 - b)*Sin[4*(e + f*x)] + 16*a*Sin[6*(e + f*x)] - 16*b*Sin[6*(e + f*x)] - 3*
b*Sin[8*(e + f*x)])/(3072*f)
```

Maple [A] time = 0.046, size = 112, normalized size = 1.

$$\frac{1}{f}\left(b\left(-\frac{\sin(fx+e)(\cos(fx+e))^7}{8} + \frac{\sin(fx+e)}{48}\left((\cos(fx+e))^5 + \frac{5(\cos(fx+e))^3}{4} + \frac{15\cos(fx+e)}{8}\right) + \frac{5fx}{128} + \frac{5}{12}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^6*(a+b*sin(f*x+e)^2),x)
```

```
[Out] 1/f*(b*(-1/8*sin(f*x+e)*cos(f*x+e)^7+1/48*(cos(f*x+e)^5+5/4*cos(f*x+e)^3+15
/8*cos(f*x+e))*sin(f*x+e)+5/128*f*x+5/128*e)+a*(1/6*(cos(f*x+e)^5+5/4*cos(f
*x+e)^3+15/8*cos(f*x+e))*sin(f*x+e)+5/16*f*x+5/16*e))
```

Maxima [A] time = 1.45466, size = 165, normalized size = 1.51

$$\frac{15(fx+e)(8a+b) + \frac{15(8a+b)\tan(fx+e)^7 + 55(8a+b)\tan(fx+e)^5 + 73(8a+b)\tan(fx+e)^3 + 3(88a-5b)\tan(fx+e)}{\tan(fx+e)^8 + 4\tan(fx+e)^6 + 6\tan(fx+e)^4 + 4\tan(fx+e)^2 + 1}}{384f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+b*sin(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/384*(15*(f*x + e)*(8*a + b) + (15*(8*a + b)*tan(f*x + e)^7 + 55*(8*a + b)
*tan(f*x + e)^5 + 73*(8*a + b)*tan(f*x + e)^3 + 3*(88*a - 5*b)*tan(f*x + e)
)/(tan(f*x + e)^8 + 4*tan(f*x + e)^6 + 6*tan(f*x + e)^4 + 4*tan(f*x + e)^2
+ 1))/f
```

Fricas [A] time = 1.94567, size = 205, normalized size = 1.88

$$\frac{15\,(8\,a+b)fx-\left(48\,b\cos\left(fx+e\right)^{7}-8\,(8\,a+b)\cos\left(fx+e\right)^{5}-10\,(8\,a+b)\cos\left(fx+e\right)^{3}-15\,(8\,a+b)\cos\left(fx+e\right)\right)}{384\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+b*sin(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/384*(15*(8*a + b)*f*x - (48*b*cos(f*x + e)^7 - 8*(8*a + b)*cos(f*x + e)^5
 - 10*(8*a + b)*cos(f*x + e)^3 - 15*(8*a + b)*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [A] time = 13.7463, size = 354, normalized size = 3.25

$$\begin{cases}\frac{5ax\sin^{6}\left(e+fx\right)}{16}+\frac{15ax\sin^{4}\left(e+fx\right)\cos^{2}\left(e+fx\right)}{16}+\frac{15ax\sin^{2}\left(e+fx\right)\cos^{4}\left(e+fx\right)}{16}+\frac{5ax\cos^{6}\left(e+fx\right)}{16}+\frac{5a\sin^{5}\left(e+fx\right)\cos\left(e+fx\right)}{16f}+\frac{5a\sin^{3}\left(e+fx\right)}{6f} \\ x\left(a+b\sin^{2}\left(e\right)\right)\cos^{6}\left(e\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**6*(a+b*sin(f*x+e)**2),x)
```

```
[Out] Piecewise((5*a*x*sin(e + f*x)**6/16 + 15*a*x*sin(e + f*x)**4*cos(e + f*x)**
2/16 + 15*a*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 5*a*x*cos(e + f*x)**6/16
 + 5*a*sin(e + f*x)**5*cos(e + f*x)/(16*f) + 5*a*sin(e + f*x)**3*cos(e + f*
x)**3/(6*f) + 11*a*sin(e + f*x)*cos(e + f*x)**5/(16*f) + 5*b*x*sin(e + f*x)
**8/128 + 5*b*x*sin(e + f*x)**6*cos(e + f*x)**2/32 + 15*b*x*sin(e + f*x)**4
*cos(e + f*x)**4/64 + 5*b*x*sin(e + f*x)**2*cos(e + f*x)**6/32 + 5*b*x*cos(
e + f*x)**8/128 + 5*b*sin(e + f*x)**7*cos(e + f*x)/(128*f) + 55*b*sin(e + f
*x)**5*cos(e + f*x)**3/(384*f) + 73*b*sin(e + f*x)**3*cos(e + f*x)**5/(384*
f) - 5*b*sin(e + f*x)*cos(e + f*x)**7/(128*f), Ne(f, 0)), (x*(a + b*sin(e)*
*2)*cos(e)**6, True))
```

Giac [A] time = 1.13745, size = 117, normalized size = 1.07

$$\frac{5}{128}\,(8\,a+b)x-\frac{b\sin\left(8\,fx+8\,e\right)}{1024\,f}+\frac{(a-b)\sin\left(6\,fx+6\,e\right)}{192\,f}+\frac{(6\,a-b)\sin\left(4\,fx+4\,e\right)}{128\,f}+\frac{(15\,a+b)\sin\left(2\,fx+2\,e\right)}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+b*sin(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 5/128*(8*a + b)*x - 1/1024*b*sin(8*f*x + 8*e)/f + 1/192*(a - b)*sin(6*f*x +
 6*e)/f + 1/128*(6*a - b)*sin(4*f*x + 4*e)/f + 1/64*(15*a + b)*sin(2*f*x +
2*e)/f
```

3.286 $\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx$

Optimal. Leaf size=83

$$\frac{(6a+b)\sin(e+fx)\cos^3(e+fx)}{24f}+\frac{(6a+b)\sin(e+fx)\cos(e+fx)}{16f}+\frac{1}{16}x(6a+b)-\frac{b\sin(e+fx)\cos^5(e+fx)}{6f}$$

```
[Out] ((6*a + b)*x)/16 + ((6*a + b)*Cos[e + f*x]*Sin[e + f*x])/(16*f) + ((6*a + b
)*Cos[e + f*x]^3*Sin[e + f*x])/(24*f) - (b*Cos[e + f*x]^5*Sin[e + f*x])/(6*
f)
```

Rubi [A] time = 0.0511575, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3191, 385, 199, 203}

$$\frac{(6a+b)\sin(e+fx)\cos^3(e+fx)}{24f}+\frac{(6a+b)\sin(e+fx)\cos(e+fx)}{16f}+\frac{1}{16}x(6a+b)-\frac{b\sin(e+fx)\cos^5(e+fx)}{6f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^2),x]
```

```
[Out] ((6*a + b)*x)/16 + ((6*a + b)*Cos[e + f*x]*Sin[e + f*x])/(16*f) + ((6*a + b
)*Cos[e + f*x]^3*Sin[e + f*x])/(24*f) - (b*Cos[e + f*x]^5*Sin[e + f*x])/(6*
f)
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)dx = \frac{\text{Subst}\left(\int \frac{a+(a+b)x^2}{(1+x^2)^4}dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cos^5(e+fx)\sin(e+fx)}{6f} + \frac{(6a+b)\,\text{Subst}\left(\int \frac{1}{(1+x^2)^3}dx, x, \tan(e+fx)\right)}{6f}$$

$$= \frac{(6a+b)\cos^3(e+fx)\sin(e+fx)}{24f} - \frac{b\cos^5(e+fx)\sin(e+fx)}{6f} + \frac{(6a+b)\,\text{Subs}}{}$$

$$= \frac{(6a+b)\cos(e+fx)\sin(e+fx)}{16f} + \frac{(6a+b)\cos^3(e+fx)\sin(e+fx)}{24f} - \frac{b\cos^5(e}{}$$

$$= \frac{1}{16}(6a+b)x + \frac{(6a+b)\cos(e+fx)\sin(e+fx)}{16f} + \frac{(6a+b)\cos^3(e+fx)\sin(e+}{24f}$$

Mathematica [A] time = 0.14942, size = 64, normalized size = 0.77

$$\frac{3(16a+b)\sin(2(e+fx)) + (6a-3b)\sin(4(e+fx)) + 72ae + 72afx - b\sin(6(e+fx)) + 12bfx}{192f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^2),x]
```

```
[Out] (72*a*e + 72*a*f*x + 12*b*f*x + 3*(16*a + b)*Sin[2*(e + f*x)] + (6*a - 3*b)
*Sin[4*(e + f*x)] - b*Sin[6*(e + f*x)])/(192*f)
```

Maple [A] time = 0.045, size = 92, normalized size = 1.1

$$\frac{1}{f}\left(b\left(-\frac{\sin(fx+e)(\cos(fx+e))^5}{6} + \frac{\sin(fx+e)}{24}\left((\cos(fx+e))^3 + \frac{3\cos(fx+e)}{2}\right) + \frac{fx}{16} + \frac{e}{16}\right) + a\left(\frac{\sin(fx+e)}{4}\left((\text{c}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(a+b*sin(f*x+e)^2),x)
```

```
[Out] 1/f*(b*(-1/6*sin(f*x+e)*cos(f*x+e)^5+1/24*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin
(f*x+e)+1/16*f*x+1/16*e)+a*(1/4*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/
8*f*x+3/8*e))
```

Maxima [A] time = 1.46822, size = 131, normalized size = 1.58

$$\frac{3(fx+e)(6a+b) + \frac{3(6a+b)\tan(fx+e)^5 + 8(6a+b)\tan(fx+e)^3 + 3(10a-b)\tan(fx+e)}{\tan(fx+e)^6 + 3\tan(fx+e)^4 + 3\tan(fx+e)^2 + 1}}{48f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/48*(3*(f*x + e)*(6*a + b) + (3*(6*a + b)*tan(f*x + e)^5 + 8*(6*a + b)*tan
(f*x + e)^3 + 3*(10*a - b)*tan(f*x + e))/(tan(f*x + e)^6 + 3*tan(f*x + e)^4
 + 3*tan(f*x + e)^2 + 1))/f
```

Fricas [A] time = 1.93832, size = 159, normalized size = 1.92

$$\frac{3\,(6\,a+b)fx-\left(8\,b\cos\left(fx+e\right)^{5}-2\,(6\,a+b)\cos\left(fx+e\right)^{3}-3\,(6\,a+b)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/48*(3*(6*a + b)*f*x - (8*b*cos(f*x + e)^5 - 2*(6*a + b)*cos(f*x + e)^3 -
3*(6*a + b)*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [A] time = 4.95717, size = 250, normalized size = 3.01

$$\begin{cases}\frac{3ax\sin^{4}\left(e+fx\right)}{8}+\frac{3ax\sin^{2}\left(e+fx\right)\cos^{2}\left(e+fx\right)}{4}+\frac{3ax\cos^{4}\left(e+fx\right)}{8}+\frac{3a\sin^{3}\left(e+fx\right)\cos\left(e+fx\right)}{8f}+\frac{5a\sin\left(e+fx\right)\cos^{3}\left(e+fx\right)}{8f}+\frac{bx\sin^{6}\left(e+fx\right)}{16}+\cdots\\ x\left(a+b\sin^{2}\left(e\right)\right)\cos^{4}\left(e\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(a+b*sin(f*x+e)**2),x)
```

```
[Out] Piecewise((3*a*x*sin(e + f*x)**4/8 + 3*a*x*sin(e + f*x)**2*cos(e + f*x)**2/
4 + 3*a*x*cos(e + f*x)**4/8 + 3*a*sin(e + f*x)**3*cos(e + f*x)/(8*f) + 5*a*
sin(e + f*x)*cos(e + f*x)**3/(8*f) + b*x*sin(e + f*x)**6/16 + 3*b*x*sin(e +
 f*x)**4*cos(e + f*x)**2/16 + 3*b*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + b*
x*cos(e + f*x)**6/16 + b*sin(e + f*x)**5*cos(e + f*x)/(16*f) + b*sin(e + f*
x)**3*cos(e + f*x)**3/(6*f) - b*sin(e + f*x)*cos(e + f*x)**5/(16*f), Ne(f,
0)), (x*(a + b*sin(e)**2)*cos(e)**4, True))
```

Giac [A] time = 1.13837, size = 90, normalized size = 1.08

$$\frac{1}{16}\,(6\,a+b)x-\frac{b\sin\left(6\,fx+6\,e\right)}{192\,f}+\frac{(2\,a-b)\sin\left(4\,fx+4\,e\right)}{64\,f}+\frac{(16\,a+b)\sin\left(2\,fx+2\,e\right)}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/16*(6*a + b)*x - 1/192*b*sin(6*f*x + 6*e)/f + 1/64*(2*a - b)*sin(4*f*x +
4*e)/f + 1/64*(16*a + b)*sin(2*f*x + 2*e)/f
```

3.287 $\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx$

Optimal. Leaf size=57

$$\frac{(4a+b)\sin(e+fx)\cos(e+fx)}{8f}+\frac{1}{8}x(4a+b)-\frac{b\sin(e+fx)\cos^3(e+fx)}{4f}$$

```
[Out] ((4*a + b)*x)/8 + ((4*a + b)*Cos[e + f*x]*Sin[e + f*x])/(8*f) - (b*Cos[e +
f*x]^3*Sin[e + f*x])/(4*f)
```

Rubi [A] time = 0.0444805, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3191, 385, 199, 203}

$$\frac{(4a+b)\sin(e+fx)\cos(e+fx)}{8f}+\frac{1}{8}x(4a+b)-\frac{b\sin(e+fx)\cos^3(e+fx)}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^2),x]
```

```
[Out] ((4*a + b)*x)/8 + ((4*a + b)*Cos[e + f*x]*Sin[e + f*x])/(8*f) - (b*Cos[e +
f*x]^3*Sin[e + f*x])/(4*f)
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)dx = \frac{\text{Subst}\left(\int \frac{a+(a+b)x^2}{(1+x^2)^3}dx, x, \tan(e+fx)\right)}{f}$$
$$= -\frac{b\cos^3(e+fx)\sin(e+fx)}{4f} + \frac{(4a+b)\,\text{Subst}\left(\int \frac{1}{(1+x^2)^2}dx, x, \tan(e+fx)\right)}{4f}$$
$$= \frac{(4a+b)\cos(e+fx)\sin(e+fx)}{8f} - \frac{b\cos^3(e+fx)\sin(e+fx)}{4f} + \frac{(4a+b)\,\text{Su}}{}$$
$$= \frac{1}{8}(4a+b)x + \frac{(4a+b)\cos(e+fx)\sin(e+fx)}{8f} - \frac{b\cos^3(e+fx)\sin(e+fx)}{4f}$$

Mathematica [A] time = 0.0799996, size = 46, normalized size = 0.81

$$\frac{4(4ae+4afx+bfx)+8a\sin(2(e+fx))-b\sin(4(e+fx))}{32f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^2),x]
```

```
[Out] (4*(4*a*e + 4*a*f*x + b*f*x) + 8*a*Sin[2*(e + f*x)] - b*Sin[4*(e + f*x)])/(
32*f)
```

Maple [A] time = 0.046, size = 70, normalized size = 1.2

$$\frac{1}{f}\left(b\left(-\frac{\sin(fx+e)(\cos(fx+e))^3}{4} + \frac{\sin(fx+e)\cos(fx+e)}{8} + \frac{fx}{8} + \frac{e}{8}\right) + a\left(\frac{\sin(fx+e)\cos(fx+e)}{2} + \frac{fx}{2} + \frac{e}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sin(f*x+e)^2),x)
```

```
[Out] 1/f*(b*(-1/4*sin(f*x+e)*cos(f*x+e)^3+1/8*sin(f*x+e)*cos(f*x+e)+1/8*f*x+1/8*
e)+a*(1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e))
```

Maxima [A] time = 1.49416, size = 93, normalized size = 1.63

$$\frac{(fx+e)(4a+b) + \frac{(4a+b)\tan(fx+e)^3+(4a-b)\tan(fx+e)}{\tan(fx+e)^4+2\tan(fx+e)^2+1}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/8*((f*x + e)*(4*a + b) + ((4*a + b)*tan(f*x + e)^3 + (4*a - b)*tan(f*x +
e))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1))/f
```

Fricas [A] time = 1.89235, size = 113, normalized size = 1.98

$$\frac{(4a+b)fx - \left(2b\cos\left(fx+e\right)^3 - (4a+b)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/8*((4*a + b)*f*x - (2*b*cos(f*x + e)^3 - (4*a + b)*cos(f*x + e))*sin(f*x
+ e))/f
```

Sympy [A] time = 1.83485, size = 150, normalized size = 2.63

$$\begin{cases} \frac{ax\sin^2\left(e+fx\right)}{2} + \frac{ax\cos^2\left(e+fx\right)}{2} + \frac{a\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f} + \frac{bx\sin^4\left(e+fx\right)}{8} + \frac{bx\sin^2\left(e+fx\right)\cos^2\left(e+fx\right)}{4} + \frac{bx\cos^4\left(e+fx\right)}{8} + \frac{b\sin^3\left(e+fx\right)\cos\left(e+f\right.}{8f} \\ x\left(a+b\sin^2\left(e\right)\right)\cos^2\left(e\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sin(f*x+e)**2),x)
```

```
[Out] Piecewise((a*x*sin(e + f*x)**2/2 + a*x*cos(e + f*x)**2/2 + a*sin(e + f*x)*c
os(e + f*x)/(2*f) + b*x*sin(e + f*x)**4/8 + b*x*sin(e + f*x)**2*cos(e + f*x
)**2/4 + b*x*cos(e + f*x)**4/8 + b*sin(e + f*x)**3*cos(e + f*x)/(8*f) - b*s
in(e + f*x)*cos(e + f*x)**3/(8*f), Ne(f, 0)), (x*(a + b*sin(e)**2)*cos(e)**
2, True))
```

Giac [A] time = 1.12607, size = 55, normalized size = 0.96

$$\frac{1}{8}(4a+b)x - \frac{b\sin\left(4fx+4e\right)}{32f} + \frac{a\sin\left(2fx+2e\right)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/8*(4*a + b)*x - 1/32*b*sin(4*f*x + 4*e)/f + 1/4*a*sin(2*f*x + 2*e)/f
```

3.288 $\int \left(a+b\sin^2(e+fx)\right)\,dx$

Optimal. Leaf size=30

$$ax-\frac{b\sin(e+fx)\cos(e+fx)}{2f}+\frac{bx}{2}$$

```
[Out] a*x + (b*x)/2 - (b*Cos[e + f*x]*Sin[e + f*x])/(2*f)
```

Rubi [A] time = 0.0153733, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2635, 8}

$$ax-\frac{b\sin(e+fx)\cos(e+fx)}{2f}+\frac{bx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Sin[e + f*x]^2,x]
```

```
[Out] a*x + (b*x)/2 - (b*Cos[e + f*x]*Sin[e + f*x])/(2*f)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\sin^2(e+fx)\right)\,dx &= ax+b\int \sin^2(e+fx)\,dx\\
&= ax-\frac{b\cos(e+fx)\sin(e+fx)}{2f}+\frac{1}{2}b\int 1\,dx\\
&= ax+\frac{bx}{2}-\frac{b\cos(e+fx)\sin(e+fx)}{2f}
\end{aligned}$$

Mathematica [A] time = 0.0289019, size = 33, normalized size = 1.1

$$ax+\frac{b(e+fx)}{2f}-\frac{b\sin(2(e+fx))}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Sin[e + f*x]^2,x]
```

```
[Out] a*x + (b*(e + f*x))/(2*f) - (b*Sin[2*(e + f*x)])/(4*f)
```

Maple [A] time = 0.02, size = 32, normalized size = 1.1

$$ax+\frac{b}{f}\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*sin(f*x+e)^2,x)
```

```
[Out] a*x+b/f*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)
```

Maxima [A] time = 0.980732, size = 39, normalized size = 1.3

$$ax+\frac{\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)b}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] a*x + 1/4*(2*f*x + 2*e - sin(2*f*x + 2*e))*b/f
```

Fricas [A] time = 1.84782, size = 72, normalized size = 2.4

$$\frac{(2\,a+b)fx-b\cos\left(fx+e\right)\sin\left(fx+e\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((2*a + b)*f*x - b*cos(f*x + e)*sin(f*x + e))/f
```

Sympy [A] time = 0.362814, size = 51, normalized size = 1.7

$$ax+b\left(\begin{cases}\frac{x\sin^2\left(e+fx\right)}{2}+\frac{x\cos^2\left(e+fx\right)}{2}-\frac{\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f} & \text{for } f\neq 0\\ x\sin^2\left(e\right) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(f*x+e)**2,x)
```

```
[Out] a*x + b*Piecewise((x*sin(e + f*x)**2/2 + x*cos(e + f*x)**2/2 - sin(e + f*x)
*cos(e + f*x)/(2*f), Ne(f, 0)), (x*sin(e)**2, True))
```

Giac [A] time = 1.08347, size = 35, normalized size = 1.17

$$\frac{1}{4}\,b\left(2\,x-\frac{\sin\left(2\,fx+2\,e\right)}{f}\right)+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(f*x+e)^2,x, algorithm="giac")
```

```
[Out] 1/4*b*(2*x - sin(2*f*x + 2*e)/f) + a*x
```

3.289 $\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)dx$

Optimal. Leaf size=18

$$\frac{(a+b)\tan(e+fx)}{f}-bx$$

```
[Out] -(b*x) + ((a + b)*Tan[e + f*x])/f
```

Rubi [A] time = 0.0325623, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3191, 388, 203}

$$\frac{(a+b)\tan(e+fx)}{f}-bx$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2),x]
```

```
[Out] -(b*x) + ((a + b)*Tan[e + f*x])/f
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)dx &= \frac{\text{Subst}\left(\int \frac{a+(a+b)x^2}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{(a+b)\tan(e+fx)}{f}-\frac{b\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}\\
&= -bx+\frac{(a+b)\tan(e+fx)}{f}
\end{aligned}$$

Mathematica [A] time = 0.0147332, size = 36, normalized size = 2.

$$\frac{a\tan(e+fx)}{f}-\frac{b\tan^{-1}(\tan(e+fx))}{f}+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2),x]
```

```
[Out] -((b*ArcTan[Tan[e + f*x]])/f) + (a*Tan[e + f*x])/f + (b*Tan[e + f*x])/f
```

Maple [A] time = 0.05, size = 30, normalized size = 1.7

$$\frac{\tan\left(fx+e\right)a+b\left(\tan\left(fx+e\right)-fx-e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+b*sin(f*x+e)^2),x)
```

```
[Out] 1/f*(tan(f*x+e)*a+b*(tan(f*x+e)-f*x-e))
```

Maxima [A] time = 1.48076, size = 41, normalized size = 2.28

$$-\frac{\left(fx+e-\tan\left(fx+e\right)\right)b-a\tan\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] -((f*x + e - tan(f*x + e))*b - a*tan(f*x + e))/f
```

Fricas [A] time = 1.75496, size = 85, normalized size = 4.72

$$-\frac{bfx\cos\left(fx+e\right)-(a+b)\sin\left(fx+e\right)}{f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] -(b*f*x*cos(f*x + e) - (a + b)*sin(f*x + e))/(f*cos(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sin^2\left(e+fx\right)\right)\sec^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+b*sin(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sin(e + f*x)**2)*sec(e + f*x)**2, x)
```

Giac [B] time = 1.12307, size = 66, normalized size = 3.67

$$-\frac{\left(fx-\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor+e-\tan\left(fx+e\right)\right)b-a\tan\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2),x, algorithm="giac")
```

```
[Out] -((f*x - pi*floor((f*x + e)/pi + 1/2) + e - tan(f*x + e))*b - a*tan(f*x + e
))/f
```

3.290 $\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)dx$

Optimal. Leaf size=30

$$\frac{(a+b)\tan^3(e+fx)}{3f}+\frac{a\tan(e+fx)}{f}$$

```
[Out] (a*Tan[e + f*x])/f + ((a + b)*Tan[e + f*x]^3)/(3*f)
```

Rubi [A] time = 0.032005, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {3191}

$$\frac{(a+b)\tan^3(e+fx)}{3f}+\frac{a\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^2),x]

[Out] (a*Tan[e + f*x])/f + ((a + b)*Tan[e + f*x]^3)/(3*f)
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)dx &= \frac{\text{Subst}\left(\int\left(a+(a+b)x^2\right)dx,x,\tan(e+fx)\right)}{f}\\ &= \frac{a\tan(e+fx)}{f}+\frac{(a+b)\tan^3(e+fx)}{3f}\end{aligned}$$

Mathematica [A] time = 0.0741321, size = 41, normalized size = 1.37

$$\frac{a\left(\frac{1}{3}\tan^3(e+fx)+\tan(e+fx)\right)}{f}+\frac{b\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^2),x]

[Out] (b*Tan[e + f*x]^3)/(3*f) + (a*(Tan[e + f*x] + Tan[e + f*x]^3/3))/f
```

Maple [A] time = 0.056, size = 46, normalized size = 1.5

$$\frac{1}{f}\left(-a\left(-\frac{2}{3}-\frac{\left(\sec\left(fx+e\right)\right)^2}{3}\right)\tan\left(fx+e\right)+\frac{b\left(\sin\left(fx+e\right)\right)^3}{3\left(\cos\left(fx+e\right)\right)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(a+b*sin(f*x+e)^2),x)
```

```
[Out] 1/f*(-a*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+1/3*b*sin(f*x+e)^3/cos(f*x+e)^3)
```

Maxima [A] time = 0.979309, size = 36, normalized size = 1.2

$$\frac{(a+b)\tan\left(fx+e\right)^3+3\,a\tan\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/3*((a + b)*tan(f*x + e)^3 + 3*a*tan(f*x + e))/f
```

Fricas [A] time = 1.87285, size = 97, normalized size = 3.23

$$\frac{\left((2\,a-b)\cos\left(fx+e\right)^2+a+b\right)\sin\left(fx+e\right)}{3\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/3*((2*a - b)*cos(f*x + e)^2 + a + b)*sin(f*x + e)/(f*cos(f*x + e)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(a+b*sin(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16849, size = 51, normalized size = 1.7

$$\frac{a\tan\left(fx+e\right)^3+b\tan\left(fx+e\right)^3+3\,a\tan\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/3*(a*tan(f*x + e)^3 + b*tan(f*x + e)^3 + 3*a*tan(f*x + e))/f
```

3.291 $\int \sec^6(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx$

Optimal. Leaf size=50

$$\frac{(a+b)\tan^5(e+fx)}{5f}+\frac{(2a+b)\tan^3(e+fx)}{3f}+\frac{a\tan(e+fx)}{f}$$

```
[Out] (a*Tan[e + f*x])/f + ((2*a + b)*Tan[e + f*x]^3)/(3*f) + ((a + b)*Tan[e + f*
x]^5)/(5*f)
```

Rubi [A] time = 0.0444607, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3191, 373}

$$\frac{(a+b)\tan^5(e+fx)}{5f}+\frac{(2a+b)\tan^3(e+fx)}{3f}+\frac{a\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^6*(a + b*Sin[e + f*x]^2),x]
```

```
[Out] (a*Tan[e + f*x])/f + ((2*a + b)*Tan[e + f*x]^3)/(3*f) + ((a + b)*Tan[e + f*
x]^5)/(5*f)
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^6(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx &= \frac{\operatorname{Subst}\left(\int \left(1+x^2\right)\left(a+(a+b)x^2\right)\,dx,x,\tan(e+fx)\right)}{f}\\ &= \frac{\operatorname{Subst}\left(\int \left(a+(2a+b)x^2+(a+b)x^4\right)\,dx,x,\tan(e+fx)\right)}{f}\\ &= \frac{a\tan(e+fx)}{f}+\frac{(2a+b)\tan^3(e+fx)}{3f}+\frac{(a+b)\tan^5(e+fx)}{5f}\end{aligned}$$

Mathematica [A] time = 0.178007, size = 64, normalized size = 1.28

$$\frac{\tan(e+fx)\left(3a\tan^4(e+fx)+10a\tan^2(e+fx)+15a+3b\sec^4(e+fx)-b\sec^2(e+fx)-2b\right)}{15f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^6*(a + b*Sin[e + f*x]^2),x]
```

```
[Out] (Tan[e + f*x]*(15*a - 2*b - b*Sec[e + f*x]^2 + 3*b*Sec[e + f*x]^4 + 10*a*Ta
n[e + f*x]^2 + 3*a*Tan[e + f*x]^4))/(15*f)
```

Maple [A] time = 0.06, size = 76, normalized size = 1.5

$$\frac{1}{f}\left(-a\left(-\frac{8}{15}-\frac{\left(\sec\left(fx+e\right)\right)^{4}}{5}-\frac{4\left(\sec\left(fx+e\right)\right)^{2}}{15}\right)\tan\left(fx+e\right)+b\left(\frac{\left(\sin\left(fx+e\right)\right)^{3}}{5\left(\cos\left(fx+e\right)\right)^{5}}+\frac{2\left(\sin\left(fx+e\right)\right)^{3}}{15\left(\cos\left(fx+e\right)\right)^{3}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6*(a+b*sin(f*x+e)^2),x)
```

```
[Out] 1/f*(-a*(-8/15-1/5*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e)+b*(1/5*sin(f*
x+e)^3/cos(f*x+e)^5+2/15*sin(f*x+e)^3/cos(f*x+e)^3))
```

Maxima [A] time = 0.992635, size = 58, normalized size = 1.16

$$\frac{3\,(a+b)\tan\left(fx+e\right)^{5}+5\,(2\,a+b)\tan\left(fx+e\right)^{3}+15\,a\tan\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sin(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*(a + b)*tan(f*x + e)^5 + 5*(2*a + b)*tan(f*x + e)^3 + 15*a*tan(f*x
+ e))/f
```

Fricas [A] time = 1.94451, size = 143, normalized size = 2.86

$$\frac{\left(2\,(4\,a-b)\cos\left(fx+e\right)^{4}+(4\,a-b)\cos\left(fx+e\right)^{2}+3\,a+3\,b\right)\sin\left(fx+e\right)}{15\,f\cos\left(fx+e\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sin(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/15*(2*(4*a - b)*cos(f*x + e)^4 + (4*a - b)*cos(f*x + e)^2 + 3*a + 3*b)*si
n(f*x + e)/(f*cos(f*x + e)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6*(a+b*sin(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13642, size = 86, normalized size = 1.72

$$\frac{3\,a\tan\left(fx+e\right)^{5}+3\,b\tan\left(fx+e\right)^{5}+10\,a\tan\left(fx+e\right)^{3}+5\,b\tan\left(fx+e\right)^{3}+15\,a\tan\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sin(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/15*(3*a*tan(f*x + e)^5 + 3*b*tan(f*x + e)^5 + 10*a*tan(f*x + e)^3 + 5*b*t
an(f*x + e)^3 + 15*a*tan(f*x + e))/f
```

3.292 $\int \sec^8(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx$

Optimal. Leaf size=72

$$\frac{(a+b)\tan^7(e+fx)}{7f}+\frac{(3a+2b)\tan^5(e+fx)}{5f}+\frac{(3a+b)\tan^3(e+fx)}{3f}+\frac{a\tan(e+fx)}{f}$$

```
[Out] (a*Tan[e + f*x])/f + ((3*a + b)*Tan[e + f*x]^3)/(3*f) + ((3*a + 2*b)*Tan[e
+ f*x]^5)/(5*f) + ((a + b)*Tan[e + f*x]^7)/(7*f)
```

Rubi [A] time = 0.0552121, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3191, 373}

$$\frac{(a+b)\tan^7(e+fx)}{7f}+\frac{(3a+2b)\tan^5(e+fx)}{5f}+\frac{(3a+b)\tan^3(e+fx)}{3f}+\frac{a\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^8*(a + b*Sin[e + f*x]^2),x]
```

```
[Out] (a*Tan[e + f*x])/f + ((3*a + b)*Tan[e + f*x]^3)/(3*f) + ((3*a + 2*b)*Tan[e
+ f*x]^5)/(5*f) + ((a + b)*Tan[e + f*x]^7)/(7*f)
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^8(e+fx)\left(a+b\sin^2(e+fx)\right)dx &= \frac{\text{Subst}\left(\int \left(1+x^2\right)^2\left(a+(a+b)x^2\right)dx,x,\tan(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left(a+(3a+b)x^2+(3a+2b)x^4+(a+b)x^6\right)dx,x,\tan(e+fx)\right)}{f}\\ &= \frac{a\tan(e+fx)}{f}+\frac{(3a+b)\tan^3(e+fx)}{3f}+\frac{(3a+2b)\tan^5(e+fx)}{5f}+\frac{(a+b)\tan^7}{7f}\end{aligned}$$

Mathematica [A] time = 0.308562, size = 86, normalized size = 1.19

$$\frac{\tan(e+fx)\left(15a\tan^6(e+fx)+63a\tan^4(e+fx)+105a\tan^2(e+fx)+105a+15b\sec^6(e+fx)-3b\sec^4(e+fx)-4b\right.}{105f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^8*(a + b*Sin[e + f*x]^2),x]
```

```
[Out] (Tan[e + f*x]*(105*a - 8*b - 4*b*Sec[e + f*x]^2 - 3*b*Sec[e + f*x]^4 + 15*b
*Sec[e + f*x]^6 + 105*a*Tan[e + f*x]^2 + 63*a*Tan[e + f*x]^4 + 15*a*Tan[e +
 f*x]^6))/(105*f)
```

Maple [A] time = 0.063, size = 104, normalized size = 1.4

$$\frac{1}{f}\left(-a\left(-\frac{16}{35}-\frac{\left(\sec\left(fx+e\right)\right)^{6}}{7}-\frac{6\left(\sec\left(fx+e\right)\right)^{4}}{35}-\frac{8\left(\sec\left(fx+e\right)\right)^{2}}{35}\right)\tan\left(fx+e\right)+b\left(\frac{\left(\sin\left(fx+e\right)\right)^{3}}{7\left(\cos\left(fx+e\right)\right)^{7}}+\frac{4\,(\mathrm{si}}{35\,(\mathrm{c}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^8*(a+b*sin(f*x+e)^2),x)
```

```
[Out] 1/f*(-a*(-16/35-1/7*sec(f*x+e)^6-6/35*sec(f*x+e)^4-8/35*sec(f*x+e)^2)*tan(f
*x+e)+b*(1/7*sin(f*x+e)^3/cos(f*x+e)^7+4/35*sin(f*x+e)^3/cos(f*x+e)^5+8/105
*sin(f*x+e)^3/cos(f*x+e)^3))
```

Maxima [A] time = 0.984819, size = 81, normalized size = 1.12

$$\frac{15\,(a+b)\tan\left(fx+e\right)^{7}+21\,(3\,a+2\,b)\tan\left(fx+e\right)^{5}+35\,(3\,a+b)\tan\left(fx+e\right)^{3}+105\,a\tan\left(fx+e\right)}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^8*(a+b*sin(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/105*(15*(a + b)*tan(f*x + e)^7 + 21*(3*a + 2*b)*tan(f*x + e)^5 + 35*(3*a
+ b)*tan(f*x + e)^3 + 105*a*tan(f*x + e))/f
```

Fricas [A] time = 1.8906, size = 189, normalized size = 2.62

$$\frac{\left(8\,(6\,a-b)\cos\left(fx+e\right)^{6}+4\,(6\,a-b)\cos\left(fx+e\right)^{4}+3\,(6\,a-b)\cos\left(fx+e\right)^{2}+15\,a+15\,b\right)\sin\left(fx+e\right)}{105\,f\cos\left(fx+e\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^8*(a+b*sin(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/105*(8*(6*a - b)*cos(f*x + e)^6 + 4*(6*a - b)*cos(f*x + e)^4 + 3*(6*a - b
)*cos(f*x + e)^2 + 15*a + 15*b)*sin(f*x + e)/(f*cos(f*x + e)^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**8*(a+b*sin(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1679, size = 119, normalized size = 1.65

$$\frac{15\,a\tan\left(fx+e\right)^{7}+15\,b\tan\left(fx+e\right)^{7}+63\,a\tan\left(fx+e\right)^{5}+42\,b\tan\left(fx+e\right)^{5}+105\,a\tan\left(fx+e\right)^{3}+35\,b\tan\left(fx+\right.}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^8*(a+b*sin(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/105*(15*a*tan(f*x + e)^7 + 15*b*tan(f*x + e)^7 + 63*a*tan(f*x + e)^5 + 42
*b*tan(f*x + e)^5 + 105*a*tan(f*x + e)^3 + 35*b*tan(f*x + e)^3 + 105*a*tan(
f*x + e))/f
```

3.293 $\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=156

$$\frac{\left(48a^2+16ab+3b^2\right)\sin(e+fx)\cos^3(e+fx)}{192f}+\frac{\left(48a^2+16ab+3b^2\right)\sin(e+fx)\cos(e+fx)}{128f}+\frac{1}{128}x\left(48a^2+16ab+\right.$$

```
[Out] ((48*a^2 + 16*a*b + 3*b^2)*x)/128 + ((48*a^2 + 16*a*b + 3*b^2)*Cos[e + f*x]
*Sin[e + f*x])/(128*f) + ((48*a^2 + 16*a*b + 3*b^2)*Cos[e + f*x]^3*Sin[e +
f*x])/(192*f) - (b*(10*a + 3*b)*Cos[e + f*x]^5*Sin[e + f*x])/(48*f) - (b*Co
s[e + f*x]^7*Sin[e + f*x]*(a + (a + b)*Tan[e + f*x]^2))/(8*f)
```

Rubi [A] time = 0.149702, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3191, 413, 385, 199, 203}

$$\frac{\left(48a^2+16ab+3b^2\right)\sin(e+fx)\cos^3(e+fx)}{192f}+\frac{\left(48a^2+16ab+3b^2\right)\sin(e+fx)\cos(e+fx)}{128f}+\frac{1}{128}x\left(48a^2+16ab+\right.$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^2)^2,x]
```

```
[Out] ((48*a^2 + 16*a*b + 3*b^2)*x)/128 + ((48*a^2 + 16*a*b + 3*b^2)*Cos[e + f*x]
*Sin[e + f*x])/(128*f) + ((48*a^2 + 16*a*b + 3*b^2)*Cos[e + f*x]^3*Sin[e +
f*x])/(192*f) - (b*(10*a + 3*b)*Cos[e + f*x]^5*Sin[e + f*x])/(48*f) - (b*Co
s[e + f*x]^7*Sin[e + f*x]*(a + (a + b)*Tan[e + f*x]^2))/(8*f)
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
```

```
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx=\frac{\text{Subst}\left(\int \frac{\left(a+(a+b)x^2\right)^2}{\left(1+x^2\right)^5}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=-\frac{b\cos^7(e+fx)\sin(e+fx)\left(a+(a+b)\tan^2(e+fx)\right)}{8f}+\frac{\text{Subst}\left(\int \frac{a(8a+b)+(a+}{(1+x}\right.}{}$$

$$=-\frac{b(10a+3b)\cos^5(e+fx)\sin(e+fx)}{48f}-\frac{b\cos^7(e+fx)\sin(e+fx)\left(a+(a+b\right.}{8f}$$

$$=\frac{\left(48a^2+16ab+3b^2\right)\cos^3(e+fx)\sin(e+fx)}{192f}-\frac{b(10a+3b)\cos^5(e+fx)\sin(}{48f}$$

$$=\frac{\left(48a^2+16ab+3b^2\right)\cos(e+fx)\sin(e+fx)}{128f}+\frac{\left(48a^2+16ab+3b^2\right)\cos^3(e+}{192f}$$

$$=\frac{1}{128}\left(48a^2+16ab+3b^2\right)x+\frac{\left(48a^2+16ab+3b^2\right)\cos(e+fx)\sin(e+fx)}{128f}+$$

Mathematica [A] time = 0.317676, size = 96, normalized size = 0.62

$$\frac{24\left(48a^2+16ab+3b^2\right)(e+fx)+24\left(4a^2-4ab-b^2\right)\sin(4(e+fx))-32ab\sin(6(e+fx))+96a(8a+b)\sin(2(e+fx))}{3072f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^2)^2,x]
```

```
[Out] (24*(48*a^2 + 16*a*b + 3*b^2)*(e + f*x) + 96*a*(8*a + b)*Sin[2*(e + f*x)] +
 24*(4*a^2 - 4*a*b - b^2)*Sin[4*(e + f*x)] - 32*a*b*Sin[6*(e + f*x)] + 3*b^
2*Sin[8*(e + f*x)])/(3072*f)
```

Maple [A] time = 0.055, size = 167, normalized size = 1.1

$$\frac{1}{f}\left(b^2\left(-\frac{\left(\sin\left(fx+e\right)\right)^3\left(\cos\left(fx+e\right)\right)^5}{8}-\frac{\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^5}{16}+\frac{\sin\left(fx+e\right)}{64}\left(\left(\cos\left(fx+e\right)\right)^3+\frac{3\cos\left(fx+e\right)}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(b^2*(-1/8*sin(f*x+e)^3*cos(f*x+e)^5-1/16*sin(f*x+e)*cos(f*x+e)^5+1/64*
(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/128*f*x+3/128*e)+2*a*b*(-1/6*sin
(f*x+e)*cos(f*x+e)^5+1/24*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+1/16*f*x
+1/16*e)+a^2*(1/4*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/8*f*x+3/8*e))
```

Maxima [A] time = 1.47152, size = 228, normalized size = 1.46

$$\frac{3\left(48\,a^2+16\,ab+3\,b^2\right)\left(fx+e\right)+\frac{3\left(48\,a^2+16\,ab+3\,b^2\right)\tan\left(fx+e\right)^7+11\left(48\,a^2+16\,ab+3\,b^2\right)\tan\left(fx+e\right)^5+\left(624\,a^2+80\,ab-33\,b^2\right)\tan\left(fx+e\right)^3+3\,(8}{\tan\left(fx+e\right)^8+4\,\tan\left(fx+e\right)^6+6\,\tan\left(fx+e\right)^4+4\,\tan\left(fx+e\right)^2+1}}{384\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/384*(3*(48*a^2 + 16*a*b + 3*b^2)*(f*x + e) + (3*(48*a^2 + 16*a*b + 3*b^2)
*tan(f*x + e)^7 + 11*(48*a^2 + 16*a*b + 3*b^2)*tan(f*x + e)^5 + (624*a^2 +
80*a*b - 33*b^2)*tan(f*x + e)^3 + 3*(80*a^2 - 16*a*b - 3*b^2)*tan(f*x + e))
/(tan(f*x + e)^8 + 4*tan(f*x + e)^6 + 6*tan(f*x + e)^4 + 4*tan(f*x + e)^2 +
 1))/f
```

Fricas [A] time = 2.30418, size = 278, normalized size = 1.78

$$\frac{3\left(48\,a^2+16\,ab+3\,b^2\right)fx+\left(48\,b^2\cos\left(fx+e\right)^7-8\left(16\,ab+9\,b^2\right)\cos\left(fx+e\right)^5+2\left(48\,a^2+16\,ab+3\,b^2\right)\cos\left(fx+\right.\right.}{384\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/384*(3*(48*a^2 + 16*a*b + 3*b^2)*f*x + (48*b^2*cos(f*x + e)^7 - 8*(16*a*b
 + 9*b^2)*cos(f*x + e)^5 + 2*(48*a^2 + 16*a*b + 3*b^2)*cos(f*x + e)^3 + 3*(
48*a^2 + 16*a*b + 3*b^2)*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [A] time = 18.2245, size = 481, normalized size = 3.08

$$\begin{cases} \frac{3a^2x\sin^4\left(e+fx\right)}{8}+\frac{3a^2x\sin^2\left(e+fx\right)\cos^2\left(e+fx\right)}{4}+\frac{3a^2x\cos^4\left(e+fx\right)}{8}+\frac{3a^2\sin^3\left(e+fx\right)\cos\left(e+fx\right)}{8f}+\frac{5a^2\sin\left(e+fx\right)\cos^3\left(e+fx\right)}{8f}+\frac{abx\sin^6\left(e+f\right.}{8} \\ x\left(a+b\sin^2\left(e\right)\right)^2\cos^4\left(e\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(a+b*sin(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((3*a**2*x*sin(e + f*x)**4/8 + 3*a**2*x*sin(e + f*x)**2*cos(e + f*
x)**2/4 + 3*a**2*x*cos(e + f*x)**4/8 + 3*a**2*sin(e + f*x)**3*cos(e + f*x)/
(8*f) + 5*a**2*sin(e + f*x)*cos(e + f*x)**3/(8*f) + a*b*x*sin(e + f*x)**6/8
 + 3*a*b*x*sin(e + f*x)**4*cos(e + f*x)**2/8 + 3*a*b*x*sin(e + f*x)**2*cos(
```

```
e + f*x)**4/8 + a*b*x*cos(e + f*x)**6/8 + a*b*sin(e + f*x)**5*cos(e + f*x)/
(8*f) + a*b*sin(e + f*x)**3*cos(e + f*x)**3/(3*f) - a*b*sin(e + f*x)*cos(e
+ f*x)**5/(8*f) + 3*b**2*x*sin(e + f*x)**8/128 + 3*b**2*x*sin(e + f*x)**6*c
os(e + f*x)**2/32 + 9*b**2*x*sin(e + f*x)**4*cos(e + f*x)**4/64 + 3*b**2*x*
sin(e + f*x)**2*cos(e + f*x)**6/32 + 3*b**2*x*cos(e + f*x)**8/128 + 3*b**2*
sin(e + f*x)**7*cos(e + f*x)/(128*f) + 11*b**2*sin(e + f*x)**5*cos(e + f*x)
**3/(128*f) - 11*b**2*sin(e + f*x)**3*cos(e + f*x)**5/(128*f) - 3*b**2*sin(
e + f*x)*cos(e + f*x)**7/(128*f), Ne(f, 0)), (x*(a + b*sin(e)**2)**2*cos(e)
**4, True))
```

Giac [A] time = 1.14959, size = 146, normalized size = 0.94

$$\frac{1}{128}\left(48\,a^2+16\,ab+3\,b^2\right)x+\frac{b^2\sin\left(8\,fx+8\,e\right)}{1024\,f}-\frac{ab\sin\left(6\,fx+6\,e\right)}{96\,f}+\frac{\left(4\,a^2-4\,ab-b^2\right)\sin\left(4\,fx+4\,e\right)}{128\,f}+\frac{\left(8\,a^2+ab\right)\cdots}{3\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/128*(48*a^2 + 16*a*b + 3*b^2)*x + 1/1024*b^2*sin(8*f*x + 8*e)/f - 1/96*a*
b*sin(6*f*x + 6*e)/f + 1/128*(4*a^2 - 4*a*b - b^2)*sin(4*f*x + 4*e)/f + 1/3
2*(8*a^2 + a*b)*sin(2*f*x + 2*e)/f
```

3.294 $\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=116

$$\frac{\left(8a^2+4ab+b^2\right)\sin(e+fx)\cos(e+fx)}{16f}+\frac{1}{16}x\left(8a^2+4ab+b^2\right)-\frac{b(8a+3b)\sin(e+fx)\cos^3(e+fx)}{24f}-\frac{b\sin(e+fx}{}$$

```
[Out] ((8*a^2 + 4*a*b + b^2)*x)/16 + ((8*a^2 + 4*a*b + b^2)*Cos[e + f*x]*Sin[e +
f*x])/(16*f) - (b*(8*a + 3*b)*Cos[e + f*x]^3*Sin[e + f*x])/(24*f) - (b*Cos[
e + f*x]^5*Sin[e + f*x]*(a + (a + b)*Tan[e + f*x]^2))/(6*f)
```

Rubi [A] time = 0.145555, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3191, 413, 385, 199, 203}

$$\frac{\left(8a^2+4ab+b^2\right)\sin(e+fx)\cos(e+fx)}{16f}+\frac{1}{16}x\left(8a^2+4ab+b^2\right)-\frac{b(8a+3b)\sin(e+fx)\cos^3(e+fx)}{24f}-\frac{b\sin(e+fx}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^2)^2,x]
```

```
[Out] ((8*a^2 + 4*a*b + b^2)*x)/16 + ((8*a^2 + 4*a*b + b^2)*Cos[e + f*x]*Sin[e +
f*x])/(16*f) - (b*(8*a + 3*b)*Cos[e + f*x]^3*Sin[e + f*x])/(24*f) - (b*Cos[
e + f*x]^5*Sin[e + f*x]*(a + (a + b)*Tan[e + f*x]^2))/(6*f)
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
```

```
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+(a+b)x^2\right)^2}{\left(1+x^2\right)^4}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cos^5(e+fx)\sin(e+fx)\left(a+(a+b)\tan^2(e+fx)\right)}{6f} + \frac{\text{Subst}\left(\int \frac{a(6a+b)+3(a\cdots}{(1+x\cdots}\right.}{\cdots}$$

$$= -\frac{b(8a+3b)\cos^3(e+fx)\sin(e+fx)}{24f} - \frac{b\cos^5(e+fx)\sin(e+fx)\left(a+(a+b\cdots\right.}{6f}$$

$$= \frac{\left(8a^2+4ab+b^2\right)\cos(e+fx)\sin(e+fx)}{16f} - \frac{b(8a+3b)\cos^3(e+fx)\sin(e+f\cdots}{24f}$$

$$= \frac{1}{16}\left(8a^2+4ab+b^2\right)x + \frac{\left(8a^2+4ab+b^2\right)\cos(e+fx)\sin(e+fx)}{16f} - \frac{b(8a+3b\cdots}{\cdots}$$

Mathematica [C] time = 0.269357, size = 79, normalized size = 0.68

$$\frac{12(b+(2-2i)a)(b+(2+2i)a)(e+fx)-3b(4a+b)\sin(4(e+fx))+3(4a-b)(4a+b)\sin(2(e+fx))+b^2\sin(6(e+fx))}{192f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^2)^2,x]
```

```
[Out] (12*((2 - 2*I)*a + b)*((2 + 2*I)*a + b)*(e + f*x) + 3*(4*a - b)*(4*a + b)*S
in[2*(e + f*x)] - 3*b*(4*a + b)*Sin[4*(e + f*x)] + b^2*Sin[6*(e + f*x)])/(1
92*f)
```

Maple [A] time = 0.05, size = 134, normalized size = 1.2

$$\frac{1}{f}\left(b^2\left(-\frac{\left(\sin\left(fx+e\right)\right)^3\left(\cos\left(fx+e\right)\right)^3}{6} - \frac{\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^3}{8} + \frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{16} + \frac{fx}{16} + \frac{e}{16}\right) + 2ab\left(-\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(b^2*(-1/6*sin(f*x+e)^3*cos(f*x+e)^3-1/8*sin(f*x+e)*cos(f*x+e)^3+1/16*s
in(f*x+e)*cos(f*x+e)+1/16*f*x+1/16*e)+2*a*b*(-1/4*sin(f*x+e)*cos(f*x+e)^3+1
/8*sin(f*x+e)*cos(f*x+e)+1/8*f*x+1/8*e)+a^2*(1/2*sin(f*x+e)*cos(f*x+e)+1/2*
```

f*x+1/2*e))

Maxima [A] time = 1.4792, size = 171, normalized size = 1.47

$$\frac{3\left(8a^2+4ab+b^2\right)(fx+e)+\frac{3\left(8a^2+4ab+b^2\right)\tan(fx+e)^5+8\left(6a^2-b^2\right)\tan(fx+e)^3+3\left(8a^2-4ab-b^2\right)\tan(fx+e)}{\tan(fx+e)^6+3\tan(fx+e)^4+3\tan(fx+e)^2+1}}{48f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/48*(3*(8*a^2 + 4*a*b + b^2)*(f*x + e) + (3*(8*a^2 + 4*a*b + b^2)*tan(f*x
+ e)^5 + 8*(6*a^2 - b^2)*tan(f*x + e)^3 + 3*(8*a^2 - 4*a*b - b^2)*tan(f*x +
 e))/(tan(f*x + e)^6 + 3*tan(f*x + e)^4 + 3*tan(f*x + e)^2 + 1))/f
```

Fricas [A] time = 1.93942, size = 204, normalized size = 1.76

$$\frac{3\left(8a^2+4ab+b^2\right)fx+\left(8b^2\cos\left(fx+e\right)^5-2\left(12ab+7b^2\right)\cos\left(fx+e\right)^3+3\left(8a^2+4ab+b^2\right)\cos\left(fx+e\right)\right)\sin\left(f\right.}{48f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/48*(3*(8*a^2 + 4*a*b + b^2)*f*x + (8*b^2*cos(f*x + e)^5 - 2*(12*a*b + 7*b
^2)*cos(f*x + e)^3 + 3*(8*a^2 + 4*a*b + b^2)*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [A] time = 5.98898, size = 314, normalized size = 2.71

$$\begin{cases}\frac{a^2x\sin^2(e+fx)}{2}+\frac{a^2x\cos^2(e+fx)}{2}+\frac{a^2\sin(e+fx)\cos(e+fx)}{2f}+\frac{abx\sin^4(e+fx)}{4}+\frac{abx\sin^2(e+fx)\cos^2(e+fx)}{2}+\frac{abx\cos^4(e+fx)}{4}+\frac{ab\sin^3(e+\\ x\left(a+b\sin^2(e)\right)^2\cos^2(e)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sin(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((a**2*x*sin(e + f*x)**2/2 + a**2*x*cos(e + f*x)**2/2 + a**2*sin(e
 + f*x)*cos(e + f*x)/(2*f) + a*b*x*sin(e + f*x)**4/4 + a*b*x*sin(e + f*x)**
2*cos(e + f*x)**2/2 + a*b*x*cos(e + f*x)**4/4 + a*b*sin(e + f*x)**3*cos(e +
 f*x)/(4*f) - a*b*sin(e + f*x)*cos(e + f*x)**3/(4*f) + b**2*x*sin(e + f*x)*
*6/16 + 3*b**2*x*sin(e + f*x)**4*cos(e + f*x)**2/16 + 3*b**2*x*sin(e + f*x)
**2*cos(e + f*x)**4/16 + b**2*x*cos(e + f*x)**6/16 + b**2*sin(e + f*x)**5*c
os(e + f*x)/(16*f) - b**2*sin(e + f*x)**3*cos(e + f*x)**3/(6*f) - b**2*sin(
e + f*x)*cos(e + f*x)**5/(16*f), Ne(f, 0)), (x*(a + b*sin(e)**2)**2*cos(e)*
*2, True))
```

Giac [A] time = 1.15296, size = 113, normalized size = 0.97

$$\frac{1}{16}\left(8\,a^2+4\,ab+b^2\right)x+\frac{b^2\sin\left(6\,fx+6\,e\right)}{192\,f}-\frac{\left(4\,ab+b^2\right)\sin\left(4\,fx+4\,e\right)}{64\,f}+\frac{\left(16\,a^2-b^2\right)\sin\left(2\,fx+2\,e\right)}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/16*(8*a^2 + 4*a*b + b^2)*x + 1/192*b^2*sin(6*f*x + 6*e)/f - 1/64*(4*a*b +
 b^2)*sin(4*f*x + 4*e)/f + 1/64*(16*a^2 - b^2)*sin(2*f*x + 2*e)/f
```

3.295 $\int \left(a+b\sin^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=72

$$\frac{1}{8}x\left(8a^2+8ab+3b^2\right)-\frac{b(8a+3b)\sin(e+fx)\cos(e+fx)}{8f}-\frac{b^2\sin^3(e+fx)\cos(e+fx)}{4f}$$

```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*x)/8 - (b*(8*a + 3*b)*Cos[e + f*x]*Sin[e + f*x])/(
8*f) - (b^2*Cos[e + f*x]*Sin[e + f*x]^3)/(4*f)
```

Rubi [A] time = 0.020182, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3179}

$$\frac{1}{8}x\left(8a^2+8ab+3b^2\right)-\frac{b(8a+3b)\sin(e+fx)\cos(e+fx)}{8f}-\frac{b^2\sin^3(e+fx)\cos(e+fx)}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x]^2)^2,x]
```

```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*x)/8 - (b*(8*a + 3*b)*Cos[e + f*x]*Sin[e + f*x])/(
8*f) - (b^2*Cos[e + f*x]*Sin[e + f*x]^3)/(4*f)
```

Rule 3179

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^2, x_Symbol] :> Simp[((8*a^2 +
 8*a*b + 3*b^2)*x)/8, x] + (-Simp[(b^2*Cos[e + f*x]*Sin[e + f*x]^3)/(4*f),
x] - Simp[(b*(8*a + 3*b)*Cos[e + f*x]*Sin[e + f*x])/(8*f), x]) /; FreeQ[{a,
 b, e, f}, x]
```

Rubi steps

$$\int \left(a+b\sin^2(e+fx)\right)^2\,dx=\frac{1}{8}\left(8a^2+8ab+3b^2\right)x-\frac{b(8a+3b)\cos(e+fx)\sin(e+fx)}{8f}-\frac{b^2\cos(e+fx)\sin^3(e+fx}{4f}$$

Mathematica [A] time = 0.116051, size = 58, normalized size = 0.81

$$\frac{4\left(8a^2+8ab+3b^2\right)(e+fx)-8b(2a+b)\sin(2(e+fx))+b^2\sin(4(e+fx))}{32f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)^2,x]
```

```
[Out] (4*(8*a^2 + 8*a*b + 3*b^2)*(e + f*x) - 8*b*(2*a + b)*Sin[2*(e + f*x)] + b^2
*Sin[4*(e + f*x)])/(32*f)
```

Maple [A] time = 0.03, size = 78, normalized size = 1.1

$$\frac{1}{f}\left(b^2\left(-\frac{\cos\left(fx+e\right)}{4}\left(\left(\sin\left(fx+e\right)\right)^3+\frac{3\sin\left(fx+e\right)}{2}\right)+\frac{3fx}{8}+\frac{3e}{8}\right)+2ab\left(-1/2\sin\left(fx+e\right)\cos\left(fx+e\right)+1/2f\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(b^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+2*a*
b*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+a^2*(f*x+e))
```

Maxima [A] time = 0.972697, size = 92, normalized size = 1.28

$$a^2x + \frac{(2\,fx + 2\,e - \sin(2\,fx + 2\,e))ab}{2\,f} + \frac{(12\,fx + 12\,e + \sin(4\,fx + 4\,e) - 8\,\sin(2\,fx + 2\,e))b^2}{32\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 1/2*(2*f*x + 2*e - sin(2*f*x + 2*e))*a*b/f + 1/32*(12*f*x + 12*e +
sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*b^2/f
```

Fricas [A] time = 1.95607, size = 143, normalized size = 1.99

$$\frac{(8\,a^2 + 8\,ab + 3\,b^2)fx + \left(2\,b^2\cos(fx + e)^3 - (8\,ab + 5\,b^2)\cos(fx + e)\right)\sin(fx + e)}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*((8*a^2 + 8*a*b + 3*b^2)*f*x + (2*b^2*cos(f*x + e)^3 - (8*a*b + 5*b^2)*
cos(f*x + e))*sin(f*x + e))/f
```

Sympy [A] time = 1.69381, size = 168, normalized size = 2.33

$$\begin{cases} a^2x + abx\sin^2(e + fx) + abx\cos^2(e + fx) - \frac{ab\sin(e+fx)\cos(e+fx)}{f} + \frac{3b^2x\sin^4(e+fx)}{8} + \frac{3b^2x\sin^2(e+fx)\cos^2(e+fx)}{4} + \frac{3b^2x\cos^4(e-}{8} \\ x\left(a + b\sin^2(e)\right)^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((a**2*x + a*b*x*sin(e + f*x)**2 + a*b*x*cos(e + f*x)**2 - a*b*sin
(e + f*x)*cos(e + f*x)/f + 3*b**2*x*sin(e + f*x)**4/8 + 3*b**2*x*sin(e + f*
x)**2*cos(e + f*x)**2/4 + 3*b**2*x*cos(e + f*x)**4/8 - 5*b**2*sin(e + f*x)*
*3*cos(e + f*x)/(8*f) - 3*b**2*sin(e + f*x)*cos(e + f*x)**3/(8*f), Ne(f, 0)
), (x*(a + b*sin(e)**2)**2, True))
```

Giac [A] time = 1.10804, size = 81, normalized size = 1.12

$$\frac{1}{8}\left(8\,a^{2}+8\,ab+3\,b^{2}\right)x+\frac{b^{2}\sin\left(4\,fx+4\,e\right)}{32\,f}-\frac{\left(2\,ab+b^{2}\right)\sin\left(2\,fx+2\,e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*(8*a^2 + 8*a*b + 3*b^2)*x + 1/32*b^2*sin(4*f*x + 4*e)/f - 1/4*(2*a*b +
b^2)*sin(2*f*x + 2*e)/f
```

3.296 $\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=51

$$\frac{(a+b)^2\tan(e+fx)}{f}-\frac{1}{2}bx(4a+3b)+\frac{b^2\sin(e+fx)\cos(e+fx)}{2f}$$

```
[Out] -(b*(4*a + 3*b)*x)/2 + (b^2*Cos[e + f*x]*Sin[e + f*x])/(2*f) + ((a + b)^2*T
an[e + f*x])/f
```

Rubi [A] time = 0.0892798, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3191, 390, 385, 203}

$$\frac{(a+b)^2\tan(e+fx)}{f}-\frac{1}{2}bx(4a+3b)+\frac{b^2\sin(e+fx)\cos(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)^2,x]
```

```
[Out] -(b*(4*a + 3*b)*x)/2 + (b^2*Cos[e + f*x]*Sin[e + f*x])/(2*f) + ((a + b)^2*T
an[e + f*x])/f
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+(a+b)x^2\right)^2}{\left(1+x^2\right)^2}\,dx,x,\tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left((a+b)^2-\frac{b(2a+b)+2b(a+b)x^2}{\left(1+x^2\right)^2}\right)dx,x,\tan(e+fx)\right)}{f}$$
$$= \frac{(a+b)^2\tan(e+fx)}{f}-\frac{\text{Subst}\left(\int \frac{b(2a+b)+2b(a+b)x^2}{\left(1+x^2\right)^2}\,dx,x,\tan(e+fx)\right)}{f}$$
$$= \frac{b^2\cos(e+fx)\sin(e+fx)}{2f}+\frac{(a+b)^2\tan(e+fx)}{f}-\frac{(b(4a+3b))\,\text{Subst}\left(\int\right.}{2}$$
$$= -\frac{1}{2}b(4a+3b)x+\frac{b^2\cos(e+fx)\sin(e+fx)}{2f}+\frac{(a+b)^2\tan(e+fx)}{f}$$

Mathematica [A] time = 0.318603, size = 48, normalized size = 0.94

$$\frac{-2b(4a+3b)(e+fx)+4(a+b)^2\tan(e+fx)+b^2\sin(2(e+fx))}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)^2,x]
```

```
[Out] (-2*b*(4*a + 3*b)*(e + f*x) + b^2*Sin[2*(e + f*x)] + 4*(a + b)^2*Tan[e + f*
x])/(4*f)
```

Maple [A] time = 0.06, size = 87, normalized size = 1.7

$$\frac{1}{f}\left(a^2\tan\left(fx+e\right)+2\,ab\left(\tan\left(fx+e\right)-fx-e\right)+b^2\left(\frac{\left(\sin\left(fx+e\right)\right)^5}{\cos\left(fx+e\right)}+\left(\left(\sin\left(fx+e\right)\right)^3+\frac{3\,\sin\left(fx+e\right)}{2}\right)\cos\left(fx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(a^2*tan(f*x+e)+2*a*b*(tan(f*x+e)-f*x-e)+b^2*(sin(f*x+e)^5/cos(f*x+e)+(
sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)-3/2*f*x-3/2*e))
```

Maxima [A] time = 1.48159, size = 100, normalized size = 1.96

$$-\frac{4\left(fx+e-\tan\left(fx+e\right)\right)ab+\left(3\,fx+3\,e-\frac{\tan\left(fx+e\right)}{\tan\left(fx+e\right)^2+1}-2\,\tan\left(fx+e\right)\right)b^2-2\,a^2\tan\left(fx+e\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(4*(f*x + e - tan(f*x + e))*a*b + (3*f*x + 3*e - tan(f*x + e)/(tan(f*x
 + e)^2 + 1) - 2*tan(f*x + e))*b^2 - 2*a^2*tan(f*x + e))/f
```

Fricas [A] time = 1.85229, size = 159, normalized size = 3.12

$$-\frac{\left(4\,ab+3\,b^2\right)fx\cos\left(fx+e\right)-\left(b^2\cos\left(fx+e\right)^2+2\,a^2+4\,ab+2\,b^2\right)\sin\left(fx+e\right)}{2\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((4*a*b + 3*b^2)*f*x*cos(f*x + e) - (b^2*cos(f*x + e)^2 + 2*a^2 + 4*a*
b + 2*b^2)*sin(f*x + e))/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+b*sin(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.13218, size = 134, normalized size = 2.63

$$\frac{2\,a^2\tan\left(fx+e\right)+4\,ab\tan\left(fx+e\right)+2\,b^2\tan\left(fx+e\right)-\left(4\,ab+3\,b^2\right)\left(fx-\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor+e\right)+\frac{b^2\tan\left(fx+e\right)}{\tan\left(fx+e\right)^2+1}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*a^2*tan(f*x + e) + 4*a*b*tan(f*x + e) + 2*b^2*tan(f*x + e) - (4*a*b
+ 3*b^2)*(f*x - pi*floor((f*x + e)/pi + 1/2) + e) + b^2*tan(f*x + e)/(tan(f
*x + e)^2 + 1))/f
```

3.297 $\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=45

$$\frac{(a^2-b^2)\tan(e+fx)}{f}+\frac{(a+b)^2\tan^3(e+fx)}{3f}+b^2x$$

```
[Out] b^2*x + ((a^2 - b^2)*Tan[e + f*x])/f + ((a + b)^2*Tan[e + f*x]^3)/(3*f)
```

Rubi [A] time = 0.062085, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3191, 390, 203}

$$\frac{(a^2-b^2)\tan(e+fx)}{f}+\frac{(a+b)^2\tan^3(e+fx)}{3f}+b^2x$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^2)^2,x]
```

```
[Out] b^2*x + ((a^2 - b^2)*Tan[e + f*x])/f + ((a + b)^2*Tan[e + f*x]^3)/(3*f)
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+(a+b)x^2\right)^2}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(a^2-b^2+(a+b)^2x^2+\frac{b^2}{1+x^2}\right)dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{\left(a^2-b^2\right)\tan(e+fx)}{f} + \frac{(a+b)^2\tan^3(e+fx)}{3f} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(e\right.}{f}$$
$$= b^2x + \frac{\left(a^2-b^2\right)\tan(e+fx)}{f} + \frac{(a+b)^2\tan^3(e+fx)}{3f}$$

Mathematica [A] time = 0.324862, size = 57, normalized size = 1.27

$$\frac{(a+b)\tan(e+fx)\sec^2(e+fx)((a-2b)\cos(2(e+fx))+2a-b)+3b^2(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^2)^2,x]
```

```
[Out] (3*b^2*(e + f*x) + (a + b)*(2*a - b + (a - 2*b)*Cos[2*(e + f*x)])*Sec[e + f
*x]^2*Tan[e + f*x])/(3*f)
```

Maple [A] time = 0.072, size = 76, normalized size = 1.7

$$\frac{1}{f}\left(-a^2\left(-\frac{2}{3}-\frac{\left(\sec\left(fx+e\right)\right)^2}{3}\right)\tan\left(fx+e\right)+\frac{2\,ab\left(\sin\left(fx+e\right)\right)^3}{3\left(\cos\left(fx+e\right)\right)^3}+b^2\left(\frac{\left(\tan\left(fx+e\right)\right)^3}{3}-\tan\left(fx+e\right)+fx+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(-a^2*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+2/3*a*b*sin(f*x+e)^3/cos(f*x+e
)^3+b^2*(1/3*tan(f*x+e)^3-tan(f*x+e)+f*x+e))
```

Maxima [A] time = 1.47153, size = 72, normalized size = 1.6

$$\frac{\left(a^2+2\,ab+b^2\right)\tan\left(fx+e\right)^3+3\left(fx+e\right)b^2+3\left(a^2-b^2\right)\tan\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*((a^2 + 2*a*b + b^2)*tan(f*x + e)^3 + 3*(f*x + e)*b^2 + 3*(a^2 - b^2)*t
an(f*x + e))/f
```

Fricas [A] time = 1.972, size = 169, normalized size = 3.76

$$\frac{3\,b^2 fx\cos\left(fx+e\right)^3+\left(2\left(a^2-ab-2\,b^2\right)\cos\left(fx+e\right)^2+a^2+2\,ab+b^2\right)\sin\left(fx+e\right)}{3\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*b^2*f*x*cos(f*x + e)^3 + (2*(a^2 - a*b - 2*b^2)*cos(f*x + e)^2 + a^2
 + 2*a*b + b^2)*sin(f*x + e))/(f*cos(f*x + e)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(a+b*sin(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14529, size = 108, normalized size = 2.4

$$\frac{a^2\tan\left(fx+e\right)^3+2\,ab\tan\left(fx+e\right)^3+b^2\tan\left(fx+e\right)^3+3\left(fx+e\right)b^2+3\,a^2\tan\left(fx+e\right)-3\,b^2\tan\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(a^2*tan(f*x + e)^3 + 2*a*b*tan(f*x + e)^3 + b^2*tan(f*x + e)^3 + 3*(f*
x + e)*b^2 + 3*a^2*tan(f*x + e) - 3*b^2*tan(f*x + e))/f
```

3.298 $\int \sec^6(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=53

$$\frac{a^2\tan(e+fx)}{f}+\frac{(a+b)^2\tan^5(e+fx)}{5f}+\frac{2a(a+b)\tan^3(e+fx)}{3f}$$

```
[Out] (a^2*Tan[e + f*x])/f + (2*a*(a + b)*Tan[e + f*x]^3)/(3*f) + ((a + b)^2*Tan[
e + f*x]^5)/(5*f)
```

Rubi [A] time = 0.0572108, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3191, 194}

$$\frac{a^2\tan(e+fx)}{f}+\frac{(a+b)^2\tan^5(e+fx)}{5f}+\frac{2a(a+b)\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^6*(a + b*Sin[e + f*x]^2)^2,x]
```

```
[Out] (a^2*Tan[e + f*x])/f + (2*a*(a + b)*Tan[e + f*x]^3)/(3*f) + ((a + b)^2*Tan[
e + f*x]^5)/(5*f)
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^6(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \left(a+(a+b)x^2\right)^2\,dx, x, \tan(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left(a^2+2a(a+b)x^2+(a+b)^2x^4\right)dx, x, \tan(e+fx)\right)}{f}\\ &= \frac{a^2\tan(e+fx)}{f}+\frac{2a(a+b)\tan^3(e+fx)}{3f}+\frac{(a+b)^2\tan^5(e+fx)}{5f}\end{aligned}$$

Mathematica [A] time = 0.353802, size = 67, normalized size = 1.26

$$\frac{\tan(e+fx)\left(\left(4a^2-2ab-6b^2\right)\sec^2(e+fx)+8a^2+3(a+b)^2\sec^4(e+fx)-4ab+3b^2\right)}{15f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^6*(a + b*Sin[e + f*x]^2)^2,x]
```

```
[Out] ((8*a^2 - 4*a*b + 3*b^2 + (4*a^2 - 2*a*b - 6*b^2)*Sec[e + f*x]^2 + 3*(a + b
)^2*Sec[e + f*x]^4)*Tan[e + f*x])/(15*f)
```

Maple [B] time = 0.072, size = 101, normalized size = 1.9

$$\frac{1}{f}\left(-a^2\left(-\frac{8}{15}-\frac{\left(\sec\left(fx+e\right)\right)^4}{5}-\frac{4\left(\sec\left(fx+e\right)\right)^2}{15}\right)\tan\left(fx+e\right)+2ab\left(1/5\frac{\left(\sin\left(fx+e\right)\right)^3}{\left(\cos\left(fx+e\right)\right)^5}+2/15\frac{\left(\sin\left(fx+e\right)\right)^3}{\left(\cos\left(fx+e\right)\right)^3}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6*(a+b*sin(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(-a^2*(-8/15-1/5*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e)+2*a*b*(1/5*
sin(f*x+e)^3/cos(f*x+e)^5+2/15*sin(f*x+e)^3/cos(f*x+e)^3)+1/5*b^2*sin(f*x+e
)^5/cos(f*x+e)^5)
```

Maxima [A] time = 1.00522, size = 74, normalized size = 1.4

$$\frac{3\left(a^2+2ab+b^2\right)\tan\left(fx+e\right)^5+10\left(a^2+ab\right)\tan\left(fx+e\right)^3+15a^2\tan\left(fx+e\right)}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sin(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/15*(3*(a^2 + 2*a*b + b^2)*tan(f*x + e)^5 + 10*(a^2 + a*b)*tan(f*x + e)^3
+ 15*a^2*tan(f*x + e))/f
```

Fricas [A] time = 1.81191, size = 194, normalized size = 3.66

$$\frac{\left(\left(8a^2-4ab+3b^2\right)\cos\left(fx+e\right)^4+2\left(2a^2-ab-3b^2\right)\cos\left(fx+e\right)^2+3a^2+6ab+3b^2\right)\sin\left(fx+e\right)}{15f\cos\left(fx+e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sin(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/15*((8*a^2 - 4*a*b + 3*b^2)*cos(f*x + e)^4 + 2*(2*a^2 - a*b - 3*b^2)*cos(
f*x + e)^2 + 3*a^2 + 6*a*b + 3*b^2)*sin(f*x + e)/(f*cos(f*x + e)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6*(a+b*sin(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16325, size = 116, normalized size = 2.19

$$\frac{3\,a^2\tan\left(fx+e\right)^5+6\,ab\tan\left(fx+e\right)^5+3\,b^2\tan\left(fx+e\right)^5+10\,a^2\tan\left(fx+e\right)^3+10\,ab\tan\left(fx+e\right)^3+15\,a^2\tan\left(fx+\right.}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sin(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/15*(3*a^2*tan(f*x + e)^5 + 6*a*b*tan(f*x + e)^5 + 3*b^2*tan(f*x + e)^5 +
10*a^2*tan(f*x + e)^3 + 10*a*b*tan(f*x + e)^3 + 15*a^2*tan(f*x + e))/f
```

3.299 $\int \sec^8(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=80

$$\frac{a^2\tan(e+fx)}{f}+\frac{(a+b)^2\tan^7(e+fx)}{7f}+\frac{(a+b)(3a+b)\tan^5(e+fx)}{5f}+\frac{a(3a+2b)\tan^3(e+fx)}{3f}$$

```
[Out] (a^2*Tan[e + f*x])/f + (a*(3*a + 2*b)*Tan[e + f*x]^3)/(3*f) + ((a + b)*(3*a
 + b)*Tan[e + f*x]^5)/(5*f) + ((a + b)^2*Tan[e + f*x]^7)/(7*f)
```

Rubi [A] time = 0.0757315, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3191, 373}

$$\frac{a^2\tan(e+fx)}{f}+\frac{(a+b)^2\tan^7(e+fx)}{7f}+\frac{(a+b)(3a+b)\tan^5(e+fx)}{5f}+\frac{a(3a+2b)\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^8*(a + b*Sin[e + f*x]^2)^2,x]
```

```
[Out] (a^2*Tan[e + f*x])/f + (a*(3*a + 2*b)*Tan[e + f*x]^3)/(3*f) + ((a + b)*(3*a
 + b)*Tan[e + f*x]^5)/(5*f) + ((a + b)^2*Tan[e + f*x]^7)/(7*f)
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \sec^8(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx=\frac{\text{Subst}\left(\int\left(1+x^2\right)\left(a+(a+b)x^2\right)^2\,dx,x,\tan(e+fx)\right)}{f}$$
$$=\frac{\text{Subst}\left(\int\left(a^2+a(3a+2b)x^2+(a+b)(3a+b)x^4+(a+b)^2x^6\right)dx,x,\tan(e\right.}{f}$$
$$=\frac{a^2\tan(e+fx)}{f}+\frac{a(3a+2b)\tan^3(e+fx)}{3f}+\frac{(a+b)(3a+b)\tan^5(e+fx)}{5f}$$

Mathematica [A] time = 0.478616, size = 92, normalized size = 1.15

$$\frac{\tan(e+fx)\left(6\left(3a^2-ab-4b^2\right)\sec^4(e+fx)+\left(24a^2-8ab+3b^2\right)\sec^2(e+fx)+48a^2+15(a+b)^2\sec^6(e+fx)-16\right.}{105f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^8*(a + b*Sin[e + f*x]^2)^2,x]
```

```
[Out] ((48*a^2 - 16*a*b + 6*b^2 + (24*a^2 - 8*a*b + 3*b^2)*Sec[e + f*x]^2 + 6*(3*
a^2 - a*b - 4*b^2)*Sec[e + f*x]^4 + 15*(a + b)^2*Sec[e + f*x]^6)*Tan[e + f*
x])/(105*f)
```

Maple [A] time = 0.075, size = 149, normalized size = 1.9

$$\frac{1}{f}\left(-a^2\left(-\frac{16}{35}-\frac{\left(\sec\left(fx+e\right)\right)^6}{7}-\frac{6\left(\sec\left(fx+e\right)\right)^4}{35}-\frac{8\left(\sec\left(fx+e\right)\right)^2}{35}\right)\tan\left(fx+e\right)+2ab\left(1/7\frac{\left(\sin\left(fx+e\right)\right)^3}{\left(\cos\left(fx+e\right)\right)^7}+\frac{4\left(\right.}{35\left(\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^8*(a+b*sin(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(-a^2*(-16/35-1/7*sec(f*x+e)^6-6/35*sec(f*x+e)^4-8/35*sec(f*x+e)^2)*tan
(f*x+e)+2*a*b*(1/7*sin(f*x+e)^3/cos(f*x+e)^7+4/35*sin(f*x+e)^3/cos(f*x+e)^5
+8/105*sin(f*x+e)^3/cos(f*x+e)^3)+b^2*(1/7*sin(f*x+e)^5/cos(f*x+e)^7+2/35*s
in(f*x+e)^5/cos(f*x+e)^5))
```

Maxima [A] time = 0.97737, size = 109, normalized size = 1.36

$$\frac{15\left(a^2+2ab+b^2\right)\tan\left(fx+e\right)^7+21\left(3a^2+4ab+b^2\right)\tan\left(fx+e\right)^5+35\left(3a^2+2ab\right)\tan\left(fx+e\right)^3+105a^2\tan\left(fx\right.}{105f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^8*(a+b*sin(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/105*(15*(a^2 + 2*a*b + b^2)*tan(f*x + e)^7 + 21*(3*a^2 + 4*a*b + b^2)*tan
(f*x + e)^5 + 35*(3*a^2 + 2*a*b)*tan(f*x + e)^3 + 105*a^2*tan(f*x + e))/f
```

Fricas [A] time = 1.95275, size = 261, normalized size = 3.26

$$\frac{\left(2\left(24a^2-8ab+3b^2\right)\cos\left(fx+e\right)^6+\left(24a^2-8ab+3b^2\right)\cos\left(fx+e\right)^4+6\left(3a^2-ab-4b^2\right)\cos\left(fx+e\right)^2+15a^2+3\right.}{105f\cos\left(fx+e\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^8*(a+b*sin(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/105*(2*(24*a^2 - 8*a*b + 3*b^2)*cos(f*x + e)^6 + (24*a^2 - 8*a*b + 3*b^2)
*cos(f*x + e)^4 + 6*(3*a^2 - a*b - 4*b^2)*cos(f*x + e)^2 + 15*a^2 + 30*a*b
+ 15*b^2)*sin(f*x + e)/(f*cos(f*x + e)^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**8*(a+b*sin(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15234, size = 171, normalized size = 2.14

$$\frac{15\,a^2\tan\left(fx+e\right)^7+30\,ab\tan\left(fx+e\right)^7+15\,b^2\tan\left(fx+e\right)^7+63\,a^2\tan\left(fx+e\right)^5+84\,ab\tan\left(fx+e\right)^5+21\,b^2\,\mathrm{ta}}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^8*(a+b*sin(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/105*(15*a^2*tan(f*x + e)^7 + 30*a*b*tan(f*x + e)^7 + 15*b^2*tan(f*x + e)^
7 + 63*a^2*tan(f*x + e)^5 + 84*a*b*tan(f*x + e)^5 + 21*b^2*tan(f*x + e)^5 +
 105*a^2*tan(f*x + e)^3 + 70*a*b*tan(f*x + e)^3 + 105*a^2*tan(f*x + e))/f
```

3.300 $\int \sec^{10}(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=106

$$\frac{\left(6a^2+6ab+b^2\right)\tan^5(e+fx)}{5f}+\frac{a^2\tan(e+fx)}{f}+\frac{(a+b)^2\tan^9(e+fx)}{9f}+\frac{2(a+b)(2a+b)\tan^7(e+fx)}{7f}+\frac{2a(2a+b)\,\mathrm{ta}}{3}$$

```
[Out] (a^2*Tan[e + f*x])/f + (2*a*(2*a + b)*Tan[e + f*x]^3)/(3*f) + ((6*a^2 + 6*a
*b + b^2)*Tan[e + f*x]^5)/(5*f) + (2*(a + b)*(2*a + b)*Tan[e + f*x]^7)/(7*f
) + ((a + b)^2*Tan[e + f*x]^9)/(9*f)
```

Rubi [A] time = 0.0927193, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3191, 373}

$$\frac{\left(6a^2+6ab+b^2\right)\tan^5(e+fx)}{5f}+\frac{a^2\tan(e+fx)}{f}+\frac{(a+b)^2\tan^9(e+fx)}{9f}+\frac{2(a+b)(2a+b)\tan^7(e+fx)}{7f}+\frac{2a(2a+b)\,\mathrm{ta}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^10*(a + b*Sin[e + f*x]^2)^2,x]
```

```
[Out] (a^2*Tan[e + f*x])/f + (2*a*(2*a + b)*Tan[e + f*x]^3)/(3*f) + ((6*a^2 + 6*a
*b + b^2)*Tan[e + f*x]^5)/(5*f) + (2*(a + b)*(2*a + b)*Tan[e + f*x]^7)/(7*f
) + ((a + b)^2*Tan[e + f*x]^9)/(9*f)
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^{10}(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \left(1+x^2\right)^2\left(a+(a+b)x^2\right)^2\,dx,x,\tan(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left(a^2+2a(2a+b)x^2+\left(6a^2+6ab+b^2\right)x^4+2(a+b)(2a+b)x^6+(a\right.\right.}{f}\\ &= \frac{a^2\tan(e+fx)}{f}+\frac{2a(2a+b)\tan^3(e+fx)}{3f}+\frac{\left(6a^2+6ab+b^2\right)\tan^5(e+fx)}{5f}+\end{aligned}$$

Mathematica [A] time = 0.505523, size = 107, normalized size = 1.01

$$\frac{\sec^9(e+fx)\left(252\left(8a^2+8ab+3b^2\right)\sin(e+fx)+336\left(4a^2-ab-b^2\right)\sin(3(e+fx))+\left(16a^2-4ab+b^2\right)(36\sin(5(e+f\right.}{10080f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^10*(a + b*Sin[e + f*x]^2)^2,x]
```

```
[Out] (Sec[e + f*x]^9*(252*(8*a^2 + 8*a*b + 3*b^2)*Sin[e + f*x] + 336*(4*a^2 - a*
b - b^2)*Sin[3*(e + f*x)] + (16*a^2 - 4*a*b + b^2)*(36*Sin[5*(e + f*x)] + 9
*Sin[7*(e + f*x)] + Sin[9*(e + f*x)])))/(10080*f)
```

Maple [A] time = 0.075, size = 195, normalized size = 1.8

$$\frac{1}{f}\left(-a^2\left(-\frac{128}{315}-\frac{(\sec(fx+e))^8}{9}-\frac{8(\sec(fx+e))^6}{63}-\frac{16(\sec(fx+e))^4}{105}-\frac{64(\sec(fx+e))^2}{315}\right)\tan(fx+e)+2ab\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^10*(a+b*sin(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(-a^2*(-128/315-1/9*sec(f*x+e)^8-8/63*sec(f*x+e)^6-16/105*sec(f*x+e)^4-
64/315*sec(f*x+e)^2)*tan(f*x+e)+2*a*b*(1/9*sin(f*x+e)^3/cos(f*x+e)^9+2/21*s
in(f*x+e)^3/cos(f*x+e)^7+8/105*sin(f*x+e)^3/cos(f*x+e)^5+16/315*sin(f*x+e)^
3/cos(f*x+e)^3)+b^2*(1/9*sin(f*x+e)^5/cos(f*x+e)^9+4/63*sin(f*x+e)^5/cos(f*
x+e)^7+8/315*sin(f*x+e)^5/cos(f*x+e)^5))
```

Maxima [A] time = 0.988156, size = 139, normalized size = 1.31

$$\frac{35\left(a^2+2ab+b^2\right)\tan(fx+e)^9+90\left(2a^2+3ab+b^2\right)\tan(fx+e)^7+63\left(6a^2+6ab+b^2\right)\tan(fx+e)^5+210\left(2a\right.}{315f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^10*(a+b*sin(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/315*(35*(a^2 + 2*a*b + b^2)*tan(f*x + e)^9 + 90*(2*a^2 + 3*a*b + b^2)*tan
(f*x + e)^7 + 63*(6*a^2 + 6*a*b + b^2)*tan(f*x + e)^5 + 210*(2*a^2 + a*b)*t
an(f*x + e)^3 + 315*a^2*tan(f*x + e))/f
```

Fricas [A] time = 1.90386, size = 316, normalized size = 2.98

$$\frac{\left(8\left(16a^2-4ab+b^2\right)\cos(fx+e)^8+4\left(16a^2-4ab+b^2\right)\cos(fx+e)^6+3\left(16a^2-4ab+b^2\right)\cos(fx+e)^4+10\left(4a\right.\right.}{315f\cos(fx+e)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^10*(a+b*sin(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/315*(8*(16*a^2 - 4*a*b + b^2)*cos(f*x + e)^8 + 4*(16*a^2 - 4*a*b + b^2)*c
os(f*x + e)^6 + 3*(16*a^2 - 4*a*b + b^2)*cos(f*x + e)^4 + 10*(4*a^2 - a*b -
 5*b^2)*cos(f*x + e)^2 + 35*a^2 + 70*a*b + 35*b^2)*sin(f*x + e)/(f*cos(f*x
```

```
+ e)^9)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**10*(a+b*sin(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17024, size = 227, normalized size = 2.14

$$\frac{35\,a^2\tan\left(fx+e\right)^9+70\,ab\tan\left(fx+e\right)^9+35\,b^2\tan\left(fx+e\right)^9+180\,a^2\tan\left(fx+e\right)^7+270\,ab\tan\left(fx+e\right)^7+90\,b^2\tan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^10*(a+b*sin(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/315*(35*a^2*tan(f*x + e)^9 + 70*a*b*tan(f*x + e)^9 + 35*b^2*tan(f*x + e)^
9 + 180*a^2*tan(f*x + e)^7 + 270*a*b*tan(f*x + e)^7 + 90*b^2*tan(f*x + e)^7
 + 378*a^2*tan(f*x + e)^5 + 378*a*b*tan(f*x + e)^5 + 63*b^2*tan(f*x + e)^5
+ 420*a^2*tan(f*x + e)^3 + 210*a*b*tan(f*x + e)^3 + 315*a^2*tan(f*x + e))/f
```

3.301 $\int \frac{\cos^7(x)}{a+b\sin^2(x)}\,dx$

Optimal. Leaf size=78

$$-\frac{\left(a^2+3ab+3b^2\right)\sin(x)}{b^3}+\frac{(a+3b)\sin^3(x)}{3b^2}+\frac{(a+b)^3\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}}-\frac{\sin^5(x)}{5b}$$

```
[Out] ((a + b)^3*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(Sqrt[a]*b^(7/2)) - ((a^2 + 3*
a*b + 3*b^2)*Sin[x])/b^3 + ((a + 3*b)*Sin[x]^3)/(3*b^2) - Sin[x]^5/(5*b)
```

Rubi [A] time = 0.0895978, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3190, 390, 205}

$$-\frac{\left(a^2+3ab+3b^2\right)\sin(x)}{b^3}+\frac{(a+3b)\sin^3(x)}{3b^2}+\frac{(a+b)^3\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}}-\frac{\sin^5(x)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^7/(a + b*Sin[x]^2),x]
```

```
[Out] ((a + b)^3*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(Sqrt[a]*b^(7/2)) - ((a^2 + 3*
a*b + 3*b^2)*Sin[x])/b^3 + ((a + 3*b)*Sin[x]^3)/(3*b^2) - Sin[x]^5/(5*b)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^7(x)}{a+b\sin^2(x)}\,dx = \text{Subst}\left(\int \frac{(1-x^2)^3}{a+bx^2}\,dx,x,\sin(x)\right)$$
$$= \text{Subst}\left(\int \left(-\frac{a^2+3ab+3b^2}{b^3}+\frac{(a+3b)x^2}{b^2}-\frac{x^4}{b}+\frac{a^3+3a^2b+3ab^2+b^3}{b^3(a+bx^2)}\right)dx,x,\sin(x)\right)$$
$$= -\frac{(a^2+3ab+3b^2)\sin(x)}{b^3}+\frac{(a+3b)\sin^3(x)}{3b^2}-\frac{\sin^5(x)}{5b}+\frac{(a+b)^3\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sin(x)\right)}{b^3}$$
$$= \frac{(a+b)^3\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}}-\frac{(a^2+3ab+3b^2)\sin(x)}{b^3}+\frac{(a+3b)\sin^3(x)}{3b^2}-\frac{\sin^5(x)}{5b}$$

Mathematica [A] time = 0.275232, size = 109, normalized size = 1.4

$$\frac{-2\sqrt{a}\sqrt{b}\sin(x)\left(120a^2+4b(5a+12b)\cos(2x)+340ab+3b^2\cos(4x)+309b^2\right)+120(a+b)^3\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)-120(a+}{240\sqrt{a}b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^7/(a + b*Sin[x]^2),x]
```

```
[Out] (-120*(a + b)^3*ArcTan[(Sqrt[a]*Csc[x])/Sqrt[b]] + 120*(a + b)^3*ArcTan[(Sq
rt[b]*Sin[x])/Sqrt[a]] - 2*Sqrt[a]*Sqrt[b]*(120*a^2 + 340*a*b + 309*b^2 + 4
*b*(5*a + 12*b)*Cos[2*x] + 3*b^2*Cos[4*x])*Sin[x])/(240*Sqrt[a]*b^(7/2))
```

Maple [B] time = 0.043, size = 136, normalized size = 1.7

$$-\frac{(\sin(x))^5}{5b}+\frac{a(\sin(x))^3}{3b^2}+\frac{(\sin(x))^3}{b}-\frac{a^2\sin(x)}{b^3}-3\frac{a\sin(x)}{b^2}-3\frac{\sin(x)}{b}+\frac{a^3}{b^3}\arctan\left(\sin(x)b\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+3\frac{a^2}{b^2\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^7/(a+b*sin(x)^2),x)
```

```
[Out] -1/5*sin(x)^5/b+1/3/b^2*sin(x)^3*a+sin(x)^3/b-1/b^3*a^2*sin(x)-3/b^2*sin(x)
*a-3*sin(x)/b+1/b^3/(a*b)^(1/2)*arctan(sin(x)*b/(a*b)^(1/2))*a^3+3/b^2/(a*b
)^(1/2)*arctan(sin(x)*b/(a*b)^(1/2))*a^2+3/b/(a*b)^(1/2)*arctan(sin(x)*b/(a
*b)^(1/2))*a+1/(a*b)^(1/2)*arctan(sin(x)*b/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^7/(a+b*sin(x)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.4446, size = 566, normalized size = 7.26

$$\left[-\frac{15\left(a^3+3a^2b+3ab^2+b^3\right)\sqrt{-ab}\log\left(-\frac{b\cos(x)^2+2\sqrt{-ab}\sin(x)+a-b}{b\cos(x)^2-a-b}\right)+2\left(3ab^3\cos(x)^4+15a^3b+40a^2b^2+33ab^3+(5\right.}{30ab^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^7/(a+b*sin(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/30*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(-a*b)*log(-(b*cos(x)^2 + 2*
sqrt(-a*b)*sin(x) + a - b)/(b*cos(x)^2 - a - b)) + 2*(3*a*b^3*cos(x)^4 + 15
*a^3*b + 40*a^2*b^2 + 33*a*b^3 + (5*a^2*b^2 + 9*a*b^3)*cos(x)^2)*sin(x))/(a
*b^4), 1/15*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b)*arctan(sqrt(a*b)*
sin(x)/a) - (3*a*b^3*cos(x)^4 + 15*a^3*b + 40*a^2*b^2 + 33*a*b^3 + (5*a^2*b
^2 + 9*a*b^3)*cos(x)^2)*sin(x))/(a*b^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**7/(a+b*sin(x)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11695, size = 132, normalized size = 1.69

$$\frac{\left(a^3+3a^2b+3ab^2+b^3\right)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{\sqrt{ab}b^3}-\frac{3b^4\sin(x)^5-5ab^3\sin(x)^3-15b^4\sin(x)^3+15a^2b^2\sin(x)+45ab^3\sin}{15b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^7/(a+b*sin(x)^2),x, algorithm="giac")
```

```
[Out] (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*arctan(b*sin(x)/sqrt(a*b))/(sqrt(a*b)*b^3)
- 1/15*(3*b^4*sin(x)^5 - 5*a*b^3*sin(x)^3 - 15*b^4*sin(x)^3 + 15*a^2*b^2*si
n(x) + 45*a*b^3*sin(x) + 45*b^4*sin(x))/b^5
```

3.302 $\int \frac{\cos^6(x)}{a+b\sin^2(x)}\,dx$

Optimal. Leaf size=87

$$-\frac{x\left(8a^2+20ab+15b^2\right)}{8b^3}+\frac{(a+b)^{5/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}b^3}-\frac{(4a+7b)\sin(x)\cos(x)}{8b^2}-\frac{\sin(x)\cos^3(x)}{4b}$$

```
[Out] -((8*a^2 + 20*a*b + 15*b^2)*x)/(8*b^3) + ((a + b)^(5/2)*ArcTan[(Sqrt[a + b]
*Tan[x])/Sqrt[a]])/(Sqrt[a]*b^3) - ((4*a + 7*b)*Cos[x]*Sin[x])/(8*b^2) - (C
os[x]^3*Sin[x])/(4*b)
```

Rubi [A] time = 0.1869, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {3191, 414, 527, 522, 203, 205}

$$-\frac{x\left(8a^2+20ab+15b^2\right)}{8b^3}+\frac{(a+b)^{5/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}b^3}-\frac{(4a+7b)\sin(x)\cos(x)}{8b^2}-\frac{\sin(x)\cos^3(x)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^6/(a + b*Sin[x]^2),x]
```

```
[Out] -((8*a^2 + 20*a*b + 15*b^2)*x)/(8*b^3) + ((a + b)^(5/2)*ArcTan[(Sqrt[a + b]
*Tan[x])/Sqrt[a]])/(Sqrt[a]*b^3) - ((4*a + 7*b)*Cos[x]*Sin[x])/(8*b^2) - (C
os[x]^3*Sin[x])/(4*b)
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^6(x)}{a+b\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^3\left(a+(a+b)x^2\right)}\,dx, x, \tan(x)\right)\\
&= -\frac{\cos^3(x)\sin(x)}{4b}+\frac{\text{Subst}\left(\int \frac{a+4b-3(a+b)x^2}{\left(1+x^2\right)^2\left(a+(a+b)x^2\right)}\,dx, x, \tan(x)\right)}{4b}\\
&= -\frac{(4a+7b)\cos(x)\sin(x)}{8b^2}-\frac{\cos^3(x)\sin(x)}{4b}+\frac{\text{Subst}\left(\int \frac{4a^2+9ab+8b^2-(a+b)(4a+7b)x^2}{\left(1+x^2\right)\left(a+(a+b)x^2\right)}\,dx, x, \tan(x)\right)}{8b^2}\\
&= -\frac{(4a+7b)\cos(x)\sin(x)}{8b^2}-\frac{\cos^3(x)\sin(x)}{4b}+\frac{(a+b)^3\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \tan(x)\right)}{b^3}-\frac{\left(8a^2+\right.}{}\\
&= -\frac{\left(8a^2+20ab+15b^2\right)x}{8b^3}+\frac{(a+b)^{5/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}b^3}-\frac{(4a+7b)\cos(x)\sin(x)}{8b^2}-\frac{\cos^3(x)\sin}{4b}\end{aligned}$$

Mathematica [A] time = 0.188778, size = 79, normalized size = 0.91

$$\frac{(a+b)^{5/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}b^3}-\frac{4x\left(8a^2+20ab+15b^2\right)+8b(a+2b)\sin(2x)+b^2\sin(4x)}{32b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^6/(a + b*Sin[x]^2),x]
```

```
[Out] ((a + b)^(5/2)*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(Sqrt[a]*b^3) - (4*(8*
a^2 + 20*a*b + 15*b^2)*x + 8*b*(a + 2*b)*Sin[2*x] + b^2*Sin[4*x])/(32*b^3)
```

Maple [B] time = 0.085, size = 202, normalized size = 2.3

$$-\frac{(\tan(x))^3 a}{2b^2\left((\tan(x))^2+1\right)^2}-\frac{7(\tan(x))^3}{8b\left((\tan(x))^2+1\right)^2}-\frac{\tan(x)\,a}{2b^2\left((\tan(x))^2+1\right)^2}-\frac{9\tan(x)}{8b\left((\tan(x))^2+1\right)^2}-\frac{\arctan(\tan(x))\,a^2}{b^3}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^6/(a+b*sin(x)^2),x)
```

```
[Out] -1/2/b^2/(tan(x)^2+1)^2*tan(x)^3*a-7/8/b/(tan(x)^2+1)^2*tan(x)^3-1/2/b^2/(t
an(x)^2+1)^2*tan(x)*a-9/8/b/(tan(x)^2+1)^2*tan(x)-1/b^3*arctan(tan(x))*a^2-
5/2/b^2*arctan(tan(x))*a-15/8/b*arctan(tan(x))+1/b^3/(a*(a+b))^(1/2)*arctan
((a+b)*tan(x)/(a*(a+b))^(1/2))*a^3+3/b^2/(a*(a+b))^(1/2)*arctan((a+b)*tan(x
)/(a*(a+b))^(1/2))*a^2+3/b/(a*(a+b))^(1/2)*arctan((a+b)*tan(x)/(a*(a+b))^(1
/2))*a+1/(a*(a+b))^(1/2)*arctan((a+b)*tan(x)/(a*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6/(a+b*sin(x)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.51328, size = 775, normalized size = 8.91

$$\left[\frac{2\left(a^2+2ab+b^2\right)\sqrt{-\frac{a+b}{a}}\log\left(\frac{\left(8a^2+8ab+b^2\right)\cos(x)^4-2\left(4a^2+5ab+b^2\right)\cos(x)^2-4\left(\left(2a^2+ab\right)\cos(x)^3-\left(a^2+ab\right)\cos(x)\right)\sqrt{-\frac{a+b}{a}}\sin(x)+a^2+2ab+b^2}{b^2\cos(x)^4-2\left(ab+b^2\right)\cos(x)^2+a^2+2ab+b^2}\right)-}{8b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6/(a+b*sin(x)^2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*(a^2 + 2*a*b + b^2)*sqrt(-(a + b)/a)*log(((8*a^2 + 8*a*b + b^2)*cos
(x)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(x)^2 - 4*((2*a^2 + a*b)*cos(x)^3 - (a^2
 + a*b)*cos(x))*sqrt(-(a + b)/a)*sin(x) + a^2 + 2*a*b + b^2)/(b^2*cos(x)^4
- 2*(a*b + b^2)*cos(x)^2 + a^2 + 2*a*b + b^2)) - (8*a^2 + 20*a*b + 15*b^2)*
x - (2*b^2*cos(x)^3 + (4*a*b + 7*b^2)*cos(x))*sin(x))/b^3, -1/8*(4*(a^2 + 2
*a*b + b^2)*sqrt((a + b)/a)*arctan(1/2*((2*a + b)*cos(x)^2 - a - b)*sqrt((a
 + b)/a)/((a + b)*cos(x)*sin(x))) + (8*a^2 + 20*a*b + 15*b^2)*x + (2*b^2*co
s(x)^3 + (4*a*b + 7*b^2)*cos(x))*sin(x))/b^3]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**6/(a+b*sin(x)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11581, size = 177, normalized size = 2.03

$$-\frac{\left(8\,a^{2}+20\,ab+15\,b^{2}\right)x}{8\,b^{3}}+\frac{\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}\right)\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(2\,a+2\,b\right)+\arctan\left(\frac{a\tan\left(x\right)+b\tan\left(x\right)}{\sqrt{a^{2}+ab}}\right)\right)}{\sqrt{a^{2}+ab}b^{3}}-\frac{4\,a\tan\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6/(a+b*sin(x)^2),x, algorithm="giac")
```

```
[Out] -1/8*(8*a^2 + 20*a*b + 15*b^2)*x/b^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(pi*
floor(x/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(x) + b*tan(x))/sqrt(a^2 +
a*b)))/(sqrt(a^2 + a*b)*b^3) - 1/8*(4*a*tan(x)^3 + 7*b*tan(x)^3 + 4*a*tan(x
) + 9*b*tan(x))/((tan(x)^2 + 1)^2*b^2)
```

3.303 $\int \frac{\cos^5(x)}{a+b\sin^2(x)}\,dx$

Optimal. Leaf size=54

$$-\frac{(a+2b)\sin(x)}{b^2}+\frac{(a+b)^2\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}}+\frac{\sin^3(x)}{3b}$$

```
[Out] ((a + b)^2*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(Sqrt[a]*b^(5/2)) - ((a + 2*b)
*Sin[x])/b^2 + Sin[x]^3/(3*b)
```

Rubi [A] time = 0.0732421, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3190, 390, 205}

$$-\frac{(a+2b)\sin(x)}{b^2}+\frac{(a+b)^2\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}}+\frac{\sin^3(x)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^5/(a + b*Sin[x]^2),x]
```

```
[Out] ((a + b)^2*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(Sqrt[a]*b^(5/2)) - ((a + 2*b)
*Sin[x])/b^2 + Sin[x]^3/(3*b)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^5(x)}{a+b\sin^2(x)}\,dx = \text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{a+bx^2}\,dx, x, \sin(x)\right)$$
$$= \text{Subst}\left(\int \left(-\frac{a+2b}{b^2}+\frac{x^2}{b}+\frac{a^2+2ab+b^2}{b^2\left(a+bx^2\right)}\right)dx, x, \sin(x)\right)$$
$$= -\frac{(a+2b)\sin(x)}{b^2}+\frac{\sin^3(x)}{3b}+\frac{(a+b)^2\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sin(x)\right)}{b^2}$$
$$= \frac{(a+b)^2\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}}-\frac{(a+2b)\sin(x)}{b^2}+\frac{\sin^3(x)}{3b}$$

Mathematica [A] time = 0.173709, size = 84, normalized size = 1.56

$$\frac{6(a+b)^2\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)-2\sqrt{a}\sqrt{b}\sin(x)(6a+b\cos(2x)+11b)-6(a+b)^2\tan^{-1}\left(\frac{\sqrt{a}\csc(x)}{\sqrt{b}}\right)}{12\sqrt{a}b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^5/(a + b*Sin[x]^2),x]
```

```
[Out] (-6*(a + b)^2*ArcTan[(Sqrt[a]*Csc[x])/Sqrt[b]] + 6*(a + b)^2*ArcTan[(Sqrt[b
]*Sin[x])/Sqrt[a]] - 2*Sqrt[a]*Sqrt[b]*(6*a + 11*b + b*Cos[2*x])*Sin[x])/(1
2*Sqrt[a]*b^(5/2))
```

Maple [A] time = 0.043, size = 85, normalized size = 1.6

$$\frac{(\sin(x))^3}{3b}-\frac{a\sin(x)}{b^2}-2\frac{\sin(x)}{b}+\frac{a^2}{b^2}\arctan\left(\sin(x)\,b\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+2\frac{a}{\sqrt{ab}b}\arctan\left(\frac{\sin(x)\,b}{\sqrt{ab}}\right)+\arctan\left(\sin(x)\,b\frac{1}{\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^5/(a+b*sin(x)^2),x)
```

```
[Out] 1/3*sin(x)^3/b-1/b^2*sin(x)*a-2*sin(x)/b+1/b^2/(a*b)^(1/2)*arctan(sin(x)*b/
(a*b)^(1/2))*a^2+2/b/(a*b)^(1/2)*arctan(sin(x)*b/(a*b)^(1/2))*a+1/(a*b)^(1/
2)*arctan(sin(x)*b/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5/(a+b*sin(x)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.30336, size = 393, normalized size = 7.28

$$\left[-\frac{3\left(a^2+2\,ab+b^2\right)\sqrt{-ab}\log\left(-\frac{b\cos(x)^2+2\sqrt{-ab}\sin(x)+a-b}{b\cos(x)^2-a-b}\right)+2\left(ab^2\cos(x)^2+3\,a^2b+5\,ab^2\right)\sin(x)}{6\,ab^3}, \frac{3\left(a^2+2\,ab+b^2\right)\sqrt{ab}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5/(a+b*sin(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(a^2 + 2*a*b + b^2)*sqrt(-a*b)*log(-(b*cos(x)^2 + 2*sqrt(-a*b)*sin
(x) + a - b)/(b*cos(x)^2 - a - b)) + 2*(a*b^2*cos(x)^2 + 3*a^2*b + 5*a*b^2)
*sin(x))/(a*b^3), 1/3*(3*(a^2 + 2*a*b + b^2)*sqrt(a*b)*arctan(sqrt(a*b)*sin
(x)/a) - (a*b^2*cos(x)^2 + 3*a^2*b + 5*a*b^2)*sin(x))/(a*b^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**5/(a+b*sin(x)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13079, size = 78, normalized size = 1.44

$$\frac{\left(a^2+2\,ab+b^2\right)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{\sqrt{ab}b^2}+\frac{b^2\sin(x)^3-3\,ab\sin(x)-6\,b^2\sin(x)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5/(a+b*sin(x)^2),x, algorithm="giac")
```

```
[Out] (a^2 + 2*a*b + b^2)*arctan(b*sin(x)/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3*(b^2*s
in(x)^3 - 3*a*b*sin(x) - 6*b^2*sin(x))/b^3
```

3.304 $\int \frac{\cos^4(x)}{a+b\sin^2(x)}\,dx$

Optimal. Leaf size=59

$$-\frac{x(2a+3b)}{2b^2}+\frac{(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}b^2}-\frac{\sin(x)\cos(x)}{2b}$$

```
[Out] -((2*a + 3*b)*x)/(2*b^2) + ((a + b)^(3/2)*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[
a]])/(Sqrt[a]*b^2) - (Cos[x]*Sin[x])/(2*b)
```

Rubi [A] time = 0.108488, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3191, 414, 522, 203, 205}

$$-\frac{x(2a+3b)}{2b^2}+\frac{(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}b^2}-\frac{\sin(x)\cos(x)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^4/(a + b*Sin[x]^2),x]
```

```
[Out] -((2*a + 3*b)*x)/(2*b^2) + ((a + b)^(3/2)*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[
a]])/(Sqrt[a]*b^2) - (Cos[x]*Sin[x])/(2*b)
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^4(x)}{a+b\sin^2(x)}\,dx = \text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^2\left(a+(a+b)x^2\right)}\,dx, x, \tan(x)\right)$$
$$= -\frac{\cos(x)\sin(x)}{2b} + \frac{\text{Subst}\left(\int \frac{a+2b+(-a-b)x^2}{\left(1+x^2\right)\left(a+(a+b)x^2\right)}\,dx, x, \tan(x)\right)}{2b}$$
$$= -\frac{\cos(x)\sin(x)}{2b} + \frac{(a+b)^2\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \tan(x)\right)}{b^2} - \frac{(2a+3b)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right)}{2b^2}$$
$$= -\frac{(2a+3b)x}{2b^2} + \frac{(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}b^2} - \frac{\cos(x)\sin(x)}{2b}$$

Mathematica [A] time = 0.151682, size = 55, normalized size = 0.93

$$\frac{\frac{4(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}} - 2(2ax+3bx+b\sin(x)\cos(x))}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^4/(a + b*Sin[x]^2),x]
```

```
[Out] ((4*(a + b)^(3/2)*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/Sqrt[a] - 2*(2*a*x
+ 3*b*x + b*Cos[x]*Sin[x]))/(4*b^2)
```

Maple [B] time = 0.079, size = 111, normalized size = 1.9

$$-\frac{\tan(x)}{2b\left((\tan(x))^2+1\right)} - \frac{3\arctan(\tan(x))}{2b} - \frac{\arctan(\tan(x))\,a}{b^2} + \frac{a^2}{b^2}\arctan\left((a+b)\tan(x)\frac{1}{\sqrt{a(a+b)}}\right)\frac{1}{\sqrt{a(a+b)}} + 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4/(a+b*sin(x)^2),x)
```

```
[Out] -1/2/b*tan(x)/(tan(x)^2+1)-3/2/b*arctan(tan(x))-1/b^2*arctan(tan(x))*a+1/b^
2/(a*(a+b))^(1/2)*arctan((a+b)*tan(x)/(a*(a+b))^(1/2))*a^2+2/b/(a*(a+b))^(1
/2)*arctan((a+b)*tan(x)/(a*(a+b))^(1/2))*a+1/(a*(a+b))^(1/2)*arctan((a+b)*t
an(x)/(a*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4/(a+b*sin(x)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.05545, size = 617, normalized size = 10.46

$$\left[-\frac{2\,b\cos\left(x\right)\sin\left(x\right)-\left(a+b\right)\sqrt{-\frac{a+b}{a}}\log\left(\frac{\left(8\,a^2+8\,ab+b^2\right)\cos(x)^4-2\left(4\,a^2+5\,ab+b^2\right)\cos(x)^2-4\left(\left(2\,a^2+ab\right)\cos(x)^3-\left(a^2+ab\right)\cos(x)\right)\sqrt{-\frac{a+b}{a}}\sin(x}{b^2\cos(x)^4-2\left(ab+b^2\right)\cos(x)^2+a^2+2\,ab+b^2}\right.}{4\,b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4/(a+b*sin(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*b*cos(x)*sin(x) - (a + b)*sqrt(-(a + b)/a)*log(((8*a^2 + 8*a*b + b
^2)*cos(x)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(x)^2 - 4*((2*a^2 + a*b)*cos(x)^3
 - (a^2 + a*b)*cos(x))*sqrt(-(a + b)/a)*sin(x) + a^2 + 2*a*b + b^2)/(b^2*co
s(x)^4 - 2*(a*b + b^2)*cos(x)^2 + a^2 + 2*a*b + b^2)) + 2*(2*a + 3*b)*x)/b^
2, -1/2*(b*cos(x)*sin(x) + (a + b)*sqrt((a + b)/a)*arctan(1/2*((2*a + b)*co
s(x)^2 - a - b)*sqrt((a + b)/a)/((a + b)*cos(x)*sin(x))) + (2*a + 3*b)*x)/b
^2]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**4/(a+b*sin(x)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12514, size = 124, normalized size = 2.1

$$-\frac{(2\,a+3\,b)x}{2\,b^2}+\frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}\left(2\,a+2\,b\right)+\arctan\left(\frac{a\tan(x)+b\tan(x)}{\sqrt{a^2+ab}}\right)\right)\left(a^2+2\,ab+b^2\right)}{\sqrt{a^2+ab}b^2}-\frac{\tan\left(x\right)}{2\left(\tan\left(x\right)^2+1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4/(a+b*sin(x)^2),x, algorithm="giac")
```

```
[Out] -1/2*(2*a + 3*b)*x/b^2 + (pi*floor(x/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*t
an(x) + b*tan(x))/sqrt(a^2 + a*b)))*(a^2 + 2*a*b + b^2)/(sqrt(a^2 + a*b)*b^
2) - 1/2*tan(x)/((tan(x)^2 + 1)*b)
```

3.305 $\int \frac{\cos^3(x)}{a+b\sin^2(x)}\,dx$

Optimal. Leaf size=36

$$\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}-\frac{\sin(x)}{b}$$

```
[Out] ((a + b)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(Sqrt[a]*b^(3/2)) - Sin[x]/b
```

Rubi [A] time = 0.0563766, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3190, 388, 205}

$$\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}-\frac{\sin(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^3/(a + b*Sin[x]^2),x]
```

```
[Out] ((a + b)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(Sqrt[a]*b^(3/2)) - Sin[x]/b
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(x)}{a+b\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{1-x^2}{a+bx^2}\,dx,x,\sin(x)\right)\\ &= -\frac{\sin(x)}{b}+\frac{(a+b)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sin(x)\right)}{b}\\ &= \frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}-\frac{\sin(x)}{b}\end{aligned}$$

Mathematica [A] time = 0.023088, size = 36, normalized size = 1.

$$\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}-\frac{\sin(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^3/(a + b*Sin[x]^2),x]
```

```
[Out] ((a + b)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(Sqrt[a]*b^(3/2)) - Sin[x]/b
```

Maple [A] time = 0.045, size = 45, normalized size = 1.3

$$-\frac{\sin(x)}{b}+\frac{a}{b}\arctan\left(\sin(x)\,b\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\arctan\left(\sin(x)\,b\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3/(a+b*sin(x)^2),x)
```

```
[Out] -sin(x)/b+1/b/(a*b)^(1/2)*arctan(sin(x)*b/(a*b)^(1/2))*a+1/(a*b)^(1/2)*arct
an(sin(x)*b/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(a+b*sin(x)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.00546, size = 262, normalized size = 7.28

$$\left[-\frac{2\,ab\sin(x)+\sqrt{-ab}(a+b)\log\left(-\frac{b\cos(x)^2+2\,\sqrt{-ab}\sin(x)+a-b}{b\cos(x)^2-a-b}\right)}{2\,ab^2},-\frac{ab\sin(x)-\sqrt{ab}(a+b)\arctan\left(\frac{\sqrt{ab}\sin(x)}{a}\right)}{ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(a+b*sin(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*b*sin(x) + sqrt(-a*b)*(a + b)*log(-(b*cos(x)^2 + 2*sqrt(-a*b)*si
n(x) + a - b)/(b*cos(x)^2 - a - b)))/(a*b^2), -(a*b*sin(x) - sqrt(a*b)*(a +
 b)*arctan(sqrt(a*b)*sin(x)/a))/(a*b^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3/(a+b*sin(x)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11156, size = 41, normalized size = 1.14

$$\frac{(a+b)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{\sqrt{ab}b}-\frac{\sin(x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(a+b*sin(x)^2),x, algorithm="giac")
```

```
[Out] (a + b)*arctan(b*sin(x)/sqrt(a*b))/(sqrt(a*b)*b) - sin(x)/b
```

3.306 $\int \frac{\cos^2(x)}{a+b\sin^2(x)}\,dx$

Optimal. Leaf size=39

$$\frac{\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}b}-\frac{x}{b}$$

```
[Out] -(x/b) + (Sqrt[a + b]*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(Sqrt[a]*b)
```

Rubi [A] time = 0.059619, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3191, 391, 203, 205}

$$\frac{\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}b}-\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2/(a + b*Sin[x]^2),x]
```

```
[Out] -(x/b) + (Sqrt[a + b]*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(Sqrt[a]*b)
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^2(x)}{a+b\sin^2(x)}\,dx = \text{Subst}\left(\int \frac{1}{(1+x^2)\left(a+(a+b)x^2\right)}\,dx, x, \tan(x)\right)$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right)}{b} + \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \tan(x)\right)}{b}$$
$$= -\frac{x}{b} + \frac{\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}b}$$

Mathematica [A] time = 0.056113, size = 39, normalized size = 1.

$$\frac{\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}b} - \frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2/(a + b*Sin[x]^2),x]
```

```
[Out] -(x/b) + (Sqrt[a + b]*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(Sqrt[a]*b)
```

Maple [A] time = 0.077, size = 58, normalized size = 1.5

$$-\frac{\arctan(\tan(x))}{b} + \frac{a}{b}\arctan\left((a+b)\tan(x)\frac{1}{\sqrt{a(a+b)}}\right)\frac{1}{\sqrt{a(a+b)}} + \arctan\left((a+b)\tan(x)\frac{1}{\sqrt{a(a+b)}}\right)\frac{1}{\sqrt{a(a+b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/(a+b*sin(x)^2),x)
```

```
[Out] -1/b*arctan(tan(x))+1/b/(a*(a+b))^(1/2)*arctan((a+b)*tan(x)/(a*(a+b))^(1/2)
)*a+1/(a*(a+b))^(1/2)*arctan((a+b)*tan(x)/(a*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a+b*sin(x)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.17185, size = 508, normalized size = 13.03

$$\left[\frac{\sqrt{-\frac{a+b}{a}}\log\left(\frac{\left(8a^2+8ab+b^2\right)\cos(x)^4-2\left(4a^2+5ab+b^2\right)\cos(x)^2-4\left(\left(2a^2+ab\right)\cos(x)^3-\left(a^2+ab\right)\cos(x)\right)\sqrt{-\frac{a+b}{a}}\sin(x)+a^2+2ab+b^2}{b^2\cos(x)^4-2\left(ab+b^2\right)\cos(x)^2+a^2+2ab+b^2}\right)-4x}{4b}, -\frac{\sqrt{\frac{a+b}{a}}\,\text{arct}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a+b*sin(x)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(-(a + b)/a)*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2 + 5*a
*b + b^2)*cos(x)^2 - 4*((2*a^2 + a*b)*cos(x)^3 - (a^2 + a*b)*cos(x))*sqrt(-
(a + b)/a)*sin(x) + a^2 + 2*a*b + b^2)/(b^2*cos(x)^4 - 2*(a*b + b^2)*cos(x)
^2 + a^2 + 2*a*b + b^2)) - 4*x)/b, -1/2*(sqrt((a + b)/a)*arctan(1/2*((2*a +
 b)*cos(x)^2 - a - b)*sqrt((a + b)/a)/((a + b)*cos(x)*sin(x))) + 2*x)/b]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2/(a+b*sin(x)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12046, size = 84, normalized size = 2.15

$$\frac{\left(\pi \left\lfloor \frac{x}{\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}\left(2\,a + 2\,b\right) + \arctan\left(\frac{a\tan\left(x\right) + b\tan\left(x\right)}{\sqrt{a^{2} + ab}}\right)\right)\left(a + b\right)}{\sqrt{a^{2} + ab}b} - \frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a+b*sin(x)^2),x, algorithm="giac")
```

```
[Out] (pi*floor(x/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(x) + b*tan(x))/sqrt(a^
2 + a*b)))*(a + b)/(sqrt(a^2 + a*b)*b) - x/b
```

3.307 $\int \frac{\cos(x)}{a+b\sin^2(x)}\,dx$

Optimal. Leaf size=25

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

```
[Out] ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Rubi [A] time = 0.0288314, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3190, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/(a + b*Sin[x]^2),x]
```

```
[Out] ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)}{a+b\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sin(x)\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.0074584, size = 25, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/(a + b*Sin[x]^2),x]
```

[Out] ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b])

Maple [A] time = 0.027, size = 17, normalized size = 0.7

$$\arctan\left(\sin(x)\,b\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)/(a+b*sin(x)^2),x)

[Out] 1/(a*b)^(1/2)*arctan(sin(x)*b/(a*b)^(1/2))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/(a+b*sin(x)^2),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 2.03186, size = 190, normalized size = 7.6

$$\left[-\frac{\sqrt{-ab}\log\left(-\frac{b\cos(x)^2+2\sqrt{-ab}\sin(x)+a-b}{b\cos(x)^2-a-b}\right)}{2\,ab},\ \frac{\sqrt{ab}\arctan\left(\frac{\sqrt{ab}\sin(x)}{a}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/(a+b*sin(x)^2),x, algorithm="fricas")

[Out] [-1/2*sqrt(-a*b)*log(-(b*cos(x)^2 + 2*sqrt(-a*b)*sin(x) + a - b)/(b*cos(x)^
2 - a - b))/(a*b), sqrt(a*b)*arctan(sqrt(a*b)*sin(x)/a)/(a*b)]

Sympy [A] time = 1.63744, size = 87, normalized size = 3.48

$$\begin{cases} \frac{\tilde{\infty}}{\sin(x)} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{\sin(x)}{a} & \text{for}\: b = 0 \\ -\frac{1}{b\sin(x)} & \text{for}\: a = 0 \\ -\frac{i\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sin(x)\right)}{2\sqrt{ab}\sqrt{\frac{1}{b}}} + \frac{i\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sin(x)\right)}{2\sqrt{ab}\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/(a+b*sin(x)**2),x)

```
[Out] Piecewise((zoo/sin(x), Eq(a, 0) & Eq(b, 0)), (sin(x)/a, Eq(b, 0)), (-1/(b*s
in(x)), Eq(a, 0)), (-I*log(-I*sqrt(a)*sqrt(1/b) + sin(x))/(2*sqrt(a)*b*sqrt
(1/b)) + I*log(I*sqrt(a)*sqrt(1/b) + sin(x))/(2*sqrt(a)*b*sqrt(1/b)), True)
)
```

Giac [A] time = 1.121, size = 22, normalized size = 0.88

$$\frac{\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a+b*sin(x)^2),x, algorithm="giac")
```

```
[Out] arctan(b*sin(x)/sqrt(a*b))/sqrt(a*b)
```

3.308 $\int \frac{\sec(x)}{a+b\sin^2(x)}\,dx$

Optimal. Leaf size=40

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}+\frac{\tanh^{-1}(\sin(x))}{a+b}$$

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(Sqrt[a]*(a + b)) + ArcTanh[Sin[
x]]/(a + b)
```

Rubi [A] time = 0.0509571, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3190, 391, 206, 205}

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}+\frac{\tanh^{-1}(\sin(x))}{a+b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]/(a + b*Sin[x]^2),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(Sqrt[a]*(a + b)) + ArcTanh[Sin[
x]]/(a + b)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec(x)}{a+b\sin^2(x)}\,dx = \text{Subst}\left(\int \frac{1}{(1-x^2)(a+bx^2)}\,dx, x, \sin(x)\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(x)\right)}{a+b} + \frac{b\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sin(x)\right)}{a+b}$$
$$= \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)} + \frac{\tanh^{-1}(\sin(x))}{a+b}$$

Mathematica [B] time = 0.124296, size = 96, normalized size = 2.4

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right) - \sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\csc(x)}{\sqrt{b}}\right) + 2\sqrt{a}\left(\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right) - \log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)\right)}{2\sqrt{a}(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]/(a + b*Sin[x]^2),x]
```

```
[Out] (-(Sqrt[b]*ArcTan[(Sqrt[a]*Csc[x])/Sqrt[b]]) + Sqrt[b]*ArcTan[(Sqrt[b]*Sin[
x])/Sqrt[a]] + 2*Sqrt[a]*(-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/
2]]))/(2*Sqrt[a]*(a + b))
```

Maple [A] time = 0.055, size = 55, normalized size = 1.4

$$-\frac{\ln(-1+\sin(x))}{2a+2b} + \frac{\ln(1+\sin(x))}{2a+2b} + \frac{b}{a+b}\arctan\left(\sin(x)\,b\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)/(a+b*sin(x)^2),x)
```

```
[Out] -1/(2*a+2*b)*ln(-1+sin(x))+1/(2*a+2*b)*ln(1+sin(x))+b/(a+b)/(a*b)^(1/2)*arc
tan(sin(x)*b/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a+b*sin(x)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.28405, size = 308, normalized size = 7.7

$$\left[\frac{\sqrt{-\frac{b}{a}}\log\left(-\frac{b\cos(x)^2-2a\sqrt{-\frac{b}{a}}\sin(x)+a-b}{b\cos(x)^2-a-b}\right) + \log(\sin(x)+1) - \log(-\sin(x)+1)}{2(a+b)}, \frac{2\sqrt{\frac{b}{a}}\arctan\left(\sqrt{\frac{b}{a}}\sin(x)\right) + \log(\sin(x)+\right.}{2(a+b)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a+b*sin(x)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-b/a)*log(-(b*cos(x)^2 - 2*a*sqrt(-b/a)*sin(x) + a - b)/(b*cos(x
)^2 - a - b)) + log(sin(x) + 1) - log(-sin(x) + 1))/(a + b), 1/2*(2*sqrt(b/
a)*arctan(sqrt(b/a)*sin(x)) + log(sin(x) + 1) - log(-sin(x) + 1))/(a + b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(x)}{a + b\sin^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a+b*sin(x)**2),x)
```

```
[Out] Integral(sec(x)/(a + b*sin(x)**2), x)
```

Giac [A] time = 1.09957, size = 66, normalized size = 1.65

$$\frac{b\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{\sqrt{ab}(a+b)} + \frac{\log(\sin(x)+1)}{2(a+b)} - \frac{\log(-\sin(x)+1)}{2(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a+b*sin(x)^2),x, algorithm="giac")
```

```
[Out] b*arctan(b*sin(x)/sqrt(a*b))/(sqrt(a*b)*(a + b)) + 1/2*log(sin(x) + 1)/(a +
 b) - 1/2*log(-sin(x) + 1)/(a + b)
```

3.309 $\int \frac{\sec^2(x)}{a+b\sin^2(x)}\,dx$

Optimal. Leaf size=39

$$\frac{b\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{3/2}}+\frac{\tan(x)}{a+b}$$

```
[Out] (b*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(3/2)) + Tan[x]/(
a + b)
```

Rubi [A] time = 0.0606788, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3191, 388, 205}

$$\frac{b\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{3/2}}+\frac{\tan(x)}{a+b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2/(a + b*Sin[x]^2),x]
```

```
[Out] (b*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(3/2)) + Tan[x]/(
a + b)
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(x)}{a+b\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{1+x^2}{a+(a+b)x^2}\,dx,x,\tan(x)\right)\\ &= \frac{\tan(x)}{a+b}+\frac{b\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx,x,\tan(x)\right)}{a+b}\\ &= \frac{b\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{3/2}}+\frac{\tan(x)}{a+b}\end{aligned}$$

Mathematica [A] time = 0.0804537, size = 39, normalized size = 1.

$$\frac{b\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{3/2}}+\frac{\tan(x)}{a+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/(a + b*Sin[x]^2),x]
```

```
[Out] (b*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(3/2)) + Tan[x]/(
a + b)
```

Maple [A] time = 0.08, size = 38, normalized size = 1.

$$\frac{\tan(x)}{a+b}+\frac{b}{a+b}\arctan\left((a+b)\tan(x)\frac{1}{\sqrt{a(a+b)}}\right)\frac{1}{\sqrt{a(a+b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(a+b*sin(x)^2),x)
```

```
[Out] tan(x)/(a+b)+b/(a+b)/(a*(a+b))^(1/2)*arctan((a+b)*tan(x)/(a*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a+b*sin(x)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.38485, size = 641, normalized size = 16.44

$$\left[-\frac{\sqrt{-a^2-ab}\,b\cos(x)\log\left(\frac{(8a^2+8ab+b^2)\cos(x)^4-2(4a^2+5ab+b^2)\cos(x)^2+4((2a+b)\cos(x)^3-(a+b)\cos(x))\sqrt{-a^2-ab}\sin(x)+a^2+2ab+b^2}{b^2\cos(x)^4-2(ab+b^2)\cos(x)^2+a^2+2ab+b^2}\right)-4(}{4(a^3+2a^2b+ab^2)\cos(x)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a+b*sin(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(-a^2 - a*b)*b*cos(x)*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4
*a^2 + 5*a*b + b^2)*cos(x)^2 + 4*((2*a + b)*cos(x)^3 - (a + b)*cos(x))*sqrt
(-a^2 - a*b)*sin(x) + a^2 + 2*a*b + b^2)/(b^2*cos(x)^4 - 2*(a*b + b^2)*cos(
x)^2 + a^2 + 2*a*b + b^2)) - 4*(a^2 + a*b)*sin(x))/((a^3 + 2*a^2*b + a*b^2)
*cos(x)), -1/2*(sqrt(a^2 + a*b)*b*arctan(1/2*((2*a + b)*cos(x)^2 - a - b)/(
sqrt(a^2 + a*b)*cos(x)*sin(x)))*cos(x) - 2*(a^2 + a*b)*sin(x))/((a^3 + 2*a^
```

```
2*b + a*b^2)*cos(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(x)}{a + b\sin^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(a+b*sin(x)**2),x)
```

```
[Out] Integral(sec(x)**2/(a + b*sin(x)**2), x)
```

Giac [A] time = 1.12551, size = 61, normalized size = 1.56

$$\frac{b\arctan\left(\frac{a\tan(x)+b\tan(x)}{\sqrt{a^2+ab}}\right)}{\sqrt{a^2+ab}(a+b)} + \frac{\tan(x)}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a+b*sin(x)^2),x, algorithm="giac")
```

```
[Out] b*arctan((a*tan(x) + b*tan(x))/sqrt(a^2 + a*b))/(sqrt(a^2 + a*b)*(a + b)) +
 tan(x)/(a + b)
```

3.310 $\int \frac{\sec^3(x)}{a+b\sin^2(x)}\,dx$

Optimal. Leaf size=61

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^2}+\frac{(a+3b)\tanh^{-1}(\sin(x))}{2(a+b)^2}+\frac{\tan(x)\sec(x)}{2(a+b)}$$

```
[Out] (b^(3/2)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(Sqrt[a]*(a + b)^2) + ((a + 3*b)
*ArcTanh[Sin[x]])/(2*(a + b)^2) + (Sec[x]*Tan[x])/(2*(a + b))
```

Rubi [A] time = 0.0867202, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3190, 414, 522, 206, 205}

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^2}+\frac{(a+3b)\tanh^{-1}(\sin(x))}{2(a+b)^2}+\frac{\tan(x)\sec(x)}{2(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^3/(a + b*Sin[x]^2),x]
```

```
[Out] (b^(3/2)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(Sqrt[a]*(a + b)^2) + ((a + 3*b)
*ArcTanh[Sin[x]])/(2*(a + b)^2) + (Sec[x]*Tan[x])/(2*(a + b))
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec^3(x)}{a+b\sin^2(x)}\,dx = \text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2\left(a+bx^2\right)}\,dx, x, \sin(x)\right)$$

$$= \frac{\sec(x)\tan(x)}{2(a+b)} + \frac{\text{Subst}\left(\int \frac{a+2b+bx^2}{\left(1-x^2\right)\left(a+bx^2\right)}\,dx, x, \sin(x)\right)}{2(a+b)}$$

$$= \frac{\sec(x)\tan(x)}{2(a+b)} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sin(x)\right)}{(a+b)^2} + \frac{(a+3b)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(x)\right)}{2(a+b)^2}$$

$$= \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^2} + \frac{(a+3b)\tanh^{-1}(\sin(x))}{2(a+b)^2} + \frac{\sec(x)\tan(x)}{2(a+b)}$$

Mathematica [B] time = 0.315132, size = 147, normalized size = 2.41

$$\frac{\frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\csc(x)}{\sqrt{b}}\right)}{\sqrt{a}} + \frac{a+b}{\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)^2} - \frac{a+b}{\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^2} - 2(a+3b)\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right) + 2(a+3b)\,\mathrm{l}}{4(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^3/(a + b*Sin[x]^2),x]
```

```
[Out] ((-2*b^(3/2)*ArcTan[(Sqrt[a]*Csc[x])/Sqrt[b]])/Sqrt[a] + (2*b^(3/2)*ArcTan[
(Sqrt[b]*Sin[x])/Sqrt[a]])/Sqrt[a] - 2*(a + 3*b)*Log[Cos[x/2] - Sin[x/2]] +
 2*(a + 3*b)*Log[Cos[x/2] + Sin[x/2]] + (a + b)/(Cos[x/2] - Sin[x/2])^2 - (
a + b)/(Cos[x/2] + Sin[x/2])^2)/(4*(a + b)^2)
```

Maple [B] time = 0.062, size = 112, normalized size = 1.8

$$-\frac{1}{(4a+4b)(-1+\sin(x))} - \frac{\ln(-1+\sin(x))\,a}{4(a+b)^2} - \frac{3\ln(-1+\sin(x))\,b}{4(a+b)^2} - \frac{1}{(4a+4b)(1+\sin(x))} + \frac{\ln(1+\sin(x))\,a}{4(a+b)^2} + \frac{3\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^3/(a+b*sin(x)^2),x)
```

```
[Out] -1/(4*a+4*b)/(-1+sin(x))-1/4/(a+b)^2*ln(-1+sin(x))*a-3/4/(a+b)^2*ln(-1+sin(
x))*b-1/(4*a+4*b)/(1+sin(x))+1/4/(a+b)^2*ln(1+sin(x))*a+3/4/(a+b)^2*ln(1+si
n(x))*b+b^2/(a+b)^2/(a*b)^(1/2)*arctan(sin(x)*b/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3/(a+b*sin(x)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.80953, size = 556, normalized size = 9.11

$$\left[\frac{2\,b\sqrt{-\frac{b}{a}}\cos\left(x\right)^{2}\log\left(-\frac{b\cos(x)^{2}-2\,a\sqrt{-\frac{b}{a}}\sin(x)+a-b}{b\cos(x)^{2}-a-b}\right)+\left(a+3\,b\right)\cos\left(x\right)^{2}\log\left(\sin\left(x\right)+1\right)-\left(a+3\,b\right)\cos\left(x\right)^{2}\log\left(-\sin\left(x\right)\right.}{4\left(a^{2}+2\,ab+b^{2}\right)\cos\left(x\right)^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3/(a+b*sin(x)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*b*sqrt(-b/a)*cos(x)^2*log(-(b*cos(x)^2 - 2*a*sqrt(-b/a)*sin(x) + a
- b)/(b*cos(x)^2 - a - b)) + (a + 3*b)*cos(x)^2*log(sin(x) + 1) - (a + 3*b)
*cos(x)^2*log(-sin(x) + 1) + 2*(a + b)*sin(x))/((a^2 + 2*a*b + b^2)*cos(x)^
2), 1/4*(4*b*sqrt(b/a)*arctan(sqrt(b/a)*sin(x))*cos(x)^2 + (a + 3*b)*cos(x)
^2*log(sin(x) + 1) - (a + 3*b)*cos(x)^2*log(-sin(x) + 1) + 2*(a + b)*sin(x)
)/((a^2 + 2*a*b + b^2)*cos(x)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec^{3}\left(x\right)}{a+b\sin^{2}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**3/(a+b*sin(x)**2),x)
```

```
[Out] Integral(sec(x)**3/(a + b*sin(x)**2), x)
```

Giac [B] time = 1.14683, size = 138, normalized size = 2.26

$$\frac{b^{2}\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{\left(a^{2}+2\,ab+b^{2}\right)\sqrt{ab}}+\frac{\left(a+3\,b\right)\log\left(\sin\left(x\right)+1\right)}{4\left(a^{2}+2\,ab+b^{2}\right)}-\frac{\left(a+3\,b\right)\log\left(-\sin\left(x\right)+1\right)}{4\left(a^{2}+2\,ab+b^{2}\right)}-\frac{\sin\left(x\right)}{2\left(\sin\left(x\right)^{2}-1\right)\left(a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3/(a+b*sin(x)^2),x, algorithm="giac")
```

```
[Out] b^2*arctan(b*sin(x)/sqrt(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*b)) + 1/4*(a + 3
*b)*log(sin(x) + 1)/(a^2 + 2*a*b + b^2) - 1/4*(a + 3*b)*log(-sin(x) + 1)/(a
^2 + 2*a*b + b^2) - 1/2*sin(x)/((sin(x)^2 - 1)*(a + b))
```

3.311 $\int \frac{\sec^4(x)}{a+b\sin^2(x)}\,dx$

Optimal. Leaf size=59

$$\frac{b^2\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{5/2}}+\frac{\tan^3(x)}{3(a+b)}+\frac{(a+2b)\tan(x)}{(a+b)^2}$$

```
[Out] (b^2*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(5/2)) + ((a +
2*b)*Tan[x])/(a + b)^2 + Tan[x]^3/(3*(a + b))
```

Rubi [A] time = 0.0813427, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3191, 390, 205}

$$\frac{b^2\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{5/2}}+\frac{\tan^3(x)}{3(a+b)}+\frac{(a+2b)\tan(x)}{(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^4/(a + b*Sin[x]^2),x]
```

```
[Out] (b^2*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(5/2)) + ((a +
2*b)*Tan[x])/(a + b)^2 + Tan[x]^3/(3*(a + b))
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec^4(x)}{a+b\sin^2(x)}\,dx = \text{Subst}\left(\int \frac{(1+x^2)^2}{a+(a+b)x^2}\,dx, x, \tan(x)\right)$$
$$= \text{Subst}\left(\int \left(\frac{a+2b}{(a+b)^2}+\frac{x^2}{a+b}+\frac{b^2}{(a+b)^2(a+(a+b)x^2)}\right)dx, x, \tan(x)\right)$$
$$= \frac{(a+2b)\tan(x)}{(a+b)^2}+\frac{\tan^3(x)}{3(a+b)}+\frac{b^2\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \tan(x)\right)}{(a+b)^2}$$
$$= \frac{b^2\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{5/2}}+\frac{(a+2b)\tan(x)}{(a+b)^2}+\frac{\tan^3(x)}{3(a+b)}$$

Mathematica [A] time = 0.2104, size = 59, normalized size = 1.

$$\frac{b^2\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{5/2}}+\frac{\tan(x)\left((a+b)\sec^2(x)+2a+5b\right)}{3(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^4/(a + b*Sin[x]^2),x]
```

```
[Out] (b^2*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(5/2)) + ((2*a
+ 5*b + (a + b)*Sec[x]^2)*Tan[x])/(3*(a + b)^2)
```

Maple [A] time = 0.099, size = 75, normalized size = 1.3

$$\frac{(\tan(x))^3 a}{3(a+b)^2}+\frac{(\tan(x))^3 b}{3(a+b)^2}+\frac{\tan(x)a}{(a+b)^2}+2\frac{\tan(x)b}{(a+b)^2}+\frac{b^2}{(a+b)^2}\arctan\left((a+b)\tan(x)\frac{1}{\sqrt{a(a+b)}}\right)\frac{1}{\sqrt{a(a+b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^4/(a+b*sin(x)^2),x)
```

```
[Out] 1/3/(a+b)^2*tan(x)^3*a+1/3/(a+b)^2*tan(x)^3*b+1/(a+b)^2*tan(x)*a+2/(a+b)^2*
tan(x)*b+b^2/(a+b)^2/(a*(a+b))^(1/2)*arctan((a+b)*tan(x)/(a*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4/(a+b*sin(x)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.1583, size = 834, normalized size = 14.14

$$\left[-\frac{3\sqrt{-a^2-ab}b^2\cos(x)^3\log\left(\frac{(8a^2+8ab+b^2)\cos(x)^4-2(4a^2+5ab+b^2)\cos(x)^2+4((2a+b)\cos(x)^3-(a+b)\cos(x))\sqrt{-a^2-ab}\sin(x)+a^2+2ab+b^2}{b^2\cos(x)^4-2(ab+b^2)\cos(x)^2+a^2+2ab+b^2}\right)-4}{12(a^4+3a^3b+3a^2b^2+ab^3)\cos(x)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4/(a+b*sin(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*sqrt(-a^2 - a*b)*b^2*cos(x)^3*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4
 - 2*(4*a^2 + 5*a*b + b^2)*cos(x)^2 + 4*((2*a + b)*cos(x)^3 - (a + b)*cos(x
))*sqrt(-a^2 - a*b)*sin(x) + a^2 + 2*a*b + b^2)/(b^2*cos(x)^4 - 2*(a*b + b^
2)*cos(x)^2 + a^2 + 2*a*b + b^2)) - 4*(a^3 + 2*a^2*b + a*b^2 + (2*a^3 + 7*a
^2*b + 5*a*b^2)*cos(x)^2)*sin(x))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cos(
x)^3), -1/6*(3*sqrt(a^2 + a*b)*b^2*arctan(1/2*((2*a + b)*cos(x)^2 - a - b)/
(sqrt(a^2 + a*b)*cos(x)*sin(x)))*cos(x)^3 - 2*(a^3 + 2*a^2*b + a*b^2 + (2*a
^3 + 7*a^2*b + 5*a*b^2)*cos(x)^2)*sin(x))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b
^3)*cos(x)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**4/(a+b*sin(x)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.1147, size = 181, normalized size = 3.07

$$\frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a+2b)+\arctan\left(\frac{a\tan(x)+b\tan(x)}{\sqrt{a^2+ab}}\right)\right)b^2}{(a^2+2ab+b^2)\sqrt{a^2+ab}}+\frac{a^2\tan(x)^3+2ab\tan(x)^3+b^2\tan(x)^3+3a^2\tan(x)+9ab\,\mathrm{t}}{3(a^3+3a^2b+3ab^2+b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4/(a+b*sin(x)^2),x, algorithm="giac")
```

```
[Out] (pi*floor(x/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(x) + b*tan(x))/sqrt(a^
2 + a*b)))*b^2/((a^2 + 2*a*b + b^2)*sqrt(a^2 + a*b)) + 1/3*(a^2*tan(x)^3 +
2*a*b*tan(x)^3 + b^2*tan(x)^3 + 3*a^2*tan(x) + 9*a*b*tan(x) + 6*b^2*tan(x))
/(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
```

3.312 $\int \frac{\sec^5(x)}{a+b\sin^2(x)}\,dx$

Optimal. Leaf size=93

$$\frac{(3a^2+10ab+15b^2)\tanh^{-1}(\sin(x))}{8(a+b)^3}+\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^3}+\frac{\tan(x)\sec^3(x)}{4(a+b)}+\frac{(3a+7b)\tan(x)\sec(x)}{8(a+b)^2}$$

```
[Out] (b^(5/2)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(Sqrt[a]*(a + b)^3) + ((3*a^2 +
10*a*b + 15*b^2)*ArcTanh[Sin[x]])/(8*(a + b)^3) + ((3*a + 7*b)*Sec[x]*Tan[x
])/(8*(a + b)^2) + (Sec[x]^3*Tan[x])/(4*(a + b))
```

Rubi [A] time = 0.145, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {3190, 414, 527, 522, 206, 205}

$$\frac{(3a^2+10ab+15b^2)\tanh^{-1}(\sin(x))}{8(a+b)^3}+\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^3}+\frac{\tan(x)\sec^3(x)}{4(a+b)}+\frac{(3a+7b)\tan(x)\sec(x)}{8(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^5/(a + b*Sin[x]^2),x]
```

```
[Out] (b^(5/2)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(Sqrt[a]*(a + b)^3) + ((3*a^2 +
10*a*b + 15*b^2)*ArcTanh[Sin[x]])/(8*(a + b)^3) + ((3*a + 7*b)*Sec[x]*Tan[x
])/(8*(a + b)^2) + (Sec[x]^3*Tan[x])/(4*(a + b))
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec^5(x)}{a+b\sin^2(x)}\,dx = \text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^3\left(a+bx^2\right)}\,dx, x, \sin(x)\right)$$
$$= \frac{\sec^3(x)\tan(x)}{4(a+b)} + \frac{\text{Subst}\left(\int \frac{3a+4b+3bx^2}{\left(1-x^2\right)^2\left(a+bx^2\right)}\,dx, x, \sin(x)\right)}{4(a+b)}$$
$$= \frac{(3a+7b)\sec(x)\tan(x)}{8(a+b)^2} + \frac{\sec^3(x)\tan(x)}{4(a+b)} + \frac{\text{Subst}\left(\int \frac{3a^2+7ab+8b^2+b(3a+7b)x^2}{\left(1-x^2\right)\left(a+bx^2\right)}\,dx, x, \sin(x)\right)}{8(a+b)^2}$$
$$= \frac{(3a+7b)\sec(x)\tan(x)}{8(a+b)^2} + \frac{\sec^3(x)\tan(x)}{4(a+b)} + \frac{b^3\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sin(x)\right)}{(a+b)^3} + \frac{\left(3a^2+10ab+15b^2\right)}{8}$$
$$= \frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^3} + \frac{\left(3a^2+10ab+15b^2\right)\tanh^{-1}(\sin(x))}{8(a+b)^3} + \frac{(3a+7b)\sec(x)\tan(x)}{8(a+b)^2} + \frac{\sec^3(x)\tan(x)}{4(a+b)}$$

Mathematica [B] time = 1.2299, size = 214, normalized size = 2.3

$$-\frac{2\left(3a^2+10ab+15b^2\right)\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-2\left(3a^2+10ab+15b^2\right)\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)-\frac{8b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{8b^{5/}}{}}{16(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^5/(a + b*Sin[x]^2),x]
```

```
[Out] -((8*b^(5/2)*ArcTan[(Sqrt[a]*Csc[x])/Sqrt[b]])/Sqrt[a] - (8*b^(5/2)*ArcTan[
(Sqrt[b]*Sin[x])/Sqrt[a]])/Sqrt[a] + 2*(3*a^2 + 10*a*b + 15*b^2)*Log[Cos[x/
2] - Sin[x/2]] - 2*(3*a^2 + 10*a*b + 15*b^2)*Log[Cos[x/2] + Sin[x/2]] - (a
+ b)^2/(Cos[x/2] - Sin[x/2])^4 + (a + b)^2/(Cos[x/2] + Sin[x/2])^4 + ((a +
b)*(3*a + 7*b))/(Cos[x/2] + Sin[x/2])^2 + ((a + b)*(3*a + 7*b))/(-1 + Sin[x
]))/(16*(a + b)^3)
```

Maple [B] time = 0.068, size = 204, normalized size = 2.2

$$\frac{1}{(16a+16b)(-1+\sin(x))^2} - \frac{3a}{16(a+b)^2(-1+\sin(x))} - \frac{7b}{16(a+b)^2(-1+\sin(x))} - \frac{3\ln(-1+\sin(x))a^2}{16(a+b)^3} - \frac{5\ln(-1+}{8(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^5/(a+b*sin(x)^2),x)
```

```
[Out] 1/2/(8*a+8*b)/(-1+sin(x))^2-3/16/(a+b)^2/(-1+sin(x))*a-7/16/(a+b)^2/(-1+sin
(x))*b-3/16/(a+b)^3*ln(-1+sin(x))*a^2-5/8/(a+b)^3*ln(-1+sin(x))*a*b-15/16/(
a+b)^3*ln(-1+sin(x))*b^2-1/2/(8*a+8*b)/(1+sin(x))^2-3/16/(a+b)^2/(1+sin(x))
*a-7/16/(a+b)^2/(1+sin(x))*b+3/16/(a+b)^3*ln(1+sin(x))*a^2+5/8/(a+b)^3*ln(1
+sin(x))*a*b+15/16/(a+b)^3*ln(1+sin(x))*b^2+b^3/(a+b)^3/(a*b)^(1/2)*arctan(
sin(x)*b/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^5/(a+b*sin(x)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.96914, size = 825, normalized size = 8.87

$$\left[\frac{8\,b^2\sqrt{-\frac{b}{a}}\cos\left(x\right)^4\log\left(-\frac{b\cos(x)^2-2\,a\sqrt{-\frac{b}{a}}\sin(x)+a-b}{b\cos(x)^2-a-b}\right)+\left(3\,a^2+10\,ab+15\,b^2\right)\cos\left(x\right)^4\log\left(\sin\left(x\right)+1\right)-\left(3\,a^2+10\,ab+1\right.}{16\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\mathrm{c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^5/(a+b*sin(x)^2),x, algorithm="fricas")
```

```
[Out] [1/16*(8*b^2*sqrt(-b/a)*cos(x)^4*log(-(b*cos(x)^2 - 2*a*sqrt(-b/a)*sin(x) +
 a - b)/(b*cos(x)^2 - a - b)) + (3*a^2 + 10*a*b + 15*b^2)*cos(x)^4*log(sin(
x) + 1) - (3*a^2 + 10*a*b + 15*b^2)*cos(x)^4*log(-sin(x) + 1) + 2*((3*a^2 +
 10*a*b + 7*b^2)*cos(x)^2 + 2*a^2 + 4*a*b + 2*b^2)*sin(x))/((a^3 + 3*a^2*b
+ 3*a*b^2 + b^3)*cos(x)^4), 1/16*(16*b^2*sqrt(b/a)*arctan(sqrt(b/a)*sin(x))
*cos(x)^4 + (3*a^2 + 10*a*b + 15*b^2)*cos(x)^4*log(sin(x) + 1) - (3*a^2 + 1
0*a*b + 15*b^2)*cos(x)^4*log(-sin(x) + 1) + 2*((3*a^2 + 10*a*b + 7*b^2)*cos
(x)^2 + 2*a^2 + 4*a*b + 2*b^2)*sin(x))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos
(x)^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**5/(a+b*sin(x)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.10426, size = 239, normalized size = 2.57

$$\frac{b^3 \arctan\left(\frac{b \sin(x)}{\sqrt{ab}}\right)}{\left(a^3+3 a^2 b+3 a b^2+b^3\right)\sqrt{ab}}+\frac{\left(3 a^2+10 a b+15 b^2\right)\log\left(\sin\left(x\right)+1\right)}{16\left(a^3+3 a^2 b+3 a b^2+b^3\right)}-\frac{\left(3 a^2+10 a b+15 b^2\right)\log\left(-\sin\left(x\right)+1\right)}{16\left(a^3+3 a^2 b+3 a b^2+b^3\right)}-\frac{3 a \sin\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^5/(a+b*sin(x)^2),x, algorithm="giac")
```

```
[Out] b^3*arctan(b*sin(x)/sqrt(a*b))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b))
+ 1/16*(3*a^2 + 10*a*b + 15*b^2)*log(sin(x) + 1)/(a^3 + 3*a^2*b + 3*a*b^2 +
 b^3) - 1/16*(3*a^2 + 10*a*b + 15*b^2)*log(-sin(x) + 1)/(a^3 + 3*a^2*b + 3*
a*b^2 + b^3) - 1/8*(3*a*sin(x)^3 + 7*b*sin(x)^3 - 5*a*sin(x) - 9*b*sin(x))/
((a^2 + 2*a*b + b^2)*(sin(x)^2 - 1)^2)
```

3.313 $\int \frac{\sec^6(x)}{a+b\sin^2(x)}\,dx$

Optimal. Leaf size=87

$$\frac{\left(a^2+3ab+3b^2\right)\tan(x)}{(a+b)^3}+\frac{b^3\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{7/2}}+\frac{\tan^5(x)}{5(a+b)}+\frac{(2a+3b)\tan^3(x)}{3(a+b)^2}$$

```
[Out] (b^3*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(7/2)) + ((a^2
+ 3*a*b + 3*b^2)*Tan[x])/(a + b)^3 + ((2*a + 3*b)*Tan[x]^3)/(3*(a + b)^2) +
 Tan[x]^5/(5*(a + b))
```

Rubi [A] time = 0.113406, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3191, 390, 205}

$$\frac{\left(a^2+3ab+3b^2\right)\tan(x)}{(a+b)^3}+\frac{b^3\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{7/2}}+\frac{\tan^5(x)}{5(a+b)}+\frac{(2a+3b)\tan^3(x)}{3(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^6/(a + b*Sin[x]^2),x]
```

```
[Out] (b^3*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(7/2)) + ((a^2
+ 3*a*b + 3*b^2)*Tan[x])/(a + b)^3 + ((2*a + 3*b)*Tan[x]^3)/(3*(a + b)^2) +
 Tan[x]^5/(5*(a + b))
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec^6(x)}{a+b\sin^2(x)}\,dx = \text{Subst}\left(\int \frac{(1+x^2)^3}{a+(a+b)x^2}\,dx, x, \tan(x)\right)$$
$$= \text{Subst}\left(\int \left(\frac{a^2+3ab+3b^2}{(a+b)^3} + \frac{(2a+3b)x^2}{(a+b)^2} + \frac{x^4}{a+b} + \frac{b^3}{(a+b)^3\left(a+(a+b)x^2\right)}\right)dx, x, \tan(x)\right)$$
$$= \frac{\left(a^2+3ab+3b^2\right)\tan(x)}{(a+b)^3} + \frac{(2a+3b)\tan^3(x)}{3(a+b)^2} + \frac{\tan^5(x)}{5(a+b)} + \frac{b^3\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \tan(x)\right)}{(a+b)^3}$$
$$= \frac{b^3\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{7/2}} + \frac{\left(a^2+3ab+3b^2\right)\tan(x)}{(a+b)^3} + \frac{(2a+3b)\tan^3(x)}{3(a+b)^2} + \frac{\tan^5(x)}{5(a+b)}$$

Mathematica [A] time = 0.370925, size = 90, normalized size = 1.03

$$\frac{\tan(x)\left(\left(4a^2+13ab+9b^2\right)\sec^2(x)+8a^2+3(a+b)^2\sec^4(x)+26ab+33b^2\right)}{15(a+b)^3} + \frac{b^3\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^6/(a + b*Sin[x]^2),x]
```

```
[Out] (b^3*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(7/2)) + ((8*a^
2 + 26*a*b + 33*b^2 + (4*a^2 + 13*a*b + 9*b^2)*Sec[x]^2 + 3*(a + b)^2*Sec[x
]^4)*Tan[x])/(15*(a + b)^3)
```

Maple [A] time = 0.089, size = 147, normalized size = 1.7

$$\frac{(\tan(x))^5 a^2}{5(a+b)^3} + \frac{2(\tan(x))^5 ab}{5(a+b)^3} + \frac{(\tan(x))^5 b^2}{5(a+b)^3} + \frac{2(\tan(x))^3 a^2}{3(a+b)^3} + \frac{5(\tan(x))^3 ab}{3(a+b)^3} + \frac{(\tan(x))^3 b^2}{(a+b)^3} + \frac{a^2\tan(x)}{(a+b)^3} + 3\frac{ab\,\text{ta}}{(a+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^6/(a+b*sin(x)^2),x)
```

```
[Out] 1/5/(a+b)^3*tan(x)^5*a^2+2/5/(a+b)^3*tan(x)^5*a*b+1/5/(a+b)^3*tan(x)^5*b^2+
2/3/(a+b)^3*tan(x)^3*a^2+5/3/(a+b)^3*tan(x)^3*a*b+1/(a+b)^3*tan(x)^3*b^2+1/
(a+b)^3*a^2*tan(x)+3/(a+b)^3*a*b*tan(x)+3/(a+b)^3*b^2*tan(x)+b^3/(a+b)^3/(a
*(a+b))^(1/2)*arctan((a+b)*tan(x)/(a*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^6/(a+b*sin(x)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.32889, size = 1098, normalized size = 12.62

$$\left[-\frac{15\sqrt{-a^2-ab}b^3\cos(x)^5\log\left(\frac{(8a^2+8ab+b^2)\cos(x)^4-2(4a^2+5ab+b^2)\cos(x)^2+4((2a+b)\cos(x)^3-(a+b)\cos(x))\sqrt{-a^2-ab}\sin(x)+a^2+2ab+b^2}{b^2\cos(x)^4-2(ab+b^2)\cos(x)^2+a^2+2ab+b^2}\right)}{60(a^5+4a^4b+\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^6/(a+b*sin(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/60*(15*sqrt(-a^2 - a*b)*b^3*cos(x)^5*log(((8*a^2 + 8*a*b + b^2)*cos(x)^
4 - 2*(4*a^2 + 5*a*b + b^2)*cos(x)^2 + 4*((2*a + b)*cos(x)^3 - (a + b)*cos(
x))*sqrt(-a^2 - a*b)*sin(x) + a^2 + 2*a*b + b^2)/(b^2*cos(x)^4 - 2*(a*b + b
^2)*cos(x)^2 + a^2 + 2*a*b + b^2)) - 4*((8*a^4 + 34*a^3*b + 59*a^2*b^2 + 33
*a*b^3)*cos(x)^4 + 3*a^4 + 9*a^3*b + 9*a^2*b^2 + 3*a*b^3 + (4*a^4 + 17*a^3*
b + 22*a^2*b^2 + 9*a*b^3)*cos(x)^2)*sin(x))/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4
*a^2*b^3 + a*b^4)*cos(x)^5), -1/30*(15*sqrt(a^2 + a*b)*b^3*arctan(1/2*((2*a
 + b)*cos(x)^2 - a - b)/(sqrt(a^2 + a*b)*cos(x)*sin(x)))*cos(x)^5 - 2*((8*a
^4 + 34*a^3*b + 59*a^2*b^2 + 33*a*b^3)*cos(x)^4 + 3*a^4 + 9*a^3*b + 9*a^2*b
^2 + 3*a*b^3 + (4*a^4 + 17*a^3*b + 22*a^2*b^2 + 9*a*b^3)*cos(x)^2)*sin(x))/
((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cos(x)^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**6/(a+b*sin(x)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.11407, size = 343, normalized size = 3.94

$$\frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a+2b)+\arctan\left(\frac{a\tan(x)+b\tan(x)}{\sqrt{a^2+ab}}\right)\right)b^3}{(a^3+3a^2b+3ab^2+b^3)\sqrt{a^2+ab}}+\frac{3a^4\tan(x)^5+12a^3b\tan(x)^5+18a^2b^2\tan(x)^5+12ab^3\tan\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^6/(a+b*sin(x)^2),x, algorithm="giac")
```

```
[Out] (pi*floor(x/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(x) + b*tan(x))/sqrt(a^
2 + a*b)))*b^3/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a^2 + a*b)) + 1/15*(3*
a^4*tan(x)^5 + 12*a^3*b*tan(x)^5 + 18*a^2*b^2*tan(x)^5 + 12*a*b^3*tan(x)^5
+ 3*b^4*tan(x)^5 + 10*a^4*tan(x)^3 + 45*a^3*b*tan(x)^3 + 75*a^2*b^2*tan(x)^
3 + 55*a*b^3*tan(x)^3 + 15*b^4*tan(x)^3 + 15*a^4*tan(x) + 75*a^3*b*tan(x) +
 150*a^2*b^2*tan(x) + 135*a*b^3*tan(x) + 45*b^4*tan(x))/(a^5 + 5*a^4*b + 10
*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)
```

3.314 $\int \frac{\cos^6(x)}{\left(a+b\sin^2(x)\right)^2}\,dx$

Optimal. Leaf size=113

$$-\frac{(4a-b)(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}b^3}+\frac{x(4a+5b)}{2b^3}+\frac{(2a+b)(a+b)\tan(x)}{2ab^2\left((a+b)\tan^2(x)+a\right)}-\frac{\sin(x)\cos(x)}{2b\left((a+b)\tan^2(x)+a\right)}$$

```
[Out] ((4*a + 5*b)*x)/(2*b^3) - ((4*a - b)*(a + b)^(3/2)*ArcTan[(Sqrt[a + b]*Tan[
x])/Sqrt[a]])/(2*a^(3/2)*b^3) - (Cos[x]*Sin[x])/(2*b*(a + (a + b)*Tan[x]^2)
) + ((a + b)*(2*a + b)*Tan[x])/(2*a*b^2*(a + (a + b)*Tan[x]^2))
```

Rubi [A] time = 0.223141, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {3191, 414, 527, 522, 203, 205}

$$-\frac{(4a-b)(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}b^3}+\frac{x(4a+5b)}{2b^3}+\frac{(2a+b)(a+b)\tan(x)}{2ab^2\left((a+b)\tan^2(x)+a\right)}-\frac{\sin(x)\cos(x)}{2b\left((a+b)\tan^2(x)+a\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^6/(a + b*Sin[x]^2)^2,x]
```

```
[Out] ((4*a + 5*b)*x)/(2*b^3) - ((4*a - b)*(a + b)^(3/2)*ArcTan[(Sqrt[a + b]*Tan[
x])/Sqrt[a]])/(2*a^(3/2)*b^3) - (Cos[x]*Sin[x])/(2*b*(a + (a + b)*Tan[x]^2)
) + ((a + b)*(2*a + b)*Tan[x])/(2*a*b^2*(a + (a + b)*Tan[x]^2))
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^6(x)}{\left(a+b\sin^2(x)\right)^2}\,dx = \text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^2\left(a+(a+b)x^2\right)^2}\,dx, x, \tan(x)\right)$$

$$= -\frac{\cos(x)\sin(x)}{2b\left(a+(a+b)\tan^2(x)\right)} + \frac{\text{Subst}\left(\int \frac{a+2b-3(a+b)x^2}{\left(1+x^2\right)\left(a+(a+b)x^2\right)^2}\,dx, x, \tan(x)\right)}{2b}$$

$$= -\frac{\cos(x)\sin(x)}{2b\left(a+(a+b)\tan^2(x)\right)} + \frac{(a+b)(2a+b)\tan(x)}{2ab^2\left(a+(a+b)\tan^2(x)\right)} - \frac{\text{Subst}\left(\int \frac{2\left(2a^2+2ab-b^2\right)-2(a+b)(2a+b)x^2}{\left(1+x^2\right)\left(a+(a+b)x^2\right)}\right.}{4ab^2}$$

$$= -\frac{\cos(x)\sin(x)}{2b\left(a+(a+b)\tan^2(x)\right)} + \frac{(a+b)(2a+b)\tan(x)}{2ab^2\left(a+(a+b)\tan^2(x)\right)} - \frac{\left((4a-b)(a+b)^2\right)\text{Subst}\left(\int \frac{1}{a+(a+b)}\right.}{2ab^3}$$

$$= \frac{(4a+5b)x}{2b^3} - \frac{(4a-b)(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}b^3} - \frac{\cos(x)\sin(x)}{2b\left(a+(a+b)\tan^2(x)\right)} + \frac{(a+b)(2a}{2ab^2\left(a+(a\right.}$$

Mathematica [A] time = 0.294315, size = 90, normalized size = 0.8

$$\frac{-\frac{2(4a-b)(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{a^{3/2}} + 2x(4a+5b) + \frac{2b(a+b)^2\sin(2x)}{a(2a-b\cos(2x)+b)} + b\sin(2x)}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^6/(a + b*Sin[x]^2)^2,x]
```

```
[Out] (2*(4*a + 5*b)*x - (2*(4*a - b)*(a + b)^(3/2)*ArcTan[(Sqrt[a + b]*Tan[x])/S
qrt[a]])/a^(3/2) + b*Sin[2*x] + (2*b*(a + b)^2*Sin[2*x])/(a*(2*a + b - b*Co
s[2*x])))/(4*b^3)
```

Maple [B] time = 0.091, size = 211, normalized size = 1.9

$$\frac{\tan(x)}{2b^2\left((\tan(x))^2+1\right)} + \frac{5\arctan(\tan(x))}{2b^2} + 2\frac{\arctan(\tan(x))\,a}{b^3} + \frac{a\tan(x)}{2b^2\left((\tan(x))^2a+(\tan(x))^2b+a\right)} + \frac{}{b\left((\tan(x)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^6/(a+b*sin(x)^2)^2,x)
```

```
[Out] 1/2/b^2*tan(x)/(tan(x)^2+1)+5/2/b^2*arctan(tan(x))+2/b^3*arctan(tan(x))*a+1
/2/b^2*a*tan(x)/(tan(x)^2*a+tan(x)^2*b+a)+1/b*tan(x)/(tan(x)^2*a+tan(x)^2*b
+a)+1/2/a*tan(x)/(tan(x)^2*a+tan(x)^2*b+a)-2/b^3/(a*(a+b))^(1/2)*arctan((a+
b)*tan(x)/(a*(a+b))^(1/2))*a^2-7/2/b^2/(a*(a+b))^(1/2)*arctan((a+b)*tan(x)/
(a*(a+b))^(1/2))*a-1/b/(a*(a+b))^(1/2)*arctan((a+b)*tan(x)/(a*(a+b))^(1/2))
+1/2/a/(a*(a+b))^(1/2)*arctan((a+b)*tan(x)/(a*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6/(a+b*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.40978, size = 1127, normalized size = 9.97

$$\left[\frac{4\left(4\,a^2b+5\,ab^2\right)x\cos\left(x\right)^2+\left(4\,a^3+7\,a^2b+2\,ab^2-b^3-\left(4\,a^2b+3\,ab^2-b^3\right)\cos\left(x\right)^2\right)\sqrt{-\frac{a+b}{a}}\log\left(\frac{\left(8\,a^2+8\,ab+b^2\right)\cos(x)^4-2\,(4}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6/(a+b*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(4*(4*a^2*b + 5*a*b^2)*x*cos(x)^2 + (4*a^3 + 7*a^2*b + 2*a*b^2 - b^3 -
 (4*a^2*b + 3*a*b^2 - b^3)*cos(x)^2)*sqrt(-(a + b)/a)*log(((8*a^2 + 8*a*b +
 b^2)*cos(x)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(x)^2 - 4*((2*a^2 + a*b)*cos(x)
^3 - (a^2 + a*b)*cos(x))*sqrt(-(a + b)/a)*sin(x) + a^2 + 2*a*b + b^2)/(b^2*
cos(x)^4 - 2*(a*b + b^2)*cos(x)^2 + a^2 + 2*a*b + b^2)) - 4*(4*a^3 + 9*a^2*
b + 5*a*b^2)*x + 4*(a*b^2*cos(x)^3 - (2*a^2*b + 3*a*b^2 + b^3)*cos(x))*sin(
x))/(a*b^4*cos(x)^2 - a^2*b^3 - a*b^4), 1/4*(2*(4*a^2*b + 5*a*b^2)*x*cos(x)
^2 - (4*a^3 + 7*a^2*b + 2*a*b^2 - b^3 - (4*a^2*b + 3*a*b^2 - b^3)*cos(x)^2)
*sqrt((a + b)/a)*arctan(1/2*((2*a + b)*cos(x)^2 - a - b)*sqrt((a + b)/a)/((
a + b)*cos(x)*sin(x))) - 2*(4*a^3 + 9*a^2*b + 5*a*b^2)*x + 2*(a*b^2*cos(x)^
3 - (2*a^2*b + 3*a*b^2 + b^3)*cos(x))*sin(x))/(a*b^4*cos(x)^2 - a^2*b^3 - a
*b^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**6/(a+b*sin(x)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12205, size = 236, normalized size = 2.09

$$\frac{(4a+5b)x}{2b^3} - \frac{(4a^3+7a^2b+2ab^2-b^3)\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a+2b)+\arctan\left(\frac{a\tan(x)+b\tan(x)}{\sqrt{a^2+ab}}\right)\right)}{2\sqrt{a^2+ab}ab^3} + \frac{2a^2\tan(x)^3+3ab}{2\left(a\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6/(a+b*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*a + 5*b)*x/b^3 - 1/2*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*(pi*floor(x/p
i + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(x) + b*tan(x))/sqrt(a^2 + a*b)))/(s
qrt(a^2 + a*b)*a*b^3) + 1/2*(2*a^2*tan(x)^3 + 3*a*b*tan(x)^3 + b^2*tan(x)^3
 + 2*a^2*tan(x) + 2*a*b*tan(x) + b^2*tan(x))/((a*tan(x)^4 + b*tan(x)^4 + 2*
a*tan(x)^2 + b*tan(x)^2 + a)*a*b^2)
```

3.315 $\int \frac{\cos^5(x)}{\left(a+b\sin^2(x)\right)^2}\,dx$

Optimal. Leaf size=72

$$-\frac{(3a-b)(a+b)\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}+\frac{(a+b)^2\sin(x)}{2ab^2\left(a+b\sin^2(x)\right)}+\frac{\sin(x)}{b^2}$$

```
[Out] -((3*a - b)*(a + b)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(2*a^(3/2)*b^(5/2)) +
 Sin[x]/b^2 + ((a + b)^2*Sin[x])/(2*a*b^2*(a + b*Sin[x]^2))
```

Rubi [A] time = 0.118633, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3190, 390, 385, 205}

$$-\frac{(3a-b)(a+b)\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}+\frac{(a+b)^2\sin(x)}{2ab^2\left(a+b\sin^2(x)\right)}+\frac{\sin(x)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^5/(a + b*Sin[x]^2)^2,x]
```


```
[Out] -((3*a - b)*(a + b)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(2*a^(3/2)*b^(5/2)) +
 Sin[x]/b^2 + ((a + b)^2*Sin[x])/(2*a*b^2*(a + b*Sin[x]^2))
```


Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^5(x)}{\left(a+b\sin^2(x)\right)^2}\,dx = \text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\left(a+bx^2\right)^2}\,dx, x, \sin(x)\right)$$
$$= \text{Subst}\left(\int \left(\frac{1}{b^2} - \frac{a^2-b^2+2b(a+b)x^2}{b^2\left(a+bx^2\right)^2}\right)dx, x, \sin(x)\right)$$
$$= \frac{\sin(x)}{b^2} - \frac{\text{Subst}\left(\int \frac{a^2-b^2+2b(a+b)x^2}{\left(a+bx^2\right)^2}\,dx, x, \sin(x)\right)}{b^2}$$
$$= \frac{\sin(x)}{b^2} + \frac{(a+b)^2\sin(x)}{2ab^2\left(a+b\sin^2(x)\right)} - \frac{((3a-b)(a+b))\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sin(x)\right)}{2ab^2}$$
$$= -\frac{(3a-b)(a+b)\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}} + \frac{\sin(x)}{b^2} + \frac{(a+b)^2\sin(x)}{2ab^2\left(a+b\sin^2(x)\right)}$$

Mathematica [A] time = 0.328314, size = 118, normalized size = 1.64

$$\frac{\frac{\left(-3a^2-2ab+b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{a^{3/2}} + \frac{\left(3a^2+2ab-b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\csc(x)}{\sqrt{b}}\right)}{a^{3/2}} + \frac{4\sqrt{b}(a+b)^2\sin(x)}{a(2a-b\cos(2x)+b)} + 4\sqrt{b}\sin(x)}{4b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^5/(a + b*Sin[x]^2)^2,x]
```

```
[Out] (((3*a^2 + 2*a*b - b^2)*ArcTan[(Sqrt[a]*Csc[x])/Sqrt[b]])/a^(3/2) + ((-3*a^
2 - 2*a*b + b^2)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/a^(3/2) + 4*Sqrt[b]*Sin[
x] + (4*Sqrt[b]*(a + b)^2*Sin[x])/(a*(2*a + b - b*Cos[2*x])))/(4*b^(5/2))
```

Maple [A] time = 0.05, size = 120, normalized size = 1.7

$$\frac{\sin(x)}{b^2} + \frac{a\sin(x)}{2b^2\left(a+b(\sin(x))^2\right)} + \frac{\sin(x)}{b\left(a+b(\sin(x))^2\right)} + \frac{\sin(x)}{2a\left(a+b(\sin(x))^2\right)} - \frac{3a}{2b^2}\arctan\left(\sin(x)b\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} - \frac{1}{b}\text{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^5/(a+b*sin(x)^2)^2,x)
```

```
[Out] sin(x)/b^2+1/2/b^2*a*sin(x)/(a+b*sin(x)^2)+1/b*sin(x)/(a+b*sin(x)^2)+1/2*si
n(x)/a/(a+b*sin(x)^2)-3/2/b^2*a/(a*b)^(1/2)*arctan(sin(x)*b/(a*b)^(1/2))-1/
b/(a*b)^(1/2)*arctan(sin(x)*b/(a*b)^(1/2))+1/2/a/(a*b)^(1/2)*arctan(sin(x)*
b/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5/(a+b*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.12144, size = 641, normalized size = 8.9

$$\left[-\frac{\left(3\,a^{3}+5\,a^{2}b+ab^{2}-b^{3}-\left(3\,a^{2}b+2\,ab^{2}-b^{3}\right)\cos\left(x\right)^{2}\right)\sqrt{-ab}\log\left(-\frac{b\cos(x)^{2}+2\,\sqrt{-ab}\sin(x)+a-b}{b\cos(x)^{2}-a-b}\right)-2\left(2\,a^{2}b^{2}\cos\left(x\right)^{2}-3\,a^{3}b\right.}{4\left(a^{2}b^{4}\cos\left(x\right)^{2}-a^{3}b^{3}-a^{2}b^{4}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5/(a+b*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*((3*a^3 + 5*a^2*b + a*b^2 - b^3 - (3*a^2*b + 2*a*b^2 - b^3)*cos(x)^2)
*sqrt(-a*b)*log(-(b*cos(x)^2 + 2*sqrt(-a*b)*sin(x) + a - b)/(b*cos(x)^2 - a
 - b)) - 2*(2*a^2*b^2*cos(x)^2 - 3*a^3*b - 4*a^2*b^2 - a*b^3)*sin(x))/(a^2*
b^4*cos(x)^2 - a^3*b^3 - a^2*b^4), 1/2*((3*a^3 + 5*a^2*b + a*b^2 - b^3 - (3
*a^2*b + 2*a*b^2 - b^3)*cos(x)^2)*sqrt(a*b)*arctan(sqrt(a*b)*sin(x)/a) + (2
*a^2*b^2*cos(x)^2 - 3*a^3*b - 4*a^2*b^2 - a*b^3)*sin(x))/(a^2*b^4*cos(x)^2
- a^3*b^3 - a^2*b^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**5/(a+b*sin(x)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12598, size = 111, normalized size = 1.54

$$\frac{\sin\left(x\right)}{b^{2}}-\frac{\left(3\,a^{2}+2\,ab-b^{2}\right)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab^{2}}+\frac{a^{2}\sin\left(x\right)+2\,ab\sin\left(x\right)+b^{2}\sin\left(x\right)}{2\left(b\sin\left(x\right)^{2}+a\right)ab^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5/(a+b*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] sin(x)/b^2 - 1/2*(3*a^2 + 2*a*b - b^2)*arctan(b*sin(x)/sqrt(a*b))/(sqrt(a*b
)*a*b^2) + 1/2*(a^2*sin(x) + 2*a*b*sin(x) + b^2*sin(x))/((b*sin(x)^2 + a)*a
*b^2)
```

3.316 $\int \frac{\cos^4(x)}{\left(a+b\sin^2(x)\right)^2} dx$

Optimal. Leaf size=75

$$-\frac{(2a-b)\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}b^2}+\frac{(a+b)\tan(x)}{2ab\left((a+b)\tan^2(x)+a\right)}+\frac{x}{b^2}$$

```
[Out] x/b^2 - ((2*a - b)*Sqrt[a + b]*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(2*a^(
3/2)*b^2) + ((a + b)*Tan[x])/(2*a*b*(a + (a + b)*Tan[x]^2))
```

Rubi [A] time = 0.105546, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3191, 414, 522, 203, 205}

$$-\frac{(2a-b)\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}b^2}+\frac{(a+b)\tan(x)}{2ab\left((a+b)\tan^2(x)+a\right)}+\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^4/(a + b*Sin[x]^2)^2,x]
```

```
[Out] x/b^2 - ((2*a - b)*Sqrt[a + b]*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(2*a^(
3/2)*b^2) + ((a + b)*Tan[x])/(2*a*b*(a + (a + b)*Tan[x]^2))
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^4(x)}{\left(a+b\sin^2(x)\right)^2}\,dx = \text{Subst}\left(\int \frac{1}{\left(1+x^2\right)\left(a+(a+b)x^2\right)^2}\,dx, x, \tan(x)\right)$$

$$= \frac{(a+b)\tan(x)}{2ab\left(a+(a+b)\tan^2(x)\right)} - \frac{\text{Subst}\left(\int \frac{a-b+(-a-b)x^2}{\left(1+x^2\right)\left(a+(a+b)x^2\right)}\,dx, x, \tan(x)\right)}{2ab}$$

$$= \frac{(a+b)\tan(x)}{2ab\left(a+(a+b)\tan^2(x)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right)}{b^2} - \frac{((2a-b)(a+b))\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\right.}{2ab^2}$$

$$= \frac{x}{b^2} - \frac{(2a-b)\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}b^2} + \frac{(a+b)\tan(x)}{2ab\left(a+(a+b)\tan^2(x)\right)}$$

Mathematica [A] time = 0.311862, size = 79, normalized size = 1.05

$$\frac{\frac{\left(-2a^2-ab+b^2\right)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a+b}} + \frac{b(a+b)\sin(2x)}{a(2a-b\cos(2x)+b)} + 2x}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^4/(a + b*Sin[x]^2)^2,x]
```

```
[Out] (2*x + ((-2*a^2 - a*b + b^2)*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(a^(3/2)
*Sqrt[a + b]) + (b*(a + b)*Sin[2*x])/(a*(2*a + b - b*Cos[2*x])))/(2*b^2)
```

Maple [B] time = 0.089, size = 132, normalized size = 1.8

$$\frac{\tan(x)}{2b\left((\tan(x))^2 a + (\tan(x))^2 b + a\right)} - \frac{a}{b^2}\arctan\left((a+b)\tan(x)\frac{1}{\sqrt{a(a+b)}}\right)\frac{1}{\sqrt{a(a+b)}} - \frac{1}{2b}\arctan\left((a+b)\tan(x)\frac{}{\sqrt{a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4/(a+b*sin(x)^2)^2,x)
```

```
[Out] 1/2/b*tan(x)/(tan(x)^2*a+tan(x)^2*b+a)-1/b^2/(a*(a+b))^(1/2)*arctan((a+b)*t
an(x)/(a*(a+b))^(1/2))*a-1/2/b/(a*(a+b))^(1/2)*arctan((a+b)*tan(x)/(a*(a+b)
)^(1/2))+1/2/a*tan(x)/(tan(x)^2*a+tan(x)^2*b+a)+1/2/a/(a*(a+b))^(1/2)*arcta
n((a+b)*tan(x)/(a*(a+b))^(1/2))+x/b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4/(a+b*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.1054, size = 882, normalized size = 11.76

$$\left[\frac{8\,abx\cos(x)^2-4\left(ab+b^2\right)\cos(x)\sin(x)-\left(\left(2\,ab-b^2\right)\cos(x)^2-2\,a^2-ab+b^2\right)\sqrt{-\frac{a+b}{a}}\log\left(\frac{\left(8\,a^2+8\,ab+b^2\right)\cos(x)^4-2\left(4\,a\right.}{}\right.}{8\left(ab^3\cos(x)^2-a^2b^2-ab^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4/(a+b*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(8*a*b*x*cos(x)^2 - 4*(a*b + b^2)*cos(x)*sin(x) - ((2*a*b - b^2)*cos(x
)^2 - 2*a^2 - a*b + b^2)*sqrt(-(a + b)/a)*log(((8*a^2 + 8*a*b + b^2)*cos(x)
^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(x)^2 - 4*((2*a^2 + a*b)*cos(x)^3 - (a^2 +
a*b)*cos(x))*sqrt(-(a + b)/a)*sin(x) + a^2 + 2*a*b + b^2)/(b^2*cos(x)^4 - 2
*(a*b + b^2)*cos(x)^2 + a^2 + 2*a*b + b^2)) - 8*(a^2 + a*b)*x)/(a*b^3*cos(x
)^2 - a^2*b^2 - a*b^3), 1/4*(4*a*b*x*cos(x)^2 - 2*(a*b + b^2)*cos(x)*sin(x)
 + ((2*a*b - b^2)*cos(x)^2 - 2*a^2 - a*b + b^2)*sqrt((a + b)/a)*arctan(1/2*
((2*a + b)*cos(x)^2 - a - b)*sqrt((a + b)/a)/((a + b)*cos(x)*sin(x))) - 4*(
a^2 + a*b)*x)/(a*b^3*cos(x)^2 - a^2*b^2 - a*b^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**4/(a+b*sin(x)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1135, size = 147, normalized size = 1.96

$$\frac{x}{b^2}-\frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(2\,a+2\,b\right)+\arctan\left(\frac{a\tan(x)+b\tan(x)}{\sqrt{a^2+ab}}\right)\right)\left(2\,a^2+ab-b^2\right)}{2\sqrt{a^2+ab}ab^2}+\frac{a\tan(x)+b\tan(x)}{2\left(a\tan(x)^2+b\tan(x)^2+a\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4/(a+b*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] x/b^2 - 1/2*(pi*floor(x/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(x) + b*tan
(x))/sqrt(a^2 + a*b)))*(2*a^2 + a*b - b^2)/(sqrt(a^2 + a*b)*a*b^2) + 1/2*(a
*tan(x) + b*tan(x))/((a*tan(x)^2 + b*tan(x)^2 + a)*a*b)
```

3.317 $\int \frac{\cos^3(x)}{\left(a+b\sin^2(x)\right)^2}\,dx$

Optimal. Leaf size=59

$$\frac{(a+b)\sin(x)}{2ab\left(a+b\sin^2(x)\right)}-\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}$$

```
[Out] -((a - b)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(2*a^(3/2)*b^(3/2)) + ((a + b)*
Sin[x])/(2*a*b*(a + b*Sin[x]^2))
```

Rubi [A] time = 0.0581525, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3190, 385, 205}

$$\frac{(a+b)\sin(x)}{2ab\left(a+b\sin^2(x)\right)}-\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^3/(a + b*Sin[x]^2)^2,x]
```

```
[Out] -((a - b)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(2*a^(3/2)*b^(3/2)) + ((a + b)*
Sin[x])/(2*a*b*(a + b*Sin[x]^2))
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^3(x)}{\left(a+b\sin^2(x)\right)^2}\,dx=\text{Subst}\left(\int \frac{1-x^2}{\left(a+bx^2\right)^2}\,dx,x,\sin(x)\right)$$
$$=\frac{(a+b)\sin(x)}{2ab\left(a+b\sin^2(x)\right)}-\frac{(a-b)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sin(x)\right)}{2ab}$$
$$=-\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}+\frac{(a+b)\sin(x)}{2ab\left(a+b\sin^2(x)\right)}$$

Mathematica [A] time = 0.0652649, size = 59, normalized size = 1.

$$\frac{(a+b)\sin(x)}{2ab\left(a+b\sin^2(x)\right)}-\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^3/(a + b*Sin[x]^2)^2,x]
```

```
[Out] -((a - b)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(2*a^(3/2)*b^(3/2)) + ((a + b)*
Sin[x])/(2*a*b*(a + b*Sin[x]^2))
```

Maple [A] time = 0.05, size = 65, normalized size = 1.1

$$\frac{(a+b)\sin(x)}{2\,ab\left(a+b\left(\sin(x)\right)^2\right)}-\frac{1}{2\,b}\arctan\left(\sin(x)\,b\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{1}{2\,a}\arctan\left(\sin(x)\,b\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3/(a+b*sin(x)^2)^2,x)
```

```
[Out] 1/2*(a+b)*sin(x)/a/b/(a+b*sin(x)^2)-1/2/b/(a*b)^(1/2)*arctan(sin(x)*b/(a*b)
^(1/2))+1/2/a/(a*b)^(1/2)*arctan(sin(x)*b/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(a+b*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.1153, size = 455, normalized size = 7.71

$$\left[\frac{\left(\left(ab-b^2\right)\cos(x)^2-a^2+b^2\right)\sqrt{-ab}\log\left(-\frac{b\cos(x)^2+2\sqrt{-ab}\sin(x)+a-b}{b\cos(x)^2-a-b}\right)-2\left(a^2b+ab^2\right)\sin(x)}{4\left(a^2b^3\cos(x)^2-a^3b^2-a^2b^3\right)},-\frac{\left(\left(ab-b^2\right)\cos(x)^2-a^2+\right.}{2\left(a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(a+b*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(((a*b - b^2)*cos(x)^2 - a^2 + b^2)*sqrt(-a*b)*log(-(b*cos(x)^2 + 2*sq
rt(-a*b)*sin(x) + a - b)/(b*cos(x)^2 - a - b)) - 2*(a^2*b + a*b^2)*sin(x))/
(a^2*b^3*cos(x)^2 - a^3*b^2 - a^2*b^3), -1/2*(((a*b - b^2)*cos(x)^2 - a^2 +
 b^2)*sqrt(a*b)*arctan(sqrt(a*b)*sin(x)/a) + (a^2*b + a*b^2)*sin(x))/(a^2*b
^3*cos(x)^2 - a^3*b^2 - a^2*b^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3/(a+b*sin(x)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11243, size = 76, normalized size = 1.29

$$-\frac{(a-b)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{2\sqrt{ab}ab}+\frac{a\sin(x)+b\sin(x)}{2\left(b\sin(x)^2+a\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(a+b*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(a - b)*arctan(b*sin(x)/sqrt(a*b))/(sqrt(a*b)*a*b) + 1/2*(a*sin(x) + b
*sin(x))/((b*sin(x)^2 + a)*a*b)
```

3.318 $\int \frac{\cos^2(x)}{\left(a+b\sin^2(x)\right)^2}\,dx$

Optimal. Leaf size=54

$$\frac{\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{a+b}}+\frac{\tan(x)}{2a\left((a+b)\tan^2(x)+a\right)}$$

```
[Out] ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[a + b]) + Tan[x]/(2*a*
(a + (a + b)*Tan[x]^2))
```

Rubi [A] time = 0.0559371, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3191, 199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{a+b}}+\frac{\tan(x)}{2a\left((a+b)\tan^2(x)+a\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2/(a + b*Sin[x]^2)^2,x]
```

```
[Out] ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[a + b]) + Tan[x]/(2*a*
(a + (a + b)*Tan[x]^2))
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^2(x)}{\left(a+b\sin^2(x)\right)^2}\,dx = \text{Subst}\left(\int \frac{1}{\left(a+(a+b)x^2\right)^2}\,dx, x, \tan(x)\right)$$
$$= \frac{\tan(x)}{2a\left(a+(a+b)\tan^2(x)\right)} + \frac{\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \tan(x)\right)}{2a}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{a+b}} + \frac{\tan(x)}{2a\left(a+(a+b)\tan^2(x)\right)}$$

Mathematica [A] time = 0.143341, size = 59, normalized size = 1.09

$$\frac{\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{a+b}} - \frac{\sin(2x)}{2a(-2a+b\cos(2x)-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2/(a + b*Sin[x]^2)^2,x]
```

```
[Out] ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[a + b]) - Sin[2*x]/(2*
a*(-2*a - b + b*Cos[2*x]))
```

Maple [A] time = 0.077, size = 51, normalized size = 0.9

$$\frac{\tan(x)}{2a\left((\tan(x))^2 a + (\tan(x))^2 b + a\right)} + \frac{1}{2a}\arctan\left((a+b)\tan(x)\frac{1}{\sqrt{a(a+b)}}\right)\frac{1}{\sqrt{a(a+b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/(a+b*sin(x)^2)^2,x)
```

```
[Out] 1/2/a*tan(x)/(tan(x)^2*a+tan(x)^2*b+a)+1/2/a/(a*(a+b))^(1/2)*arctan((a+b)*t
an(x)/(a*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a+b*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.19838, size = 749, normalized size = 13.87

$$\left[\frac{4\left(a^2+ab\right)\cos(x)\sin(x) + \left(b\cos(x)^2 - a - b\right)\sqrt{-a^2-ab}\log\left(\frac{\left(8a^2+8ab+b^2\right)\cos(x)^4 - 2\left(4a^2+5ab+b^2\right)\cos(x)^2 + 4\left((2a+b)\cos(x)^3 - (a+b)\right.}{b^2\cos(x)^4 - 2\left(ab+b^2\right)\cos(x)^2 + a^2 + 2ab+}\right.}{8\left(a^4+2a^3b+a^2b^2 - \left(a^3b+a^2b^2\right)\cos(x)^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a+b*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a^2 + a*b)*cos(x)*sin(x) + (b*cos(x)^2 - a - b)*sqrt(-a^2 - a*b)*l
og(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(x)^2 + 4*(
(2*a + b)*cos(x)^3 - (a + b)*cos(x))*sqrt(-a^2 - a*b)*sin(x) + a^2 + 2*a*b
+ b^2)/(b^2*cos(x)^4 - 2*(a*b + b^2)*cos(x)^2 + a^2 + 2*a*b + b^2)))/(a^4 +
 2*a^3*b + a^2*b^2 - (a^3*b + a^2*b^2)*cos(x)^2), 1/4*(2*(a^2 + a*b)*cos(x)
*sin(x) + (b*cos(x)^2 - a - b)*sqrt(a^2 + a*b)*arctan(1/2*((2*a + b)*cos(x)
^2 - a - b)/(sqrt(a^2 + a*b)*cos(x)*sin(x))))/(a^4 + 2*a^3*b + a^2*b^2 - (a
^3*b + a^2*b^2)*cos(x)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2/(a+b*sin(x)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11575, size = 104, normalized size = 1.93

$$\frac{\pi \left\lfloor \frac{x}{\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}\left(2\,a + 2\,b\right) + \arctan\left(\frac{a\tan(x) + b\tan(x)}{\sqrt{a^2 + ab}}\right)}{2\,\sqrt{a^2 + ab}a} + \frac{\tan\left(x\right)}{2\,\left(a\tan\left(x\right)^2 + b\tan\left(x\right)^2 + a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a+b*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(pi*floor(x/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(x) + b*tan(x))/sqr
t(a^2 + a*b)))/(sqrt(a^2 + a*b)*a) + 1/2*tan(x)/((a*tan(x)^2 + b*tan(x)^2 +
 a)*a)
```

3.319 $\int \frac{\cos(x)}{\left(a+b\sin^2(x)\right)^2}\,dx$

Optimal. Leaf size=48

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{\sin(x)}{2a\left(a+b\sin^2(x)\right)}$$

```
[Out] ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]) + Sin[x]/(2*a*(a + b*S
in[x]^2))
```

Rubi [A] time = 0.0348803, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3190, 199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{\sin(x)}{2a\left(a+b\sin^2(x)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/(a + b*Sin[x]^2)^2,x]
```

```
[Out] ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]) + Sin[x]/(2*a*(a + b*S
in[x]^2))
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos(x)}{\left(a+b\sin^2(x)\right)^2}\,dx = \text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^2}\,dx, x, \sin(x)\right)$$
$$= \frac{\sin(x)}{2a\left(a+b\sin^2(x)\right)} + \frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sin(x)\right)}{2a}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}} + \frac{\sin(x)}{2a\left(a+b\sin^2(x)\right)}$$

Mathematica [A] time = 0.0582955, size = 48, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}} + \frac{\sin(x)}{2a\left(a+b\sin^2(x)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/(a + b*Sin[x]^2)^2,x]
```

```
[Out] ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]) + Sin[x]/(2*a*(a + b*S
in[x]^2))
```

Maple [A] time = 0.035, size = 39, normalized size = 0.8

$$\frac{\sin(x)}{2a\left(a+b(\sin(x))^2\right)} + \frac{1}{2a}\arctan\left(\sin(x)\,b\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(a+b*sin(x)^2)^2,x)
```

```
[Out] 1/2*sin(x)/a/(a+b*sin(x)^2)+1/2/a/(a*b)^(1/2)*arctan(sin(x)*b/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a+b*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.02593, size = 381, normalized size = 7.94

$$\left[-\frac{2\,ab\sin(x) + \left(b\cos(x)^2 - a - b\right)\sqrt{-ab}\log\left(-\frac{b\cos(x)^2 + 2\sqrt{-ab}\sin(x) + a - b}{b\cos(x)^2 - a - b}\right)}{4\left(a^2b^2\cos(x)^2 - a^3b - a^2b^2\right)}, -\frac{ab\sin(x) - \left(b\cos(x)^2 - a - b\right)\sqrt{ab}\arctan}{2\left(a^2b^2\cos(x)^2 - a^3b - a^2b^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a+b*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*b*sin(x) + (b*cos(x)^2 - a - b)*sqrt(-a*b)*log(-(b*cos(x)^2 + 2*
sqrt(-a*b)*sin(x) + a - b)/(b*cos(x)^2 - a - b)))/(a^2*b^2*cos(x)^2 - a^3*b
 - a^2*b^2), -1/2*(a*b*sin(x) - (b*cos(x)^2 - a - b)*sqrt(a*b)*arctan(sqrt(
a*b)*sin(x)/a))/(a^2*b^2*cos(x)^2 - a^3*b - a^2*b^2)]
```

Sympy [A] time = 31.5219, size = 340, normalized size = 7.08

$$\begin{cases} \frac{\tilde{\infty}}{\sin^3(x)} \\ \frac{\sin(x)}{a^2} \\ -\frac{1}{3b^2\sin^3(x)} \\ \frac{2i\sqrt{a}b\sqrt{\frac{1}{b}}\sin(x)}{4ia^{\frac{5}{2}}b\sqrt{\frac{1}{b}}+4ia^{\frac{3}{2}}b^2\sqrt{\frac{1}{b}}\sin^2(x)} + \frac{a\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sin(x)\right)}{4ia^{\frac{5}{2}}b\sqrt{\frac{1}{b}}+4ia^{\frac{3}{2}}b^2\sqrt{\frac{1}{b}}\sin^2(x)} - \frac{a\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sin(x)\right)}{4ia^{\frac{5}{2}}b\sqrt{\frac{1}{b}}+4ia^{\frac{3}{2}}b^2\sqrt{\frac{1}{b}}\sin^2(x)} + \frac{b\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sin(x)\right)\sin^2(x)}{4ia^{\frac{5}{2}}b\sqrt{\frac{1}{b}}+4ia^{\frac{3}{2}}b^2\sqrt{\frac{1}{b}}\sin^2(x)} - \frac{b\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sin\right.}{4ia^{\frac{5}{2}}b\sqrt{\frac{1}{b}}+4ia^{\frac{3}{2}}b} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a+b*sin(x)**2)**2,x)
```

```
[Out] Piecewise((zoo/sin(x)**3, Eq(a, 0) & Eq(b, 0)), (sin(x)/a**2, Eq(b, 0)), (-
1/(3*b**2*sin(x)**3), Eq(a, 0)), (2*I*sqrt(a)*b*sqrt(1/b)*sin(x)/(4*I*a**(5
/2)*b*sqrt(1/b) + 4*I*a**(3/2)*b**2*sqrt(1/b)*sin(x)**2) + a*log(-I*sqrt(a)
*sqrt(1/b) + sin(x))/(4*I*a**(5/2)*b*sqrt(1/b) + 4*I*a**(3/2)*b**2*sqrt(1/b
)*sin(x)**2) - a*log(I*sqrt(a)*sqrt(1/b) + sin(x))/(4*I*a**(5/2)*b*sqrt(1/b
) + 4*I*a**(3/2)*b**2*sqrt(1/b)*sin(x)**2) + b*log(-I*sqrt(a)*sqrt(1/b) + s
in(x))*sin(x)**2/(4*I*a**(5/2)*b*sqrt(1/b) + 4*I*a**(3/2)*b**2*sqrt(1/b)*si
n(x)**2) - b*log(I*sqrt(a)*sqrt(1/b) + sin(x))*sin(x)**2/(4*I*a**(5/2)*b*sq
rt(1/b) + 4*I*a**(3/2)*b**2*sqrt(1/b)*sin(x)**2), True))
```

Giac [A] time = 1.15537, size = 51, normalized size = 1.06

$$\frac{\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{2\sqrt{ab}a} + \frac{\sin(x)}{2\left(b\sin(x)^2 + a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a+b*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*arctan(b*sin(x)/sqrt(a*b))/(sqrt(a*b)*a) + 1/2*sin(x)/((b*sin(x)^2 + a)
*a)
```

3.320 $\int \frac{\sec(x)}{\left(a+b\sin^2(x)\right)^2}\,dx$

Optimal. Leaf size=73

$$\frac{\sqrt{b}(3a+b)\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^2}+\frac{b\sin(x)}{2a(a+b)\left(a+b\sin^2(x)\right)}+\frac{\tanh^{-1}(\sin(x))}{(a+b)^2}$$

```
[Out] (Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(2*a^(3/2)*(a + b)^2)
+ ArcTanh[Sin[x]]/(a + b)^2 + (b*Sin[x])/(2*a*(a + b)*(a + b*Sin[x]^2))
```

Rubi [A] time = 0.0903967, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3190, 414, 522, 206, 205}

$$\frac{\sqrt{b}(3a+b)\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^2}+\frac{b\sin(x)}{2a(a+b)\left(a+b\sin^2(x)\right)}+\frac{\tanh^{-1}(\sin(x))}{(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]/(a + b*Sin[x]^2)^2,x]
```

```
[Out] (Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(2*a^(3/2)*(a + b)^2)
+ ArcTanh[Sin[x]]/(a + b)^2 + (b*Sin[x])/(2*a*(a + b)*(a + b*Sin[x]^2))
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec(x)}{\left(a+b\sin^2(x)\right)^2}\,dx = \text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\left(a+bx^2\right)^2}\,dx, x, \sin(x)\right)$$
$$= \frac{b\sin(x)}{2a(a+b)\left(a+b\sin^2(x)\right)} - \frac{\text{Subst}\left(\int \frac{b-2(a+b)+bx^2}{(1-x^2)(a+bx^2)}\,dx, x, \sin(x)\right)}{2a(a+b)}$$
$$= \frac{b\sin(x)}{2a(a+b)\left(a+b\sin^2(x)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(x)\right)}{(a+b)^2} + \frac{(b(3a+b))\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sin(x)\right)}{2a(a+b)^2}$$
$$= \frac{\sqrt{b}(3a+b)\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^2} + \frac{\tanh^{-1}(\sin(x))}{(a+b)^2} + \frac{b\sin(x)}{2a(a+b)\left(a+b\sin^2(x)\right)}$$

Mathematica [A] time = 0.561215, size = 130, normalized size = 1.78

$$\frac{\frac{\sqrt{b}(3a+b)\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{a^{3/2}} - \frac{\sqrt{b}(3a+b)\tan^{-1}\left(\frac{\sqrt{a}\csc(x)}{\sqrt{b}}\right)}{a^{3/2}} + 4\left(\frac{b(a+b)\sin(x)}{a(2a-b\cos(2x)+b)} - \log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right) + \log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\right)}{4(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]/(a + b*Sin[x]^2)^2,x]
```

```
[Out] (-((Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[a]*Csc[x])/Sqrt[b]])/a^(3/2)) + (Sqrt[b]
*(3*a + b)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/a^(3/2) + 4*(-Log[Cos[x/2] - S
in[x/2]] + Log[Cos[x/2] + Sin[x/2]] + (b*(a + b)*Sin[x])/(a*(2*a + b - b*Co
s[2*x]))))/(4*(a + b)^2)
```

Maple [A] time = 0.067, size = 122, normalized size = 1.7

$$-\frac{\ln(-1+\sin(x))}{2(a+b)^2} + \frac{\ln(1+\sin(x))}{2(a+b)^2} + \frac{b\sin(x)}{2(a+b)^2\left(a+b(\sin(x))^2\right)} + \frac{b^2\sin(x)}{2(a+b)^2 a\left(a+b(\sin(x))^2\right)} + \frac{3b}{2(a+b)^2}\arctan\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)/(a+b*sin(x)^2)^2,x)
```

```
[Out] -1/2/(a+b)^2*ln(-1+sin(x))+1/2/(a+b)^2*ln(1+sin(x))+1/2*b/(a+b)^2*sin(x)/(a
+b*sin(x)^2)+1/2*b^2/(a+b)^2/a*sin(x)/(a+b*sin(x)^2)+3/2*b/(a+b)^2/(a*b)^(1
/2)*arctan(sin(x)*b/(a*b)^(1/2))+1/2*b^2/(a+b)^2/a/(a*b)^(1/2)*arctan(sin(x
)*b/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a+b*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.55647, size = 826, normalized size = 11.32

$$\left[-\frac{\left(\left(3\,ab+b^2\right)\cos\left(x\right)^2-3\,a^2-4\,ab-b^2\right)\sqrt{-\frac{b}{a}}\log\left(-\frac{b\cos(x)^2-2\,a\sqrt{-\frac{b}{a}}\sin(x)+a-b}{b\cos(x)^2-a-b}\right)+2\left(ab\cos\left(x\right)^2-a^2-ab\right)\log\left(\sin\left(x\right)\right.}{4\left(a^4+3\,a^3b+3\,a^2b^2+ab^3-\left(a^3b+2\,a^2b^2+ab^3\right)\mathrm{co}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a+b*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(((3*a*b + b^2)*cos(x)^2 - 3*a^2 - 4*a*b - b^2)*sqrt(-b/a)*log(-(b*co
s(x)^2 - 2*a*sqrt(-b/a)*sin(x) + a - b)/(b*cos(x)^2 - a - b)) + 2*(a*b*cos(
x)^2 - a^2 - a*b)*log(sin(x) + 1) - 2*(a*b*cos(x)^2 - a^2 - a*b)*log(-sin(x
) + 1) - 2*(a*b + b^2)*sin(x))/(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 - (a^3*b
+ 2*a^2*b^2 + a*b^3)*cos(x)^2), -1/2*(((3*a*b + b^2)*cos(x)^2 - 3*a^2 - 4*a
*b - b^2)*sqrt(b/a)*arctan(sqrt(b/a)*sin(x)) + (a*b*cos(x)^2 - a^2 - a*b)*l
og(sin(x) + 1) - (a*b*cos(x)^2 - a^2 - a*b)*log(-sin(x) + 1) - (a*b + b^2)*
sin(x))/(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 - (a^3*b + 2*a^2*b^2 + a*b^3)*co
s(x)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a+b*sin(x)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10629, size = 147, normalized size = 2.01

$$\frac{\left(3\,ab+b^2\right)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{2\left(a^3+2\,a^2b+ab^2\right)\sqrt{ab}}+\frac{\log\left(\sin\left(x\right)+1\right)}{2\left(a^2+2\,ab+b^2\right)}-\frac{\log\left(-\sin\left(x\right)+1\right)}{2\left(a^2+2\,ab+b^2\right)}+\frac{b\sin\left(x\right)}{2\left(b\sin\left(x\right)^2+a\right)\left(a^2+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a+b*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*a*b + b^2)*arctan(b*sin(x)/sqrt(a*b))/((a^3 + 2*a^2*b + a*b^2)*sqrt(
a*b)) + 1/2*log(sin(x) + 1)/(a^2 + 2*a*b + b^2) - 1/2*log(-sin(x) + 1)/(a^2
 + 2*a*b + b^2) + 1/2*b*sin(x)/((b*sin(x)^2 + a)*(a^2 + a*b))
```

3.321 $\int \frac{\sec^2(x)}{\left(a+b\sin^2(x)\right)^2}\,dx$

Optimal. Leaf size=76

$$\frac{b(4a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{5/2}}+\frac{b^2\tan(x)}{2a(a+b)^2\left((a+b)\tan^2(x)+a\right)}+\frac{\tan(x)}{(a+b)^2}$$

```
[Out] (b*(4*a + b)*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(2*a^(3/2)*(a + b)^(5/2)
) + Tan[x]/(a + b)^2 + (b^2*Tan[x])/(2*a*(a + b)^2*(a + (a + b)*Tan[x]^2))
```

Rubi [A] time = 0.123771, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3191, 390, 385, 205}

$$\frac{b(4a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{5/2}}+\frac{b^2\tan(x)}{2a(a+b)^2\left((a+b)\tan^2(x)+a\right)}+\frac{\tan(x)}{(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2/(a + b*Sin[x]^2)^2,x]
```

```
[Out] (b*(4*a + b)*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(2*a^(3/2)*(a + b)^(5/2)
) + Tan[x]/(a + b)^2 + (b^2*Tan[x])/(2*a*(a + b)^2*(a + (a + b)*Tan[x]^2))
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(x)}{\left(a+b\sin^2(x)\right)^2}\,dx &= \text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{\left(a+(a+b)x^2\right)^2}\,dx, x, \tan(x)\right)\\ &= \text{Subst}\left(\int \left(\frac{1}{(a+b)^2}+\frac{b(2a+b)+2b(a+b)x^2}{(a+b)^2\left(a+(a+b)x^2\right)^2}\right)dx, x, \tan(x)\right)\\ &= \frac{\tan(x)}{(a+b)^2}+\frac{\text{Subst}\left(\int \frac{b(2a+b)+2b(a+b)x^2}{\left(a+(a+b)x^2\right)^2}\,dx, x, \tan(x)\right)}{(a+b)^2}\\ &= \frac{\tan(x)}{(a+b)^2}+\frac{b^2\tan(x)}{2a(a+b)^2\left(a+(a+b)\tan^2(x)\right)}+\frac{(b(4a+b))\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \tan(x)\right)}{2a(a+b)^2}\\ &= \frac{b(4a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{5/2}}+\frac{\tan(x)}{(a+b)^2}+\frac{b^2\tan(x)}{2a(a+b)^2\left(a+(a+b)\tan^2(x)\right)}\end{aligned}$$

Mathematica [A] time = 0.518547, size = 76, normalized size = 1.

$$\frac{1}{2}\left(\frac{b(4a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{a^{3/2}(a+b)^{5/2}}+\frac{\frac{b^2\sin(2x)}{a(2a-b\cos(2x)+b)}+2\tan(x)}{(a+b)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/(a + b*Sin[x]^2)^2,x]
```

```
[Out] ((b*(4*a + b)*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(a^(3/2)*(a + b)^(5/2))
 + ((b^2*Sin[2*x])/(a*(2*a + b - b*Cos[2*x])) + 2*Tan[x])/(a + b)^2)/2
```

Maple [A] time = 0.091, size = 112, normalized size = 1.5

$$\frac{\tan(x)}{a^2+2ab+b^2}+\frac{b^2\tan(x)}{2(a+b)^2 a\left((\tan(x))^2 a+(\tan(x))^2 b+a\right)}+2\frac{b}{(a+b)^2\sqrt{a(a+b)}}\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{a(a+b)}}\right)+\frac{b^2}{2(a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(a+b*sin(x)^2)^2,x)
```

```
[Out] 1/(a^2+2*a*b+b^2)*tan(x)+1/2*b^2/(a+b)^2/a*tan(x)/(tan(x)^2*a+tan(x)^2*b+a)
+2*b/(a+b)^2/(a*(a+b))^(1/2)*arctan((a+b)*tan(x)/(a*(a+b))^(1/2))+1/2*b^2/(
a+b)^2/a/(a*(a+b))^(1/2)*arctan((a+b)*tan(x)/(a*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a+b*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.31892, size = 1152, normalized size = 15.16

$$\Bigg[-\frac{((4ab^2+b^3)\cos(x)^3-(4a^2b+5ab^2+b^3)\cos(x))\sqrt{-a^2-ab}\log\Bigg(\frac{(8a^2+8ab+b^2)\cos(x)^4-2(4a^2+5ab+b^2)\cos(x)^2+4((2a+b)\cos \cdots}{b^2\cos(x)^4-2(ab+b^2)\cos(x) \cdots}}{8((a^5b+3a^4b^2+3a^3b^3+a^2b^4)\cos(x)^3-(a^6+ \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a+b*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(((4*a*b^2 + b^3)*cos(x)^3 - (4*a^2*b + 5*a*b^2 + b^3)*cos(x))*sqrt(-
a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2 + 5*a*b + b^2)*co
s(x)^2 + 4*((2*a + b)*cos(x)^3 - (a + b)*cos(x))*sqrt(-a^2 - a*b)*sin(x) +
a^2 + 2*a*b + b^2)/(b^2*cos(x)^4 - 2*(a*b + b^2)*cos(x)^2 + a^2 + 2*a*b + b
^2)) + 4*(2*a^4 + 4*a^3*b + 2*a^2*b^2 - (2*a^3*b + a^2*b^2 - a*b^3)*cos(x)^
2)*sin(x))/((a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*cos(x)^3 - (a^6 + 4*a
^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*cos(x)), -1/4*(((4*a*b^2 + b^3)*cos
(x)^3 - (4*a^2*b + 5*a*b^2 + b^3)*cos(x))*sqrt(a^2 + a*b)*arctan(1/2*((2*a
+ b)*cos(x)^2 - a - b)/(sqrt(a^2 + a*b)*cos(x)*sin(x))) + 2*(2*a^4 + 4*a^3*
b + 2*a^2*b^2 - (2*a^3*b + a^2*b^2 - a*b^3)*cos(x)^2)*sin(x))/((a^5*b + 3*a
^4*b^2 + 3*a^3*b^3 + a^2*b^4)*cos(x)^3 - (a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3
*b^3 + a^2*b^4)*cos(x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(a+b*sin(x)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14809, size = 153, normalized size = 2.01

$$\frac{b^2\tan(x)}{2(a^3+2a^2b+ab^2)(a\tan(x)^2+b\tan(x)^2+a)}+\frac{(4ab+b^2)\arctan\left(\frac{a\tan(x)+b\tan(x)}{\sqrt{a^2+ab}}\right)}{2(a^3+2a^2b+ab^2)\sqrt{a^2+ab}}+\frac{\tan(x)}{a^2+2ab+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a+b*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*b^2*tan(x)/((a^3 + 2*a^2*b + a*b^2)*(a*tan(x)^2 + b*tan(x)^2 + a)) + 1/
2*(4*a*b + b^2)*arctan((a*tan(x) + b*tan(x))/sqrt(a^2 + a*b))/((a^3 + 2*a^2
*b + a*b^2)*sqrt(a^2 + a*b)) + tan(x)/(a^2 + 2*a*b + b^2)
```

3.322 $\int \frac{\sec^3(x)}{(a+b\sin^2(x))^2}\,dx$

Optimal. Leaf size=109

$$\frac{b^{3/2}(5a+b)\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^3}-\frac{b(a-b)\sin(x)}{2a(a+b)^2\left(a+b\sin^2(x)\right)}+\frac{(a+5b)\tanh^{-1}(\sin(x))}{2(a+b)^3}+\frac{\tan(x)\sec(x)}{2(a+b)\left(a+b\sin^2(x)\right)}$$

```
[Out] (b^(3/2)*(5*a + b)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(2*a^(3/2)*(a + b)^3)
+ ((a + 5*b)*ArcTanh[Sin[x]])/(2*(a + b)^3) - ((a - b)*b*Sin[x])/(2*a*(a +
b)^2*(a + b*Sin[x]^2)) + (Sec[x]*Tan[x])/(2*(a + b)*(a + b*Sin[x]^2))
```

Rubi [A] time = 0.163165, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {3190, 414, 527, 522, 206, 205}

$$\frac{b^{3/2}(5a+b)\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^3}-\frac{b(a-b)\sin(x)}{2a(a+b)^2\left(a+b\sin^2(x)\right)}+\frac{(a+5b)\tanh^{-1}(\sin(x))}{2(a+b)^3}+\frac{\tan(x)\sec(x)}{2(a+b)\left(a+b\sin^2(x)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^3/(a + b*Sin[x]^2)^2,x]
```

```
[Out] (b^(3/2)*(5*a + b)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(2*a^(3/2)*(a + b)^3)
+ ((a + 5*b)*ArcTanh[Sin[x]])/(2*(a + b)^3) - ((a - b)*b*Sin[x])/(2*a*(a +
b)^2*(a + b*Sin[x]^2)) + (Sec[x]*Tan[x])/(2*(a + b)*(a + b*Sin[x]^2))
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^3(x)}{\left(a+b\sin^2(x)\right)^2}\,dx &= \text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2\left(a+bx^2\right)^2}\,dx, x, \sin(x)\right)\\
&= \frac{\sec(x)\tan(x)}{2(a+b)\left(a+b\sin^2(x)\right)} + \frac{\text{Subst}\left(\int \frac{a+2b+3bx^2}{\left(1-x^2\right)\left(a+bx^2\right)^2}\,dx, x, \sin(x)\right)}{2(a+b)}\\
&= -\frac{(a-b)b\sin(x)}{2a(a+b)^2\left(a+b\sin^2(x)\right)} + \frac{\sec(x)\tan(x)}{2(a+b)\left(a+b\sin^2(x)\right)} - \frac{\text{Subst}\left(\int \frac{-2\left(a^2+4ab+b^2\right)-2(a-b)bx^2}{\left(1-x^2\right)\left(a+bx^2\right)}\,dx, x
\end{aligned}$$

$$\begin{aligned}
&= -\frac{(a-b)b\sin(x)}{2a(a+b)^2\left(a+b\sin^2(x)\right)} + \frac{\sec(x)\tan(x)}{2(a+b)\left(a+b\sin^2(x)\right)} + \frac{\left(b^2(5a+b)\right)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \text{si}\right.}{2a(a+b)^3}\\
&= \frac{b^{3/2}(5a+b)\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^3} + \frac{(a+5b)\tanh^{-1}(\sin(x))}{2(a+b)^3} - \frac{(a-b)b\sin(x)}{2a(a+b)^2\left(a+b\sin^2(x)\right)} + \frac{\text{s}}{2(a+}
\end{aligned}$$

Mathematica [A] time = 1.01182, size = 183, normalized size = 1.68

$$\frac{\frac{b^{3/2}(5a+b)\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{a^{3/2}} - \frac{b^{3/2}(5a+b)\tan^{-1}\left(\frac{\sqrt{a}\csc(x)}{\sqrt{b}}\right)}{a^{3/2}} + \frac{4b^2(a+b)\sin(x)}{a(2a-b\cos(2x)+b)} + \frac{a+b}{\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)^2} - \frac{a+b}{\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^2} - 2(a+5b)\log\left(\text{co}\right.}{4(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^3/(a + b*Sin[x]^2)^2,x]
```

```
[Out] (-((b^(3/2)*(5*a + b)*ArcTan[(Sqrt[a]*Csc[x])/Sqrt[b]])/a^(3/2)) + (b^(3/2)
*(5*a + b)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/a^(3/2) - 2*(a + 5*b)*Log[Cos[
x/2] - Sin[x/2]] + 2*(a + 5*b)*Log[Cos[x/2] + Sin[x/2]] + (a + b)/(Cos[x/2]
 - Sin[x/2])^2 - (a + b)/(Cos[x/2] + Sin[x/2])^2 + (4*b^2*(a + b)*Sin[x])/(
a*(2*a + b - b*Cos[2*x])))/(4*(a + b)^3)
```

Maple [A] time = 0.079, size = 180, normalized size = 1.7

$$-\frac{1}{4\,(a+b)^2\,(-1+\sin(x))}-\frac{\ln(-1+\sin(x))\,a}{4\,(a+b)^3}-\frac{5\,\ln(-1+\sin(x))\,b}{4\,(a+b)^3}-\frac{1}{4\,(a+b)^2\,(1+\sin(x))}+\frac{\ln(1+\sin(x))\,a}{4\,(a+b)^3}+\frac{5\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^3/(a+b*sin(x)^2)^2,x)
```

```
[Out] -1/4/(a+b)^2/(-1+sin(x))-1/4/(a+b)^3*ln(-1+sin(x))*a-5/4/(a+b)^3*ln(-1+sin(
x))*b-1/4/(a+b)^2/(1+sin(x))+1/4/(a+b)^3*ln(1+sin(x))*a+5/4/(a+b)^3*ln(1+si
n(x))*b+1/2*b^2/(a+b)^3*sin(x)/(a+b*sin(x)^2)+1/2*b^3/(a+b)^3/a*sin(x)/(a+b
*sin(x)^2)+5/2*b^2/(a+b)^3/(a*b)^(1/2)*arctan(sin(x)*b/(a*b)^(1/2))+1/2*b^3
/(a+b)^3/a/(a*b)^(1/2)*arctan(sin(x)*b/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3/(a+b*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.14437, size = 1274, normalized size = 11.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3/(a+b*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(((5*a*b^2 + b^3)*cos(x)^4 - (5*a^2*b + 6*a*b^2 + b^3)*cos(x)^2)*sqrt(
-b/a)*log(-(b*cos(x)^2 - 2*a*sqrt(-b/a)*sin(x) + a - b)/(b*cos(x)^2 - a - b
)) + ((a^2*b + 5*a*b^2)*cos(x)^4 - (a^3 + 6*a^2*b + 5*a*b^2)*cos(x)^2)*log(
sin(x) + 1) - ((a^2*b + 5*a*b^2)*cos(x)^4 - (a^3 + 6*a^2*b + 5*a*b^2)*cos(x
)^2)*log(-sin(x) + 1) - 2*(a^3 + 2*a^2*b + a*b^2 - (a^2*b - b^3)*cos(x)^2)*
sin(x))/((a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(x)^4 - (a^5 + 4*a^4*b
+ 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cos(x)^2), 1/4*(2*((5*a*b^2 + b^3)*cos(x)^
4 - (5*a^2*b + 6*a*b^2 + b^3)*cos(x)^2)*sqrt(b/a)*arctan(sqrt(b/a)*sin(x))
+ ((a^2*b + 5*a*b^2)*cos(x)^4 - (a^3 + 6*a^2*b + 5*a*b^2)*cos(x)^2)*log(sin
(x) + 1) - ((a^2*b + 5*a*b^2)*cos(x)^4 - (a^3 + 6*a^2*b + 5*a*b^2)*cos(x)^2
)*log(-sin(x) + 1) - 2*(a^3 + 2*a^2*b + a*b^2 - (a^2*b - b^3)*cos(x)^2)*sin
(x))/((a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(x)^4 - (a^5 + 4*a^4*b + 6
*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cos(x)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**3/(a+b*sin(x)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.14059, size = 262, normalized size = 2.4

$$\frac{(a+5\,b)\log\left(\sin\left(x\right)+1\right)}{4\left(a^3+3\,a^2b+3\,ab^2+b^3\right)}-\frac{(a+5\,b)\log\left(-\sin\left(x\right)+1\right)}{4\left(a^3+3\,a^2b+3\,ab^2+b^3\right)}+\frac{\left(5\,ab^2+b^3\right)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{2\left(a^4+3\,a^3b+3\,a^2b^2+ab^3\right)\sqrt{ab}}-\frac{ab\sin\left(x\right)^3-b^2\,\mathrm{s}}{2\left(b\sin\left(x\right)^4+a\sin\left(x\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3/(a+b*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/4*(a + 5*b)*log(sin(x) + 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 1/4*(a + 5*
b)*log(-sin(x) + 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 1/2*(5*a*b^2 + b^3)*a
rctan(b*sin(x)/sqrt(a*b))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sqrt(a*b)) -
 1/2*(a*b*sin(x)^3 - b^2*sin(x)^3 + a^2*sin(x) + b^2*sin(x))/((b*sin(x)^4 +
 a*sin(x)^2 - b*sin(x)^2 - a)*(a^3 + 2*a^2*b + a*b^2))
```

3.323 $\int \frac{\sec^4(x)}{\left(a+b\sin^2(x)\right)^2}\,dx$

Optimal. Leaf size=96

$$\frac{b^2(6a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{7/2}}+\frac{b^3\tan(x)}{2a(a+b)^3\left((a+b)\tan^2(x)+a\right)}+\frac{\tan^3(x)}{3(a+b)^2}+\frac{(a+3b)\tan(x)}{(a+b)^3}$$

```
[Out] (b^2*(6*a + b)*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(2*a^(3/2)*(a + b)^(7/
2)) + ((a + 3*b)*Tan[x])/(a + b)^3 + Tan[x]^3/(3*(a + b)^2) + (b^3*Tan[x])/
(2*a*(a + b)^3*(a + (a + b)*Tan[x]^2))
```

Rubi [A] time = 0.147673, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3191, 390, 385, 205}

$$\frac{b^2(6a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{7/2}}+\frac{b^3\tan(x)}{2a(a+b)^3\left((a+b)\tan^2(x)+a\right)}+\frac{\tan^3(x)}{3(a+b)^2}+\frac{(a+3b)\tan(x)}{(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^4/(a + b*Sin[x]^2)^2,x]
```

```
[Out] (b^2*(6*a + b)*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(2*a^(3/2)*(a + b)^(7/
2)) + ((a + 3*b)*Tan[x])/(a + b)^3 + Tan[x]^3/(3*(a + b)^2) + (b^3*Tan[x])/
(2*a*(a + b)^3*(a + (a + b)*Tan[x]^2))
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec^4(x)}{\left(a+b\sin^2(x)\right)^2}\,dx = \text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{\left(a+(a+b)x^2\right)^2}\,dx, x, \tan(x)\right)$$

$$= \text{Subst}\left(\int \left(\frac{a+3b}{(a+b)^3}+\frac{x^2}{(a+b)^2}+\frac{b^2(3a+b)+3b^2(a+b)x^2}{(a+b)^3\left(a+(a+b)x^2\right)^2}\right)dx, x, \tan(x)\right)$$

$$= \frac{(a+3b)\tan(x)}{(a+b)^3}+\frac{\tan^3(x)}{3(a+b)^2}+\frac{\text{Subst}\left(\int \frac{b^2(3a+b)+3b^2(a+b)x^2}{\left(a+(a+b)x^2\right)^2}\,dx, x, \tan(x)\right)}{(a+b)^3}$$

$$= \frac{(a+3b)\tan(x)}{(a+b)^3}+\frac{\tan^3(x)}{3(a+b)^2}+\frac{b^3\tan(x)}{2a(a+b)^3\left(a+(a+b)\tan^2(x)\right)}+\frac{\left(b^2(6a+b)\right)\text{Subst}\left(\int \frac{}{a+(a\cdot}\right.}{2a(a+b)}$$

$$= \frac{b^2(6a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{7/2}}+\frac{(a+3b)\tan(x)}{(a+b)^3}+\frac{\tan^3(x)}{3(a+b)^2}+\frac{b^3\tan(x)}{2a(a+b)^3\left(a+(a+b)\tan^2(x\right.}$$

Mathematica [A] time = 0.970912, size = 97, normalized size = 1.01

$$\frac{1}{6}\left(\frac{3b^2(6a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{a^{3/2}(a+b)^{7/2}}+\frac{\frac{3b^3\sin(2x)}{a(2a-b\cos(2x)+b)}+2(a+b)\tan(x)\sec^2(x)+4a\tan(x)+16b\tan(x)}{(a+b)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^4/(a + b*Sin[x]^2)^2,x]
```

```
[Out] ((3*b^2*(6*a + b)*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(a^(3/2)*(a + b)^(7
/2)) + ((3*b^3*Sin[2*x])/(a*(2*a + b - b*Cos[2*x])) + 4*a*Tan[x] + 16*b*Tan
[x] + 2*(a + b)*Sec[x]^2*Tan[x])/(a + b)^3)/6
```

Maple [B] time = 0.102, size = 193, normalized size = 2.

$$\frac{(\tan(x))^3 a}{\left(3a^2+6ab+3b^2\right)(a+b)}+\frac{(\tan(x))^3 b}{\left(3a^2+6ab+3b^2\right)(a+b)}+\frac{\tan(x)a}{\left(a^2+2ab+b^2\right)(a+b)}+3\frac{\tan(x)b}{\left(a^2+2ab+b^2\right)(a+b)}+\frac{}{2\,(\cdot}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^4/(a+b*sin(x)^2)^2,x)
```

```
[Out] 1/3/(a^2+2*a*b+b^2)/(a+b)*tan(x)^3*a+1/3/(a^2+2*a*b+b^2)/(a+b)*tan(x)^3*b+1
/(a^2+2*a*b+b^2)/(a+b)*tan(x)*a+3/(a^2+2*a*b+b^2)/(a+b)*tan(x)*b+1/2*b^3/(a
+b)^3/a*tan(x)/(tan(x)^2*a+tan(x)^2*b+a)+3*b^2/(a+b)^3/(a*(a+b))^(1/2)*arct
an((a+b)*tan(x)/(a*(a+b))^(1/2))+1/2*b^3/(a+b)^3/a/(a*(a+b))^(1/2)*arctan((
a+b)*tan(x)/(a*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4/(a+b*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.45104, size = 1481, normalized size = 15.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4/(a+b*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((6*a*b^3 + b^4)*cos(x)^5 - (6*a^2*b^2 + 7*a*b^3 + b^4)*cos(x)^3)
*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2 + 5*a*b +
b^2)*cos(x)^2 + 4*((2*a + b)*cos(x)^3 - (a + b)*cos(x))*sqrt(-a^2 - a*b)*si
n(x) + a^2 + 2*a*b + b^2)/(b^2*cos(x)^4 - 2*(a*b + b^2)*cos(x)^2 + a^2 + 2*
a*b + b^2)) + 4*(2*a^5 + 6*a^4*b + 6*a^3*b^2 + 2*a^2*b^3 - (4*a^4*b + 20*a^
3*b^2 + 13*a^2*b^3 - 3*a*b^4)*cos(x)^4 + 2*(2*a^5 + 11*a^4*b + 16*a^3*b^2 +
 7*a^2*b^3)*cos(x)^2)*sin(x))/((a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 +
 a^2*b^5)*cos(x)^5 - (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 +
 a^2*b^5)*cos(x)^3), -1/12*(3*((6*a*b^3 + b^4)*cos(x)^5 - (6*a^2*b^2 + 7*a*
b^3 + b^4)*cos(x)^3)*sqrt(a^2 + a*b)*arctan(1/2*((2*a + b)*cos(x)^2 - a - b
)/(sqrt(a^2 + a*b)*cos(x)*sin(x))) + 2*(2*a^5 + 6*a^4*b + 6*a^3*b^2 + 2*a^2
*b^3 - (4*a^4*b + 20*a^3*b^2 + 13*a^2*b^3 - 3*a*b^4)*cos(x)^4 + 2*(2*a^5 +
11*a^4*b + 16*a^3*b^2 + 7*a^2*b^3)*cos(x)^2)*sin(x))/((a^6*b + 4*a^5*b^2 +
6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*cos(x)^5 - (a^7 + 5*a^6*b + 10*a^5*b^2 + 1
0*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*cos(x)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**4/(a+b*sin(x)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.14682, size = 365, normalized size = 3.8

$$\frac{b^3\tan(x)}{2\left(a^4+3a^3b+3a^2b^2+ab^3\right)\left(a\tan(x)^2+b\tan(x)^2+a\right)}+\frac{\left(6ab^2+b^3\right)\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a+2b)+\arctan\left(\frac{a\tan(x)+b\tan}{\sqrt{a^2+ab}}\right.\right.}{2\left(a^4+3a^3b+3a^2b^2+ab^3\right)\sqrt{a^2+ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4/(a+b*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*b^3*tan(x)/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*(a*tan(x)^2 + b*tan(x)^
2 + a)) + 1/2*(6*a*b^2 + b^3)*(pi*floor(x/pi + 1/2)*sgn(2*a + 2*b) + arctan
```

```
((a*tan(x) + b*tan(x))/sqrt(a^2 + a*b)))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^
3)*sqrt(a^2 + a*b)) + 1/3*(a^4*tan(x)^3 + 4*a^3*b*tan(x)^3 + 6*a^2*b^2*tan(
x)^3 + 4*a*b^3*tan(x)^3 + b^4*tan(x)^3 + 3*a^4*tan(x) + 18*a^3*b*tan(x) + 3
6*a^2*b^2*tan(x) + 30*a*b^3*tan(x) + 9*b^4*tan(x))/(a^6 + 6*a^5*b + 15*a^4*
b^2 + 20*a^3*b^3 + 15*a^2*b^4 + 6*a*b^5 + b^6)
```

3.324 $\int \cos^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=117

$$\frac{a(a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{8b^{3/2}f}-\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4bf}+\frac{(a+4b)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8bf}$$

```
[Out] (a*(a + 4*b)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/(8
*b^(3/2)*f) + ((a + 4*b)*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(8*b*f) -
 (Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2))/(4*b*f)
```

Rubi [A] time = 0.118236, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3190, 388, 195, 217, 206}

$$\frac{a(a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{8b^{3/2}f}-\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4bf}+\frac{(a+4b)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8bf}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (a*(a + 4*b)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/(8
*b^(3/2)*f) + ((a + 4*b)*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(8*b*f) -
 (Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2))/(4*b*f)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \left(1-x^2\right)\sqrt{a+bx^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= -\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4bf} + \frac{(a+4b)\,\text{Subst}\left(\int \sqrt{a+bx^2}\,dx, x, \sin\right.}{4bf}$$

$$= \frac{(a+4b)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8bf} - \frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/}}{4bf}$$

$$= \frac{(a+4b)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8bf} - \frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/}}{4bf}$$

$$= \frac{a(a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{8b^{3/2}f} + \frac{(a+4b)\sin(e+fx)\sqrt{a+b\sin^2(e+fx}}{8bf}$$

Mathematica [A] time = 0.464168, size = 125, normalized size = 1.07

$$\frac{\sqrt{a+b\sin^2(e+fx)}\left(\sqrt{a}(a+4b)\sinh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a}}\right)-\sqrt{b}\sin(e+fx)\left(a+2b\sin^2(e+fx)-4b\right)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\right)}{8b^{3/2}f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]^2]*(Sqrt[a]*(a + 4*b)*ArcSinh[(Sqrt[b]*Sin[e + f*x
])/Sqrt[a]] - Sqrt[b]*Sin[e + f*x]*(a - 4*b + 2*b*Sin[e + f*x]^2)*Sqrt[1 +
(b*Sin[e + f*x]^2)/a]))/(8*b^(3/2)*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a])
```

Maple [A] time = 1.184, size = 155, normalized size = 1.3

$$-\frac{\left(\sin\left(fx+e\right)\right)^3}{4f}\sqrt{a+b\left(\sin\left(fx+e\right)\right)^2}-\frac{a\sin\left(fx+e\right)}{8bf}\sqrt{a+b\left(\sin\left(fx+e\right)\right)^2}+\frac{a^2}{8f}\ln\left(\sin\left(fx+e\right)\sqrt{b}+\sqrt{a+b(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/4/f*sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2)-1/8/f*a/b*sin(f*x+e)*(a+b*sin(
f*x+e)^2)^(1/2)+1/8/f/b^(3/2)*a^2*ln(sin(f*x+e)*b^(1/2)+(a+b*sin(f*x+e)^2)^
(1/2))+1/2*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f+1/2/f*a*ln(sin(f*x+e)*b^(1
```

```
/2)+(a+b*sin(f*x+e)^2)^(1/2))/b^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 5.15004, size = 1239, normalized size = 10.59

$$\left[\frac{\left(a^2+4\,ab\right)\sqrt{b}\log\left(128\,b^4\cos\left(fx+e\right)^8-256\left(ab^3+2\,b^4\right)\cos\left(fx+e\right)^6+32\left(5\,a^2b^2+24\,ab^3+24\,b^4\right)\cos\left(fx+e\right)^4+a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/64*((a^2 + 4*a*b)*sqrt(b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + 2*b^
4)*cos(f*x + e)^6 + 32*(5*a^2*b^2 + 24*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^4
 + 32*a^3*b + 160*a^2*b^2 + 256*a*b^3 + 128*b^4 - 32*(a^3*b + 10*a^2*b^2 +
24*a*b^3 + 16*b^4)*cos(f*x + e)^2 - 8*(16*b^3*cos(f*x + e)^6 - 24*(a*b^2 +
2*b^3)*cos(f*x + e)^4 - a^3 - 10*a^2*b - 24*a*b^2 - 16*b^3 + 2*(5*a^2*b + 2
4*a*b^2 + 24*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(b)*s
in(f*x + e)) + 8*(2*b^2*cos(f*x + e)^2 - a*b + 2*b^2)*sqrt(-b*cos(f*x + e)^
2 + a + b)*sin(f*x + e))/(b^2*f), -1/32*((a^2 + 4*a*b)*sqrt(-b)*arctan(1/4*
(8*b^2*cos(f*x + e)^4 - 8*(a*b + 2*b^2)*cos(f*x + e)^2 + a^2 + 8*a*b + 8*b^
2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)/((2*b^3*cos(f*x + e)^4 + a^2*b
+ 3*a*b^2 + 2*b^3 - (3*a*b^2 + 4*b^3)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(2
*b^2*cos(f*x + e)^2 - a*b + 2*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sin(f*x
+ e))/(b^2*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18608, size = 130, normalized size = 1.11

$$-\frac{\sqrt{b\sin\left(fx+e\right)^{2}+a}\left(2\,\sin\left(fx+e\right)^{2}+\frac{ab-4\,b^{2}}{b^{2}}\right)\sin\left(fx+e\right)+\frac{\left(a^{2}+4\,ab\right)\log\left(\left|-\sqrt{b}\sin\left(fx+e\right)+\sqrt{b\sin\left(fx+e\right)^{2}+a}\right|\right)}{b^{\frac{3}{2}}}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*(sqrt(b*sin(f*x + e)^2 + a)*(2*sin(f*x + e)^2 + (a*b - 4*b^2)/b^2)*sin
(f*x + e) + (a^2 + 4*a*b)*log(abs(-sqrt(b)*sin(f*x + e) + sqrt(b*sin(f*x +
e)^2 + a)))/b^(3/2))/f
```

3.325 $\int \cos(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=72

$$\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2f}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2\sqrt{b}f}$$

```
[Out] (a*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/(2*Sqrt[b]*f
) + (Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(2*f)
```

Rubi [A] time = 0.0547196, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3190, 195, 217, 206}

$$\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2f}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2\sqrt{b}f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (a*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/(2*Sqrt[b]*f
) + (Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(2*f)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \sqrt{a+bx^2}\,dx, x, \sin(e+fx)\right)}{f}$$
$$= \frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2f} + \frac{a\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sin(e+fx)\right)}{2f}$$
$$= \frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2f} + \frac{a\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2f}$$
$$= \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2\sqrt{b}f} + \frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2f}$$

Mathematica [A] time = 0.271186, size = 96, normalized size = 1.33

$$\frac{a^{3/2}\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a}}\right)+\sqrt{b}\sin(e+fx)\left(a+b\sin^2(e+fx)\right)}{2\sqrt{b}f\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (Sqrt[b]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2) + a^(3/2)*ArcSinh[(Sqrt[b]*Sin
[e + f*x])/Sqrt[a]]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(2*Sqrt[b]*f*Sqrt[a + b
*Sin[e + f*x]^2])
```

Maple [A] time = 0.088, size = 62, normalized size = 0.9

$$\frac{\sin(fx+e)}{2f}\sqrt{a+b\left(\sin(fx+e)\right)^2}+\frac{a}{2f}\ln\left(\sin(fx+e)\sqrt{b}+\sqrt{a+b\left(\sin(fx+e)\right)^2}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/2*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f+1/2/f*a*ln(sin(f*x+e)*b^(1/2)+(a+
b*sin(f*x+e)^2)^(1/2))/b^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.82302, size = 1106, normalized size = 15.36

$$\left[\frac{a\sqrt{b}\log\left(128\,b^4\cos\left(fx+e\right)^8-256\left(ab^3+2\,b^4\right)\cos\left(fx+e\right)^6+32\left(5\,a^2b^2+24\,ab^3+24\,b^4\right)\cos\left(fx+e\right)^4+a^4+32\,a^3b\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(a*sqrt(b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + 2*b^4)*cos(f*x +
 e)^6 + 32*(5*a^2*b^2 + 24*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^4 + 32*a^3*b
+ 160*a^2*b^2 + 256*a*b^3 + 128*b^4 - 32*(a^3*b + 10*a^2*b^2 + 24*a*b^3 + 1
6*b^4)*cos(f*x + e)^2 - 8*(16*b^3*cos(f*x + e)^6 - 24*(a*b^2 + 2*b^3)*cos(f
*x + e)^4 - a^3 - 10*a^2*b - 24*a*b^2 - 16*b^3 + 2*(5*a^2*b + 24*a*b^2 + 24
*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(b)*sin(f*x + e))
 + 8*sqrt(-b*cos(f*x + e)^2 + a + b)*b*sin(f*x + e))/(b*f), -1/8*(a*sqrt(-b
)*arctan(1/4*(8*b^2*cos(f*x + e)^4 - 8*(a*b + 2*b^2)*cos(f*x + e)^2 + a^2 +
 8*a*b + 8*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)/((2*b^3*cos(f*x +
e)^4 + a^2*b + 3*a*b^2 + 2*b^3 - (3*a*b^2 + 4*b^3)*cos(f*x + e)^2)*sin(f*x
+ e))) - 4*sqrt(-b*cos(f*x + e)^2 + a + b)*b*sin(f*x + e))/(b*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18914, size = 88, normalized size = 1.22

$$-\frac{\frac{a\log\left(\left|-\sqrt{b}\sin\left(fx+e\right)+\sqrt{b\sin\left(fx+e\right)^2+a}\right|\right)}{\sqrt{b}}-\sqrt{b\sin\left(fx+e\right)^2+a}\sin\left(fx+e\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(a*log(abs(-sqrt(b)*sin(f*x + e) + sqrt(b*sin(f*x + e)^2 + a)))/sqrt(b
) - sqrt(b*sin(f*x + e)^2 + a)*sin(f*x + e))/f
```

3.326 $\int \sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=82

$$\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f}$$

```
[Out] -((Sqrt[b]*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/f) +
 (Sqrt[a + b]*ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]
])/f
```

Rubi [A] time = 0.0923954, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3190, 402, 217, 206, 377}

$$\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((Sqrt[b]*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/f) +
 (Sqrt[a + b]*ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]
])/f
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\int \sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{1-x^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= -\frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sin(e+fx)\right)}{f} + \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \sin\right.}{f}$$

$$= -\frac{b\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f} + \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\,dx, x, \frac{\sin(}{\sqrt{a+b\,\text{s}}}\right.}{f}$$

$$= -\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f} + \frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f}$$

Mathematica [A] time = 0.278162, size = 129, normalized size = 1.57

$$\frac{\frac{\sqrt{a}\sqrt{-b}\sin^{-1}\left(\frac{\sqrt{-b}\sin(e+fx)}{\sqrt{a}}\right)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}}{\sqrt{2a-b\cos(2(e+fx))+b}} + \sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{2a+2b}\sin(e+fx)}{\sqrt{2a-b\cos(2(e+fx))+b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (Sqrt[a + b]*ArcTanh[(Sqrt[2*a + 2*b]*Sin[e + f*x])/Sqrt[2*a + b - b*Cos[2*
(e + f*x)]]] + (Sqrt[a]*Sqrt[-b]*ArcSin[(Sqrt[-b]*Sin[e + f*x])/Sqrt[a]]*Sq
rt[(2*a + b - b*Cos[2*(e + f*x)])/a])/Sqrt[2*a + b - b*Cos[2*(e + f*x)]])/f
```

Maple [B] time = 3.635, size = 155, normalized size = 1.9

$$-\frac{1}{f}\sqrt{b}\ln\left(\left(\sqrt{a+b-b\left(\cos\left(fx+e\right)\right)^2}\sqrt{b}+b\sin\left(fx+e\right)\right)\frac{1}{\sqrt{b}}\right)+\frac{1}{2f}\sqrt{a+b}\ln\left(2\,\frac{\sqrt{a+b}\sqrt{a+b-b\left(\cos\left(fx+e\right)\right)^2}+b}{-1+\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/f*b^(1/2)*ln(((a+b-b*cos(f*x+e)^2)^(1/2)*b^(1/2)+b*sin(f*x+e))/b^(1/2))+
1/2/f*(a+b)^(1/2)*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1
/2)+b*sin(f*x+e)+a))-1/2/f*(a+b)^(1/2)*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+
b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.97599, size = 3027, normalized size = 36.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + 2*b^4)*cos(f*x + e)
^6 + 32*(5*a^2*b^2 + 24*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^4 + 32*a^3*b + 1
60*a^2*b^2 + 256*a*b^3 + 128*b^4 - 32*(a^3*b + 10*a^2*b^2 + 24*a*b^3 + 16*b
^4)*cos(f*x + e)^2 + 8*(16*b^3*cos(f*x + e)^6 - 24*(a*b^2 + 2*b^3)*cos(f*x
+ e)^4 - a^3 - 10*a^2*b - 24*a*b^2 - 16*b^3 + 2*(5*a^2*b + 24*a*b^2 + 24*b^
3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(b)*sin(f*x + e)) +
2*sqrt(a + b)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 8*(a^2 + 3*a*b +
2*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*cos(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*co
s(f*x + e)^2 + a + b)*sqrt(a + b)*sin(f*x + e) + 8*a^2 + 16*a*b + 8*b^2)/co
s(f*x + e)^4))/f, -1/8*(4*sqrt(-a - b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2
 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(((a*b + b^2)*co
s(f*x + e)^2 - a^2 - 2*a*b - b^2)*sin(f*x + e))) - sqrt(b)*log(128*b^4*cos(
f*x + e)^8 - 256*(a*b^3 + 2*b^4)*cos(f*x + e)^6 + 32*(5*a^2*b^2 + 24*a*b^3
+ 24*b^4)*cos(f*x + e)^4 + a^4 + 32*a^3*b + 160*a^2*b^2 + 256*a*b^3 + 128*b
^4 - 32*(a^3*b + 10*a^2*b^2 + 24*a*b^3 + 16*b^4)*cos(f*x + e)^2 + 8*(16*b^3
*cos(f*x + e)^6 - 24*(a*b^2 + 2*b^3)*cos(f*x + e)^4 - a^3 - 10*a^2*b - 24*a
*b^2 - 16*b^3 + 2*(5*a^2*b + 24*a*b^2 + 24*b^3)*cos(f*x + e)^2)*sqrt(-b*cos
(f*x + e)^2 + a + b)*sqrt(b)*sin(f*x + e)))/f, 1/4*(sqrt(-b)*arctan(1/4*(8*
b^2*cos(f*x + e)^4 - 8*(a*b + 2*b^2)*cos(f*x + e)^2 + a^2 + 8*a*b + 8*b^2)*
sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)/((2*b^3*cos(f*x + e)^4 + a^2*b + 3
*a*b^2 + 2*b^3 - (3*a*b^2 + 4*b^3)*cos(f*x + e)^2)*sin(f*x + e))) + sqrt(a
+ b)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 8*(a^2 + 3*a*b + 2*b^2)*co
s(f*x + e)^2 - 4*((a + 2*b)*cos(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x + e
)^2 + a + b)*sqrt(a + b)*sin(f*x + e) + 8*a^2 + 16*a*b + 8*b^2)/cos(f*x + e
)^4))/f, -1/4*(2*sqrt(-a - b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - 2*a -
2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(((a*b + b^2)*cos(f*x + e
)^2 - a^2 - 2*a*b - b^2)*sin(f*x + e))) - sqrt(-b)*arctan(1/4*(8*b^2*cos(f*
x + e)^4 - 8*(a*b + 2*b^2)*cos(f*x + e)^2 + a^2 + 8*a*b + 8*b^2)*sqrt(-b*co
s(f*x + e)^2 + a + b)*sqrt(-b)/((2*b^3*cos(f*x + e)^4 + a^2*b + 3*a*b^2 + 2
*b^3 - (3*a*b^2 + 4*b^3)*cos(f*x + e)^2)*sin(f*x + e))))/f]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\sin^2\left(e + fx\right)}\sec\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*sec(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin\left(fx+e\right)^2+a}\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*sec(f*x + e), x)
```

3.327 $\int \sec^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=82

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2f\sqrt{a+b}}+\frac{\tan(e+fx)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2f}$$

```
[Out] (a*ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/(2*Sqrt[
a + b]*f) + (Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x])/(2*f)
```

Rubi [A] time = 0.0941853, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3190, 378, 377, 206}

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2f\sqrt{a+b}}+\frac{\tan(e+fx)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (a*ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/(2*Sqrt[
a + b]*f) + (Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x])/(2*f)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{\left(1-x^2\right)^2}dx, x, \sin(e+fx)\right)}{f}$$
$$= \frac{\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{2f} + \frac{a\,\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\sqrt{a+bx^2}}dx, x, \sin\right.}{2f}$$
$$= \frac{\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{2f} + \frac{a\,\text{Subst}\left(\int \frac{1}{1-(a+b)x^2}dx, x, \frac{\sin(}{\sqrt{a+b\,\text{s}}}\right.}{2f}$$
$$= \frac{a\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2\sqrt{a+b}f} + \frac{\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{2f}$$

Mathematica [A] time = 2.329, size = 164, normalized size = 2.

$$\frac{\sin(e+fx)\left(\sqrt{2}a\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}\tanh^{-1}\left(\frac{\sqrt{\frac{(a+b)\sin^2(e+fx)}{a}}}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}\right) + \sec^2(e+fx)\sqrt{\frac{(a+b)\sin^2(e+fx)}{a}}(2a-b\cos(2(e+fx))+b)\right)}{4f\sqrt{\frac{(a+b)\sin^2(e+fx)}{a}}\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^3*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (Sin[e + f*x]*(Sqrt[2]*a*ArcTanh[Sqrt[((a + b)*Sin[e + f*x]^2)/a]/Sqrt[1 +
(b*Sin[e + f*x]^2)/a]]*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a] + (2*a + b -
b*Cos[2*(e + f*x)])*Sec[e + f*x]^2*Sqrt[((a + b)*Sin[e + f*x]^2)/a]))/(4*f*
Sqrt[((a + b)*Sin[e + f*x]^2)/a]*Sqrt[a + b*Sin[e + f*x]^2])
```

Maple [B] time = 3.897, size = 290, normalized size = 3.5

$$\frac{1}{4\left(\cos\left(fx+e\right)\right)^2 f}\left(2\sqrt{a+b}\sqrt{a+b-b\left(\cos\left(fx+e\right)\right)^2}b\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^2 + 2\left(a+b-b\left(\cos\left(fx+e\right)\right)^2\right)^{3/2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/4*(2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)*b*sin(f*x+e)*cos(f*x+e)^2+2*(
a+b-b*cos(f*x+e)^2)^(3/2)*(a+b)^(1/2)*sin(f*x+e)+a*(ln(2/(-1+sin(f*x+e))*((
a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a+ln(2/(-1+sin(f*x+e
))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*b-ln(2/(1+sin(f
*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a-ln(2/(1+s
in(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*b)*cos(
f*x+e)^2)/(a+b)^(3/2)/cos(f*x+e)^2/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin\left(fx+e\right)^2+a}\sec\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*sec(f*x + e)^3, x)
```

Fricas [B] time = 2.74399, size = 852, normalized size = 10.39

$$\left[\frac{\sqrt{a+b}a\cos\left(fx+e\right)^2\log\left(\frac{\left(a^2+8\,ab+8\,b^2\right)\cos\left(fx+e\right)^4-8\left(a^2+3\,ab+2\,b^2\right)\cos\left(fx+e\right)^2-4\left(\left(a+2\,b\right)\cos\left(fx+e\right)^2-2\,a-2\,b\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}\sqrt{a+b}}{\cos\left(fx+e\right)^4}\right.}{8\left(a+b\right)f\cos\left(fx+e\right)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a + b)*a*cos(f*x + e)^2*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^
4 - 8*(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*cos(f*x + e)^2 -
2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b)*sin(f*x + e) + 8*a^2
 + 16*a*b + 8*b^2)/cos(f*x + e)^4) + 4*sqrt(-b*cos(f*x + e)^2 + a + b)*(a +
 b)*sin(f*x + e))/((a + b)*f*cos(f*x + e)^2), -1/4*(a*sqrt(-a - b)*arctan(1
/2*((a + 2*b)*cos(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*s
qrt(-a - b)/(((a*b + b^2)*cos(f*x + e)^2 - a^2 - 2*a*b - b^2)*sin(f*x + e))
)*cos(f*x + e)^2 - 2*sqrt(-b*cos(f*x + e)^2 + a + b)*(a + b)*sin(f*x + e))/
((a + b)*f*cos(f*x + e)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin\left(fx+e\right)^2+a}\sec\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*sec(f*x + e)^3, x)
```

3.328 $\int \sec^5(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=143

$$\frac{a(3a+4b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{8f(a+b)^{3/2}}+\frac{\tan(e+fx)\sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4f(a+b)}+\frac{(3a+4b)\tan(e+fx)\sec(e+}{8f(a+}$$

```
[Out] (a*(3*a + 4*b)*ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2
]])/(8*(a + b)^(3/2)*f) + ((3*a + 4*b)*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]
^2]*Tan[e + f*x])/(8*(a + b)*f) + (Sec[e + f*x]^3*(a + b*Sin[e + f*x]^2)^(3
/2)*Tan[e + f*x])/(4*(a + b)*f)
```

Rubi [A] time = 0.139333, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3190, 382, 378, 377, 206}

$$\frac{a(3a+4b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{8f(a+b)^{3/2}}+\frac{\tan(e+fx)\sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4f(a+b)}+\frac{(3a+4b)\tan(e+fx)\sec(e+}{8f(a+}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^5*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (a*(3*a + 4*b)*ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2
]])/(8*(a + b)^(3/2)*f) + ((3*a + 4*b)*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]
^2]*Tan[e + f*x])/(8*(a + b)*f) + (Sec[e + f*x]^3*(a + b*Sin[e + f*x]^2)^(3
/2)*Tan[e + f*x])/(4*(a + b)*f)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^5(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{\left(1-x^2\right)^3}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\tan(e+fx)}{4(a+b)f} + \frac{(3a+4b)\,\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{\left(1-x^2\right)^2}\,dx\right.}{4(a+b)f}$$

$$= \frac{(3a+4b)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{8(a+b)f} + \frac{\sec^3(e+fx)\left(a+b\sin\right.}{4(a}$$

$$= \frac{(3a+4b)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{8(a+b)f} + \frac{\sec^3(e+fx)\left(a+b\sin\right.}{4(a}$$

$$= \frac{a(3a+4b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{8(a+b)^{3/2}f} + \frac{(3a+4b)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8(a+b)f}$$

Mathematica [C] time = 14.3778, size = 669, normalized size = 4.68

$$-\frac{\tan(e+fx)\sec^3(e+fx)\left(\frac{b\sin^2(e+fx)}{a}+1\right)\left(10b\sin^2(e+fx)\sqrt{-\frac{(a+b)\tan^2(e+fx)\sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)}{a^2}}+15a\sqrt{-\frac{(a+b)\tan^2(e\cdots}{}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^5*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -(Sec[e + f*x]^3*(1 + (b*Sin[e + f*x]^2)/a)*Tan[e + f*x]*(-15*a*ArcSin[Sqrt
[-(((a + b)*Tan[e + f*x]^2)/a)]] - 10*b*ArcSin[Sqrt[-(((a + b)*Tan[e + f*x]
^2)/a)]]*Sin[e + f*x]^2 - 30*a*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))
/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(3/2) - 20*b*Sin[e + f*x]^2*Sqrt[(Sec[e
 + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(3/2)
- 32*a*Hypergeometric2F1[2, 4, 7/2, -(((a + b)*Tan[e + f*x]^2)/a)]*Sqrt[(Se
c[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(5/
2) - 32*b*Hypergeometric2F1[2, 4, 7/2, -(((a + b)*Tan[e + f*x]^2)/a)]*Sin[e
 + f*x]^2*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*Tan[e
 + f*x]^2)/a))^(5/2) + 32*a*Hypergeometric2F1[2, 4, 7/2, -(((a + b)*Tan[e +
 f*x]^2)/a)]*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*Ta
n[e + f*x]^2)/a))^(7/2) + 32*b*Hypergeometric2F1[2, 4, 7/2, -(((a + b)*Tan[
e + f*x]^2)/a)]*Sin[e + f*x]^2*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))
/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(7/2) + 15*a*Sqrt[-(((a + b)*Sec[e + f*
```

```
x]^2*(a + b*Sin[e + f*x]^2)*Tan[e + f*x]^2)/a^2)] + 10*b*Sin[e + f*x]^2*Sqr
t[-(((a + b)*Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)*Tan[e + f*x]^2)/a^2)]))/
(40*f*Sqrt[a + b*Sin[e + f*x]^2]*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2
))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(3/2))
```

Maple [B] time = 4.52, size = 570, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^5*(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/16*(2*(a+b)^(3/2)*(a+b-b*cos(f*x+e)^2)^(1/2)*b*(3*a+4*b)*sin(f*x+e)*cos(f
*x+e)^4+2*(a+b)^(3/2)*(a+b-b*cos(f*x+e)^2)^(3/2)*(3*a+4*b)*cos(f*x+e)^2*sin
(f*x+e)+4*(a+b)^(5/2)*(a+b-b*cos(f*x+e)^2)^(3/2)*sin(f*x+e)+a*(3*ln(2/(-1+s
in(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^3+10*
ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a
))*a^2*b+11*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*
sin(f*x+e)+a))*a*b^2+4*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^
2)^(1/2)+b*sin(f*x+e)+a))*b^3-3*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos
(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^3-10*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(
a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^2*b-11*ln(2/(1+sin(f*x+e))*((a
+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a*b^2-4*ln(2/(1+sin(f
*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*b^3)*cos(f*
x+e)^4)/(a+b)^(3/2)/cos(f*x+e)^4/(a^2+2*a*b+b^2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sin\left(fx+e\right)^2 + a} \sec\left(fx+e\right)^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*sec(f*x + e)^5, x)
```

Fricas [A] time = 5.38284, size = 1079, normalized size = 7.55

$$\left[\frac{\left(3\,a^2+4\,ab\right)\sqrt{a+b}\cos\left(fx+e\right)^4\log\left(\frac{\left(a^2+8\,ab+8\,b^2\right)\cos\left(fx+e\right)^4-8\left(a^2+3\,ab+2\,b^2\right)\cos\left(fx+e\right)^2-4\left(\left(a+2\,b\right)\cos\left(fx+e\right)^2-2\,a-2\,b\right)\sqrt{-b\cos\left(fx\right.}}{\cos\left(fx+e\right)^4}\right.}{32\left(a^2+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*((3*a^2 + 4*a*b)*sqrt(a + b)*cos(f*x + e)^4*log(((a^2 + 8*a*b + 8*b^2
)*cos(f*x + e)^4 - 8*(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*co
s(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b)*sin(f
*x + e) + 8*a^2 + 16*a*b + 8*b^2)/cos(f*x + e)^4) + 4*((3*a^2 + 5*a*b + 2*b
^2)*cos(f*x + e)^2 + 2*a^2 + 4*a*b + 2*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)
*sin(f*x + e))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^4), -1/16*((3*a^2 + 4*a*
b)*sqrt(-a - b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*c
os(f*x + e)^2 + a + b)*sqrt(-a - b)/(((a*b + b^2)*cos(f*x + e)^2 - a^2 - 2*
a*b - b^2)*sin(f*x + e)))*cos(f*x + e)^4 - 2*((3*a^2 + 5*a*b + 2*b^2)*cos(f
*x + e)^2 + 2*a^2 + 4*a*b + 2*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sin(f*x
+ e))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**5*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin\left(fx+e\right)^{2}+a}\sec\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*sec(f*x + e)^5, x)
```

3.329 $\int \cos^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=220

$$-\frac{\left(2a^2+7ab-3b^2\right)\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{15b^2f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{2a(a+b)(a+3b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(e+fx\middle|-\frac{b}{a}\right)}{15b^2f\sqrt{a+b\sin^2(e+fx)}}-\frac{\sin(e+\ldots}{}$$

```
[Out] (2*(a + 3*b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(15*b*f)
 - (Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2))/(5*b*f) - ((2*a
^2 + 7*a*b - 3*b^2)*EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/
(15*b^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (2*a*(a + b)*(a + 3*b)*Elliptic
F[e + f*x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(15*b^2*f*Sqrt[a + b*Sin
[e + f*x]^2])
```

Rubi [A] time = 0.275173, antiderivative size = 260, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3192, 416, 528, 524, 426, 424, 421, 419}

$$-\frac{\left(2a^2+7ab-3b^2\right)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{15b^2f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{2a(a+b)(a+3b)\sqrt{\cos\ldots}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (2*(a + 3*b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(15*b*f)
 - (Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2))/(5*b*f) - ((2*a
^2 + 7*a*b - 3*b^2)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(
b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(15*b^2*f*Sqrt[1 + (b*Sin[e
+ f*x]^2)/a]) + (2*a*(a + b)*(a + 3*b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSi
n[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(15*b
^2*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \cos^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int\left(1-x^2\right)^{3/2}\sqrt{a+bx^2}\,dx,x,\sin(e+fx\right.}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{5bf} + \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+f\right.}{}$$

$$= \frac{2(a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15bf} - \frac{\cos(e+fx)\sin(e+}{}$$

$$= \frac{2(a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15bf} - \frac{\cos(e+fx)\sin(e+}{}$$

$$= \frac{2(a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15bf} - \frac{\cos(e+fx)\sin(e+}{}$$

$$= \frac{2(a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15bf} - \frac{\cos(e+fx)\sin(e+}{}$$

Mathematica [A] time = 1.44337, size = 199, normalized size = 0.9

$$\frac{-\sqrt{2}b\sin(2(e+fx))\left(8a^2-4b(4a-3b)\cos(2(e+fx))-32ab+3b^2\cos(4(e+fx))-15b^2\right)+32a\left(a^2+4ab+3b^2\right)\sqrt{}}{240b^2f\sqrt{2a-b\cos(2(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-16*a*(2*a^2 + 7*a*b - 3*b^2)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*Ellip
ticE[e + f*x, -(b/a)] + 32*a*(a^2 + 4*a*b + 3*b^2)*Sqrt[(2*a + b - b*Cos[2*
(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)] - Sqrt[2]*b*(8*a^2 - 32*a*b - 15*
b^2 - 4*(4*a - 3*b)*b*Cos[2*(e + f*x)] + 3*b^2*Cos[4*(e + f*x)])*Sin[2*(e +
 f*x)])/(240*b^2*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A] time = 1.106, size = 432, normalized size = 2.

$$\frac{1}{15\,b^2\cos\left(fx+e\right)f}\left(-3\,b^3\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^6+4\,ab^2\left(\cos\left(fx+e\right)\right)^4\sin\left(fx+e\right)+\left(-a^2b+2\,ab^2+3\,b^3\right)\left(c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/15*(-3*b^3*sin(f*x+e)*cos(f*x+e)^6+4*a*b^2*cos(f*x+e)^4*sin(f*x+e)+(-a^2*
b+2*a*b^2+3*b^3)*cos(f*x+e)^2*sin(f*x+e)+2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f
*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3+8*(cos(f*x+
```

```
e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)
^(1/2))*a^2*b+6*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*Elli
pticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2-2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x
+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3-7*(cos(f*x+e)
^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(
1/2))*a^2*b+3*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*Ellipt
icE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2)/b^2/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/
2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin\left(fx+e\right)^2+a}\cos\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*cos(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{-b\cos\left(fx+e\right)^2+a+b}\cos\left(fx+e\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)*cos(f*x + e)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin\left(fx+e\right)^2+a}\cos\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*cos(f*x + e)^4, x)
```

3.330 $\int \cos^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=159

$$\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{3bf\sqrt{a+b\sin^2(e+fx)}}-\frac{(a-b)\sqrt{a+b\sin^2(e+fx)}E}{3bf\sqrt{\frac{b\sin^2(e+fx)}{a}+}}$$

```
[Out] (Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f) - ((a - b)*Ell
ipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(3*b*f*Sqrt[1 + (b*Sin[
e + f*x]^2)/a]) + (a*(a + b)*EllipticF[e + f*x, -(b/a)]*Sqrt[1 + (b*Sin[e +
 f*x]^2)/a])/(3*b*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.183763, antiderivative size = 199, normalized size of antiderivative = 1.25, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3192, 417, 524, 426, 424, 421, 419}

$$\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{a(a+b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\sin^{-1}(\sin(e+fx\right.}{3bf\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f) - ((a - b)*Sqr
t[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt
[a + b*Sin[e + f*x]^2])/(3*b*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (a*(a + b)
*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*
Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*b*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 417

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(x*(a + b*x^n)^p*(c + d*x^n)^q)/(n*(p + q) + 1), x] + Dist[n/(n*(p
+ q) + 1), Int[(a + b*x^n)^(p - 1)*(c + d*x^n)^(q - 1)*Simp[a*c*(p + q) + (
q*(b*c - a*d) + a*d*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] &
& NeQ[b*c - a*d, 0] && GtQ[q, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, n
, p, q, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
```

```
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \cos^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int\sqrt{1-x^2}\sqrt{a+bx^2}\,dx,x,\sin(e+fx)\right)}{f}$$

$$= \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} + \frac{\left(2\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Su}}{}$$

$$= \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} - \frac{\left((a-b)\sqrt{\cos^2(e+fx)}\sec(e+fx\right.}{}$$

$$= \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} - \frac{\left((a-b)\sqrt{\cos^2(e+fx)}\sec(e+fx\right.}{}$$

$$= \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} - \frac{(a-b)\sqrt{\cos^2(e+fx)}E\left(\sin^{-1}(\sin\right.}{3b}$$

Mathematica [A] time = 0.882895, size = 158, normalized size = 0.99

$$\frac{b\sin(2(e+fx))(2a-b\cos(2(e+fx))+b)+2\sqrt{2}a(a+b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}F\left(e+fx\middle|-\frac{b}{a}\right)-2\sqrt{2}a(a-b)\sqrt{\frac{2a-b\cos(2(e+fx}{a}}}{6\sqrt{2}bf\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-2*Sqrt[2]*a*(a - b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e +
f*x, -(b/a)] + 2*Sqrt[2]*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*E
llipticF[e + f*x, -(b/a)] + b*(2*a + b - b*Cos[2*(e + f*x)])*Sin[2*(e + f*x
)])/(6*Sqrt[2]*b*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A] time = 1.167, size = 265, normalized size = 1.7

$$\frac{1}{3b\cos\left(fx+e\right)f}\left(-\left(\sin\left(fx+e\right)\right)^{5}b^{2}+\sqrt{\left(\cos\left(fx+e\right)\right)^{2}}\sqrt{\frac{a+b\left(\sin\left(fx+e\right)\right)^{2}}{a}}\mathrm{EllipticF}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)a^{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3*(-sin(f*x+e)^5*b^2+(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*El
lipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2+a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x
+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-(cos(f*x+e)^2)^(1/2)
*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2+(cos
(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)
^(1/2))*a*b-sin(f*x+e)^3*a*b+sin(f*x+e)^3*b^2+sin(f*x+e)*a*b)/b/cos(f*x+e)/
(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin\left(fx+e\right)^{2}+a}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*cos(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\cos\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)*cos(f*x + e)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sin \left(fx + e\right)^2 + a} \cos \left(fx + e\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*cos(f*x + e)^2, x)
```

3.331 $\int \sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=51

$$\frac{\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

```
[Out] (EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(f*Sqrt[1 + (b*Sin[
e + f*x]^2)/a])
```

Rubi [A] time = 0.0352502, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3178, 3177}

$$\frac{\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(f*Sqrt[1 + (b*Sin[
e + f*x]^2)/a])
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+b\sin^2(e+fx)}\,dx &= \frac{\sqrt{a+b\sin^2(e+fx)}\int \sqrt{1+\frac{b\sin^2(e+fx)}{a}}\,dx}{\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}\\ &= \frac{E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}\end{aligned}$$

Mathematica [A] time = 0.0856124, size = 61, normalized size = 1.2

$$\frac{a\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)])/(f*Sq
rt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A] time = 0.635, size = 71, normalized size = 1.4

$$\frac{a}{f\cos\left(fx+e\right)}\sqrt{\left(\cos\left(fx+e\right)\right)^{2}}\sqrt{\frac{a+b\left(\sin\left(fx+e\right)\right)^{2}}{a}}\mathrm{EllipticE}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)\frac{1}{\sqrt{a+b\left(\sin\left(fx+e\right)\right)^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-
1/a*b)^(1/2))/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sin\left(fx+e\right)^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{-b\cos\left(fx+e\right)^{2}+a+b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a+b\sin^{2}\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sin \left(fx + e\right)^{2} + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a), x)
```

3.332 $\int \sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=131

$$\frac{\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f}+\frac{a\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}-\frac{\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

```
[Out] -((EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(f*Sqrt[1 + (b*Si
n[e + f*x]^2)/a])) + (a*EllipticF[e + f*x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]
^2)/a])/(f*Sqrt[a + b*Sin[e + f*x]^2]) + (Sqrt[a + b*Sin[e + f*x]^2]*Tan[e
+ f*x])/f
```

Rubi [A] time = 0.165278, antiderivative size = 171, normalized size of antiderivative = 1.31, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3192, 412, 12, 493, 426, 424, 421, 419}

$$\frac{\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f}+\frac{a\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}-\frac{\sqrt{\cos^2(e+}\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x
]*Sqrt[a + b*Sin[e + f*x]^2])/(f*Sqrt[1 + (b*Sin[e + f*x]^2)/a])) + (a*Sqrt
[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[
1 + (b*Sin[e + f*x]^2)/a])/(f*Sqrt[a + b*Sin[e + f*x]^2]) + (Sqrt[a + b*Sin
[e + f*x]^2]*Tan[e + f*x])/f
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] && !IntegerQ[p]
```

Rule 412

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] + Dist[1/(
a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p + 1)
 + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x]
&& NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 493

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) &&  !(EqQ[n, 2] && Simp
lerSqrtQ[-(b/a), -(d/c)])
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{\left(1-x^2\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{f} - \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{b}{\sqrt{1-x^2}\ldots}\right.}{f}$$

$$= \frac{\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{f} - \frac{\left(b\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{\ldots}{\sqrt{1-\ldots}}\right.}{f}$$

$$= \frac{\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{f} - \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{a+b\ldots}}{\sqrt{1-x\ldots}}\right.}{f}$$

$$= \frac{\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{f} - \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+\ldots}\right.}{f\sqrt{1+\frac{b\sin\ldots}{\ldots}}}$$

$$= -\frac{\sqrt{\cos^2(e+fx)}E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} + \ldots$$

Mathematica [A] time = 0.487736, size = 134, normalized size = 1.02

$$\frac{\sqrt{2}\tan(e+fx)(2a-b\cos(2(e+fx))+b)+2a\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}F\left(e+fx\middle|-\frac{b}{a}\right)-2a\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)}{2f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-2*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] + 2
*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)] + Sqrt
[2]*(2*a + b - b*Cos[2*(e + f*x)])*Tan[e + f*x])/(2*f*Sqrt[2*a + b - b*Cos[
2*(e + f*x)]])
```

Maple [A] time = 1.91, size = 294, normalized size = 2.2

$$\frac{1}{f\cos(fx+e)}\left(-\sqrt{-b\left(\cos(fx+e)\right)^4+(a+b)\left(\cos(fx+e)\right)^2}\,b\sin(fx+e)\left(\cos(fx+e)\right)^2+\sqrt{-b\left(\cos(fx+e)\right)^4+(a\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] (-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*sin(f*x+e)*cos(f*x+e)^2+(-b*
cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a+b)*sin(f*x+e)+a*(cos(f*x+e)^2)^(1
/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^
(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))-a*(cos(f*x+e)^2)^(1/2)*(-b/a*cos
```

```
(f*x+e)^2+(a+b)/a)^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*Ellipti
cE(sin(f*x+e),(-1/a*b)^(1/2)))/(-(a+b*sin(f*x+e)^2)*(-1+sin(f*x+e))*(1+sin(
f*x+e)))^(1/2)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin\left(fx+e\right)^2+a}\sec\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*sec(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-b\cos\left(fx+e\right)^2+a+b}\sec\left(fx+e\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)*sec(f*x + e)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\sin^2\left(e+fx\right)}\sec^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*sec(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin\left(fx+e\right)^2+a}\sec\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*sec(f*x + e)^2, x)
```

3.333 $\int \sec^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=196

$$\frac{(2a+b)\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f(a+b)}+\frac{2a\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{3f\sqrt{a+b\sin^2(e+fx)}}-\frac{(2a+b)\sqrt{a+b\sin^2(e+fx)}\,E\left(e+fx\middle|-\right.}{3f(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

```
[Out] -((2*a + b)*EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(3*(a +
b)*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (2*a*EllipticF[e + f*x, -(b/a)]*Sqrt
[1 + (b*Sin[e + f*x]^2)/a])/(3*f*Sqrt[a + b*Sin[e + f*x]^2]) + ((2*a + b)*S
qrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x])/(3*(a + b)*f) + (Sec[e + f*x]^2*Sqr
t[a + b*Sin[e + f*x]^2]*Tan[e + f*x])/(3*f)
```

Rubi [A] time = 0.223134, antiderivative size = 236, normalized size of antiderivative = 1.2, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3192, 412, 527, 524, 426, 424, 421, 419}

$$\frac{(2a+b)\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f(a+b)}+\frac{\tan(e+fx)\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{2a\sqrt{\cos^2(e+fx)}\sec(e+fx)}{3f\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((2*a + b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Se
c[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*(a + b)*f*Sqrt[1 + (b*Sin[e + f*x
]^2)/a]) + (2*a*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)
]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*f*Sqrt[a + b*Sin[e + f*x]
^2]) + ((2*a + b)*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x])/(3*(a + b)*f) +
(Sec[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x])/(3*f)
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 412

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] + Dist[1/(
a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p + 1)
 + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x]
&& NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
```

```
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \sec^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{(1-x^2)^{5/2}}dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{3f} - \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Su}}{}$$

$$= \frac{(2a+b)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{3(a+b)f} + \frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\tan}{3f}$$

$$= \frac{(2a+b)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{3(a+b)f} + \frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\tan}{3f}$$

$$= \frac{(2a+b)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{3(a+b)f} + \frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\tan}{3f}$$

$$= -\frac{(2a+b)\sqrt{\cos^2(e+fx)}E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b\sin^2(e+}}{3(a+b)f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

Mathematica [A] time = 1.88943, size = 187, normalized size = 0.95

$$\frac{\frac{\tan(e+fx)\sec^2(e+fx)\left(\left(8a^2-4b^2\right)\cos(2(e+fx))+(2a+b)(8a-b\cos(4(e+fx))+5b)\right)}{2\sqrt{2}} + 4a(a+b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}F\left(e+fx\middle|-\frac{b}{a}\right) - 2a(2a+b)\sqrt{}}{6f(a+b)\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-2*a*(2*a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -
(b/a)] + 4*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f
*x, -(b/a)] + (((8*a^2 - 4*b^2)*Cos[2*(e + f*x)] + (2*a + b)*(8*a + 5*b - b
*Cos[4*(e + f*x)]))*Sec[e + f*x]^2*Tan[e + f*x])/(2*Sqrt[2]))/(6*(a + b)*f*
Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A] time = 2.336, size = 368, normalized size = 1.9

$$\frac{1}{(3a+3b)\left(-1+\sin\left(fx+e\right)\right)\left(1+\sin\left(fx+e\right)\right)\cos\left(fx+e\right)f}\left(\sqrt{-b\left(\cos\left(fx+e\right)\right)^4+(a+b)\left(\cos\left(fx+e\right)\right)^2}b(2a+b)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3*((-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*(2*a+b)*sin(f*x+e)*cos(f*
x+e)^4-2*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*a*(a+b)*cos(f*x+e)^2*si
```

```
n(f*x+e)-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a^2+2*a*b+b^2)*sin(f*x
+e)-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^
(1/2)*(cos(f*x+e)^2)^(1/2)*a*(2*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a+2*El
lipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-2*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))
*a-EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*b)*cos(f*x+e)^2)/(-(a+b*sin(f*x+e)^
2)*(-1+sin(f*x+e))*(1+sin(f*x+e)))^(1/2)/(a+b)/(-1+sin(f*x+e))/(1+sin(f*x+e
))/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin\left(fx+e\right)^2+a}\sec\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*sec(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-b\cos\left(fx+e\right)^2+a+b}\sec\left(fx+e\right)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)*sec(f*x + e)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(a+b*sin(f*x+e)**2)**(1/2),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin\left(fx+e\right)^2+a}\sec\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*sec(f*x + e)^4, x)
```

3.334 $\int \cos^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=157

$$\frac{a^2(a+6b)\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{16b^{3/2}f}-\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{6bf}+\frac{(a+6b)\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{24bf}+$$

```
[Out] (a^2*(a + 6*b)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/
(16*b^(3/2)*f) + (a*(a + 6*b)*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(16*
b*f) + ((a + 6*b)*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2))/(24*b*f) - (Si
n[e + f*x]*(a + b*Sin[e + f*x]^2)^(5/2))/(6*b*f)
```

Rubi [A] time = 0.137606, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3190, 388, 195, 217, 206}

$$\frac{a^2(a+6b)\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{16b^{3/2}f}-\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{6bf}+\frac{(a+6b)\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{24bf}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (a^2*(a + 6*b)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/
(16*b^(3/2)*f) + (a*(a + 6*b)*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(16*
b*f) + ((a + 6*b)*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2))/(24*b*f) - (Si
n[e + f*x]*(a + b*Sin[e + f*x]^2)^(5/2))/(6*b*f)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \left(1-x^2\right)\left(a+bx^2\right)^{3/2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= -\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{6bf} + \frac{(a+6b)\,\text{Subst}\left(\int \left(a+bx^2\right)^{3/2}\,dx,\right.}{6bf}$$

$$= \frac{(a+6b)\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{24bf} - \frac{\sin(e+fx)\left(a+b\sin^2(e+\right.}{6bf}$$

$$= \frac{a(a+6b)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{16bf} + \frac{(a+6b)\sin(e+fx)\left(a+b\,\mathrm{si}\right.}{24bf}$$

$$= \frac{a(a+6b)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{16bf} + \frac{(a+6b)\sin(e+fx)\left(a+b\,\mathrm{si}\right.}{24bf}$$

$$= \frac{a^2(a+6b)\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{16b^{3/2}f} + \frac{a(a+6b)\sin(e+fx)\sqrt{a+b\sin^2(e}}{16bf}$$

Mathematica [A] time = 0.84795, size = 149, normalized size = 0.95

$$\frac{\sqrt{a+b\sin^2(e+fx)}\left(3a^{3/2}(a+6b)\sinh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a}}\right)+\sqrt{b}\sin(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\left(-2b(7a-6b)\sin^2(e+fx)-\right.\right.}{48b^{3/2}f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]^2]*(3*a^(3/2)*(a + 6*b)*ArcSinh[(Sqrt[b]*Sin[e + f
*x])/Sqrt[a]] + Sqrt[b]*Sin[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a]*(-3*a*(
a - 10*b) - 2*(7*a - 6*b)*b*Sin[e + f*x]^2 - 8*b^2*Sin[e + f*x]^4)))/(48*b^
(3/2)*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a])
```

Maple [B] time = 1.391, size = 277, normalized size = 1.8

$$-\frac{b\sin(fx+e)\left(\cos(fx+e)\right)^4}{6f}\sqrt{a+b-b\left(\cos(fx+e)\right)^2}+\frac{7\left(\cos(fx+e)\right)^2\sin(fx+e)\,a}{24f}\sqrt{a+b-b\left(\cos(fx+e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/6*b/f*(a+b-b*cos(f*x+e)^2)^(1/2)*sin(f*x+e)*cos(f*x+e)^4+7/24/f*(a+b-b*c
os(f*x+e)^2)^(1/2)*cos(f*x+e)^2*sin(f*x+e)*a+1/12/f*(a+b-b*cos(f*x+e)^2)^(1
/2)*cos(f*x+e)^2*sin(f*x+e)*b-1/16/b/f*(a+b-b*cos(f*x+e)^2)^(1/2)*sin(f*x+e
)*a^2+1/3/f*(a+b-b*cos(f*x+e)^2)^(1/2)*sin(f*x+e)*a+1/12*b/f*(a+b-b*cos(f*x
+e)^2)^(1/2)*sin(f*x+e)+1/16/b^(3/2)/f*a^3*ln(sin(f*x+e)*b^(1/2)+(a+b-b*cos
(f*x+e)^2)^(1/2))+3/8/b^(1/2)/f*a^2*ln(sin(f*x+e)*b^(1/2)+(a+b-b*cos(f*x+e)
^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 15.8315, size = 1393, normalized size = 8.87

$$\left[\frac{3\left(a^3+6a^2b\right)\sqrt{b}\log\left(128b^4\cos\left(fx+e\right)^8-256\left(ab^3+2b^4\right)\cos\left(fx+e\right)^6+32\left(5a^2b^2+24ab^3+24b^4\right)\cos\left(fx+e\right)^4\cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/384*(3*(a^3 + 6*a^2*b)*sqrt(b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 +
 2*b^4)*cos(f*x + e)^6 + 32*(5*a^2*b^2 + 24*a*b^3 + 24*b^4)*cos(f*x + e)^4
+ a^4 + 32*a^3*b + 160*a^2*b^2 + 256*a*b^3 + 128*b^4 - 32*(a^3*b + 10*a^2*b
^2 + 24*a*b^3 + 16*b^4)*cos(f*x + e)^2 - 8*(16*b^3*cos(f*x + e)^6 - 24*(a*b
^2 + 2*b^3)*cos(f*x + e)^4 - a^3 - 10*a^2*b - 24*a*b^2 - 16*b^3 + 2*(5*a^2*
b + 24*a*b^2 + 24*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt
(b)*sin(f*x + e)) - 8*(8*b^3*cos(f*x + e)^4 + 3*a^2*b - 16*a*b^2 - 4*b^3 -
2*(7*a*b^2 + 2*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sin(f*x
 + e))/(b^2*f), -1/192*(3*(a^3 + 6*a^2*b)*sqrt(-b)*arctan(1/4*(8*b^2*cos(f*
x + e)^4 - 8*(a*b + 2*b^2)*cos(f*x + e)^2 + a^2 + 8*a*b + 8*b^2)*sqrt(-b*co
s(f*x + e)^2 + a + b)*sqrt(-b)/((2*b^3*cos(f*x + e)^4 + a^2*b + 3*a*b^2 + 2
*b^3 - (3*a*b^2 + 4*b^3)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(8*b^3*cos(f*x
+ e)^4 + 3*a^2*b - 16*a*b^2 - 4*b^3 - 2*(7*a*b^2 + 2*b^3)*cos(f*x + e)^2)*s
qrt(-b*cos(f*x + e)^2 + a + b)*sin(f*x + e))/(b^2*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23859, size = 181, normalized size = 1.15

$$-\frac{\left(2\left(4\,b\sin\left(fx+e\right)^2+\frac{7\,ab^4-6\,b^5}{b^4}\right)\sin\left(fx+e\right)^2+\frac{3\left(a^2b^3-10\,ab^4\right)}{b^4}\right)\sqrt{b\sin\left(fx+e\right)^2+a}\,\sin\left(fx+e\right)+\frac{3\left(a^3+6\,a^2b\right)\log\left(\left|-\sqrt{b}\,\mathrm{s}\right.\right.}{}}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/48*((2*(4*b*sin(f*x + e)^2 + (7*a*b^4 - 6*b^5)/b^4)*sin(f*x + e)^2 + 3*(
a^2*b^3 - 10*a*b^4)/b^4)*sqrt(b*sin(f*x + e)^2 + a)*sin(f*x + e) + 3*(a^3 +
 6*a^2*b)*log(abs(-sqrt(b)*sin(f*x + e) + sqrt(b*sin(f*x + e)^2 + a)))/b^(3
/2))/f
```

3.335 $\int \cos(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=104

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{8\sqrt{b}f}+\frac{3a\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8f}+\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4f}$$

```
[Out] (3*a^2*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/(8*Sqrt[
b]*f) + (3*a*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(8*f) + (Sin[e + f*x]
*(a + b*Sin[e + f*x]^2)^(3/2))/(4*f)
```

Rubi [A] time = 0.0692238, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3190, 195, 217, 206}

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{8\sqrt{b}f}+\frac{3a\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8f}+\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*a^2*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/(8*Sqrt[
b]*f) + (3*a*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(8*f) + (Sin[e + f*x]
*(a + b*Sin[e + f*x]^2)^(3/2))/(4*f)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \left(a+bx^2\right)^{3/2}\,dx,x,\sin(e+fx)\right)}{f}$$
$$= \frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4f} + \frac{(3a)\,\text{Subst}\left(\int \sqrt{a+bx^2}\,dx,x,\sin(e+\right.}{4f}$$
$$= \frac{3a\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8f} + \frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4f} +$$
$$= \frac{3a\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8f} + \frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4f} +$$
$$= \frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{8\sqrt{b}f} + \frac{3a\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8f} + \sin(e$$

Mathematica [A] time = 0.511889, size = 93, normalized size = 0.89

$$\frac{\sqrt{a+b\sin^2(e+fx)}\left(\frac{3a^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+5a\sin(e+fx)+2b\sin^3(e+fx)\right)}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]^2]*(5*a*Sin[e + f*x] + 2*b*Sin[e + f*x]^3 + (3*a^(
3/2)*ArcSinh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a]])/(Sqrt[b]*Sqrt[1 + (b*Sin[e +
f*x]^2)/a])))/(8*f)
```

Maple [A] time = 0.089, size = 90, normalized size = 0.9

$$\frac{\sin(fx+e)}{4f}\left(a+b\left(\sin(fx+e)\right)^2\right)^{\frac{3}{2}}+\frac{3a\sin(fx+e)}{8f}\sqrt{a+b\left(\sin(fx+e)\right)^2}+\frac{3a^2}{8f}\ln\left(\sin(fx+e)\sqrt{b}+\sqrt{a+b(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/4*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2)/f+3/8*a*sin(f*x+e)*(a+b*sin(f*x+e)^
2)^(1/2)/f+3/8/f*a^2*ln(sin(f*x+e)*b^(1/2)+(a+b*sin(f*x+e)^2)^(1/2))/b^(1/2
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.07956, size = 1218, normalized size = 11.71

$$\left[\frac{3\,a^2\sqrt{b}\log\left(128\,b^4\cos\left(fx+e\right)^8 - 256\left(ab^3+2\,b^4\right)\cos\left(fx+e\right)^6 + 32\left(5\,a^2b^2+24\,ab^3+24\,b^4\right)\cos\left(fx+e\right)^4 + a^4 + 32\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/64*(3*a^2*sqrt(b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + 2*b^4)*cos(f
*x + e)^6 + 32*(5*a^2*b^2 + 24*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^4 + 32*a^
3*b + 160*a^2*b^2 + 256*a*b^3 + 128*b^4 - 32*(a^3*b + 10*a^2*b^2 + 24*a*b^3
 + 16*b^4)*cos(f*x + e)^2 - 8*(16*b^3*cos(f*x + e)^6 - 24*(a*b^2 + 2*b^3)*c
os(f*x + e)^4 - a^3 - 10*a^2*b - 24*a*b^2 - 16*b^3 + 2*(5*a^2*b + 24*a*b^2
+ 24*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(b)*sin(f*x +
 e)) - 8*(2*b^2*cos(f*x + e)^2 - 5*a*b - 2*b^2)*sqrt(-b*cos(f*x + e)^2 + a
+ b)*sin(f*x + e))/(b*f), -1/32*(3*a^2*sqrt(-b)*arctan(1/4*(8*b^2*cos(f*x +
 e)^4 - 8*(a*b + 2*b^2)*cos(f*x + e)^2 + a^2 + 8*a*b + 8*b^2)*sqrt(-b*cos(f
*x + e)^2 + a + b)*sqrt(-b)/((2*b^3*cos(f*x + e)^4 + a^2*b + 3*a*b^2 + 2*b^
3 - (3*a*b^2 + 4*b^3)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(2*b^2*cos(f*x + e
)^2 - 5*a*b - 2*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sin(f*x + e))/(b*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19605, size = 113, normalized size = 1.09

$$-\frac{\frac{3\,a^2\log\left(\left|-\sqrt{b}\sin\left(fx+e\right)+\sqrt{b\sin\left(fx+e\right)^2+a}\right|\right)}{\sqrt{b}} - \left(2\,b\sin\left(fx+e\right)^2+5\,a\right)\sqrt{b\sin\left(fx+e\right)^2+a}\sin\left(fx+e\right)}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/8*(3*a^2*log(abs(-sqrt(b)*sin(f*x + e) + sqrt(b*sin(f*x + e)^2 + a)))/sq
rt(b) - (2*b*sin(f*x + e)^2 + 5*a)*sqrt(b*sin(f*x + e)^2 + a)*sin(f*x + e))
/f
```

3.336 $\int \sec(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=121

$$-\frac{b\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2f}+\frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f}-\frac{\sqrt{b}(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2f}$$

```
[Out] -(Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x
]^2]])/(2*f) + ((a + b)^(3/2)*ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b
*Sin[e + f*x]^2]])/f - (b*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(2*f)
```

Rubi [A] time = 0.143921, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3190, 416, 523, 217, 206, 377}

$$-\frac{b\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2f}+\frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f}-\frac{\sqrt{b}(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x
]^2]])/(2*f) + ((a + b)^(3/2)*ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b
*Sin[e + f*x]^2]])/f - (b*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(2*f)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\int \sec(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{1-x^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= -\frac{b\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2f} - \frac{\text{Subst}\left(\int \frac{-a(2a+b)-b(3a+2b)x^2}{\left(1-x^2\right)\sqrt{a+bx^2}}\,dx, x, \sin\right.}{2f}$$

$$= -\frac{b\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2f} + \frac{(a+b)^2\,\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\sqrt{a+bx^2}}\,dx, x,\right.}{f}$$

$$= -\frac{b\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2f} + \frac{(a+b)^2\,\text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\,dx, x, \frac{}{\sqrt{a+}}\right.}{f}$$

$$= -\frac{\sqrt{b}(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2f} + \frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f}$$

Mathematica [A] time = 0.661287, size = 233, normalized size = 1.93

$$\frac{\sqrt{2}\left(4a^2+5ab+2b^2\right)\tanh^{-1}\left(\frac{\sqrt{2a+2b}\sin(e+fx)}{\sqrt{2a-b\cos(2(e+fx))+b}}\right)-2\sqrt{b}\sqrt{a+b}\left(\sqrt{b}\sin(e+fx)\sqrt{2a-b\cos(2(e+fx))+b}+\sqrt{2}(3a\right.}{4\sqrt{2}f\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[2]*b*(3*a + 2*b)*ArcTanh[(Sqrt[2]*Sqrt[a + b]*Sin[e + f*x])/Sqrt[2*a
+ b - b*Cos[2*(e + f*x)]]] + Sqrt[2]*(4*a^2 + 5*a*b + 2*b^2)*ArcTanh[(Sqrt[
2*a + 2*b]*Sin[e + f*x])/Sqrt[2*a + b - b*Cos[2*(e + f*x)]]] - 2*Sqrt[b]*Sq
rt[a + b]*(Sqrt[2]*(3*a + 2*b)*Log[Sqrt[2*a + b - b*Cos[2*(e + f*x)]] + Sqr
t[2]*Sqrt[b]*Sin[e + f*x]] + Sqrt[b]*Sqrt[2*a + b - b*Cos[2*(e + f*x)]]*Sin
[e + f*x]))/(4*Sqrt[2]*Sqrt[a + b]*f)
```

Maple [B] time = 3.106, size = 451, normalized size = 3.7

$$-\frac{1}{f}b^{\frac{3}{2}}\ln\left(\sin\left(fx+e\right)\sqrt{b}+\sqrt{a+b-b\left(\cos\left(fx+e\right)\right)^{2}}\right)-\frac{b\sin\left(fx+e\right)}{2f}\sqrt{a+b-b\left(\cos\left(fx+e\right)\right)^{2}}-\frac{3a}{2f}\sqrt{b}\ln\left(\sin\left(fx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/f*b^(3/2)*ln(sin(f*x+e)*b^(1/2)+(a+b-b*cos(f*x+e)^2)^(1/2))-1/2*b/f*(a+b
-b*cos(f*x+e)^2)^(1/2)*sin(f*x+e)-3/2/f*a*b^(1/2)*ln(sin(f*x+e)*b^(1/2)+(a+
b-b*cos(f*x+e)^2)^(1/2))-1/2/(a+b)^(1/2)/f*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)
*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^2-1/(a+b)^(1/2)/f*ln(2/(1+si
n(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a*b-1/2/
(a+b)^(1/2)/f*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b
*sin(f*x+e)+a))*b^2+1/2/(a+b)^(1/2)/f*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+
b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^2+1/(a+b)^(1/2)/f*ln(2/(-1+sin(f
*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a*b+1/2/(a+
b)^(1/2)/f*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*s
in(f*x+e)+a))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 6.54467, size = 3399, normalized size = 28.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*((3*a + 2*b)*sqrt(b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + 2*b^4)
*cos(f*x + e)^6 + 32*(5*a^2*b^2 + 24*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^4 +
 32*a^3*b + 160*a^2*b^2 + 256*a*b^3 + 128*b^4 - 32*(a^3*b + 10*a^2*b^2 + 24
*a*b^3 + 16*b^4)*cos(f*x + e)^2 + 8*(16*b^3*cos(f*x + e)^6 - 24*(a*b^2 + 2*
b^3)*cos(f*x + e)^4 - a^3 - 10*a^2*b - 24*a*b^2 - 16*b^3 + 2*(5*a^2*b + 24*
a*b^2 + 24*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(b)*sin
(f*x + e)) + 4*(a + b)^(3/2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 8*
(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*cos(f*x + e)^2 - 2*a -
2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b)*sin(f*x + e) + 8*a^2 + 16*
a*b + 8*b^2)/cos(f*x + e)^4) - 8*sqrt(-b*cos(f*x + e)^2 + a + b)*b*sin(f*x
+ e))/f, -1/16*(8*(a + b)*sqrt(-a - b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2
 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(((a*b + b^2)*co
s(f*x + e)^2 - a^2 - 2*a*b - b^2)*sin(f*x + e))) - (3*a + 2*b)*sqrt(b)*log(
128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + 2*b^4)*cos(f*x + e)^6 + 32*(5*a^2*b^2
```

```
 + 24*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^4 + 32*a^3*b + 160*a^2*b^2 + 256*a
*b^3 + 128*b^4 - 32*(a^3*b + 10*a^2*b^2 + 24*a*b^3 + 16*b^4)*cos(f*x + e)^2
 + 8*(16*b^3*cos(f*x + e)^6 - 24*(a*b^2 + 2*b^3)*cos(f*x + e)^4 - a^3 - 10*
a^2*b - 24*a*b^2 - 16*b^3 + 2*(5*a^2*b + 24*a*b^2 + 24*b^3)*cos(f*x + e)^2)
*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(b)*sin(f*x + e)) + 8*sqrt(-b*cos(f*x
+ e)^2 + a + b)*b*sin(f*x + e))/f, 1/8*((3*a + 2*b)*sqrt(-b)*arctan(1/4*(8*
b^2*cos(f*x + e)^4 - 8*(a*b + 2*b^2)*cos(f*x + e)^2 + a^2 + 8*a*b + 8*b^2)*
sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)/((2*b^3*cos(f*x + e)^4 + a^2*b + 3
*a*b^2 + 2*b^3 - (3*a*b^2 + 4*b^3)*cos(f*x + e)^2)*sin(f*x + e))) + 2*(a +
b)^(3/2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 8*(a^2 + 3*a*b + 2*b^2
)*cos(f*x + e)^2 - 4*((a + 2*b)*cos(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x
 + e)^2 + a + b)*sqrt(a + b)*sin(f*x + e) + 8*a^2 + 16*a*b + 8*b^2)/cos(f*x
 + e)^4) - 4*sqrt(-b*cos(f*x + e)^2 + a + b)*b*sin(f*x + e))/f, -1/8*(4*(a
+ b)*sqrt(-a - b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - 2*a - 2*b)*sqrt(-b
*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(((a*b + b^2)*cos(f*x + e)^2 - a^2 -
2*a*b - b^2)*sin(f*x + e))) - (3*a + 2*b)*sqrt(-b)*arctan(1/4*(8*b^2*cos(f*
x + e)^4 - 8*(a*b + 2*b^2)*cos(f*x + e)^2 + a^2 + 8*a*b + 8*b^2)*sqrt(-b*co
s(f*x + e)^2 + a + b)*sqrt(-b)/((2*b^3*cos(f*x + e)^4 + a^2*b + 3*a*b^2 + 2
*b^3 - (3*a*b^2 + 4*b^3)*cos(f*x + e)^2)*sin(f*x + e))) + 4*sqrt(-b*cos(f*x
 + e)^2 + a + b)*b*sin(f*x + e))/f]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*sec(f*x + e), x)
```

3.337 $\int \sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=127

$$\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f}+\frac{(a-2b)\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2f}+\frac{(a+b)\tan(e+fx)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2f}$$

```
[Out] (b^(3/2)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/f + ((
a - 2*b)*Sqrt[a + b]*ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e +
f*x]^2]])/(2*f) + ((a + b)*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e +
f*x])/(2*f)
```

Rubi [A] time = 0.145834, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3190, 413, 523, 217, 206, 377}

$$\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f}+\frac{(a-2b)\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2f}+\frac{(a+b)\tan(e+fx)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (b^(3/2)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/f + ((
a - 2*b)*Sqrt[a + b]*ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e +
f*x]^2]])/(2*f) + ((a + b)*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e +
f*x])/(2*f)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\int \sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{\left(1-x^2\right)^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{(a+b)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{2f} - \frac{\text{Subst}\left(\int \frac{-a(a-b)+2l}{\left(1-x^2\right)\sqrt{a+}}\right.}{}$$

$$= \frac{(a+b)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{2f} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\right.}{}$$

$$= \frac{(a+b)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{2f} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{1-bx^2}\right.}{}$$

$$= \frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f} + \frac{(a-2b)\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2f} +$$

Mathematica [A] time = 0.969861, size = 210, normalized size = 1.65

$$\frac{2\left(a^2-ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{2a+2b}\sin(e+fx)}{\sqrt{2a-b\cos(2(e+fx))+b}}\right) - 2b^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a+b}\sin(e+fx)}{\sqrt{2a-b\cos(2(e+fx))+b}}\right) + \sqrt{a+b}\left(4b^{3/2}\log\left(\sqrt{2a-b\cos(2(e+\right.\right.}{4f\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^3*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-2*b^2*ArcTanh[(Sqrt[2]*Sqrt[a + b]*Sin[e + f*x])/Sqrt[2*a + b - b*Cos[2*(
e + f*x)]]] + 2*(a^2 - a*b - b^2)*ArcTanh[(Sqrt[2*a + 2*b]*Sin[e + f*x])/Sq
rt[2*a + b - b*Cos[2*(e + f*x)]]] + Sqrt[a + b]*(4*b^(3/2)*Log[Sqrt[2*a + b
 - b*Cos[2*(e + f*x)]] + Sqrt[2]*Sqrt[b]*Sin[e + f*x]] + Sqrt[2]*(a + b)*Sq
rt[2*a + b - b*Cos[2*(e + f*x)]]*Sec[e + f*x]*Tan[e + f*x]))/(4*Sqrt[a + b]
*f)
```

Maple [B] time = 3.408, size = 402, normalized size = 3.2

$$\frac{1}{4\left(\cos\left(fx+e\right)\right)^{2}f}\left(2\sin\left(fx+e\right)\sqrt{a+b-b\left(\cos\left(fx+e\right)\right)^{2}}\left(a+b\right)^{5/2}-\left(-4\,b^{3/2}\ln\left(\sin\left(fx+e\right)\sqrt{b}+\sqrt{a+b-b\left(\cos\right.}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/4*(2*sin(f*x+e)*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(5/2)-(-4*b^(3/2)*ln(sin
(f*x+e)*b^(1/2)+(a+b-b*cos(f*x+e)^2)^(1/2))*(a+b)^(3/2)-ln(2/(-1+sin(f*x+e)
)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^3+3*ln(2/(-1+s
in(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a*b^2+2
*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+
a))*b^3+ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f
*x+e)+a))*a^3-3*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)
-b*sin(f*x+e)+a))*a*b^2-2*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e
)^2)^(1/2)-b*sin(f*x+e)+a))*b^3)*cos(f*x+e)^2)/(a+b)^(3/2)/cos(f*x+e)^2/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^3, x)
```

Fricas [B] time = 7.19163, size = 3646, normalized size = 28.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(b^(3/2)*cos(f*x + e)^2*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + 2*b^
4)*cos(f*x + e)^6 + 32*(5*a^2*b^2 + 24*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^4
 + 32*a^3*b + 160*a^2*b^2 + 256*a*b^3 + 128*b^4 - 32*(a^3*b + 10*a^2*b^2 +
24*a*b^3 + 16*b^4)*cos(f*x + e)^2 - 8*(16*b^3*cos(f*x + e)^6 - 24*(a*b^2 +
2*b^3)*cos(f*x + e)^4 - a^3 - 10*a^2*b - 24*a*b^2 - 16*b^3 + 2*(5*a^2*b + 2
4*a*b^2 + 24*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(b)*s
in(f*x + e)) - sqrt(a + b)*(a - 2*b)*cos(f*x + e)^2*log(((a^2 + 8*a*b + 8*b
^2)*cos(f*x + e)^4 - 8*(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^2 + 4*((a + 2*b)*
cos(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b)*sin
(f*x + e) + 8*a^2 + 16*a*b + 8*b^2)/cos(f*x + e)^4) + 4*sqrt(-b*cos(f*x + e
)^2 + a + b)*(a + b)*sin(f*x + e))/(f*cos(f*x + e)^2), -1/8*(2*(a - 2*b)*sq
rt(-a - b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*
x + e)^2 + a + b)*sqrt(-a - b)/(((a*b + b^2)*cos(f*x + e)^2 - a^2 - 2*a*b -
 b^2)*sin(f*x + e)))*cos(f*x + e)^2 - b^(3/2)*cos(f*x + e)^2*log(128*b^4*co
s(f*x + e)^8 - 256*(a*b^3 + 2*b^4)*cos(f*x + e)^6 + 32*(5*a^2*b^2 + 24*a*b^
```

```
3 + 24*b^4)*cos(f*x + e)^4 + a^4 + 32*a^3*b + 160*a^2*b^2 + 256*a*b^3 + 128
*b^4 - 32*(a^3*b + 10*a^2*b^2 + 24*a*b^3 + 16*b^4)*cos(f*x + e)^2 - 8*(16*b
^3*cos(f*x + e)^6 - 24*(a*b^2 + 2*b^3)*cos(f*x + e)^4 - a^3 - 10*a^2*b - 24
*a*b^2 - 16*b^3 + 2*(5*a^2*b + 24*a*b^2 + 24*b^3)*cos(f*x + e)^2)*sqrt(-b*c
os(f*x + e)^2 + a + b)*sqrt(b)*sin(f*x + e)) - 4*sqrt(-b*cos(f*x + e)^2 + a
 + b)*(a + b)*sin(f*x + e))/(f*cos(f*x + e)^2), -1/8*(2*sqrt(-b)*b*arctan(1
/4*(8*b^2*cos(f*x + e)^4 - 8*(a*b + 2*b^2)*cos(f*x + e)^2 + a^2 + 8*a*b + 8
*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)/((2*b^3*cos(f*x + e)^4 + a^2
*b + 3*a*b^2 + 2*b^3 - (3*a*b^2 + 4*b^3)*cos(f*x + e)^2)*sin(f*x + e)))*cos
(f*x + e)^2 + sqrt(a + b)*(a - 2*b)*cos(f*x + e)^2*log(((a^2 + 8*a*b + 8*b^
2)*cos(f*x + e)^4 - 8*(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^2 + 4*((a + 2*b)*c
os(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b)*sin(
f*x + e) + 8*a^2 + 16*a*b + 8*b^2)/cos(f*x + e)^4) - 4*sqrt(-b*cos(f*x + e)
^2 + a + b)*(a + b)*sin(f*x + e))/(f*cos(f*x + e)^2), -1/4*((a - 2*b)*sqrt(
-a - b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x +
 e)^2 + a + b)*sqrt(-a - b)/(((a*b + b^2)*cos(f*x + e)^2 - a^2 - 2*a*b - b^
2)*sin(f*x + e)))*cos(f*x + e)^2 + sqrt(-b)*b*arctan(1/4*(8*b^2*cos(f*x + e
)^4 - 8*(a*b + 2*b^2)*cos(f*x + e)^2 + a^2 + 8*a*b + 8*b^2)*sqrt(-b*cos(f*x
 + e)^2 + a + b)*sqrt(-b)/((2*b^3*cos(f*x + e)^4 + a^2*b + 3*a*b^2 + 2*b^3
- (3*a*b^2 + 4*b^3)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e)^2 - 2*sqrt(
-b*cos(f*x + e)^2 + a + b)*(a + b)*sin(f*x + e))/(f*cos(f*x + e)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^3, x)
```

3.338 $\int \sec^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=122

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{8f\sqrt{a+b}}+\frac{\tan(e+fx)\sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4f}+\frac{3a\tan(e+fx)\sec(e+fx)\sqrt{a+b\sin^2(}}{8f}$$

```
[Out] (3*a^2*ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/(8*S
qrt[a + b]*f) + (3*a*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x])/
(8*f) + (Sec[e + f*x]^3*(a + b*Sin[e + f*x]^2)^(3/2)*Tan[e + f*x])/(4*f)
```

Rubi [A] time = 0.120778, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3190, 378, 377, 206}

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{8f\sqrt{a+b}}+\frac{\tan(e+fx)\sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4f}+\frac{3a\tan(e+fx)\sec(e+fx)\sqrt{a+b\sin^2(}}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^5*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*a^2*ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/(8*S
qrt[a + b]*f) + (3*a*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x])/
(8*f) + (Sec[e + f*x]^3*(a + b*Sin[e + f*x]^2)^(3/2)*Tan[e + f*x])/(4*f)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx=\frac{\text{Subst}\left(\int\frac{\left(a+bx^2\right)^{3/2}}{\left(1-x^2\right)^3}dx,x,\sin(e+fx)\right)}{f}$$

$$=\frac{\sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\tan(e+fx)}{4f}+\frac{(3a)\,\text{Subst}\left(\int\frac{\sqrt{a+bx^2}}{\left(1-x^2\right)^2}d\right.}{4f}$$

$$=\frac{3a\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{8f}+\frac{\sec^3(e+fx)\left(a+b\sin^2\right.}{4}$$

$$=\frac{3a\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{8f}+\frac{\sec^3(e+fx)\left(a+b\sin^2\right.}{4}$$

$$=\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{8\sqrt{a+b}f}+\frac{3a\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\tan(e+}{8f}$$

Mathematica [C] time = 0.130156, size = 63, normalized size = 0.52

$$\frac{a^2\sin(e+fx)\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{(a+b)\sin^2(e+fx)}{b\sin^2(e+fx)+a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^5*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (a^2*Hypergeometric2F1[1/2, 3, 3/2, ((a + b)*Sin[e + f*x]^2)/(a + b*Sin[e +
 f*x]^2)]*Sin[e + f*x])/(f*Sqrt[a + b*Sin[e + f*x]^2])
```

Maple [B] time = 2.654, size = 406, normalized size = 3.3

$$\frac{1}{16\left(\cos\left(fx+e\right)\right)^4f}\left(2\sqrt{a+b-b\left(\cos\left(fx+e\right)\right)^2}(a+b)^{5/2}(3a-2b)\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^2+4\sqrt{a+b-b\left(c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^5*(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/16*(2*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(5/2)*(3*a-2*b)*sin(f*x+e)*cos(f*x
+e)^2+4*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(7/2)*sin(f*x+e)-3*a^2*(ln(2/(1+si
n(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^2+2*ln
(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*
a*b+ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e
)+a))*b^2-ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*si
n(f*x+e)+a))*a^2-2*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(
1/2)+b*sin(f*x+e)+a))*a*b-ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+
e)^2)^(1/2)+b*sin(f*x+e)+a))*b^2)*cos(f*x+e)^4)/(a+b)^(5/2)/cos(f*x+e)^4/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^5, x)
```

Fricas [A] time = 6.28442, size = 1014, normalized size = 8.31

$$\left[\frac{3\sqrt{a+b}a^{2}\cos\left(fx+e\right)^{4}\log\left(\frac{\left(a^{2}+8\,ab+8\,b^{2}\right)\cos\left(fx+e\right)^{4}-8\left(a^{2}+3\,ab+2\,b^{2}\right)\cos\left(fx+e\right)^{2}-4\left(\left(a+2\,b\right)\cos\left(fx+e\right)^{2}-2\,a-2\,b\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\sqrt{a+b}}{\cos\left(fx+e\right)^{4}}\right.}{32\,(a+b)f\,\mathrm{c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(3*sqrt(a + b)*a^2*cos(f*x + e)^4*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x
+ e)^4 - 8*(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*cos(f*x + e)
^2 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b)*sin(f*x + e) +
8*a^2 + 16*a*b + 8*b^2)/cos(f*x + e)^4) + 4*((3*a^2 + a*b - 2*b^2)*cos(f*x
+ e)^2 + 2*a^2 + 4*a*b + 2*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sin(f*x + e
))/((a + b)*f*cos(f*x + e)^4), -1/16*(3*a^2*sqrt(-a - b)*arctan(1/2*((a + 2
*b)*cos(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b
)/(((a*b + b^2)*cos(f*x + e)^2 - a^2 - 2*a*b - b^2)*sin(f*x + e)))*cos(f*x
+ e)^4 - 2*((3*a^2 + a*b - 2*b^2)*cos(f*x + e)^2 + 2*a^2 + 4*a*b + 2*b^2)*s
qrt(-b*cos(f*x + e)^2 + a + b)*sin(f*x + e))/((a + b)*f*cos(f*x + e)^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**5*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^5, x)
```

3.339 $\int \sec^7(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=195

$$\frac{a^2(5a+6b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{16f(a+b)^{3/2}}+\frac{\tan(e+fx)\sec^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{6f(a+b)}+\frac{(5a+6b)\tan(e+fx)\sec^3(e+\ldots}{24f(a+\ldots}$$

```
[Out] (a^2*(5*a + 6*b)*ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]
^2]])/(16*(a + b)^(3/2)*f) + (a*(5*a + 6*b)*Sec[e + f*x]*Sqrt[a + b*Sin[e +
 f*x]^2]*Tan[e + f*x])/(16*(a + b)*f) + ((5*a + 6*b)*Sec[e + f*x]^3*(a + b*
Sin[e + f*x]^2)^(3/2)*Tan[e + f*x])/(24*(a + b)*f) + (Sec[e + f*x]^5*(a + b
*Sin[e + f*x]^2)^(5/2)*Tan[e + f*x])/(6*(a + b)*f)
```

Rubi [A] time = 0.176011, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3190, 382, 378, 377, 206}

$$\frac{a^2(5a+6b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{16f(a+b)^{3/2}}+\frac{\tan(e+fx)\sec^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{6f(a+b)}+\frac{(5a+6b)\tan(e+fx)\sec^3(e+\ldots}{24f(a+\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^7*(a + b*Sin[e + f*x]^2)^(3/2),x]
```


```
[Out] (a^2*(5*a + 6*b)*ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]
^2]])/(16*(a + b)^(3/2)*f) + (a*(5*a + 6*b)*Sec[e + f*x]*Sqrt[a + b*Sin[e +
 f*x]^2]*Tan[e + f*x])/(16*(a + b)*f) + ((5*a + 6*b)*Sec[e + f*x]^3*(a + b*
Sin[e + f*x]^2)^(3/2)*Tan[e + f*x])/(24*(a + b)*f) + (Sec[e + f*x]^5*(a + b
*Sin[e + f*x]^2)^(5/2)*Tan[e + f*x])/(6*(a + b)*f)
```


Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec^7(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{\left(1-x^2\right)^4}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sec^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}\tan(e+fx)}{6(a+b)f} + \frac{(5a+6b)\,\text{Subst}\left(\int \frac{(a+}{(1}\right.}{6(a}$$

$$= \frac{(5a+6b)\sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\tan(e+fx)}{24(a+b)f} + \frac{\sec^5(e+fx)}{}$$

$$= \frac{a(5a+6b)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{16(a+b)f} + \frac{(5a+6b)\sec^3(}{}$$

$$= \frac{a(5a+6b)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{16(a+b)f} + \frac{(5a+6b)\sec^3(}{}$$

$$= \frac{a^2(5a+6b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{16(a+b)^{3/2}f} + \frac{a(5a+6b)\sec(e+fx)\sqrt{a+b\sin^2}}{16(a+b)f}$$

Mathematica [C] time = 15.9361, size = 938, normalized size = 4.81

$$a^2\sec^3(e+fx)\left(\frac{b\sin^2(e+fx)}{a}+1\right)^2\tan(e+fx)\left(256b\,{}_2F_1\left(2,5;\frac{7}{2};-\frac{(a+b)\tan^2(e+fx)}{a}\right)\sin^2(e+fx)\sqrt{\frac{\sec^2(e+fx)\left(b\sin^2(e+fx)+a\right)}{a}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^7*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (a^2*Sec[e + f*x]^3*(1 + (b*Sin[e + f*x]^2)/a)^2*Tan[e + f*x]*(45*a*ArcSin[
Sqrt[-(((a + b)*Tan[e + f*x]^2)/a)]] + 30*b*ArcSin[Sqrt[-(((a + b)*Tan[e +
f*x]^2)/a)]]*Sin[e + f*x]^2 + 210*a*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x
]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(3/2) + 140*b*Sin[e + f*x]^2*Sqrt[
(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^
(3/2) - 120*a*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*T
an[e + f*x]^2)/a))^(5/2) + 256*a*Hypergeometric2F1[2, 5, 7/2, -(((a + b)*Ta
n[e + f*x]^2)/a)]*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a +
b)*Tan[e + f*x]^2)/a))^(5/2) - 80*b*Sin[e + f*x]^2*Sqrt[(Sec[e + f*x]^2*(a
```

```
+ b*Sin[e + f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(5/2) + 256*b*Hyper
geometric2F1[2, 5, 7/2, -(((a + b)*Tan[e + f*x]^2)/a)]*Sin[e + f*x]^2*Sqrt[
(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^
(5/2) - 512*a*Hypergeometric2F1[2, 5, 7/2, -(((a + b)*Tan[e + f*x]^2)/a)]*S
qrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/
a))^(7/2) - 512*b*Hypergeometric2F1[2, 5, 7/2, -(((a + b)*Tan[e + f*x]^2)/a
)]*Sin[e + f*x]^2*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a +
b)*Tan[e + f*x]^2)/a))^(7/2) + 256*a*Hypergeometric2F1[2, 5, 7/2, -(((a + b
)*Tan[e + f*x]^2)/a)]*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((
a + b)*Tan[e + f*x]^2)/a))^(9/2) + 256*b*Hypergeometric2F1[2, 5, 7/2, -(((a
 + b)*Tan[e + f*x]^2)/a)]*Sin[e + f*x]^2*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e
+ f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(9/2) - 45*a*Sqrt[-(((a + b)*
Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)*Tan[e + f*x]^2)/a^2)] - 30*b*Sin[e +
f*x]^2*Sqrt[-(((a + b)*Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)*Tan[e + f*x]^2
)/a^2)]))/(240*f*(a + b*Sin[e + f*x]^2)^(3/2)*Sqrt[(Sec[e + f*x]^2*(a + b*S
in[e + f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(3/2))
```

Maple [B] time = 3.241, size = 693, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^7*(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/96*(2*(a+b)^(7/2)*(a+b-b*cos(f*x+e)^2)^(1/2)*(15*a^2+8*a*b-4*b^2)*sin(f*x
+e)*cos(f*x+e)^4+4*(a+b)^(7/2)*(a+b-b*cos(f*x+e)^2)^(1/2)*(5*a^2+3*a*b-2*b^
2)*cos(f*x+e)^2*sin(f*x+e)+16*(a+b)^(7/2)*(a+b-b*cos(f*x+e)^2)^(1/2)*(a^2+2
*a*b+b^2)*sin(f*x+e)+3*a^2*(5*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(
f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^4+21*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(
a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^3*b+33*ln(2/(-1+sin(f*x+e))*((
a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^2*b^2+23*ln(2/(-1+
sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a*b^3+
6*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)
+a))*b^4-5*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*si
n(f*x+e)+a))*a^4-21*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(
1/2)-b*sin(f*x+e)+a))*a^3*b-33*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(
f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^2*b^2-23*ln(2/(1+sin(f*x+e))*((a+b)^(1/2
)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a*b^3-6*ln(2/(1+sin(f*x+e))*(
(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*b^4)*cos(f*x+e)^6)/
(a+b)^(9/2)/cos(f*x+e)^6/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^{7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^7*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^7, x)
```

Fricas [A] time = 25.5334, size = 1310, normalized size = 6.72

$$\left[\frac{3\left(5a^3+6a^2b\right)\sqrt{a+b}\cos\left(fx+e\right)^6\log\left(\frac{\left(a^2+8ab+8b^2\right)\cos\left(fx+e\right)^4-8\left(a^2+3ab+2b^2\right)\cos\left(fx+e\right)^2-4\left((a+2b)\cos\left(fx+e\right)^2-2a-2b\right)\sqrt{-b\cos}}{\cos\left(fx+e\right)^4}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^7*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(5*a^3 + 6*a^2*b)*sqrt(a + b)*cos(f*x + e)^6*log(((a^2 + 8*a*b +
8*b^2)*cos(f*x + e)^4 - 8*(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^2 - 4*((a + 2*
b)*cos(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b)*
sin(f*x + e) + 8*a^2 + 16*a*b + 8*b^2)/cos(f*x + e)^4) + 4*((15*a^3 + 23*a^
2*b + 4*a*b^2 - 4*b^3)*cos(f*x + e)^4 + 8*a^3 + 24*a^2*b + 24*a*b^2 + 8*b^3
 + 2*(5*a^3 + 8*a^2*b + a*b^2 - 2*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)
^2 + a + b)*sin(f*x + e))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^6), -1/96*(3*
(5*a^3 + 6*a^2*b)*sqrt(-a - b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - 2*a -
 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(((a*b + b^2)*cos(f*x +
e)^2 - a^2 - 2*a*b - b^2)*sin(f*x + e)))*cos(f*x + e)^6 - 2*((15*a^3 + 23*a
^2*b + 4*a*b^2 - 4*b^3)*cos(f*x + e)^4 + 8*a^3 + 24*a^2*b + 24*a*b^2 + 8*b^
3 + 2*(5*a^3 + 8*a^2*b + a*b^2 - 2*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e
)^2 + a + b)*sin(f*x + e))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^6)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**7*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^7\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^7*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^7, x)
```

3.340 $\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=321

$$-\frac{\left(a^2-9ab-2b^2\right)\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{35bf}+\frac{a(a+b)\left(2a^2+9ab-b^2\right)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b}{\cdots}}}{35b^2f\sqrt{a+b\sin^2(e+\cdots}}$$

```
[Out] -((a^2 - 9*a*b - 2*b^2)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2
])/(35*b*f) + (2*(4*a + b)*Cos[e + f*x]^3*Sin[e + f*x]*Sqrt[a + b*Sin[e + f
*x]^2])/(35*f) - (b*Cos[e + f*x]^5*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])
/(7*f) - (2*(a - b)*(a^2 + 6*a*b + b^2)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcS
in[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(35*b^2*
f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (a*(a + b)*(2*a^2 + 9*a*b - b^2)*Sqrt[C
os[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1
+ (b*Sin[e + f*x]^2)/a])/(35*b^2*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.389175, antiderivative size = 321, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3192, 416, 528, 524, 426, 424, 421, 419}

$$-\frac{\left(a^2-9ab-2b^2\right)\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{35bf}+\frac{a(a+b)\left(2a^2+9ab-b^2\right)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b}{\cdots}}}{35b^2f\sqrt{a+b\sin^2(e+\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a^2 - 9*a*b - 2*b^2)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2
])/(35*b*f) + (2*(4*a + b)*Cos[e + f*x]^3*Sin[e + f*x]*Sqrt[a + b*Sin[e + f
*x]^2])/(35*f) - (b*Cos[e + f*x]^5*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])
/(7*f) - (2*(a - b)*(a^2 + 6*a*b + b^2)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcS
in[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(35*b^2*
f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (a*(a + b)*(2*a^2 + 9*a*b - b^2)*Sqrt[C
os[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1
+ (b*Sin[e + f*x]^2)/a])/(35*b^2*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
```

```
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\mathrm{Subst}\left(\int\left(1-x^2\right)^{3/2}\left(a+bx^2\right)^{3/2}dx,x,\sin(e+}{f}$$

$$= -\frac{b\cos^5(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{7f} - \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+f}{}$$

$$= \frac{2(4a+b)\cos^3(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{35f} - \frac{b\cos^5(e+fx)\sin(}{}$$

$$= -\frac{\left(a^2-9ab-2b^2\right)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{35bf} + \frac{2(4a+b)c}{}$$

$$= -\frac{\left(a^2-9ab-2b^2\right)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{35bf} + \frac{2(4a+b)c}{}$$

$$= -\frac{\left(a^2-9ab-2b^2\right)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{35bf} + \frac{2(4a+b)c}{}$$

$$= -\frac{\left(a^2-9ab-2b^2\right)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{35bf} + \frac{2(4a+b)c}{}$$

Mathematica [A] time = 2.6002, size = 247, normalized size = 0.77

$$\frac{\sqrt{2}b\sin(2(e+fx))\left(b\left(144a^2-192ab-37b^2\right)\cos(2(e+fx))+400a^2b-32a^3+2b^2(b-26a)\cos(4(e+fx))+212ab^2+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-128*a*(a^3 + 5*a^2*b - 5*a*b^2 - b^3)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])
/a]*EllipticE[e + f*x, -(b/a)] + 64*a*(2*a^3 + 11*a^2*b + 8*a*b^2 - b^3)*Sq
rt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)] + Sqrt[2]*b
*(-32*a^3 + 400*a^2*b + 212*a*b^2 + 30*b^3 + b*(144*a^2 - 192*a*b - 37*b^2)
*Cos[2*(e + f*x)] + 2*b^2*(-26*a + b)*Cos[4*(e + f*x)] + 5*b^3*Cos[6*(e + f
*x)])*Sin[2*(e + f*x)])/(2240*b^2*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A] time = 1.266, size = 590, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/35*(5*b^4*sin(f*x+e)*cos(f*x+e)^8+(-13*a*b^3-7*b^4)*cos(f*x+e)^6*sin(f*x+
e)+(9*a^2*b^2+a*b^3)*cos(f*x+e)^4*sin(f*x+e)+(-a^3*b+8*a^2*b^2+11*a*b^3+2*b
```

```
^4)*cos(f*x+e)^2*sin(f*x+e)+2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)
/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^4+11*(cos(f*x+e)^2)^(1/2)*
(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3*
b+8*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*
x+e),(-1/a*b)^(1/2))*a^2*b^2-(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/
a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b^3-2*(cos(f*x+e)^2)^(1/2)*
(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^4-
10*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x
+e),(-1/a*b)^(1/2))*a^3*b+10*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/
a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b^2+2*(cos(f*x+e)^2)^(1/2
)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*
b^3)/b^2/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(b\cos\left(fx+e\right)^6-(a+b)\cos\left(fx+e\right)^4\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(b*cos(f*x + e)^6 - (a + b)*cos(f*x + e)^4)*sqrt(-b*cos(f*x + e)^
2 + a + b), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^4, x)
```

3.341 $\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=259

$$-\frac{\left(3a^2-7ab-2b^2\right)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{15bf\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{b\sin(e+fx)\cos^3(e+\ldots}{}$$

```
[Out] (2*(3*a + b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(15*f) -
 (b*Cos[e + f*x]^3*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(5*f) - ((3*a^2
 - 7*a*b - 2*b^2)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/
a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(15*b*f*Sqrt[1 + (b*Sin[e + f*
x]^2)/a]) + (a*(3*a - b)*(a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[
e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(15*b*f*Sqr
t[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.284961, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3192, 416, 528, 524, 426, 424, 421, 419}

$$-\frac{\left(3a^2-7ab-2b^2\right)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{15bf\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{b\sin(e+fx)\cos^3(e+\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (2*(3*a + b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(15*f) -
 (b*Cos[e + f*x]^3*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(5*f) - ((3*a^2
 - 7*a*b - 2*b^2)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/
a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(15*b*f*Sqrt[1 + (b*Sin[e + f*
x]^2)/a]) + (a*(3*a - b)*(a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[
e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(15*b*f*Sqr
t[a + b*Sin[e + f*x]^2])
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] && !IntegerQ[p]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] && !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \sqrt{1-x^2}\left(a+bx^2\right)^{3/2}dx, x, \sin(e+\right.}{f}$$

$$= -\frac{b\cos^3(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{5f} - \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e\right.}{}$$

$$= \frac{2(3a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15f} - \frac{b\cos^3(e+fx)\sin}{}$$

$$= \frac{2(3a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15f} - \frac{b\cos^3(e+fx)\sin}{}$$

$$= \frac{2(3a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15f} - \frac{b\cos^3(e+fx)\sin}{}$$

$$= \frac{2(3a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15f} - \frac{b\cos^3(e+fx)\sin}{}$$

Mathematica [A] time = 1.32096, size = 200, normalized size = 0.77

$$\frac{\sqrt{2}b\sin(2(e+fx))\left(48a^2-4b(9a+2b)\cos(2(e+fx))+28ab+3b^2\cos(4(e+fx))+5b^2\right)+16a\left(3a^2+2ab-b^2\right)\sqrt{\frac{2}{}}}{240bf\sqrt{2a-b\cos(2(e+fx))+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-16*a*(3*a^2 - 7*a*b - 2*b^2)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*Ellip
ticE[e + f*x, -(b/a)] + 16*a*(3*a^2 + 2*a*b - b^2)*Sqrt[(2*a + b - b*Cos[2*
(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)] + Sqrt[2]*b*(48*a^2 + 28*a*b + 5*
b^2 - 4*b*(9*a + 2*b)*Cos[2*(e + f*x)] + 3*b^2*Cos[4*(e + f*x)])*Sin[2*(e +
 f*x)])/(240*b*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A] time = 1.421, size = 429, normalized size = 1.7

$$\frac{1}{15b\cos(fx+e)f}\left(-3b^3\left(\sin(fx+e)\right)^7-9ab^2\left(\sin(fx+e)\right)^5+4b^3\left(\sin(fx+e)\right)^5+3\sqrt{\left(\cos(fx+e)\right)^2}\sqrt{\frac{a+b(}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/15*(-3*b^3*sin(f*x+e)^7-9*a*b^2*sin(f*x+e)^5+4*b^3*sin(f*x+e)^5+3*(cos(f*
x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1
/2))*a^3+2*a^2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(
```

```
sin(f*x+e),(-1/a*b)^(1/2))*b-a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^
(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^2-3*(cos(f*x+e)^2)^(1/2)*((a+b
*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3+7*(cos(f*x
+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/
2))*a^2*b+2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin
(f*x+e),(-1/a*b)^(1/2))*a*b^2-6*a^2*b*sin(f*x+e)^3+10*a*b^2*sin(f*x+e)^3-si
n(f*x+e)^3*b^3+6*sin(f*x+e)*a^2*b-a*b^2*sin(f*x+e))/b/cos(f*x+e)/(a+b*sin(f
*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(b\cos\left(fx+e\right)^4-(a+b)\cos\left(fx+e\right)^2\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(b*cos(f*x + e)^4 - (a + b)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^
2 + a + b), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^2, x)
```

3.342 $\int \left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=154

$$-\frac{b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(e+fx\middle|-\frac{b}{a}\right)}{3f\sqrt{a+b\sin^2(e+fx)}}+\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx}}{3f\sqrt{\frac{b\sin^2(e+fx)}{a}}}$$

```
[Out] -(b*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f) + (2*(2*a +
 b)*EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f*Sqrt[1 + (b
*Sin[e + f*x]^2)/a]) - (a*(a + b)*EllipticF[e + f*x, -(b/a)]*Sqrt[1 + (b*Si
n[e + f*x]^2)/a])/(3*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.1651, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3180, 3172, 3178, 3177, 3183, 3182}

$$-\frac{b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(e+fx\middle|-\frac{b}{a}\right)}{3f\sqrt{a+b\sin^2(e+fx)}}+\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx}}{3f\sqrt{\frac{b\sin^2(e+fx)}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -(b*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f) + (2*(2*a +
 b)*EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f*Sqrt[1 + (b
*Sin[e + f*x]^2)/a]) - (a*(a + b)*EllipticF[e + f*x, -(b/a)]*Sqrt[1 + (b*Si
n[e + f*x]^2)/a])/(3*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 3180

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p - 1))/(2*f*p), x] + Dist[
1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a + b
)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a +
 b, 0] && GtQ[p, 1]
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rubi steps

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}dx = -\frac{b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{1}{3}\int \frac{a(3a+b)+2b(2a+b)\sin^2(e+}{\sqrt{a+b\sin^2(e+fx)}}$$

$$= -\frac{b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{1}{3}(a(a+b))\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,d$$

$$= -\frac{b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{\left(2(2a+b)\sqrt{a+b\sin^2(e+fx)}\right)\int\sqrt{}}{3\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

$$= -\frac{b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{2(2a+b)E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2}}{3f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

Mathematica [A] time = 0.730871, size = 156, normalized size = 1.01

$$\frac{b\sin(2(e+fx))(-2a+b\cos(2(e+fx))-b)-2\sqrt{2}a(a+b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}F\left(e+fx\middle|-\frac{b}{a}\right)+4\sqrt{2}a(2a+b)\sqrt{\frac{2a-b\cos}{}}}{6\sqrt{2}f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (4*Sqrt[2]*a*(2*a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e +
 f*x, -(b/a)] - 2*Sqrt[2]*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*
EllipticF[e + f*x, -(b/a)] + b*(-2*a - b + b*Cos[2*(e + f*x)])*Sin[2*(e + f
*x)])/(6*Sqrt[2]*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A] time = 1.099, size = 266, normalized size = 1.7

$$\frac{1}{f\cos\left(fx+e\right)}\left(-\frac{a^2}{3}\sqrt{\left(\cos\left(fx+e\right)\right)^2}\sqrt{\frac{a+b\left(\sin\left(fx+e\right)\right)^2}{a}}\mathrm{EllipticF}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)-\frac{ab}{3}\sqrt{\left(\cos\left(fx+e\right)\right)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] (-1/3*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e
),(-1/a*b)^(1/2))*a^2-1/3*a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/
2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b+4/3*(cos(f*x+e)^2)^(1/2)*((a+b*si
n(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2+2/3*(cos(f*x+
e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2
))*a*b+1/3*sin(f*x+e)^5*b^2+1/3*sin(f*x+e)^3*a*b-1/3*sin(f*x+e)^3*b^2-1/3*s
in(f*x+e)*a*b)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(-b\cos\left(fx+e\right)^2+a+b\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((-b*cos(f*x + e)^2 + a + b)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.343 $\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=182

$$\frac{(a+b)\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f}+\frac{a(a+b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] -(((a + 2*b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*S
ec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]))
 + (a*(a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*
Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(f*Sqrt[a + b*Sin[e + f*x]^2])
 + ((a + b)*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x])/f
```

Rubi [A] time = 0.180064, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3192, 413, 524, 426, 424, 421, 419}

$$\frac{(a+b)\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f}+\frac{a(a+b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -(((a + 2*b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*S
ec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]))
 + (a*(a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*
Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(f*Sqrt[a + b*Sin[e + f*x]^2])
 + ((a + b)*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x])/f
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
```

```
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int\frac{\left(a+bx^2\right)^{3/2}}{\left(1-x^2\right)^{3/2}}\,dx,x,\sin(e+fx)\right)}{f}$$

$$= \frac{(a+b)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{f} - \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Su}}{}$$

$$= \frac{(a+b)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{f} + \frac{\left(a(a+b)\sqrt{\cos^2(e+fx)}\sec(e+\right.}{}$$

$$= \frac{(a+b)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{f} - \frac{\left(\left((a+2b)\sqrt{\cos^2(e+fx)}\sec(e+\right.\right.}{}$$

$$= -\frac{(a+2b)\sqrt{\cos^2(e+fx)}E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b\sin}}{f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

Mathematica [A] time = 0.840234, size = 144, normalized size = 0.79

$$\frac{(a+b)\left(\sqrt{2}\tan(e+fx)(2a-b\cos(2(e+fx))+b)+2a\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}F\left(e+fx\middle|-\frac{b}{a}\right)\right)-2a(a+2b)\sqrt{\frac{2a-b\cos(2(e+f}{a}}}{2f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-2*a*(a + 2*b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -
(b/a)] + (a + b)*(2*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e +
f*x, -(b/a)] + Sqrt[2]*(2*a + b - b*Cos[2*(e + f*x)])*Tan[e + f*x]))/(2*f*S
qrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [B] time = 2.187, size = 466, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] (-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*(a+b)*sin(f*x+e)*cos(f*x+e)^
2+(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a^2+2*a*b+b^2)*sin(f*x+e)+(co
s(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-
1/a*b)^(1/2))*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*a^2+a*b*(cos(f*x+e
)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^
(1/2))*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)-(cos(f*x+e)^2)^(1/2)*(-b/
a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*(-b*cos(
f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*a^2-2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x
+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*(-b*cos(f*x+e)^4+
(a+b)*cos(f*x+e)^2)^(1/2)*a*b)/(-(a+b*sin(f*x+e)^2)*(-1+sin(f*x+e))*(1+sin(
f*x+e)))^(1/2)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(b\cos\left(fx+e\right)^2-a-b\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}\sec\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(b*cos(f*x + e)^2 - a - b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sec(f*
x + e)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^2, x)
```

3.344 $\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=236

$$\frac{2(a-b)\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{(a+b)\tan(e+fx)\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{a(2a-b)\sqrt{\cos^2(e+f}{}$$

```
[Out] (-2*(a - b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Se
c[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a])
 + (a*(2*a - b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)
]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*f*Sqrt[a + b*Sin[e + f*x]
^2]) + (2*(a - b)*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x])/(3*f) + ((a + b)
*Sec[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x])/(3*f)
```

Rubi [A] time = 0.256409, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3192, 413, 527, 524, 426, 424, 421, 419}

$$\frac{2(a-b)\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{(a+b)\tan(e+fx)\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{a(2a-b)\sqrt{\cos^2(e+f}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-2*(a - b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Se
c[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a])
 + (a*(2*a - b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)
]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*f*Sqrt[a + b*Sin[e + f*x]
^2]) + (2*(a - b)*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x])/(3*f) + ((a + b)
*Sec[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x])/(3*f)
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{(a+bx^2)^{3/2}}{(1-x^2)^{5/2}}dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{(a+b)\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{3f} - \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e\right.}{}$$

$$= \frac{2(a-b)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{3f} + \frac{(a+b)\sec^2(e+fx)\sqrt{a+b\sin^2(e}}{3f}$$

$$= \frac{2(a-b)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{3f} + \frac{(a+b)\sec^2(e+fx)\sqrt{a+b\sin^2(e}}{3f}$$

$$= \frac{2(a-b)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{3f} + \frac{(a+b)\sec^2(e+fx)\sqrt{a+b\sin^2(e}}{3f}$$

$$= -\frac{2(a-b)\sqrt{\cos^2(e+fx)}E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b\sin^2(e}}{3f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

Mathematica [A] time = 2.03666, size = 190, normalized size = 0.81

$$\frac{\frac{\tan(e+fx)\sec^2(e+fx)\left(\left(4a^2-6ab-2b^2\right)\cos(2(e+fx))+8a^2+b(b-a)\cos(4(e+fx))+3ab+b^2\right)}{\sqrt{2}} + 2a(2a-b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}F\left(e+fx\middle|-\frac{b}{a}\right) - 4a(a}{6f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-4*a*(a - b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b
/a)] + 2*a*(2*a - b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f
*x, -(b/a)] + ((8*a^2 + 3*a*b + b^2 + (4*a^2 - 6*a*b - 2*b^2)*Cos[2*(e + f*
x)] + b*(-a + b)*Cos[4*(e + f*x)])*Sec[e + f*x]^2*Tan[e + f*x])/Sqrt[2])/(6
*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A] time = 2.043, size = 375, normalized size = 1.6

$$\frac{1}{\left(-3+3\sin\left(fx+e\right)\right)\left(1+\sin\left(fx+e\right)\right)\cos\left(fx+e\right)f}\left(2\sqrt{-b\left(\cos\left(fx+e\right)\right)^4+(a+b)\left(\cos\left(fx+e\right)\right)^2}b(a-b)\sin\left(fx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/3*(2*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*(a-b)*sin(f*x+e)*cos(f*
x+e)^4-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(2*a^2-a*b-3*b^2)*cos(f*x
```

```
+e)^2*sin(f*x+e)-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a^2+2*a*b+b^2)
*sin(f*x+e)-(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(cos(f*x+e)^2)^(1/2)*(-b*cos(
f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*a*(2*EllipticF(sin(f*x+e),(-1/a*b)^(1/2)
)*a-EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-2*EllipticE(sin(f*x+e),(-1/a*b)^
(1/2))*a+2*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*b)*cos(f*x+e)^2)/(-(a+b*sin
(f*x+e)^2)*(-1+sin(f*x+e))*(1+sin(f*x+e)))^(1/2)/(-1+sin(f*x+e))/(1+sin(f*x
+e))/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(b\cos\left(fx+e\right)^2-a-b\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}\sec\left(fx+e\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(b*cos(f*x + e)^2 - a - b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sec(f*
x + e)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^4, x)
```

3.345 $\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=79

$$\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2b^{3/2}f}-\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2bf}$$

```
[Out] ((a + 2*b)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/(2*b
^(3/2)*f) - (Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(2*b*f)
```

Rubi [A] time = 0.0931914, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3190, 388, 217, 206}

$$\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2b^{3/2}f}-\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2bf}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] ((a + 2*b)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/(2*b
^(3/2)*f) - (Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(2*b*f)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1-x^2}{\sqrt{a+bx^2}}\,dx, x, \sin(e+fx)\right)}{f}$$
$$= -\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2bf} + \frac{(a+2b)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sin(e+fx)\right)}{2bf}$$
$$= -\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2bf} + \frac{(a+2b)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2bf}$$
$$= \frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2b^{3/2}f} - \frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2bf}$$

Mathematica [A] time = 0.114194, size = 79, normalized size = 1.

$$\frac{-\frac{(-a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2b^{3/2}} - \frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2b}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-((-a - 2*b)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/(
2*b^(3/2)) - (Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(2*b))/f
```

Maple [A] time = 1.06, size = 98, normalized size = 1.2

$$-\frac{\sin(fx+e)}{2bf}\sqrt{a+b\left(\sin(fx+e)\right)^2} + \frac{a}{2f}\ln\left(\sin(fx+e)\sqrt{b} + \sqrt{a+b\left(\sin(fx+e)\right)^2}\right)b^{-\frac{3}{2}} + \frac{1}{f}\ln\left(\sin(fx+e)\sqrt{b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/2*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/b/f+1/2/f*a/b^(3/2)*ln(sin(f*x+e)*
b^(1/2)+(a+b*sin(f*x+e)^2)^(1/2))+1/f*ln(sin(f*x+e)*b^(1/2)+(a+b*sin(f*x+e)
^2)^(1/2))/b^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.99992, size = 1133, normalized size = 14.34

$$\left[\frac{(a+2\,b)\sqrt{b}\log\left(128\,b^4\cos\left(fx+e\right)^8-256\left(ab^3+2\,b^4\right)\cos\left(fx+e\right)^6+32\left(5\,a^2b^2+24\,ab^3+24\,b^4\right)\cos\left(fx+e\right)^4+a^4\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((a + 2*b)*sqrt(b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + 2*b^4)*c
os(f*x + e)^6 + 32*(5*a^2*b^2 + 24*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^4 + 3
2*a^3*b + 160*a^2*b^2 + 256*a*b^3 + 128*b^4 - 32*(a^3*b + 10*a^2*b^2 + 24*a
*b^3 + 16*b^4)*cos(f*x + e)^2 - 8*(16*b^3*cos(f*x + e)^6 - 24*(a*b^2 + 2*b^
3)*cos(f*x + e)^4 - a^3 - 10*a^2*b - 24*a*b^2 - 16*b^3 + 2*(5*a^2*b + 24*a*
b^2 + 24*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(b)*sin(f
*x + e)) - 8*sqrt(-b*cos(f*x + e)^2 + a + b)*b*sin(f*x + e))/(b^2*f), -1/8*
((a + 2*b)*sqrt(-b)*arctan(1/4*(8*b^2*cos(f*x + e)^4 - 8*(a*b + 2*b^2)*cos(
f*x + e)^2 + a^2 + 8*a*b + 8*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)/
((2*b^3*cos(f*x + e)^4 + a^2*b + 3*a*b^2 + 2*b^3 - (3*a*b^2 + 4*b^3)*cos(f*
x + e)^2)*sin(f*x + e))) + 4*sqrt(-b*cos(f*x + e)^2 + a + b)*b*sin(f*x + e)
)/(b^2*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32863, size = 96, normalized size = 1.22

$$-\frac{\frac{(a+2\,b)\log\left(\left|-\sqrt{b}\sin\left(fx+e\right)+\sqrt{b\sin\left(fx+e\right)^2+a}\right|\right)}{b^{\frac{3}{2}}}+\frac{\sqrt{b\sin\left(fx+e\right)^2+a}\sin\left(fx+e\right)}{b}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*((a + 2*b)*log(abs(-sqrt(b)*sin(f*x + e) + sqrt(b*sin(f*x + e)^2 + a))
)/b^(3/2) + sqrt(b*sin(f*x + e)^2 + a)*sin(f*x + e)/b)/f
```

3.346 $\int \frac{\cos(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=38

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{\sqrt{b}f}$$

```
[Out] ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]]/(Sqrt[b]*f)
```

Rubi [A] time = 0.046163, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3190, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{\sqrt{b}f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]]/(Sqrt[b]*f)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{\sqrt{b}f}$$

Mathematica [A] time = 0.0156675, size = 38, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{\sqrt{b}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]]/(Sqrt[b]*f)
```

Maple [A] time = 0.088, size = 34, normalized size = 0.9

$$\frac{1}{f}\ln\left(\sin\left(fx+e\right)\sqrt{b}+\sqrt{a+b\left(\sin\left(fx+e\right)\right)^2}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/f*ln(sin(f*x+e)*b^(1/2)+(a+b*sin(f*x+e)^2)^(1/2))/b^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.49464, size = 953, normalized size = 25.08

$$\left[\frac{\log\left(128\,b^4\cos\left(fx+e\right)^8-256\left(ab^3+2\,b^4\right)\cos\left(fx+e\right)^6+32\left(5\,a^2b^2+24\,ab^3+24\,b^4\right)\cos\left(fx+e\right)^4+a^4+32\,a^3b\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + 2*b^4)*cos(f*x + e)^6 + 32*(
5*a^2*b^2 + 24*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^4 + 32*a^3*b + 160*a^2*b^
2 + 256*a*b^3 + 128*b^4 - 32*(a^3*b + 10*a^2*b^2 + 24*a*b^3 + 16*b^4)*cos(f
*x + e)^2 - 8*(16*b^3*cos(f*x + e)^6 - 24*(a*b^2 + 2*b^3)*cos(f*x + e)^4 -
a^3 - 10*a^2*b - 24*a*b^2 - 16*b^3 + 2*(5*a^2*b + 24*a*b^2 + 24*b^3)*cos(f*
x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(b)*sin(f*x + e))/(sqrt(b)*f)
, -1/4*sqrt(-b)*arctan(1/4*(8*b^2*cos(f*x + e)^4 - 8*(a*b + 2*b^2)*cos(f*x
+ e)^2 + a^2 + 8*a*b + 8*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)/((2*
b^3*cos(f*x + e)^4 + a^2*b + 3*a*b^2 + 2*b^3 - (3*a*b^2 + 4*b^3)*cos(f*x +
e)^2)*sin(f*x + e)))/(b*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18072, size = 51, normalized size = 1.34

$$-\frac{\log\left(\left|-\sqrt{b}\sin\left(fx+e\right)+\sqrt{b\sin\left(fx+e\right)^2+a}\right|\right)}{\sqrt{b}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-sqrt(b)*sin(f*x + e) + sqrt(b*sin(f*x + e)^2 + a)))/(sqrt(b)*f)
```

3.347 $\int \frac{\sec(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=42

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f\sqrt{a+b}}$$

```
[Out] ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]]/(Sqrt[a + b]
*f)
```

Rubi [A] time = 0.0720426, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3190, 377, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]]/(Sqrt[a + b]
*f)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\,dx, x, \frac{\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{\sqrt{a+b}f}$$

Mathematica [A] time = 0.0315527, size = 42, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]]/(Sqrt[a + b]
*f)
```

Maple [B] time = 2.483, size = 113, normalized size = 2.7

$$\frac{1}{2f}\ln\left(2\frac{\sqrt{a+b}\sqrt{a+b-b\left(\cos\left(fx+e\right)\right)^2}+b\sin\left(fx+e\right)+a}{-1+\sin\left(fx+e\right)}\right)\frac{1}{\sqrt{a+b}}-\frac{1}{2f}\ln\left(2\frac{\sqrt{a+b}\sqrt{a+b-b\left(\cos\left(fx+e\right)\right)}}{1+\sin\left(fx+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/2/(a+b)^(1/2)/f*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1
/2)+b*sin(f*x+e)+a))-1/2/(a+b)^(1/2)/f*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+
b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.7494, size = 595, normalized size = 14.17

$$\left[\frac{\log\left(\frac{\left(a^2+8ab+8b^2\right)\cos\left(fx+e\right)^4-8\left(a^2+3ab+2b^2\right)\cos\left(fx+e\right)^2-4\left(\left(a+2b\right)\cos\left(fx+e\right)^2-2a-2b\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}\sqrt{a+b}\sin\left(fx+e\right)+8a^2+16ab+8b^2}{\cos\left(fx+e\right)^4}\right)}{4\sqrt{a+b}f}, -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 8*(a^2 + 3*a*b + 2*b^2)*co
s(f*x + e)^2 - 4*((a + 2*b)*cos(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x + e
)^2 + a + b)*sqrt(a + b)*sin(f*x + e) + 8*a^2 + 16*a*b + 8*b^2)/cos(f*x + e
)^4)/(sqrt(a + b)*f), -1/2*sqrt(-a - b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^
2 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(((a*b + b^2)*c
os(f*x + e)^2 - a^2 - 2*a*b - b^2)*sin(f*x + e)))/((a + b)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(e+fx\right)}{\sqrt{a+b\sin^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/sqrt(a + b*sin(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)/sqrt(b*sin(f*x + e)^2 + a), x)
```

3.348 $\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=91

$$\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2f(a+b)^{3/2}}+\frac{\tan(e+fx)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2f(a+b)}$$

```
[Out] ((a + 2*b)*ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/
(2*(a + b)^(3/2)*f) + (Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x]
)/(2*(a + b)*f)
```

Rubi [A] time = 0.111315, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3190, 382, 377, 206}

$$\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2f(a+b)^{3/2}}+\frac{\tan(e+fx)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2f(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] ((a + 2*b)*ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/
(2*(a + b)^(3/2)*f) + (Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x]
)/(2*(a + b)*f)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2\sqrt{a+bx^2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{2(a+b)f} + \frac{(a+2b)\,\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \sin(e+\right.}{2(a+b)f}$$

$$= \frac{\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{2(a+b)f} + \frac{(a+2b)\,\text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\,dx, x, \frac{\sin(e+fx)}{\sqrt{a+b\sin^2(e+}}\right.}{2(a+b)f}$$

$$= \frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2(a+b)^{3/2}f} + \frac{\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{2(a+b)f}$$

Mathematica [C] time = 9.84732, size = 436, normalized size = 4.79

$$\frac{\tan(e+fx)\sec^3(e+fx)\left(\frac{b\sin^2(e+fx)}{a}+1\right)\left(-30b\sin^2(e+fx)\sqrt{-\frac{\tan^2(e+fx)\sec^2(e+fx)\left(a^2+ab\left(\sin^2(e+fx)+1\right)+b^2\sin^2(e+fx)\right)}{a^2}} - 45a\sqrt{\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^3/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (Sec[e + f*x]^3*(1 + (b*Sin[e + f*x]^2)/a)*Tan[e + f*x]*(45*a*ArcSin[Sqrt[-
(((a + b)*Tan[e + f*x]^2)/a)]] + 30*b*ArcSin[Sqrt[-(((a + b)*Tan[e + f*x]^2
)/a)]]*Sin[e + f*x]^2 + 16*a*Hypergeometric2F1[2, 3, 7/2, -(((a + b)*Tan[e
+ f*x]^2)/a)]*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*T
an[e + f*x]^2)/a))^(5/2) + 16*b*Hypergeometric2F1[2, 3, 7/2, -(((a + b)*Tan
[e + f*x]^2)/a)]*Sin[e + f*x]^2*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)
)/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(5/2) - 45*a*Sqrt[-((Sec[e + f*x]^2*(a
^2 + b^2*Sin[e + f*x]^2 + a*b*(1 + Sin[e + f*x]^2))*Tan[e + f*x]^2)/a^2)] -
 30*b*Sin[e + f*x]^2*Sqrt[-((Sec[e + f*x]^2*(a^2 + b^2*Sin[e + f*x]^2 + a*b
*(1 + Sin[e + f*x]^2))*Tan[e + f*x]^2)/a^2)]))/(30*a*f*Sqrt[a + b*Sin[e + f
*x]^2]*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*Tan[e +
f*x]^2)/a))^(3/2))
```

Maple [B] time = 2.79, size = 360, normalized size = 4.

$$\frac{1}{4\left(\cos\left(fx+e\right)\right)^2 f}\left(2\sin\left(fx+e\right)\sqrt{a+b-b\left(\cos\left(fx+e\right)\right)^2}(a+b)^{3/2} - \left(\ln\left(2\frac{\sqrt{a+b}\sqrt{a+b-b\left(\cos\left(fx+e\right)\right)^2}-b\,\text{s}}{1+\sin\left(fx+e\right)}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/4*(2*sin(f*x+e)*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(3/2)-(ln(2/(1+sin(f*x+e
))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^2+3*ln(2/(1+s
in(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a*b+2*l
n(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))
*b^2-ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x
+e)+a))*a^2-3*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+
b*sin(f*x+e)+a))*a*b-2*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^
2)^(1/2)+b*sin(f*x+e)+a))*b^2)*cos(f*x+e)^2)/(a+b)^(5/2)/cos(f*x+e)^2/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)^{3}}{\sqrt{b\sin\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^3/sqrt(b*sin(f*x + e)^2 + a), x)
```

Fricas [B] time = 3.2708, size = 906, normalized size = 9.96

$$\left[\frac{(a+2b)\sqrt{a+b}\cos\left(fx+e\right)^{2}\log\left(\frac{\left(a^{2}+8ab+8b^{2}\right)\cos\left(fx+e\right)^{4}-8\left(a^{2}+3ab+2b^{2}\right)\cos\left(fx+e\right)^{2}-4\left((a+2b)\cos\left(fx+e\right)^{2}-2a-2b\right)\sqrt{-b\cos\left(fx+e\right)^{2}+\cdots}}{\cos\left(fx+e\right)^{4}}\right.}{8\left(a^{2}+2ab+b^{2}\right)f\cos\left(fx+e\right)^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*((a + 2*b)*sqrt(a + b)*cos(f*x + e)^2*log(((a^2 + 8*a*b + 8*b^2)*cos(f
*x + e)^4 - 8*(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*cos(f*x +
 e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b)*sin(f*x + e)
 + 8*a^2 + 16*a*b + 8*b^2)/cos(f*x + e)^4) + 4*sqrt(-b*cos(f*x + e)^2 + a +
 b)*(a + b)*sin(f*x + e))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2), -1/4*((a
+ 2*b)*sqrt(-a - b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - 2*a - 2*b)*sqrt(
-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(((a*b + b^2)*cos(f*x + e)^2 - a^2
- 2*a*b - b^2)*sin(f*x + e)))*cos(f*x + e)^2 - 2*sqrt(-b*cos(f*x + e)^2 + a
 + b)*(a + b)*sin(f*x + e))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^{3}\left(e+fx\right)}{\sqrt{a+b\sin^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)**3/sqrt(a + b*sin(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)^{3}}{\sqrt{b\sin\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^3/sqrt(b*sin(f*x + e)^2 + a), x)
```

3.349 $\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=168

$$\frac{(a+b)(2a+3b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(e+fx\middle|-\frac{b}{a}\right)}{3b^2f\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a+2b)\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{3b^2f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{\sin(e+fx)\cos(e+}{}$$

```
[Out] -(Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*b*f) - (2*(a + 2
*b)*EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(3*b^2*f*Sqrt[1
+ (b*Sin[e + f*x]^2)/a]) + ((a + b)*(2*a + 3*b)*EllipticF[e + f*x, -(b/a)]*
Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*b^2*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.194184, antiderivative size = 208, normalized size of antiderivative = 1.24, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3192, 416, 524, 426, 424, 421, 419}

$$\frac{(a+b)(2a+3b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{3b^2f\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a+2b)\sqrt{\cos^2(e+fx)}\sec(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -(Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*b*f) - (2*(a + 2
*b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*
x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*b^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (
(a + b)*(2*a + 3*b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(
b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*b^2*f*Sqrt[a + b*Sin[
e + f*x]^2])
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{\left(1-x^2\right)^{3/2}}{\sqrt{a+bx^2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3bf} + \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{a+3b}{\sqrt{1-}}\right.}{3bf}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3bf} - \frac{\left(2(a+2b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subs}}{3b^2f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3bf} - \frac{\left(2(a+2b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+}\right.}{3b^2f\sqrt{}}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3bf} - \frac{2(a+2b)\sqrt{\cos^2(e+fx)}E\left(\sin^{-1}(\sin(e+fx\right.}{3b^2f\sqrt{1+\frac{b}{}}}$$

Mathematica [A] time = 0.85441, size = 170, normalized size = 1.01

$$\frac{2\sqrt{2}\left(2a^2+5ab+3b^2\right)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}F\left(e+fx\middle|-\frac{b}{a}\right)+b\sin(2(e+fx))(-2a+b\cos(2(e+fx))-b)-4\sqrt{2}a(a+2}{6\sqrt{2}b^2f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-4*Sqrt[2]*a*(a + 2*b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e
+ f*x, -(b/a)] + 2*Sqrt[2]*(2*a^2 + 5*a*b + 3*b^2)*Sqrt[(2*a + b - b*Cos[2*
(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)] + b*(-2*a - b + b*Cos[2*(e + f*x)
])*Sin[2*(e + f*x)])/(6*Sqrt[2]*b^2*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A] time = 1.154, size = 316, normalized size = 1.9

$$\frac{1}{3\,b^2\cos\left(fx+e\right)f}\left(\left(\sin\left(fx+e\right)\right)^5b^2+2\sqrt{\left(\cos\left(fx+e\right)\right)^2}\sqrt{\frac{a+b\left(\sin\left(fx+e\right)\right)^2}{a}}\mathrm{EllipticF}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3*(sin(f*x+e)^5*b^2+2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*E
llipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2+5*a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(
f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b+3*(cos(f*x+e)^2)^
(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^2
-2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(
-1/a*b)^(1/2))*a^2-4*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*Elli
pticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b+sin(f*x+e)^3*a*b-sin(f*x+e)^3*b^2-sin(
f*x+e)*a*b)/b^2/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cos\left(fx+e\right)^4}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^4/sqrt(b*sin(f*x + e)^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\cos\left(fx+e\right)^4}{b\cos\left(fx+e\right)^2-a-b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)*cos(f*x + e)^4/(b*cos(f*x + e)^2
- a - b), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^4}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^4/sqrt(b*sin(f*x + e)^2 + a), x)
```

3.350 $\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}} \, dx$

Optimal. Leaf size=114

$$\frac{(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(e+fx\left|-\frac{b}{a}\right.\right)}{bf\sqrt{a+b\sin^2(e+fx)}}-\frac{a\sqrt{\frac{b\sin^2(e+fx)}{a}+1}E\left(e+fx\left|-\frac{b}{a}\right.\right)}{bf\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] -((a*EllipticE[e + f*x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(b*f*Sqrt[a
 + b*Sin[e + f*x]^2])) + ((a + b)*EllipticF[e + f*x, -(b/a)]*Sqrt[1 + (b*Si
n[e + f*x]^2)/a])/(b*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.140654, antiderivative size = 153, normalized size of antiderivative = 1.34, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3192, 423, 426, 424, 421, 419}

$$\frac{(a+b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{bf\sqrt{a+b\sin^2(e+fx)}}-\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2}}{bf\sqrt{\frac{b\sin^2(e}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x
]*Sqrt[a + b*Sin[e + f*x]^2])/(b*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a])) + ((a +
 b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*
x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(b*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 423

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{\sqrt{a+bx^2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= -\frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^2}}\,dx, x, \sin(e+fx)\right)}{bf} + \frac{\left((a+b)\sqrt{\cos^2(e+fx)}\right.}{}$$

$$= -\frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{bx^2}{a}}}{\sqrt{1-x^2}}\,dx, x, \sin(e+fx)\right)}{bf\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} + -$$

$$= -\frac{\sqrt{\cos^2(e+fx)}E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{bf\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} + \frac{(a+b)\sqrt{\cos^2}}{}$$

Mathematica [A] time = 0.200491, size = 83, normalized size = 0.73

$$\frac{\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}\left((a+b)F\left(e+fx\middle|-\frac{b}{a}\right)-aE\left(e+fx\middle|-\frac{b}{a}\right)\right)}{bf\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*(-(a*EllipticE[e + f*x, -(b/a)]) +
(a + b)*EllipticF[e + f*x, -(b/a)]))/(b*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]
])
```

Maple [A] time = 1.05, size = 113, normalized size = 1.

$$-\frac{1}{b\cos\left(fx+e\right)f}\sqrt{\left(\cos\left(fx+e\right)\right)^{2}}\sqrt{\frac{a+b\left(\sin\left(fx+e\right)\right)^{2}}{a}}\left(\mathrm{EllipticE}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)a-\mathrm{EllipticF}\left(\sin\left(fx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] -(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*(EllipticE(sin(f*x+e),(-
1/a*b)^(1/2))*a-EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a-EllipticF(sin(f*x+e)
,(-1/a*b)^(1/2))*b)/b/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^{2}}{\sqrt{b\sin\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^2/sqrt(b*sin(f*x + e)^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\cos\left(fx+e\right)^{2}}{b\cos\left(fx+e\right)^{2}-a-b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)*cos(f*x + e)^2/(b*cos(f*x + e)^2
- a - b), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^2}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^2/sqrt(b*sin(f*x + e)^2 + a), x)
```

3.351 $\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=51

$$\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] (EllipticF[e + f*x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(f*Sqrt[a + b*S
in[e + f*x]^2])
```

Rubi [A] time = 0.0347665, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3183, 3182}

$$\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (EllipticF[e + f*x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(f*Sqrt[a + b*S
in[e + f*x]^2])
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx &= \frac{\sqrt{1+\frac{b\sin^2(e+fx)}{a}}\int \frac{1}{\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}\,dx}{\sqrt{a+b\sin^2(e+fx)}}\\ &= \frac{F\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{f\sqrt{a+b\sin^2(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.0746555, size = 60, normalized size = 1.18

$$\frac{\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}F\left(e+fx\left|-\frac{b}{a}\right.\right)}{f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)])/(f*Sqrt
[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [C] time = 0.236, size = 60, normalized size = 1.2

$$\frac{1}{f}\sqrt{-\frac{b\left(\cos\left(fx+e\right)\right)^{2}-a-b}{a}}\mathrm{InverseJacobiAM}\left(fx+e,i\sqrt{b}\frac{1}{\sqrt{a}}\right)\frac{1}{\sqrt{a+b-b\left(\cos\left(fx+e\right)\right)^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/f/(a+b-b*cos(f*x+e)^2)^(1/2)*(-(b*cos(f*x+e)^2-a-b)/a)^(1/2)*InverseJacob
iAM(f*x+e,I/a^(1/2)*b^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sin\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sin(f*x + e)^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}}{b\cos\left(fx+e\right)^{2}-a-b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)/(b*cos(f*x + e)^2 - a - b), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + b\sin^2\left(e + fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sin(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sin\left(fx + e\right)^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sin(f*x + e)^2 + a), x)
```

3.352 $\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=140

$$\frac{\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f(a+b)}+\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}-\frac{\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{f(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

```
[Out] -((EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/((a + b)*f*Sqrt[1
 + (b*Sin[e + f*x]^2)/a])) + (EllipticF[e + f*x, -(b/a)]*Sqrt[1 + (b*Sin[e
+ f*x]^2)/a])/(f*Sqrt[a + b*Sin[e + f*x]^2]) + (Sqrt[a + b*Sin[e + f*x]^2]*
Tan[e + f*x])/((a + b)*f)
```

Rubi [A] time = 0.168916, antiderivative size = 180, normalized size of antiderivative = 1.29, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3192, 414, 21, 423, 426, 424, 421, 419}

$$\frac{\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f(a+b)}+\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}-\frac{\sqrt{\cos^2(e+f}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x
]*Sqrt[a + b*Sin[e + f*x]^2])/((a + b)*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a])) +
 (Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]
*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(f*Sqrt[a + b*Sin[e + f*x]^2]) + (Sqrt[a +
 b*Sin[e + f*x]^2]*Tan[e + f*x])/((a + b)*f)
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 423

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{1}{(1-x^2)^{3/2}\sqrt{a+bx^2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{(a+b)f} + \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{b-bx^2}{\sqrt{1-x^2}\sqrt{a+bx^2}}\,dx, x\right.}{(a+b)f}$$

$$= \frac{\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{(a+b)f} + \frac{\left(b\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{\sqrt{a+bx^2}}\,dx, x, \sin\right.}{(a+b)f}$$

$$= \frac{\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{(a+b)f} + \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{a+bx^2}}\,dx, x\right.}{f}$$

$$= \frac{\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{(a+b)f} - \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\right)\text{Subst}}{(a+b)f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

$$= -\frac{\sqrt{\cos^2(e+fx)}E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{(a+b)f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} + \frac{\sqrt{\cos^2(e+fx}}{}$$

Mathematica [A] time = 0.609255, size = 141, normalized size = 1.01

$$\frac{\sqrt{2}\tan(e+fx)(2a-b\cos(2(e+fx))+b)+2(a+b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}F\left(e+fx\middle|-\frac{b}{a}\right)-2a\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}E\left(e+fx\right|}{2f(a+b)\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-2*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] + 2
*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)]
+ Sqrt[2]*(2*a + b - b*Cos[2*(e + f*x)])*Tan[e + f*x])/(2*(a + b)*f*Sqrt[2*
a + b - b*Cos[2*(e + f*x)]])
```

Maple [A] time = 2.016, size = 278, normalized size = 2.

$$\frac{1}{(a+b)\cos(fx+e)f}\sqrt{-b\left(\cos(fx+e)\right)^4+(a+b)\left(\cos(fx+e)\right)^2}\left(a\sqrt{\left(\cos(fx+e)\right)^2}\sqrt{-\frac{b\left(\cos(fx+e)\right)^2}{a}+\frac{a+b}{a}}\text{El}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] (-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a*(cos(f*x+e)^2)^(1/2)*(-b/a*co
s(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))+b*(cos(f*x+e
)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^
```

```
(1/2))-a*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(s
in(f*x+e),(-1/a*b)^(1/2))-cos(f*x+e)^2*sin(f*x+e)*b+a*sin(f*x+e)+b*sin(f*x+
e))/(a+b)/(-(a+b*sin(f*x+e)^2)*(-1+sin(f*x+e))*(1+sin(f*x+e)))^(1/2)/cos(f*
x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)^2}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^2/sqrt(b*sin(f*x + e)^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\sec\left(fx+e\right)^2}{b\cos\left(fx+e\right)^2-a-b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)*sec(f*x + e)^2/(b*cos(f*x + e)^2
- a - b), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2\left(e+fx\right)}{\sqrt{a+b\sin^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)**2/sqrt(a + b*sin(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)^2}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")

[Out] integrate(sec(f*x + e)^2/sqrt(b*sin(f*x + e)^2 + a), x)
```

3.353 $\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}} \, dx$

Optimal. Leaf size=212

$$\frac{2(a+2b)\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f(a+b)^2}+\frac{(2a+3b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(e+fx\middle|-\frac{b}{a}\right)}{3f(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a+2b)\sqrt{a+b\sin^2(e+fx)}}{3f(a+b)^2\sqrt{\frac{b\sin^2(e+}{a}}}$$

```
[Out] (-2*(a + 2*b)*EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(3*(a
+ b)^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + ((2*a + 3*b)*EllipticF[e + f*x,
-(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*(a + b)*f*Sqrt[a + b*Sin[e + f*x
]^2]) + (2*(a + 2*b)*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x])/(3*(a + b)^2*
f) + (Sec[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x])/(3*(a + b)*f)
```

Rubi [A] time = 0.248134, antiderivative size = 252, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3192, 414, 527, 524, 426, 424, 421, 419}

$$\frac{2(a+2b)\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f(a+b)^2}+\frac{\tan(e+fx)\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f(a+b)}+\frac{(2a+3b)\sqrt{\cos^2(e+fx)}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-2*(a + 2*b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*
Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*(a + b)^2*f*Sqrt[1 + (b*Sin[e +
 f*x]^2)/a]) + ((2*a + 3*b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f
*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*(a + b)*f*Sqr
t[a + b*Sin[e + f*x]^2]) + (2*(a + 2*b)*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e +
f*x])/(3*(a + b)^2*f) + (Sec[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e +
f*x])/(3*(a + b)*f)
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{1}{(1-x^2)^{5/2}\sqrt{a+bx^2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{3(a+b)f} + \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \right.}{3(a+b)f}$$

$$= \frac{2(a+2b)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{3(a+b)^2f} + \frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\tan(e+f}{3(a+b)f}$$

$$= \frac{2(a+2b)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{3(a+b)^2f} + \frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\tan(e+f}{3(a+b)f}$$

$$= \frac{2(a+2b)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{3(a+b)^2f} + \frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\tan(e+f}{3(a+b)f}$$

$$= -\frac{2(a+2b)\sqrt{\cos^2(e+fx)}E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3(a+b)^2f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} + \cdots$$

Mathematica [A] time = 2.17632, size = 205, normalized size = 0.97

$$\frac{2\left(2a^2+5ab+3b^2\right)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}F\left(e+fx\middle|-\frac{b}{a}\right)+\frac{\tan(e+fx)\sec^2(e+fx)\left(\left(4a^2+6ab-2b^2\right)\cos(2(e+fx))+8a^2-b(a+2b)\cos(4(e+fx))+\right.}{\sqrt{2}}}{6f(a+b)^2\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-4*a*(a + 2*b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -
(b/a)] + 2*(2*a^2 + 5*a*b + 3*b^2)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*E
llipticF[e + f*x, -(b/a)] + ((8*a^2 + 15*a*b + 4*b^2 + (4*a^2 + 6*a*b - 2*b
^2)*Cos[2*(e + f*x)] - b*(a + 2*b)*Cos[4*(e + f*x)])*Sec[e + f*x]^2*Tan[e +
 f*x])/Sqrt[2])/(6*(a + b)^2*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A] time = 2.41, size = 405, normalized size = 1.9

$$\frac{1}{\left(-3+3\sin\left(fx+e\right)\right)\left(1+\sin\left(fx+e\right)\right)(a+b)^2\cos\left(fx+e\right)f}\left(2\sqrt{-b\left(\cos\left(fx+e\right)\right)^4+(a+b)\left(\cos\left(fx+e\right)\right)^2}b(a+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3*(2*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*(a+2*b)*sin(f*x+e)*cos(
f*x+e)^4-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(2*a^2+5*a*b+3*b^2)*cos
```

```
(f*x+e)^2*sin(f*x+e)-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a^2+2*a*b+
b^2)*sin(f*x+e)-(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(-b*
cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(2*EllipticF(sin(f*x+e),(-1/a*b)^(1/
2))*a^2+5*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b+3*EllipticF(sin(f*x+e),(
-1/a*b)^(1/2))*b^2-2*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2-4*EllipticE(s
in(f*x+e),(-1/a*b)^(1/2))*a*b)*cos(f*x+e)^2)/(-1+sin(f*x+e))/(1+sin(f*x+e))
/(a+b)^2/(-(a+b*sin(f*x+e)^2)*(-1+sin(f*x+e))*(1+sin(f*x+e)))^(1/2)/cos(f*x
+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)^4}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^4/sqrt(b*sin(f*x + e)^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sec\left(fx+e\right)^4}{b\cos\left(fx+e\right)^2-a-b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)*sec(f*x + e)^4/(b*cos(f*x + e)^2
- a - b), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^4\left(e+fx\right)}{\sqrt{a+b\sin^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)**4/sqrt(a + b*sin(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)^4}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^4/sqrt(b*sin(f*x + e)^2 + a), x)
```

3.354 $\int \frac{\cos^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=75

$$\frac{(a+b)\sin(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{b^{3/2}f}$$

```
[Out] -(ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]]/(b^(3/2)*f)) +
 ((a + b)*Sin[e + f*x])/(a*b*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.10049, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3190, 385, 217, 206}

$$\frac{(a+b)\sin(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{b^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]]/(b^(3/2)*f)) +
 ((a + b)*Sin[e + f*x])/(a*b*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1-x^2}{\left(a+bx^2\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{(a+b)\sin(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sin(e+fx)\right)}{bf}$$

$$= \frac{(a+b)\sin(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{bf}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{b^{3/2}f} + \frac{(a+b)\sin(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}}$$

Mathematica [A] time = 0.178865, size = 88, normalized size = 1.17

$$\frac{\sqrt{b}(a+b)\sin(e+fx) - a^{3/2}\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a}}\right)}{ab^{3/2}f\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*(a + b)*Sin[e + f*x] - a^(3/2)*ArcSinh[(Sqrt[b]*Sin[e + f*x])/Sqrt
[a]]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(a*b^(3/2)*f*Sqrt[a + b*Sin[e + f*x]^2
])
```

Maple [A] time = 0.946, size = 90, normalized size = 1.2

$$\frac{\sin(fx+e)}{bf}\frac{1}{\sqrt{a+b\left(\sin(fx+e)\right)^2}} - \frac{1}{f}\ln\left(\sin(fx+e)\sqrt{b} + \sqrt{a+b\left(\sin(fx+e)\right)^2}\right)b^{-\frac{3}{2}} + \frac{\sin(fx+e)}{af}\frac{1}{\sqrt{a+b\left(\sin\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/f*sin(f*x+e)/b/(a+b*sin(f*x+e)^2)^(1/2)-1/f/b^(3/2)*ln(sin(f*x+e)*b^(1/2)
+(a+b*sin(f*x+e)^2)^(1/2))+sin(f*x+e)/a/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.04989, size = 1324, normalized size = 17.65

$$\left[\frac{\left(ab\cos\left(fx+e\right)^2-a^2-ab\right)\sqrt{b}\log\left(128\,b^4\cos\left(fx+e\right)^8-256\left(ab^3+2\,b^4\right)\cos\left(fx+e\right)^6+32\left(5\,a^2b^2+24\,ab^3+24\,b^4\right)\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*((a*b*cos(f*x + e)^2 - a^2 - a*b)*sqrt(b)*log(128*b^4*cos(f*x + e)^8 -
 256*(a*b^3 + 2*b^4)*cos(f*x + e)^6 + 32*(5*a^2*b^2 + 24*a*b^3 + 24*b^4)*co
s(f*x + e)^4 + a^4 + 32*a^3*b + 160*a^2*b^2 + 256*a*b^3 + 128*b^4 - 32*(a^3
*b + 10*a^2*b^2 + 24*a*b^3 + 16*b^4)*cos(f*x + e)^2 + 8*(16*b^3*cos(f*x + e
)^6 - 24*(a*b^2 + 2*b^3)*cos(f*x + e)^4 - a^3 - 10*a^2*b - 24*a*b^2 - 16*b^
3 + 2*(5*a^2*b + 24*a*b^2 + 24*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2
+ a + b)*sqrt(b)*sin(f*x + e)) - 8*sqrt(-b*cos(f*x + e)^2 + a + b)*(a*b + b
^2)*sin(f*x + e))/(a*b^3*f*cos(f*x + e)^2 - (a^2*b^2 + a*b^3)*f), 1/4*((a*b
*cos(f*x + e)^2 - a^2 - a*b)*sqrt(-b)*arctan(1/4*(8*b^2*cos(f*x + e)^4 - 8*
(a*b + 2*b^2)*cos(f*x + e)^2 + a^2 + 8*a*b + 8*b^2)*sqrt(-b*cos(f*x + e)^2
+ a + b)*sqrt(-b)/((2*b^3*cos(f*x + e)^4 + a^2*b + 3*a*b^2 + 2*b^3 - (3*a*b
^2 + 4*b^3)*cos(f*x + e)^2)*sin(f*x + e))) - 4*sqrt(-b*cos(f*x + e)^2 + a +
 b)*(a*b + b^2)*sin(f*x + e))/(a*b^3*f*cos(f*x + e)^2 - (a^2*b^2 + a*b^3)*f
)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.37571, size = 96, normalized size = 1.28

$$\frac{\frac{\log\left(\left|-\sqrt{b}\sin\left(fx+e\right)+\sqrt{b\sin\left(fx+e\right)^2+a}\right|\right)}{b^{\frac{3}{2}}}+\frac{(a+b)\sin\left(fx+e\right)}{\sqrt{b\sin\left(fx+e\right)^2+a}ab}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] (log(abs(-sqrt(b)*sin(f*x + e) + sqrt(b*sin(f*x + e)^2 + a)))/b^(3/2) + (a
+ b)*sin(f*x + e)/(sqrt(b*sin(f*x + e)^2 + a)*a*b))/f
```

3.355 $\int \frac{\cos(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=29

$$\frac{\sin(e+fx)}{af\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] Sin[e + f*x]/(a*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.0439352, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3190, 191}

$$\frac{\sin(e+fx)}{af\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] Sin[e + f*x]/(a*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}\\ &= \frac{\sin(e+fx)}{af\sqrt{a+b\sin^2(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.0302894, size = 29, normalized size = 1.

$$\frac{\sin(e+fx)}{af\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] Sin[e + f*x]/(a*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Maple [A] time = 0.085, size = 28, normalized size = 1.

$$\frac{\sin\left(fx+e\right)}{af}\frac{1}{\sqrt{a+b\left(\sin\left(fx+e\right)\right)^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] sin(f*x+e)/a/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Maxima [A] time = 0.969579, size = 36, normalized size = 1.24

$$\frac{\sin\left(fx+e\right)}{\sqrt{b\sin\left(fx+e\right)^{2}+a}af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] sin(f*x + e)/(sqrt(b*sin(f*x + e)^2 + a)*a*f)
```

Fricas [A] time = 2.43475, size = 116, normalized size = 4.

$$-\frac{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\sin\left(fx+e\right)}{abf\cos\left(fx+e\right)^{2}-\left(a^{2}+ab\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-b*cos(f*x + e)^2 + a + b)*sin(f*x + e)/(a*b*f*cos(f*x + e)^2 - (a^2
+ a*b)*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36706, size = 39, normalized size = 1.34

$$\frac{\sin\left(fx+e\right)}{\sqrt{b\sin\left(fx+e\right)^{2}+a}af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] sin(f*x + e)/(sqrt(b*sin(f*x + e)^2 + a)*a*f)
```

3.356 $\int \frac{\sec(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=78

$$\frac{b\sin(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f(a+b)^{3/2}}$$

```
[Out] ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]]/((a + b)^(3/
2)*f) + (b*Sin[e + f*x])/(a*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.101223, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3190, 382, 377, 206}

$$\frac{b\sin(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]]/((a + b)^(3/
2)*f) + (b*Sin[e + f*x])/(a*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a+bx^2)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{b\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \sin(e+fx)\right)}{(a+b)f}$$

$$= \frac{b\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\,dx, x, \frac{\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{(a+b)f}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{(a+b)^{3/2}f} + \frac{b\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}}$$

Mathematica [C] time = 7.44037, size = 480, normalized size = 6.15

$$\frac{\tan(e+fx)\sec(e+fx)\left(-\frac{30b(a+b)\sin^2(e+fx)\tan^2(e+fx)\sin^{-1}\left(\sqrt{-\frac{(a+b)\tan^2(e+fx)}{a}}\right)}{a^2} + \frac{30b\sin^2(e+fx)\sqrt{-\frac{(a+b)\tan^2(e+fx)\sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)}{a^2}}}{a}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sec[e + f*x]*Tan[e + f*x]*(-45*ArcSin[Sqrt[-(((a + b)*Tan[e + f*x]^2)/a)]]
 - (30*b*ArcSin[Sqrt[-(((a + b)*Tan[e + f*x]^2)/a)]]*Sin[e + f*x]^2)/a - (4
5*(a + b)*ArcSin[Sqrt[-(((a + b)*Tan[e + f*x]^2)/a)]]*Tan[e + f*x]^2)/a - (
30*b*(a + b)*ArcSin[Sqrt[-(((a + b)*Tan[e + f*x]^2)/a)]]*Sin[e + f*x]^2*Tan
[e + f*x]^2)/a^2 + 4*Hypergeometric2F1[2, 2, 7/2, -(((a + b)*Tan[e + f*x]^2
)/a)]*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*Tan[e + f
*x]^2)/a))^(5/2) + (4*b*Hypergeometric2F1[2, 2, 7/2, -(((a + b)*Tan[e + f*x
]^2)/a)]*Sin[e + f*x]^2*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(
((a + b)*Tan[e + f*x]^2)/a))^(5/2))/a + 45*Sqrt[-(((a + b)*Sec[e + f*x]^2*(
a + b*Sin[e + f*x]^2)*Tan[e + f*x]^2)/a^2)] + (30*b*Sin[e + f*x]^2*Sqrt[-((
(a + b)*Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)*Tan[e + f*x]^2)/a^2)])/a))/(1
5*a*f*Sqrt[a + b*Sin[e + f*x]^2]*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2
))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(3/2))
```

Maple [B] time = 3.955, size = 398, normalized size = 5.1

$$\frac{1}{2a\left(-ab\left(\cos\left(fx+e\right)\right)^2 - b^2\left(\cos\left(fx+e\right)\right)^2 + a^2 + 2ab + b^2\right)f}\left(2\sqrt{a+b}\sqrt{-b\left(\cos\left(fx+e\right)\right)^2 + \frac{ab^2+b^3}{b^2}}b\sin\left(fx+e\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/2/(a+b)^(1/2)/a/(-a*b*cos(f*x+e)^2-b^2*cos(f*x+e)^2+a^2+2*a*b+b^2)*(2*(a+
b)^(1/2)*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*b*sin(f*x+e)-a*b*(ln(2/(-1
+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))-ln(2/
(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a)))*co
s(f*x+e)^2+ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*s
in(f*x+e)+a))*a^2+ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1
/2)+b*sin(f*x+e)+a))*a*b-ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)
^2)^(1/2)-b*sin(f*x+e)+a))*a^2-ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(
f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a*b)/f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.16569, size = 1068, normalized size = 13.69

$$\left[\frac{\left(ab\cos\left(fx+e\right)^2 - a^2 - ab\right)\sqrt{a+b}\log\left(\frac{\left(a^2+8ab+8b^2\right)\cos\left(fx+e\right)^4 - 8\left(a^2+3ab+2b^2\right)\cos\left(fx+e\right)^2 - 4\left(\left(a+2b\right)\cos\left(fx+e\right)^2 - 2a - 2b\right)\sqrt{-b\cos(\ldots}}{\cos\left(fx+e\right)^4}\right)}{4\left(\left(a^3b+2a^2b^2+ab^3\right)f\cos\left(fx+e\right)^2 - \left(a^4+3a\ldots\right)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*b*cos(f*x + e)^2 - a^2 - a*b)*sqrt(a + b)*log(((a^2 + 8*a*b + 8*b^
2)*cos(f*x + e)^4 - 8*(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*c
os(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b)*sin(
f*x + e) + 8*a^2 + 16*a*b + 8*b^2)/cos(f*x + e)^4) - 4*sqrt(-b*cos(f*x + e)
^2 + a + b)*(a*b + b^2)*sin(f*x + e))/((a^3*b + 2*a^2*b^2 + a*b^3)*f*cos(f*
x + e)^2 - (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f), -1/2*((a*b*cos(f*x + e)^
2 - a^2 - a*b)*sqrt(-a - b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - 2*a - 2*
b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(((a*b + b^2)*cos(f*x + e)^
2 - a^2 - 2*a*b - b^2)*sin(f*x + e))) + 2*sqrt(-b*cos(f*x + e)^2 + a + b)*(
a*b + b^2)*sin(f*x + e))/((a^3*b + 2*a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 - (a
^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(e+fx\right)}{\left(a+b\sin^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(sec(e + f*x)/(a + b*sin(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.357 $\int \frac{\sec^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=134

$$-\frac{b(a-2b)\sin(e+fx)}{2af(a+b)^2\sqrt{a+b\sin^2(e+fx)}}+\frac{(a+4b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2f(a+b)^{5/2}}+\frac{\tan(e+fx)\sec(e+fx)}{2f(a+b)\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] ((a + 4*b)*ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/
(2*(a + b)^(5/2)*f) - ((a - 2*b)*b*Sin[e + f*x])/(2*a*(a + b)^2*f*Sqrt[a +
b*Sin[e + f*x]^2]) + (Sec[e + f*x]*Tan[e + f*x])/(2*(a + b)*f*Sqrt[a + b*Si
n[e + f*x]^2])
```

Rubi [A] time = 0.175953, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3190, 414, 527, 12, 377, 206}

$$-\frac{b(a-2b)\sin(e+fx)}{2af(a+b)^2\sqrt{a+b\sin^2(e+fx)}}+\frac{(a+4b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2f(a+b)^{5/2}}+\frac{\tan(e+fx)\sec(e+fx)}{2f(a+b)\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + 4*b)*ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/
(2*(a + b)^(5/2)*f) - ((a - 2*b)*b*Sin[e + f*x])/(2*a*(a + b)^2*f*Sqrt[a +
b*Sin[e + f*x]^2]) + (Sec[e + f*x]*Tan[e + f*x])/(2*(a + b)*f*Sqrt[a + b*Si
n[e + f*x]^2])
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
```

```
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2\left(a+bx^2\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sec(e+fx)\tan(e+fx)}{2(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a+2b+2bx^2}{\left(1-x^2\right)\left(a+bx^2\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{2(a+b)f}$$

$$= -\frac{(a-2b)b\sin(e+fx)}{2a(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\sec(e+fx)\tan(e+fx)}{2(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a(a+4b)}{\left(1-x^2\right)\sqrt{a+b}\cdots}\right.}{2a(a\cdots}$$

$$= -\frac{(a-2b)b\sin(e+fx)}{2a(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\sec(e+fx)\tan(e+fx)}{2(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{(a+4b)\,\text{Subst}\left(\int \cdots\right.}{2\cdots}$$

$$= -\frac{(a-2b)b\sin(e+fx)}{2a(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\sec(e+fx)\tan(e+fx)}{2(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{(a+4b)\,\text{Subst}\left(\int \frac{\cdots}{1\cdots}\right.}{2\cdots}$$

$$= \frac{(a+4b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2(a+b)^{5/2}f} - \frac{(a-2b)b\sin(e+fx)}{2a(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\sec(e+fx)\tan\cdots}{2(a+b)f\sqrt{a+b\sin\cdots}}$$

Mathematica [C] time = 7.46951, size = 224, normalized size = 1.67

$$-\frac{\tan(e+fx)\sec^5(e+fx)\left(16(a+b)\sin^2(e+fx)\left(a+b\sin^2(e+fx)\right)^2 {}_3F_2\left(2,2,3;1,\frac{9}{2};-\frac{(a+b)\tan^2(e+fx)}{a}\right)+16(a+b)\sin^2\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Sec[e + f*x]^5*(16*(a + b)*HypergeometricPFQ[{2, 2, 3}, {1, 9/2}, -(((a +
 b)*Tan[e + f*x]^2)/a)]*Sin[e + f*x]^2*(a + b*Sin[e + f*x]^2)^2 + 16*(a + b
)*Hypergeometric2F1[2, 3, 9/2, -(((a + b)*Tan[e + f*x]^2)/a)]*Sin[e + f*x]^
2*(4*a^2 + 7*a*b*Sin[e + f*x]^2 + 3*b^2*Sin[e + f*x]^4) - 7*a*Cos[e + f*x]^
2*Hypergeometric2F1[1, 2, 7/2, -(((a + b)*Tan[e + f*x]^2)/a)]*(15*a^2 + 20*
a*b*Sin[e + f*x]^2 + 8*b^2*Sin[e + f*x]^4))*Tan[e + f*x])/(105*a^4*f*Sqrt[a
 + b*Sin[e + f*x]^2])
```

Maple [B] time = 9.885, size = 3219, normalized size = 24.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/4/(a+b)^(1/2)/a/b^5/cos(f*x+e)^2/(a^4*b^2*cos(f*x+e)^4+4*a^3*b^3*cos(f*x+
e)^4+6*a^2*b^4*cos(f*x+e)^4+4*a*b^5*cos(f*x+e)^4+b^6*cos(f*x+e)^4-2*a^5*b*c
os(f*x+e)^2-10*a^4*b^2*cos(f*x+e)^2-20*a^3*b^3*cos(f*x+e)^2-20*a^2*b^4*cos(
f*x+e)^2-10*a*b^5*cos(f*x+e)^2-2*b^6*cos(f*x+e)^2+a^6+6*a^5*b+15*a^4*b^2+20
*a^3*b^3+15*a^2*b^4+6*a*b^5+b^6)*(-a*(8*ln(((a+b-b*cos(f*x+e)^2)^(1/2)*b^(1
/2)+b*sin(f*x+e))/b^(1/2))*b^(19/2)*(a+b)^(1/2)-8*ln(((-b*cos(f*x+e)^2+(a*b
^2+b^3)/b^2)^(1/2)*b^(3/2)+sin(f*x+e)*b^2)/b^(3/2))*b^(19/2)*(a+b)^(1/2)+24
*ln(((a+b-b*cos(f*x+e)^2)^(1/2)*b^(1/2)+b*sin(f*x+e))/b^(1/2))*b^(17/2)*(a+
b)^(1/2)*a-24*ln(((-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*b^(3/2)+sin(f*x+e
)*b^2)/b^(3/2))*b^(17/2)*(a+b)^(1/2)*a+24*ln(((a+b-b*cos(f*x+e)^2)^(1/2)*b^
(1/2)+b*sin(f*x+e))/b^(1/2))*b^(15/2)*(a+b)^(1/2)*a^2-24*ln(((-b*cos(f*x+e)
^2+(a*b^2+b^3)/b^2)^(1/2)*b^(3/2)+sin(f*x+e)*b^2)/b^(3/2))*b^(15/2)*(a+b)^(
1/2)*a^2+8*ln(((a+b-b*cos(f*x+e)^2)^(1/2)*b^(1/2)+b*sin(f*x+e))/b^(1/2))*b^
(13/2)*(a+b)^(1/2)*a^3-8*ln(((-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*b^(3/2
)+sin(f*x+e)*b^2)/b^(3/2))*b^(13/2)*(a+b)^(1/2)*a^3+ln(2/(1+sin(f*x+e))*((a
+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^5*b^5+8*ln(2/(1+sin
(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^4*b^6+2
2*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+
a))*a^3*b^7+28*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-
b*sin(f*x+e)+a))*a^2*b^8+17*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x
+e)^2)^(1/2)-b*sin(f*x+e)+a))*a*b^9+4*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b
-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*b^10-ln(2/(-1+sin(f*x+e))*((a+b)^(1
/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^5*b^5-8*ln(2/(-1+sin(f*x+
e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^4*b^6-22*ln(
2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*
a^3*b^7-28*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*s
in(f*x+e)+a))*a^2*b^8-17*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e
)^2)^(1/2)+b*sin(f*x+e)+a))*a*b^9-4*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-
b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*b^10)*cos(f*x+e)^2+2*sin(f*x+e)*(a+b
-b*cos(f*x+e)^2)^(3/2)*(a+b)^(1/2)*a*b^5*(a^3+3*a^2*b+3*a*b^2+b^3)-a*(8*ln(
((a+b-b*cos(f*x+e)^2)^(1/2)*b^(1/2)+b*sin(f*x+e))/b^(1/2))*b^(19/2)*(a+b)^(
1/2)-8*ln(((-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*b^(3/2)+sin(f*x+e)*b^2)/
b^(3/2))*b^(19/2)*(a+b)^(1/2)+8*ln(((a+b-b*cos(f*x+e)^2)^(1/2)*b^(1/2)+b*si
n(f*x+e))/b^(1/2))*b^(17/2)*(a+b)^(1/2)*a-8*ln(((-b*cos(f*x+e)^2+(a*b^2+b^3
)/b^2)^(1/2)*b^(3/2)+sin(f*x+e)*b^2)/b^(3/2))*b^(17/2)*(a+b)^(1/2)*a+ln(2/(
1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^3*
b^7+6*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x
+e)+a))*a^2*b^8+9*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/
2)-b*sin(f*x+e)+a))*a*b^9+4*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x
```

```
+e)^2)^(1/2)-b*sin(f*x+e)+a))*b^10-ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b
*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^3*b^7-6*ln(2/(-1+sin(f*x+e))*((a+b)
^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^2*b^8-9*ln(2/(-1+sin(f
*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a*b^9-4*ln(
2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*
b^10)*cos(f*x+e)^6+2*a*(8*ln(((a+b-b*cos(f*x+e)^2)^(1/2)*b^(1/2)+b*sin(f*x+
e))/b^(1/2))*b^(19/2)*(a+b)^(1/2)-8*ln(((-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(
1/2)*b^(3/2)+sin(f*x+e)*b^2)/b^(3/2))*b^(19/2)*(a+b)^(1/2)+16*ln(((a+b-b*co
s(f*x+e)^2)^(1/2)*b^(1/2)+b*sin(f*x+e))/b^(1/2))*b^(17/2)*(a+b)^(1/2)*a-16*
ln(((-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*b^(3/2)+sin(f*x+e)*b^2)/b^(3/2)
)*b^(17/2)*(a+b)^(1/2)*a+8*ln(((a+b-b*cos(f*x+e)^2)^(1/2)*b^(1/2)+b*sin(f*x
+e))/b^(1/2))*b^(15/2)*(a+b)^(1/2)*a^2-8*ln(((-b*cos(f*x+e)^2+(a*b^2+b^3)/b
^2)^(1/2)*b^(3/2)+sin(f*x+e)*b^2)/b^(3/2))*b^(15/2)*(a+b)^(1/2)*a^2+ln(2/(1
+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^4*b
^6+7*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+
e)+a))*a^3*b^7+15*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/
2)-b*sin(f*x+e)+a))*a^2*b^8+13*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(
f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a*b^9+4*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(
a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*b^10-ln(2/(-1+sin(f*x+e))*((a+b)
^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^4*b^6-7*ln(2/(-1+sin(f
*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^3*b^7-15*
ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a
))*a^2*b^8-13*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+
b*sin(f*x+e)+a))*a*b^9-4*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e
)^2)^(1/2)+b*sin(f*x+e)+a))*b^10)*cos(f*x+e)^4+2*sin(f*x+e)*cos(f*x+e)^6*(a
+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(3/2)*a*b^8-2*cos(f*x+e)^2*sin(f*x+e)*(a+b)^
(1/2)*b^6*(2*(a+b-b*cos(f*x+e)^2)^(3/2)*a^3+4*(a+b-b*cos(f*x+e)^2)^(3/2)*a^
2*b+2*(a+b-b*cos(f*x+e)^2)^(3/2)*a*b^2-2*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^
(3/2)*a^2*b-4*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(3/2)*a*b^2-2*(-b*cos(f*x+e
)^2+(a*b^2+b^3)/b^2)^(3/2)*b^3-(a+b-b*cos(f*x+e)^2)^(1/2)*a^4-3*(a+b-b*cos(
f*x+e)^2)^(1/2)*a^3*b-3*(a+b-b*cos(f*x+e)^2)^(1/2)*a^2*b^2-(a+b-b*cos(f*x+e
)^2)^(1/2)*a*b^3)-2*cos(f*x+e)^4*sin(f*x+e)*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b
)^(1/2)*a*b^7*(a*b*cos(f*x+e)^2+b^2*cos(f*x+e)^2+a^2+2*a*b+b^2))/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 6.26998, size = 1438, normalized size = 10.73

$$\left[\frac{\left(\left(a^{2} b+4\, a b^{2}\right) \cos\left(f x+e\right)^{4}-\left(a^{3}+5\, a^{2} b+4\, a b^{2}\right) \cos\left(f x+e\right)^{2}\right) \sqrt{a+b} \log\left(\frac{\left(a^{2}+8\, a b+8\, b^{2}\right) \cos\left(f x+e\right)^{4}-8\left(a^{2}+3\, a b+2\, b^{2}\right) \cos\left(f x+e\right.}{}\right.}{8\left(\left(a^{4} b+3\, a^{3} b^{2}+3\, a^{2} b^{3}+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(((a^2*b + 4*a*b^2)*cos(f*x + e)^4 - (a^3 + 5*a^2*b + 4*a*b^2)*cos(f*x
 + e)^2)*sqrt(a + b)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 8*(a^2 + 3
*a*b + 2*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*cos(f*x + e)^2 - 2*a - 2*b)*sqr
t(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b)*sin(f*x + e) + 8*a^2 + 16*a*b + 8*
b^2)/cos(f*x + e)^4) - 4*(a^3 + 2*a^2*b + a*b^2 - (a^2*b - a*b^2 - 2*b^3)*c
os(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sin(f*x + e))/((a^4*b + 3*a^
3*b^2 + 3*a^2*b^3 + a*b^4)*f*cos(f*x + e)^4 - (a^5 + 4*a^4*b + 6*a^3*b^2 +
4*a^2*b^3 + a*b^4)*f*cos(f*x + e)^2), -1/4*(((a^2*b + 4*a*b^2)*cos(f*x + e)
^4 - (a^3 + 5*a^2*b + 4*a*b^2)*cos(f*x + e)^2)*sqrt(-a - b)*arctan(1/2*((a
+ 2*b)*cos(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a
- b)/(((a*b + b^2)*cos(f*x + e)^2 - a^2 - 2*a*b - b^2)*sin(f*x + e))) + 2*(
a^3 + 2*a^2*b + a*b^2 - (a^2*b - a*b^2 - 2*b^3)*cos(f*x + e)^2)*sqrt(-b*cos
(f*x + e)^2 + a + b)*sin(f*x + e))/((a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)
*f*cos(f*x + e)^4 - (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*f*cos(f
*x + e)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(sec(e + f*x)**3/(a + b*sin(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(fx+e)^3}{\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^3/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.358 $\int \frac{\cos^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=274

$$\frac{\left(8a^2+13ab+3b^2\right)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}E\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{3ab^3f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{(4a+3b)\sin(e+fx)\cos}{3}$$

```
[Out] ((a + b)*Cos[e + f*x]^3*Sin[e + f*x])/(a*b*f*Sqrt[a + b*Sin[e + f*x]^2]) +
((4*a + 3*b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a*b^2
*f) + ((8*a^2 + 13*a*b + 3*b^2)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e
 + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a*b^3*f*Sqrt[
1 + (b*Sin[e + f*x]^2)/a]) - ((a + b)*(8*a + 9*b)*Sqrt[Cos[e + f*x]^2]*Elli
pticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2
)/a])/(3*b^3*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.289935, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3192, 413, 528, 524, 426, 424, 421, 419}

$$\frac{\left(8a^2+13ab+3b^2\right)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}E\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{3ab^3f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{(4a+3b)\sin(e+fx)\cos}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^6/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + b)*Cos[e + f*x]^3*Sin[e + f*x])/(a*b*f*Sqrt[a + b*Sin[e + f*x]^2]) +
((4*a + 3*b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a*b^2
*f) + ((8*a^2 + 13*a*b + 3*b^2)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e
 + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a*b^3*f*Sqrt[
1 + (b*Sin[e + f*x]^2)/a]) - ((a + b)*(8*a + 9*b)*Sqrt[Cos[e + f*x]^2]*Elli
pticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2
)/a])/(3*b^3*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\cos^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{\left(1-x^2\right)^{5/2}}{\left(a+bx^2\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{(a+b)\cos^3(e+fx)\sin(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}} + \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{1-x^2}\left(-a+(4a+3b)\right.}{\sqrt{a+bx^2}}\right.}{abf}$$

$$= \frac{(a+b)\cos^3(e+fx)\sin(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}} + \frac{(4a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3ab^2f}$$

$$= \frac{(a+b)\cos^3(e+fx)\sin(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}} + \frac{(4a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3ab^2f}$$

$$= \frac{(a+b)\cos^3(e+fx)\sin(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}} + \frac{(4a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3ab^2f}$$

$$= \frac{(a+b)\cos^3(e+fx)\sin(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}} + \frac{(4a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3ab^2f}$$

Mathematica [A] time = 1.10885, size = 184, normalized size = 0.67

$$\frac{\sqrt{2}b\sin(2(e+fx))\left(8a^2-ab\cos(2(e+fx))+13ab+6b^2\right)-4a\left(8a^2+17ab+9b^2\right)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}F\left(e+fx\middle|-\frac{b}{a}\right)+4}{12ab^3f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^6/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (4*a*(8*a^2 + 13*a*b + 3*b^2)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*Ellipt
icE[e + f*x, -(b/a)] - 4*a*(8*a^2 + 17*a*b + 9*b^2)*Sqrt[(2*a + b - b*Cos[2
*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)] + Sqrt[2]*b*(8*a^2 + 13*a*b + 6*
b^2 - a*b*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/(12*a*b^3*f*Sqrt[2*a + b - b*
Cos[2*(e + f*x)]])
```

Maple [A] time = 1.301, size = 415, normalized size = 1.5

$$-\frac{1}{3\,ab^3\cos\left(fx+e\right)f}\left(ab^2\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^4+\left(-4\,a^2b-7\,ab^2-3\,b^3\right)\left(\cos\left(fx+e\right)\right)^2\sin\left(fx+e\right)+8\sqrt{\left(\cos\left(\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^6/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/3*(a*b^2*sin(f*x+e)*cos(f*x+e)^4+(-4*a^2*b-7*a*b^2-3*b^3)*cos(f*x+e)^2*s
in(f*x+e)+8*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*Elliptic
F(sin(f*x+e),(-1/a*b)^(1/2))*a^3+17*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2
+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b+9*(cos(f*x+e)^2)
^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2
))*a*b^2-8*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE
(sin(f*x+e),(-1/a*b)^(1/2))*a^3-13*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+
(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b-3*(cos(f*x+e)^2)^
(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2)
)*a*b^2)/a/b^3/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^6}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^6/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\cos\left(fx+e\right)^6}{b^2\cos\left(fx+e\right)^4-2\left(ab+b^2\right)\cos\left(fx+e\right)^2+a^2+2\,ab+b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)*cos(f*x + e)^6/(b^2*cos(f*x + e)^4
 - 2*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**6/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^6}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^6/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.359 $\int \frac{\cos^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=202

$$-\frac{2(a+b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{b^2f\sqrt{a+b\sin^2(e+fx)}}+\frac{(2a+b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\cdots}{ab^2f}$$

```
[Out] ((a + b)*Cos[e + f*x]*Sin[e + f*x])/(a*b*f*Sqrt[a + b*Sin[e + f*x]^2]) + ((
2*a + b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e
 + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(a*b^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]
) - (2*(a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]
*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(b^2*f*Sqrt[a + b*Sin[e + f*x
]^2])
```

Rubi [A] time = 0.205915, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3192, 413, 524, 426, 424, 421, 419}

$$-\frac{2(a+b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{b^2f\sqrt{a+b\sin^2(e+fx)}}+\frac{(2a+b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\cdots}{ab^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(3/2),x]
```


```
[Out] ((a + b)*Cos[e + f*x]*Sin[e + f*x])/(a*b*f*Sqrt[a + b*Sin[e + f*x]^2]) + ((
2*a + b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e
 + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(a*b^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]
) - (2*(a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]
*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(b^2*f*Sqrt[a + b*Sin[e + f*x
]^2])
```


Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\cos^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{\left(1-x^2\right)^{3/2}}{\left(a+bx^2\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{(a+b)\cos(e+fx)\sin(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}} + \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{-a+(2a+b)x^2}{\sqrt{1-x^2}\sqrt{a+bx^2}}\,dx, x \ldots\right.}{abf}$$

$$= \frac{(a+b)\cos(e+fx)\sin(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(2(a+b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{a+\ldots}}\right.}{b^2f}$$

$$= \frac{(a+b)\cos(e+fx)\sin(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}} + \frac{\left((2a+b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\ldots\right.}{ab^2f\sqrt{1+\frac{b\sin^2(e+f\ldots)}{a}}}$$

$$= \frac{(a+b)\cos(e+fx)\sin(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}} + \frac{(2a+b)\sqrt{\cos^2(e+fx)}E\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)\sec(e\ldots}{ab^2f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

Mathematica [A] time = 0.556972, size = 139, normalized size = 0.69

$$\frac{2a(2a+b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)-(a+b)\left(4a\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}F\left(e+fx\middle|-\frac{b}{a}\right)-\sqrt{2}b\sin(2(e+fx))\right)}{2ab^2f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (2*a*(2*a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(
b/a)] - (a + b)*(4*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f
*x, -(b/a)] - Sqrt[2]*b*Sin[2*(e + f*x)]))/(2*a*b^2*f*Sqrt[2*a + b - b*Cos[
2*(e + f*x)]])
```

Maple [A] time = 1.103, size = 274, normalized size = 1.4

$$-\frac{1}{ab^2\cos\left(fx+e\right)f}\left(\left(-ab-b^2\right)\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^2+2\sqrt{\left(\cos\left(fx+e\right)\right)^2}\sqrt{-\frac{b\left(\cos\left(fx+e\right)\right)^2}{a}+\frac{a+b}{a}}\text{Ellip}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] -((-a*b-b^2)*sin(f*x+e)*cos(f*x+e)^2+2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e
)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2+2*(cos(f*x+e)^2
)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/
2))*a*b-2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(
sin(f*x+e),(-1/a*b)^(1/2))*a^2-(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b
)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b)/a/b^2/cos(f*x+e)/(a+b*
sin(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^4}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^4/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\cos\left(fx+e\right)^4}{b^2\cos\left(fx+e\right)^4-2\left(ab+b^2\right)\cos\left(fx+e\right)^2+a^2+2ab+b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)*cos(f*x + e)^4/(b^2*cos(f*x + e)^4
 - 2*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^4}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^4/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.360 $\int \frac{\cos^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=188

$$\frac{\sin(e+fx)\cos(e+fx)}{af\sqrt{a+b\sin^2(e+fx)}}-\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{bf\sqrt{a+b\sin^2(e+fx)}}+\frac{\sqrt{\cos^2(e+fx)}\sec$$

```
[Out] (Cos[e + f*x]*Sin[e + f*x])/(a*f*Sqrt[a + b*Sin[e + f*x]^2]) + (Sqrt[Cos[e
+ f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*S
in[e + f*x]^2])/(a*b*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) - (Sqrt[Cos[e + f*x]
^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e
+ f*x]^2)/a])/(b*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.183802, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3192, 412, 493, 426, 424, 421, 419}

$$\frac{\sin(e+fx)\cos(e+fx)}{af\sqrt{a+b\sin^2(e+fx)}}-\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{bf\sqrt{a+b\sin^2(e+fx)}}+\frac{\sqrt{\cos^2(e+fx)}\sec$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (Cos[e + f*x]*Sin[e + f*x])/(a*f*Sqrt[a + b*Sin[e + f*x]^2]) + (Sqrt[Cos[e
+ f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*S
in[e + f*x]^2])/(a*b*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) - (Sqrt[Cos[e + f*x]
^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e
+ f*x]^2)/a])/(b*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 412

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] + Dist[1/(
a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p + 1)
 + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x]
&& NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 493

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
```

```
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) &&  !(EqQ[n, 2] && Simp
lerSqrtQ[-(b/a), -(d/c)])
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{\left(a+bx^2\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{af\sqrt{a+b\sin^2(e+fx)}} + \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-x^2}\sqrt{a+bx^2}}\,dx, x, \sin(e\right.}{af}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{af\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{a+bx^2}}\,dx, x, \sin(e\right.}{bf}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{af\sqrt{a+b\sin^2(e+fx)}} + \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{1}}{\sqrt{1}}\right.}{abf\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{af\sqrt{a+b\sin^2(e+fx)}} + \frac{\sqrt{\cos^2(e+fx)}E\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b\,\text{si}}}{abf\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

Mathematica [A] time = 0.301733, size = 133, normalized size = 0.71

$$\frac{-\sqrt{2}a\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}F\left(e+fx\middle|-\frac{b}{a}\right)+\sqrt{2}a\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)+b\sin(2(e+fx))}{\sqrt{2}abf\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[2]*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)
] - Sqrt[2]*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b
/a)] + b*Sin[2*(e + f*x)])/(Sqrt[2]*a*b*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]
])
```

Maple [A] time = 1.058, size = 145, normalized size = 0.8

$$-\frac{1}{ab\cos\left(fx+e\right)f}\left(\sqrt{\left(\cos\left(fx+e\right)\right)^{2}}\sqrt{\frac{a+b\left(\sin\left(fx+e\right)\right)^{2}}{a}}\mathrm{EllipticF}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)a-\sqrt{\left(\cos\left(fx+e\right)\right)^{2}}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] -((cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-
1/a*b)^(1/2))*a-(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*a*Ellipti
cE(sin(f*x+e),(-1/a*b)^(1/2))+b*sin(f*x+e)^3-b*sin(f*x+e))/a/b/cos(f*x+e)/(
a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^{2}}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\cos\left(fx+e\right)^{2}}{b^{2}\cos\left(fx+e\right)^{4}-2\left(ab+b^{2}\right)\cos\left(fx+e\right)^{2}+a^{2}+2\,ab+b^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)*cos(f*x + e)^2/(b^2*cos(f*x + e)^4
 - 2*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^2}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.361 $\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=101

$$\frac{b\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}+\frac{\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{af(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

```
[Out] (b*Cos[e + f*x]*Sin[e + f*x])/(a*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2]) + (E
llipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(a*(a + b)*f*Sqrt[1 +
 (b*Sin[e + f*x]^2)/a])
```

Rubi [A] time = 0.0599659, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3184, 21, 3178, 3177}

$$\frac{b\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}+\frac{\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{af(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x]^2)^(-3/2),x]
```

```
[Out] (b*Cos[e + f*x]*Sin[e + f*x])/(a*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2]) + (E
llipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(a*(a + b)*f*Sqrt[1 +
 (b*Sin[e + f*x]^2)/a])
```

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a +
 b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)
*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{b\cos(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\int \frac{-a-b\sin^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx}{a(a+b)}$$

$$= \frac{b\cos(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\int \sqrt{a+b\sin^2(e+fx)}\,dx}{a(a+b)}$$

$$= \frac{b\cos(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\sqrt{a+b\sin^2(e+fx)}\int \sqrt{1+\frac{b\sin^2(e+fx)}{a}}\,dx}{a(a+b)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

$$= \frac{b\cos(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{a(a+b)f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

Mathematica [A] time = 0.14595, size = 90, normalized size = 0.89

$$\frac{2a\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)+\sqrt{2}b\sin(2(e+fx))}{2af(a+b)\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)^(-3/2),x]
```

```
[Out] (2*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] + Sq
rt[2]*b*Sin[2*(e + f*x)])/(2*a*(a + b)*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]]
)
```

Maple [A] time = 1.351, size = 103, normalized size = 1.

$$\frac{1}{a(a+b)\cos(fx+e)f}\left(a\sqrt{\left(\cos(fx+e)\right)^2}\sqrt{-\frac{b\left(\cos(fx+e)\right)^2}{a}+\frac{a+b}{a}}\text{EllipticE}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)+\left(\cos(fx+e\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] (a*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x
+e),(-1/a*b)^(1/2))+cos(f*x+e)^2*sin(f*x+e)*b)/a/(a+b)/cos(f*x+e)/(a+b*sin(
f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}}{b^2\cos\left(fx+e\right)^4-2\left(ab+b^2\right)\cos\left(fx+e\right)^2+a^2+2\,ab+b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)/(b^2*cos(f*x + e)^4 - 2*(a*b + b^2
)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(-3/2), x)
```

3.362 $\int \frac{\sec^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=240

$$\frac{\tan(e+fx)}{f(a+b)\sqrt{a+b\sin^2(e+fx)}} - \frac{b(a-b)\sin(e+fx)\cos(e+fx)}{af(a+b)^2\sqrt{a+b\sin^2(e+fx)}} + \frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(\sin^{-1}(\sin\right.}{f(a+b)\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] -(((a - b)*b*Cos[e + f*x]*Sin[e + f*x])/(a*(a + b)^2*f*Sqrt[a + b*Sin[e + f
*x]^2])) - ((a - b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(
b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(a*(a + b)^2*f*Sqrt[1 + (b*S
in[e + f*x]^2)/a]) + (Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]],
-(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/((a + b)*f*Sqrt[a + b*
Sin[e + f*x]^2]) + Tan[e + f*x]/((a + b)*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.239238, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3192, 414, 527, 524, 426, 424, 421, 419}

$$\frac{\tan(e+fx)}{f(a+b)\sqrt{a+b\sin^2(e+fx)}} - \frac{b(a-b)\sin(e+fx)\cos(e+fx)}{af(a+b)^2\sqrt{a+b\sin^2(e+fx)}} + \frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(\sin^{-1}(\sin\right.}{f(a+b)\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(3/2),x]
```


```
[Out] -(((a - b)*b*Cos[e + f*x]*Sin[e + f*x])/(a*(a + b)^2*f*Sqrt[a + b*Sin[e + f
*x]^2])) - ((a - b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(
b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(a*(a + b)^2*f*Sqrt[1 + (b*S
in[e + f*x]^2)/a]) + (Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]],
-(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/((a + b)*f*Sqrt[a + b*
Sin[e + f*x]^2]) + Tan[e + f*x]/((a + b)*f*Sqrt[a + b*Sin[e + f*x]^2])
```


Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] && !IntegerQ[p]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\sec^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{1}{(1-x^2)^{3/2}(a+bx^2)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)}{(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{b+bx^2}{\sqrt{1-x^2}(a+bx^2)^{3/2}}\,dx, x\right.}{(a+b)f}$$

$$= -\frac{(a-b)b\cos(e+fx)\sin(e+fx)}{a(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\tan(e+fx)}{(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e\right.}{}$$

$$= -\frac{(a-b)b\cos(e+fx)\sin(e+fx)}{a(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\tan(e+fx)}{(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left((a-b)\sqrt{\cos^2(e+fx\right.}{}$$

$$= -\frac{(a-b)b\cos(e+fx)\sin(e+fx)}{a(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\tan(e+fx)}{(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(\left((a-b)\sqrt{\cos^2(e+fx\right.\right.}{}$$

$$= -\frac{(a-b)b\cos(e+fx)\sin(e+fx)}{a(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{(a-b)\sqrt{\cos^2(e+fx)}E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)\sec(}{a(a+b)^2f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

Mathematica [A] time = 1.19473, size = 167, normalized size = 0.7

$$\frac{\tan(e+fx)\left(2a^2+b(b-a)\cos(2(e+fx))+ab+b^2\right)+\sqrt{2}a(a+b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}F\left(e+fx\middle|-\frac{b}{a}\right)-\sqrt{2}a(a-b)\sqrt{\frac{2a-b}{}}}{\sqrt{2}af(a+b)^2\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-(Sqrt[2]*a*(a - b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f
*x, -(b/a)]) + Sqrt[2]*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*Ell
ipticF[e + f*x, -(b/a)] + (2*a^2 + a*b + b^2 + b*(-a + b)*Cos[2*(e + f*x)])
*Tan[e + f*x])/(Sqrt[2]*a*(a + b)^2*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [B] time = 2.345, size = 468, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] (-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*(a-b)*sin(f*x+e)*cos(f*x+e)^
2+(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*a*(a+b)*sin(f*x+e)+(cos(f*x+e)
^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(
1/2))*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*a^2+a*b*(cos(f*x+e)^2)^(1/
```

```
2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*(
-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)-(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*
x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*(-b*cos(f*x+e)^4
+(a+b)*cos(f*x+e)^2)^(1/2)*a^2+(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b
)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*(-b*cos(f*x+e)^4+(a+b)*cos(
f*x+e)^2)^(1/2)*a*b)/(a+b)^2/(-(a+b*sin(f*x+e)^2)*(-1+sin(f*x+e))*(1+sin(f*
x+e)))^(1/2)/a/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)^2}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\sec\left(fx+e\right)^2}{b^2\cos\left(fx+e\right)^4-2\left(ab+b^2\right)\cos\left(fx+e\right)^2+a^2+2\,ab+b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)*sec(f*x + e)^2/(b^2*cos(f*x + e)^4
 - 2*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2\left(e+fx\right)}{\left(a+b\sin^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(sec(e + f*x)**2/(a + b*sin(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)^2}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.363 $\int \frac{\cos^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=130

$$-\frac{(3a-2b)(a+b)\sin(e+fx)}{3a^2b^2f\sqrt{a+b\sin^2(e+fx)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{b^{5/2}f}+\frac{(a+b)\sin(e+fx)\cos^2(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

```
[Out] ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]]/(b^(5/2)*f) + ((
a + b)*Cos[e + f*x]^2*Sin[e + f*x])/(3*a*b*f*(a + b*Sin[e + f*x]^2)^(3/2))
- ((3*a - 2*b)*(a + b)*Sin[e + f*x])/(3*a^2*b^2*f*Sqrt[a + b*Sin[e + f*x]^2
])
```

Rubi [A] time = 0.134544, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3190, 413, 385, 217, 206}

$$-\frac{(3a-2b)(a+b)\sin(e+fx)}{3a^2b^2f\sqrt{a+b\sin^2(e+fx)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{b^{5/2}f}+\frac{(a+b)\sin(e+fx)\cos^2(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^5/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]]/(b^(5/2)*f) + ((
a + b)*Cos[e + f*x]^2*Sin[e + f*x])/(3*a*b*f*(a + b*Sin[e + f*x]^2)^(3/2))
- ((3*a - 2*b)*(a + b)*Sin[e + f*x])/(3*a^2*b^2*f*Sqrt[a + b*Sin[e + f*x]^2
])
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
```

```
p, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\left(a+bx^2\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{(a+b)\cos^2(e+fx)\sin(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-a+2b+3ax^2}{\left(a+bx^2\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{3abf}$$

$$= \frac{(a+b)\cos^2(e+fx)\sin(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{(3a-2b)(a+b)\sin(e+fx)}{3a^2b^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \text{s}\right.}{b^2f}$$

$$= \frac{(a+b)\cos^2(e+fx)\sin(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{(3a-2b)(a+b)\sin(e+fx)}{3a^2b^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \right.}{b^2f}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{b^{5/2}f} + \frac{(a+b)\cos^2(e+fx)\sin(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{(3a-2b)(a+b)\sin(e+fx)}{3a^2b^2f\sqrt{a+b\sin^2(e+fx)}}$$

Mathematica [A] time = 0.809614, size = 128, normalized size = 0.98

$$\frac{\frac{2\sqrt{2}(a+b)\sin(e+fx)\left(-3a^2+b(2a-b)\cos(2(e+fx))+ab+b^2\right)}{a^2(2a-b\cos(2(e+fx))+b)^{3/2}} + \frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt{-b}\sin(e+fx)}{\sqrt{2a-b\cos(2(e+fx))+b}}\right)}{\sqrt{-b}}}{3b^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^5/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] ((3*ArcTan[(Sqrt[2]*Sqrt[-b]*Sin[e + f*x])/Sqrt[2*a + b - b*Cos[2*(e + f*x)
]]])/Sqrt[-b] + (2*Sqrt[2]*(a + b)*(-3*a^2 + a*b + b^2 + (2*a - b)*b*Cos[2*
(e + f*x)])*Sin[e + f*x])/(a^2*(2*a + b - b*Cos[2*(e + f*x)])^(3/2)))/(3*b^
2*f)
```

Maple [B] time = 3.361, size = 383, normalized size = 3.

$$\frac{1}{3\,a^{2}\left(b^{2}\left(\cos\left(fx+e\right)\right)^{4}-2\,ab\left(\cos\left(fx+e\right)\right)^{2}-2\,b^{2}\left(\cos\left(fx+e\right)\right)^{2}+a^{2}+2\,ab+b^{2}\right)f}\left(3\,\ln\left(\sin\left(fx+e\right)\sqrt{b}+\sqrt{a}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3/b^(13/2)/a^2/(b^2*cos(f*x+e)^4-2*a*b*cos(f*x+e)^2-2*b^2*cos(f*x+e)^2+a^
2+2*a*b+b^2)*(3*ln(sin(f*x+e)*b^(1/2)+(a+b-b*cos(f*x+e)^2)^(1/2))*a^4*b^4+6
*ln(sin(f*x+e)*b^(1/2)+(a+b-b*cos(f*x+e)^2)^(1/2))*a^3*b^5+3*ln(sin(f*x+e)*
b^(1/2)+(a+b-b*cos(f*x+e)^2)^(1/2))*a^2*b^6+3*ln(sin(f*x+e)*b^(1/2)+(a+b-b*
cos(f*x+e)^2)^(1/2))*a^2*b^6*cos(f*x+e)^4-6*ln(sin(f*x+e)*b^(1/2)+(a+b-b*co
s(f*x+e)^2)^(1/2))*a^2*b^5*(a+b)*cos(f*x+e)^2+2*b^(11/2)*(-b*cos(f*x+e)^2+(
a*b^2+b^3)/b^2)^(1/2)*(2*a^2+a*b-b^2)*sin(f*x+e)*cos(f*x+e)^2-sin(f*x+e)*b^
(9/2)*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*(3*a^3+4*a^2*b-a*b^2-2*b^3))/
f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 12.5564, size = 1828, normalized size = 14.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*(a^2*b^2*cos(f*x + e)^4 + a^4 + 2*a^3*b + a^2*b^2 - 2*(a^3*b + a^2
*b^2)*cos(f*x + e)^2)*sqrt(b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + 2*b
^4)*cos(f*x + e)^6 + 32*(5*a^2*b^2 + 24*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^
4 + 32*a^3*b + 160*a^2*b^2 + 256*a*b^3 + 128*b^4 - 32*(a^3*b + 10*a^2*b^2 +
 24*a*b^3 + 16*b^4)*cos(f*x + e)^2 - 8*(16*b^3*cos(f*x + e)^6 - 24*(a*b^2 +
 2*b^3)*cos(f*x + e)^4 - a^3 - 10*a^2*b - 24*a*b^2 - 16*b^3 + 2*(5*a^2*b +
24*a*b^2 + 24*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(b)*
sin(f*x + e)) - 8*(3*a^3*b + 4*a^2*b^2 - a*b^3 - 2*b^4 - 2*(2*a^2*b^2 + a*b
^3 - b^4)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sin(f*x + e))/(a^
2*b^5*f*cos(f*x + e)^4 - 2*(a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^2 + (a^4*b^3
+ 2*a^3*b^4 + a^2*b^5)*f), -1/12*(3*(a^2*b^2*cos(f*x + e)^4 + a^4 + 2*a^3*b
 + a^2*b^2 - 2*(a^3*b + a^2*b^2)*cos(f*x + e)^2)*sqrt(-b)*arctan(1/4*(8*b^2
*cos(f*x + e)^4 - 8*(a*b + 2*b^2)*cos(f*x + e)^2 + a^2 + 8*a*b + 8*b^2)*sqr
t(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)/((2*b^3*cos(f*x + e)^4 + a^2*b + 3*a*
b^2 + 2*b^3 - (3*a*b^2 + 4*b^3)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(3*a^3*b
 + 4*a^2*b^2 - a*b^3 - 2*b^4 - 2*(2*a^2*b^2 + a*b^3 - b^4)*cos(f*x + e)^2)*
```

```
sqrt(-b*cos(f*x + e)^2 + a + b)*sin(f*x + e))/(a^2*b^5*f*cos(f*x + e)^4 - 2
*(a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^2 + (a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**5/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^5}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^5/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

3.364 $\int \frac{\cos^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=73

$$\frac{2\sin(e+fx)}{3a^2f\sqrt{a+b\sin^2(e+fx)}}+\frac{\sin(e+fx)\cos^2(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

```
[Out] (Cos[e + f*x]^2*Sin[e + f*x])/(3*a*f*(a + b*Sin[e + f*x]^2)^(3/2)) + (2*Sin
[e + f*x])/(3*a^2*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.0922958, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3190, 378, 191}

$$\frac{2\sin(e+fx)}{3a^2f\sqrt{a+b\sin^2(e+fx)}}+\frac{\sin(e+fx)\cos^2(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (Cos[e + f*x]^2*Sin[e + f*x])/(3*a*f*(a + b*Sin[e + f*x]^2)^(3/2)) + (2*Sin
[e + f*x])/(3*a^2*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\cos^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1-x^2}{\left(a+bx^2\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\cos^2(e+fx)\sin(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{3af}$$

$$= \frac{\cos^2(e+fx)\sin(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2\sin(e+fx)}{3a^2f\sqrt{a+b\sin^2(e+fx)}}$$

Mathematica [A] time = 0.106822, size = 51, normalized size = 0.7

$$\frac{3a\sin(e+fx) - (a-2b)\sin^3(e+fx)}{3a^2f\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (3*a*Sin[e + f*x] - (a - 2*b)*Sin[e + f*x]^3)/(3*a^2*f*(a + b*Sin[e + f*x]^
2)^(3/2))
```

Maple [A] time = 3.362, size = 120, normalized size = 1.6

$$\frac{\sin(fx+e)\left(a\left(\cos(fx+e)\right)^2 - 2b\left(\cos(fx+e)\right)^2 + 2a + 2b\right)}{3a^2\left(b^2\left(\cos(fx+e)\right)^4 - 2ab\left(\cos(fx+e)\right)^2 - 2b^2\left(\cos(fx+e)\right)^2 + a^2 + 2ab + b^2\right)f}\sqrt{-b\left(\cos(fx+e)\right)^2 + \frac{ab^2+b^3}{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3/a^2/(b^2*cos(f*x+e)^4-2*a*b*cos(f*x+e)^2-2*b^2*cos(f*x+e)^2+a^2+2*a*b+b
^2)*sin(f*x+e)*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*(a*cos(f*x+e)^2-2*b*
cos(f*x+e)^2+2*a+2*b)/f
```

Maxima [A] time = 0.97476, size = 144, normalized size = 1.97

$$\frac{\frac{2\sin(fx+e)}{\sqrt{b\sin(fx+e)^2+a}a^2} + \frac{\sin(fx+e)}{\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}a} + \frac{\sin(fx+e)}{\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}b} - \frac{\sin(fx+e)}{\sqrt{b\sin(fx+e)^2+a}ab}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*sin(f*x + e)/(sqrt(b*sin(f*x + e)^2 + a)*a^2) + sin(f*x + e)/((b*sin
(f*x + e)^2 + a)^(3/2)*a) + sin(f*x + e)/((b*sin(f*x + e)^2 + a)^(3/2)*b) -
```

```
sin(f*x + e)/(sqrt(b*sin(f*x + e)^2 + a)*a*b))/f
```

Fricas [A] time = 4.28294, size = 250, normalized size = 3.42

$$\frac{\left((a-2b)\cos\left(fx+e\right)^2+2a+2b\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}\sin\left(fx+e\right)}{3\left(a^2b^2f\cos\left(fx+e\right)^4-2\left(a^3b+a^2b^2\right)f\cos\left(fx+e\right)^2+\left(a^4+2a^3b+a^2b^2\right)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*((a - 2*b)*cos(f*x + e)^2 + 2*a + 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*
sin(f*x + e)/(a^2*b^2*f*cos(f*x + e)^4 - 2*(a^3*b + a^2*b^2)*f*cos(f*x + e)
^2 + (a^4 + 2*a^3*b + a^2*b^2)*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.39039, size = 78, normalized size = 1.07

$$-\frac{\left(\frac{\left(ab-2b^2\right)\sin\left(fx+e\right)^2}{a^2b}-\frac{3}{a}\right)\sin\left(fx+e\right)}{3\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*((a*b - 2*b^2)*sin(f*x + e)^2/(a^2*b) - 3/a)*sin(f*x + e)/((b*sin(f*x
+ e)^2 + a)^(3/2)*f)
```

3.365 $\int \frac{\cos(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=65

$$\frac{2\sin(e+fx)}{3a^2f\sqrt{a+b\sin^2(e+fx)}}+\frac{\sin(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

```
[Out] Sin[e + f*x]/(3*a*f*(a + b*Sin[e + f*x]^2)^(3/2)) + (2*Sin[e + f*x])/(3*a^2
*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.0552965, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3190, 192, 191}

$$\frac{2\sin(e+fx)}{3a^2f\sqrt{a+b\sin^2(e+fx)}}+\frac{\sin(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] Sin[e + f*x]/(3*a*f*(a + b*Sin[e + f*x]^2)^(3/2)) + (2*Sin[e + f*x])/(3*a^2
*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\cos(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(a+bx^2)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sin(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2\,\text{Subst}\left(\int \frac{1}{(a+bx^2)^{3/2}}\,dx, x, \sin(e+fx)\right)}{3af}$$

$$= \frac{\sin(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2\sin(e+fx)}{3a^2f\sqrt{a+b\sin^2(e+fx)}}$$

Mathematica [A] time = 0.0467914, size = 47, normalized size = 0.72

$$\frac{\sin(e+fx)\left(3a+2b\sin^2(e+fx)\right)}{3a^2f\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (Sin[e + f*x]*(3*a + 2*b*Sin[e + f*x]^2))/(3*a^2*f*(a + b*Sin[e + f*x]^2)^(
3/2))
```

Maple [A] time = 0.087, size = 56, normalized size = 0.9

$$\frac{1}{f}\left(\frac{\sin\left(fx+e\right)}{3\,a}\left(a+b\left(\sin\left(fx+e\right)\right)^2\right)^{-\frac{3}{2}}+\frac{2\,\sin\left(fx+e\right)}{3\,a^2}\frac{1}{\sqrt{a+b\left(\sin\left(fx+e\right)\right)^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/f*(1/3*sin(f*x+e)/a/(a+b*sin(f*x+e)^2)^(3/2)+2/3/a^2*sin(f*x+e)/(a+b*sin(
f*x+e)^2)^(1/2))
```

Maxima [A] time = 0.97846, size = 74, normalized size = 1.14

$$\frac{\frac{2\,\sin\left(fx+e\right)}{\sqrt{b\sin\left(fx+e\right)^2+a}a^2}+\frac{\sin\left(fx+e\right)}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}a}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*sin(f*x + e)/(sqrt(b*sin(f*x + e)^2 + a)*a^2) + sin(f*x + e)/((b*sin
(f*x + e)^2 + a)^(3/2)*a))/f
```

Fricas [A] time = 3.64586, size = 243, normalized size = 3.74

$$-\frac{\left(2\,b\cos\left(fx+e\right)^{2}-3\,a-2\,b\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\sin\left(fx+e\right)}{3\left(a^{2}b^{2}f\cos\left(fx+e\right)^{4}-2\left(a^{3}b+a^{2}b^{2}\right)f\cos\left(fx+e\right)^{2}+\left(a^{4}+2\,a^{3}b+a^{2}b^{2}\right)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*b*cos(f*x + e)^2 - 3*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sin(f
*x + e)/(a^2*b^2*f*cos(f*x + e)^4 - 2*(a^3*b + a^2*b^2)*f*cos(f*x + e)^2 +
(a^4 + 2*a^3*b + a^2*b^2)*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.35162, size = 65, normalized size = 1.

$$\frac{\left(\frac{2\,b\sin\left(fx+e\right)^{2}}{a^{2}}+\frac{3}{a}\right)\sin\left(fx+e\right)}{3\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(2*b*sin(f*x + e)^2/a^2 + 3/a)*sin(f*x + e)/((b*sin(f*x + e)^2 + a)^(3/
2)*f)
```

3.366 $\int \frac{\sec(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=126

$$\frac{b(5a+2b)\sin(e+fx)}{3a^2f(a+b)^2\sqrt{a+b\sin^2(e+fx)}}+\frac{b\sin(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f(a+b)^{5/2}}$$

```
[Out] ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]]/((a + b)^(5/
2)*f) + (b*Sin[e + f*x])/(3*a*(a + b)*f*(a + b*Sin[e + f*x]^2)^(3/2)) + (b*
(5*a + 2*b)*Sin[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.158976, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3190, 414, 527, 12, 377, 206}

$$\frac{b(5a+2b)\sin(e+fx)}{3a^2f(a+b)^2\sqrt{a+b\sin^2(e+fx)}}+\frac{b\sin(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]]/((a + b)^(5/
2)*f) + (b*Sin[e + f*x])/(3*a*(a + b)*f*(a + b*Sin[e + f*x]^2)^(3/2)) + (b*
(5*a + 2*b)*Sin[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a+bx^2)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{b\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{b-3(a+b)+2bx^2}{(1-x^2)(a+bx^2)^{3/2}}\,dx, x, \sin(e+fx)\right)}{3a(a+b)f}$$

$$= \frac{b\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{b(5a+2b)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{3a}{(1-x^2)\sqrt{}}\right.}{3a}$$

$$= \frac{b\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{b(5a+2b)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{}}\right.}{(}$$

$$= \frac{b\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{b(5a+2b)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1-(a+b)}\right.}{(a}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{(a+b)^{5/2}f} + \frac{b\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{b(5a+2b)\sin(e+f}{3a^2(a+b)^2f\sqrt{a+b\sin^2(}}$$

Mathematica [C] time = 9.31516, size = 1291, normalized size = 10.25

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (Sec[e + f*x]*Tan[e + f*x]*(1575*ArcSin[Sqrt[-(((a + b)*Tan[e + f*x]^2)/a)]
] + (2100*b*ArcSin[Sqrt[-(((a + b)*Tan[e + f*x]^2)/a)]]*Sin[e + f*x]^2)/a +
 (840*b^2*ArcSin[Sqrt[-(((a + b)*Tan[e + f*x]^2)/a)]]*Sin[e + f*x]^4)/a^2 +
 (3150*(a + b)*ArcSin[Sqrt[-(((a + b)*Tan[e + f*x]^2)/a)]]*Tan[e + f*x]^2)/
```

```
a + (4200*b*(a + b)*ArcSin[Sqrt[-(((a + b)*Tan[e + f*x]^2)/a)]]*Sin[e + f*x
]^2*Tan[e + f*x]^2)/a^2 + (1680*b^2*(a + b)*ArcSin[Sqrt[-(((a + b)*Tan[e +
f*x]^2)/a)]]*Sin[e + f*x]^4*Tan[e + f*x]^2)/a^3 + (1575*(a + b)^2*ArcSin[Sq
rt[-(((a + b)*Tan[e + f*x]^2)/a)]]*Tan[e + f*x]^4)/a^2 + (2100*b*(a + b)^2*
ArcSin[Sqrt[-(((a + b)*Tan[e + f*x]^2)/a)]]*Sin[e + f*x]^2*Tan[e + f*x]^4)/
a^3 + (840*b^2*(a + b)^2*ArcSin[Sqrt[-(((a + b)*Tan[e + f*x]^2)/a)]]*Sin[e
+ f*x]^4*Tan[e + f*x]^4)/a^4 + 2100*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x
]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(3/2) + (2800*b*Sin[e + f*x]^2*Sqr
t[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a)
)^(3/2))/a + (1120*b^2*Sin[e + f*x]^4*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f
*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(3/2))/a^2 + 96*Hypergeometric2F
1[2, 2, 9/2, -(((a + b)*Tan[e + f*x]^2)/a)]*Sqrt[(Sec[e + f*x]^2*(a + b*Sin
[e + f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(7/2) + 24*HypergeometricP
FQ[{2, 2, 2}, {1, 9/2}, -(((a + b)*Tan[e + f*x]^2)/a)]*Sqrt[(Sec[e + f*x]^2
*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(7/2) + (168*b*
Hypergeometric2F1[2, 2, 9/2, -(((a + b)*Tan[e + f*x]^2)/a)]*Sin[e + f*x]^2*
Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)
/a))^(7/2))/a + (48*b*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, -(((a + b)*Tan
[e + f*x]^2)/a)]*Sin[e + f*x]^2*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)
)/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(7/2))/a + (72*b^2*Hypergeometric2F1[2
, 2, 9/2, -(((a + b)*Tan[e + f*x]^2)/a)]*Sin[e + f*x]^4*Sqrt[(Sec[e + f*x]^
2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(7/2))/a^2 + (
24*b^2*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, -(((a + b)*Tan[e + f*x]^2)/a)
]*Sin[e + f*x]^4*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b
)*Tan[e + f*x]^2)/a))^(7/2))/a^2 - 1575*Sqrt[-(((a + b)*Sec[e + f*x]^2*(a +
 b*Sin[e + f*x]^2)*Tan[e + f*x]^2)/a^2)] - (2100*b*Sin[e + f*x]^2*Sqrt[-(((
a + b)*Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)*Tan[e + f*x]^2)/a^2)])/a - (84
0*b^2*Sin[e + f*x]^4*Sqrt[-(((a + b)*Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)*
Tan[e + f*x]^2)/a^2)])/a^2))/(315*a^2*f*Sqrt[a + b*Sin[e + f*x]^2]*Sqrt[(Se
c[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(1 + (b*Sin[e + f*x]^2)/a)*(-(((a +
 b)*Tan[e + f*x]^2)/a))^(5/2))
```

Maple [B] time = 4.599, size = 899, normalized size = 7.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/6/b^2/a^2/(a+b)^(1/2)/(a^2*b^2*cos(f*x+e)^4+2*a*b^3*cos(f*x+e)^4+b^4*cos(
f*x+e)^4-2*a^3*b*cos(f*x+e)^2-6*a^2*b^2*cos(f*x+e)^2-6*a*b^3*cos(f*x+e)^2-2
*b^4*cos(f*x+e)^2+a^4+4*a^3*b+6*a^2*b^2+4*a*b^3+b^4)*(3*a^4*b^2*ln(2/(-1+si
n(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))-3*ln(2/(
1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^4*
b^2+3*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*
x+e)+a))*a^2*b^4-3*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1
/2)-b*sin(f*x+e)+a))*a^2*b^4+6*a^3*b^3*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a
+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))-6*ln(2/(1+sin(f*x+e))*((a+b)^(1/2
)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^3*b^3-3*a^2*b^4*(ln(2/(1+si
n(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))-ln(2/(-1
+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a)))*cos(
f*x+e)^4-2*sin(f*x+e)*cos(f*x+e)^2*(a+b)^(1/2)*(-b*cos(f*x+e)^2+(a*b^2+b^3)
/b^2)^(1/2)*b^4*(5*a+2*b)+4*sin(f*x+e)*(a+b)^(1/2)*(-b*cos(f*x+e)^2+(a*b^2+
b^3)/b^2)^(1/2)*b^3*(3*a^2+4*a*b+b^2)+6*a^2*b^3*(ln(2/(1+sin(f*x+e))*((a+b)
^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a+ln(2/(1+sin(f*x+e))*((
a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*b-ln(2/(-1+sin(f*x+e
```

```
))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a-ln(2/(-1+sin(
f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*b)*cos(f*x
+e)^2)/f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 6.39366, size = 1752, normalized size = 13.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a^2*b^2*cos(f*x + e)^4 + a^4 + 2*a^3*b + a^2*b^2 - 2*(a^3*b + a^2
*b^2)*cos(f*x + e)^2)*sqrt(a + b)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4
 - 8*(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*cos(f*x + e)^2 - 2
*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b)*sin(f*x + e) + 8*a^2
+ 16*a*b + 8*b^2)/cos(f*x + e)^4) + 4*(6*a^3*b + 14*a^2*b^2 + 10*a*b^3 + 2*
b^4 - (5*a^2*b^2 + 7*a*b^3 + 2*b^4)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2
+ a + b)*sin(f*x + e))/((a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*f*cos(f
*x + e)^4 - 2*(a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*f*cos(f
*x + e)^2 + (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)
*f), -1/6*(3*(a^2*b^2*cos(f*x + e)^4 + a^4 + 2*a^3*b + a^2*b^2 - 2*(a^3*b +
 a^2*b^2)*cos(f*x + e)^2)*sqrt(-a - b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2
 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(((a*b + b^2)*co
s(f*x + e)^2 - a^2 - 2*a*b - b^2)*sin(f*x + e))) - 2*(6*a^3*b + 14*a^2*b^2
+ 10*a*b^3 + 2*b^4 - (5*a^2*b^2 + 7*a*b^3 + 2*b^4)*cos(f*x + e)^2)*sqrt(-b*
cos(f*x + e)^2 + a + b)*sin(f*x + e))/((a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a
^2*b^5)*f*cos(f*x + e)^4 - 2*(a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a
^2*b^5)*f*cos(f*x + e)^2 + (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3
*b^4 + a^2*b^5)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

3.367 $\int \frac{\cos^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=243

$$-\frac{2(2a-b)(a+b)\sin(e+fx)\cos(e+fx)}{3a^2b^2f\sqrt{a+b\sin^2(e+fx)}}-\frac{\left(8a^2+3ab-2b^2\right)\sqrt{a+b\sin^2(e+fx)}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{3a^2b^3f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{(8a-b)(a+b)\sqrt{\frac{b\sin}{}}}{3ab^3f\sqrt{a}}$$

```
[Out] ((a + b)*Cos[e + f*x]^3*Sin[e + f*x])/(3*a*b*f*(a + b*Sin[e + f*x]^2)^(3/2)
) - (2*(2*a - b)*(a + b)*Cos[e + f*x]*Sin[e + f*x])/(3*a^2*b^2*f*Sqrt[a + b
*Sin[e + f*x]^2]) - ((8*a^2 + 3*a*b - 2*b^2)*EllipticE[e + f*x, -(b/a)]*Sqr
t[a + b*Sin[e + f*x]^2])/(3*a^2*b^3*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + ((8
*a - b)*(a + b)*EllipticF[e + f*x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/
(3*a*b^3*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.309695, antiderivative size = 283, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3192, 413, 526, 524, 426, 424, 421, 419}

$$-\frac{2(2a-b)(a+b)\sin(e+fx)\cos(e+fx)}{3a^2b^2f\sqrt{a+b\sin^2(e+fx)}}-\frac{\left(8a^2+3ab-2b^2\right)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\sin^{-1}(\sin\right.}{3a^2b^3f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^6/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] ((a + b)*Cos[e + f*x]^3*Sin[e + f*x])/(3*a*b*f*(a + b*Sin[e + f*x]^2)^(3/2)
) - (2*(2*a - b)*(a + b)*Cos[e + f*x]*Sin[e + f*x])/(3*a^2*b^2*f*Sqrt[a + b
*Sin[e + f*x]^2]) - ((8*a^2 + 3*a*b - 2*b^2)*Sqrt[Cos[e + f*x]^2]*EllipticE
[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*
a^2*b^3*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + ((8*a - b)*(a + b)*Sqrt[Cos[e +
 f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*S
in[e + f*x]^2)/a])/(3*a*b^3*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\cos^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{\left(1-x^2\right)^{5/2}}{\left(a+bx^2\right)^{5/2}}dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{(a+b)\cos^3(e+fx)\sin(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{1-x^2}\left(-a+2b+(4a+ \right.}{\left(a+bx^2\right)^{3/2}}\right.}{3abf}$$

$$= \frac{(a+b)\cos^3(e+fx)\sin(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{2(2a-b)(a+b)\cos(e+fx)\sin(e+fx)}{3a^2b^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(\sqrt{\cos^2(e+\right.}{}$$

$$= \frac{(a+b)\cos^3(e+fx)\sin(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{2(2a-b)(a+b)\cos(e+fx)\sin(e+fx)}{3a^2b^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\left((8a-b)(a\right.}{}$$

$$= \frac{(a+b)\cos^3(e+fx)\sin(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{2(2a-b)(a+b)\cos(e+fx)\sin(e+fx)}{3a^2b^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(\left(8a^2+3ab\right.\right.}{}$$

$$= \frac{(a+b)\cos^3(e+fx)\sin(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{2(2a-b)(a+b)\cos(e+fx)\sin(e+fx)}{3a^2b^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(8a^2+3ab\right.}{}$$

Mathematica [A] time = 2.08649, size = 194, normalized size = 0.8

$$\frac{\frac{1}{2}(a+b)\left(4a^2(8a-b)\left(\frac{2a-b\cos(2(e+fx))+b}{a}\right)^{3/2}F\left(e+fx\middle|-\frac{b}{a}\right)-2\sqrt{2}b\sin(2(e+fx))\left(8a^2+b(2b-5a)\cos(2(e+fx))-ab-\right.\right.}{6a^2b^3f(2a-b\cos(2(e+fx))+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^6/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (-2*a^2*(8*a^2 + 3*a*b - 2*b^2)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*El
lipticE[e + f*x, -(b/a)] + ((a + b)*(4*a^2*(8*a - b)*((2*a + b - b*Cos[2*(e
 + f*x)])/a)^(3/2)*EllipticF[e + f*x, -(b/a)] - 2*Sqrt[2]*b*(8*a^2 - a*b -
2*b^2 + b*(-5*a + 2*b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)]))/2)/(6*a^2*b^3*f
*(2*a + b - b*Cos[2*(e + f*x)])^(3/2))
```

Maple [B] time = 1.57, size = 712, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^6/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3*((5*a^2*b^2+3*a*b^3-2*b^4)*sin(f*x+e)*cos(f*x+e)^4+(-4*a^3*b-6*a^2*b^2+
2*b^4)*cos(f*x+e)^2*sin(f*x+e)-(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b
```

```
)/a)^(1/2)*a*b*(8*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2+7*EllipticF(sin(
f*x+e),(-1/a*b)^(1/2))*a*b-EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^2-8*Ellip
ticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2-3*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*
a*b+2*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*b^2)*cos(f*x+e)^2+8*(cos(f*x+e)^
2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1
/2))*a^4+15*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*Elliptic
F(sin(f*x+e),(-1/a*b)^(1/2))*a^3*b+6*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^
2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b^2-(cos(f*x+e)^2
)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/
2))*a*b^3-8*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*Elliptic
E(sin(f*x+e),(-1/a*b)^(1/2))*a^4-11*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2
+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3*b-(cos(f*x+e)^2)^(
1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))
*a^2*b^2+2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE
(sin(f*x+e),(-1/a*b)^(1/2))*a*b^3)/a^2/(a+b*sin(f*x+e)^2)^(3/2)/b^3/cos(f*x
+e)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^{6}}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^6/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\cos\left(fx+e\right)^{6}}{b^{3}\cos\left(fx+e\right)^{6}-3\left(ab^{2}+b^{3}\right)\cos\left(fx+e\right)^{4}-a^{3}-3\,a^{2}b-3\,ab^{2}-b^{3}+3\left(a^{2}b+2\,ab^{2}+b^{3}\right)\cos\left(fx+e\right)^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)*cos(f*x + e)^6/(b^3*cos(f*x + e)^
6 - 3*(a*b^2 + b^3)*cos(f*x + e)^4 - a^3 - 3*a^2*b - 3*a*b^2 - b^3 + 3*(a^2
*b + 2*a*b^2 + b^3)*cos(f*x + e)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**6/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(fx+e)^6}{\left(b\sin(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^6/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

3.368 $\int \frac{\cos^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=223

$$-\frac{2(a-b)\sin(e+fx)\cos(e+fx)}{3a^2bf\sqrt{a+b\sin^2(e+fx)}}+\frac{(2a-b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(e+fx\middle|-\frac{b}{a}\right)}{3ab^2f\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a-b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}E\left(e+fx\middle|-\frac{b}{a}\right)}{3ab^2f\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] ((a + b)*Cos[e + f*x]*Sin[e + f*x])/(3*a*b*f*(a + b*Sin[e + f*x]^2)^(3/2))
- (2*(a - b)*Cos[e + f*x]*Sin[e + f*x])/(3*a^2*b*f*Sqrt[a + b*Sin[e + f*x]^
2]) - (2*(a - b)*EllipticE[e + f*x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])
/(3*a*b^2*f*Sqrt[a + b*Sin[e + f*x]^2]) + ((2*a - b)*EllipticF[e + f*x, -(b
/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*a*b^2*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.283316, antiderivative size = 263, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3192, 413, 527, 524, 426, 424, 421, 419}

$$-\frac{2(a-b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}E\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{3a^2b^2f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{2(a-b)\sin(e+fx)\cos(e+fx)}{3a^2bf\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] ((a + b)*Cos[e + f*x]*Sin[e + f*x])/(3*a*b*f*(a + b*Sin[e + f*x]^2)^(3/2))
- (2*(a - b)*Cos[e + f*x]*Sin[e + f*x])/(3*a^2*b*f*Sqrt[a + b*Sin[e + f*x]^
2]) - (2*(a - b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a
)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a^2*b^2*f*Sqrt[1 + (b*Sin[e
+ f*x]^2)/a]) + ((2*a - b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*
x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*a*b^2*f*Sqrt[a
 + b*Sin[e + f*x]^2])
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\cos^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{\left(1-x^2\right)^{3/2}}{\left(a+bx^2\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{(a+b)\cos(e+fx)\sin(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{-a+2b+(2a-b)x^2}{\sqrt{1-x^2}\left(a+bx^2\right)^{3/2}}\right.}{3abf}$$

$$= \frac{(a+b)\cos(e+fx)\sin(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{2(a-b)\cos(e+fx)\sin(e+fx)}{3a^2bf\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\text{se}\right.}{}$$

$$= \frac{(a+b)\cos(e+fx)\sin(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{2(a-b)\cos(e+fx)\sin(e+fx)}{3a^2bf\sqrt{a+b\sin^2(e+fx)}} + \frac{\left((2a-b)\sqrt{\cos^2(e\right.}{}$$

$$= \frac{(a+b)\cos(e+fx)\sin(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{2(a-b)\cos(e+fx)\sin(e+fx)}{3a^2bf\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(2\left(a^2-b^2\right)\sqrt{\cos\right.}{}$$

$$= \frac{(a+b)\cos(e+fx)\sin(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{2(a-b)\cos(e+fx)\sin(e+fx)}{3a^2bf\sqrt{a+b\sin^2(e+fx)}} - \frac{2(a-b)\sqrt{\cos^2(e}}{}$$

Mathematica [A] time = 1.38757, size = 171, normalized size = 0.77

$$\frac{-\sqrt{2}b\sin(2(e+fx))\left(a^2+b(b-a)\cos(2(e+fx))-2ab-b^2\right)+a^2(2a-b)\left(\frac{2a-b\cos(2(e+fx))+b}{a}\right)^{3/2}F\left(e+fx\middle|-\frac{b}{a}\right)-2a^2(}{3a^2b^2f(2a-b\cos(2(e+fx))+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (-2*a^2*(a - b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticE[e + f*x,
 -(b/a)] + a^2*(2*a - b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticF
[e + f*x, -(b/a)] - Sqrt[2]*b*(a^2 - 2*a*b - b^2 + b*(-a + b)*Cos[2*(e + f*
x)])*Sin[2*(e + f*x)])/(3*a^2*b^2*f*(2*a + b - b*Cos[2*(e + f*x)])^(3/2))
```

Maple [A] time = 1.336, size = 485, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3*((2*a*b^2-2*b^3)*sin(f*x+e)*cos(f*x+e)^4+(-a^2*b+a*b^2+2*b^3)*cos(f*x+e
)^2*sin(f*x+e)-(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*a*b*(
2*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a-EllipticF(sin(f*x+e),(-1/a*b)^(1/2
```

```
))*b-2*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a+2*EllipticE(sin(f*x+e),(-1/a*
b)^(1/2))*b)*cos(f*x+e)^2+2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a
)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3+(cos(f*x+e)^2)^(1/2)*(-b/a
*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b-(co
s(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-
1/a*b)^(1/2))*a*b^2-2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2
)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3+2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos
(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2)/a^2/(a
+b*sin(f*x+e)^2)^(3/2)/b^2/cos(f*x+e)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^4}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^4/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\cos\left(fx+e\right)^4}{b^3\cos\left(fx+e\right)^6-3\left(ab^2+b^3\right)\cos\left(fx+e\right)^4-a^3-3a^2b-3ab^2-b^3+3\left(a^2b+2ab^2+b^3\right)\cos\left(fx+e\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)*cos(f*x + e)^4/(b^3*cos(f*x + e)^
6 - 3*(a*b^2 + b^3)*cos(f*x + e)^4 - a^3 - 3*a^2*b - 3*a*b^2 - b^3 + 3*(a^2
*b + 2*a*b^2 + b^3)*cos(f*x + e)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^4}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^4/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

3.369 $\int \frac{\cos^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=217

$$\frac{(a+2b)\sin(e+fx)\cos(e+fx)}{3a^2f(a+b)\sqrt{a+b\sin^2(e+fx)}}+\frac{\sin(e+fx)\cos(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(e+fx\middle|-\frac{b}{a}\right)}{3abf\sqrt{a+b\sin^2(e+fx)}}+\frac{(a+2b)\sqrt{\frac{b\sin^2(e+fx)}{a}}}{3abf(a+b)\sqrt{a+}}$$

```
[Out] (Cos[e + f*x]*Sin[e + f*x])/(3*a*f*(a + b*Sin[e + f*x]^2)^(3/2)) + ((a + 2*
b)*Cos[e + f*x]*Sin[e + f*x])/(3*a^2*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2])
+ ((a + 2*b)*EllipticE[e + f*x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*
a*b*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2]) - (EllipticF[e + f*x, -(b/a)]*Sqr
t[1 + (b*Sin[e + f*x]^2)/a])/(3*a*b*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.241758, antiderivative size = 257, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3192, 412, 527, 524, 426, 424, 421, 419}

$$\frac{(a+2b)\sin(e+fx)\cos(e+fx)}{3a^2f(a+b)\sqrt{a+b\sin^2(e+fx)}}+\frac{(a+2b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{3a^2bf(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (Cos[e + f*x]*Sin[e + f*x])/(3*a*f*(a + b*Sin[e + f*x]^2)^(3/2)) + ((a + 2*
b)*Cos[e + f*x]*Sin[e + f*x])/(3*a^2*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2])
+ ((a + 2*b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*S
ec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a^2*b*(a + b)*f*Sqrt[1 + (b*Sin[
e + f*x]^2)/a]) - (Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b
/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*a*b*f*Sqrt[a + b*Sin[e
 + f*x]^2])
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 412

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] + Dist[1/(
a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p + 1)
 + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x]
&& NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\mathrm{Subst}\left(\int \frac{\sqrt{1-x^2}}{\left(a+bx^2\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\mathrm{Subst}\left(\int \frac{-2+x^2}{\sqrt{1-x^2}\left(a+bx^2\right)^{3/2}}\,dx, x, \mathrm{s}\right.}{3af}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{(a+2b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+\right.}{}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{(a+2b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+\right.}{}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{(a+2b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\left((a+2b)\sqrt{\cos^2(e+fx)}\right.}{}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{(a+2b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{(a+2b)\sqrt{\cos^2(e+fx)}E}{}$$

Mathematica [A] time = 1.42125, size = 175, normalized size = 0.81

$$\frac{-\sqrt{2}b\sin(2(e+fx))\left(-4a^2+b(a+2b)\cos(2(e+fx))-7ab-2b^2\right)-2a^2(a+b)\left(\frac{2a-b\cos(2(e+fx))+b}{a}\right)^{3/2}F\left(e+fx\middle|-\frac{b}{a}\right)+2a}{6a^2bf(a+b)(2a-b\cos(2(e+fx))+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (2*a^2*(a + 2*b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticE[e + f*x
, -(b/a)] - 2*a^2*(a + b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*Elliptic
F[e + f*x, -(b/a)] - Sqrt[2]*b*(-4*a^2 - 7*a*b - 2*b^2 + b*(a + 2*b)*Cos[2*
(e + f*x)])*Sin[2*(e + f*x)])/(6*a^2*b*(a + b)*f*(2*a + b - b*Cos[2*(e + f*
x)])^(3/2))
```

Maple [B] time = 1.753, size = 549, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/3*((cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e
),(-1/a*b)^(1/2))*a^2*b*sin(f*x+e)^2+(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^
2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2*sin(f*x+e)^2-(cos(f*
```

```
x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1
/2))*a^2*b*sin(f*x+e)^2-2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)
*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2*sin(f*x+e)^2+a*b^2*sin(f*x+e)^5
+2*b^3*sin(f*x+e)^5+(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*Ellip
ticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3+a^2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+
e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-(cos(f*x+e)^2)^(1/2)*
((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3-2*(co
s(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b
)^(1/2))*a^2*b+2*a^2*b*sin(f*x+e)^3+2*a*b^2*sin(f*x+e)^3-2*sin(f*x+e)^3*b^3
-2*sin(f*x+e)*a^2*b-3*a*b^2*sin(f*x+e))/a^2/(a+b)/(a+b*sin(f*x+e)^2)^(3/2)/
b/cos(f*x+e)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^2}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\cos\left(fx+e\right)^2}{b^3\cos\left(fx+e\right)^6-3\left(ab^2+b^3\right)\cos\left(fx+e\right)^4-a^3-3\,a^2b-3\,ab^2-b^3+3\left(a^2b+2\,ab^2+b^3\right)\cos\left(fx+e\right)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)*cos(f*x + e)^2/(b^3*cos(f*x + e)^
6 - 3*(a*b^2 + b^3)*cos(f*x + e)^4 - a^3 - 3*a^2*b - 3*a*b^2 - b^3 + 3*(a^2
*b + 2*a*b^2 + b^3)*cos(f*x + e)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(fx+e\right)^{2}}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

3.370 $\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=223

$$\frac{2b(2a+b)\sin(e+fx)\cos(e+fx)}{3a^2f(a+b)^2\sqrt{a+b\sin^2(e+fx)}}+\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{3a^2f(a+b)^2\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{b\sin(e+fx)\cos(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

```
[Out] (b*Cos[e + f*x]*Sin[e + f*x])/(3*a*(a + b)*f*(a + b*Sin[e + f*x]^2)^(3/2))
+ (2*b*(2*a + b)*Cos[e + f*x]*Sin[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[a + b*S
in[e + f*x]^2]) + (2*(2*a + b)*EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e
+ f*x]^2])/(3*a^2*(a + b)^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) - (EllipticF[
e + f*x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*a*(a + b)*f*Sqrt[a + b*
Sin[e + f*x]^2])
```

Rubi [A] time = 0.265512, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3184, 3173, 3172, 3178, 3177, 3183, 3182}

$$\frac{2b(2a+b)\sin(e+fx)\cos(e+fx)}{3a^2f(a+b)^2\sqrt{a+b\sin^2(e+fx)}}+\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{3a^2f(a+b)^2\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{b\sin(e+fx)\cos(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x]^2)^(-5/2),x]
```

```
[Out] (b*Cos[e + f*x]*Sin[e + f*x])/(3*a*(a + b)*f*(a + b*Sin[e + f*x]^2)^(3/2))
+ (2*b*(2*a + b)*Cos[e + f*x]*Sin[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[a + b*S
in[e + f*x]^2]) + (2*(2*a + b)*EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e
+ f*x]^2])/(3*a^2*(a + b)^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) - (EllipticF[
e + f*x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*a*(a + b)*f*Sqrt[a + b*
Sin[e + f*x]^2])
```

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a +
 b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)
*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rule 3173

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b - a*B)*Cos[e + f*x]*Sin[e + f*x]
*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(a + b)*(p + 1)), x] - Dist[1/(2*a*
(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p +
 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{b\cos(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\int \frac{-3a-2b+b\sin^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx}{3a(a+b)}$$

$$= \frac{b\cos(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(2a+b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{\int \frac{-a(3a+b)-2b(2a\cdots}{\sqrt{a+b\sin\cdots}}}{3a^2(a\cdots}$$

$$= \frac{b\cos(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(2a+b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{\int \frac{1}{\sqrt{a+b\sin^2(e+fx\cdots}}}{3a(a+b)}$$

$$= \frac{b\cos(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(2a+b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\left(2(2a+b)\sqrt{a\cdots}\right.}{3a\cdots}$$

$$= \frac{b\cos(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(2a+b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{2(2a+b)E\left(e+\cdots\right.}{3a^2(a+\cdots}$$

Mathematica [A] time = 1.22509, size = 172, normalized size = 0.77

$$\frac{-\sqrt{2}b\sin(2(e+fx))\left(-5a^2+b(2a+b)\cos(2(e+fx))-5ab-b^2\right)-a^2(a+b)\left(\frac{2a-b\cos(2(e+fx))+b}{a}\right)^{3/2}F\left(e+fx\middle|-\frac{b}{a}\right)+2a^2(\cdots}{3a^2f(a+b)^2(2a-b\cos(2(e+fx))+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)^(-5/2),x]
```

```
[Out] (2*a^2*(2*a + b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticE[e + f*x
, -(b/a)] - a^2*(a + b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticF[
e + f*x, -(b/a)] - Sqrt[2]*b*(-5*a^2 - 5*a*b - b^2 + b*(2*a + b)*Cos[2*(e +
 f*x)])*Sin[2*(e + f*x)])/(3*a^2*(a + b)^2*f*(2*a + b - b*Cos[2*(e + f*x)])
^(3/2))
```

Maple [B] time = 1.664, size = 547, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/3*((cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e
),(-1/a*b)^(1/2))*a^2*b*sin(f*x+e)^2+(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^
2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2*sin(f*x+e)^2-4*(cos(
f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^
(1/2))*a^2*b*sin(f*x+e)^2-2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/
2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2*sin(f*x+e)^2+4*a*b^2*sin(f*x+
e)^5+2*b^3*sin(f*x+e)^5+(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*E
llipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3+a^2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(
f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-4*(cos(f*x+e)^2)^
(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3
-2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(
-1/a*b)^(1/2))*a^2*b+5*a^2*b*sin(f*x+e)^3-a*b^2*sin(f*x+e)^3-2*sin(f*x+e)^3
*b^3-5*sin(f*x+e)*a^2*b-3*a*b^2*sin(f*x+e))/(a+b*sin(f*x+e)^2)^(3/2)/a^2/(a
+b)^2/cos(f*x+e)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(-5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}}{b^3\cos\left(fx+e\right)^6-3\left(ab^2+b^3\right)\cos\left(fx+e\right)^4-a^3-3\,a^2b-3\,ab^2-b^3+3\left(a^2b+2\,ab^2+b^3\right)\cos\left(fx+e\right)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)/(b^3*cos(f*x + e)^6 - 3*(a*b^2 +
b^3)*cos(f*x + e)^4 - a^3 - 3*a^2*b - 3*a*b^2 - b^3 + 3*(a^2*b + 2*a*b^2 +
b^3)*cos(f*x + e)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(-5/2), x)
```

3.371 $\int \frac{\sec^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=288

$$-\frac{b\left(3a^2-7ab-2b^2\right)\sin(e+fx)\cos(e+fx)}{3a^2f(a+b)^3\sqrt{a+b\sin^2(e+fx)}}-\frac{\left(3a^2-7ab-2b^2\right)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}E\left(e+fx\middle|-\frac{b}{a}\right)}{3af(a+b)^3\sqrt{a+b\sin^2(e+fx)}}+\frac{\tan(e+}{f(a+b)\left(a+b\sin\right.}$$

```
[Out] -((3*a - b)*b*Cos[e + f*x]*Sin[e + f*x])/(3*a*(a + b)^2*f*(a + b*Sin[e + f*
x]^2)^(3/2)) - (b*(3*a^2 - 7*a*b - 2*b^2)*Cos[e + f*x]*Sin[e + f*x])/(3*a^2
*(a + b)^3*f*Sqrt[a + b*Sin[e + f*x]^2]) - ((3*a^2 - 7*a*b - 2*b^2)*Ellipti
cE[e + f*x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*a*(a + b)^3*f*Sqrt[a
 + b*Sin[e + f*x]^2]) + ((3*a - b)*EllipticF[e + f*x, -(b/a)]*Sqrt[1 + (b*S
in[e + f*x]^2)/a])/(3*a*(a + b)^2*f*Sqrt[a + b*Sin[e + f*x]^2]) + Tan[e + f
*x]/((a + b)*f*(a + b*Sin[e + f*x]^2)^(3/2))
```

Rubi [A] time = 0.356559, antiderivative size = 328, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3192, 414, 527, 524, 426, 424, 421, 419}

$$-\frac{b\left(3a^2-7ab-2b^2\right)\sin(e+fx)\cos(e+fx)}{3a^2f(a+b)^3\sqrt{a+b\sin^2(e+fx)}}-\frac{\left(3a^2-7ab-2b^2\right)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}E\left(\sin\right.}{3a^2f(a+b)^3\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] -((3*a - b)*b*Cos[e + f*x]*Sin[e + f*x])/(3*a*(a + b)^2*f*(a + b*Sin[e + f*
x]^2)^(3/2)) - (b*(3*a^2 - 7*a*b - 2*b^2)*Cos[e + f*x]*Sin[e + f*x])/(3*a^2
*(a + b)^3*f*Sqrt[a + b*Sin[e + f*x]^2]) - ((3*a^2 - 7*a*b - 2*b^2)*Sqrt[Co
s[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a +
 b*Sin[e + f*x]^2])/(3*a^2*(a + b)^3*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + ((
3*a - b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e
 + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*a*(a + b)^2*f*Sqrt[a + b*Sin[e +
 f*x]^2]) + Tan[e + f*x]/((a + b)*f*(a + b*Sin[e + f*x]^2)^(3/2))
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
```

```
d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\sec^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\operatorname{Subst}\left(\int \frac{1}{(1-x^2)^{3/2}(a+bx^2)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)}{(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\operatorname{Subst}\left(\int \frac{b+3bx^2}{\sqrt{1-x^2}(a+bx^2)^{5/}}\right.}{(a+b)f}$$

$$= -\frac{(3a-b)b\cos(e+fx)\sin(e+fx)}{3a(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{\tan(e+fx)}{(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\left(\sqrt{\cos^2(e+\right.}{}$$

$$= -\frac{(3a-b)b\cos(e+fx)\sin(e+fx)}{3a(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{b\left(3a^2-7ab-2b^2\right)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)^3f\sqrt{a+b\sin^2(e+fx)}} + \cdots$$

$$= -\frac{(3a-b)b\cos(e+fx)\sin(e+fx)}{3a(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{b\left(3a^2-7ab-2b^2\right)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)^3f\sqrt{a+b\sin^2(e+fx)}} + \cdots$$

$$= -\frac{(3a-b)b\cos(e+fx)\sin(e+fx)}{3a(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{b\left(3a^2-7ab-2b^2\right)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)^3f\sqrt{a+b\sin^2(e+fx)}} + \cdots$$

$$= -\frac{(3a-b)b\cos(e+fx)\sin(e+fx)}{3a(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{b\left(3a^2-7ab-2b^2\right)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)^3f\sqrt{a+b\sin^2(e+fx)}} - \cdots$$

Mathematica [A] time = 3.29956, size = 245, normalized size = 0.85

$$\frac{\frac{\tan(e+fx)\left(-4ab\left(6a^2-5ab-3b^2\right)\cos(2(e+fx))+b^2\left(3a^2-7ab-2b^2\right)\cos(4(e+fx))+41a^2b^2+24a^3b+24a^4+19ab^3+2b^4\right)}{\sqrt{2}} + 2a^2\left(3a^2+2ab-b^2\right)\left(\frac{2a-b\cos}{}\right.\cdots}{6a^2f(a+b)^3(2a-b\cos(2(e+fx))\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (-2*a^2*(3*a^2 - 7*a*b - 2*b^2)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*El
lipticE[e + f*x, -(b/a)] + 2*a^2*(3*a^2 + 2*a*b - b^2)*((2*a + b - b*Cos[2*
(e + f*x)])/a)^(3/2)*EllipticF[e + f*x, -(b/a)] + ((24*a^4 + 24*a^3*b + 41*
a^2*b^2 + 19*a*b^3 + 2*b^4 - 4*a*b*(6*a^2 - 5*a*b - 3*b^2)*Cos[2*(e + f*x)]
 + b^2*(3*a^2 - 7*a*b - 2*b^2)*Cos[4*(e + f*x)])*Tan[e + f*x])/Sqrt[2])/(6*
a^2*(a + b)^3*f*(2*a + b - b*Cos[2*(e + f*x)])^(3/2))
```

Maple [B] time = 2.833, size = 1082, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3*((-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b^2*(3*a^2-7*a*b-2*b^2)*sin
(f*x+e)*cos(f*x+e)^4-2*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*(3*a^3-
a^2*b-5*a*b^2-b^3)*cos(f*x+e)^2*sin(f*x+e)+3*(-b*cos(f*x+e)^4+(a+b)*cos(f*x
+e)^2)^(1/2)*a^2*(a^2+2*a*b+b^2)*sin(f*x+e)-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+
e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(cos(f*x+e)^2)^(1/2)*a*b*(3*E
llipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2+2*EllipticF(sin(f*x+e),(-1/a*b)^(1/
2))*a*b-EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^2-3*EllipticE(sin(f*x+e),(-1
/a*b)^(1/2))*a^2+7*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b+2*EllipticE(sin
(f*x+e),(-1/a*b)^(1/2))*b^2)*cos(f*x+e)^2+3*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(
f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*(-b*cos(f*x+e)
^4+(a+b)*cos(f*x+e)^2)^(1/2)*a^4+5*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+
(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*(-b*cos(f*x+e)^4+(a+b)*
cos(f*x+e)^2)^(1/2)*a^3*b+(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^
(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e
)^2)^(1/2)*a^2*b^2-(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*E
llipticF(sin(f*x+e),(-1/a*b)^(1/2))*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1
/2)*a*b^3-3*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*Elliptic
E(sin(f*x+e),(-1/a*b)^(1/2))*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*a^4
+4*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x
+e),(-1/a*b)^(1/2))*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*a^3*b+9*(cos
(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1
/a*b)^(1/2))*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*a^2*b^2+2*(cos(f*x+
e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*
x+e)^2)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^3)/(-(a+b*sin(f*x+e)
^2)*(-1+sin(f*x+e))*(1+sin(f*x+e)))^(1/2)/(a+b*sin(f*x+e)^2)^(3/2)/a^2/(a+b
)^3/cos(f*x+e)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\sec\left(fx+e\right)^{2}}{b^{3}\cos\left(fx+e\right)^{6}-3\left(ab^{2}+b^{3}\right)\cos\left(fx+e\right)^{4}-a^{3}-3\,a^{2}b-3\,ab^{2}-b^{3}+3\left(a^{2}b+2\,ab^{2}+b^{3}\right)\cos\left(fx+e\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)*sec(f*x + e)^2/(b^3*cos(f*x + e)^
6 - 3*(a*b^2 + b^3)*cos(f*x + e)^4 - a^3 - 3*a^2*b - 3*a*b^2 - b^3 + 3*(a^2
*b + 2*a*b^2 + b^3)*cos(f*x + e)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec \left(fx+e\right)^{2}}{\left(b\sin \left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

3.372 $\int (d\cos(e+fx))^m \left(a+b\sin^2(e+fx)\right)^p \, dx$

Optimal. Leaf size=115

$$\frac{d\sin(e+fx)\cos^2(e+fx)^{\frac{1-m}{2}}(d\cos(e+fx))^{m-1}\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};\frac{1-m}{2},-p;\frac{3}{2};\sin^2(e+fx),-\right.}{f}$$

```
[Out] (d*AppellF1[1/2, (1 - m)/2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a
)]*(d*Cos[e + f*x])^(-1 + m)*(Cos[e + f*x]^2)^((1 - m)/2)*Sin[e + f*x]*(a +
 b*Sin[e + f*x]^2)^p)/(f*(1 + (b*Sin[e + f*x]^2)/a)^p)
```

Rubi [A] time = 0.0900169, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3193, 430, 429}

$$\frac{d\sin(e+fx)\cos^2(e+fx)^{\frac{1-m}{2}}(d\cos(e+fx))^{m-1}\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};\frac{1-m}{2},-p;\frac{3}{2};\sin^2(e+fx),-\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^m*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (d*AppellF1[1/2, (1 - m)/2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a
)]*(d*Cos[e + f*x])^(-1 + m)*(Cos[e + f*x]^2)^((1 - m)/2)*Sin[e + f*x]*(a +
 b*Sin[e + f*x]^2)^p)/(f*(1 + (b*Sin[e + f*x]^2)/a)^p)
```

Rule 3193

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(
ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Cos[e + f*x])^(2*FracPart[(m - 1)/2]))/(
f*(Cos[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/
2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f
, m, p}, x] && !IntegerQ[m]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int (d\cos(e+fx))^m \left(a+b\sin^2(e+fx)\right)^p dx = \frac{\left(d(d\cos(e+fx))^{2\left(-\frac{1}{2}+\frac{m}{2}\right)}\cos^2(e+fx)^{\frac{1}{2}-\frac{m}{2}}\right)\text{Subst}\left(\int \left(1-x^2\right)^{\frac{1}{2}(-1+m)}\left(\right.\right.}{f}$$

$$= \frac{\left(d(d\cos(e+fx))^{2\left(-\frac{1}{2}+\frac{m}{2}\right)}\cos^2(e+fx)^{\frac{1}{2}-\frac{m}{2}}\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin}{}\right.\right.}{f}$$

$$= \frac{dF_1\left(\frac{1}{2};\frac{1-m}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)(d\cos(e+fx))^{-1+m}\cos^2(}{f}$$

Mathematica [A] time = 0.833344, size = 228, normalized size = 1.98

$$\frac{3a\tan(e+fx)(d\cos(e+fx))^m\left(a+b\sin^2(e+fx)\right)^p F_1\left(\frac{1}{2};\frac{1-m}{2},-p;\frac{3}{2};\sin^2(e+\right.}{f\left(\sin^2(e+fx)\left(2bpF_1\left(\frac{3}{2};\frac{1-m}{2},1-p;\frac{5}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)-a(m-1)F_1\left(\frac{3}{2};\frac{3-m}{2},-p;\frac{5}{2};\sin^2(e+fx),-\frac{b\sin^2(e+}{a}\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Cos[e + f*x])^m*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (3*a*AppellF1[1/2, (1 - m)/2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)
/a)]*(d*Cos[e + f*x])^m*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x])/(f*(3*a*Appe
llF1[1/2, (1 - m)/2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)] + (2
*b*p*AppellF1[3/2, (1 - m)/2, 1 - p, 5/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]
^2)/a)] - a*(-1 + m)*AppellF1[3/2, (3 - m)/2, -p, 5/2, Sin[e + f*x]^2, -((b
*Sin[e + f*x]^2)/a)])*Sin[e + f*x]^2))
```

Maple [F] time = 1.342, size = 0, normalized size = 0.

$$\int \left(d\cos\left(fx+e\right)\right)^m \left(a+b\left(\sin\left(fx+e\right)\right)^2\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^m*(a+b*sin(f*x+e)^2)^p,x)
```

```
[Out] int((d*cos(f*x+e))^m*(a+b*sin(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p \left(d\cos\left(fx+e\right)\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*(d*cos(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-b\cos\left(fx+e\right)^{2}+a+b\right)^{p}\left(d\cos\left(fx+e\right)\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*(d*cos(f*x + e))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**m*(a+b*sin(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sin\left(fx+e\right)^{2}+a\right)^{p}\left(d\cos\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*(d*cos(f*x + e))^m, x)
```

3.373 $\int \cos^5(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=214

$$\frac{\left(3a^2+2ab(2p+5)+b^2\left(4p^2+16p+15\right)\right)\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sin^2(e+fx)}{a}\right)}{b^2f(2p+3)(2p+5)}$$

```
[Out] -(((3*a + b*(7 + 2*p))*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(1 + p))/(b^2*f*
(3 + 2*p)*(5 + 2*p))) - (Cos[e + f*x]^2*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)
^(1 + p))/(b*f*(5 + 2*p)) + ((3*a^2 + 2*a*b*(5 + 2*p) + b^2*(15 + 16*p + 4*
p^2))*Hypergeometric2F1[1/2, -p, 3/2, -((b*Sin[e + f*x]^2)/a)]*Sin[e + f*x]
*(a + b*Sin[e + f*x]^2)^p)/(b^2*f*(3 + 2*p)*(5 + 2*p)*(1 + (b*Sin[e + f*x]^
2)/a)^p)
```

Rubi [A] time = 0.209068, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3190, 416, 388, 246, 245}

$$\frac{\left(3a^2+2ab(2p+5)+b^2\left(4p^2+16p+15\right)\right)\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sin^2(e+fx)}{a}\right)}{b^2f(2p+3)(2p+5)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^5*(a + b*Sin[e + f*x]^2)^p,x]
```


```
[Out] -(((3*a + b*(7 + 2*p))*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(1 + p))/(b^2*f*
(3 + 2*p)*(5 + 2*p))) - (Cos[e + f*x]^2*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)
^(1 + p))/(b*f*(5 + 2*p)) + ((3*a^2 + 2*a*b*(5 + 2*p) + b^2*(15 + 16*p + 4*
p^2))*Hypergeometric2F1[1/2, -p, 3/2, -((b*Sin[e + f*x]^2)/a)]*Sin[e + f*x]
*(a + b*Sin[e + f*x]^2)^p)/(b^2*f*(3 + 2*p)*(5 + 2*p)*(1 + (b*Sin[e + f*x]^
2)/a)^p)
```


Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \cos^5(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \left(1-x^2\right)^2\left(a+bx^2\right)^p\,dx, x, \sin(e+fx)\right)}{f}$$

$$= -\frac{\cos^2(e+fx)\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{1+p}}{bf(5+2p)} + \frac{\text{Subst}\left(\int \left(a+bx^2\right)^p\left(a+\right.\right.}{}$$

$$= -\frac{(3a+b(7+2p))\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)} - \frac{\cos^2(e+fx)\sin(e+}{bf(}$$

$$= -\frac{(3a+b(7+2p))\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)} - \frac{\cos^2(e+fx)\sin(e+}{bf(}$$

$$= -\frac{(3a+b(7+2p))\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)} - \frac{\cos^2(e+fx)\sin(e+}{bf(}$$

Mathematica [C] time = 0.402542, size = 191, normalized size = 0.89

$$\frac{3a\sin(e+fx)\cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p F_1\left(\frac{1}{2};-2,-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(}{a}\right.}{f\left(2\sin^2(e+fx)\left(bpF_1\left(\frac{3}{2};-2,1-p;\frac{5}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)-2aF_1\left(\frac{3}{2};-1,-p;\frac{5}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\right)+3aF\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^5*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (3*a*AppellF1[1/2, -2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Co
s[e + f*x]^4*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^p)/(f*(3*a*AppellF1[1/2, -
2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)] + 2*(b*p*AppellF1[3/2,
 -2, 1 - p, 5/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)] - 2*a*AppellF1[3/
2, -1, -p, 5/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)])*Sin[e + f*x]^2))
```

Maple [F] time = 1.368, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^5\left(a+b\left(\sin\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x)
```

```
[Out] int(cos(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*cos(f*x + e)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(-b\cos\left(fx+e\right)^2+a+b\right)^p\cos\left(fx+e\right)^5,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*cos(f*x + e)^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**5*(a+b*sin(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*cos(f*x + e)^5, x)
```

3.374 $\int \cos^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=124

$$\frac{(a+b(2p+3))\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sin^2(e+fx)}{a}\right)}{bf(2p+3)}-\frac{\sin(e+fx)\left(a+b\sin^2(e\right.}{bf(2p+3)}$$

```
[Out] -((Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(1 + p))/(b*f*(3 + 2*p))) + ((a + b*
(3 + 2*p))*Hypergeometric2F1[1/2, -p, 3/2, -((b*Sin[e + f*x]^2)/a)]*Sin[e +
 f*x]*(a + b*Sin[e + f*x]^2)^p)/(b*f*(3 + 2*p)*(1 + (b*Sin[e + f*x]^2)/a)^p
)
```

Rubi [A] time = 0.104095, antiderivative size = 119, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3190, 388, 246, 245}

$$\frac{\left(\frac{a}{2bp+3b}+1\right)\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sin^2(e+fx)}{a}\right)}{f}-\frac{\sin(e+fx)\left(a+b\sin^2(e+\right.}{bf(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] -((Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(1 + p))/(b*f*(3 + 2*p))) + ((1 + a/
(3*b + 2*b*p))*Hypergeometric2F1[1/2, -p, 3/2, -((b*Sin[e + f*x]^2)/a)]*Sin
[e + f*x]*(a + b*Sin[e + f*x]^2)^p)/(f*(1 + (b*Sin[e + f*x]^2)/a)^p)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \cos^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p dx = \frac{\text{Subst}\left(\int \left(1-x^2\right)\left(a+bx^2\right)^p dx, x, \sin(e+fx)\right)}{f}$$
$$= -\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{1+p}}{bf(3+2p)} + \frac{\left(1+\frac{a}{3b+2bp}\right)\text{Subst}\left(\int \left(a+bx^2\right)^p d\right.}{f}$$
$$= -\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{1+p}}{bf(3+2p)} + \frac{\left(\left(1+\frac{a}{3b+2bp}\right)\left(a+b\sin^2(e+fx)\right)^p\right.}{}$$
$$= -\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{1+p}}{bf(3+2p)} + \frac{\left(1+\frac{a}{3b+2bp}\right){}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sin^2(e-}{a}\right.}{}$$

Mathematica [A] time = 0.194375, size = 120, normalized size = 0.97

$$-\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}\left(\left(a+b\sin^2(e+fx)\right)\left(\frac{b\sin^2(e+fx)}{a}+1\right)^p-(a+b(2p+3))\,{}_2F_1\left(\frac{1}{2},\right.\right.}{bf(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] -((Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^p*(-((a + b*(3 + 2*p))*Hypergeometri
c2F1[1/2, -p, 3/2, -((b*Sin[e + f*x]^2)/a)]) + (a + b*Sin[e + f*x]^2)*(1 +
(b*Sin[e + f*x]^2)/a)^p))/(b*f*(3 + 2*p)*(1 + (b*Sin[e + f*x]^2)/a)^p))
```

Maple [F] time = 2.313, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^3\left(a+b\left(\sin\left(fx+e\right)\right)^2\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x)
```

```
[Out] int(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*cos(f*x + e)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-b\cos\left(fx+e\right)^{2}+a+b\right)^{p}\cos\left(fx+e\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*cos(f*x + e)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3*(a+b*sin(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sin\left(fx+e\right)^{2}+a\right)^{p}\cos\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*cos(f*x + e)^3, x)
```

3.375 $\int \cos(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=67

$$\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sin^2(e+fx)}{a}\right)}{f}$$

```
[Out] (Hypergeometric2F1[1/2, -p, 3/2, -((b*Sin[e + f*x]^2)/a)]*Sin[e + f*x]*(a +
 b*Sin[e + f*x]^2)^p)/(f*(1 + (b*Sin[e + f*x]^2)/a)^p)
```

Rubi [A] time = 0.0440752, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3190, 246, 245}

$$\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sin^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (Hypergeometric2F1[1/2, -p, 3/2, -((b*Sin[e + f*x]^2)/a)]*Sin[e + f*x]*(a +
 b*Sin[e + f*x]^2)^p)/(f*(1 + (b*Sin[e + f*x]^2)/a)^p)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \cos(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \left(a+bx^2\right)^p\,dx, x, \sin(e+fx)\right)}{f}$$
$$= \frac{\left(\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int\left(1+\frac{bx^2}{a}\right)^p\,dx, x, \sin(e+f\right.}{f}$$
$$= \frac{{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sin^2(e+fx)}{a}\right)\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+fx)}{a}\right)^{-p}}{f}$$

Mathematica [A] time = 0.0274024, size = 67, normalized size = 1.

$$\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sin^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (Hypergeometric2F1[1/2, -p, 3/2, -((b*Sin[e + f*x]^2)/a)]*Sin[e + f*x]*(a +
 b*Sin[e + f*x]^2)^p)/(f*(1 + (b*Sin[e + f*x]^2)/a)^p)
```

Maple [F] time = 1.371, size = 0, normalized size = 0.

$$\int \cos(fx+e)\left(a+b\left(\sin(fx+e)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+b*sin(f*x+e)^2)^p,x)
```

```
[Out] int(cos(f*x+e)*(a+b*sin(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin(fx+e)^2+a\right)^p\cos(fx+e)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*cos(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-b\cos(fx+e)^2+a+b\right)^p\cos(fx+e), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*cos(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sin \left(fx + e\right)^2 + a\right)^p \cos \left(fx + e\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*cos(f*x + e), x)
```

3.376 $\int \sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=76

$$\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)}{f}$$

```
[Out] (AppellF1[1/2, 1, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sin[e +
 f*x]*(a + b*Sin[e + f*x]^2)^p)/(f*(1 + (b*Sin[e + f*x]^2)/a)^p)
```

Rubi [A] time = 0.0741071, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3190, 430, 429}

$$\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 1, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sin[e +
 f*x]*(a + b*Sin[e + f*x]^2)^p)/(f*(1 + (b*Sin[e + f*x]^2)/a)^p)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^p}{1-x^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{1-x^2}\,dx, x, \sin(e+f\right.}{f}$$

$$= \frac{F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\right.}{f}$$

Mathematica [F] time = 4.41652, size = 0, normalized size = 0.

$$\int \sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] Integrate[Sec[e + f*x]*(a + b*Sin[e + f*x]^2)^p, x]
```

Maple [F] time = 0.878, size = 0, normalized size = 0.

$$\int \sec\left(fx+e\right)\left(a+b\left(\sin\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sin(f*x+e)^2)^p,x)
```

```
[Out] int(sec(f*x+e)*(a+b*sin(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sec(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-b\cos\left(fx+e\right)^2+a+b\right)^p\sec\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*sec(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^{2}+a\right)^{p}\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sec(f*x + e), x)
```

3.377 $\int \sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=76

$$\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};2,-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)}{f}$$

```
[Out] (AppellF1[1/2, 2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sin[e +
 f*x]*(a + b*Sin[e + f*x]^2)^p)/(f*(1 + (b*Sin[e + f*x]^2)/a)^p)
```

Rubi [A] time = 0.0820807, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3190, 430, 429}

$$\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};2,-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sin[e +
 f*x]*(a + b*Sin[e + f*x]^2)^p)/(f*(1 + (b*Sin[e + f*x]^2)/a)^p)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^p}{(1-x^2)^2}dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{(1-x^2)^2}dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{F_1\left(\frac{1}{2};2,-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(1 \right.}{f}$$

Mathematica [F] time = 6.20837, size = 0, normalized size = 0.

$$\int \sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^3*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] Integrate[Sec[e + f*x]^3*(a + b*Sin[e + f*x]^2)^p, x]
```

Maple [F] time = 0.901, size = 0, normalized size = 0.

$$\int \left(\sec\left(fx+e\right)\right)^3\left(a+b\left(\sin\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x)
```

```
[Out] int(sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p\sec\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sec(f*x + e)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-b\cos\left(fx+e\right)^2+a+b\right)^p\sec\left(fx+e\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*sec(f*x + e)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3*(a+b*sin(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p\sec\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sec(f*x + e)^3, x)
```

3.378 $\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=90

$$\frac{\sqrt{\cos^2(e+fx)}\tan(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-\frac{3}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)}{f}$$

```
[Out] (AppellF1[1/2, -3/2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sqrt
[Cos[e + f*x]^2]*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x])/(f*(1 + (b*Sin[e +
f*x]^2)/a)^p)
```

Rubi [A] time = 0.082417, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3192, 430, 429}

$$\frac{\sqrt{\cos^2(e+fx)}\tan(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-\frac{3}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, -3/2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sqrt
[Cos[e + f*x]^2]*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x])/(f*(1 + (b*Sin[e +
f*x]^2)/a)^p)
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int\left(1-x^2\right)^{3/2}\left(a+bx^2\right)^p\,dx,x,\sin(e+\right.}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+fx)}{a}\right)^{-p}\right)\text{Subs}}{f}$$

$$= \frac{F_1\left(\frac{1}{2};-\frac{3}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sqrt{\cos^2(e+fx)}\left(a+b\sin^2(e+f\right.}{f}$$

Mathematica [B] time = 0.615708, size = 199, normalized size = 2.21

$$\frac{3a\sin(e+fx)\cos^3(e+fx)\left(a+b\sin^2(e+fx)\right)^pF_1\left(\frac{1}{2};-\frac{3}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b}{}\right.}{f\left(\sin^2(e+fx)\left(2bpF_1\left(\frac{3}{2};-\frac{3}{2},1-p;\frac{5}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)-3aF_1\left(\frac{3}{2};-\frac{1}{2},-p;\frac{5}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\right)+3\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (3*a*AppellF1[1/2, -3/2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*
Cos[e + f*x]^3*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^p)/(f*(3*a*AppellF1[1/2,
 -3/2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)] + (2*b*p*AppellF1[
3/2, -3/2, 1 - p, 5/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)] - 3*a*Appel
lF1[3/2, -1/2, -p, 5/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)])*Sin[e + f
*x]^2))
```

Maple [F] time = 1.292, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^4\left(a+b\left(\sin\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x)
```

```
[Out] int(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*cos(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(-b\cos\left(fx+e\right)^{2}+a+b\right)^{p}\cos\left(fx+e\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*cos(f*x + e)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(a+b*sin(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sin\left(fx+e\right)^{2}+a\right)^{p}\cos\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*cos(f*x + e)^4, x)
```

3.379 $\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=90

$$\frac{\sqrt{\cos^2(e+fx)}\tan(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-\frac{1}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)}{f}$$

```
[Out] (AppellF1[1/2, -1/2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sqrt
[Cos[e + f*x]^2]*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x])/(f*(1 + (b*Sin[e +
f*x]^2)/a)^p)
```

Rubi [A] time = 0.0815529, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3192, 430, 429}

$$\frac{\sqrt{\cos^2(e+fx)}\tan(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-\frac{1}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, -1/2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sqrt
[Cos[e + f*x]^2]*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x])/(f*(1 + (b*Sin[e +
f*x]^2)/a)^p)
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int\sqrt{1-x^2}\left(a+bx^2\right)^p dx,x,\sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\right.}{f}$$

$$= \frac{F_1\left(\frac{1}{2};-\frac{1}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sqrt{\cos^2(e+fx)}\left(a+b\sin^2(e+fx)\right)}{f}$$

Mathematica [B] time = 0.506849, size = 195, normalized size = 2.17

$$\frac{3a\sin(2(e+fx))\left(a+b\sin^2(e+fx)\right)^p F_1\left(\frac{1}{2};-\frac{1}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)}{2f\left(\sin^2(e+fx)\left(2bpF_1\left(\frac{3}{2};-\frac{1}{2},1-p;\frac{5}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)-aF_1\left(\frac{3}{2};\frac{1}{2},-p;\frac{5}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\right)+3aF_1\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (3*a*AppellF1[1/2, -1/2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*
(a + b*Sin[e + f*x]^2)^p*Sin[2*(e + f*x)])/(2*f*(3*a*AppellF1[1/2, -1/2, -p
, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)] + (2*b*p*AppellF1[3/2, -1/2
, 1 - p, 5/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)] - a*AppellF1[3/2, 1/
2, -p, 5/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)])*Sin[e + f*x]^2))
```

Maple [F] time = 1.817, size = 0, normalized size = 0.

$$\int\left(\cos\left(fx+e\right)\right)^2\left(a+b\left(\sin\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x)
```

```
[Out] int(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sin\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*cos(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-b\cos\left(fx+e\right)^{2}+a+b\right)^{p}\cos\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*cos(f*x + e)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sin(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sin\left(fx+e\right)^{2}+a\right)^{p}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*cos(f*x + e)^2, x)
```

3.380 $\int \left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=90

$$\frac{\sqrt{\cos^2(e+fx)}\tan(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};\frac{1}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)}{f}$$

```
[Out] (AppellF1[1/2, 1/2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sqrt[
Cos[e + f*x]^2]*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x])/(f*(1 + (b*Sin[e + f
*x]^2)/a)^p)
```

Rubi [A] time = 0.052151, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3185, 430, 429}

$$\frac{\sqrt{\cos^2(e+fx)}\tan(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};\frac{1}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 1/2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sqrt[
Cos[e + f*x]^2]*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x])/(f*(1 + (b*Sin[e + f
*x]^2)/a)^p)
```

Rule 3185

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f
*x]), Subst[Int[(a + b*ff^2*x^2)^p/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] &&  !IntegerQ[p]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \left(a+b\sin^2(e+fx)\right)^p dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{\left(a+bx^2\right)^p}{\sqrt{1-x^2}}dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{\sqrt{1-x^2}}\right.}{f}$$

$$= \frac{F_1\left(\frac{1}{2};\frac{1}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sqrt{\cos^2(e+fx)}\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2}{}\right.}{f}$$

Mathematica [A] time = 0.522443, size = 145, normalized size = 1.61

$$\frac{2^{-p-1}\csc(2(e+fx))\sqrt{-\frac{b\sin^2(e+fx)}{a}}\sqrt{\frac{b\cos^2(e+fx)}{a+b}}(2a-b\cos(2(e+fx))+b)^{p+1}F_1\left(p+1;\frac{1}{2},\frac{1}{2};p+2;\frac{2a+b-b\cos(2(e+fx))}{2(a+b)},\frac{2a}{}\right.}{bf(p+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (2^(-1 - p)*AppellF1[1 + p, 1/2, 1/2, 2 + p, (2*a + b - b*Cos[2*(e + f*x)])
/(2*(a + b)), (2*a + b - b*Cos[2*(e + f*x)])/(2*a)]*Sqrt[(b*Cos[e + f*x]^2)
/(a + b)]*(2*a + b - b*Cos[2*(e + f*x)])^(1 + p)*Csc[2*(e + f*x)]*Sqrt[-((b
*Sin[e + f*x]^2)/a)])/(b*f*(1 + p))
```

Maple [F] time = 0.546, size = 0, normalized size = 0.

$$\int \left(a+b\left(\sin\left(fx+e\right)\right)^2\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^p,x)
```

```
[Out] int((a+b*sin(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-b\cos\left(fx+e\right)^2+a+b\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sin\left(fx+e\right)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p, x)
```

3.381 $\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=90

$$\frac{\sqrt{\cos^2(e+fx)}\tan(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};\frac{3}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)}{f}$$

```
[Out] (AppellF1[1/2, 3/2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sqrt[
Cos[e + f*x]^2]*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x])/(f*(1 + (b*Sin[e + f
*x]^2)/a)^p)
```

Rubi [A] time = 0.080179, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3192, 430, 429}

$$\frac{\sqrt{\cos^2(e+fx)}\tan(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};\frac{3}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 3/2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sqrt[
Cos[e + f*x]^2]*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x])/(f*(1 + (b*Sin[e + f
*x]^2)/a)^p)
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int\frac{\left(a+bx^2\right)^p}{\left(1-x^2\right)^{3/2}}\,dx,x,\sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\Big(\int}{f}$$

$$= \frac{F_1\left(\frac{1}{2};\frac{3}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sqrt{\cos^2(e+fx)}\left(a+b\sin^2(e+fx)\right)^p}{f}$$

Mathematica [F] time = 4.37178, size = 0, normalized size = 0.

$$\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] Integrate[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)^p, x]
```

Maple [F] time = 0.786, size = 0, normalized size = 0.

$$\int \left(\sec\left(fx+e\right)\right)^2\left(a+b\left(\sin\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x)
```

```
[Out] int(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p\sec\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sec(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-b\cos\left(fx+e\right)^2+a+b\right)^p\sec\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*sec(f*x + e)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+b*sin(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sin \left(fx + e\right)^2 + a\right)^p \sec \left(fx + e\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sec(f*x + e)^2, x)
```

3.382 $\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=90

$$\frac{\sqrt{\cos^2(e+fx)}\tan(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};\frac{5}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)}{f}$$

```
[Out] (AppellF1[1/2, 5/2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sqrt[
Cos[e + f*x]^2]*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x])/(f*(1 + (b*Sin[e + f
*x]^2)/a)^p)
```

Rubi [A] time = 0.0812078, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3192, 430, 429}

$$\frac{\sqrt{\cos^2(e+fx)}\tan(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};\frac{5}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^2)^p,x]
```


```
[Out] (AppellF1[1/2, 5/2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sqrt[
Cos[e + f*x]^2]*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x])/(f*(1 + (b*Sin[e + f
*x]^2)/a)^p)
```


Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\mathrm{Subst}\left(\int \frac{(a+bx^2)^p}{(1-x^2)^{5/2}}\,dx,x,\sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+fx)}{a}\right)^{-p}\right)\mathrm{Subs}}{f}$$

$$= \frac{F_1\left(\frac{1}{2};\frac{5}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sqrt{\cos^2(e+fx)}\left(a+b\sin^2(e+fx\right.}{f}$$

Mathematica [F] time = 5.66315, size = 0, normalized size = 0.

$$\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] Integrate[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^2)^p, x]
```

Maple [F] time = 0.639, size = 0, normalized size = 0.

$$\int \left(\sec\left(fx+e\right)\right)^4\left(a+b\left(\sin\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x)
```

```
[Out] int(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p\sec\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sec(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(-b\cos\left(fx+e\right)^2+a+b\right)^p\sec\left(fx+e\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*sec(f*x + e)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(a+b*sin(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^{2}+a\right)^{p}\sec\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sec(f*x + e)^4, x)
```

3.383 $\int \frac{\cos^5(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=219

$$-\frac{\left(a^{4/3}+b^{4/3}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{6a^{2/3}b^{5/3}d}+\frac{\left(a^{4/3}+b^{4/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}b^{5/3}d}+\frac{\left(a^{4/3}-b^4\right.}{}$$

```
[Out] ((a^(4/3) - b^(4/3))*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(
1/3))])/(Sqrt[3]*a^(2/3)*b^(5/3)*d) + ((a^(4/3) + b^(4/3))*Log[a^(1/3) + b^
(1/3)*Sin[c + d*x]])/(3*a^(2/3)*b^(5/3)*d) - ((a^(4/3) + b^(4/3))*Log[a^(2/
3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2])/(6*a^(2/3)*b^(
5/3)*d) - (2*Log[a + b*Sin[c + d*x]^3])/(3*b*d) + Sin[c + d*x]^2/(2*b*d)
```

Rubi [A] time = 0.289502, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {3223, 1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\left(a^{4/3}+b^{4/3}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{6a^{2/3}b^{5/3}d}+\frac{\left(a^{4/3}+b^{4/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}b^{5/3}d}+\frac{\left(a^{4/3}-b^4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] ((a^(4/3) - b^(4/3))*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(
1/3))])/(Sqrt[3]*a^(2/3)*b^(5/3)*d) + ((a^(4/3) + b^(4/3))*Log[a^(1/3) + b^
(1/3)*Sin[c + d*x]])/(3*a^(2/3)*b^(5/3)*d) - ((a^(4/3) + b^(4/3))*Log[a^(2/
3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2])/(6*a^(2/3)*b^(
5/3)*d) - (2*Log[a + b*Sin[c + d*x]^3])/(3*b*d) + Sin[c + d*x]^2/(2*b*d)
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
```

```
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{a+b\sin^3(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{a+bx^3}\,dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{x}{b}+\frac{b-ax-2bx^2}{b(a+bx^3)}\right)dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{\sin^2(c+dx)}{2bd} + \frac{\text{Subst}\left(\int \frac{b-ax-2bx^2}{a+bx^3}\,dx, x, \sin(c+dx)\right)}{bd}$$
$$= \frac{\sin^2(c+dx)}{2bd} - \frac{2\,\text{Subst}\left(\int \frac{x^2}{a+bx^3}\,dx, x, \sin(c+dx)\right)}{d} + \frac{\text{Subst}\left(\int \frac{b-ax}{a+bx^3}\,dx, x, \sin(c+dx)\right)}{bd}$$
$$= -\frac{2\log\left(a+b\sin^3(c+dx)\right)}{3bd} + \frac{\sin^2(c+dx)}{2bd} + \frac{\left(\frac{1}{a^{2/3}}+\frac{a^{2/3}}{b^{4/3}}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx, x, \sin(c+d$$
$$= \frac{\left(a^{4/3}+b^{4/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}b^{5/3}d} - \frac{2\log\left(a+b\sin^3(c+dx)\right)}{3bd} + \frac{\sin^2(c+dx)}{2bd} - \frac{(a^{4/3}}{}$$
$$= \frac{\left(a^{4/3}+b^{4/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}b^{5/3}d} - \frac{\left(a^{4/3}+b^{4/3}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\right.}{6a^{2/3}b^{5/3}d}$$
$$= \frac{\left(a^{4/3}-b^{4/3}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{2/3}b^{5/3}d} + \frac{\left(a^{4/3}+b^{4/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}b^{5/3}d} - \frac{\left(a^{4/3}+b^{4/3}\right)}{}$$

Mathematica [C] time = 0.242779, size = 203, normalized size = 0.93

$$\frac{-b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)-3a^{2/3}\sin^2(c+dx)\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{b\sin^3(c+dx)}{a}\right)-4a^{2/3}\log\left(a+b\right.}{6a^{2/3}bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (-2*Sqrt[3]*b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1
/3))] + 2*b^(2/3)*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]] - b^(2/3)*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2] - 4*a^(2/3)*Log[a
 + b*Sin[c + d*x]^3] + 3*a^(2/3)*Sin[c + d*x]^2 - 3*a^(2/3)*Hypergeometric2
F1[2/3, 1, 5/3, -((b*Sin[c + d*x]^3)/a)]*Sin[c + d*x]^2)/(6*a^(2/3)*b*d)
```

Maple [A] time = 0.101, size = 278, normalized size = 1.3

$$\frac{(\sin(dx+c))^2}{2bd} + \frac{1}{3bd}\ln\left(\sin(dx+c)+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{1}{6bd}\ln\left((\sin(dx+c))^2-\sqrt[3]{\frac{a}{b}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{\sqrt{}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+b*sin(d*x+c)^3),x)
```

```
[Out] 1/2*sin(d*x+c)^2/b/d+1/3/d/b/(a/b)^(2/3)*ln(sin(d*x+c)+(a/b)^(1/3))-1/6/d/b
/(a/b)^(2/3)*ln(sin(d*x+c)^2-(a/b)^(1/3)*sin(d*x+c)+(a/b)^(2/3))+1/3/d/b/(a
/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*sin(d*x+c)-1))+1/3/d/b^
2*a/(a/b)^(1/3)*ln(sin(d*x+c)+(a/b)^(1/3))-1/6/d/b^2*a/(a/b)^(1/3)*ln(sin(d
*x+c)^2-(a/b)^(1/3)*sin(d*x+c)+(a/b)^(2/3))-1/3/d/b^2*a*3^(1/2)/(a/b)^(1/3)
*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*sin(d*x+c)-1))-2/3*ln(a+b*sin(d*x+c)^3)/
b/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 12.5217, size = 7471, normalized size = 34.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/12*(2*((1/2)^(1/3)*(I*sqrt(3) + 1)*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4
)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3) + 4/(b*d) + 2*(1/2)^(2/3
)*(-I*sqrt(3) + 1)/(b^2*d^2*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5
*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3)))*b*d*log(1/4*((1/2)^(1/3)*(I*sqrt
(3) + 1)*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)
/(a^2*b^5*d^3))^(1/3) + 4/(b*d) + 2*(1/2)^(2/3)*(-I*sqrt(3) + 1)/(b^2*d^2*(
2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*
d^3))^(1/3)))^2*a^3*b^3*d^2 + 2*a^3*b - 2*a*b^3 - 1/2*(4*a^3*b^2 - a*b^4)*(
(1/2)^(1/3)*(I*sqrt(3) + 1)*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5
*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3) + 4/(b*d) + 2*(1/2)^(2/3)*(-I*sqrt
(3) + 1)/(b^2*d^2*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a
^4 - b^4)/(a^2*b^5*d^3))^(1/3)))*d + (a^4 - b^4)*sin(d*x + c)) + 6*cos(d*x
+ c)^2 - (((1/2)^(1/3)*(I*sqrt(3) + 1)*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^
4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3) + 4/(b*d) + 2*(1/2)^(2/
3)*(-I*sqrt(3) + 1)/(b^2*d^2*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^
5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3)))*b*d + 3*sqrt(1/3)*b*d*sqrt(-(((
1/2)^(1/3)*(I*sqrt(3) + 1)*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*
d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3) + 4/(b*d) + 2*(1/2)^(2/3)*(-I*sqrt(
3) + 1)/(b^2*d^2*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a^
4 - b^4)/(a^2*b^5*d^3))^(1/3)))^2*b*d - 8*(1/2)^(1/3)*(I*sqrt(3) + 1)*(2/(b
^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3)
)^(1/3) - 32/(b*d) - 16*(1/2)^(2/3)*(-I*sqrt(3) + 1)/(b^2*d^2*(2/(b^3*d^3)
+ (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3))
)/(b*d)) - 12)*log(1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*(2/(b^3*d^3) + (a^4 - 2
*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3) + 4/(b*d)
+ 2*(1/2)^(2/3)*(-I*sqrt(3) + 1)/(b^2*d^2*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 +
 b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3)))^2*a^3*b^3*d^2 + 2*
a^3*b - 2*a*b^3 - 1/2*(4*a^3*b^2 - a*b^4)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(2/(
b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3
```

```
))^(1/3) + 4/(b*d) + 2*(1/2)^(2/3)*(-I*sqrt(3) + 1)/(b^2*d^2*(2/(b^3*d^3) +
 (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3)))
*d - 3/4*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*(2/(b^3*d^3) + (a^4 - 2*a^
2*b^2 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3) + 4/(b*d) + 2
*(1/2)^(2/3)*(-I*sqrt(3) + 1)/(b^2*d^2*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^
4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3)))*a^3*b^3*d^2 - 2*(2*a^
3*b^2 + a*b^4)*d)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*(2/(b^3*d^3) + (a^4 -
 2*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3) + 4/(b*d
) + 2*(1/2)^(2/3)*(-I*sqrt(3) + 1)/(b^2*d^2*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2
 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3)))^2*b*d - 8*(1/2)^
(1/3)*(I*sqrt(3) + 1)*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3)
- (a^4 - b^4)/(a^2*b^5*d^3))^(1/3) - 32/(b*d) - 16*(1/2)^(2/3)*(-I*sqrt(3)
+ 1)/(b^2*d^2*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a^4 -
 b^4)/(a^2*b^5*d^3))^(1/3)))/(b*d)) - 2*(a^4 - b^4)*sin(d*x + c)) - (((1/2)
^(1/3)*(I*sqrt(3) + 1)*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3)
 - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3) + 4/(b*d) + 2*(1/2)^(2/3)*(-I*sqrt(3) +
 1)/(b^2*d^2*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a^4 -
b^4)/(a^2*b^5*d^3))^(1/3)))*b*d - 3*sqrt(1/3)*b*d*sqrt(-(((1/2)^(1/3)*(I*sq
rt(3) + 1)*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a^4 - b^
4)/(a^2*b^5*d^3))^(1/3) + 4/(b*d) + 2*(1/2)^(2/3)*(-I*sqrt(3) + 1)/(b^2*d^2
*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^
5*d^3))^(1/3)))^2*b*d - 8*(1/2)^(1/3)*(I*sqrt(3) + 1)*(2/(b^3*d^3) + (a^4 -
 2*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3) - 32/(b*
d) - 16*(1/2)^(2/3)*(-I*sqrt(3) + 1)/(b^2*d^2*(2/(b^3*d^3) + (a^4 - 2*a^2*b
^2 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3)))/(b*d)) - 12)*l
og(-1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)
/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3) + 4/(b*d) + 2*(1/2)^(2/3)
*(-I*sqrt(3) + 1)/(b^2*d^2*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*
d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3)))^2*a^3*b^3*d^2 - 2*a^3*b + 2*a*b^3
 + 1/2*(4*a^3*b^2 - a*b^4)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(2/(b^3*d^3) + (a^4
 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3) + 4/(b
*d) + 2*(1/2)^(2/3)*(-I*sqrt(3) + 1)/(b^2*d^2*(2/(b^3*d^3) + (a^4 - 2*a^2*b
^2 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3)))*d - 3/4*sqrt(1
/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a
^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3) + 4/(b*d) + 2*(1/2)^(2/3)*(-
I*sqrt(3) + 1)/(b^2*d^2*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3
) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3)))*a^3*b^3*d^2 - 2*(2*a^3*b^2 + a*b^4)*
d)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^
4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3) + 4/(b*d) + 2*(1/2)^(2/
3)*(-I*sqrt(3) + 1)/(b^2*d^2*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^
5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3)))^2*b*d - 8*(1/2)^(1/3)*(I*sqrt(3
) + 1)*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(
a^2*b^5*d^3))^(1/3) - 32/(b*d) - 16*(1/2)^(2/3)*(-I*sqrt(3) + 1)/(b^2*d^2*(
2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*
d^3))^(1/3)))/(b*d)) + 2*(a^4 - b^4)*sin(d*x + c)))/(b*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19711, size = 298, normalized size = 1.36

$$\frac{\frac{3\sin(dx+c)^2}{b} - \frac{4\log\left(\left|b\sin(dx+c)^3+a\right|\right)}{b} + \frac{2\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^2+\left(-ab^2\right)^{\frac{2}{3}}a\right)\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\sin(dx+c)\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab^3} + \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}b^2-\left(-ab^2\right)^{\frac{2}{3}}a\right)\log\left(\sin(dx+c)^2+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right.}{ab^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 1/6*(3*sin(d*x + c)^2/b - 4*log(abs(b*sin(d*x + c)^3 + a))/b + 2*sqrt(3)*((
-a*b^2)^(1/3)*b^2 + (-a*b^2)^(2/3)*a)*arctan(1/3*sqrt(3)*((-a/b)^(1/3) + 2*
sin(d*x + c))/(-a/b)^(1/3))/(a*b^3) + ((-a*b^2)^(1/3)*b^2 - (-a*b^2)^(2/3)*
a)*log(sin(d*x + c)^2 + (-a/b)^(1/3)*sin(d*x + c) + (-a/b)^(2/3))/(a*b^3) +
 2*(a*b^4*(-a/b)^(1/3) - b^5)*(-a/b)^(1/3)*log(abs(-(-a/b)^(1/3) + sin(d*x
+ c)))/(a*b^5))/d
```

3.384 $\int \frac{\cos^3(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=167

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{6a^{2/3}\sqrt[3]{b}d}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}d}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}d}-\frac{\log\left(a+\right.}{}$$

```
[Out] -(ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(
2/3)*b^(1/3)*d)) + Log[a^(1/3) + b^(1/3)*Sin[c + d*x]]/(3*a^(2/3)*b^(1/3)*d
) - Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2]/(6
*a^(2/3)*b^(1/3)*d) - Log[a + b*Sin[c + d*x]^3]/(3*b*d)
```

Rubi [A] time = 0.148356, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3223, 1871, 200, 31, 634, 617, 204, 628, 260}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{6a^{2/3}\sqrt[3]{b}d}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}d}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}d}-\frac{\log\left(a+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] -(ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(
2/3)*b^(1/3)*d)) + Log[a^(1/3) + b^(1/3)*Sin[c + d*x]]/(3*a^(2/3)*b^(1/3)*d
) - Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2]/(6
*a^(2/3)*b^(1/3)*d) - Log[a + b*Sin[c + d*x]^3]/(3*b*d)
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{a+b\sin^3(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1-x^2}{a+bx^3}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{a+bx^3}\,dx, x, \sin(c+dx)\right)}{d} - \frac{\text{Subst}\left(\int \frac{x^2}{a+bx^3}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= -\frac{\log\left(a+b\sin^3(c+dx)\right)}{3bd} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx, x, \sin(c+dx)\right)}{3a^{2/3}d} + \frac{\text{Subst}\left(\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\right.}{3a^{2/3}d}$$

$$= \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}d} - \frac{\log\left(a+b\sin^3(c+dx)\right)}{3bd} + \frac{\text{Subst}\left(\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, \sin(c\right.}{2\sqrt[3]{a}d}$$

$$= \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}d} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{6a^{2/3}\sqrt[3]{b}d} - \frac{\log\left(a+b\,\text{si}\right.}{3b}$$

$$= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}d} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}d} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\,\text{si}\right.}{6a^{2/3}\sqrt[3]{b}d}$$

Mathematica [A] time = 0.135558, size = 139, normalized size = 0.83

$$\frac{\left((-1)^{2/3}b^{2/3}-a^{2/3}\right)\log\left(-(-1)^{2/3}\sqrt[3]{a}-\sqrt[3]{b}\sin(c+dx)\right)+\left(b^{2/3}-a^{2/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)-\left(a^{2/3}+\sqrt[3]{-1}b^{2/3}\right)\log\left(\sqrt[3]{\right.}{3a^{2/3}bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] ((-a^(2/3) + (-1)^(2/3)*b^(2/3))*Log[-((-1)^(2/3)*a^(1/3)) - b^(1/3)*Sin[c
+ d*x]] + (-a^(2/3) + b^(2/3))*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]] - (a^(2/
3) + (-1)^(1/3)*b^(2/3))*Log[a^(1/3) + (-1)^(2/3)*b^(1/3)*Sin[c + d*x]])/(3
*a^(2/3)*b*d)
```

Maple [A] time = 0.099, size = 141, normalized size = 0.8

$$\frac{1}{3\,bd}\ln\left(\sin\left(dx+c\right)+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{1}{6\,bd}\ln\left(\left(\sin\left(dx+c\right)\right)^{2}-\sqrt[3]{\frac{a}{b}}\sin\left(dx+c\right)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{\sqrt{3}}{3\,bd}\arctan\left(\frac{\sqrt{3}}{3}\left(2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*sin(d*x+c)^3),x)
```

```
[Out] 1/3/d/b/(a/b)^(2/3)*ln(sin(d*x+c)+(a/b)^(1/3))-1/6/d/b/(a/b)^(2/3)*ln(sin(d
*x+c)^2-(a/b)^(1/3)*sin(d*x+c)+(a/b)^(2/3))+1/3/d/b/(a/b)^(2/3)*3^(1/2)*arc
tan(1/3*3^(1/2)*(2/(a/b)^(1/3)*sin(d*x+c)-1))-1/3*ln(a+b*sin(d*x+c)^3)/b/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14128, size = 211, normalized size = 1.26

$$-\frac{\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\sin(dx+c)\right|\right)}{a}+\frac{2\log\left(\left|b\sin(dx+c)^{3}+a\right|\right)}{b}-\frac{2\sqrt{3}\left(-ab^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\sin(dx+c)\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab}-\frac{\left(-ab^{2}\right)^{\frac{1}{3}}\log\left(\sin(dx+c)^{2}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx\right.}{ab}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] -1/6*(2*(-a/b)^(1/3)*log(abs(-(-a/b)^(1/3) + sin(d*x + c)))/a + 2*log(abs(b
*sin(d*x + c)^3 + a))/b - 2*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*((-a/
b)^(1/3) + 2*sin(d*x + c))/(-a/b)^(1/3))/(a*b) - (-a*b^2)^(1/3)*log(sin(d*x
 + c)^2 + (-a/b)^(1/3)*sin(d*x + c) + (-a/b)^(2/3))/(a*b))/d
```

3.385 $\int \frac{\cos(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=144

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{6a^{2/3}\sqrt[3]{b}d}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}d}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}d}$$

```
[Out] -(ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(
2/3)*b^(1/3)*d)) + Log[a^(1/3) + b^(1/3)*Sin[c + d*x]]/(3*a^(2/3)*b^(1/3)*d
) - Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2]/(6
*a^(2/3)*b^(1/3)*d)
```

Rubi [A] time = 0.0999976, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3223, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{6a^{2/3}\sqrt[3]{b}d}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}d}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Sin[c + d*x]^3),x]
```


```
[Out] -(ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(
2/3)*b^(1/3)*d)) + Log[a^(1/3) + b^(1/3)*Sin[c + d*x]]/(3*a^(2/3)*b^(1/3)*d
) - Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2]/(6
*a^(2/3)*b^(1/3)*d)
```


Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{a+b\sin^3(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{a+bx^3}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx, x, \sin(c+dx)\right)}{3a^{2/3}d} + \frac{\text{Subst}\left(\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, \sin(c+dx)\right)}{3a^{2/3}d}$$

$$= \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}d} + \frac{\text{Subst}\left(\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, \sin(c+dx)\right)}{2\sqrt[3]{a}d} - \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{a}\ldots}{a^{2/3}-\sqrt[3]{\ldots}}\ldots\right.}{\ldots}$$

$$= \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}d} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{6a^{2/3}\sqrt[3]{b}d} + \frac{\text{Subst}\left(\int \frac{\ldots}{-\ldots}\right.}{\ldots}$$

$$= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}d} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}d} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin\ldots\right.}{6a^{2/3}\sqrt[3]{b}d}$$

Mathematica [A]   time = 0.0560145, size = 116, normalized size = 0.81

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)+2\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{6a^{2/3}\sqrt[3]{b}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] -(2*Sqrt[3]*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1/3))] -
2*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]] + Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c
 + d*x] + b^(2/3)*Sin[c + d*x]^2])/(6*a^(2/3)*b^(1/3)*d)
```

Maple [A] time = 0.047, size = 120, normalized size = 0.8

$$\frac{1}{3\,bd}\ln\left(\sin\left(dx+c\right)+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{1}{6\,bd}\ln\left(\left(\sin\left(dx+c\right)\right)^{2}-\sqrt[3]{\frac{a}{b}}\sin\left(dx+c\right)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{\sqrt{3}}{3\,bd}\arctan\left(\frac{\sqrt{3}}{3}\left(2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*sin(d*x+c)^3),x)
```

```
[Out] 1/3/d/b/(a/b)^(2/3)*ln(sin(d*x+c)+(a/b)^(1/3))-1/6/d/b/(a/b)^(2/3)*ln(sin(d
*x+c)^2-(a/b)^(1/3)*sin(d*x+c)+(a/b)^(2/3))+1/3/d/b/(a/b)^(2/3)*3^(1/2)*arc
tan(1/3*3^(1/2)*(2/(a/b)^(1/3)*sin(d*x+c)-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.8406, size = 1018, normalized size = 7.07

$$\left[\frac{3\sqrt{\frac{1}{3}}ab\sqrt{-\frac{\left(a^{2}b\right)^{\frac{1}{3}}}{b}}\log\left(-\frac{3\left(a^{2}b\right)^{\frac{1}{3}}a\sin(dx+c)+a^{2}+3\sqrt{\frac{1}{3}}\left(2\,ab\cos(dx+c)^{2}-2\,ab-\left(a^{2}b\right)^{\frac{2}{3}}\sin(dx+c)+\left(a^{2}b\right)^{\frac{1}{3}}a\right)\sqrt{-\frac{\left(a^{2}b\right)^{\frac{1}{3}}}{b}}+2\left(ab\cos(dx+c)^{2}-ab\right)\sin(d}{\left(b\cos(dx+c)^{2}-b\right)\sin(dx+c)-a}\right.}{6\,a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*sqrt(-(a^2*b)^(1/3)/b)*log(-(3*(a^2*b)^(1/3)*a*sin(d*
x + c) + a^2 + 3*sqrt(1/3)*(2*a*b*cos(d*x + c)^2 - 2*a*b - (a^2*b)^(2/3)*si
n(d*x + c) + (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b) + 2*(a*b*cos(d*x + c)^
2 - a*b)*sin(d*x + c))/((b*cos(d*x + c)^2 - b)*sin(d*x + c) - a)) - (a^2*b)
^(2/3)*log(-a*b*cos(d*x + c)^2 + a*b - (a^2*b)^(2/3)*sin(d*x + c) + (a^2*b)
^(1/3)*a) + 2*(a^2*b)^(2/3)*log(a*b*sin(d*x + c) + (a^2*b)^(2/3)))/(a^2*b*d
), 1/6*(6*sqrt(1/3)*a*b*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(
2/3)*sin(d*x + c) - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - (a^2*b)^(
2/3)*log(-a*b*cos(d*x + c)^2 + a*b - (a^2*b)^(2/3)*sin(d*x + c) + (a^2*b)^(
1/3)*a) + 2*(a^2*b)^(2/3)*log(a*b*sin(d*x + c) + (a^2*b)^(2/3)))/(a^2*b*d)]
```

Sympy [A] time = 15.9635, size = 260, normalized size = 1.81

$$\begin{cases} \frac{\tilde{\infty} x \cos{(c)}}{\sin^{3}{(c)}} \\ \frac{\sin{(c+dx)}}{ad} \\ -\frac{1}{2bd\sin^{2}{(c+dx)}} \\ \frac{x\cos{(c)}}{a+b\sin^{3}{(c)}} \\ -\frac{\sqrt[3]{-1}\log{\left(-\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}+\sin{(c+dx)}\right)}}{3a^{\frac{2}{3}}b^{7}d\left(\frac{1}{b}\right)^{\frac{20}{3}}} + \frac{\sqrt[3]{-1}\log{\left(4(-1)^{\frac{2}{3}}a^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}+4\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}\sin{(c+dx)}+4\sin^{2}{(c+dx)}\right)}}{6a^{\frac{2}{3}}b^{7}d\left(\frac{1}{b}\right)^{\frac{20}{3}}} + \frac{\sqrt[3]{-1}\sqrt{3}\operatorname{atan}{\left(\frac{\sqrt{3}}{3}-\frac{2(-1)^{\frac{2}{3}}\sqrt{3}\sin{(c+dx)}}{3\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}}\right)}}{3a^{\frac{2}{3}}b^{7}d\left(\frac{1}{b}\right)^{\frac{20}{3}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Piecewise((zoo*x*cos(c)/sin(c)**3, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (sin(c
+ d*x)/(a*d), Eq(b, 0)), (-1/(2*b*d*sin(c + d*x)**2), Eq(a, 0)), (x*cos(c)/
(a + b*sin(c)**3), Eq(d, 0)), (-(-1)**(1/3)*log(-(-1)**(1/3)*a**(1/3)*(1/b)
**(1/3) + sin(c + d*x))/(3*a**(2/3)*b**7*d*(1/b)**(20/3)) + (-1)**(1/3)*log
(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1/3)*
sin(c + d*x) + 4*sin(c + d*x)**2)/(6*a**(2/3)*b**7*d*(1/b)**(20/3)) + (-1)*
*(1/3)*sqrt(3)*atan(sqrt(3)/3 - 2*(-1)**(2/3)*sqrt(3)*sin(c + d*x)/(3*a**(1
/3)*(1/b)**(1/3)))/(3*a**(2/3)*b**7*d*(1/b)**(20/3)), True))
```

Giac [A] time = 1.13095, size = 185, normalized size = 1.28

$$-\frac{\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\sin(dx+c)\right|\right)}{a} - \frac{2\sqrt{3}\left(-ab^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\sin(dx+c)\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab} - \frac{\left(-ab^{2}\right)^{\frac{1}{3}}\log\left(\sin(dx+c)^{2}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{ab}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] -1/6*(2*(-a/b)^(1/3)*log(abs(-(-a/b)^(1/3) + sin(d*x + c)))/a - 2*sqrt(3)*(
-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*((-a/b)^(1/3) + 2*sin(d*x + c))/(-a/b)^(1/
3))/(a*b) - (-a*b^2)^(1/3)*log(sin(d*x + c)^2 + (-a/b)^(1/3)*sin(d*x + c) +
 (-a/b)^(2/3))/(a*b))/d
```

3.386 $\int \frac{\sec(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=290

$$\frac{\sqrt[3]{b}\left(a^{4/3}+b^{4/3}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{6a^{2/3}d\left(a^2-b^2\right)}-\frac{b\log\left(a+b\sin^3(c+dx)\right)}{3d\left(a^2-b^2\right)}-\frac{\sqrt[3]{b}\left(a^{4/3}+b^{4/3}\right)\log\left(\cdots\right.}{3a^{2/3}d\left(\cdots\right.}$$

```
[Out] -((b^(1/3)*(a^(4/3) - b^(4/3))*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(S
qrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*(a^2 - b^2)*d)) - Log[1 - Sin[c + d*x]]/
(2*(a + b)*d) + Log[1 + Sin[c + d*x]]/(2*(a - b)*d) - (b^(1/3)*(a^(4/3) + b
^(4/3))*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]])/(3*a^(2/3)*(a^2 - b^2)*d) + (b
^(1/3)*(a^(4/3) + b^(4/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(
2/3)*Sin[c + d*x]^2])/(6*a^(2/3)*(a^2 - b^2)*d) - (b*Log[a + b*Sin[c + d*x]
^3])/(3*(a^2 - b^2)*d)
```

Rubi [A] time = 0.329331, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {3223, 2074, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\sqrt[3]{b}\left(a^{4/3}+b^{4/3}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{6a^{2/3}d\left(a^2-b^2\right)}-\frac{b\log\left(a+b\sin^3(c+dx)\right)}{3d\left(a^2-b^2\right)}-\frac{\sqrt[3]{b}\left(a^{4/3}+b^{4/3}\right)\log\left(\cdots\right.}{3a^{2/3}d\left(\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sin[c + d*x]^3),x]
```


```
[Out] -((b^(1/3)*(a^(4/3) - b^(4/3))*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(S
qrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*(a^2 - b^2)*d)) - Log[1 - Sin[c + d*x]]/
(2*(a + b)*d) + Log[1 + Sin[c + d*x]]/(2*(a - b)*d) - (b^(1/3)*(a^(4/3) + b
^(4/3))*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]])/(3*a^(2/3)*(a^2 - b^2)*d) + (b
^(1/3)*(a^(4/3) + b^(4/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(
2/3)*Sin[c + d*x]^2])/(6*a^(2/3)*(a^2 - b^2)*d) - (b*Log[a + b*Sin[c + d*x]
^3])/(3*(a^2 - b^2)*d)
```


Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
```

```
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{a+b\sin^3(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a+bx^3)}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{2(a+b)(-1+x)} + \frac{1}{2(a-b)(1+x)} + \frac{b(b-ax+bx^2)}{(-a^2+b^2)(a+bx^3)}\right) dx, x, \sin(c+dx)\right)}{d}$$

$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)d} + \frac{\log(1+\sin(c+dx))}{2(a-b)d} - \frac{b\,\text{Subst}\left(\int \frac{b-ax+bx^2}{a+bx^3}\,dx, x, \sin(c+dx)\right)}{(a^2-b^2)\,d}$$

$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)d} + \frac{\log(1+\sin(c+dx))}{2(a-b)d} - \frac{b\,\text{Subst}\left(\int \frac{b-ax}{a+bx^3}\,dx, x, \sin(c+dx)\right)}{(a^2-b^2)\,d} - \frac{b^2\,\text{S}}{}$$

$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)d} + \frac{\log(1+\sin(c+dx))}{2(a-b)d} - \frac{b\log\left(a+b\sin^3(c+dx)\right)}{3(a^2-b^2)\,d} - \frac{b^{2/3}\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)d} + \frac{\log(1+\sin(c+dx))}{2(a-b)d} - \frac{\sqrt[3]{b}\left(a^{4/3}+b^{4/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}(a^2-b^2)\,d}$$

$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)d} + \frac{\log(1+\sin(c+dx))}{2(a-b)d} - \frac{\sqrt[3]{b}\left(a^{4/3}+b^{4/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}(a^2-b^2)\,d}$$

$$= -\frac{\sqrt[3]{b}\left(a^{4/3}-b^{4/3}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{2/3}(a^2-b^2)\,d} - \frac{\log(1-\sin(c+dx))}{2(a+b)d} + \frac{\log(1+\sin(c+dx))}{2(a-b)d} - \frac{\sqrt[3]{b}}{}$$

Mathematica [C] time = 0.213135, size = 268, normalized size = 0.92

$$\frac{b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)+3a^{2/3}b\sin^2(c+dx)\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{b\sin^3(c+dx)}{a}\right)-2a^{2/3}b\log\left(a+b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (2*Sqrt[3]*b^(5/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1/
3))] - 3*a^(5/3)*Log[1 - Sin[c + d*x]] + 3*a^(2/3)*b*Log[1 - Sin[c + d*x]]
+ 3*a^(5/3)*Log[1 + Sin[c + d*x]] + 3*a^(2/3)*b*Log[1 + Sin[c + d*x]] - 2*b
^(5/3)*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]] + b^(5/3)*Log[a^(2/3) - a^(1/3)*
b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2] - 2*a^(2/3)*b*Log[a + b*Sin[
c + d*x]^3] + 3*a^(2/3)*b*Hypergeometric2F1[2/3, 1, 5/3, -((b*Sin[c + d*x]^
3)/a)]*Sin[c + d*x]^2)/(6*a^(2/3)*(a - b)*(a + b)*d)
```

Maple [A] time = 0.128, size = 374, normalized size = 1.3

$$-\frac{\ln(\sin(dx+c)-1)}{d(2a+2b)} - \frac{b}{3d(a-b)(a+b)}\ln\left(\sin(dx+c)+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{b}{6d(a-b)(a+b)}\ln\left((\sin(dx+c))^2-\sqrt[3]{\frac{a}{b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sin(d*x+c)^3),x)
```

```
[Out] -1/d/(2*a+2*b)*ln(sin(d*x+c)-1)-1/3/d*b/(a-b)/(a+b)/(a/b)^(2/3)*ln(sin(d*x+
c)+(a/b)^(1/3))+1/6/d*b/(a-b)/(a+b)/(a/b)^(2/3)*ln(sin(d*x+c)^2-(a/b)^(1/3)
*sin(d*x+c)+(a/b)^(2/3))-1/3/d*b/(a-b)/(a+b)/(a/b)^(2/3)*3^(1/2)*arctan(1/3
*3^(1/2)*(2/(a/b)^(1/3)*sin(d*x+c)-1))-1/3/d/(a-b)/(a+b)*a/(a/b)^(1/3)*ln(s
in(d*x+c)+(a/b)^(1/3))+1/6/d/(a-b)/(a+b)*a/(a/b)^(1/3)*ln(sin(d*x+c)^2-(a/b
)^(1/3)*sin(d*x+c)+(a/b)^(2/3))+1/3/d/(a-b)/(a+b)*a*3^(1/2)/(a/b)^(1/3)*arc
tan(1/3*3^(1/2)*(2/(a/b)^(1/3)*sin(d*x+c)-1))-1/3/d*b/(a-b)/(a+b)*ln(a+b*si
n(d*x+c)^3)+1/d/(2*a-2*b)*ln(1+sin(d*x+c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 14.5048, size = 9420, normalized size = 32.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/36*(2*(a^2 - b^2)*(9*(I*sqrt(3) + 1)*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) -
1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1
/3) + b^2*(-I*sqrt(3) + 1)/((a^2*d - b^2*d)^2*(-1/54*b/(a^4*d^3 - a^2*b^2*d
^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^
3))^(1/3)) + 6*b/(a^2*d - b^2*d))*d*log(-1/36*(a^5 - a^3*b^2)*(9*(I*sqrt(3)
 + 1)*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*
(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3) + b^2*(-I*sqrt(3) + 1)/((a^2*d
 - b^2*d)^2*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 +
 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3)) + 6*b/(a^2*d - b^2*d))^
2*d^2 + a*b^2 + 1/6*(2*a^3*b + a*b^3)*(9*(I*sqrt(3) + 1)*(-1/54*b/(a^4*d^3
- a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^
2)^2*a^2*d^3))^(1/3) + b^2*(-I*sqrt(3) + 1)/((a^2*d - b^2*d)^2*(-1/54*b/(a^
4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^
2 - b^2)^2*a^2*d^3))^(1/3)) + 6*b/(a^2*d - b^2*d))*d - (a^2*b + b^3)*sin(d*
x + c)) - ((a^2 - b^2)*(9*(I*sqrt(3) + 1)*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3)
- 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^
(1/3) + b^2*(-I*sqrt(3) + 1)/((a^2*d - b^2*d)^2*(-1/54*b/(a^4*d^3 - a^2*b^2
*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*
d^3))^(1/3)) + 6*b/(a^2*d - b^2*d))*d - 3*sqrt(1/3)*(a^2 - b^2)*d*sqrt(-((a
^4 - 2*a^2*b^2 + b^4)*(9*(I*sqrt(3) + 1)*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) -
 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(
1/3) + b^2*(-I*sqrt(3) + 1)/((a^2*d - b^2*d)^2*(-1/54*b/(a^4*d^3 - a^2*b^2*
d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d
^3))^(1/3)) + 6*b/(a^2*d - b^2*d))^2*d^2 - 12*(a^2*b - b^3)*(9*(I*sqrt(3) +
 1)*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a
```

```
^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3) + b^2*(-I*sqrt(3) + 1)/((a^2*d -
 b^2*d)^2*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1
/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3)) + 6*b/(a^2*d - b^2*d))*d
- 108*b^2)/((a^4 - 2*a^2*b^2 + b^4)*d^2)) - 18*b)*log(1/36*(a^5 - a^3*b^2)*
(9*(I*sqrt(3) + 1)*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2
*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3) + b^2*(-I*sqrt(3)
 + 1)/((a^2*d - b^2*d)^2*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d
 - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3)) + 6*b/(a^2
*d - b^2*d))^2*d^2 - a*b^2 - 1/6*(2*a^3*b + a*b^3)*(9*(I*sqrt(3) + 1)*(-1/5
4*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)
*b/((a^2 - b^2)^2*a^2*d^3))^(1/3) + b^2*(-I*sqrt(3) + 1)/((a^2*d - b^2*d)^2
*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2
+ b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3)) + 6*b/(a^2*d - b^2*d))*d + 1/12*sq
rt(1/3)*((a^5 - a^3*b^2)*(9*(I*sqrt(3) + 1)*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3
) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3)
)^(1/3) + b^2*(-I*sqrt(3) + 1)/((a^2*d - b^2*d)^2*(-1/54*b/(a^4*d^3 - a^2*b
^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^
2*d^3))^(1/3)) + 6*b/(a^2*d - b^2*d))*d^2 - 6*(a^3*b - a*b^3)*d)*sqrt(-((a^
4 - 2*a^2*b^2 + b^4)*(9*(I*sqrt(3) + 1)*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) -
1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1
/3) + b^2*(-I*sqrt(3) + 1)/((a^2*d - b^2*d)^2*(-1/54*b/(a^4*d^3 - a^2*b^2*d
^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^
3))^(1/3)) + 6*b/(a^2*d - b^2*d))^2*d^2 - 12*(a^2*b - b^3)*(9*(I*sqrt(3) +
1)*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^
2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3) + b^2*(-I*sqrt(3) + 1)/((a^2*d -
b^2*d)^2*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/
54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3)) + 6*b/(a^2*d - b^2*d))*d -
 108*b^2)/((a^4 - 2*a^2*b^2 + b^4)*d^2)) - 2*(a^2*b + b^3)*sin(d*x + c)) -
((a^2 - b^2)*(9*(I*sqrt(3) + 1)*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3
/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3) + b^
2*(-I*sqrt(3) + 1)/((a^2*d - b^2*d)^2*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/
27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3
)) + 6*b/(a^2*d - b^2*d))*d + 3*sqrt(1/3)*(a^2 - b^2)*d*sqrt(-((a^4 - 2*a^2
*b^2 + b^4)*(9*(I*sqrt(3) + 1)*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/
(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3) + b^2
*(-I*sqrt(3) + 1)/((a^2*d - b^2*d)^2*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/2
7*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3)
) + 6*b/(a^2*d - b^2*d))^2*d^2 - 12*(a^2*b - b^3)*(9*(I*sqrt(3) + 1)*(-1/54
*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*
b/((a^2 - b^2)^2*a^2*d^3))^(1/3) + b^2*(-I*sqrt(3) + 1)/((a^2*d - b^2*d)^2*
(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 +
 b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3)) + 6*b/(a^2*d - b^2*d))*d - 108*b^2)
/((a^4 - 2*a^2*b^2 + b^4)*d^2)) - 18*b)*log(-1/36*(a^5 - a^3*b^2)*(9*(I*sqr
t(3) + 1)*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1
/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3) + b^2*(-I*sqrt(3) + 1)/((a
^2*d - b^2*d)^2*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)
^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3)) + 6*b/(a^2*d - b^2*
d))^2*d^2 + a*b^2 + 1/6*(2*a^3*b + a*b^3)*(9*(I*sqrt(3) + 1)*(-1/54*b/(a^4*
d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2
- b^2)^2*a^2*d^3))^(1/3) + b^2*(-I*sqrt(3) + 1)/((a^2*d - b^2*d)^2*(-1/54*b
/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/
((a^2 - b^2)^2*a^2*d^3))^(1/3)) + 6*b/(a^2*d - b^2*d))*d + 1/12*sqrt(1/3)*(
(a^5 - a^3*b^2)*(9*(I*sqrt(3) + 1)*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*
b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3) +
 b^2*(-I*sqrt(3) + 1)/((a^2*d - b^2*d)^2*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) -
 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(
1/3)) + 6*b/(a^2*d - b^2*d))*d^2 - 6*(a^3*b - a*b^3)*d)*sqrt(-((a^4 - 2*a^2
*b^2 + b^4)*(9*(I*sqrt(3) + 1)*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/
(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3) + b^2
```

```
*(-I*sqrt(3) + 1)/((a^2*d - b^2*d)^2*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/2
7*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3)
) + 6*b/(a^2*d - b^2*d))^2*d^2 - 12*(a^2*b - b^3)*(9*(I*sqrt(3) + 1)*(-1/54
*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*
b/((a^2 - b^2)^2*a^2*d^3))^(1/3) + b^2*(-I*sqrt(3) + 1)/((a^2*d - b^2*d)^2*
(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 +
 b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3)) + 6*b/(a^2*d - b^2*d))*d - 108*b^2)
/((a^4 - 2*a^2*b^2 + b^4)*d^2)) + 2*(a^2*b + b^3)*sin(d*x + c)) - 18*(a + b
)*log(sin(d*x + c) + 1) + 18*(a - b)*log(-sin(d*x + c) + 1))/((a^2 - b^2)*d
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18017, size = 417, normalized size = 1.44

$$-\frac{\frac{2\left(a^3b^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^4\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^2b^3+b^5\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\sin(dx+c)\right|\right)}{a^5b-2a^3b^3+ab^5}+\frac{6\left(\left(-ab^2\right)^{\frac{1}{3}}b^2+\left(-ab^2\right)^{\frac{2}{3}}a\right)\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\sin(dx+c)\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}a^3b-\sqrt{3}ab^3}+\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}b^2-\left(-ab^2\right)\cdots}{}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] -1/6*(2*(a^3*b^2*(-a/b)^(1/3) - a*b^4*(-a/b)^(1/3) - a^2*b^3 + b^5)*(-a/b)^
(1/3)*log(abs(-(-a/b)^(1/3) + sin(d*x + c)))/(a^5*b - 2*a^3*b^3 + a*b^5) +
6*((-a*b^2)^(1/3)*b^2 + (-a*b^2)^(2/3)*a)*arctan(1/3*sqrt(3)*((-a/b)^(1/3)
+ 2*sin(d*x + c))/(-a/b)^(1/3))/(sqrt(3)*a^3*b - sqrt(3)*a*b^3) + ((-a*b^2)
^(1/3)*b^2 - (-a*b^2)^(2/3)*a)*log(sin(d*x + c)^2 + (-a/b)^(1/3)*sin(d*x +
c) + (-a/b)^(2/3))/(a^3*b - a*b^3) + 2*b*log(abs(b*sin(d*x + c)^3 + a))/(a^
2 - b^2) - 3*log(abs(sin(d*x + c) + 1))/(a - b) + 3*log(abs(sin(d*x + c) -
1))/(a + b))/d
```

3.387 $\int \frac{\sec^3(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=385

$$-\frac{b^{5/3}\left(3a^{4/3}b^{2/3}+2a^2+b^2\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{6a^{2/3}d\left(a^2-b^2\right)^2}+\frac{b\left(a^2+2b^2\right)\log\left(a+b\sin^3(c+dx)\right)}{3d\left(a^2-b^2\right)^2}+$$

```
[Out] -((b^(5/3)*(2*a^2 - 3*a^(4/3)*b^(2/3) + b^2)*ArcTan[(a^(1/3) - 2*b^(1/3)*Si
n[c + d*x])/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*(a^2 - b^2)^2*d)) - ((a +
4*b)*Log[1 - Sin[c + d*x]])/(4*(a + b)^2*d) + ((a - 4*b)*Log[1 + Sin[c + d*
x]])/(4*(a - b)^2*d) + (b^(5/3)*(2*a^2 + 3*a^(4/3)*b^(2/3) + b^2)*Log[a^(1/
3) + b^(1/3)*Sin[c + d*x]])/(3*a^(2/3)*(a^2 - b^2)^2*d) - (b^(5/3)*(2*a^2 +
 3*a^(4/3)*b^(2/3) + b^2)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2
/3)*Sin[c + d*x]^2])/(6*a^(2/3)*(a^2 - b^2)^2*d) + (b*(a^2 + 2*b^2)*Log[a +
 b*Sin[c + d*x]^3])/(3*(a^2 - b^2)^2*d) + 1/(4*(a + b)*d*(1 - Sin[c + d*x])
) - 1/(4*(a - b)*d*(1 + Sin[c + d*x]))
```

Rubi [A] time = 0.504331, antiderivative size = 385, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {3223, 2074, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{b^{5/3}\left(3a^{4/3}b^{2/3}+2a^2+b^2\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{6a^{2/3}d\left(a^2-b^2\right)^2}+\frac{b\left(a^2+2b^2\right)\log\left(a+b\sin^3(c+dx)\right)}{3d\left(a^2-b^2\right)^2}+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Sin[c + d*x]^3),x]
```


```
[Out] -((b^(5/3)*(2*a^2 - 3*a^(4/3)*b^(2/3) + b^2)*ArcTan[(a^(1/3) - 2*b^(1/3)*Si
n[c + d*x])/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*(a^2 - b^2)^2*d)) - ((a +
4*b)*Log[1 - Sin[c + d*x]])/(4*(a + b)^2*d) + ((a - 4*b)*Log[1 + Sin[c + d*
x]])/(4*(a - b)^2*d) + (b^(5/3)*(2*a^2 + 3*a^(4/3)*b^(2/3) + b^2)*Log[a^(1/
3) + b^(1/3)*Sin[c + d*x]])/(3*a^(2/3)*(a^2 - b^2)^2*d) - (b^(5/3)*(2*a^2 +
 3*a^(4/3)*b^(2/3) + b^2)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2
/3)*Sin[c + d*x]^2])/(6*a^(2/3)*(a^2 - b^2)^2*d) + (b*(a^2 + 2*b^2)*Log[a +
 b*Sin[c + d*x]^3])/(3*(a^2 - b^2)^2*d) + 1/(4*(a + b)*d*(1 - Sin[c + d*x])
) - 1/(4*(a - b)*d*(1 + Sin[c + d*x]))
```


Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{a+b\sin^3(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2(a+bx^3)}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{4(a+b)(-1+x)^2} + \frac{-a-4b}{4(a+b)^2(-1+x)} + \frac{1}{4(a-b)(1+x)^2} + \frac{a-4b}{4(a-b)^2(1+x)} + \frac{b^2(2a^2+b^2-3abx+(a^2+2b^2)x^2)}{(a^2-b^2)^2(a+bx^3)}\right.\right.}{d}$$

$$= -\frac{(a+4b)\log(1-\sin(c+dx))}{4(a+b)^2d} + \frac{(a-4b)\log(1+\sin(c+dx))}{4(a-b)^2d} + \frac{1}{4(a+b)d(1-\sin(c+dx))}$$

$$= -\frac{(a+4b)\log(1-\sin(c+dx))}{4(a+b)^2d} + \frac{(a-4b)\log(1+\sin(c+dx))}{4(a-b)^2d} + \frac{1}{4(a+b)d(1-\sin(c+dx))}$$

$$= -\frac{(a+4b)\log(1-\sin(c+dx))}{4(a+b)^2d} + \frac{(a-4b)\log(1+\sin(c+dx))}{4(a-b)^2d} + \frac{b\left(a^2+2b^2\right)\log\left(a+b\sin\right.}{3\left(a^2-b^2\right)^2 d}$$

$$= -\frac{(a+4b)\log(1-\sin(c+dx))}{4(a+b)^2d} + \frac{(a-4b)\log(1+\sin(c+dx))}{4(a-b)^2d} + \frac{b^{5/3}\left(2a^2+3a^{4/3}b^{2/3}+b^2\right)}{3a^{2/3}\left(a\right.}$$

$$= -\frac{(a+4b)\log(1-\sin(c+dx))}{4(a+b)^2d} + \frac{(a-4b)\log(1+\sin(c+dx))}{4(a-b)^2d} + \frac{b^{5/3}\left(2a^2+3a^{4/3}b^{2/3}+b^2\right)}{3a^{2/3}\left(a\right.}$$

$$= -\frac{b^{5/3}\left(2a^2-3a^{4/3}b^{2/3}+b^2\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{2/3}\left(a^2-b^2\right)^2 d} - \frac{(a+4b)\log(1-\sin(c+dx))}{4(a+b)^2d} + \frac{(a-4b}{}$$

Mathematica [C] time = 2.32315, size = 333, normalized size = 0.86

$$-\frac{\frac{18b^3\sin^2(c+dx)\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{b\sin^3(c+dx)}{a}\right)}{(a^2-b^2)^2} - \frac{4b(a^2+2b^2)\log(a+b\sin^3(c+dx))}{(a^2-b^2)^2} - \frac{4b^{5/3}(2a^2+b^2)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{a^{2/3}(a^2-b^2)^2} + \frac{2b^{5/3}(2a^2+b^2)\left(\log\left(a^{2/3}-\sqrt[3]{a}\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] -((3*(a + 4*b)*Log[1 - Sin[c + d*x]])/(a + b)^2 - (3*(a - 4*b)*Log[1 + Sin[
c + d*x]])/(a - b)^2 - (4*b^(5/3)*(2*a^2 + b^2)*Log[a^(1/3) + b^(1/3)*Sin[c
 + d*x]])/(a^(2/3)*(a^2 - b^2)^2) + (2*b^(5/3)*(2*a^2 + b^2)*(2*Sqrt[3]*Arc
Tan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1/3))] + Log[a^(2/3) - a
^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2]))/(a^(2/3)*(a^2 - b^2
)^2) - (4*b*(a^2 + 2*b^2)*Log[a + b*Sin[c + d*x]^3])/(a^2 - b^2)^2 + 3/((a
+ b)*(-1 + Sin[c + d*x])) + (18*b^3*Hypergeometric2F1[2/3, 1, 5/3, -((b*Sin
[c + d*x]^3)/a)]*Sin[c + d*x]^2)/(a^2 - b^2)^2 + 3/((a - b)*(1 + Sin[c + d*
x])))/(12*d)
```

Maple [B] time = 0.146, size = 668, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*sin(d*x+c)^3),x)
```

```
[Out] -1/d/(4*a+4*b)/(sin(d*x+c)-1)-1/4/d/(a+b)^2*ln(sin(d*x+c)-1)*a-1/d/(a+b)^2*
ln(sin(d*x+c)-1)*b+2/3/d*b/(a-b)^2/(a+b)^2/(a/b)^(2/3)*ln(sin(d*x+c)+(a/b)^
(1/3))*a^2+1/3/d*b^3/(a-b)^2/(a+b)^2/(a/b)^(2/3)*ln(sin(d*x+c)+(a/b)^(1/3))
-1/3/d*b/(a-b)^2/(a+b)^2/(a/b)^(2/3)*ln(sin(d*x+c)^2-(a/b)^(1/3)*sin(d*x+c)
+(a/b)^(2/3))*a^2-1/6/d*b^3/(a-b)^2/(a+b)^2/(a/b)^(2/3)*ln(sin(d*x+c)^2-(a/
b)^(1/3)*sin(d*x+c)+(a/b)^(2/3))+2/3/d*b/(a-b)^2/(a+b)^2/(a/b)^(2/3)*3^(1/2
)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*sin(d*x+c)-1))*a^2+1/3/d*b^3/(a-b)^2/(a
+b)^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*sin(d*x+c)-1))+
1/d*b^2/(a-b)^2/(a+b)^2*a/(a/b)^(1/3)*ln(sin(d*x+c)+(a/b)^(1/3))-1/2/d*b^2/
(a-b)^2/(a+b)^2*a/(a/b)^(1/3)*ln(sin(d*x+c)^2-(a/b)^(1/3)*sin(d*x+c)+(a/b)^
(2/3))-1/d*b^2/(a-b)^2/(a+b)^2*a*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/
(a/b)^(1/3)*sin(d*x+c)-1))+1/3/d*b/(a-b)^2/(a+b)^2*ln(a+b*sin(d*x+c)^3)*a^2
+2/3/d*b^3/(a-b)^2/(a+b)^2*ln(a+b*sin(d*x+c)^3)-1/d/(4*a-4*b)/(1+sin(d*x+c)
)-1/d/(a-b)^2*ln(1+sin(d*x+c))*b+1/4/d/(a-b)^2*ln(1+sin(d*x+c))*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 25.194, size = 20282, normalized size = 52.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/12*(2*(a^4 - 2*a^2*b^2 + b^4)*(2*(1/2)^(2/3)*(b^2/(a^4*d^2 - 2*a^2*b^2*d^
2 + b^4*d^2) - (a^2*b + 2*b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2)*(-I*sqrt(
3) + 1)/(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^
2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 2*(
a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^5/((a^2
- b^2)^4*a^2*d^3))^(1/3) - (1/2)^(1/3)*(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*
b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*
d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*
d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3)*(I*sqrt(3) + 1) + 2
*(a^2*b + 2*b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))*d*cos(d*x + c)^2*log(7*a^3*
b^2 + 2*a*b^4 + 3/4*(a^7 - 2*a^5*b^2 + a^3*b^4)*(2*(1/2)^(2/3)*(b^2/(a^4*d^
2 - 2*a^2*b^2*d^2 + b^4*d^2) - (a^2*b + 2*b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4
*d)^2)*(-I*sqrt(3) + 1)/(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a
^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d
 + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2 +
 b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3) - (1/2)^(1/3)*(b^3/(a^6*d^3 - 2*a^
4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2
```

```
+ b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*
a^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3)*(I*
sqrt(3) + 1) + 2*(a^2*b + 2*b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))^2*d^2 - 1/2
*(10*a^5*b + 16*a^3*b^3 + a*b^5)*(2*(1/2)^(2/3)*(b^2/(a^4*d^2 - 2*a^2*b^2*d
^2 + b^4*d^2) - (a^2*b + 2*b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2)*(-I*sqrt
(3) + 1)/(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b
^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 2*
(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^5/((a^2
 - b^2)^4*a^2*d^3))^(1/3) - (1/2)^(1/3)*(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2
*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4
*d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4
*d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3)*(I*sqrt(3) + 1) +
2*(a^2*b + 2*b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))*d - (8*a^2*b^3 + b^5)*sin(
d*x + c)) + 3*(a^3 - 2*a^2*b - 7*a*b^2 - 4*b^3)*cos(d*x + c)^2*log(sin(d*x
+ c) + 1) - 3*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*cos(d*x + c)^2*log(-sin(d*x
 + c) + 1) - 6*a^2*b + 6*b^3 - ((a^4 - 2*a^2*b^2 + b^4)*(2*(1/2)^(2/3)*(b^2
/(a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2) - (a^2*b + 2*b^3)^2/(a^4*d - 2*a^2*b^2
*d + b^4*d)^2)*(-I*sqrt(3) + 1)/(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3
) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a
^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 +
(8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3) - (1/2)^(1/3)*(b^3/(a^6*d^
3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*
b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^
4*d - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(
1/3)*(I*sqrt(3) + 1) + 2*(a^2*b + 2*b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))*d*c
os(d*x + c)^2 - 3*sqrt(1/3)*(a^4 - 2*a^2*b^2 + b^4)*d*sqrt(-(4*a^4*b^2 - 80
*a^2*b^4 - 32*b^6 + (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(2*(1/2
)^(2/3)*(b^2/(a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2) - (a^2*b + 2*b^3)^2/(a^4*d
 - 2*a^2*b^2*d + b^4*d)^2)*(-I*sqrt(3) + 1)/(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 +
 a^2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*
(a^4*d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d +
 b^4*d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3) - (1/2)^(1/3)*
(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*
d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b +
2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4
*a^2*d^3))^(1/3)*(I*sqrt(3) + 1) + 2*(a^2*b + 2*b^3)/(a^4*d - 2*a^2*b^2*d +
 b^4*d))^2*d^2 - 4*(a^6*b - 3*a^2*b^5 + 2*b^7)*(2*(1/2)^(2/3)*(b^2/(a^4*d^2
 - 2*a^2*b^2*d^2 + b^4*d^2) - (a^2*b + 2*b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*
d)^2)*(-I*sqrt(3) + 1)/(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^
2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d
+ b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2 +
b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3) - (1/2)^(1/3)*(b^3/(a^6*d^3 - 2*a^4
*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 +
 b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a
^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3)*(I*s
qrt(3) + 1) + 2*(a^2*b + 2*b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))*d)/((a^8 - 4
*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*d^2))*cos(d*x + c)^2 - 6*(a^2*b + 2
*b^3)*cos(d*x + c)^2)*log(7*a^3*b^2 + 2*a*b^4 + 3/4*(a^7 - 2*a^5*b^2 + a^3*
b^4)*(2*(1/2)^(2/3)*(b^2/(a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2) - (a^2*b + 2*b
^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2)*(-I*sqrt(3) + 1)/(b^3/(a^6*d^3 - 2*a
^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2
 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2
*a^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3) -
(1/2)^(1/3)*(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2*b^3
)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) +
 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^5/((
a^2 - b^2)^4*a^2*d^3))^(1/3)*(I*sqrt(3) + 1) + 2*(a^2*b + 2*b^3)/(a^4*d - 2
*a^2*b^2*d + b^4*d))^2*d^2 - 1/2*(10*a^5*b + 16*a^3*b^3 + a*b^5)*(2*(1/2)^(
2/3)*(b^2/(a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2) - (a^2*b + 2*b^3)^2/(a^4*d -
```

```
2*a^2*b^2*d + b^4*d)^2)*(-I*sqrt(3) + 1)/(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^
2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^
4*d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^
4*d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3) - (1/2)^(1/3)*(b^
3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2
 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b
^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^
2*d^3))^(1/3)*(I*sqrt(3) + 1) + 2*(a^2*b + 2*b^3)/(a^4*d - 2*a^2*b^2*d + b^
4*d))*d + 3/4*sqrt(1/3)*(3*(a^7 - 2*a^5*b^2 + a^3*b^4)*(2*(1/2)^(2/3)*(b^2/
(a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2) - (a^2*b + 2*b^3)^2/(a^4*d - 2*a^2*b^2*
d + b^4*d)^2)*(-I*sqrt(3) + 1)/(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3)
 - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^
2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + (
8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3) - (1/2)^(1/3)*(b^3/(a^6*d^3
 - 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b
^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4
*d - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1
/3)*(I*sqrt(3) + 1) + 2*(a^2*b + 2*b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))*d^2
 + 2*(a^5*b - 2*a^3*b^3 + a*b^5)*d)*sqrt(-(4*a^4*b^2 - 80*a^2*b^4 - 32*b^6 +
 (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(2*(1/2)^(2/3)*(b^2/(a^4*d
^2 - 2*a^2*b^2*d^2 + b^4*d^2) - (a^2*b + 2*b^3)^2/(a^4*d - 2*a^2*b^2*d + b^
4*d)^2)*(-I*sqrt(3) + 1)/(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(
a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*
d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2
 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3) - (1/2)^(1/3)*(b^3/(a^6*d^3 - 2*a
^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2
 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2
*a^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3)*(I
*sqrt(3) + 1) + 2*(a^2*b + 2*b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))^2*d^2 - 4*
(a^6*b - 3*a^2*b^5 + 2*b^7)*(2*(1/2)^(2/3)*(b^2/(a^4*d^2 - 2*a^2*b^2*d^2 +
b^4*d^2) - (a^2*b + 2*b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2)*(-I*sqrt(3) +
 1)/(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((
a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*
b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^
2)^4*a^2*d^3))^(1/3) - (1/2)^(1/3)*(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*
d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d -
2*a^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3
 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3)*(I*sqrt(3) + 1) + 2*(a^
2*b + 2*b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))*d)/((a^8 - 4*a^6*b^2 + 6*a^4*b^
4 - 4*a^2*b^6 + b^8)*d^2)) + 2*(8*a^2*b^3 + b^5)*sin(d*x + c)) - ((a^4 - 2*
a^2*b^2 + b^4)*(2*(1/2)^(2/3)*(b^2/(a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2) - (a
^2*b + 2*b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2)*(-I*sqrt(3) + 1)/(b^3/(a^6
*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a
^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/
(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3)
)^(1/3) - (1/2)^(1/3)*(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2
*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d +
 b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2 + b
^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3)*(I*sqrt(3) + 1) + 2*(a^2*b + 2*b^3)/
(a^4*d - 2*a^2*b^2*d + b^4*d))*d*cos(d*x + c)^2 + 3*sqrt(1/3)*(a^4 - 2*a^2*
b^2 + b^4)*d*sqrt(-(4*a^4*b^2 - 80*a^2*b^4 - 32*b^6 + (a^8 - 4*a^6*b^2 + 6*
a^4*b^4 - 4*a^2*b^6 + b^8)*(2*(1/2)^(2/3)*(b^2/(a^4*d^2 - 2*a^2*b^2*d^2 + b
^4*d^2) - (a^2*b + 2*b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2)*(-I*sqrt(3) +
1)/(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a
^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b
 + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2
)^4*a^2*d^3))^(1/3) - (1/2)^(1/3)*(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d
^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2
*a^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3
```

```
+ (8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3)*(I*sqrt(3) + 1) + 2*(a^2
*b + 2*b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))^2*d^2 - 4*(a^6*b - 3*a^2*b^5 + 2
*b^7)*(2*(1/2)^(2/3)*(b^2/(a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2) - (a^2*b + 2*
b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2)*(-I*sqrt(3) + 1)/(b^3/(a^6*d^3 - 2*
a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^
2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d -
2*a^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3) -
 (1/2)^(1/3)*(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2*b^
3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d))
+ 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^5/(
(a^2 - b^2)^4*a^2*d^3))^(1/3)*(I*sqrt(3) + 1) + 2*(a^2*b + 2*b^3)/(a^4*d -
2*a^2*b^2*d + b^4*d))*d)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*d
^2))*cos(d*x + c)^2 - 6*(a^2*b + 2*b^3)*cos(d*x + c)^2)*log(-7*a^3*b^2 - 2*
a*b^4 - 3/4*(a^7 - 2*a^5*b^2 + a^3*b^4)*(2*(1/2)^(2/3)*(b^2/(a^4*d^2 - 2*a^
2*b^2*d^2 + b^4*d^2) - (a^2*b + 2*b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2)*(
-I*sqrt(3) + 1)/(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2
*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d
)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^
5/((a^2 - b^2)^4*a^2*d^3))^(1/3) - (1/2)^(1/3)*(b^3/(a^6*d^3 - 2*a^4*b^2*d^
3 + a^2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^
2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*
d + b^4*d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3)*(I*sqrt(3)
+ 1) + 2*(a^2*b + 2*b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))^2*d^2 + 1/2*(10*a^5
*b + 16*a^3*b^3 + a*b^5)*(2*(1/2)^(2/3)*(b^2/(a^4*d^2 - 2*a^2*b^2*d^2 + b^4
*d^2) - (a^2*b + 2*b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2)*(-I*sqrt(3) + 1)
/(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4
*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b +
 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^
4*a^2*d^3))^(1/3) - (1/2)^(1/3)*(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3
) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a
^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 +
(8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3)*(I*sqrt(3) + 1) + 2*(a^2*b
 + 2*b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))*d + 3/4*sqrt(1/3)*(3*(a^7 - 2*a^5*
b^2 + a^3*b^4)*(2*(1/2)^(2/3)*(b^2/(a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2) - (a
^2*b + 2*b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2)*(-I*sqrt(3) + 1)/(b^3/(a^6
*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a
^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/
(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3)
)^(1/3) - (1/2)^(1/3)*(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2
*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d +
 b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2 + b
^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3)*(I*sqrt(3) + 1) + 2*(a^2*b + 2*b^3)/
(a^4*d - 2*a^2*b^2*d + b^4*d))*d^2 + 2*(a^5*b - 2*a^3*b^3 + a*b^5)*d)*sqrt(
-(4*a^4*b^2 - 80*a^2*b^4 - 32*b^6 + (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^
6 + b^8)*(2*(1/2)^(2/3)*(b^2/(a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2) - (a^2*b +
 2*b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2)*(-I*sqrt(3) + 1)/(b^3/(a^6*d^3 -
 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2
*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d
 - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3
) - (1/2)^(1/3)*(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2
*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d
)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^
5/((a^2 - b^2)^4*a^2*d^3))^(1/3)*(I*sqrt(3) + 1) + 2*(a^2*b + 2*b^3)/(a^4*d
 - 2*a^2*b^2*d + b^4*d))^2*d^2 - 4*(a^6*b - 3*a^2*b^5 + 2*b^7)*(2*(1/2)^(2/
3)*(b^2/(a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2) - (a^2*b + 2*b^3)^2/(a^4*d - 2*
a^2*b^2*d + b^4*d)^2)*(-I*sqrt(3) + 1)/(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*
b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*
d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*
d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3) - (1/2)^(1/3)*(b^3/
```

```
(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 -
 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3
)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*
d^3))^(1/3)*(I*sqrt(3) + 1) + 2*(a^2*b + 2*b^3)/(a^4*d - 2*a^2*b^2*d + b^4*
d))*d)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*d^2)) - 2*(8*a^2*b^
3 + b^5)*sin(d*x + c)) + 6*(a^3 - a*b^2)*sin(d*x + c))/((a^4 - 2*a^2*b^2 +
b^4)*d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22952, size = 689, normalized size = 1.79

$$\frac{4\left(3a^5b^4\left(-\frac{a}{b}\right)^{\frac{1}{3}}-6a^3b^6\left(-\frac{a}{b}\right)^{\frac{1}{3}}+3ab^8\left(-\frac{a}{b}\right)^{\frac{1}{3}}-2a^6b^3+3a^4b^5-b^9\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\sin(dx+c)\right|\right)}{a^9b-4a^7b^3+6a^5b^5-4a^3b^7+ab^9}+\frac{12\left(3\left(-ab^2\right)^{\frac{2}{3}}ab+\left(2a^2b+b^3\right)\left(-ab^2\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\cdots\right)}{3\left(-\cdots\right)}\right)}{\sqrt{3}a^5-2\sqrt{3}a^3b^2+\sqrt{3}ab^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 1/12*(4*(3*a^5*b^4*(-a/b)^(1/3) - 6*a^3*b^6*(-a/b)^(1/3) + 3*a*b^8*(-a/b)^(
1/3) - 2*a^6*b^3 + 3*a^4*b^5 - b^9)*(-a/b)^(1/3)*log(abs(-(-a/b)^(1/3) + si
n(d*x + c)))/(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9) + 12*(3*(-
a*b^2)^(2/3)*a*b + (2*a^2*b + b^3)*(-a*b^2)^(1/3))*arctan(1/3*sqrt(3)*((-a/
b)^(1/3) + 2*sin(d*x + c))/(-a/b)^(1/3))/(sqrt(3)*a^5 - 2*sqrt(3)*a^3*b^2 +
 sqrt(3)*a*b^4) - 2*(3*(-a*b^2)^(2/3)*a*b - (2*a^2*b + b^3)*(-a*b^2)^(1/3))
*log(sin(d*x + c)^2 + (-a/b)^(1/3)*sin(d*x + c) + (-a/b)^(2/3))/(a^5 - 2*a^
3*b^2 + a*b^4) + 4*(a^2*b + 2*b^3)*log(abs(b*sin(d*x + c)^3 + a))/(a^4 - 2*
a^2*b^2 + b^4) + 3*(a - 4*b)*log(abs(sin(d*x + c) + 1))/(a^2 - 2*a*b + b^2)
 - 3*(a + 4*b)*log(abs(sin(d*x + c) - 1))/(a^2 + 2*a*b + b^2) + 6*(a^2*b*si
n(d*x + c)^2 + 2*b^3*sin(d*x + c)^2 - a^3*sin(d*x + c) + a*b^2*sin(d*x + c)
 - 3*b^3)/((a^4 - 2*a^2*b^2 + b^4)*(sin(d*x + c)^2 - 1)))/d
```

3.388 $\int \frac{\cos^4(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=764

$$-\frac{2(-1)^{2/3}a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3b^{4/3}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}+\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}-b^{2/3}}}-\frac{4\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3b^{2/3}d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2a^{2/3}\tan}{3b}$$

```
[Out] (-2*(-1)^(2/3)*a^(2/3)*ArcTan[((-1)^(1/3)*b^(1/3) - a^(1/3)*Tan[(c + d*x)/2
])/Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3
)]*b^(4/3)*d) + (2*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3)
 - b^(2/3)]])/(3*a^(2/3)*Sqrt[a^(2/3) - b^(2/3)]*d) + (2*a^(2/3)*ArcTan[(b^
(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b^(2/3)]])/(3*Sqrt[a^(2/3)
 - b^(2/3)]*b^(4/3)*d) - (4*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqr
t[a^(2/3) - b^(2/3)]])/(3*Sqrt[a^(2/3) - b^(2/3)]*b^(2/3)*d) + (2*ArcTan[((
-1)^(2/3)*b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) + (-1)^(1/3)*b^(
2/3)]])/(3*a^(2/3)*Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*d) - (2*(-1)^(1/3)*a^
(2/3)*ArcTan[((-1)^(2/3)*b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) +
 (-1)^(1/3)*b^(2/3)]])/(3*Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*b^(4/3)*d) - (
2*ArcTan[((-1)^(1/3)*(b^(1/3) + (-1)^(2/3)*a^(1/3)*Tan[(c + d*x)/2]))/Sqrt[
a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(2/3)*Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3
)]*d) + (4*ArcTanh[(b^(1/3) - (-1)^(1/3)*a^(1/3)*Tan[(c + d*x)/2])/Sqrt[-((
-1)^(2/3)*a^(2/3)) + b^(2/3)]])/(3*Sqrt[-((-1)^(2/3)*a^(2/3)) + b^(2/3)]*b^
(2/3)*d) + (4*ArcTanh[(b^(1/3) + (-1)^(2/3)*a^(1/3)*Tan[(c + d*x)/2])/Sqrt[
(-1)^(1/3)*a^(2/3) + b^(2/3)]])/(3*Sqrt[(-1)^(1/3)*a^(2/3) + b^(2/3)]*b^(2/
3)*d) - Cos[c + d*x]/(b*d)
```

Rubi [A] time = 1.53042, antiderivative size = 764, normalized size of antiderivative = 1., number of steps used = 38, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3226, 3213, 2660, 618, 204, 3220, 206, 2638}

$$-\frac{2(-1)^{2/3}a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3b^{4/3}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}+\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}-b^{2/3}}}-\frac{4\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3b^{2/3}d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2a^{2/3}\tan}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (-2*(-1)^(2/3)*a^(2/3)*ArcTan[((-1)^(1/3)*b^(1/3) - a^(1/3)*Tan[(c + d*x)/2
])/Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3
)]*b^(4/3)*d) + (2*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3)
 - b^(2/3)]])/(3*a^(2/3)*Sqrt[a^(2/3) - b^(2/3)]*d) + (2*a^(2/3)*ArcTan[(b^
(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b^(2/3)]])/(3*Sqrt[a^(2/3)
 - b^(2/3)]*b^(4/3)*d) - (4*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqr
t[a^(2/3) - b^(2/3)]])/(3*Sqrt[a^(2/3) - b^(2/3)]*b^(2/3)*d) + (2*ArcTan[((
-1)^(2/3)*b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) + (-1)^(1/3)*b^(
2/3)]])/(3*a^(2/3)*Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*d) - (2*(-1)^(1/3)*a^
(2/3)*ArcTan[((-1)^(2/3)*b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) +
 (-1)^(1/3)*b^(2/3)]])/(3*Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*b^(4/3)*d) - (
2*ArcTan[((-1)^(1/3)*(b^(1/3) + (-1)^(2/3)*a^(1/3)*Tan[(c + d*x)/2]))/Sqrt[
a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(2/3)*Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3
)]*d) + (4*ArcTanh[(b^(1/3) - (-1)^(1/3)*a^(1/3)*Tan[(c + d*x)/2])/Sqrt[-((
```

```
-1)^(2/3)*a^(2/3)) + b^(2/3)]])/(3*Sqrt[-((-1)^(2/3)*a^(2/3)) + b^(2/3)]*b^
(2/3)*d) + (4*ArcTanh[(b^(1/3) + (-1)^(2/3)*a^(1/3)*Tan[(c + d*x)/2])/Sqrt[
(-1)^(1/3)*a^(2/3) + b^(2/3)]])/(3*Sqrt[(-1)^(1/3)*a^(2/3) + b^(2/3)]*b^(2/
3)*d) - Cos[c + d*x]/(b*d)
```

Rule 3226

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_)
), x_Symbol] :> Int[Expand[(1 - Sin[e + f*x]^2)^(m/2)/(a + b*Sin[e + f*x]^n
), x], x] /; FreeQ[{a, b, e, f}, x] && IGtQ[m/2, 0] && IntegerQ[(n - 1)/2]
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \left(\frac{1}{a+b\sin^3(c+dx)} - \frac{2\sin^2(c+dx)}{a+b\sin^3(c+dx)} + \frac{\sin^4(c+dx)}{a+b\sin^3(c+dx)}\right)dx$$

$$= -\left(2\int \frac{\sin^2(c+dx)}{a+b\sin^3(c+dx)}\,dx\right) + \int \frac{1}{a+b\sin^3(c+dx)}\,dx + \int \frac{\sin^4(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

$$= -\left(2\int \left(\frac{1}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)} + \frac{1}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)} + \frac{}{3b^{2/3}\left((-1)^{2/3}\right.}\right.\right.$$

$$= -\frac{\int \frac{1}{-\sqrt[3]{a}-\sqrt[3]{b}\sin(c+dx)}\,dx}{3a^{2/3}} - \frac{\int \frac{1}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}\sin(c+dx)}\,dx}{3a^{2/3}} - \frac{\int \frac{1}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\sin(c+dx)}\,dx}{3a^{2/3}} + \frac{\int \sin(c+}{b}$$

$$= -\frac{\cos(c+dx)}{bd} - \frac{a\int \left(-\frac{1}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)} - \frac{(-1)^{2/3}}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}\sin(c+dx)\right)} + \frac{\sqrt[3]{-1}}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+(-1)^{2/}\right.}\right.}{b}$$

$$= -\frac{\cos(c+dx)}{bd} + \frac{a^{2/3}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)}\,dx}{3b^{4/3}} - \frac{\left(\sqrt[3]{-1}a^{2/3}\right)\int \frac{1}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\sin(c+dx)}\,dx}{3b^{4/3}} + \frac{\left((-1)^{2/3}a\right.}{}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}d} + \frac{2\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-b^{2/3}}d} - \frac{4\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+\right.}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3\sqrt{a^{2/3}-b^{2/3}}b^{2/3}d}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}d} + \frac{2\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-b^{2/3}}d} - \frac{4\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+\right.}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3\sqrt{a^{2/3}-b^{2/3}}b^{2/3}d}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}d} - \frac{2(-1)^{2/3}a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}b^{4/3}d} + \frac{2\tan^{-1}\left(\right.}{3a^{2/3}}$$

Mathematica [C] time = 0.141628, size = 300, normalized size = 0.39

$$-\frac{3\cos(c+dx) + i\text{RootSum}\left[8\#1^3a + i\#1^6b - 3i\#1^4b + 3i\#1^2b - ib\&, \frac{\#1^3a\log\left(\#1^2-2\#1\cos(c+dx)+1\right) - \#1a\log\left(\#1^2-2\#1\cos(c+dx)\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] -(3*Cos[c + d*x] + I*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^
4 + I*b*#1^6 & , (2*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - I*b*Log[1
- 2*Cos[c + d*x]*#1 + #1^2] - (2*I)*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #
1)]*#1 - a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1 + (2*I)*a*ArcTan[Sin[c + d*
x]/(Cos[c + d*x] - #1)]*#1^3 + a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^3 + 2
*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - I*b*Log[1 - 2*Cos[c + d*
x]*#1 + #1^2]*#1^4)/(b*#1 - (4*I)*a*#1^2 - 2*b*#1^3 + b*#1^5) & ])/(3*b*d)
```

Maple [C] time = 0.214, size = 123, normalized size = 0.2

$$\frac{1}{3bd}\sum_{\_R=\text{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)} \frac{\_R^4b - 2\_R^3a - 6\_R^2b - 2\_Ra + b}{\_R^5a + 2\_R^3a + 4\_R^2b + \_Ra}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right) - \_R\right) - 2\frac{}{bd\left(1+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*sin(d*x+c)^3),x)
```

```
[Out] 1/3/d/b*sum((_R^4*b-2*_R^3*a-6*_R^2*b-2*_R*a+b)/(_R^5*a+2*_R^3*a+4*_R^2*b+_
R*a)*ln(tan(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+
a))-2/d/b/(1+tan(1/2*d*x+1/2*c)^2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^4(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Integral(cos(c + d*x)**4/(a + b*sin(c + d*x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^4}{b\sin(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^4/(b*sin(d*x + c)^3 + a), x)
```

3.389 $\int \frac{\cos^2(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=484

$$\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}-b^{2/3}}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3b^{2/3}d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{-1}\left((-1)^{2/3}\cdots\right.}{\sqrt{a^{2}\cdots}}\right.}{3a^{2/3}d\sqrt{a^{2/3}\cdots}}$$

```
[Out] (2*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b^(2/3)]])/(3
*a^(2/3)*Sqrt[a^(2/3) - b^(2/3)]*d) - (2*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c +
 d*x)/2])/Sqrt[a^(2/3) - b^(2/3)]])/(3*Sqrt[a^(2/3) - b^(2/3)]*b^(2/3)*d) +
 (2*ArcTan[((-1)^(2/3)*b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) + (
-1)^(1/3)*b^(2/3)]])/(3*a^(2/3)*Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*d) - (2*
ArcTan[((-1)^(1/3)*(b^(1/3) + (-1)^(2/3)*a^(1/3)*Tan[(c + d*x)/2]))/Sqrt[a^
(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(2/3)*Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]
*d) + (2*ArcTanh[(b^(1/3) - (-1)^(1/3)*a^(1/3)*Tan[(c + d*x)/2])/Sqrt[-((-1
)^(2/3)*a^(2/3)) + b^(2/3)]])/(3*Sqrt[-((-1)^(2/3)*a^(2/3)) + b^(2/3)]*b^(2
/3)*d) + (2*ArcTanh[(b^(1/3) + (-1)^(2/3)*a^(1/3)*Tan[(c + d*x)/2])/Sqrt[(-
1)^(1/3)*a^(2/3) + b^(2/3)]])/(3*Sqrt[(-1)^(1/3)*a^(2/3) + b^(2/3)]*b^(2/3)
*d)
```

Rubi [A] time = 0.634457, antiderivative size = 484, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3226, 3213, 2660, 618, 204, 3220, 206}

$$\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}-b^{2/3}}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3b^{2/3}d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{-1}\left((-1)^{2/3}\cdots\right.}{\sqrt{a^{2}\cdots}}\right.}{3a^{2/3}d\sqrt{a^{2/3}\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (2*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b^(2/3)]])/(3
*a^(2/3)*Sqrt[a^(2/3) - b^(2/3)]*d) - (2*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c +
 d*x)/2])/Sqrt[a^(2/3) - b^(2/3)]])/(3*Sqrt[a^(2/3) - b^(2/3)]*b^(2/3)*d) +
 (2*ArcTan[((-1)^(2/3)*b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) + (
-1)^(1/3)*b^(2/3)]])/(3*a^(2/3)*Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*d) - (2*
ArcTan[((-1)^(1/3)*(b^(1/3) + (-1)^(2/3)*a^(1/3)*Tan[(c + d*x)/2]))/Sqrt[a^
(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(2/3)*Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]
*d) + (2*ArcTanh[(b^(1/3) - (-1)^(1/3)*a^(1/3)*Tan[(c + d*x)/2])/Sqrt[-((-1
)^(2/3)*a^(2/3)) + b^(2/3)]])/(3*Sqrt[-((-1)^(2/3)*a^(2/3)) + b^(2/3)]*b^(2
/3)*d) + (2*ArcTanh[(b^(1/3) + (-1)^(2/3)*a^(1/3)*Tan[(c + d*x)/2])/Sqrt[(-
1)^(1/3)*a^(2/3) + b^(2/3)]])/(3*Sqrt[(-1)^(1/3)*a^(2/3) + b^(2/3)]*b^(2/3)
*d)
```

Rule 3226

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_)
), x_Symbol] :> Int[Expand[(1 - Sin[e + f*x]^2)^(m/2)/(a + b*Sin[e + f*x]^n
), x], x] /; FreeQ[{a, b, e, f}, x] && IGtQ[m/2, 0] && IntegerQ[(n - 1)/2]
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \left(\frac{1}{a+b\sin^3(c+dx)} - \frac{\sin^2(c+dx)}{a+b\sin^3(c+dx)}\right) dx$$

$$= \int \frac{1}{a+b\sin^3(c+dx)}\,dx - \int \frac{\sin^2(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

$$= -\int \left(\frac{1}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)} + \frac{1}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)} + \frac{}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}\right.}\right.$$

$$= -\frac{\int \frac{1}{-\sqrt[3]{a}-\sqrt[3]{b}\sin(c+dx)}\,dx}{3a^{2/3}} - \frac{\int \frac{1}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}\sin(c+dx)}\,dx}{3a^{2/3}} - \frac{\int \frac{1}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\sin(c+dx)}\,dx}{3a^{2/3}} - \frac{\int \frac{}{\sqrt[3]{a}+\sqrt[3]{b}}}{3}$$

$$= -\frac{2\,\mathrm{Subst}\left(\int \frac{1}{-\sqrt[3]{a}-2\sqrt[3]{b}x-\sqrt[3]{a}x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{3a^{2/3}d} - \frac{2\,\mathrm{Subst}\left(\int \frac{1}{-\sqrt[3]{a}+2\sqrt[3]{-1}\sqrt[3]{b}x-\sqrt[3]{a}x^2}\,dx, x,\right.}{3a^{2/3}d}$$

$$= \frac{4\,\mathrm{Subst}\left(\int \frac{1}{-4\left(a^{2/3}-b^{2/3}\right)-x^2}\,dx, x, -2\sqrt[3]{b}-2\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3a^{2/3}d} + \frac{4\,\mathrm{Subst}\left(\int \frac{1}{-4\left(a^{2/3}+\sqrt[3]{-1}b^{2/3}\right.}\right.}{}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,d} + \frac{2\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-b^{2/3}}\,d} - \frac{2\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+\right.}{\sqrt{a^{2/3}-b^{2/3}}}\right.}{3\sqrt{a^{2/3}-b^{2/3}}\,b^{2/3}d}$$

Mathematica [C] time = 0.101202, size = 231, normalized size = 0.48

$$-\frac{i\mathrm{RootSum}\left[8\#1^3a+i\#1^6b-3i\#1^4b+3i\#1^2b-ib\&, \frac{-i\#1^4\log\left(\#1^2-2\#1\cos(c+dx)+1\right)-2i\#1^2\log\left(\#1^2-2\#1\cos(c+dx)+1\right)-i\log\left(\#1^2-2\#\right.}{-4i\#1^2}\right.}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] ((-I/6)*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#1^6
& , (2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - I*Log[1 - 2*Cos[c + d*x]*
#1 + #1^2] + 4*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 - (2*I)*Log[1
- 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1
)]*#1^4 - I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4)/(b*#1 - (4*I)*a*#1^2 -
2*b*#1^3 + b*#1^5) & ])/d
```

Maple [C] time = 0.191, size = 83, normalized size = 0.2

$$\frac{1}{3d}\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)} \frac{\_R^4-2\_R^2+1}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*sin(d*x+c)^3),x)
```

```
[Out] 1/3/d*sum((_R^4-2*_R^2+1)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/
2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2}{b\sin(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/(b*sin(d*x + c)^3 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Integral(cos(c + d*x)**2/(a + b*sin(c + d*x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2}{b\sin(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/(b*sin(d*x + c)^3 + a), x)
```

3.390 $\int \frac{1}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=245

$$\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{-1}\left((-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}$$

```
[Out] (2*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b^(2/3)]])/(3
*a^(2/3)*Sqrt[a^(2/3) - b^(2/3)]*d) + (2*ArcTan[((-1)^(2/3)*b^(1/3) + a^(1/
3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]])/(3*a^(2/3)*Sqrt[a
^(2/3) + (-1)^(1/3)*b^(2/3)]*d) - (2*ArcTan[((-1)^(1/3)*(b^(1/3) + (-1)^(2/
3)*a^(1/3)*Tan[(c + d*x)/2]))/Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(2/
3)*Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]*d)
```

Rubi [A] time = 0.251439, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3213, 2660, 618, 204}

$$\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{-1}\left((-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x]^3)^(-1),x]
```

```
[Out] (2*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b^(2/3)]])/(3
*a^(2/3)*Sqrt[a^(2/3) - b^(2/3)]*d) + (2*ArcTan[((-1)^(2/3)*b^(1/3) + a^(1/
3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]])/(3*a^(2/3)*Sqrt[a
^(2/3) + (-1)^(1/3)*b^(2/3)]*d) - (2*ArcTan[((-1)^(1/3)*(b^(1/3) + (-1)^(2/
3)*a^(1/3)*Tan[(c + d*x)/2]))/Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(2/
3)*Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]*d)
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{a+b\sin^3(c+dx)}\,dx = \int \left(-\frac{1}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}\sin(c+dx)\right)}-\frac{1}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}\sin(c+dx)\right)}-\frac{1}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2}\right.}$$

$$= -\frac{\int \frac{1}{-\sqrt[3]{a}-\sqrt[3]{b}\sin(c+dx)}\,dx}{3a^{2/3}}-\frac{\int \frac{1}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}\sin(c+dx)}\,dx}{3a^{2/3}}-\frac{\int \frac{1}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\sin(c+dx)}\,dx}{3a^{2/3}}$$

$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{-\sqrt[3]{a}-2\sqrt[3]{b}x-\sqrt[3]{a}x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{3a^{2/3}d}-\frac{2\,\text{Subst}\left(\int \frac{1}{-\sqrt[3]{a}+2\sqrt[3]{-1}\sqrt[3]{b}x-\sqrt[3]{a}x^2}\,dx, x, \text{ta}\right.}{3a^{2/3}d}$$

$$= \frac{4\,\text{Subst}\left(\int \frac{1}{-4\left(a^{2/3}-b^{2/3}\right)-x^2}\,dx, x, -2\sqrt[3]{b}-2\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3a^{2/3}d}+\frac{4\,\text{Subst}\left(\int \frac{1}{-4\left(a^{2/3}+\sqrt[3]{-1}b^{2/3}\right)-x^2}\right.}{}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,d}+\frac{2\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-b^{2/3}}\,d}+\frac{2\tan^{-1}\left(\frac{(-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}\right.}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right.}{3a^{2/3}\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}$$

Mathematica [C] time = 0.158277, size = 126, normalized size = 0.51

$$-\frac{2i\,\text{RootSum}\left[8\#1^3a+i\#1^6b-3i\#1^4b+3i\#1^2b-ib\&,\frac{2\#1\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)-i\#1\log\left(\#1^2-2\#1\cos(c+dx)+1\right)}{\#1^4b-2\#1^2b-4i\#1a+b}\&\right]}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x]^3)^(-1),x]
```

```
[Out] (((-2*I)/3)*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#
1^6 & , (2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 - I*Log[1 - 2*Cos[c
+ d*x]*#1 + #1^2]*#1)/(b - (4*I)*a*#1 - 2*b*#1^2 + b*#1^4) & ])/d
```

Maple [C] time = 0.161, size = 83, normalized size = 0.3

$$\frac{1}{3d}\sum_{\_R=\text{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\_R^4+2\_R^2+1}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(d*x+c)^3),x)
```

```
[Out] 1/3/d*sum((_R^4+2*_R^2+1)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/
2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b\sin(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*sin(d*x + c)^3 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{a+b\sin^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Integral(1/(a + b*sin(c + d*x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b\sin(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(1/(b*sin(d*x + c)^3 + a), x)
```

3.391 $\int \frac{\sec^2(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=299

$$\frac{2(-1)^{2/3}b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}d\left(a^{2/3}-(-1)^{2/3}b^{2/3}\right)^{3/2}}-\frac{2b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}d\left(a^{2/3}-b^{2/3}\right)^{3/2}}+\frac{2\sqrt[3]{-1}b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3a^{2/3}d\left(a^{2/3}+\sqrt[3]{-1}b^{2/3}\right)^{3/2}}$$

```
[Out] (2*(-1)^(2/3)*b^(2/3)*ArcTan[((-1)^(1/3)*b^(1/3) - a^(1/3)*Tan[(c + d*x)/2]
)/Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(2/3)*(a^(2/3) - (-1)^(2/3)*b^(
2/3))^(3/2)*d) - (2*b^(2/3)*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqr
t[a^(2/3) - b^(2/3)]])/(3*a^(2/3)*(a^(2/3) - b^(2/3))^(3/2)*d) + (2*(-1)^(1
/3)*b^(2/3)*ArcTan[((-1)^(2/3)*b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(
2/3) + (-1)^(1/3)*b^(2/3)]])/(3*a^(2/3)*(a^(2/3) + (-1)^(1/3)*b^(2/3))^(3/2
)*d) + (Sec[c + d*x]*(b - a*Sin[c + d*x]))/((-a^2 + b^2)*d)
```

Rubi [F] time = 0.0453848, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sec^2(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^2/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] Defer[Int][Sec[c + d*x]^2/(a + b*Sin[c + d*x]^3), x]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{a+b\sin^3(c+dx)}\,dx=\int \frac{\sec^2(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

Mathematica [C] time = 0.253172, size = 432, normalized size = 1.44

$$-ib\cos(c+dx)\text{RootSum}\left[8\#1^3a+i\#1^6b-3i\#1^4b+3i\#1^2b-ib\&,\frac{-2\#1^3a\log\left(\#1^2-2\#1\cos(c+dx)+1\right)+2\#1a\log\left(\#1^2-2\#1\cos(c+dx)+1\right)}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (-6*b + 6*b*Cos[c + d*x] - I*b*Cos[c + d*x]*RootSum[(-I)*b + (3*I)*b*#1^2 +
 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#1^6 & , (2*b*ArcTan[Sin[c + d*x]/(Cos[c + d
*x] - #1)] - I*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] + (4*I)*a*ArcTan[Sin[c +
 d*x]/(Cos[c + d*x] - #1)]*#1 + 2*a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1 -
12*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 + (6*I)*b*Log[1 - 2*Cos[
c + d*x]*#1 + #1^2]*#1^2 - (4*I)*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]
```

```
*#1^3 - 2*a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^3 + 2*b*ArcTan[Sin[c + d*x
]/(Cos[c + d*x] - #1)]*#1^4 - I*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4)/(
b*#1 - (4*I)*a*#1^2 - 2*b*#1^3 + b*#1^5) & ] + 6*a*Sin[c + d*x])/(6*(a - b)
*(a + b)*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2]))
```

Maple [C] time = 0.213, size = 164, normalized size = 0.6

$$-\frac{b}{3d(a-b)(a+b)}\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\_R^4b-2\_R^3a+6\_R^2b-2\_Ra+b}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sin(d*x+c)^3),x)
```

```
[Out] -1/3/d*b/(a-b)/(a+b)*sum((_R^4*b-2*_R^3*a+6*_R^2*b-2*_R*a+b)/(_R^5*a+2*_R^3
*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^
3*b+3*_Z^2*a+a))-2/d/(2*a-2*b)/(tan(1/2*d*x+1/2*c)+1)-2/d/(2*a+2*b)/(tan(1/
2*d*x+1/2*c)-1)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sin(d*x+c)**3),x)
```

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^2}{b\sin(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/(b*sin(d*x + c)^3 + a), x)
```

3.392 $\int \frac{\sec^4(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=1093

result too large to display

```
[Out] (-2*(-1)^(2/3)*a^(2/3)*b^(8/3)*ArcTan[((-1)^(1/3)*b^(1/3) - a^(1/3)*Tan[(c
+ d*x)/2])/Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(Sqrt[a^(2/3) - (-1)^(2/3)*
b^(2/3)]*(a^2 - b^2)^2*d) - (2*b^2*(2*a^2 + b^2)*ArcTan[((-1)^(1/3)*b^(1/3)
 - a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(2/3
)*Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]*(a^2 - b^2)^2*d) + (2*a^(2/3)*b^(8/3)*
ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b^(2/3)]])/(Sqrt
[a^(2/3) - b^(2/3)]*(a^2 - b^2)^2*d) + (2*b^2*(2*a^2 + b^2)*ArcTan[(b^(1/3)
 + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b^(2/3)]])/(3*a^(2/3)*Sqrt[a^(2
/3) - b^(2/3)]*(a^2 - b^2)^2*d) + (2*b^(4/3)*(a^2 + 2*b^2)*ArcTan[(b^(1/3)
+ a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b^(2/3)]])/(3*Sqrt[a^(2/3) - b^(
2/3)]*(a^2 - b^2)^2*d) - (2*(-1)^(1/3)*a^(2/3)*b^(8/3)*ArcTan[((-1)^(2/3)*b
^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]])/(Sq
rt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*(a^2 - b^2)^2*d) + (2*b^2*(2*a^2 + b^2)*Ar
cTan[((-1)^(2/3)*b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) + (-1)^(1
/3)*b^(2/3)]])/(3*a^(2/3)*Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*(a^2 - b^2)^2*
d) - (2*b^(4/3)*(a^2 + 2*b^2)*ArcTanh[(b^(1/3) - (-1)^(1/3)*a^(1/3)*Tan[(c
+ d*x)/2])/Sqrt[-((-1)^(2/3)*a^(2/3)) + b^(2/3)]])/(3*Sqrt[-((-1)^(2/3)*a^(
2/3)) + b^(2/3)]*(a^2 - b^2)^2*d) - (2*b^(4/3)*(a^2 + 2*b^2)*ArcTanh[(b^(1/
3) + (-1)^(2/3)*a^(1/3)*Tan[(c + d*x)/2])/Sqrt[(-1)^(1/3)*a^(2/3) + b^(2/3)
]])/(3*Sqrt[(-1)^(1/3)*a^(2/3) + b^(2/3)]*(a^2 - b^2)^2*d) + Cos[c + d*x]/(
12*(a + b)*d*(1 - Sin[c + d*x])^2) + Cos[c + d*x]/(12*(a + b)*d*(1 - Sin[c
+ d*x])) + ((a + 4*b)*Cos[c + d*x])/(4*(a + b)^2*d*(1 - Sin[c + d*x])) - Co
s[c + d*x]/(12*(a - b)*d*(1 + Sin[c + d*x])^2) - ((a - 4*b)*Cos[c + d*x])/(
4*(a - b)^2*d*(1 + Sin[c + d*x])) - Cos[c + d*x]/(12*(a - b)*d*(1 + Sin[c +
 d*x]))
```

Rubi [F] time = 0.0462909, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sec^4(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^4/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] Defer[Int][Sec[c + d*x]^4/(a + b*Sin[c + d*x]^3), x]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \frac{\sec^4(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

Mathematica [C] time = 1.69115, size = 679, normalized size = 0.62

$$\sec^3(c+dx)\left(-3b\left(5a^2+13b^2\right)\cos(c+dx)+12b\left(a^2+2b^2\right)\cos(2(c+dx))-5a^2b\cos(3(c+dx))+4a^2b+12a^3\sin(c\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] ((4*I)*b^2*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#1
^6 & , (2*a^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + 4*b^2*ArcTan[Sin[c
 + d*x]/(Cos[c + d*x] - #1)] - I*a^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] - (2
*I)*b^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] + (12*I)*a*b*ArcTan[Sin[c + d*x]/
(Cos[c + d*x] - #1)]*#1 + 6*a*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1 - 20*a
^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 - 16*b^2*ArcTan[Sin[c + d*
x]/(Cos[c + d*x] - #1)]*#1^2 + (10*I)*a^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]
*#1^2 + (8*I)*b^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (12*I)*a*b*ArcTa
n[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^3 - 6*a*b*Log[1 - 2*Cos[c + d*x]*#1
+ #1^2]*#1^3 + 2*a^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 + 4*b^2*
ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - I*a^2*Log[1 - 2*Cos[c + d*x
]*#1 + #1^2]*#1^4 - (2*I)*b^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4)/(b*#1
 - (4*I)*a*#1^2 - 2*b*#1^3 + b*#1^5) & ] + Sec[c + d*x]^3*(4*a^2*b + 32*b^3
 - 3*b*(5*a^2 + 13*b^2)*Cos[c + d*x] + 12*b*(a^2 + 2*b^2)*Cos[2*(c + d*x)]
- 5*a^2*b*Cos[3*(c + d*x)] - 13*b^3*Cos[3*(c + d*x)] + 12*a^3*Sin[c + d*x]
- 30*a*b^2*Sin[c + d*x] + 4*a^3*Sin[3*(c + d*x)] - 22*a*b^2*Sin[3*(c + d*x)
]))/(24*(a - b)^2*(a + b)^2*d)
```

Maple [C] time = 0.241, size = 346, normalized size = 0.3

$$\frac{b^2}{3\,d\,(a-b)^2\,(a+b)^2}\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3\,a\_Z^4+8\,b\_Z^3+3\,a\_Z^2+a\right)}\frac{\left(2\,a^2+b^2\right)\_R^4-6\_R^3ab+2\left(4\,a^2+5\,b^2\right)\_R^2-6\_R\,ab+2\,a}{\_R^5a+2\_R^3a+4\_R^2b+\_R\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*sin(d*x+c)^3),x)
```

```
[Out] 1/3/d*b^2/(a-b)^2/(a+b)^2*sum(((2*a^2+b^2)*_R^4-6*_R^3*a*b+2*(4*a^2+5*b^2)*
_R^2-6*_R*a*b+2*a^2+b^2)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2
*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))-2/3/d/(tan(1/2*d*x+
1/2*c)-1)^3/(2*a+2*b)-1/d/(2*a+2*b)/(tan(1/2*d*x+1/2*c)-1)^2-1/d/(a+b)^2/(t
an(1/2*d*x+1/2*c)-1)*a-5/2/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)*b-2/3/d/(tan(1/
2*d*x+1/2*c)+1)^3/(2*a-2*b)+1/d/(2*a-2*b)/(tan(1/2*d*x+1/2*c)+1)^2-1/d/(a-b
)^2/(tan(1/2*d*x+1/2*c)+1)*a+5/2/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)*b
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (dx+c)^4}{b\sin (dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^4/(b*sin(d*x + c)^3 + a), x)
```

3.393 $\int \frac{\cos^7(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$

Optimal. Leaf size=288

$$-\frac{\sin(c+dx)\left(a^2+3ab\sin(c+dx)+3b^2\sin^2(c+dx)-b^2\right)}{3ab^2d\left(a+b\sin^3(c+dx)\right)}-\frac{\left(-3a^{4/3}b^{2/3}+2a^2+b^2\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\right.}{9a^{5/3}b^{7/3}d}$$

```
[Out] (-2*(2*a^2 + 3*a^(4/3)*b^(2/3) + b^2)*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d
*x])/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(7/3)*d) + (2*(2*a^2 - 3*a^(4
/3)*b^(2/3) + b^2)*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]])/(9*a^(5/3)*b^(7/3)*
d) - ((2*a^2 - 3*a^(4/3)*b^(2/3) + b^2)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c
 + d*x] + b^(2/3)*Sin[c + d*x]^2])/(9*a^(5/3)*b^(7/3)*d) - Sin[c + d*x]/(b^
2*d) - (Sin[c + d*x]*(a^2 - b^2 + 3*a*b*Sin[c + d*x] + 3*b^2*Sin[c + d*x]^2
))/(3*a*b^2*d*(a + b*Sin[c + d*x]^3))
```

Rubi [A] time = 0.335041, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3223, 1858, 1887, 1860, 31, 634, 617, 204, 628}

$$-\frac{\sin(c+dx)\left(a^2+3ab\sin(c+dx)+3b^2\sin^2(c+dx)-b^2\right)}{3ab^2d\left(a+b\sin^3(c+dx)\right)}-\frac{\left(-3a^{4/3}b^{2/3}+2a^2+b^2\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\right.}{9a^{5/3}b^{7/3}d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] (-2*(2*a^2 + 3*a^(4/3)*b^(2/3) + b^2)*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d
*x])/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(7/3)*d) + (2*(2*a^2 - 3*a^(4
/3)*b^(2/3) + b^2)*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]])/(9*a^(5/3)*b^(7/3)*
d) - ((2*a^2 - 3*a^(4/3)*b^(2/3) + b^2)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c
 + d*x] + b^(2/3)*Sin[c + d*x]^2])/(9*a^(5/3)*b^(7/3)*d) - Sin[c + d*x]/(b^
2*d) - (Sin[c + d*x]*(a^2 - b^2 + 3*a*b*Sin[c + d*x] + 3*b^2*Sin[c + d*x]^2
))/(3*a*b^2*d*(a + b*Sin[c + d*x]^3))
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\cos^7(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^3}{(a+bx^3)^2}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= -\frac{\sin(c+dx)\left(a^2-b^2+3ab\sin(c+dx)+3b^2\sin^2(c+dx)\right)}{3ab^2d\left(a+b\sin^3(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-a^2-2b^2-6abx+3abx^3}{a+bx^3}\,d\right.}{3ab^2d}$$

$$= -\frac{\sin(c+dx)\left(a^2-b^2+3ab\sin(c+dx)+3b^2\sin^2(c+dx)\right)}{3ab^2d\left(a+b\sin^3(c+dx)\right)} - \frac{\text{Subst}\left(\int \left(3a - \frac{2(2a^2+b^2+3abx}{a+bx^3}\right.\right.}{3ab^2d}$$

$$= -\frac{\sin(c+dx)}{b^2d} - \frac{\sin(c+dx)\left(a^2-b^2+3ab\sin(c+dx)+3b^2\sin^2(c+dx)\right)}{3ab^2d\left(a+b\sin^3(c+dx)\right)} + \frac{2\,\text{Subst}\left(\int \frac{2a^2}{}\right.}{}$$

$$= -\frac{\sin(c+dx)}{b^2d} - \frac{\sin(c+dx)\left(a^2-b^2+3ab\sin(c+dx)+3b^2\sin^2(c+dx)\right)}{3ab^2d\left(a+b\sin^3(c+dx)\right)} + \frac{2\,\text{Subst}\left(\int \frac{\sqrt[3]{a}}{}\right.}{}$$

$$= \frac{2\left(2a^2-3a^{4/3}b^{2/3}+b^2\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}b^{7/3}d} - \frac{\sin(c+dx)}{b^2d} - \frac{\sin(c+dx)\left(a^2-b^2+3\right.}{3ab^2d\left(\right.}$$

$$= \frac{2\left(2a^2-3a^{4/3}b^{2/3}+b^2\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}b^{7/3}d} - \frac{\left(2a^2-3a^{4/3}b^{2/3}+b^2\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{}\right.}{9a^{5/3}b^{7}}$$

$$= -\frac{2\left(2a^2+3a^{4/3}b^{2/3}+b^2\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}a^{5/3}b^{7/3}d} + \frac{2\left(2a^2-3a^{4/3}b^{2/3}+b^2\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin\right.}{9a^{5/3}b^{7/3}d}$$

Mathematica [C] time = 3.59769, size = 402, normalized size = 1.4

$$\frac{\frac{6\left(1-\frac{a^2}{b^2}\right)\sin(c+dx)}{a\left(a+b\sin^3(c+dx)\right)} + \frac{6\sqrt[3]{-1}\left(2\sqrt[3]{-1}a^{2/3}+3b^{2/3}\right)\log\left(-(-1)^{2/3}\sqrt[3]{a}-\sqrt[3]{b}\sin(c+dx)\right)}{\sqrt[3]{a}b^{7/3}} + \frac{6\left(2a^{2/3}-3b^{2/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{\sqrt[3]{a}b^{7/3}} - \frac{6\sqrt[3]{-1}\left(2a^{2/3}+3\sqrt[3]{-1}b^{2/3}\right)\log\left(\sqrt[3]{}\right.}{\sqrt[3]{a}b^{7/3}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] ((6*(-1)^(1/3)*(2*(-1)^(1/3)*a^(2/3) + 3*b^(2/3))*Log[-((-1)^(2/3)*a^(1/3))
 - b^(1/3)*Sin[c + d*x]])/(a^(1/3)*b^(7/3)) + (6*(2*a^(2/3) - 3*b^(2/3))*Lo
g[a^(1/3) + b^(1/3)*Sin[c + d*x]])/(a^(1/3)*b^(7/3)) - (6*(-1)^(1/3)*(2*a^(
2/3) + 3*(-1)^(1/3)*b^(2/3))*Log[a^(1/3) + (-1)^(2/3)*b^(1/3)*Sin[c + d*x]]
)/(a^(1/3)*b^(7/3)) + (2*(a^2 - b^2)*(2*Sqrt[3]*ArcTan[(a^(1/3) - 2*b^(1/3)
*Sin[c + d*x])/(Sqrt[3]*a^(1/3))] - 2*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]] +
 Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2]))/(a^
(5/3)*b^(7/3)) - (18*Sin[c + d*x])/b^2 - (27*Hypergeometric2F1[2/3, 2, 5/3,
 -((b*Sin[c + d*x]^3)/a)]*Sin[c + d*x]^2)/(a*b) + 18/(b*(a + b*Sin[c + d*x]
^3)) + (6*(1 - a^2/b^2)*Sin[c + d*x])/(a*(a + b*Sin[c + d*x]^3)))/(18*d)
```

Maple [B] time = 0.145, size = 490, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7/(a+b*sin(d*x+c)^3)^2,x)
```

```
[Out] -sin(d*x+c)/b^2/d-1/d/b/(a+b*sin(d*x+c)^3)*sin(d*x+c)^2-1/3/d/b^2/(a+b*sin(
d*x+c)^3)*sin(d*x+c)*a+1/3*sin(d*x+c)/a/d/(a+b*sin(d*x+c)^3)+1/d/b/(a+b*sin
(d*x+c)^3)+4/9/d/b^3*a/(a/b)^(2/3)*ln(sin(d*x+c)+(a/b)^(1/3))+2/9/d/b/a/(a/
b)^(2/3)*ln(sin(d*x+c)+(a/b)^(1/3))-2/9/d/b^3*a/(a/b)^(2/3)*ln(sin(d*x+c)^2
-(a/b)^(1/3)*sin(d*x+c)+(a/b)^(2/3))-1/9/d/b/a/(a/b)^(2/3)*ln(sin(d*x+c)^2-
(a/b)^(1/3)*sin(d*x+c)+(a/b)^(2/3))+4/9/d/b^3*a/(a/b)^(2/3)*3^(1/2)*arctan(
1/3*3^(1/2)*(2/(a/b)^(1/3)*sin(d*x+c)-1))+2/9/d/b/a/(a/b)^(2/3)*3^(1/2)*arc
tan(1/3*3^(1/2)*(2/(a/b)^(1/3)*sin(d*x+c)-1))-2/3/d/b^2/(a/b)^(1/3)*ln(sin(
d*x+c)+(a/b)^(1/3))+1/3/d/b^2/(a/b)^(1/3)*ln(sin(d*x+c)^2-(a/b)^(1/3)*sin(d
*x+c)+(a/b)^(2/3))+2/3/d/b^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b
)^(1/3)*sin(d*x+c)-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+b*sin(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+b*sin(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7/(a+b*sin(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2355, size = 374, normalized size = 1.3

$$-\frac{\frac{9\sin(dx+c)}{b^2}+\frac{2\left(3ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2a^2+b^2\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\sin(dx+c)\right|\right)}{a^2b^2}+\frac{2\sqrt{3}\left(3\left(-ab^2\right)^{\frac{2}{3}}a-\left(-ab^2\right)^{\frac{1}{3}}\left(2a^2+b^2\right)\right)\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\sin(dx+c)\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^2b^3}+\frac{3(3a\cdots}{\cdots}}{9d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+b*sin(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] -1/9*(9*sin(d*x + c)/b^2 + 2*(3*a*b*(-a/b)^(1/3) + 2*a^2 + b^2)*(-a/b)^(1/3
)*log(abs(-(-a/b)^(1/3) + sin(d*x + c)))/(a^2*b^2) + 2*sqrt(3)*(3*(-a*b^2)^
(2/3)*a - (-a*b^2)^(1/3)*(2*a^2 + b^2))*arctan(1/3*sqrt(3)*((-a/b)^(1/3) +
2*sin(d*x + c))/(-a/b)^(1/3))/(a^2*b^3) + 3*(3*a*b*sin(d*x + c)^2 + a^2*sin
(d*x + c) - b^2*sin(d*x + c) - 3*a*b)/((b*sin(d*x + c)^3 + a)*a*b^2) - (3*(
-a*b^2)^(2/3)*a + (-a*b^2)^(1/3)*(2*a^2 + b^2))*log(sin(d*x + c)^2 + (-a/b)
^(1/3)*sin(d*x + c) + (-a/b)^(2/3))/(a^2*b^3))/d
```

3.394 $\int \frac{\cos^5(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$

Optimal. Leaf size=238

$$\frac{\left(a^{4/3}-b^{4/3}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{9a^{5/3}b^{5/3}d}-\frac{2\left(a^{4/3}-b^{4/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}b^{5/3}d}-\frac{2\left(a^{4/3}+b\right.}{}$$

```
[Out] (-2*(a^(4/3) + b^(4/3))*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*
a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(5/3)*d) - (2*(a^(4/3) - b^(4/3))*Log[a^(1/
3) + b^(1/3)*Sin[c + d*x]])/(9*a^(5/3)*b^(5/3)*d) + ((a^(4/3) - b^(4/3))*Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2])/(9*a^(5
/3)*b^(5/3)*d) + (Sin[c + d*x]*(b - a*Sin[c + d*x] - 2*b*Sin[c + d*x]^2))/(
3*a*b*d*(a + b*Sin[c + d*x]^3))
```

Rubi [A] time = 0.224561, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3223, 1858, 1860, 31, 634, 617, 204, 628}

$$\frac{\left(a^{4/3}-b^{4/3}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{9a^{5/3}b^{5/3}d}-\frac{2\left(a^{4/3}-b^{4/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}b^{5/3}d}-\frac{2\left(a^{4/3}+b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] (-2*(a^(4/3) + b^(4/3))*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*
a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(5/3)*d) - (2*(a^(4/3) - b^(4/3))*Log[a^(1/
3) + b^(1/3)*Sin[c + d*x]])/(9*a^(5/3)*b^(5/3)*d) + ((a^(4/3) - b^(4/3))*Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2])/(9*a^(5
/3)*b^(5/3)*d) + (Sin[c + d*x]*(b - a*Sin[c + d*x] - 2*b*Sin[c + d*x]^2))/(
3*a*b*d*(a + b*Sin[c + d*x]^3))
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{(a+bx^3)^2}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\sin(c+dx)\left(b-a\sin(c+dx)-2b\sin^2(c+dx)\right)}{3abd\left(a+b\sin^3(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-2b^2-2abx}{a+bx^3}\,dx, x, \sin(c+dx)\right)}{3ab^2d}$$

$$= \frac{\sin(c+dx)\left(b-a\sin(c+dx)-2b\sin^2(c+dx)\right)}{3abd\left(a+b\sin^3(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a}\left(-2a^{4/3}b-4b^{7/3}\right)+\sqrt[3]{b}\left(-2a^{4/3}b+\right.}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\right.}{9a^{5/3}b^{7/3}d}$$

$$= -\frac{2\left(a^{4/3}-b^{4/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}b^{5/3}d} + \frac{\sin(c+dx)\left(b-a\sin(c+dx)-2b\sin^2(c+\right.}{3abd\left(a+b\sin^3(c+dx)\right)}$$

$$= -\frac{2\left(a^{4/3}-b^{4/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}b^{5/3}d} + \frac{\left(a^{4/3}-b^{4/3}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)\right.}{9a^{5/3}b^{5/3}d}$$

$$= -\frac{2\left(a^{4/3}+b^{4/3}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}a^{5/3}b^{5/3}d} - \frac{2\left(a^{4/3}-b^{4/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}b^{5/3}d} + \frac{\left(a^{4/3}\right.}{}$$

Mathematica [C] time = 1.03697, size = 258, normalized size = 1.08

$$\frac{-\frac{2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{a^{5/3}\sqrt[3]{b}} + \frac{4\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{a^{5/3}\sqrt[3]{b}} - \frac{4\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{5/3}\sqrt[3]{b}} + \frac{9\sin^2(c+dx)\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{b\sin^3(c+dx)}{a}\right)}{ab}}{18d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] ((-4*Sqrt[3]*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1/3))])/
(a^(5/3)*b^(1/3)) + (4*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]])/(a^(5/3)*b^(1/3
)) - (2*Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2
])/(a^(5/3)*b^(1/3)) + (9*Hypergeometric2F1[2/3, 1, 5/3, -((b*Sin[c + d*x]^
3)/a)]*Sin[c + d*x]^2)/(a*b) - (9*Hypergeometric2F1[2/3, 2, 5/3, -((b*Sin[c
 + d*x]^3)/a)]*Sin[c + d*x]^2)/(a*b) + 12/(b*(a + b*Sin[c + d*x]^3)) + (6*S
in[c + d*x])/(a*(a + b*Sin[c + d*x]^3)))/(18*d)
```

Maple [A] time = 0.145, size = 327, normalized size = 1.4

$$-\frac{(\sin(dx+c))^2}{3bd\left(a+b(\sin(dx+c))^3\right)} + \frac{\sin(dx+c)}{3da\left(a+b(\sin(dx+c))^3\right)} + \frac{2}{3bd\left(a+b(\sin(dx+c))^3\right)} + \frac{2}{9abd}\ln\left(\sin(dx+c)+\sqrt[3]{\frac{a}{b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+b*sin(d*x+c)^3)^2,x)
```

```
[Out] -1/3/d/b/(a+b*sin(d*x+c)^3)*sin(d*x+c)^2+1/3*sin(d*x+c)/a/d/(a+b*sin(d*x+c)
^3)+2/3/d/b/(a+b*sin(d*x+c)^3)+2/9/d/b/a/(a/b)^(2/3)*ln(sin(d*x+c)+(a/b)^(1
```

```
/3))-1/9/d/b/a/(a/b)^(2/3)*ln(sin(d*x+c)^2-(a/b)^(1/3)*sin(d*x+c)+(a/b)^(2/
3))+2/9/d/b/a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*sin(d*x
+c)-1))-2/9/d/b^2/(a/b)^(1/3)*ln(sin(d*x+c)+(a/b)^(1/3))+1/9/d/b^2/(a/b)^(1
/3)*ln(sin(d*x+c)^2-(a/b)^(1/3)*sin(d*x+c)+(a/b)^(2/3))+2/9/d/b^2*3^(1/2)/(
a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*sin(d*x+c)-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+b*sin(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19461, size = 308, normalized size = 1.29

$$-\frac{\frac{2\left(a\left(-\frac{a}{b}\right)^{\frac{1}{3}}+b\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\sin(dx+c)\right|\right)}{a^2b}+\frac{3\left(a\sin(dx+c)^2-b\sin(dx+c)-2a\right)}{\left(b\sin(dx+c)^3+a\right)ab}-\frac{2\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^2-\left(-ab^2\right)^{\frac{2}{3}}a\right)\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\sin(dx+c)\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^2b^3}-\cdots}{9d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] -1/9*(2*(a*(-a/b)^(1/3) + b)*(-a/b)^(1/3)*log(abs(-(-a/b)^(1/3) + sin(d*x +
 c)))/(a^2*b) + 3*(a*sin(d*x + c)^2 - b*sin(d*x + c) - 2*a)/((b*sin(d*x + c
)^3 + a)*a*b) - 2*sqrt(3)*((-a*b^2)^(1/3)*b^2 - (-a*b^2)^(2/3)*a)*arctan(1/
3*sqrt(3)*((-a/b)^(1/3) + 2*sin(d*x + c))/(-a/b)^(1/3))/(a^2*b^3) - ((-a*b^
2)^(1/3)*b^2 + (-a*b^2)^(2/3)*a)*log(sin(d*x + c)^2 + (-a/b)^(1/3)*sin(d*x
+ c) + (-a/b)^(2/3))/(a^2*b^3))/d
```

3.395 $\int \frac{\cos^3(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$

Optimal. Leaf size=183

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}d}+\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}d}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}\sqrt[3]{b}d}+\frac{a+}{3abd\,(a}$$

```
[Out] (-2*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3
]*a^(5/3)*b^(1/3)*d) + (2*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]])/(9*a^(5/3)*b
^(1/3)*d) - Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*
x]^2]/(9*a^(5/3)*b^(1/3)*d) + (a + b*Sin[c + d*x])/(3*a*b*d*(a + b*Sin[c +
d*x]^3))
```

Rubi [A] time = 0.162416, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3223, 1854, 12, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}d}+\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}d}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}\sqrt[3]{b}d}+\frac{a+}{3abd\,(a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] (-2*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3
]*a^(5/3)*b^(1/3)*d) + (2*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]])/(9*a^(5/3)*b
^(1/3)*d) - Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*
x]^2]/(9*a^(5/3)*b^(1/3)*d) + (a + b*Sin[c + d*x])/(3*a*b*d*(a + b*Sin[c +
d*x]^3))
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1-x^2}{\left(a+bx^3\right)^2}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{a+b\sin(c+dx)}{3abd\left(a+b\sin^3(c+dx)\right)} - \frac{\text{Subst}\left(\int -\frac{2}{a+bx^3}\,dx, x, \sin(c+dx)\right)}{3ad}$$

$$= \frac{a+b\sin(c+dx)}{3abd\left(a+b\sin^3(c+dx)\right)} + \frac{2\,\text{Subst}\left(\int \frac{1}{a+bx^3}\,dx, x, \sin(c+dx)\right)}{3ad}$$

$$= \frac{a+b\sin(c+dx)}{3abd\left(a+b\sin^3(c+dx)\right)} + \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx, x, \sin(c+dx)\right)}{9a^{5/3}d} + \frac{2\,\text{Subst}\left(\int \frac{2\sqrt[3]{a}-}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}}\right.}{9}$$

$$= \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}d} + \frac{a+b\sin(c+dx)}{3abd\left(a+b\sin^3(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x\right.}{3a^{4/3}d}$$

$$= \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}d} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}d} + \frac{a+}{3abd\,(a}$$

$$= -\frac{2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}a^{5/3}\sqrt[3]{b}d} + \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}d} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+\right.}{9a^{5/3}\sqrt[3]{b}d}$$

Mathematica [A] time = 0.842328, size = 184, normalized size = 1.01

$$\frac{\frac{-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{a^{5/3}}+\frac{2b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{a^{5/3}}+\frac{3}{a+b\sin^3(c+dx)}}{b}-\frac{2\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{5/3}\sqrt[3]{b}}+\frac{3\sin(c+dx)}{a\left(a+b\sin^3(c+dx)\right)}}{9d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] ((-2*Sqrt[3]*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1/3))])/
(a^(5/3)*b^(1/3)) + (3*Sin[c + d*x])/(a*(a + b*Sin[c + d*x]^3)) + ((2*b^(2/
3)*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]])/a^(5/3) - (b^(2/3)*Log[a^(2/3) - a^
(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2])/a^(5/3) + 3/(a + b*Si
n[c + d*x]^3))/b)/(9*d)
```

Maple [A] time = 0.141, size = 179, normalized size = 1.

$$\frac{\sin(dx+c)}{3\,da\left(a+b\left(\sin(dx+c)\right)^3\right)} + \frac{2}{9\,abd}\ln\left(\sin(dx+c)+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{1}{9\,abd}\ln\left(\left(\sin(dx+c)\right)^2-\sqrt[3]{\frac{a}{b}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*sin(d*x+c)^3)^2,x)
```

```
[Out] 1/3*sin(d*x+c)/a/d/(a+b*sin(d*x+c)^3)+2/9/d/b/a/(a/b)^(2/3)*ln(sin(d*x+c)+(
a/b)^(1/3))-1/9/d/b/a/(a/b)^(2/3)*ln(sin(d*x+c)^2-(a/b)^(1/3)*sin(d*x+c)+(a
/b)^(2/3))+2/9/d/b/a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*
sin(d*x+c)-1))+1/3/d/b/(a+b*sin(d*x+c)^3)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.70672, size = 1609, normalized size = 8.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] [1/9*(3*a^2*b*sin(d*x + c) + 3*a^3 + 3*sqrt(1/3)*(a^2*b - (a*b^2*cos(d*x +
c)^2 - a*b^2)*sin(d*x + c))*sqrt(-(a^2*b)^(1/3)/b)*log(-(3*(a^2*b)^(1/3)*a*
sin(d*x + c) + a^2 + 3*sqrt(1/3)*(2*a*b*cos(d*x + c)^2 - 2*a*b - (a^2*b)^(2
/3)*sin(d*x + c) + (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b) + 2*(a*b*cos(d*x
 + c)^2 - a*b)*sin(d*x + c))/((b*cos(d*x + c)^2 - b)*sin(d*x + c) - a)) + (
a^2*b)^(2/3)*((b*cos(d*x + c)^2 - b)*sin(d*x + c) - a)*log(-a*b*cos(d*x + c
)^2 + a*b - (a^2*b)^(2/3)*sin(d*x + c) + (a^2*b)^(1/3)*a) - 2*(a^2*b)^(2/3)
*((b*cos(d*x + c)^2 - b)*sin(d*x + c) - a)*log(a*b*sin(d*x + c) + (a^2*b)^(
2/3)))/(a^4*b*d - (a^3*b^2*d*cos(d*x + c)^2 - a^3*b^2*d)*sin(d*x + c)), 1/9
*(3*a^2*b*sin(d*x + c) + 3*a^3 + 6*sqrt(1/3)*(a^2*b - (a*b^2*cos(d*x + c)^2
 - a*b^2)*sin(d*x + c))*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(
2/3)*sin(d*x + c) - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) + (a^2*b)^(
2/3)*((b*cos(d*x + c)^2 - b)*sin(d*x + c) - a)*log(-a*b*cos(d*x + c)^2 + a*
b - (a^2*b)^(2/3)*sin(d*x + c) + (a^2*b)^(1/3)*a) - 2*(a^2*b)^(2/3)*((b*cos
(d*x + c)^2 - b)*sin(d*x + c) - a)*log(a*b*sin(d*x + c) + (a^2*b)^(2/3)))/(
a^4*b*d - (a^3*b^2*d*cos(d*x + c)^2 - a^3*b^2*d)*sin(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*sin(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18884, size = 228, normalized size = 1.25

$$-\frac{\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\sin(dx+c)\right|\right)}{a^2}-\frac{2\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\sin(dx+c)\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^2b}-\frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(\sin(dx+c)^2+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{a^2b}-\frac{3\,(b\sin(dx+c)+a)}{(b\sin(dx+c)^3+a)ab}}{9\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] -1/9*(2*(-a/b)^(1/3)*log(abs(-(-a/b)^(1/3) + sin(d*x + c)))/a^2 - 2*sqrt(3)
*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*((-a/b)^(1/3) + 2*sin(d*x + c))/(-a/b)^(
1/3))/(a^2*b) - (-a*b^2)^(1/3)*log(sin(d*x + c)^2 + (-a/b)^(1/3)*sin(d*x +
c) + (-a/b)^(2/3))/(a^2*b) - 3*(b*sin(d*x + c) + a)/((b*sin(d*x + c)^3 + a)
*a*b))/d
```

3.396 $\int \frac{\cos(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$

Optimal. Leaf size=176

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}d}+\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}d}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}\sqrt[3]{b}d}+\frac{}{3ad}$$

```
[Out] (-2*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3
]*a^(5/3)*b^(1/3)*d) + (2*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]])/(9*a^(5/3)*b
^(1/3)*d) - Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*
x]^2]/(9*a^(5/3)*b^(1/3)*d) + Sin[c + d*x]/(3*a*d*(a + b*Sin[c + d*x]^3))
```

Rubi [A] time = 0.115185, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3223, 199, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}d}+\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}d}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}\sqrt[3]{b}d}+\frac{}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] (-2*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3
]*a^(5/3)*b^(1/3)*d) + (2*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]])/(9*a^(5/3)*b
^(1/3)*d) - Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*
x]^2]/(9*a^(5/3)*b^(1/3)*d) + Sin[c + d*x]/(3*a*d*(a + b*Sin[c + d*x]^3))
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+bx^3\right)^2}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\sin(c+dx)}{3ad\left(a+b\sin^3(c+dx)\right)} + \frac{2\,\text{Subst}\left(\int \frac{1}{a+bx^3}\,dx, x, \sin(c+dx)\right)}{3ad}$$

$$= \frac{\sin(c+dx)}{3ad\left(a+b\sin^3(c+dx)\right)} + \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx, x, \sin(c+dx)\right)}{9a^{5/3}d} + \frac{2\,\text{Subst}\left(\int \frac{2\sqrt[3]{a}-\sqrt[3]{\cdots}}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x\cdots}\cdots\right.}{9a\cdots}$$

$$= \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}d} + \frac{\sin(c+dx)}{3ad\left(a+b\sin^3(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, \cdots\right.}{3a^{4/3}d}$$

$$= \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}d} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}d} + \frac{\text{s}\cdots}{3ad\left(a\cdots\right.}$$

$$= -\frac{2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}a^{5/3}\sqrt[3]{b}d} + \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}d} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+\cdots\right.}{9a^{5/3}\sqrt[3]{b}d}$$

Mathematica [A] time = 0.474991, size = 152, normalized size = 0.86

$$\frac{\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)-\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{\sqrt[3]{b}}+\frac{3a^{2/3}\sin(c+dx)}{a+b\sin^3(c+dx)}-\frac{2\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{9a^{5/3}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] ((-2*Sqrt[3]*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1/3))])/
b^(1/3) + (2*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]] - Log[a^(2/3) - a^(1/3)*b^
(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2])/b^(1/3) + (3*a^(2/3)*Sin[c +
d*x])/(a + b*Sin[c + d*x]^3))/(9*a^(5/3)*d)
```

Maple [A] time = 0.064, size = 157, normalized size = 0.9

$$\frac{\sin(dx+c)}{3\,da\left(a+b\left(\sin(dx+c)\right)^3\right)}+\frac{2}{9\,abd}\ln\left(\sin(dx+c)+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{1}{9\,abd}\ln\left(\left(\sin(dx+c)\right)^2-\sqrt[3]{\frac{a}{b}}\sin(dx+c)+\left(\frac{a}{b}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*sin(d*x+c)^3)^2,x)
```

```
[Out] 1/3*sin(d*x+c)/a/d/(a+b*sin(d*x+c)^3)+2/9/d/b/a/(a/b)^(2/3)*ln(sin(d*x+c)+(
a/b)^(1/3))-1/9/d/b/a/(a/b)^(2/3)*ln(sin(d*x+c)^2-(a/b)^(1/3)*sin(d*x+c)+(a
/b)^(2/3))+2/9/d/b/a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*
sin(d*x+c)-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.73682, size = 1588, normalized size = 9.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] [1/9*(3*a^2*b*sin(d*x + c) + 3*sqrt(1/3)*(a^2*b - (a*b^2*cos(d*x + c)^2 - a
*b^2)*sin(d*x + c))*sqrt(-(a^2*b)^(1/3)/b)*log(-(3*(a^2*b)^(1/3)*a*sin(d*x
+ c) + a^2 + 3*sqrt(1/3)*(2*a*b*cos(d*x + c)^2 - 2*a*b - (a^2*b)^(2/3)*sin(
```

```
d*x + c) + (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b) + 2*(a*b*cos(d*x + c)^2
- a*b)*sin(d*x + c))/((b*cos(d*x + c)^2 - b)*sin(d*x + c) - a)) + (a^2*b)^(
2/3)*((b*cos(d*x + c)^2 - b)*sin(d*x + c) - a)*log(-a*b*cos(d*x + c)^2 + a*
b - (a^2*b)^(2/3)*sin(d*x + c) + (a^2*b)^(1/3)*a) - 2*(a^2*b)^(2/3)*((b*cos
(d*x + c)^2 - b)*sin(d*x + c) - a)*log(a*b*sin(d*x + c) + (a^2*b)^(2/3)))/(
a^4*b*d - (a^3*b^2*d*cos(d*x + c)^2 - a^3*b^2*d)*sin(d*x + c)), 1/9*(3*a^2*
b*sin(d*x + c) + 6*sqrt(1/3)*(a^2*b - (a*b^2*cos(d*x + c)^2 - a*b^2)*sin(d*
x + c))*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*sin(d*x + c
) - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) + (a^2*b)^(2/3)*((b*cos(d*x
 + c)^2 - b)*sin(d*x + c) - a)*log(-a*b*cos(d*x + c)^2 + a*b - (a^2*b)^(2/3
)*sin(d*x + c) + (a^2*b)^(1/3)*a) - 2*(a^2*b)^(2/3)*((b*cos(d*x + c)^2 - b)
*sin(d*x + c) - a)*log(a*b*sin(d*x + c) + (a^2*b)^(2/3)))/(a^4*b*d - (a^3*b
^2*d*cos(d*x + c)^2 - a^3*b^2*d)*sin(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22908, size = 219, normalized size = 1.24

$$-\frac{\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\sin(dx+c)\right|\right)}{a^{2}}-\frac{3\sin(dx+c)}{\left(b\sin(dx+c)^{3}+a\right)a}-\frac{2\sqrt{3}\left(-ab^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\sin(dx+c)\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^{2}b}-\frac{\left(-ab^{2}\right)^{\frac{1}{3}}\log\left(\sin(dx+c)^{2}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\right.\right.}{a^{2}b}}{9d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] -1/9*(2*(-a/b)^(1/3)*log(abs(-(-a/b)^(1/3) + sin(d*x + c)))/a^2 - 3*sin(d*x
 + c)/((b*sin(d*x + c)^3 + a)*a) - 2*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt
(3)*((-a/b)^(1/3) + 2*sin(d*x + c))/(-a/b)^(1/3))/(a^2*b) - (-a*b^2)^(1/3)*
log(sin(d*x + c)^2 + (-a/b)^(1/3)*sin(d*x + c) + (-a/b)^(2/3))/(a^2*b))/d
```

3.397 $\int \frac{\sec(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$

Optimal. Leaf size=587

$$\frac{b(a-\sin(c+dx)(b-a\sin(c+dx)))}{3ad\left(a^2-b^2\right)\left(a+b\sin^3(c+dx)\right)}+\frac{\sqrt[3]{b}\left(2a^{2/3}b^{4/3}+a^2+b^2\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{6\sqrt[3]{a}d\left(a^2-b^2\right)^2}+\ldots$$

```
[Out] -(b^(1/3)*(a^(4/3) - 2*b^(4/3))*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(
Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*(a^2 - b^2)*d) - (b^(1/3)*(a^2 - 2*a^
(2/3)*b^(4/3) + b^2)*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(
1/3))])/(Sqrt[3]*a^(1/3)*(a^2 - b^2)^2*d) - Log[1 - Sin[c + d*x]]/(2*(a + b
)^2*d) + Log[1 + Sin[c + d*x]]/(2*(a - b)^2*d) - (b^(1/3)*(a^(4/3) + 2*b^(4
/3))*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]])/(9*a^(5/3)*(a^2 - b^2)*d) - (b^(1
/3)*(a^2 + 2*a^(2/3)*b^(4/3) + b^2)*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]])/(3
*a^(1/3)*(a^2 - b^2)^2*d) + (b^(1/3)*(a^(4/3) + 2*b^(4/3))*Log[a^(2/3) - a^
(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2])/(18*a^(5/3)*(a^2 - b^
2)*d) + (b^(1/3)*(a^2 + 2*a^(2/3)*b^(4/3) + b^2)*Log[a^(2/3) - a^(1/3)*b^(1
/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2])/(6*a^(1/3)*(a^2 - b^2)^2*d) - (
2*a*b*Log[a + b*Sin[c + d*x]^3])/(3*(a^2 - b^2)^2*d) + (b*(a - Sin[c + d*x]
*(b - a*Sin[c + d*x])))/(3*a*(a^2 - b^2)*d*(a + b*Sin[c + d*x]^3))
```

Rubi [A] time = 0.687527, antiderivative size = 587, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {3223, 2074, 1854, 1860, 31, 634, 617, 204, 628, 1871, 260}

$$\frac{b(a-\sin(c+dx)(b-a\sin(c+dx)))}{3ad\left(a^2-b^2\right)\left(a+b\sin^3(c+dx)\right)}+\frac{\sqrt[3]{b}\left(2a^{2/3}b^{4/3}+a^2+b^2\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{6\sqrt[3]{a}d\left(a^2-b^2\right)^2}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] -(b^(1/3)*(a^(4/3) - 2*b^(4/3))*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(
Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*(a^2 - b^2)*d) - (b^(1/3)*(a^2 - 2*a^
(2/3)*b^(4/3) + b^2)*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(
1/3))])/(Sqrt[3]*a^(1/3)*(a^2 - b^2)^2*d) - Log[1 - Sin[c + d*x]]/(2*(a + b
)^2*d) + Log[1 + Sin[c + d*x]]/(2*(a - b)^2*d) - (b^(1/3)*(a^(4/3) + 2*b^(4
/3))*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]])/(9*a^(5/3)*(a^2 - b^2)*d) - (b^(1
/3)*(a^2 + 2*a^(2/3)*b^(4/3) + b^2)*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]])/(3
*a^(1/3)*(a^2 - b^2)^2*d) + (b^(1/3)*(a^(4/3) + 2*b^(4/3))*Log[a^(2/3) - a^
(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2])/(18*a^(5/3)*(a^2 - b^
2)*d) + (b^(1/3)*(a^2 + 2*a^(2/3)*b^(4/3) + b^2)*Log[a^(2/3) - a^(1/3)*b^(1
/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2])/(6*a^(1/3)*(a^2 - b^2)^2*d) - (
2*a*b*Log[a + b*Sin[c + d*x]^3])/(3*(a^2 - b^2)^2*d) + (b*(a - Sin[c + d*x]
*(b - a*Sin[c + d*x])))/(3*a*(a^2 - b^2)*d*(a + b*Sin[c + d*x]^3))
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
```

```
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\left(a+bx^3\right)^2}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{2(a+b)^2(-1+x)} + \frac{1}{2(a-b)^2(1+x)} + \frac{b\left(b-ax+bx^2\right)}{\left(-a^2+b^2\right)\left(a+bx^3\right)^2} + \frac{b\left(-2ab+\left(a^2+b^2\right)x-2abx^2\right)}{\left(a^2-b^2\right)^2\left(a+bx^3\right)}\right) dx, x, \text{si}\right.}{d}$$

$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)^2d} + \frac{\log(1+\sin(c+dx))}{2(a-b)^2d} + \frac{b\,\text{Subst}\left(\int \frac{-2ab+\left(a^2+b^2\right)x-2abx^2}{a+bx^3}\,dx, x, \sin\right.}{\left(a^2-b^2\right)^2 d}$$

$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)^2d} + \frac{\log(1+\sin(c+dx))}{2(a-b)^2d} + \frac{b(a-\sin(c+dx)(b-a\sin(c+dx)))}{3a\left(a^2-b^2\right)d\left(a+b\sin^3(c+dx)\right)} +$$

$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)^2d} + \frac{\log(1+\sin(c+dx))}{2(a-b)^2d} - \frac{2ab\log\left(a+b\sin^3(c+dx)\right)}{3\left(a^2-b^2\right)^2 d} + \frac{b(a-\sin}{3a\left(a^2-\right.}$$

$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)^2d} + \frac{\log(1+\sin(c+dx))}{2(a-b)^2d} - \frac{\sqrt[3]{b}\left(a^{4/3}+2b^{4/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+\right.}{9a^{5/3}\left(a^2-b^2\right)d}$$

$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)^2d} + \frac{\log(1+\sin(c+dx))}{2(a-b)^2d} - \frac{\sqrt[3]{b}\left(a^{4/3}+2b^{4/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+\right.}{9a^{5/3}\left(a^2-b^2\right)d}$$

$$= -\frac{\sqrt[3]{b}\left(a^{4/3}-2b^{4/3}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}a^{5/3}\left(a^2-b^2\right)d} - \frac{\sqrt[3]{b}\left(a^2-2a^{2/3}b^{4/3}+b^2\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right.}{\sqrt{3}\sqrt[3]{a}\left(a^2-b^2\right)^2 d}$$

Mathematica [C] time = 4.36755, size = 503, normalized size = 0.86

$$\frac{\frac{9b\left(a^2+b^2\right)\sin^2(c+dx)\,_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{b\sin^3(c+dx)}{a}\right)}{a\left(a^2-b^2\right)^2} + \frac{9b\sin^2(c+dx)\,_2F_1\left(\frac{2}{3},2;\frac{5}{3};-\frac{b\sin^3(c+dx)}{a}\right)}{a^3-ab^2} - \frac{6b^2\sin(c+dx)}{a\left(a^2-b^2\right)\left(a+b\sin^3(c+dx)\right)} + \frac{6b}{\left(a^2-b^2\right)\left(a+b\sin^3(c+dx)\right)} - \cdots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] ((-9*Log[1 - Sin[c + d*x]])/(a + b)^2 + (9*Log[1 + Sin[c + d*x]])/(a - b)^2
 - (12*a^(1/3)*b^(5/3)*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]])/(a^2 - b^2)^2 +
```

```
 (6*a^(1/3)*b^(5/3)*(2*Sqrt[3]*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(S
qrt[3]*a^(1/3))] + Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin
[c + d*x]^2]))/(a^2 - b^2)^2 + (2*b^(5/3)*(2*Sqrt[3]*ArcTan[(a^(1/3) - 2*b^
(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1/3))] - 2*Log[a^(1/3) + b^(1/3)*Sin[c + d*
x]] + Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2])
)/(a^(5/3)*(a^2 - b^2)) - (12*a*b*Log[a + b*Sin[c + d*x]^3])/(a^2 - b^2)^2
+ (9*b*(a^2 + b^2)*Hypergeometric2F1[2/3, 1, 5/3, -((b*Sin[c + d*x]^3)/a)]*
Sin[c + d*x]^2)/(a*(a^2 - b^2)^2) + (9*b*Hypergeometric2F1[2/3, 2, 5/3, -((
b*Sin[c + d*x]^3)/a)]*Sin[c + d*x]^2)/(a^3 - a*b^2) + (6*b)/((a^2 - b^2)*(a
 + b*Sin[c + d*x]^3)) - (6*b^2*Sin[c + d*x])/(a*(a^2 - b^2)*(a + b*Sin[c +
d*x]^3)))/(18*d)
```

Maple [A] time = 0.182, size = 934, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sin(d*x+c)^3)^2,x)
```

```
[Out] -1/2/d/(a+b)^2*ln(sin(d*x+c)-1)+1/3/d*b/(a-b)^2/(a+b)^2/(a+b*sin(d*x+c)^3)*
sin(d*x+c)^2*a^2-1/3/d*b^3/(a-b)^2/(a+b)^2/(a+b*sin(d*x+c)^3)*sin(d*x+c)^2-
1/3/d*b^2/(a-b)^2/(a+b)^2/(a+b*sin(d*x+c)^3)*sin(d*x+c)*a+1/3/d*b^4/(a-b)^2
/(a+b)^2/(a+b*sin(d*x+c)^3)/a*sin(d*x+c)+1/3/d*b/(a-b)^2/(a+b)^2/(a+b*sin(d
*x+c)^3)*a^2-1/3/d*b^3/(a-b)^2/(a+b)^2/(a+b*sin(d*x+c)^3)-8/9/d*b/(a-b)^2/(
a+b)^2*a/(a/b)^(2/3)*ln(sin(d*x+c)+(a/b)^(1/3))+2/9/d*b^3/(a-b)^2/(a+b)^2/a
/(a/b)^(2/3)*ln(sin(d*x+c)+(a/b)^(1/3))+4/9/d*b/(a-b)^2/(a+b)^2*a/(a/b)^(2/
3)*ln(sin(d*x+c)^2-(a/b)^(1/3)*sin(d*x+c)+(a/b)^(2/3))-1/9/d*b^3/(a-b)^2/(a
+b)^2/a/(a/b)^(2/3)*ln(sin(d*x+c)^2-(a/b)^(1/3)*sin(d*x+c)+(a/b)^(2/3))-8/9
/d*b/(a-b)^2/(a+b)^2*a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3
)*sin(d*x+c)-1))+2/9/d*b^3/(a-b)^2/(a+b)^2/a/(a/b)^(2/3)*3^(1/2)*arctan(1/3
*3^(1/2)*(2/(a/b)^(1/3)*sin(d*x+c)-1))-4/9/d/(a-b)^2/(a+b)^2*a^2/(a/b)^(1/3
)*ln(sin(d*x+c)+(a/b)^(1/3))-2/9/d*b^2/(a-b)^2/(a+b)^2/(a/b)^(1/3)*ln(sin(d
*x+c)+(a/b)^(1/3))+2/9/d/(a-b)^2/(a+b)^2*a^2/(a/b)^(1/3)*ln(sin(d*x+c)^2-(a
/b)^(1/3)*sin(d*x+c)+(a/b)^(2/3))+1/9/d*b^2/(a-b)^2/(a+b)^2/(a/b)^(1/3)*ln(
sin(d*x+c)^2-(a/b)^(1/3)*sin(d*x+c)+(a/b)^(2/3))+4/9/d/(a-b)^2/(a+b)^2*a^2*
3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*sin(d*x+c)-1))+2/9/d*
b^2/(a-b)^2/(a+b)^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*s
in(d*x+c)-1))-2/3/d*b/(a-b)^2/(a+b)^2*a*ln(a+b*sin(d*x+c)^3)+1/2*ln(1+sin(d
*x+c))/(a-b)^2/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 22.6878, size = 22565, normalized size = 38.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/324*(216*a^3*b - 216*a*b^3 - 108*(a^3*b - a*b^3)*cos(d*x + c)^2 - 2*((a^6
 - 2*a^4*b^2 + a^2*b^4)*d - ((a^5*b - 2*a^3*b^3 + a*b^5)*d*cos(d*x + c)^2 -
 (a^5*b - 2*a^3*b^3 + a*b^5)*d)*sin(d*x + c))*(4*(9*a^2*b^2/(a^4*d - 2*a^2*
b^2*d + b^4*d)^2 - b^2/(a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3)
 + 1)/(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2
 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a
^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4
*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*b
^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 +
 a^2*b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d
^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4
+ b^6)*b/((a^2 - b^2)^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d -
2*a^2*b^2*d + b^4*d))*log(-56*a^5*b^2 + 20*a^3*b^4 + 1/324*(2*a^11 - 3*a^9*
b^2 + a^5*b^6)*(4*(9*a^2*b^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2
 - 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d - 2
*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)
*(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^
2*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^
2 - b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d
)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^
2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^
3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3
))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d - 2*a^2*b^2*d + b^4*d))^2*d^2 - 1
/9*(12*a^8*b + 22*a^6*b^3 - 8*a^4*b^5 + a^2*b^7)*(4*(9*a^2*b^2/(a^4*d - 2*a
^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt
(3) + 1)/(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*
d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(
8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*
a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^
3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^
2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^
9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b
^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d
 - 2*a^2*b^2*d + b^4*d))*d + 4*(8*a^6*b + 28*a^4*b^3 - 10*a^2*b^5 + b^7)*si
n(d*x + c)) - (324*a^3*b - ((a^6 - 2*a^4*b^2 + a^2*b^4)*d - ((a^5*b - 2*a^3
*b^3 + a*b^5)*d*cos(d*x + c)^2 - (a^5*b - 2*a^3*b^3 + a*b^5)*d)*sin(d*x + c
))*(4*(9*a^2*b^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 - 2*a^4*b^2
*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d +
 b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*
a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*
b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4*a
^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 4/81*a
*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)
) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^3) + 4/729*(
8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3))^(1/3)*(I*
sqrt(3) + 1) + 108*a*b/(a^4*d - 2*a^2*b^2*d + b^4*d)) + 3*sqrt(1/3)*((a^6 -
 2*a^4*b^2 + a^2*b^4)*d - ((a^5*b - 2*a^3*b^3 + a*b^5)*d*cos(d*x + c)^2 - (
a^5*b - 2*a^3*b^3 + a*b^5)*d)*sin(d*x + c))*sqrt((29808*a^4*b^2 + 10368*a^2
*b^4 - 5184*b^6 - (a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8)*(4*(
9*a^2*b^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 - 2*a^4*b^2*d^2 +
a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d)
^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^2
*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^3
) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3)
)^(1/3) + 81*(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((
```

a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/7
29*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6 +
 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3))^(1/3)*(I*sqrt(3)
 + 1) + 108*a*b/(a^4*d - 2*a^2*b^2*d + b^4*d))^2*d^2 + 216*(a^7*b - 2*a^5*b
^3 + a^3*b^5)*(4*(9*a^2*b^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2
- 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d - 2*
a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*
(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2
*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2
 - b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d)
^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^2
*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^3
) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3)
)^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d - 2*a^2*b^2*d + b^4*d))*d)/((a^10
- 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8)*d^2)) - 324*(a^2*b^2*cos(d*x
 + c)^2 - a^2*b^2)*sin(d*x + c))*log(56*a^5*b^2 - 20*a^3*b^4 - 1/324*(2*a^1
1 - 3*a^9*b^2 + a^5*b^6)*(4*(9*a^2*b^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2 - b^
2/(a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/
(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^
2*b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3
- 2*a^7*b^2*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b
^6)*b/((a^2 - b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2
*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d
- 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 +
a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 - b^2)
^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d - 2*a^2*b^2*d + b^4*d))
^2*d^2 + 1/9*(12*a^8*b + 22*a^6*b^3 - 8*a^4*b^5 + a^2*b^7)*(4*(9*a^2*b^2/(a
^4*d - 2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)
)*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*
b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d))
 - 4/729*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^3) + 4/729*(8
*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3))^(1/3) + 81
*(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*
a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b
- b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2
- 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*
a*b/(a^4*d - 2*a^2*b^2*d + b^4*d))*d - 1/108*sqrt(1/3)*((2*a^11 - 3*a^9*b^2
 + a^5*b^6)*(4*(9*a^2*b^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 -
2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d - 2*a^
2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a
^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d
^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 -
 b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3
 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^2*d
 + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^3)
+ 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3))^
(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d - 2*a^2*b^2*d + b^4*d))*d^2 - 36*(6*
a^8*b - 13*a^6*b^3 + 8*a^4*b^5 - a^2*b^7)*d)*sqrt((29808*a^4*b^2 + 10368*a^
2*b^4 - 5184*b^6 - (a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8)*(4*
(9*a^2*b^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 - 2*a^4*b^2*d^2 +
 a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d
)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^
2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^
3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3
))^(1/3) + 81*(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/(
(a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/
729*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6
+ 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3))^(1/3)*(I*sqrt(3
) + 1) + 108*a*b/(a^4*d - 2*a^2*b^2*d + b^4*d))^2*d^2 + 216*(a^7*b - 2*a^5*

b^3 + a^3*b^5)*(4*(9*a^2*b^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d - 2*a^2*b^2*d + b^4*d))*d)/((a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8)*d^2)) + 8*(8*a^6*b + 28*a^4*b^3 - 10*a^2*b^5 + b^7)*sin(d*x + c)) - (324*a^3*b - ((a^6 - 2*a^4*b^2 + a^2*b^4)*d - ((a^5*b - 2*a^3*b^3 + a*b^5)*d*cos(d*x + c)^2 - (a^5*b - 2*a^3*b^3 + a*b^5)*d)*sin(d*x + c))*(4*(9*a^2*b^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d - 2*a^2*b^2*d + b^4*d)) - 3*sqrt(1/3)*((a^6 - 2*a^4*b^2 + a^2*b^4)*d - ((a^5*b - 2*a^3*b^3 + a*b^5)*d*cos(d*x + c)^2 - (a^5*b - 2*a^3*b^3 + a*b^5)*d)*sin(d*x + c))*sqrt((29808*a^4*b^2 + 10368*a^2*b^4 - 5184*b^6 - (a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8)*(4*(9*a^2*b^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d - 2*a^2*b^2*d + b^4*d))^2*d^2 + 216*(a^7*b - 2*a^5*b^3 + a^3*b^5)*(4*(9*a^2*b^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d - 2*a^2*b^2*d + b^4*d))*d)/((a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8)*d^2)) - 324*(a^2*b^2*cos(d*x + c)^2 - a^2*b^2)*sin(d*x + c))*log(-56*a^5*b^2 + 20*a^3*b^4 + 1/324*(2*a^11 - 3*a^9*b^2 + a^5*b^6)*(4*(9*a^2*b^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d - 2*a^2*b^2*d + b^4*d))^2*d^2 - 1/9*(12*a^8*b + 22*a^6*b^3 - 8*a^4*b^5

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 + a^2*b^7)*(4*(9*a^2*b^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 -
2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d - 2*a^
2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a
^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d
^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 -
 b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3
 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^2*d
 + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^3)
+ 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3))^
(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d - 2*a^2*b^2*d + b^4*d))*d - 1/108*sq
rt(1/3)*((2*a^11 - 3*a^9*b^2 + a^5*b^6)*(4*(9*a^2*b^2/(a^4*d - 2*a^2*b^2*d
+ b^4*d)^2 - b^2/(a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/
(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a
^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b -
 b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2 -
 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^
4*d - 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b
^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 - 2
*a^7*b^2*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6)
*b/((a^2 - b^2)^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d - 2*a^2*
b^2*d + b^4*d))*d^2 - 36*(6*a^8*b - 13*a^6*b^3 + 8*a^4*b^5 - a^2*b^7)*d)*sq
rt((29808*a^4*b^2 + 10368*a^2*b^4 - 5184*b^6 - (a^10 - 4*a^8*b^2 + 6*a^6*b^
4 - 4*a^4*b^6 + a^2*b^8)*(4*(9*a^2*b^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2 - b^
2/(a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/
(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^
2*b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3
- 2*a^7*b^2*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b
^6)*b/((a^2 - b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2
*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d
- 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 +
a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 - b^2)
^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d - 2*a^2*b^2*d + b^4*d))
^2*d^2 + 216*(a^7*b - 2*a^5*b^3 + a^3*b^5)*(4*(9*a^2*b^2/(a^4*d - 2*a^2*b^2
*d + b^4*d)^2 - b^2/(a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) +
1)/(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 -
2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*
b - b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^
2 - 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/
(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^
2*b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3
- 2*a^7*b^2*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b
^6)*b/((a^2 - b^2)^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d - 2*a
^2*b^2*d + b^4*d))*d)/((a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8)
*d^2)) - 8*(8*a^6*b + 28*a^4*b^3 - 10*a^2*b^5 + b^7)*sin(d*x + c)) + 162*(a
^4 + 2*a^3*b + a^2*b^2 + (a^3*b + 2*a^2*b^2 + a*b^3 - (a^3*b + 2*a^2*b^2 +
a*b^3)*cos(d*x + c)^2)*sin(d*x + c))*log(sin(d*x + c) + 1) - 162*(a^4 - 2*a
^3*b + a^2*b^2 + (a^3*b - 2*a^2*b^2 + a*b^3 - (a^3*b - 2*a^2*b^2 + a*b^3)*c
os(d*x + c)^2)*sin(d*x + c))*log(-sin(d*x + c) + 1) - 108*(a^2*b^2 - b^4)*s
in(d*x + c))/((a^6 - 2*a^4*b^2 + a^2*b^4)*d - ((a^5*b - 2*a^3*b^3 + a*b^5)*
d*cos(d*x + c)^2 - (a^5*b - 2*a^3*b^3 + a*b^5)*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c)**3)**2,x)
```

[Out] Timed out

Giac [A] time = 1.26537, size = 760, normalized size = 1.29

$$-\frac{\frac{12\,ab\log\left(\left|b\sin(dx+c)^3+a\right|\right)}{a^4-2\,a^2b^2+b^4}+\frac{4\left(2\,a^8b^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}-3\,a^6b^4\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^2b^8\left(-\frac{a}{b}\right)^{\frac{1}{3}}-4\,a^7b^3+9\,a^5b^5-6\,a^3b^7+ab^9\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\sin(dx+c)\right|\right)}{a^{11}b-4\,a^9b^3+6\,a^7b^5-4\,a^5b^7+a^3b^9}+\frac{12\left(\left(2\,a^3+a\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] -1/18*(12*a*b*log(abs(b*sin(d*x + c)^3 + a))/(a^4 - 2*a^2*b^2 + b^4) + 4*(2
*a^8*b^2*(-a/b)^(1/3) - 3*a^6*b^4*(-a/b)^(1/3) + a^2*b^8*(-a/b)^(1/3) - 4*a
^7*b^3 + 9*a^5*b^5 - 6*a^3*b^7 + a*b^9)*(-a/b)^(1/3)*log(abs(-(-a/b)^(1/3)
+ sin(d*x + c)))/(a^11*b - 4*a^9*b^3 + 6*a^7*b^5 - 4*a^5*b^7 + a^3*b^9) + 1
2*((2*a^3 + a*b^2)*(-a*b^2)^(2/3) + (4*a^2*b^2 - b^4)*(-a*b^2)^(1/3))*arcta
n(1/3*sqrt(3)*((-a/b)^(1/3) + 2*sin(d*x + c))/(-a/b)^(1/3))/(sqrt(3)*a^6*b
- 2*sqrt(3)*a^4*b^3 + sqrt(3)*a^2*b^5) - 2*((2*a^3 + a*b^2)*(-a*b^2)^(2/3)
- (4*a^2*b^2 - b^4)*(-a*b^2)^(1/3))*log(sin(d*x + c)^2 + (-a/b)^(1/3)*sin(d
*x + c) + (-a/b)^(2/3))/(a^6*b - 2*a^4*b^3 + a^2*b^5) - 9*log(abs(sin(d*x +
 c) + 1))/(a^2 - 2*a*b + b^2) + 9*log(abs(sin(d*x + c) - 1))/(a^2 + 2*a*b +
 b^2) - 6*(2*a^2*b^2*sin(d*x + c)^3 + a^3*b*sin(d*x + c)^2 - a*b^3*sin(d*x
+ c)^2 - a^2*b^2*sin(d*x + c) + b^4*sin(d*x + c) + 3*a^3*b - a*b^3)/((a^5 -
 2*a^3*b^2 + a*b^4)*(b*sin(d*x + c)^3 + a)))/d
```

3.398 $\int \frac{\sec^3(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$

Optimal. Leaf size=747

$$-\frac{b\left(a\left(a^2+2b^2\right)-b\sin(c+dx)\left(2a^2-3ab\sin(c+dx)+b^2\right)\right)}{3ad\left(a^2-b^2\right)^2\left(a+b\sin^3(c+dx)\right)}-\frac{b^{5/3}\left(3a^{4/3}b^{2/3}+4a^2+2b^2\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)\right.}{18a^{5/3}d\left(a^2-b^2\right)^2}$$

```
[Out] -(b^(5/3)*(4*a^2 - 3*a^(4/3)*b^(2/3) + 2*b^2)*ArcTan[(a^(1/3) - 2*b^(1/3)*S
in[c + d*x])/(Sqrt[3]*a^(1/3))]])/(3*Sqrt[3]*a^(5/3)*(a^2 - b^2)^2*d) - (b^(
5/3)*(4*a^(8/3) - 9*a^2*b^(2/3) + 8*a^(2/3)*b^2 - 3*b^(8/3))*ArcTan[(a^(1/3
) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1/3))]])/(Sqrt[3]*a^(1/3)*(a^2 - b^2
)^3*d) - ((a + 7*b)*Log[1 - Sin[c + d*x]])/(4*(a + b)^3*d) + ((a - 7*b)*Log
[1 + Sin[c + d*x]])/(4*(a - b)^3*d) + (b^(5/3)*(4*a^2 + 3*a^(4/3)*b^(2/3) +
 2*b^2)*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]])/(9*a^(5/3)*(a^2 - b^2)^2*d) +
(b^(5/3)*(3*b^(2/3)*(3*a^2 + b^2) + 4*a^(2/3)*(a^2 + 2*b^2))*Log[a^(1/3) +
b^(1/3)*Sin[c + d*x]])/(3*a^(1/3)*(a^2 - b^2)^3*d) - (b^(5/3)*(4*a^2 + 3*a^
(4/3)*b^(2/3) + 2*b^2)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)
*Sin[c + d*x]^2])/(18*a^(5/3)*(a^2 - b^2)^2*d) - (b^(5/3)*(3*b^(2/3)*(3*a^2
 + b^2) + 4*a^(2/3)*(a^2 + 2*b^2))*Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*
x] + b^(2/3)*Sin[c + d*x]^2])/(6*a^(1/3)*(a^2 - b^2)^3*d) + (2*a*b*(a^2 + 5
*b^2)*Log[a + b*Sin[c + d*x]^3])/(3*(a^2 - b^2)^3*d) + 1/(4*(a + b)^2*d*(1
- Sin[c + d*x])) - 1/(4*(a - b)^2*d*(1 + Sin[c + d*x])) - (b*(a*(a^2 + 2*b^
2) - b*Sin[c + d*x]*(2*a^2 + b^2 - 3*a*b*Sin[c + d*x])))/(3*a*(a^2 - b^2)^2
*d*(a + b*Sin[c + d*x]^3))
```

Rubi [A] time = 1.02152, antiderivative size = 747, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3223, 2074, 1854, 1860, 31, 634, 617, 204, 628, 1871, 260}

$$-\frac{b\left(a\left(a^2+2b^2\right)-b\sin(c+dx)\left(2a^2-3ab\sin(c+dx)+b^2\right)\right)}{3ad\left(a^2-b^2\right)^2\left(a+b\sin^3(c+dx)\right)}-\frac{b^{5/3}\left(3a^{4/3}b^{2/3}+4a^2+2b^2\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)\right.}{18a^{5/3}d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Sin[c + d*x]^3)^2,x]
```


```
[Out] -(b^(5/3)*(4*a^2 - 3*a^(4/3)*b^(2/3) + 2*b^2)*ArcTan[(a^(1/3) - 2*b^(1/3)*S
in[c + d*x])/(Sqrt[3]*a^(1/3))]])/(3*Sqrt[3]*a^(5/3)*(a^2 - b^2)^2*d) - (b^(
5/3)*(4*a^(8/3) - 9*a^2*b^(2/3) + 8*a^(2/3)*b^2 - 3*b^(8/3))*ArcTan[(a^(1/3
) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1/3))]])/(Sqrt[3]*a^(1/3)*(a^2 - b^2
)^3*d) - ((a + 7*b)*Log[1 - Sin[c + d*x]])/(4*(a + b)^3*d) + ((a - 7*b)*Log
[1 + Sin[c + d*x]])/(4*(a - b)^3*d) + (b^(5/3)*(4*a^2 + 3*a^(4/3)*b^(2/3) +
 2*b^2)*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]])/(9*a^(5/3)*(a^2 - b^2)^2*d) +
(b^(5/3)*(3*b^(2/3)*(3*a^2 + b^2) + 4*a^(2/3)*(a^2 + 2*b^2))*Log[a^(1/3) +
b^(1/3)*Sin[c + d*x]])/(3*a^(1/3)*(a^2 - b^2)^3*d) - (b^(5/3)*(4*a^2 + 3*a^
(4/3)*b^(2/3) + 2*b^2)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)
*Sin[c + d*x]^2])/(18*a^(5/3)*(a^2 - b^2)^2*d) - (b^(5/3)*(3*b^(2/3)*(3*a^2
 + b^2) + 4*a^(2/3)*(a^2 + 2*b^2))*Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*
x] + b^(2/3)*Sin[c + d*x]^2])/(6*a^(1/3)*(a^2 - b^2)^3*d) + (2*a*b*(a^2 + 5
*b^2)*Log[a + b*Sin[c + d*x]^3])/(3*(a^2 - b^2)^3*d) + 1/(4*(a + b)^2*d*(1
- Sin[c + d*x])) - 1/(4*(a - b)^2*d*(1 + Sin[c + d*x])) - (b*(a*(a^2 + 2*b^
2) - b*Sin[c + d*x]*(2*a^2 + b^2 - 3*a*b*Sin[c + d*x])))/(3*a*(a^2 - b^2)^2
```

```
*d*(a + b*Sin[c + d*x]^3))
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2\left(a+bx^3\right)^2}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{4(a+b)^2(-1+x)^2} + \frac{-a-7b}{4(a+b)^3(-1+x)} + \frac{1}{4(a-b)^2(1+x)^2} + \frac{a-7b}{4(a-b)^3(1+x)} + \frac{b^2\left(2a^2+b^2-3abx+\left(a^2+2b^2\right)x^2\right)}{\left(a^2-b^2\right)^2\left(a+bx^3\right)^2}\right.\right.}{d}$$

$$= -\frac{(a+7b)\log(1-\sin(c+dx))}{4(a+b)^3d} + \frac{(a-7b)\log(1+\sin(c+dx))}{4(a-b)^3d} + \frac{1}{4(a+b)^2d(1-\sin(c+dx)}$$

$$= -\frac{(a+7b)\log(1-\sin(c+dx))}{4(a+b)^3d} + \frac{(a-7b)\log(1+\sin(c+dx))}{4(a-b)^3d} + \frac{1}{4(a+b)^2d(1-\sin(c+dx)}$$

$$= -\frac{(a+7b)\log(1-\sin(c+dx))}{4(a+b)^3d} + \frac{(a-7b)\log(1+\sin(c+dx))}{4(a-b)^3d} + \frac{2ab\left(a^2+5b^2\right)\log\left(a+b\,s\right.}{3\left(a^2-b^2\right)^3 d}$$

$$= -\frac{(a+7b)\log(1-\sin(c+dx))}{4(a+b)^3d} + \frac{(a-7b)\log(1+\sin(c+dx))}{4(a-b)^3d} + \frac{b^{5/3}\left(4a^2+3a^{4/3}b^{2/3}+2b^2\right.}{9a^{5/3}\left(a\right.}$$

$$= -\frac{(a+7b)\log(1-\sin(c+dx))}{4(a+b)^3d} + \frac{(a-7b)\log(1+\sin(c+dx))}{4(a-b)^3d} + \frac{b^{5/3}\left(4a^2+3a^{4/3}b^{2/3}+2b^2\right.}{9a^{5/3}\left(a\right.}$$

$$= -\frac{b^{5/3}\left(4a^2-3a^{4/3}b^{2/3}+2b^2\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}a^{5/3}\left(a^2-b^2\right)^2 d} - \frac{b^{5/3}\left(4a^{8/3}-9a^2b^{2/3}+8a^{2/3}b^2-3b^{8/3}\right)}{\sqrt{3}\sqrt[3]{a}\left(a^2-b^2\right)^3}$$

Mathematica [C] time = 6.37817, size = 657, normalized size = 0.88

$$-\frac{3b^3\left(3a^2+b^2\right)\sin^2(c+dx)\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{b\sin^3(c+dx)}{a}\right)}{2a\left(a^2-b^2\right)^3} - \frac{3b^3\sin^2(c+dx)\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};-\frac{b\sin^3(c+dx)}{a}\right)}{2a\left(a^2-b^2\right)^2} + \frac{ab^2\left(\frac{b^2}{a^2}+2\right)\sin(c+dx)}{3\left(a^2-b^2\right)^2\left(a+b\sin^3(c+dx)\right)} - \frac{b\left(a^2+2b^2\right)}{3\left(a^2-b^2\right)^2\left(a+b\sin^3(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] (-((a + 7*b)*Log[1 - Sin[c + d*x]])/(4*(a + b)^3) + ((a - 7*b)*Log[1 + Sin[
c + d*x]])/(4*(a - b)^3) + (4*a^(1/3)*b^(5/3)*(a^2 + 2*b^2)*Log[a^(1/3) + b
^(1/3)*Sin[c + d*x]])/(3*(a^2 - b^2)^3) - (2*a^(1/3)*(a^2 + 2*b^2)*(2*Sqrt[
3]*b^(5/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1/3))] + b
^(5/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2]
))/(3*(a^2 - b^2)^3) + ((2 + b^2/a^2)*(2*a^(1/3)*b^(5/3)*Log[a^(1/3) + b^(1
/3)*Sin[c + d*x]] - a^(1/3)*(2*Sqrt[3]*b^(5/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*
Sin[c + d*x])/(Sqrt[3]*a^(1/3))] + b^(5/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Si
n[c + d*x] + b^(2/3)*Sin[c + d*x]^2])))/(9*(a^2 - b^2)^2) + (2*a*b*(a^2 + 5
*b^2)*Log[a + b*Sin[c + d*x]^3])/(3*(a^2 - b^2)^3) + 1/(4*(a + b)^2*(1 - Si
n[c + d*x])) - (3*b^3*(3*a^2 + b^2)*Hypergeometric2F1[2/3, 1, 5/3, -((b*Sin
[c + d*x]^3)/a)]*Sin[c + d*x]^2)/(2*a*(a^2 - b^2)^3) - (3*b^3*Hypergeometri
c2F1[2/3, 2, 5/3, -((b*Sin[c + d*x]^3)/a)]*Sin[c + d*x]^2)/(2*a*(a^2 - b^2)
^2) - 1/(4*(a - b)^2*(1 + Sin[c + d*x])) - (b*(a^2 + 2*b^2))/(3*(a^2 - b^2)
^2*(a + b*Sin[c + d*x]^3)) + (a*b^2*(2 + b^2/a^2)*Sin[c + d*x])/(3*(a^2 - b
^2)^2*(a + b*Sin[c + d*x]^3)))/d
```

Maple [B] time = 0.208, size = 1309, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*sin(d*x+c)^3)^2,x)
```

```
[Out] -5/3/d*b^2/(a-b)^3/(a+b)^3*a^2/(a/b)^(1/3)*ln(sin(d*x+c)^2-(a/b)^(1/3)*sin(
d*x+c)+(a/b)^(2/3))+2/3/d*b^2/(a-b)^3/(a+b)^3/(a+b*sin(d*x+c)^3)*a^3*sin(d*
x+c)-2/3/d*b^4/(a-b)^3/(a+b)^3*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a
/b)^(1/3)*sin(d*x+c)-1))-1/4/d/(a+b)^2/(sin(d*x+c)-1)-1/4/(a-b)^2/d/(1+sin(
d*x+c))-2/9/d*b^5/(a-b)^3/(a+b)^3/a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*
(2/(a/b)^(1/3)*sin(d*x+c)-1))+16/9/d*b/(a-b)^3/(a+b)^3*a^3/(a/b)^(2/3)*3^(1
/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*sin(d*x+c)-1))-10/3/d*b^2/(a-b)^3/(a+
b)^3*a^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*sin(d*x+c)-1
))+22/9/d*b^3/(a-b)^3/(a+b)^3*a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(
a/b)^(1/3)*sin(d*x+c)-1))-1/4/d/(a+b)^3*ln(sin(d*x+c)-1)*a-7/4/d/(a+b)^3*ln
(sin(d*x+c)-1)*b+1/4/d/(a-b)^3*ln(1+sin(d*x+c))*a-7/4/d/(a-b)^3*ln(1+sin(d*
x+c))*b-1/3/d*b^3/(a-b)^3/(a+b)^3/(a+b*sin(d*x+c)^3)*a^2+2/3/d*b^4/(a-b)^3/
(a+b)^3/(a/b)^(1/3)*ln(sin(d*x+c)+(a/b)^(1/3))-1/3/d*b^4/(a-b)^3/(a+b)^3/(a
/b)^(1/3)*ln(sin(d*x+c)^2-(a/b)^(1/3)*sin(d*x+c)+(a/b)^(2/3))+2/3/d*b/(a-b)
^3/(a+b)^3*a^3*ln(a+b*sin(d*x+c)^3)+10/3/d*b^3/(a-b)^3/(a+b)^3*a*ln(a+b*sin
(d*x+c)^3)+1/d*b^5/(a-b)^3/(a+b)^3/(a+b*sin(d*x+c)^3)*sin(d*x+c)^2-1/3/d*b/
(a-b)^3/(a+b)^3/(a+b*sin(d*x+c)^3)*a^4+16/9/d*b/(a-b)^3/(a+b)^3*a^3/(a/b)^(
2/3)*ln(sin(d*x+c)+(a/b)^(1/3))+22/9/d*b^3/(a-b)^3/(a+b)^3*a/(a/b)^(2/3)*ln
(sin(d*x+c)+(a/b)^(1/3))-2/9/d*b^5/(a-b)^3/(a+b)^3/a/(a/b)^(2/3)*ln(sin(d*x
+c)+(a/b)^(1/3))-8/9/d*b/(a-b)^3/(a+b)^3*a^3/(a/b)^(2/3)*ln(sin(d*x+c)^2-(a
/b)^(1/3)*sin(d*x+c)+(a/b)^(2/3))-11/9/d*b^3/(a-b)^3/(a+b)^3*a/(a/b)^(2/3)*
ln(sin(d*x+c)^2-(a/b)^(1/3)*sin(d*x+c)+(a/b)^(2/3))+1/9/d*b^5/(a-b)^3/(a+b)
^3/a/(a/b)^(2/3)*ln(sin(d*x+c)^2-(a/b)^(1/3)*sin(d*x+c)+(a/b)^(2/3))-1/d*b^
3/(a-b)^3/(a+b)^3/(a+b*sin(d*x+c)^3)*sin(d*x+c)^2*a^2-1/3/d*b^4/(a-b)^3/(a+
b)^3/(a+b*sin(d*x+c)^3)*a*sin(d*x+c)-1/3/d*b^6/(a-b)^3/(a+b)^3/(a+b*sin(d*x
+c)^3)/a*sin(d*x+c)+10/3/d*b^2/(a-b)^3/(a+b)^3*a^2/(a/b)^(1/3)*ln(sin(d*x+c
)+(a/b)^(1/3))+2/3/d*b^5/(a-b)^3/(a+b)^3/(a+b*sin(d*x+c)^3)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sin(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27934, size = 1064, normalized size = 1.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 1/36*(8*(15*a^10*b^4*(-a/b)^(1/3) - 42*a^8*b^6*(-a/b)^(1/3) + 36*a^6*b^8*(-
a/b)^(1/3) - 6*a^4*b^10*(-a/b)^(1/3) - 3*a^2*b^12*(-a/b)^(1/3) - 8*a^11*b^3
 + 13*a^9*b^5 + 10*a^7*b^7 - 28*a^5*b^9 + 14*a^3*b^11 - a*b^13)*(-a/b)^(1/3
)*log(abs(-(-a/b)^(1/3) + sin(d*x + c)))/(a^15*b - 6*a^13*b^3 + 15*a^11*b^5
 - 20*a^9*b^7 + 15*a^7*b^9 - 6*a^5*b^11 + a^3*b^13) + 24*(3*(5*a^3*b + a*b^
3)*(-a*b^2)^(2/3) + (8*a^4*b + 11*a^2*b^3 - b^5)*(-a*b^2)^(1/3))*arctan(1/3
*sqrt(3)*((-a/b)^(1/3) + 2*sin(d*x + c))/(-a/b)^(1/3))/(sqrt(3)*a^8 - 3*sqr
t(3)*a^6*b^2 + 3*sqrt(3)*a^4*b^4 - sqrt(3)*a^2*b^6) - 4*(3*(5*a^3*b + a*b^3
)*(-a*b^2)^(2/3) - (8*a^4*b + 11*a^2*b^3 - b^5)*(-a*b^2)^(1/3))*log(sin(d*x
 + c)^2 + (-a/b)^(1/3)*sin(d*x + c) + (-a/b)^(2/3))/(a^8 - 3*a^6*b^2 + 3*a^
4*b^4 - a^2*b^6) + 24*(a^3*b + 5*a*b^3)*log(abs(b*sin(d*x + c)^3 + a))/(a^6
 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) + 9*(a - 7*b)*log(abs(sin(d*x + c) + 1))/(a
```

```
^3 - 3*a^2*b + 3*a*b^2 - b^3) - 9*(a + 7*b)*log(abs(sin(d*x + c) - 1))/(a^3
 + 3*a^2*b + 3*a*b^2 + b^3) - 6*(3*a^3*b*sin(d*x + c)^4 + 9*a*b^3*sin(d*x +
 c)^4 - 10*a^2*b^2*sin(d*x + c)^3 - 2*b^4*sin(d*x + c)^3 + 2*a^3*b*sin(d*x
+ c)^2 - 2*a*b^3*sin(d*x + c)^2 + 3*a^4*sin(d*x + c) + 7*a^2*b^2*sin(d*x +
c) + 2*b^4*sin(d*x + c) - 8*a^3*b - 4*a*b^3)/((b*sin(d*x + c)^5 - b*sin(d*x
 + c)^3 + a*sin(d*x + c)^2 - a)*(a^5 - 2*a^3*b^2 + a*b^4)))/d
```

3.399 $\int \frac{\cos^4(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{\cos^4(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2},x\right)$$

```
[Out] Unintegrable[Cos[c + d*x]^4/(a + b*Sin[c + d*x]^3)^2, x]
```

Rubi [A] time = 0.0437858, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cos^4(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[c + d*x]^4/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] Defer[Int][Cos[c + d*x]^4/(a + b*Sin[c + d*x]^3)^2, x]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx = \int \frac{\cos^4(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$$

Mathematica [A] time = 0.351766, size = 394, normalized size = 15.76

$$\frac{\frac{24\cos(c+dx)(a+b\sin(c+dx))}{4a+3b\sin(c+dx)-b\sin(3(c+dx))} - i\text{RootSum}\left[8\#1^3a+i\#1^6b-3i\#1^4b+3i\#1^2b-ib\&, \frac{-2\#1^3a\log\left(\#1^2-2\#1\cos(c+dx)+1\right)+2\#1a\log\left(\#1^2-2\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] ((-I)*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#1^6 &
, (2*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - I*b*Log[1 - 2*Cos[c + d*x
]*#1 + #1^2] + (4*I)*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 + 2*a*Lo
g[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1 + 12*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x]
 - #1)]*#1^2 - (6*I)*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (4*I)*a*Arc
Tan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^3 - 2*a*Log[1 - 2*Cos[c + d*x]*#1
+ #1^2]*#1^3 + 2*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - I*b*Log[
1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4)/(b*#1 - (4*I)*a*#1^2 - 2*b*#1^3 + b*#1^
5) & ] + (24*Cos[c + d*x]*(a + b*Sin[c + d*x]))/(4*a + 3*b*Sin[c + d*x] - b
*Sin[3*(c + d*x)]))/(18*a*b*d)
```

Maple [A] time = 0.244, size = 550, normalized size = 22.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*sin(d*x+c)^3)^2,x)
```

```
[Out] -2/3/d/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8*b*tan(1/2*d*x+1/2
*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)/a*tan(1/2*d*x+1/2*c)^5+2/3/d/(tan(1/2*d*x
+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x
+1/2*c)^2*a+a)/b*tan(1/2*d*x+1/2*c)^4+8/3/d/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1
/2*d*x+1/2*c)^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)/a*ta
n(1/2*d*x+1/2*c)^3+4/3/d/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8
*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)/b*tan(1/2*d*x+1/2*c)^2+
2/3/d/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8*b*tan(1/2*d*x+1/2*
c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)/a*tan(1/2*d*x+1/2*c)+2/3/d/(tan(1/2*d*x+1/
2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/
2*c)^2*a+a)/b+2/9/d/a/b*sum((_R^4*b+_R^3*a+_R*a+b)/(_R^5*a+2*_R^3*a+4*_R^2*
b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2
*a+a))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*sin(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.400 $\int \frac{\cos^2(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{\cos^2(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2},x\right)$$

```
[Out] Unintegrable[Cos[c + d*x]^2/(a + b*Sin[c + d*x]^3)^2, x]
```

Rubi [A] time = 0.0425309, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cos^2(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[c + d*x]^2/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] Defer[Int][Cos[c + d*x]^2/(a + b*Sin[c + d*x]^3)^2, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx=\int \frac{\cos^2(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$$

Mathematica [A] time = 0.245492, size = 273, normalized size = 10.92

$$\frac{\frac{12\sin(2(c+dx))}{4a+3b\sin(c+dx)-b\sin(3(c+dx))}-i\text{RootSum}\left[8\#1^3a+i\#1^6b-3i\#1^4b+3i\#1^2b-ib\&,\frac{-i\#1^4\log\left(\#1^2-2\#1\cos(c+dx)+1\right)-6i\#1^2\log\left(\#1^2\right.}{}\right.}{18ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] ((-I)*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#1^6 &
, (2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - I*Log[1 - 2*Cos[c + d*x]*#1
 + #1^2] + 12*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 - (6*I)*Log[1 -
 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)
]*#1^4 - I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4)/(b*#1 - (4*I)*a*#1^2 - 2
*b*#1^3 + b*#1^5) & ] + (12*Sin[2*(c + d*x)])/(4*a + 3*b*Sin[c + d*x] - b*S
in[3*(c + d*x)]))/(18*a*d)
```

Maple [A] time = 0.247, size = 236, normalized size = 9.4

$$-\frac{2}{3\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5\left(\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6a+3\,(\tan(1/2\,dx+c/2))^4a+8\,b\,(\tan(1/2\,dx+c/2))^3+3\,(\tan(1/2\,dx+c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*sin(d*x+c)^3)^2,x)
```

```
[Out] -2/3/d/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8*b*tan(1/2*d*x+1/2
*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)/a*tan(1/2*d*x+1/2*c)^5+2/3/d/(tan(1/2*d*x
+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x
+1/2*c)^2*a+a)/a*tan(1/2*d*x+1/2*c)+2/9/d/a*sum((_R^4+1)/(_R^5*a+2*_R^3*a+4
*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+
3*_Z^2*a+a))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*sin(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2}{\left(b\sin(dx+c)^3+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/(b*sin(d*x + c)^3 + a)^2, x)
```

3.401 $\int \frac{1}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{1}{\left(a+b\sin^3(c+dx)\right)^2},x\right)$$

```
[Out] Unintegrable[(a + b*Sin[c + d*x]^3)^(-2), x]
```

Rubi [A] time = 0.0118343, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[c + d*x]^3)^(-2),x]
```

```
[Out] Defer[Int][(a + b*Sin[c + d*x]^3)^(-2), x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sin^3(c+dx)\right)^2}\,dx = \int \frac{1}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$$

Mathematica [A] time = 0.459565, size = 502, normalized size = 31.38

$$-\frac{12b\cos(c+dx)(a\cos(2(c+dx))-3a+2b\sin(c+dx))}{(a-b)(a+b)(4a+3b\sin(c+dx)-b\sin(3(c+dx)))}+\frac{i\text{RootSum}\left[8\#1^3a+i\#1^6b-3i\#1^4b+3i\#1^2b-ib\&,\frac{12i\#1^2a^2\log\left(\#1^2-2\#1\cos(c+dx)+1\right)-24\#1^2a^2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#}\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x]^3)^(-2),x]
```

```
[Out] ((I*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#1^6 & ,
(2*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - I*b^2*Log[1 - 2*Cos[c + d
*x]*#1 + #1^2] + (4*I)*a*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 + 2*
a*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1 - 24*a^2*ArcTan[Sin[c + d*x]/(Cos[
c + d*x] - #1)]*#1^2 + 12*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2
 + (12*I)*a^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (6*I)*b^2*Log[1 - 2*
Cos[c + d*x]*#1 + #1^2]*#1^2 - (4*I)*a*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x]
- #1)]*#1^3 - 2*a*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^3 + 2*b^2*ArcTan[S
in[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - I*b^2*Log[1 - 2*Cos[c + d*x]*#1 + #
1^2]*#1^4)/(b*#1 - (4*I)*a*#1^2 - 2*b*#1^3 + b*#1^5) & ])/(a^2 - b^2) - (12
*b*Cos[c + d*x]*(-3*a + a*Cos[2*(c + d*x)] + 2*b*Sin[c + d*x]))/((a - b)*(a
 + b)*(4*a + 3*b*Sin[c + d*x] - b*Sin[3*(c + d*x)])))/(18*a*d)
```

Maple [A] time = 0.198, size = 658, normalized size = 41.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(d*x+c)^3)^2,x)
```

```
[Out] 2/3/d/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8*b*tan(1/2*d*x+1/2*
c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*b^2/a/(a^2-b^2)*tan(1/2*d*x+1/2*c)^5-2/3/d
/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8*b*tan(1/2*d*x+1/2*c)^3+
3*tan(1/2*d*x+1/2*c)^2*a+a)/(a^2-b^2)*b*tan(1/2*d*x+1/2*c)^4+8/3/d/(tan(1/2
*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2
*d*x+1/2*c)^2*a+a)*b^2/a/(a^2-b^2)*tan(1/2*d*x+1/2*c)^3+8/3/d/(tan(1/2*d*x+
1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+
1/2*c)^2*a+a)/(a^2-b^2)*b*tan(1/2*d*x+1/2*c)^2-2/3/d/(tan(1/2*d*x+1/2*c)^6*
a+3*tan(1/2*d*x+1/2*c)^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*
a+a)*b^2/a/(a^2-b^2)*tan(1/2*d*x+1/2*c)+2/3/d/(tan(1/2*d*x+1/2*c)^6*a+3*tan
(1/2*d*x+1/2*c)^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)/(a
^2-b^2)*b+1/9/d/a/(a^2-b^2)*sum(((3*a^2-2*b^2)*_R^4-2*_R^3*a*b+6*_R^2*a^2-2
*_R*a*b+3*a^2-2*b^2)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-
_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(dx+c\right)^{3}+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^3 + a)^(-2), x)
```

3.402 $\int \frac{\sec^2(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{\sec^2(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2},x\right)$$

```
[Out] Unintegrable[Sec[c + d*x]^2/(a + b*Sin[c + d*x]^3)^2, x]
```

Rubi [A] time = 0.0432045, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sec^2(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^2/(a + b*Sin[c + d*x]^3)^2,x]

[Out] Defer[Int][Sec[c + d*x]^2/(a + b*Sin[c + d*x]^3)^2, x]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx = \int \frac{\sec^2(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$$

Mathematica [A] time = 1.59132, size = 845, normalized size = 33.8

$$-\frac{ib\text{RootSum}\left[ib\#1^6-3ib\#1^4+8a\#1^3+3ib\#1^2-ib\&,\frac{2b^3\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^4+16a^2b\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^4-ib^3\log\left(\#1^2-2\cos(c+dx)\#1+1\right)\#1^4-8ia^2b\log\left(\#1^2-2\text{co}\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Sin[c + d*x]^3)^2,x]

[Out] (((-I)*b*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#1^6
 & , (16*a^2*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + 2*b^3*ArcTan[Sin[
c + d*x]/(Cos[c + d*x] - #1)] - (8*I)*a^2*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^
2] - I*b^3*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] + (20*I)*a^3*ArcTan[Sin[c + d*
x]/(Cos[c + d*x] - #1)]*#1 + (16*I)*a*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x]
 - #1)]*#1 + 10*a^3*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1 + 8*a*b^2*Log[1 -
2*Cos[c + d*x]*#1 + #1^2]*#1 - 120*a^2*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x]
- #1)]*#1^2 + 12*b^3*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 + (60*I)
*a^2*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (6*I)*b^3*Log[1 - 2*Cos[c +
 d*x]*#1 + #1^2]*#1^2 - (20*I)*a^3*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]
*#1^3 - (16*I)*a*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^3 - 10*a^3
```

```
*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^3 - 8*a*b^2*Log[1 - 2*Cos[c + d*x]*#1
 + #1^2]*#1^3 + 16*a^2*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 + 2*
b^3*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - (8*I)*a^2*b*Log[1 - 2*C
os[c + d*x]*#1 + #1^2]*#1^4 - I*b^3*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4)
/(b*#1 - (4*I)*a*#1^2 - 2*b*#1^3 + b*#1^5) & ])/(a*(a^2 - b^2)^2) + (18*Sin
[(c + d*x)/2])/((a + b)^2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (18*Sin[
(c + d*x)/2])/((a - b)^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (12*b*Cos
[c + d*x]*(-2*a^3 - 7*a*b^2 + 3*a*b^2*Cos[2*(c + d*x)] + 2*b*(2*a^2 + b^2)*
Sin[c + d*x]))/(a*(a - b)^2*(a + b)^2*(4*a + 3*b*Sin[c + d*x] - b*Sin[3*(c
+ d*x)])))/(18*d)
```

Maple [A] time = 0.269, size = 1276, normalized size = 51.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sin(d*x+c)^3)^2,x)
```

```
[Out] -4/3/d*b^2/(a-b)^2/(a+b)^2/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a
+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*a*tan(1/2*d*x+1/2*c)^
5-2/3/d*b^4/(a-b)^2/(a+b)^2/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*
a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)/a*tan(1/2*d*x+1/2*c)
^5-2/3/d*b/(a-b)^2/(a+b)^2/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a
+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*tan(1/2*d*x+1/2*c)^4*
a^2+8/3/d*b^3/(a-b)^2/(a+b)^2/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^
4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*tan(1/2*d*x+1/2*c)
^4-8/3/d*b^2/(a-b)^2/(a+b)^2/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4
*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*a*tan(1/2*d*x+1/2*c
)^3-16/3/d*b^4/(a-b)^2/(a+b)^2/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)
^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)/a*tan(1/2*d*x+1/2
*c)^3-4/3/d*b/(a-b)^2/(a+b)^2/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^
4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*tan(1/2*d*x+1/2*c)
^2*a^2-20/3/d*b^3/(a-b)^2/(a+b)^2/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2
*c)^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*tan(1/2*d*x+1/
2*c)^2+4/3/d*b^2/(a-b)^2/(a+b)^2/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*
c)^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*a*tan(1/2*d*x+1
/2*c)+2/3/d*b^4/(a-b)^2/(a+b)^2/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c
)^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)/a*tan(1/2*d*x+1/
2*c)-2/3/d*b/(a-b)^2/(a+b)^2/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4
*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*a^2-4/3/d*b^3/(a-b)
^2/(a+b)^2/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8*b*tan(1/2*d*x
+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)-1/9/d*b/(a-b)^2/(a+b)^2/a*sum((b*(11*
a^2-2*b^2)*_R^4+2*a*(-5*a^2-4*b^2)*_R^3+54*_R^2*a^2*b+2*a*(-5*a^2-4*b^2)*_R
+11*a^2*b-2*b^3)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_R),
_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))-1/d/(a+b)^2/(tan(1/2*d*x+1/
2*c)-1)-1/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sin(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^2}{\left(b\sin(dx+c)^3+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/(b*sin(d*x + c)^3 + a)^2, x)
```

3.403 $\int \frac{\sec^4(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{\sec^4(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2},x\right)$$

```
[Out] Unintegrable[Sec[c + d*x]^4/(a + b*Sin[c + d*x]^3)^2, x]
```

Rubi [A] time = 0.0430803, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sec^4(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^4/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] Defer[Int][Sec[c + d*x]^4/(a + b*Sin[c + d*x]^3)^2, x]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx=\int \frac{\sec^4(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$$

Mathematica [A] time = 1.70294, size = 1158, normalized size = 46.32

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] ((4*I)*b^2*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#1
^6 & , (14*a^4*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + 74*a^2*b^2*ArcTan
[Sin[c + d*x]/(Cos[c + d*x] - #1)] + 2*b^4*ArcTan[Sin[c + d*x]/(Cos[c + d*x
] - #1)] - (7*I)*a^4*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] - (37*I)*a^2*b^2*Log
[1 - 2*Cos[c + d*x]*#1 + #1^2] - I*b^4*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] +
(144*I)*a^3*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 + (36*I)*a*b^3*Ar
cTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 + 72*a^3*b*Log[1 - 2*Cos[c + d*x]
*#1 + #1^2]*#1 + 18*a*b^3*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1 - 180*a^4*Ar
cTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 - 372*a^2*b^2*ArcTan[Sin[c + d*
x]/(Cos[c + d*x] - #1)]*#1^2 + 12*b^4*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #
1)]*#1^2 + (90*I)*a^4*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + (186*I)*a^2*
b^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (6*I)*b^4*Log[1 - 2*Cos[c + d*
x]*#1 + #1^2]*#1^2 - (144*I)*a^3*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]
*#1^3 - (36*I)*a*b^3*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^3 - 72*a^3
*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^3 - 18*a*b^3*Log[1 - 2*Cos[c + d*x]
```

```
*#1 + #1^2]*#1^3 + 14*a^4*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 + 7
4*a^2*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 + 2*b^4*ArcTan[Sin[
c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - (7*I)*a^4*Log[1 - 2*Cos[c + d*x]*#1 +
#1^2]*#1^4 - (37*I)*a^2*b^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 - I*b^4*
Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4)/(b*#1 - (4*I)*a*#1^2 - 2*b*#1^3 + b
*#1^5) & ] + (3*Sec[c + d*x]^3*(48*a^5*b + 568*a^3*b^3 + 14*a*b^5 + (78*a^5
*b + 606*a^3*b^3 + 81*a*b^5)*Cos[2*(c + d*x)] + 18*a*b^3*(4*a^2 + b^2)*Cos[
4*(c + d*x)] + 2*a^5*b*Cos[6*(c + d*x)] - 30*a^3*b^3*Cos[6*(c + d*x)] - 17*
a*b^5*Cos[6*(c + d*x)] + 48*a^6*Sin[c + d*x] - 244*a^4*b^2*Sin[c + d*x] + 2
0*a^2*b^4*Sin[c + d*x] - 4*b^6*Sin[c + d*x] + 16*a^6*Sin[3*(c + d*x)] - 194
*a^4*b^2*Sin[3*(c + d*x)] - 86*a^2*b^4*Sin[3*(c + d*x)] - 6*b^6*Sin[3*(c +
d*x)] - 14*a^4*b^2*Sin[5*(c + d*x)] - 74*a^2*b^4*Sin[5*(c + d*x)] - 2*b^6*S
in[5*(c + d*x)]))/(4*a + 3*b*Sin[c + d*x] - b*Sin[3*(c + d*x)]))/(72*a*(a^2
 - b^2)^3*d)
```

Maple [A] time = 0.323, size = 1549, normalized size = 62.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*sin(d*x+c)^3)^2,x)
```

```
[Out] 2/3/d*b^2/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+
8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*a^3*tan(1/2*d*x+1/2*c)
^5+14/3/d*b^4/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^
4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*a*tan(1/2*d*x+1/2*
c)^5+2/3/d*b^6/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)
^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)/a*tan(1/2*d*x+1/2
*c)^5-6/d*b^5/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^
4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*tan(1/2*d*x+1/2*c)
^4+16/d*b^4/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*
a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*a*tan(1/2*d*x+1/2*c)
^3+8/d*b^6/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a
+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)/a*tan(1/2*d*x+1/2*c)^
3+12/d*b^3/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a
+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*tan(1/2*d*x+1/2*c)^2*
a^2+12/d*b^5/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4
*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*tan(1/2*d*x+1/2*c)^
2-2/3/d*b^2/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*
a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*a^3*tan(1/2*d*x+1/2*
c)-14/3/d*b^4/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^
4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*a*tan(1/2*d*x+1/2*
c)-2/3/d*b^6/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4
*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)/a*tan(1/2*d*x+1/2*c
)+4/d*b^3/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+
8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*a^2+2/d*b^5/(a-b)^3/(a
+b)^3/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8*b*tan(1/2*d*x+1/2*
c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)+1/9/d*b^2/(a-b)^3/(a+b)^3/a*sum(((19*a^4+2
8*a^2*b^2-2*b^4)*_R^4+18*a*b*(-4*a^2-b^2)*_R^3+6*a^2*(11*a^2+34*b^2)*_R^2+1
8*a*b*(-4*a^2-b^2)*_R+19*a^4+28*a^2*b^2-2*b^4)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R
*a)*ln(tan(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a
))-1/3/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)^3-1/2/d/(a+b)^2/(tan(1/2*d*x+1/2*c)
-1)^2-1/d/(a+b)^3/(tan(1/2*d*x+1/2*c)-1)*a-4/d/(a+b)^3/(tan(1/2*d*x+1/2*c)-
1)*b-1/3/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)^3+1/2/d/(a-b)^2/(tan(1/2*d*x+1/2*
c)+1)^2+4/d/(a-b)^3/(tan(1/2*d*x+1/2*c)+1)*b-1/d/(a-b)^3/(tan(1/2*d*x+1/2*c
)+1)*a
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sin(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^4}{\left(b\sin(dx+c)^3+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^4/(b*sin(d*x + c)^3 + a)^2, x)
```

3.404 $\int \frac{\cos^7(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=131

$$\frac{\left(\sqrt{a}+\sqrt{b}\right)^3 \tan^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{7/4}d}-\frac{\left(\sqrt{a}-\sqrt{b}\right)^3 \tanh^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{7/4}d}+\frac{\sin^3(c+dx)}{3bd}-\frac{3\sin(c+dx)}{bd}$$

```
[Out] ((Sqrt[a] + Sqrt[b])^3*ArcTan[(b^(1/4)*Sin[c + d*x])/a^(1/4)])/(2*a^(3/4)*b
^(7/4)*d) - ((Sqrt[a] - Sqrt[b])^3*ArcTanh[(b^(1/4)*Sin[c + d*x])/a^(1/4)])
/(2*a^(3/4)*b^(7/4)*d) - (3*Sin[c + d*x])/(b*d) + Sin[c + d*x]^3/(3*b*d)
```

Rubi [A] time = 0.189984, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3223, 1171, 1167, 205, 208}

$$\frac{\left(\sqrt{a}+\sqrt{b}\right)^3 \tan^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{7/4}d}-\frac{\left(\sqrt{a}-\sqrt{b}\right)^3 \tanh^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{7/4}d}+\frac{\sin^3(c+dx)}{3bd}-\frac{3\sin(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] ((Sqrt[a] + Sqrt[b])^3*ArcTan[(b^(1/4)*Sin[c + d*x])/a^(1/4)])/(2*a^(3/4)*b
^(7/4)*d) - ((Sqrt[a] - Sqrt[b])^3*ArcTanh[(b^(1/4)*Sin[c + d*x])/a^(1/4)])
/(2*a^(3/4)*b^(7/4)*d) - (3*Sin[c + d*x])/(b*d) + Sin[c + d*x]^3/(3*b*d)
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos^7(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^3}{a-bx^4}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{3}{b}+\frac{x^2}{b}+\frac{3a+b-(a+3b)x^2}{b(a-bx^4)}\right)dx, x, \sin(c+dx)\right)}{d}$$

$$= -\frac{3\sin(c+dx)}{bd}+\frac{\sin^3(c+dx)}{3bd}+\frac{\text{Subst}\left(\int \frac{3a+b+(-a-3b)x^2}{a-bx^4}\,dx, x, \sin(c+dx)\right)}{bd}$$

$$= -\frac{3\sin(c+dx)}{bd}+\frac{\sin^3(c+dx)}{3bd}-\frac{\left(\sqrt{a}-\sqrt{b}\right)^3\text{Subst}\left(\int \frac{1}{\sqrt{a}\sqrt{b}-bx^2}\,dx, x, \sin(c+dx)\right)}{2\sqrt{a}bd}-\frac{\left(\sqrt{a}+\sqrt{\ldots}\right.}{}$$

$$= \frac{\left(\sqrt{a}+\sqrt{b}\right)^3\tan^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{7/4}d}-\frac{\left(\sqrt{a}-\sqrt{b}\right)^3\tanh^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{7/4}d}-\frac{3\sin(c+dx)}{bd}+\frac{\sin^3(c\ldots}{3b\ldots}$$

Mathematica [C] time = 0.26784, size = 207, normalized size = 1.58

$$\frac{4a^{3/4}b^{3/4}\sin^3(c+dx)-36a^{3/4}b^{3/4}\sin(c+dx)+3\left(\sqrt{a}-\sqrt{b}\right)^3\log\left(\sqrt[4]{a}-\sqrt[4]{b}\sin(c+dx)\right)-3\left(\sqrt{a}-\sqrt{b}\right)^3\log\left(\sqrt[4]{a}+\sqrt[4]{b}\sin\ldots\right.}{12a^{3/4}b^{7/4}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (3*(Sqrt[a] - Sqrt[b])^3*Log[a^(1/4) - b^(1/4)*Sin[c + d*x]] + (3*I)*(Sqrt[
a] + Sqrt[b])^3*Log[a^(1/4) - I*b^(1/4)*Sin[c + d*x]] - (3*I)*(Sqrt[a] + Sq
rt[b])^3*Log[a^(1/4) + I*b^(1/4)*Sin[c + d*x]] - 3*(Sqrt[a] - Sqrt[b])^3*Lo
g[a^(1/4) + b^(1/4)*Sin[c + d*x]] - 36*a^(3/4)*b^(3/4)*Sin[c + d*x] + 4*a^(
3/4)*b^(3/4)*Sin[c + d*x]^3)/(12*a^(3/4)*b^(7/4)*d)
```

Maple [B] time = 0.084, size = 350, normalized size = 2.7

$$\frac{(\sin(dx+c))^3}{3bd}-3\frac{\sin(dx+c)}{bd}+\frac{3}{2bd}\sqrt[4]{\frac{a}{b}}\arctan\left(\sin(dx+c)\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)+\frac{1}{2da}\sqrt[4]{\frac{a}{b}}\arctan\left(\sin(dx+c)\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)+\frac{3}{4bd}\sqrt[4]{\frac{a}{b}}\ln\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7/(a-b*sin(d*x+c)^4),x)
```

```
[Out] 1/3*sin(d*x+c)^3/b/d-3*sin(d*x+c)/b/d+3/2/d/b*(a/b)^(1/4)*arctan(sin(d*x+c)
/(a/b)^(1/4))+1/2/d*(a/b)^(1/4)/a*arctan(sin(d*x+c)/(a/b)^(1/4))+3/4/d/b*(a
/b)^(1/4)*ln((sin(d*x+c)+(a/b)^(1/4))/(sin(d*x+c)-(a/b)^(1/4)))+1/4/d*(a/b)
^(1/4)/a*ln((sin(d*x+c)+(a/b)^(1/4))/(sin(d*x+c)-(a/b)^(1/4)))+1/2/d/b^2/(a
/b)^(1/4)*a*arctan(sin(d*x+c)/(a/b)^(1/4))+3/2/d/b/(a/b)^(1/4)*arctan(sin(d
*x+c)/(a/b)^(1/4))-1/4/d/b^2/(a/b)^(1/4)*a*ln((sin(d*x+c)+(a/b)^(1/4))/(sin
```

```
(d*x+c)-(a/b)^(1/4)))-3/4/d/b/(a/b)^(1/4)*ln((sin(d*x+c)+(a/b)^(1/4))/(sin(
d*x+c)-(a/b)^(1/4)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.49865, size = 3235, normalized size = 24.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b*d*sqrt(-(a*b^3*d^2*sqrt((a^6 + 30*a^5*b + 255*a^4*b^2 + 452*a^3*b
^3 + 255*a^2*b^4 + 30*a*b^5 + b^6)/(a^3*b^7*d^4)) + 6*a^2 + 20*a*b + 6*b^2)
/(a*b^3*d^2))*log(1/2*(a^6 + 12*a^5*b - 27*a^4*b^2 + 27*a^2*b^4 - 12*a*b^5
- b^6)*sin(d*x + c) + 1/2*((a^4*b^5 + 3*a^3*b^6)*d^3*sqrt((a^6 + 30*a^5*b +
 255*a^4*b^2 + 452*a^3*b^3 + 255*a^2*b^4 + 30*a*b^5 + b^6)/(a^3*b^7*d^4)) -
 (3*a^5*b^2 + 46*a^4*b^3 + 60*a^3*b^4 + 18*a^2*b^5 + a*b^6)*d)*sqrt(-(a*b^3
*d^2*sqrt((a^6 + 30*a^5*b + 255*a^4*b^2 + 452*a^3*b^3 + 255*a^2*b^4 + 30*a*
b^5 + b^6)/(a^3*b^7*d^4)) + 6*a^2 + 20*a*b + 6*b^2)/(a*b^3*d^2))) - 3*b*d*s
qrt((a*b^3*d^2*sqrt((a^6 + 30*a^5*b + 255*a^4*b^2 + 452*a^3*b^3 + 255*a^2*b
^4 + 30*a*b^5 + b^6)/(a^3*b^7*d^4)) - 6*a^2 - 20*a*b - 6*b^2)/(a*b^3*d^2))*
log(1/2*(a^6 + 12*a^5*b - 27*a^4*b^2 + 27*a^2*b^4 - 12*a*b^5 - b^6)*sin(d*x
 + c) + 1/2*((a^4*b^5 + 3*a^3*b^6)*d^3*sqrt((a^6 + 30*a^5*b + 255*a^4*b^2 +
 452*a^3*b^3 + 255*a^2*b^4 + 30*a*b^5 + b^6)/(a^3*b^7*d^4)) + (3*a^5*b^2 +
46*a^4*b^3 + 60*a^3*b^4 + 18*a^2*b^5 + a*b^6)*d)*sqrt((a*b^3*d^2*sqrt((a^6
+ 30*a^5*b + 255*a^4*b^2 + 452*a^3*b^3 + 255*a^2*b^4 + 30*a*b^5 + b^6)/(a^3
*b^7*d^4)) - 6*a^2 - 20*a*b - 6*b^2)/(a*b^3*d^2))) - 3*b*d*sqrt(-(a*b^3*d^2
*sqrt((a^6 + 30*a^5*b + 255*a^4*b^2 + 452*a^3*b^3 + 255*a^2*b^4 + 30*a*b^5
+ b^6)/(a^3*b^7*d^4)) + 6*a^2 + 20*a*b + 6*b^2)/(a*b^3*d^2))*log(-1/2*(a^6
+ 12*a^5*b - 27*a^4*b^2 + 27*a^2*b^4 - 12*a*b^5 - b^6)*sin(d*x + c) + 1/2*(
(a^4*b^5 + 3*a^3*b^6)*d^3*sqrt((a^6 + 30*a^5*b + 255*a^4*b^2 + 452*a^3*b^3
+ 255*a^2*b^4 + 30*a*b^5 + b^6)/(a^3*b^7*d^4)) - (3*a^5*b^2 + 46*a^4*b^3 +
60*a^3*b^4 + 18*a^2*b^5 + a*b^6)*d)*sqrt(-(a*b^3*d^2*sqrt((a^6 + 30*a^5*b +
 255*a^4*b^2 + 452*a^3*b^3 + 255*a^2*b^4 + 30*a*b^5 + b^6)/(a^3*b^7*d^4)) +
 6*a^2 + 20*a*b + 6*b^2)/(a*b^3*d^2))) + 3*b*d*sqrt((a*b^3*d^2*sqrt((a^6 +
30*a^5*b + 255*a^4*b^2 + 452*a^3*b^3 + 255*a^2*b^4 + 30*a*b^5 + b^6)/(a^3*b
^7*d^4)) - 6*a^2 - 20*a*b - 6*b^2)/(a*b^3*d^2))*log(-1/2*(a^6 + 12*a^5*b -
27*a^4*b^2 + 27*a^2*b^4 - 12*a*b^5 - b^6)*sin(d*x + c) + 1/2*((a^4*b^5 + 3*
a^3*b^6)*d^3*sqrt((a^6 + 30*a^5*b + 255*a^4*b^2 + 452*a^3*b^3 + 255*a^2*b^4
 + 30*a*b^5 + b^6)/(a^3*b^7*d^4)) + (3*a^5*b^2 + 46*a^4*b^3 + 60*a^3*b^4 +
18*a^2*b^5 + a*b^6)*d)*sqrt((a*b^3*d^2*sqrt((a^6 + 30*a^5*b + 255*a^4*b^2 +
 452*a^3*b^3 + 255*a^2*b^4 + 30*a*b^5 + b^6)/(a^3*b^7*d^4)) - 6*a^2 - 20*a*
b - 6*b^2)/(a*b^3*d^2))) - 4*(cos(d*x + c)^2 + 8)*sin(d*x + c))/(b*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [B] time = 6.4702, size = 486, normalized size = 3.71

$$\frac{\frac{8\left(b^2\sin(dx+c)^3-9\,b^2\sin(dx+c)\right)}{b^3}-\frac{6\sqrt{2}\left(\left(-ab^3\right)^{\frac{3}{4}}(a+3\,b)-\left(-ab^3\right)^{\frac{1}{4}}\left(3\,ab^2+b^3\right)\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sin(dx+c)\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{ab^4}-\frac{6\sqrt{2}\left(\left(-ab^3\right)^{\frac{3}{4}}(a+3\,b)-\left(-ab^3\right)^{\frac{1}{4}}\left(3\,ab^2+b^3\right.\right.}{ab^{\cdots}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/24*(8*(b^2*sin(d*x + c)^3 - 9*b^2*sin(d*x + c))/b^3 - 6*sqrt(2)*((-a*b^3)
^(3/4)*(a + 3*b) - (-a*b^3)^(1/4)*(3*a*b^2 + b^3))*arctan(1/2*sqrt(2)*(sqrt
(2)*(-a/b)^(1/4) + 2*sin(d*x + c))/(-a/b)^(1/4))/(a*b^4) - 6*sqrt(2)*((-a*b
^3)^(3/4)*(a + 3*b) - (-a*b^3)^(1/4)*(3*a*b^2 + b^3))*arctan(-1/2*sqrt(2)*(
sqrt(2)*(-a/b)^(1/4) - 2*sin(d*x + c))/(-a/b)^(1/4))/(a*b^4) + 3*sqrt(2)*((
-a*b^3)^(3/4)*(a + 3*b) + (-a*b^3)^(1/4)*(3*a*b^2 + b^3))*log(sin(d*x + c)^
2 + sqrt(2)*(-a/b)^(1/4)*sin(d*x + c) + sqrt(-a/b))/(a*b^4) - 3*sqrt(2)*((-
a*b^3)^(3/4)*(a + 3*b) + (-a*b^3)^(1/4)*(3*a*b^2 + b^3))*log(sin(d*x + c)^2
 - sqrt(2)*(-a/b)^(1/4)*sin(d*x + c) + sqrt(-a/b))/(a*b^4))/d
```

3.405 $\int \frac{\cos^5(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=113

$$\frac{\left(\sqrt{a}+\sqrt{b}\right)^2\tan^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/4}d}+\frac{\left(-2\sqrt{a}\sqrt{b}+a+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/4}d}-\frac{\sin(c+dx)}{bd}$$

```
[Out] ((Sqrt[a] + Sqrt[b])^2*ArcTan[(b^(1/4)*Sin[c + d*x])/a^(1/4)])/(2*a^(3/4)*b
^(5/4)*d) + ((a - 2*Sqrt[a]*Sqrt[b] + b)*ArcTanh[(b^(1/4)*Sin[c + d*x])/a^(
1/4)])/(2*a^(3/4)*b^(5/4)*d) - Sin[c + d*x]/(b*d)
```

Rubi [A] time = 0.161679, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3223, 1171, 1167, 205, 208}

$$\frac{\left(\sqrt{a}+\sqrt{b}\right)^2\tan^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/4}d}+\frac{\left(-2\sqrt{a}\sqrt{b}+a+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/4}d}-\frac{\sin(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] ((Sqrt[a] + Sqrt[b])^2*ArcTan[(b^(1/4)*Sin[c + d*x])/a^(1/4)])/(2*a^(3/4)*b
^(5/4)*d) + ((a - 2*Sqrt[a]*Sqrt[b] + b)*ArcTanh[(b^(1/4)*Sin[c + d*x])/a^(
1/4)])/(2*a^(3/4)*b^(5/4)*d) - Sin[c + d*x]/(b*d)
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{a-bx^4}\,dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{b}+\frac{a+b-2bx^2}{b(a-bx^4)}\right)dx, x, \sin(c+dx)\right)}{d}$$
$$= -\frac{\sin(c+dx)}{bd}+\frac{\text{Subst}\left(\int \frac{a+b-2bx^2}{a-bx^4}\,dx, x, \sin(c+dx)\right)}{bd}$$
$$= -\frac{\sin(c+dx)}{bd}-\frac{\left(2\sqrt{b}-\frac{a+b}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a}\sqrt{b}-bx^2}\,dx, x, \sin(c+dx)\right)}{2\sqrt{b}d}-\frac{\left(2\sqrt{b}+\frac{a+b}{\sqrt{a}}\right)\text{Subst}\left(\int -\right.}{2}$$
$$= \frac{\left(\sqrt{a}+\sqrt{b}\right)^2\tan^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/4}d}+\frac{\left(\sqrt{a}-\sqrt{b}\right)^2\tanh^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/4}d}-\frac{\sin(c+dx)}{bd}$$

Mathematica [C] time = 0.210647, size = 189, normalized size = 1.67

$$\frac{-4a^{3/4}\sqrt[4]{b}\sin(c+dx)+\left(\sqrt{a}-\sqrt{b}\right)^2\left(-\log\left(\sqrt[4]{a}-\sqrt[4]{b}\sin(c+dx)\right)\right)+i\left(-i\left(\sqrt{a}-\sqrt{b}\right)^2\log\left(\sqrt[4]{a}+\sqrt[4]{b}\sin(c+dx)\right)+\left(\sqrt{a}+\right.\right.}{4a^{3/4}b^{5/4}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (-((Sqrt[a] - Sqrt[b])^2*Log[a^(1/4) - b^(1/4)*Sin[c + d*x]]) + I*((Sqrt[a]
 + Sqrt[b])^2*Log[a^(1/4) - I*b^(1/4)*Sin[c + d*x]] - (Sqrt[a] + Sqrt[b])^2
*Log[a^(1/4) + I*b^(1/4)*Sin[c + d*x]] - I*(Sqrt[a] - Sqrt[b])^2*Log[a^(1/4
) + b^(1/4)*Sin[c + d*x]]) - 4*a^(3/4)*b^(1/4)*Sin[c + d*x])/(4*a^(3/4)*b^(
5/4)*d)
```

Maple [B] time = 0.069, size = 252, normalized size = 2.2

$$-\frac{\sin(dx+c)}{bd}+\frac{1}{2bd}\sqrt[4]{\frac{a}{b}}\arctan\left(\sin(dx+c)\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)+\frac{1}{2da}\sqrt[4]{\frac{a}{b}}\arctan\left(\sin(dx+c)\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)+\frac{1}{4bd}\sqrt[4]{\frac{a}{b}}\ln\left(\left(\sin(dx+c)+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a-b*sin(d*x+c)^4),x)
```

```
[Out] -sin(d*x+c)/b/d+1/2/d/b*(a/b)^(1/4)*arctan(sin(d*x+c)/(a/b)^(1/4))+1/2/d*(a
/b)^(1/4)/a*arctan(sin(d*x+c)/(a/b)^(1/4))+1/4/d/b*(a/b)^(1/4)*ln((sin(d*x+
c)+(a/b)^(1/4))/(sin(d*x+c)-(a/b)^(1/4)))+1/4/d*(a/b)^(1/4)/a*ln((sin(d*x+c
)+(a/b)^(1/4))/(sin(d*x+c)-(a/b)^(1/4)))+1/d/b/(a/b)^(1/4)*arctan(sin(d*x+c
)/(a/b)^(1/4))-1/2/d/b/(a/b)^(1/4)*ln((sin(d*x+c)+(a/b)^(1/4))/(sin(d*x+c)-
(a/b)^(1/4)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.8742, size = 2337, normalized size = 20.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/4*(b*d*sqrt(-(a*b^2*d^2*sqrt((a^4 + 12*a^3*b + 38*a^2*b^2 + 12*a*b^3 + b
^4)/(a^3*b^5*d^4)) + 4*a + 4*b)/(a*b^2*d^2))*log(1/2*(a^4 + 4*a^3*b - 10*a^
2*b^2 + 4*a*b^3 + b^4)*sin(d*x + c) + 1/2*(2*a^3*b^4*d^3*sqrt((a^4 + 12*a^3
*b + 38*a^2*b^2 + 12*a*b^3 + b^4)/(a^3*b^5*d^4)) - (a^4*b + 7*a^3*b^2 + 7*a
^2*b^3 + a*b^4)*d)*sqrt(-(a*b^2*d^2*sqrt((a^4 + 12*a^3*b + 38*a^2*b^2 + 12*
a*b^3 + b^4)/(a^3*b^5*d^4)) + 4*a + 4*b)/(a*b^2*d^2))) - b*d*sqrt((a*b^2*d^
2*sqrt((a^4 + 12*a^3*b + 38*a^2*b^2 + 12*a*b^3 + b^4)/(a^3*b^5*d^4)) - 4*a
- 4*b)/(a*b^2*d^2))*log(1/2*(a^4 + 4*a^3*b - 10*a^2*b^2 + 4*a*b^3 + b^4)*si
n(d*x + c) + 1/2*(2*a^3*b^4*d^3*sqrt((a^4 + 12*a^3*b + 38*a^2*b^2 + 12*a*b^
3 + b^4)/(a^3*b^5*d^4)) + (a^4*b + 7*a^3*b^2 + 7*a^2*b^3 + a*b^4)*d)*sqrt((
a*b^2*d^2*sqrt((a^4 + 12*a^3*b + 38*a^2*b^2 + 12*a*b^3 + b^4)/(a^3*b^5*d^4)
) - 4*a - 4*b)/(a*b^2*d^2))) - b*d*sqrt(-(a*b^2*d^2*sqrt((a^4 + 12*a^3*b +
38*a^2*b^2 + 12*a*b^3 + b^4)/(a^3*b^5*d^4)) + 4*a + 4*b)/(a*b^2*d^2))*log(-
1/2*(a^4 + 4*a^3*b - 10*a^2*b^2 + 4*a*b^3 + b^4)*sin(d*x + c) + 1/2*(2*a^3*
b^4*d^3*sqrt((a^4 + 12*a^3*b + 38*a^2*b^2 + 12*a*b^3 + b^4)/(a^3*b^5*d^4))
- (a^4*b + 7*a^3*b^2 + 7*a^2*b^3 + a*b^4)*d)*sqrt(-(a*b^2*d^2*sqrt((a^4 + 1
2*a^3*b + 38*a^2*b^2 + 12*a*b^3 + b^4)/(a^3*b^5*d^4)) + 4*a + 4*b)/(a*b^2*d
^2))) + b*d*sqrt((a*b^2*d^2*sqrt((a^4 + 12*a^3*b + 38*a^2*b^2 + 12*a*b^3 +
b^4)/(a^3*b^5*d^4)) - 4*a - 4*b)/(a*b^2*d^2))*log(-1/2*(a^4 + 4*a^3*b - 10*
a^2*b^2 + 4*a*b^3 + b^4)*sin(d*x + c) + 1/2*(2*a^3*b^4*d^3*sqrt((a^4 + 12*a
^3*b + 38*a^2*b^2 + 12*a*b^3 + b^4)/(a^3*b^5*d^4)) + (a^4*b + 7*a^3*b^2 + 7
*a^2*b^3 + a*b^4)*d)*sqrt((a*b^2*d^2*sqrt((a^4 + 12*a^3*b + 38*a^2*b^2 + 12
*a*b^3 + b^4)/(a^3*b^5*d^4)) - 4*a - 4*b)/(a*b^2*d^2))) + 4*sin(d*x + c))/(
b*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [B] time = 6.39496, size = 420, normalized size = 3.72

$$-\frac{\frac{8\sin(dx+c)}{b}-\frac{2\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}\left(ab+b^2\right)-2\left(-ab^3\right)^{\frac{3}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}+2\sin(dx+c)\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{ab^3}-\frac{2\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}\left(ab+b^2\right)-2\left(-ab^3\right)^{\frac{3}{4}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}-2\sin(dx+c)\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{ab^3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -1/8*(8*sin(d*x + c)/b - 2*sqrt(2)*((-a*b^3)^(1/4)*(a*b + b^2) - 2*(-a*b^3)
^(3/4))*arctan(1/2*sqrt(2)*(sqrt(2)*(-a/b)^(1/4) + 2*sin(d*x + c))/(-a/b)^(
1/4))/(a*b^3) - 2*sqrt(2)*((-a*b^3)^(1/4)*(a*b + b^2) - 2*(-a*b^3)^(3/4))*a
rctan(-1/2*sqrt(2)*(sqrt(2)*(-a/b)^(1/4) - 2*sin(d*x + c))/(-a/b)^(1/4))/(a
*b^3) - sqrt(2)*((-a*b^3)^(1/4)*(a*b + b^2) + 2*(-a*b^3)^(3/4))*log(sin(d*x
 + c)^2 + sqrt(2)*(-a/b)^(1/4)*sin(d*x + c) + sqrt(-a/b))/(a*b^3) + sqrt(2)
*((-a*b^3)^(1/4)*(a*b + b^2) + 2*(-a*b^3)^(3/4))*log(sin(d*x + c)^2 - sqrt(
2)*(-a/b)^(1/4)*sin(d*x + c) + sqrt(-a/b))/(a*b^3))/d
```

3.406 $\int \frac{\cos^3(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=95

$$\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}d}-\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}d}$$

```
[Out] ((Sqrt[a] + Sqrt[b])*ArcTan[(b^(1/4)*Sin[c + d*x])/a^(1/4)])/(2*a^(3/4)*b^(
3/4)*d) - ((Sqrt[a] - Sqrt[b])*ArcTanh[(b^(1/4)*Sin[c + d*x])/a^(1/4)])/(2*
a^(3/4)*b^(3/4)*d)
```

Rubi [A] time = 0.101772, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3223, 1167, 205, 208}

$$\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}d}-\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] ((Sqrt[a] + Sqrt[b])*ArcTan[(b^(1/4)*Sin[c + d*x])/a^(1/4)])/(2*a^(3/4)*b^(
3/4)*d) - ((Sqrt[a] - Sqrt[b])*ArcTanh[(b^(1/4)*Sin[c + d*x])/a^(1/4)])/(2*
a^(3/4)*b^(3/4)*d)
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1-x^2}{a-bx^4}\,dx, x, \sin(c+dx)\right)}{d}$$
$$= -\frac{\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a}\sqrt{b}-bx^2}\,dx, x, \sin(c+dx)\right)}{2d} - \frac{\left(1+\frac{\sqrt{b}}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{1}{-\sqrt{a}\sqrt{b}-bx^2}\,dx, x, \sin(c+\right.}{2d}$$
$$= \frac{\left(\sqrt{a}+\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}d} - \frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}d}$$

Mathematica [C] time = 0.081646, size = 160, normalized size = 1.68

$$\frac{\left(\sqrt{a}-\sqrt{b}\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}\sin(c+dx)\right)+i\left(\sqrt{a}+\sqrt{b}\right)\log\left(\sqrt[4]{a}-i\sqrt[4]{b}\sin(c+dx)\right)-i\left(\sqrt{a}+\sqrt{b}\right)\log\left(\sqrt[4]{a}+i\sqrt[4]{b}\sin(c+dx)\right.}{4a^{3/4}b^{3/4}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] ((Sqrt[a] - Sqrt[b])*Log[a^(1/4) - b^(1/4)*Sin[c + d*x]] + I*(Sqrt[a] + Sqr
t[b])*Log[a^(1/4) - I*b^(1/4)*Sin[c + d*x]] - I*(Sqrt[a] + Sqrt[b])*Log[a^(
1/4) + I*b^(1/4)*Sin[c + d*x]] - (Sqrt[a] - Sqrt[b])*Log[a^(1/4) + b^(1/4)*
Sin[c + d*x]])/(4*a^(3/4)*b^(3/4)*d)
```

Maple [B] time = 0.071, size = 160, normalized size = 1.7

$$\frac{1}{4\,da}\sqrt[4]{\frac{a}{b}}\ln\left(\left(\sin(dx+c)+\sqrt[4]{\frac{a}{b}}\right)\left(\sin(dx+c)-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)+\frac{1}{2\,da}\sqrt[4]{\frac{a}{b}}\arctan\left(\sin(dx+c)\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)+\frac{1}{2\,bd}\arctan\left(\sin(dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a-b*sin(d*x+c)^4),x)
```

```
[Out] 1/4/d*(a/b)^(1/4)/a*ln((sin(d*x+c)+(a/b)^(1/4))/(sin(d*x+c)-(a/b)^(1/4)))+1
/2/d*(a/b)^(1/4)/a*arctan(sin(d*x+c)/(a/b)^(1/4))+1/2/d/b/(a/b)^(1/4)*arcta
n(sin(d*x+c)/(a/b)^(1/4))-1/4/d/b/(a/b)^(1/4)*ln((sin(d*x+c)+(a/b)^(1/4))/(
sin(d*x+c)-(a/b)^(1/4)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.87559, size = 1411, normalized size = 14.85

$$\frac{1}{4}\sqrt{-\frac{abd^2\sqrt{\frac{a^2+2\,ab+b^2}{a^3b^3d^4}}+2}{abd^2}}\log\left(\frac{1}{2}\left(a^2-b^2\right)\sin\left(dx+c\right)+\frac{1}{2}\left(a^3b^2d^3\sqrt{\frac{a^2+2\,ab+b^2}{a^3b^3d^4}}-\left(a^2b+ab^2\right)d\right)\sqrt{-\frac{abd^2\sqrt{\frac{a^2+2\,ab}{a^3b^3}}}{abd^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(-(a*b*d^2*sqrt((a^2 + 2*a*b + b^2)/(a^3*b^3*d^4)) + 2)/(a*b*d^2))*
log(1/2*(a^2 - b^2)*sin(d*x + c) + 1/2*(a^3*b^2*d^3*sqrt((a^2 + 2*a*b + b^2
)/(a^3*b^3*d^4)) - (a^2*b + a*b^2)*d)*sqrt(-(a*b*d^2*sqrt((a^2 + 2*a*b + b^
2)/(a^3*b^3*d^4)) + 2)/(a*b*d^2))) - 1/4*sqrt((a*b*d^2*sqrt((a^2 + 2*a*b +
b^2)/(a^3*b^3*d^4)) - 2)/(a*b*d^2))*log(1/2*(a^2 - b^2)*sin(d*x + c) + 1/2*
(a^3*b^2*d^3*sqrt((a^2 + 2*a*b + b^2)/(a^3*b^3*d^4)) + (a^2*b + a*b^2)*d)*s
qrt((a*b*d^2*sqrt((a^2 + 2*a*b + b^2)/(a^3*b^3*d^4)) - 2)/(a*b*d^2))) - 1/4
*sqrt(-(a*b*d^2*sqrt((a^2 + 2*a*b + b^2)/(a^3*b^3*d^4)) + 2)/(a*b*d^2))*log
(-1/2*(a^2 - b^2)*sin(d*x + c) + 1/2*(a^3*b^2*d^3*sqrt((a^2 + 2*a*b + b^2)/
(a^3*b^3*d^4)) - (a^2*b + a*b^2)*d)*sqrt(-(a*b*d^2*sqrt((a^2 + 2*a*b + b^2)
/(a^3*b^3*d^4)) + 2)/(a*b*d^2))) + 1/4*sqrt((a*b*d^2*sqrt((a^2 + 2*a*b + b^
2)/(a^3*b^3*d^4)) - 2)/(a*b*d^2))*log(-1/2*(a^2 - b^2)*sin(d*x + c) + 1/2*(
a^3*b^2*d^3*sqrt((a^2 + 2*a*b + b^2)/(a^3*b^3*d^4)) + (a^2*b + a*b^2)*d)*sq
rt((a*b*d^2*sqrt((a^2 + 2*a*b + b^2)/(a^3*b^3*d^4)) - 2)/(a*b*d^2)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [B] time = 5.90468, size = 378, normalized size = 3.98

$$\frac{\frac{2\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2-\left(-ab^3\right)^{\frac{3}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sin\left(dx+c\right)\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{ab^3}+\frac{2\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2-\left(-ab^3\right)^{\frac{3}{4}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sin\left(dx+c\right)\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{ab^3}+\frac{\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2+\left(-a\right.\right.}{}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/8*(2*sqrt(2)*((-a*b^3)^(1/4)*b^2 - (-a*b^3)^(3/4))*arctan(1/2*sqrt(2)*(sq
rt(2)*(-a/b)^(1/4) + 2*sin(d*x + c))/(-a/b)^(1/4))/(a*b^3) + 2*sqrt(2)*((-a
*b^3)^(1/4)*b^2 - (-a*b^3)^(3/4))*arctan(-1/2*sqrt(2)*(sqrt(2)*(-a/b)^(1/4)
 - 2*sin(d*x + c))/(-a/b)^(1/4))/(a*b^3) + sqrt(2)*((-a*b^3)^(1/4)*b^2 + (-
a*b^3)^(3/4))*log(sin(d*x + c)^2 + sqrt(2)*(-a/b)^(1/4)*sin(d*x + c) + sqrt
(-a/b))/(a*b^3) - sqrt(2)*((-a*b^3)^(1/4)*b^2 + (-a*b^3)^(3/4))*log(sin(d*x
 + c)^2 - sqrt(2)*(-a/b)^(1/4)*sin(d*x + c) + sqrt(-a/b))/(a*b^3))/d
```

3.407 $\int \frac{\cos(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=71

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}d}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}d}$$

```
[Out] ArcTan[(b^(1/4)*Sin[c + d*x])/a^(1/4)]/(2*a^(3/4)*b^(1/4)*d) + ArcTanh[(b^(
1/4)*Sin[c + d*x])/a^(1/4)]/(2*a^(3/4)*b^(1/4)*d)
```

Rubi [A] time = 0.0673509, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3223, 212, 208, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}d}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] ArcTan[(b^(1/4)*Sin[c + d*x])/a^(1/4)]/(2*a^(3/4)*b^(1/4)*d) + ArcTanh[(b^(
1/4)*Sin[c + d*x])/a^(1/4)]/(2*a^(3/4)*b^(1/4)*d)
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{a-bx^4}\,dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a}-\sqrt{b}x^2}\,dx, x, \sin(c+dx)\right)}{2\sqrt{a}d} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a}+\sqrt{b}x^2}\,dx, x, \sin(c+dx)\right)}{2\sqrt{a}d}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}d} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}d}$$

Mathematica [A] time = 0.0235646, size = 54, normalized size = 0.76

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right) + \tanh^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (ArcTan[(b^(1/4)*Sin[c + d*x])/a^(1/4)] + ArcTanh[(b^(1/4)*Sin[c + d*x])/a^
(1/4)])/(2*a^(3/4)*b^(1/4)*d)
```

Maple [A] time = 0.033, size = 81, normalized size = 1.1

$$\frac{1}{4da}\sqrt[4]{\frac{a}{b}}\ln\left(\left(\sin(dx+c)+\sqrt[4]{\frac{a}{b}}\right)\left(\sin(dx+c)-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right)+\frac{1}{2da}\sqrt[4]{\frac{a}{b}}\arctan\left(\sin(dx+c)\frac{1}{\sqrt[4]{\frac{a}{b}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a-b*sin(d*x+c)^4),x)
```

```
[Out] 1/4/d*(a/b)^(1/4)/a*ln((sin(d*x+c)+(a/b)^(1/4))/(sin(d*x+c)-(a/b)^(1/4)))+1
/2/d*(a/b)^(1/4)/a*arctan(sin(d*x+c)/(a/b)^(1/4))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 16.0517, size = 165, normalized size = 2.32

$$\begin{cases} \frac{\tilde{\infty} x \cos(c)}{\sin^4(c)} & \text{for } a = 0 \wedge b = 0 \wedge d = 0 \\ \frac{\sin(c+dx)}{ad} & \text{for } b = 0 \\ \frac{x\cos(c)}{a-b\sin^4(c)} & \text{for } d = 0 \\ \frac{1}{3bd\sin^3(c+dx)} & \text{for } a = 0 \\ -\frac{\log\left(-\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sin(c+dx)\right)}{4a^{\frac{3}{4}}b^2d\left(\frac{1}{b}\right)^{\frac{7}{4}}} + \frac{\log\left(\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sin(c+dx)\right)}{4a^{\frac{3}{4}}b^2d\left(\frac{1}{b}\right)^{\frac{7}{4}}} + \frac{\operatorname{atan}\left(\frac{\sin(c+dx)}{\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}}\right)}{2a^{\frac{3}{4}}b^2d\left(\frac{1}{b}\right)^{\frac{7}{4}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Piecewise((zoo*x*cos(c)/sin(c)**4, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (sin(c
+ d*x)/(a*d), Eq(b, 0)), (x*cos(c)/(a - b*sin(c)**4), Eq(d, 0)), (1/(3*b*d*
sin(c + d*x)**3), Eq(a, 0)), (-log(-a**(1/4)*(1/b)**(1/4) + sin(c + d*x))/(
4*a**(3/4)*b**2*d*(1/b)**(7/4)) + log(a**(1/4)*(1/b)**(1/4) + sin(c + d*x))
/(4*a**(3/4)*b**2*d*(1/b)**(7/4)) + atan(sin(c + d*x)/(a**(1/4)*(1/b)**(1/4
)))/(2*a**(3/4)*b**2*d*(1/b)**(7/4)), True))
```

Giac [B] time = 6.58507, size = 302, normalized size = 4.25

$$\frac{\frac{2\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}+2\sin(dx+c)\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{ab} + \frac{2\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}-2\sin(dx+c)\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{ab} + \frac{\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}\log\left(\sin(dx+c)^2+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\sin(dx+c)+\sqrt{\cdots}\right.}{ab}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/8*(2*sqrt(2)*(-a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a/b)^(1/4) + 2*
sin(d*x + c))/(-a/b)^(1/4))/(a*b) + 2*sqrt(2)*(-a*b^3)^(1/4)*arctan(-1/2*sq
rt(2)*(sqrt(2)*(-a/b)^(1/4) - 2*sin(d*x + c))/(-a/b)^(1/4))/(a*b) + sqrt(2)
*(-a*b^3)^(1/4)*log(sin(d*x + c)^2 + sqrt(2)*(-a/b)^(1/4)*sin(d*x + c) + sq
rt(-a/b))/(a*b) - sqrt(2)*(-a*b^3)^(1/4)*log(sin(d*x + c)^2 - sqrt(2)*(-a/b
)^(1/4)*sin(d*x + c) + sqrt(-a/b))/(a*b))/d
```

3.408 $\int \frac{\sec(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=117

$$\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}+\sqrt{b}\right)}-\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}-\sqrt{b}\right)}+\frac{\tanh^{-1}(\sin(c+dx))}{d(a-b)}$$

```
[Out] (b^(1/4)*ArcTan[(b^(1/4)*Sin[c + d*x])/a^(1/4)])/(2*a^(3/4)*(Sqrt[a] + Sqrt
[b])*d) + ArcTanh[Sin[c + d*x]]/((a - b)*d) - (b^(1/4)*ArcTanh[(b^(1/4)*Sin
[c + d*x])/a^(1/4)])/(2*a^(3/4)*(Sqrt[a] - Sqrt[b])*d)
```

Rubi [A] time = 0.153484, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3223, 1171, 207, 1167, 205, 208}

$$\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}+\sqrt{b}\right)}-\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}-\sqrt{b}\right)}+\frac{\tanh^{-1}(\sin(c+dx))}{d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (b^(1/4)*ArcTan[(b^(1/4)*Sin[c + d*x])/a^(1/4)])/(2*a^(3/4)*(Sqrt[a] + Sqrt
[b])*d) + ArcTanh[Sin[c + d*x]]/((a - b)*d) - (b^(1/4)*ArcTanh[(b^(1/4)*Sin
[c + d*x])/a^(1/4)])/(2*a^(3/4)*(Sqrt[a] - Sqrt[b])*d)
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a-bx^4)}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a-b)(-1+x^2)} - \frac{b(1+x^2)}{(a-b)(a-bx^4)}\right) dx, x, \sin(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sin(c+dx)\right)}{(a-b)d} - \frac{b\,\text{Subst}\left(\int \frac{1+x^2}{a-bx^4}\,dx, x, \sin(c+dx)\right)}{(a-b)d}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{(a-b)d} - \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a}\sqrt{b}-bx^2}\,dx, x, \sin(c+dx)\right)}{2\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)d} - \frac{b\,\text{Subst}\left(\int \frac{1}{-\sqrt{a}\sqrt{b}-bx^2}\,dx, x, s\right.}{2\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)d}$$

$$= \frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)d} + \frac{\tanh^{-1}(\sin(c+dx))}{(a-b)d} - \frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)d}$$

Mathematica [C] time = 0.181685, size = 184, normalized size = 1.57

$$\frac{4a^{3/4}\tanh^{-1}(\sin(c+dx)) + \sqrt[4]{b}\left(\left(\sqrt{a}+\sqrt{b}\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}\sin(c+dx)\right) + i\left(\left(\sqrt{a}-\sqrt{b}\right)\log\left(\sqrt[4]{a}-i\sqrt[4]{b}\sin(c+dx)\right) + \left(\sqrt{b}\right.\right.\right.}{4a^{3/4}d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (4*a^(3/4)*ArcTanh[Sin[c + d*x]] + b^(1/4)*((Sqrt[a] + Sqrt[b])*Log[a^(1/4)
 - b^(1/4)*Sin[c + d*x]] + I*((Sqrt[a] - Sqrt[b])*Log[a^(1/4) - I*b^(1/4)*S
in[c + d*x]] + (-Sqrt[a] + Sqrt[b])*Log[a^(1/4) + I*b^(1/4)*Sin[c + d*x]] +
 I*(Sqrt[a] + Sqrt[b])*Log[a^(1/4) + b^(1/4)*Sin[c + d*x]]))))/(4*a^(3/4)*(a
 - b)*d)
```

Maple [B] time = 0.095, size = 229, normalized size = 2.

$$-\frac{\ln(\sin(dx+c)-1)}{d(2a-2b)} + \frac{\ln(1+\sin(dx+c))}{d(2a-2b)} - \frac{b}{4d(a-b)a}\sqrt[4]{\frac{a}{b}}\ln\left(\left(\sin(dx+c)+\sqrt[4]{\frac{a}{b}}\right)\left(\sin(dx+c)-\sqrt[4]{\frac{a}{b}}\right)^{-1}\right) - \frac{b}{2d(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a-b*sin(d*x+c)^4),x)
```

```
[Out] -1/d/(2*a-2*b)*ln(sin(d*x+c)-1)+1/d/(2*a-2*b)*ln(1+sin(d*x+c))-1/4/d*b/(a-b
)*(a/b)^(1/4)/a*ln((sin(d*x+c)+(a/b)^(1/4))/(sin(d*x+c)-(a/b)^(1/4)))-1/2/d
*b/(a-b)*(a/b)^(1/4)/a*arctan(sin(d*x+c)/(a/b)^(1/4))+1/2/d/(a-b)/(a/b)^(1/
4)*arctan(sin(d*x+c)/(a/b)^(1/4))-1/4/d/(a-b)/(a/b)^(1/4)*ln((sin(d*x+c)+(a
/b)^(1/4))/(sin(d*x+c)-(a/b)^(1/4)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.22071, size = 2836, normalized size = 24.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/4*((a - b)*d*sqrt(((a^3 - 2*a^2*b + a*b^2)*d^2*sqrt((a^2*b + 2*a*b^2 + b
^3)/((a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*d^4)) + 2*b)/((a^3 -
 2*a^2*b + a*b^2)*d^2))*log(1/2*(a*b + b^2)*sin(d*x + c) + 1/2*((a^5 - 2*a^
4*b + a^3*b^2)*d^3*sqrt((a^2*b + 2*a*b^2 + b^3)/((a^7 - 4*a^6*b + 6*a^5*b^2
 - 4*a^4*b^3 + a^3*b^4)*d^4)) - (a^2*b + a*b^2)*d)*sqrt(((a^3 - 2*a^2*b + a
*b^2)*d^2*sqrt((a^2*b + 2*a*b^2 + b^3)/((a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*
b^3 + a^3*b^4)*d^4)) + 2*b)/((a^3 - 2*a^2*b + a*b^2)*d^2))) - (a - b)*d*sqr
t(-((a^3 - 2*a^2*b + a*b^2)*d^2*sqrt((a^2*b + 2*a*b^2 + b^3)/((a^7 - 4*a^6*
b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*d^4)) - 2*b)/((a^3 - 2*a^2*b + a*b^2)*
d^2))*log(1/2*(a*b + b^2)*sin(d*x + c) + 1/2*((a^5 - 2*a^4*b + a^3*b^2)*d^3
*sqrt((a^2*b + 2*a*b^2 + b^3)/((a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3
*b^4)*d^4)) + (a^2*b + a*b^2)*d)*sqrt(-((a^3 - 2*a^2*b + a*b^2)*d^2*sqrt((a
^2*b + 2*a*b^2 + b^3)/((a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*d^
4)) - 2*b)/((a^3 - 2*a^2*b + a*b^2)*d^2))) - (a - b)*d*sqrt(((a^3 - 2*a^2*b
 + a*b^2)*d^2*sqrt((a^2*b + 2*a*b^2 + b^3)/((a^7 - 4*a^6*b + 6*a^5*b^2 - 4*
a^4*b^3 + a^3*b^4)*d^4)) + 2*b)/((a^3 - 2*a^2*b + a*b^2)*d^2))*log(-1/2*(a*
b + b^2)*sin(d*x + c) + 1/2*((a^5 - 2*a^4*b + a^3*b^2)*d^3*sqrt((a^2*b + 2*
a*b^2 + b^3)/((a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*d^4)) - (a^
2*b + a*b^2)*d)*sqrt(((a^3 - 2*a^2*b + a*b^2)*d^2*sqrt((a^2*b + 2*a*b^2 + b
^3)/((a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*d^4)) + 2*b)/((a^3 -
 2*a^2*b + a*b^2)*d^2))) + (a - b)*d*sqrt(-((a^3 - 2*a^2*b + a*b^2)*d^2*sqr
t((a^2*b + 2*a*b^2 + b^3)/((a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4
)*d^4)) - 2*b)/((a^3 - 2*a^2*b + a*b^2)*d^2))*log(-1/2*(a*b + b^2)*sin(d*x
+ c) + 1/2*((a^5 - 2*a^4*b + a^3*b^2)*d^3*sqrt((a^2*b + 2*a*b^2 + b^3)/((a^
7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*d^4)) + (a^2*b + a*b^2)*d)*s
qrt(-((a^3 - 2*a^2*b + a*b^2)*d^2*sqrt((a^2*b + 2*a*b^2 + b^3)/((a^7 - 4*a^
6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*d^4)) - 2*b)/((a^3 - 2*a^2*b + a*b^2
)*d^2))) - 2*log(sin(d*x + c) + 1) + 2*log(-sin(d*x + c) + 1))/((a - b)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [B] time = 6.21232, size = 500, normalized size = 4.27

$$-\frac{\frac{2\left(\left(-ab^3\right)^{\frac{1}{4}}b^2+\left(-ab^3\right)^{\frac{3}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}+2\sin(dx+c)\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}a^2b^2-\sqrt{2}ab^3}+\frac{2\left(\left(-ab^3\right)^{\frac{1}{4}}b^2+\left(-ab^3\right)^{\frac{3}{4}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}-2\sin(dx+c)\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}a^2b^2-\sqrt{2}ab^3}+\frac{\left(\left(-ab^3\right)^{\frac{1}{4}}b^2-\left(-ab^3\right)^{\frac{3}{4}}\right)\log\left(\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -1/4*(2*((-a*b^3)^(1/4)*b^2 + (-a*b^3)^(3/4))*arctan(1/2*sqrt(2)*(sqrt(2)*(
-a/b)^(1/4) + 2*sin(d*x + c))/(-a/b)^(1/4))/(sqrt(2)*a^2*b^2 - sqrt(2)*a*b^
3) + 2*((-a*b^3)^(1/4)*b^2 + (-a*b^3)^(3/4))*arctan(-1/2*sqrt(2)*(sqrt(2)*(
-a/b)^(1/4) - 2*sin(d*x + c))/(-a/b)^(1/4))/(sqrt(2)*a^2*b^2 - sqrt(2)*a*b^
3) + ((-a*b^3)^(1/4)*b^2 - (-a*b^3)^(3/4))*log(sin(d*x + c)^2 + sqrt(2)*(-a
/b)^(1/4)*sin(d*x + c) + sqrt(-a/b))/(sqrt(2)*a^2*b^2 - sqrt(2)*a*b^3) - ((
-a*b^3)^(1/4)*b^2 - (-a*b^3)^(3/4))*log(sin(d*x + c)^2 - sqrt(2)*(-a/b)^(1/
4)*sin(d*x + c) + sqrt(-a/b))/(sqrt(2)*a^2*b^2 - sqrt(2)*a*b^3) - 2*log(abs
(sin(d*x + c) + 1))/(a - b) + 2*log(abs(sin(d*x + c) - 1))/(a - b))/d
```

3.409 $\int \frac{\sec^3(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=175

$$\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}+\sqrt{b}\right)^2}+\frac{b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}-\sqrt{b}\right)^2}+\frac{1}{4d(a-b)(1-\sin(c+dx))}-\frac{1}{4d(a-b)(\sin(c+dx)+1)}+\frac{(a-\ldots}{\ldots}$$

```
[Out] (b^(3/4)*ArcTan[(b^(1/4)*Sin[c + d*x])/a^(1/4)])/(2*a^(3/4)*(Sqrt[a] + Sqrt
[b])^2*d) + ((a - 5*b)*ArcTanh[Sin[c + d*x]])/(2*(a - b)^2*d) + (b^(3/4)*Ar
cTanh[(b^(1/4)*Sin[c + d*x])/a^(1/4)])/(2*a^(3/4)*(Sqrt[a] - Sqrt[b])^2*d)
+ 1/(4*(a - b)*d*(1 - Sin[c + d*x])) - 1/(4*(a - b)*d*(1 + Sin[c + d*x]))
```

Rubi [A] time = 0.213722, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3223, 1171, 207, 1167, 205, 208}

$$\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}+\sqrt{b}\right)^2}+\frac{b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}-\sqrt{b}\right)^2}+\frac{1}{4d(a-b)(1-\sin(c+dx))}-\frac{1}{4d(a-b)(\sin(c+dx)+1)}+\frac{(a-\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (b^(3/4)*ArcTan[(b^(1/4)*Sin[c + d*x])/a^(1/4)])/(2*a^(3/4)*(Sqrt[a] + Sqrt
[b])^2*d) + ((a - 5*b)*ArcTanh[Sin[c + d*x]])/(2*(a - b)^2*d) + (b^(3/4)*Ar
cTanh[(b^(1/4)*Sin[c + d*x])/a^(1/4)])/(2*a^(3/4)*(Sqrt[a] - Sqrt[b])^2*d)
+ 1/(4*(a - b)*d*(1 - Sin[c + d*x])) - 1/(4*(a - b)*d*(1 + Sin[c + d*x]))
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
```

```
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2(a-bx^4)}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{4(a-b)(-1+x)^2} + \frac{1}{4(a-b)(1+x)^2} + \frac{-a+5b}{2(a-b)^2(-1+x^2)} + \frac{b(a+b+2bx^2)}{(a-b)^2(a-bx^4)}\right) dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{1}{4(a-b)d(1-\sin(c+dx))} - \frac{1}{4(a-b)d(1+\sin(c+dx))} - \frac{(a-5b)\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sin(c\right.}{2(a-b)^2d}$$

$$= \frac{(a-5b)\tanh^{-1}(\sin(c+dx))}{2(a-b)^2d} + \frac{1}{4(a-b)d(1-\sin(c+dx))} - \frac{1}{4(a-b)d(1+\sin(c+dx))} + \frac{b^{3/2}}{}$$

$$= \frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^2 d} + \frac{(a-5b)\tanh^{-1}(\sin(c+dx))}{2(a-b)^2d} + \frac{b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^2 d} + \frac{}{4(a-b}$$

Mathematica [C] time = 1.00978, size = 255, normalized size = 1.46

$$-\frac{\frac{b^{3/4}\log\left(\sqrt[4]{a}-\sqrt[4]{b}\sin(c+dx)\right)}{a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^2} - \frac{ib^{3/4}\log\left(\sqrt[4]{a}-i\sqrt[4]{b}\sin(c+dx)\right)}{a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^2} + \frac{ib^{3/4}\log\left(\sqrt[4]{a}+i\sqrt[4]{b}\sin(c+dx)\right)}{a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^2} - \frac{b^{3/4}\log\left(\sqrt[4]{a}+\sqrt[4]{b}\sin(c+dx)\right)}{a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^2} + \frac{1}{(a-b)(\sin(c+dx)-1)} + \frac{}{(a}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -((-2*(a - 5*b)*ArcTanh[Sin[c + d*x]])/(a - b)^2 + (b^(3/4)*Log[a^(1/4) - b
^(1/4)*Sin[c + d*x]])/(a^(3/4)*(Sqrt[a] - Sqrt[b])^2) - (I*b^(3/4)*Log[a^(1
/4) - I*b^(1/4)*Sin[c + d*x]])/(a^(3/4)*(Sqrt[a] + Sqrt[b])^2) + (I*b^(3/4)
*Log[a^(1/4) + I*b^(1/4)*Sin[c + d*x]])/(a^(3/4)*(Sqrt[a] + Sqrt[b])^2) - (
b^(3/4)*Log[a^(1/4) + b^(1/4)*Sin[c + d*x]])/(a^(3/4)*(Sqrt[a] - Sqrt[b])^2
) + 1/((a - b)*(-1 + Sin[c + d*x])) + 1/((a - b)*(1 + Sin[c + d*x])))/(4*d)
```

Maple [B] time = 0.108, size = 415, normalized size = 2.4

$$-\frac{1}{d(4a-4b)(\sin(dx+c)-1)} - \frac{\ln(\sin(dx+c)-1)\,a}{4d(a-b)^2} + \frac{5\ln(\sin(dx+c)-1)\,b}{4d(a-b)^2} - \frac{1}{d(4a-4b)(1+\sin(dx+c))} + \frac{\ln(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a-b*sin(d*x+c)^4),x)
```

```
[Out] -1/d/(4*a-4*b)/(sin(d*x+c)-1)-1/4/d/(a-b)^2*ln(sin(d*x+c)-1)*a+5/4/d/(a-b)^
2*ln(sin(d*x+c)-1)*b-1/d/(4*a-4*b)/(1+sin(d*x+c))+1/4/d/(a-b)^2*ln(1+sin(d*
x+c))*a-5/4/d/(a-b)^2*ln(1+sin(d*x+c))*b+1/2/d*b/(a-b)^2*(a/b)^(1/4)*arctan
(sin(d*x+c)/(a/b)^(1/4))+1/2/d*b^2/(a-b)^2*(a/b)^(1/4)/a*arctan(sin(d*x+c)/
(a/b)^(1/4))+1/4/d*b/(a-b)^2*(a/b)^(1/4)*ln((sin(d*x+c)+(a/b)^(1/4))/(sin(d
*x+c)-(a/b)^(1/4)))+1/4/d*b^2/(a-b)^2*(a/b)^(1/4)/a*ln((sin(d*x+c)+(a/b)^(1
/4))/(sin(d*x+c)-(a/b)^(1/4)))-1/d*b/(a-b)^2/(a/b)^(1/4)*arctan(sin(d*x+c)/
(a/b)^(1/4))+1/2/d*b/(a-b)^2/(a/b)^(1/4)*ln((sin(d*x+c)+(a/b)^(1/4))/(sin(d
*x+c)-(a/b)^(1/4)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 9.82203, size = 5469, normalized size = 31.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/4*((a^2 - 2*a*b + b^2)*d*sqrt(((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 +
a*b^4)*d^2*sqrt((a^4*b^3 + 12*a^3*b^4 + 38*a^2*b^5 + 12*a*b^6 + b^7)/((a^11
 - 8*a^10*b + 28*a^9*b^2 - 56*a^8*b^3 + 70*a^7*b^4 - 56*a^6*b^5 + 28*a^5*b^
6 - 8*a^4*b^7 + a^3*b^8)*d^4)) + 4*a*b^2 + 4*b^3)/((a^5 - 4*a^4*b + 6*a^3*b
^2 - 4*a^2*b^3 + a*b^4)*d^2))*cos(d*x + c)^2*log(1/2*(a^2*b^2 + 6*a*b^3 + b
^4)*sin(d*x + c) + 1/2*(2*(a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)
*d^3*sqrt((a^4*b^3 + 12*a^3*b^4 + 38*a^2*b^5 + 12*a*b^6 + b^7)/((a^11 - 8*a
^10*b + 28*a^9*b^2 - 56*a^8*b^3 + 70*a^7*b^4 - 56*a^6*b^5 + 28*a^5*b^6 - 8*
a^4*b^7 + a^3*b^8)*d^4)) - (a^4*b + 7*a^3*b^2 + 7*a^2*b^3 + a*b^4)*d)*sqrt(
((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d^2*sqrt((a^4*b^3 + 12*a^3
*b^4 + 38*a^2*b^5 + 12*a*b^6 + b^7)/((a^11 - 8*a^10*b + 28*a^9*b^2 - 56*a^8
*b^3 + 70*a^7*b^4 - 56*a^6*b^5 + 28*a^5*b^6 - 8*a^4*b^7 + a^3*b^8)*d^4)) +
4*a*b^2 + 4*b^3)/((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d^2))) -
(a^2 - 2*a*b + b^2)*d*sqrt(-((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4
)*d^2*sqrt((a^4*b^3 + 12*a^3*b^4 + 38*a^2*b^5 + 12*a*b^6 + b^7)/((a^11 - 8*
a^10*b + 28*a^9*b^2 - 56*a^8*b^3 + 70*a^7*b^4 - 56*a^6*b^5 + 28*a^5*b^6 - 8
*a^4*b^7 + a^3*b^8)*d^4)) - 4*a*b^2 - 4*b^3)/((a^5 - 4*a^4*b + 6*a^3*b^2 -
4*a^2*b^3 + a*b^4)*d^2))*cos(d*x + c)^2*log(1/2*(a^2*b^2 + 6*a*b^3 + b^4)*s
in(d*x + c) + 1/2*(2*(a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*d^3*
sqrt((a^4*b^3 + 12*a^3*b^4 + 38*a^2*b^5 + 12*a*b^6 + b^7)/((a^11 - 8*a^10*b
 + 28*a^9*b^2 - 56*a^8*b^3 + 70*a^7*b^4 - 56*a^6*b^5 + 28*a^5*b^6 - 8*a^4*b
^7 + a^3*b^8)*d^4)) + (a^4*b + 7*a^3*b^2 + 7*a^2*b^3 + a*b^4)*d)*sqrt(-((a^
```

```
5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d^2*sqrt((a^4*b^3 + 12*a^3*b^4
 + 38*a^2*b^5 + 12*a*b^6 + b^7)/((a^11 - 8*a^10*b + 28*a^9*b^2 - 56*a^8*b^3
 + 70*a^7*b^4 - 56*a^6*b^5 + 28*a^5*b^6 - 8*a^4*b^7 + a^3*b^8)*d^4)) - 4*a*
b^2 - 4*b^3)/((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d^2))) - (a^2
 - 2*a*b + b^2)*d*sqrt(((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d^2
*sqrt((a^4*b^3 + 12*a^3*b^4 + 38*a^2*b^5 + 12*a*b^6 + b^7)/((a^11 - 8*a^10*
b + 28*a^9*b^2 - 56*a^8*b^3 + 70*a^7*b^4 - 56*a^6*b^5 + 28*a^5*b^6 - 8*a^4*
b^7 + a^3*b^8)*d^4)) + 4*a*b^2 + 4*b^3)/((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2
*b^3 + a*b^4)*d^2))*cos(d*x + c)^2*log(-1/2*(a^2*b^2 + 6*a*b^3 + b^4)*sin(d
*x + c) + 1/2*(2*(a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*d^3*sqrt
((a^4*b^3 + 12*a^3*b^4 + 38*a^2*b^5 + 12*a*b^6 + b^7)/((a^11 - 8*a^10*b + 2
8*a^9*b^2 - 56*a^8*b^3 + 70*a^7*b^4 - 56*a^6*b^5 + 28*a^5*b^6 - 8*a^4*b^7 +
 a^3*b^8)*d^4)) - (a^4*b + 7*a^3*b^2 + 7*a^2*b^3 + a*b^4)*d)*sqrt(((a^5 - 4
*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d^2*sqrt((a^4*b^3 + 12*a^3*b^4 + 38
*a^2*b^5 + 12*a*b^6 + b^7)/((a^11 - 8*a^10*b + 28*a^9*b^2 - 56*a^8*b^3 + 70
*a^7*b^4 - 56*a^6*b^5 + 28*a^5*b^6 - 8*a^4*b^7 + a^3*b^8)*d^4)) + 4*a*b^2 +
 4*b^3)/((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d^2))) + (a^2 - 2*
a*b + b^2)*d*sqrt(-((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d^2*sqr
t((a^4*b^3 + 12*a^3*b^4 + 38*a^2*b^5 + 12*a*b^6 + b^7)/((a^11 - 8*a^10*b +
28*a^9*b^2 - 56*a^8*b^3 + 70*a^7*b^4 - 56*a^6*b^5 + 28*a^5*b^6 - 8*a^4*b^7
+ a^3*b^8)*d^4)) - 4*a*b^2 - 4*b^3)/((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3
 + a*b^4)*d^2))*cos(d*x + c)^2*log(-1/2*(a^2*b^2 + 6*a*b^3 + b^4)*sin(d*x +
 c) + 1/2*(2*(a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*d^3*sqrt((a^
4*b^3 + 12*a^3*b^4 + 38*a^2*b^5 + 12*a*b^6 + b^7)/((a^11 - 8*a^10*b + 28*a^
9*b^2 - 56*a^8*b^3 + 70*a^7*b^4 - 56*a^6*b^5 + 28*a^5*b^6 - 8*a^4*b^7 + a^3
*b^8)*d^4)) + (a^4*b + 7*a^3*b^2 + 7*a^2*b^3 + a*b^4)*d)*sqrt(-((a^5 - 4*a^
4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d^2*sqrt((a^4*b^3 + 12*a^3*b^4 + 38*a^
2*b^5 + 12*a*b^6 + b^7)/((a^11 - 8*a^10*b + 28*a^9*b^2 - 56*a^8*b^3 + 70*a^
7*b^4 - 56*a^6*b^5 + 28*a^5*b^6 - 8*a^4*b^7 + a^3*b^8)*d^4)) - 4*a*b^2 - 4*
b^3)/((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d^2))) - (a - 5*b)*co
s(d*x + c)^2*log(sin(d*x + c) + 1) + (a - 5*b)*cos(d*x + c)^2*log(-sin(d*x
+ c) + 1) - 2*(a - b)*sin(d*x + c))/((a^2 - 2*a*b + b^2)*d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [B] time = 5.73722, size = 635, normalized size = 3.63

$$\frac{2\left(\left(-ab^3\right)^{\frac{1}{4}}\left(ab+b^2\right)+2\left(-ab^3\right)^{\frac{3}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}+2\sin(dx+c)\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}a^3b-2\sqrt{2}a^2b^2+\sqrt{2}ab^3}+\frac{2\left(\left(-ab^3\right)^{\frac{1}{4}}\left(ab+b^2\right)+2\left(-ab^3\right)^{\frac{3}{4}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}-2\sin(dx+c)\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}a^3b-2\sqrt{2}a^2b^2+\sqrt{2}ab^3}+\frac{\left(\left(-ab^3\right)^{\frac{1}{4}}\left(ab+b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/4*(2*((-a*b^3)^(1/4)*(a*b + b^2) + 2*(-a*b^3)^(3/4))*arctan(1/2*sqrt(2)*(
sqrt(2)*(-a/b)^(1/4) + 2*sin(d*x + c))/(-a/b)^(1/4))/(sqrt(2)*a^3*b - 2*sqr
t(2)*a^2*b^2 + sqrt(2)*a*b^3) + 2*((-a*b^3)^(1/4)*(a*b + b^2) + 2*(-a*b^3)^
(3/4))*arctan(-1/2*sqrt(2)*(sqrt(2)*(-a/b)^(1/4) - 2*sin(d*x + c))/(-a/b)^(
1/4))/(sqrt(2)*a^3*b - 2*sqrt(2)*a^2*b^2 + sqrt(2)*a*b^3) + ((-a*b^3)^(1/4)
*(a*b + b^2) - 2*(-a*b^3)^(3/4))*log(sin(d*x + c)^2 + sqrt(2)*(-a/b)^(1/4)*
sin(d*x + c) + sqrt(-a/b))/(sqrt(2)*a^3*b - 2*sqrt(2)*a^2*b^2 + sqrt(2)*a*b
^3) - ((-a*b^3)^(1/4)*(a*b + b^2) - 2*(-a*b^3)^(3/4))*log(sin(d*x + c)^2 -
sqrt(2)*(-a/b)^(1/4)*sin(d*x + c) + sqrt(-a/b))/(sqrt(2)*a^3*b - 2*sqrt(2)*
a^2*b^2 + sqrt(2)*a*b^3) + (a - 5*b)*log(abs(sin(d*x + c) + 1))/(a^2 - 2*a*
b + b^2) - (a - 5*b)*log(abs(sin(d*x + c) - 1))/(a^2 - 2*a*b + b^2) - 2*sin
(d*x + c)/((sin(d*x + c)^2 - 1)*(a - b)))/d
```

3.410 $\int \frac{\sec^5(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=249

$$\frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}+\sqrt{b}\right)^3}-\frac{b^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}-\sqrt{b}\right)^3}+\frac{\left(3a^2-6ab+35b^2\right)\tanh^{-1}(\sin(c+dx))}{8d(a-b)^3}+\frac{3a-11b}{16d(a-b)^2(1-\sin(c+$$

```
[Out] (b^(5/4)*ArcTan[(b^(1/4)*Sin[c + d*x])/a^(1/4)])/(2*a^(3/4)*(Sqrt[a] + Sqrt
[b])^3*d) + ((3*a^2 - 6*a*b + 35*b^2)*ArcTanh[Sin[c + d*x]])/(8*(a - b)^3*d
) - (b^(5/4)*ArcTanh[(b^(1/4)*Sin[c + d*x])/a^(1/4)])/(2*a^(3/4)*(Sqrt[a] -
 Sqrt[b])^3*d) + 1/(16*(a - b)*d*(1 - Sin[c + d*x])^2) + (3*a - 11*b)/(16*(
a - b)^2*d*(1 - Sin[c + d*x])) - 1/(16*(a - b)*d*(1 + Sin[c + d*x])^2) - (3
*a - 11*b)/(16*(a - b)^2*d*(1 + Sin[c + d*x]))
```

Rubi [A] time = 0.296795, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3223, 1171, 207, 1167, 205, 208}

$$\frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}+\sqrt{b}\right)^3}-\frac{b^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}-\sqrt{b}\right)^3}+\frac{\left(3a^2-6ab+35b^2\right)\tanh^{-1}(\sin(c+dx))}{8d(a-b)^3}+\frac{3a-11b}{16d(a-b)^2(1-\sin(c+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a - b*Sin[c + d*x]^4),x]
```


```
[Out] (b^(5/4)*ArcTan[(b^(1/4)*Sin[c + d*x])/a^(1/4)])/(2*a^(3/4)*(Sqrt[a] + Sqrt
[b])^3*d) + ((3*a^2 - 6*a*b + 35*b^2)*ArcTanh[Sin[c + d*x]])/(8*(a - b)^3*d
) - (b^(5/4)*ArcTanh[(b^(1/4)*Sin[c + d*x])/a^(1/4)])/(2*a^(3/4)*(Sqrt[a] -
 Sqrt[b])^3*d) + 1/(16*(a - b)*d*(1 - Sin[c + d*x])^2) + (3*a - 11*b)/(16*(
a - b)^2*d*(1 - Sin[c + d*x])) - 1/(16*(a - b)*d*(1 + Sin[c + d*x])^2) - (3
*a - 11*b)/(16*(a - b)^2*d*(1 + Sin[c + d*x]))
```


Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^3\left(a-bx^4\right)}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{8(a-b)(-1+x)^3}+\frac{3a-11b}{16(a-b)^2(-1+x)^2}+\frac{1}{8(a-b)(1+x)^3}+\frac{3a-11b}{16(a-b)^2(1+x)^2}+\frac{-3a^2+6ab-35b^2}{8(a-b)^3\left(-1+x^2\right)}+\frac{b^2(\cdots}{(\cdots}\right.\right.}{d}$$

$$= \frac{1}{16(a-b)d(1-\sin(c+dx))^2}+\frac{3a-11b}{16(a-b)^2d(1-\sin(c+dx))}-\frac{1}{16(a-b)d(1+\sin(c+dx))^2}\cdots$$

$$= \frac{\left(3a^2-6ab+35b^2\right)\tanh^{-1}(\sin(c+dx))}{8(a-b)^3d}+\frac{1}{16(a-b)d(1-\sin(c+dx))^2}+\frac{3a-11\cdots}{16(a-b)^2d(1-\cdots}$$

$$= \frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^3d}+\frac{\left(3a^2-6ab+35b^2\right)\tanh^{-1}(\sin(c+dx))}{8(a-b)^3d}-\frac{b^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sin(c+\cdots}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^3\cdots}$$

Mathematica [C] time = 5.65029, size = 317, normalized size = 1.27

$$\frac{\frac{4b^{5/4}\log\left(\sqrt[4]{a}-\sqrt[4]{b}\sin(c+dx)\right)}{a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^3}+\frac{4ib^{5/4}\log\left(\sqrt[4]{a}-i\sqrt[4]{b}\sin(c+dx)\right)}{a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^3}-\frac{4ib^{5/4}\log\left(\sqrt[4]{a}+i\sqrt[4]{b}\sin(c+dx)\right)}{a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^3}-\frac{4b^{5/4}\log\left(\sqrt[4]{a}+\sqrt[4]{b}\sin(c+dx)\right)}{a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^3}+\frac{2\left(3a^2-6ab+35b^2\right)\cdots}{(a-\cdots}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] ((2*(3*a^2 - 6*a*b + 35*b^2)*ArcTanh[Sin[c + d*x]])/(a - b)^3 + (4*b^(5/4)*
Log[a^(1/4) - b^(1/4)*Sin[c + d*x]])/(a^(3/4)*(Sqrt[a] - Sqrt[b])^3) + ((4*
I)*b^(5/4)*Log[a^(1/4) - I*b^(1/4)*Sin[c + d*x]])/(a^(3/4)*(Sqrt[a] + Sqrt[
b])^3) - ((4*I)*b^(5/4)*Log[a^(1/4) + I*b^(1/4)*Sin[c + d*x]])/(a^(3/4)*(Sq
rt[a] + Sqrt[b])^3) - (4*b^(5/4)*Log[a^(1/4) + b^(1/4)*Sin[c + d*x]])/(a^(3
/4)*(Sqrt[a] - Sqrt[b])^3) + 1/((a - b)*(-1 + Sin[c + d*x])^2) + (-3*a + 11
*b)/((a - b)^2*(-1 + Sin[c + d*x])) - 1/((a - b)*(1 + Sin[c + d*x])^2) + (-
3*a + 11*b)/((a - b)^2*(1 + Sin[c + d*x])))/(16*d)
```

Maple [B] time = 0.119, size = 660, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a-b*sin(d*x+c)^4),x)
```

```
[Out] 1/2/d/(8*a-8*b)/(sin(d*x+c)-1)^2-3/16/d/(a-b)^2/(sin(d*x+c)-1)*a+11/16/d/(a
-b)^2/(sin(d*x+c)-1)*b-3/16/d/(a-b)^3*ln(sin(d*x+c)-1)*a^2+3/8/d/(a-b)^3*ln
(sin(d*x+c)-1)*a*b-35/16/d/(a-b)^3*ln(sin(d*x+c)-1)*b^2-1/2/d/(8*a-8*b)/(1+
sin(d*x+c))^2-3/16/d/(a-b)^2/(1+sin(d*x+c))*a+11/16/d/(a-b)^2/(1+sin(d*x+c)
)*b+3/16/d/(a-b)^3*ln(1+sin(d*x+c))*a^2-3/8/d/(a-b)^3*ln(1+sin(d*x+c))*a*b+
35/16/d/(a-b)^3*ln(1+sin(d*x+c))*b^2-3/2/d*b^2/(a-b)^3*(a/b)^(1/4)*arctan(s
in(d*x+c)/(a/b)^(1/4))-1/2/d*b^3/(a-b)^3*(a/b)^(1/4)/a*arctan(sin(d*x+c)/(a
/b)^(1/4))-3/4/d*b^2/(a-b)^3*(a/b)^(1/4)*ln((sin(d*x+c)+(a/b)^(1/4))/(sin(d
*x+c)-(a/b)^(1/4)))-1/4/d*b^3/(a-b)^3*(a/b)^(1/4)/a*ln((sin(d*x+c)+(a/b)^(1
/4))/(sin(d*x+c)-(a/b)^(1/4)))+1/2/d*b/(a-b)^3/(a/b)^(1/4)*a*arctan(sin(d*x
+c)/(a/b)^(1/4))+3/2/d*b^2/(a-b)^3/(a/b)^(1/4)*arctan(sin(d*x+c)/(a/b)^(1/4
))-1/4/d*b/(a-b)^3/(a/b)^(1/4)*a*ln((sin(d*x+c)+(a/b)^(1/4))/(sin(d*x+c)-(a
/b)^(1/4)))-3/4/d*b^2/(a-b)^3/(a/b)^(1/4)*ln((sin(d*x+c)+(a/b)^(1/4))/(sin(
d*x+c)-(a/b)^(1/4)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 17.9734, size = 8404, normalized size = 33.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/16*(4*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sqrt((6*a^2*b^3 + 20*a*b^4 + 6*b
^5 + (a^7 - 6*a^6*b + 15*a^5*b^2 - 20*a^4*b^3 + 15*a^3*b^4 - 6*a^2*b^5 + a*
b^6)*d^2*sqrt((a^6*b^5 + 30*a^5*b^6 + 255*a^4*b^7 + 452*a^3*b^8 + 255*a^2*b
^9 + 30*a*b^10 + b^11)/((a^15 - 12*a^14*b + 66*a^13*b^2 - 220*a^12*b^3 + 49
5*a^11*b^4 - 792*a^10*b^5 + 924*a^9*b^6 - 792*a^8*b^7 + 495*a^7*b^8 - 220*a
^6*b^9 + 66*a^5*b^10 - 12*a^4*b^11 + a^3*b^12)*d^4)))/((a^7 - 6*a^6*b + 15*
a^5*b^2 - 20*a^4*b^3 + 15*a^3*b^4 - 6*a^2*b^5 + a*b^6)*d^2))*cos(d*x + c)^4
*log(1/2*(a^3*b^4 + 15*a^2*b^5 + 15*a*b^6 + b^7)*sin(d*x + c) + 1/2*((a^10
- 3*a^9*b - 3*a^8*b^2 + 25*a^7*b^3 - 45*a^6*b^4 + 39*a^5*b^5 - 17*a^4*b^6 +
 3*a^3*b^7)*d^3*sqrt((a^6*b^5 + 30*a^5*b^6 + 255*a^4*b^7 + 452*a^3*b^8 + 25
5*a^2*b^9 + 30*a*b^10 + b^11)/((a^15 - 12*a^14*b + 66*a^13*b^2 - 220*a^12*b
```

```
^3 + 495*a^11*b^4 - 792*a^10*b^5 + 924*a^9*b^6 - 792*a^8*b^7 + 495*a^7*b^8
- 220*a^6*b^9 + 66*a^5*b^10 - 12*a^4*b^11 + a^3*b^12)*d^4)) - (3*a^5*b^3 +
46*a^4*b^4 + 60*a^3*b^5 + 18*a^2*b^6 + a*b^7)*d)*sqrt((6*a^2*b^3 + 20*a*b^4
 + 6*b^5 + (a^7 - 6*a^6*b + 15*a^5*b^2 - 20*a^4*b^3 + 15*a^3*b^4 - 6*a^2*b^
5 + a*b^6)*d^2*sqrt((a^6*b^5 + 30*a^5*b^6 + 255*a^4*b^7 + 452*a^3*b^8 + 255
*a^2*b^9 + 30*a*b^10 + b^11)/((a^15 - 12*a^14*b + 66*a^13*b^2 - 220*a^12*b^
3 + 495*a^11*b^4 - 792*a^10*b^5 + 924*a^9*b^6 - 792*a^8*b^7 + 495*a^7*b^8 -
 220*a^6*b^9 + 66*a^5*b^10 - 12*a^4*b^11 + a^3*b^12)*d^4)))/((a^7 - 6*a^6*b
 + 15*a^5*b^2 - 20*a^4*b^3 + 15*a^3*b^4 - 6*a^2*b^5 + a*b^6)*d^2))) - 4*(a^
3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sqrt((6*a^2*b^3 + 20*a*b^4 + 6*b^5 - (a^7 -
6*a^6*b + 15*a^5*b^2 - 20*a^4*b^3 + 15*a^3*b^4 - 6*a^2*b^5 + a*b^6)*d^2*sqr
t((a^6*b^5 + 30*a^5*b^6 + 255*a^4*b^7 + 452*a^3*b^8 + 255*a^2*b^9 + 30*a*b^
10 + b^11)/((a^15 - 12*a^14*b + 66*a^13*b^2 - 220*a^12*b^3 + 495*a^11*b^4 -
 792*a^10*b^5 + 924*a^9*b^6 - 792*a^8*b^7 + 495*a^7*b^8 - 220*a^6*b^9 + 66*
a^5*b^10 - 12*a^4*b^11 + a^3*b^12)*d^4)))/((a^7 - 6*a^6*b + 15*a^5*b^2 - 20
*a^4*b^3 + 15*a^3*b^4 - 6*a^2*b^5 + a*b^6)*d^2))*cos(d*x + c)^4*log(1/2*(a^
3*b^4 + 15*a^2*b^5 + 15*a*b^6 + b^7)*sin(d*x + c) + 1/2*((a^10 - 3*a^9*b -
3*a^8*b^2 + 25*a^7*b^3 - 45*a^6*b^4 + 39*a^5*b^5 - 17*a^4*b^6 + 3*a^3*b^7)*
d^3*sqrt((a^6*b^5 + 30*a^5*b^6 + 255*a^4*b^7 + 452*a^3*b^8 + 255*a^2*b^9 +
30*a*b^10 + b^11)/((a^15 - 12*a^14*b + 66*a^13*b^2 - 220*a^12*b^3 + 495*a^1
1*b^4 - 792*a^10*b^5 + 924*a^9*b^6 - 792*a^8*b^7 + 495*a^7*b^8 - 220*a^6*b^
9 + 66*a^5*b^10 - 12*a^4*b^11 + a^3*b^12)*d^4)) + (3*a^5*b^3 + 46*a^4*b^4 +
 60*a^3*b^5 + 18*a^2*b^6 + a*b^7)*d)*sqrt((6*a^2*b^3 + 20*a*b^4 + 6*b^5 - (
a^7 - 6*a^6*b + 15*a^5*b^2 - 20*a^4*b^3 + 15*a^3*b^4 - 6*a^2*b^5 + a*b^6)*d
^2*sqrt((a^6*b^5 + 30*a^5*b^6 + 255*a^4*b^7 + 452*a^3*b^8 + 255*a^2*b^9 + 3
0*a*b^10 + b^11)/((a^15 - 12*a^14*b + 66*a^13*b^2 - 220*a^12*b^3 + 495*a^11
*b^4 - 792*a^10*b^5 + 924*a^9*b^6 - 792*a^8*b^7 + 495*a^7*b^8 - 220*a^6*b^9
 + 66*a^5*b^10 - 12*a^4*b^11 + a^3*b^12)*d^4)))/((a^7 - 6*a^6*b + 15*a^5*b^
2 - 20*a^4*b^3 + 15*a^3*b^4 - 6*a^2*b^5 + a*b^6)*d^2))) - 4*(a^3 - 3*a^2*b
+ 3*a*b^2 - b^3)*d*sqrt((6*a^2*b^3 + 20*a*b^4 + 6*b^5 + (a^7 - 6*a^6*b + 15
*a^5*b^2 - 20*a^4*b^3 + 15*a^3*b^4 - 6*a^2*b^5 + a*b^6)*d^2*sqrt((a^6*b^5 +
 30*a^5*b^6 + 255*a^4*b^7 + 452*a^3*b^8 + 255*a^2*b^9 + 30*a*b^10 + b^11)/(
(a^15 - 12*a^14*b + 66*a^13*b^2 - 220*a^12*b^3 + 495*a^11*b^4 - 792*a^10*b^
5 + 924*a^9*b^6 - 792*a^8*b^7 + 495*a^7*b^8 - 220*a^6*b^9 + 66*a^5*b^10 - 1
2*a^4*b^11 + a^3*b^12)*d^4)))/((a^7 - 6*a^6*b + 15*a^5*b^2 - 20*a^4*b^3 + 1
5*a^3*b^4 - 6*a^2*b^5 + a*b^6)*d^2))*cos(d*x + c)^4*log(-1/2*(a^3*b^4 + 15*
a^2*b^5 + 15*a*b^6 + b^7)*sin(d*x + c) + 1/2*((a^10 - 3*a^9*b - 3*a^8*b^2 +
 25*a^7*b^3 - 45*a^6*b^4 + 39*a^5*b^5 - 17*a^4*b^6 + 3*a^3*b^7)*d^3*sqrt((a
^6*b^5 + 30*a^5*b^6 + 255*a^4*b^7 + 452*a^3*b^8 + 255*a^2*b^9 + 30*a*b^10 +
 b^11)/((a^15 - 12*a^14*b + 66*a^13*b^2 - 220*a^12*b^3 + 495*a^11*b^4 - 792
*a^10*b^5 + 924*a^9*b^6 - 792*a^8*b^7 + 495*a^7*b^8 - 220*a^6*b^9 + 66*a^5*
b^10 - 12*a^4*b^11 + a^3*b^12)*d^4)) - (3*a^5*b^3 + 46*a^4*b^4 + 60*a^3*b^5
 + 18*a^2*b^6 + a*b^7)*d)*sqrt((6*a^2*b^3 + 20*a*b^4 + 6*b^5 + (a^7 - 6*a^6
*b + 15*a^5*b^2 - 20*a^4*b^3 + 15*a^3*b^4 - 6*a^2*b^5 + a*b^6)*d^2*sqrt((a^
6*b^5 + 30*a^5*b^6 + 255*a^4*b^7 + 452*a^3*b^8 + 255*a^2*b^9 + 30*a*b^10 +
b^11)/((a^15 - 12*a^14*b + 66*a^13*b^2 - 220*a^12*b^3 + 495*a^11*b^4 - 792*
a^10*b^5 + 924*a^9*b^6 - 792*a^8*b^7 + 495*a^7*b^8 - 220*a^6*b^9 + 66*a^5*b
^10 - 12*a^4*b^11 + a^3*b^12)*d^4)))/((a^7 - 6*a^6*b + 15*a^5*b^2 - 20*a^4*
b^3 + 15*a^3*b^4 - 6*a^2*b^5 + a*b^6)*d^2))) + 4*(a^3 - 3*a^2*b + 3*a*b^2 -
 b^3)*d*sqrt((6*a^2*b^3 + 20*a*b^4 + 6*b^5 - (a^7 - 6*a^6*b + 15*a^5*b^2 -
20*a^4*b^3 + 15*a^3*b^4 - 6*a^2*b^5 + a*b^6)*d^2*sqrt((a^6*b^5 + 30*a^5*b^6
 + 255*a^4*b^7 + 452*a^3*b^8 + 255*a^2*b^9 + 30*a*b^10 + b^11)/((a^15 - 12*
a^14*b + 66*a^13*b^2 - 220*a^12*b^3 + 495*a^11*b^4 - 792*a^10*b^5 + 924*a^9
*b^6 - 792*a^8*b^7 + 495*a^7*b^8 - 220*a^6*b^9 + 66*a^5*b^10 - 12*a^4*b^11
+ a^3*b^12)*d^4)))/((a^7 - 6*a^6*b + 15*a^5*b^2 - 20*a^4*b^3 + 15*a^3*b^4 -
 6*a^2*b^5 + a*b^6)*d^2))*cos(d*x + c)^4*log(-1/2*(a^3*b^4 + 15*a^2*b^5 + 1
5*a*b^6 + b^7)*sin(d*x + c) + 1/2*((a^10 - 3*a^9*b - 3*a^8*b^2 + 25*a^7*b^3
 - 45*a^6*b^4 + 39*a^5*b^5 - 17*a^4*b^6 + 3*a^3*b^7)*d^3*sqrt((a^6*b^5 + 30
*a^5*b^6 + 255*a^4*b^7 + 452*a^3*b^8 + 255*a^2*b^9 + 30*a*b^10 + b^11)/((a^
```

```
15 - 12*a^14*b + 66*a^13*b^2 - 220*a^12*b^3 + 495*a^11*b^4 - 792*a^10*b^5 +
 924*a^9*b^6 - 792*a^8*b^7 + 495*a^7*b^8 - 220*a^6*b^9 + 66*a^5*b^10 - 12*a
^4*b^11 + a^3*b^12)*d^4)) + (3*a^5*b^3 + 46*a^4*b^4 + 60*a^3*b^5 + 18*a^2*b
^6 + a*b^7)*d)*sqrt((6*a^2*b^3 + 20*a*b^4 + 6*b^5 - (a^7 - 6*a^6*b + 15*a^5
*b^2 - 20*a^4*b^3 + 15*a^3*b^4 - 6*a^2*b^5 + a*b^6)*d^2*sqrt((a^6*b^5 + 30*
a^5*b^6 + 255*a^4*b^7 + 452*a^3*b^8 + 255*a^2*b^9 + 30*a*b^10 + b^11)/((a^1
5 - 12*a^14*b + 66*a^13*b^2 - 220*a^12*b^3 + 495*a^11*b^4 - 792*a^10*b^5 +
924*a^9*b^6 - 792*a^8*b^7 + 495*a^7*b^8 - 220*a^6*b^9 + 66*a^5*b^10 - 12*a^
4*b^11 + a^3*b^12)*d^4)))/((a^7 - 6*a^6*b + 15*a^5*b^2 - 20*a^4*b^3 + 15*a^
3*b^4 - 6*a^2*b^5 + a*b^6)*d^2))) - (3*a^2 - 6*a*b + 35*b^2)*cos(d*x + c)^4
*log(sin(d*x + c) + 1) + (3*a^2 - 6*a*b + 35*b^2)*cos(d*x + c)^4*log(-sin(d
*x + c) + 1) - 2*((3*a^2 - 14*a*b + 11*b^2)*cos(d*x + c)^2 + 2*a^2 - 4*a*b
+ 2*b^2)*sin(d*x + c))/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a-b*sin(d*x+c)**4),x)

[Out] Timed out
```

Giac [B] time = 6.37874, size = 851, normalized size = 3.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a-b*sin(d*x+c)^4),x, algorithm="giac")

[Out] -1/16*(8*((-a*b^3)^(3/4)*(a + 3*b) + (-a*b^3)^(1/4)*(3*a*b^2 + b^3))*arctan
(1/2*sqrt(2)*(sqrt(2)*(-a/b)^(1/4) + 2*sin(d*x + c))/(-a/b)^(1/4))/(sqrt(2)
*a^4*b - 3*sqrt(2)*a^3*b^2 + 3*sqrt(2)*a^2*b^3 - sqrt(2)*a*b^4) + 8*((-a*b^
3)^(3/4)*(a + 3*b) + (-a*b^3)^(1/4)*(3*a*b^2 + b^3))*arctan(-1/2*sqrt(2)*(s
qrt(2)*(-a/b)^(1/4) - 2*sin(d*x + c))/(-a/b)^(1/4))/(sqrt(2)*a^4*b - 3*sqrt
(2)*a^3*b^2 + 3*sqrt(2)*a^2*b^3 - sqrt(2)*a*b^4) - 4*((-a*b^3)^(3/4)*(a + 3
*b) - (-a*b^3)^(1/4)*(3*a*b^2 + b^3))*log(sin(d*x + c)^2 + sqrt(2)*(-a/b)^(
1/4)*sin(d*x + c) + sqrt(-a/b))/(sqrt(2)*a^4*b - 3*sqrt(2)*a^3*b^2 + 3*sqrt
(2)*a^2*b^3 - sqrt(2)*a*b^4) + 4*((-a*b^3)^(3/4)*(a + 3*b) - (-a*b^3)^(1/4)
*(3*a*b^2 + b^3))*log(sin(d*x + c)^2 - sqrt(2)*(-a/b)^(1/4)*sin(d*x + c) +
sqrt(-a/b))/(sqrt(2)*a^4*b - 3*sqrt(2)*a^3*b^2 + 3*sqrt(2)*a^2*b^3 - sqrt(2
)*a*b^4) - (3*a^2 - 6*a*b + 35*b^2)*log(abs(sin(d*x + c) + 1))/(a^3 - 3*a^2
*b + 3*a*b^2 - b^3) + (3*a^2 - 6*a*b + 35*b^2)*log(abs(sin(d*x + c) - 1))/(
a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 2*(3*a*sin(d*x + c)^3 - 11*b*sin(d*x + c)^
3 - 5*a*sin(d*x + c) + 13*b*sin(d*x + c))/((a^2 - 2*a*b + b^2)*(sin(d*x + c
)^2 - 1)^2))/d
```

3.411 $\int \frac{\cos^{10}(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=252

$$-\frac{\left(\sqrt{a}-\sqrt{b}\right)^{9/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/2}d}+\frac{\left(\sqrt{a}+\sqrt{b}\right)^{9/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/2}d}-\frac{(a+3b)\sin(c+dx)\cos(c+dx)}{2b^2d}-$$

```
[Out] (-17*x)/(16*b) - (4*(a + b)*x)/b^2 - ((a + 3*b)*x)/(2*b^2) - ((Sqrt[a] - Sq
rt[b])^(9/2)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(
3/4)*b^(5/2)*d) + ((Sqrt[a] + Sqrt[b])^(9/2)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]
]*Tan[c + d*x])/a^(1/4)])/(2*a^(3/4)*b^(5/2)*d) - (17*Cos[c + d*x]*Sin[c +
d*x])/(16*b*d) - ((a + 3*b)*Cos[c + d*x]*Sin[c + d*x])/(2*b^2*d) - (17*Cos[
c + d*x]^3*Sin[c + d*x])/(24*b*d) - (Cos[c + d*x]^5*Sin[c + d*x])/(6*b*d)
```

Rubi [A] time = 0.439522, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3224, 1170, 199, 203, 1166, 205}

$$-\frac{\left(\sqrt{a}-\sqrt{b}\right)^{9/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/2}d}+\frac{\left(\sqrt{a}+\sqrt{b}\right)^{9/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/2}d}-\frac{(a+3b)\sin(c+dx)\cos(c+dx)}{2b^2d}-$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^10/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (-17*x)/(16*b) - (4*(a + b)*x)/b^2 - ((a + 3*b)*x)/(2*b^2) - ((Sqrt[a] - Sq
rt[b])^(9/2)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(
3/4)*b^(5/2)*d) + ((Sqrt[a] + Sqrt[b])^(9/2)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]
]*Tan[c + d*x])/a^(1/4)])/(2*a^(3/4)*b^(5/2)*d) - (17*Cos[c + d*x]*Sin[c +
d*x])/(16*b*d) - ((a + 3*b)*Cos[c + d*x]*Sin[c + d*x])/(2*b^2*d) - (17*Cos[
c + d*x]^3*Sin[c + d*x])/(24*b*d) - (Cos[c + d*x]^5*Sin[c + d*x])/(6*b*d)
```

Rule 3224

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(m/2 + 2*p +
1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2]
 && IntegerQ[p]
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
```

```
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^{10}(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^4\left(a+2ax^2+(a-b)x^4\right)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{b\left(1+x^2\right)^4}-\frac{2}{b\left(1+x^2\right)^3}+\frac{-a-3b}{b^2\left(1+x^2\right)^2}-\frac{4(a+b)}{b^2\left(1+x^2\right)}+\frac{5a^2+10ab+b^2+4\left(a^2-b^2\right)x^2}{b^2\left(a+2ax^2+(a-b)x^4\right)}\right)dx, x, \tan(c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{5a^2+10ab+b^2+4\left(a^2-b^2\right)x^2}{a+2ax^2+(a-b)x^4}\,dx, x, \tan(c+dx)\right)}{b^2d} - \frac{\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^4}\,dx, x, \tan(c+dx)\right)}{bd} - \frac{2\,\cdots}{\cdots}$$

$$= -\frac{4(a+b)x}{b^2} - \frac{(a+3b)\cos(c+dx)\sin(c+dx)}{2b^2d} - \frac{\cos^3(c+dx)\sin(c+dx)}{2bd} - \frac{\cos^5(c+dx)\sin(c\cdots}{6bd}$$

$$= -\frac{4(a+b)x}{b^2} - \frac{(a+3b)x}{2b^2} - \frac{\left(\sqrt{a}-\sqrt{b}\right)^{9/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/2}d} + \frac{\left(\sqrt{a}+\sqrt{b}\right)^{9/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\cdots}}{\cdots}\right)}{2a^{3/4}b^{5/2}d}$$

$$= -\frac{3x}{4b} - \frac{4(a+b)x}{b^2} - \frac{(a+3b)x}{2b^2} - \frac{\left(\sqrt{a}-\sqrt{b}\right)^{9/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/2}d} + \frac{\left(\sqrt{a}+\sqrt{b}\right)^{9/2}\tan^{-1}\left(\cdots\right)}{2a^{3/4}b\cdots}$$

$$= -\frac{17x}{16b} - \frac{4(a+b)x}{b^2} - \frac{(a+3b)x}{2b^2} - \frac{\left(\sqrt{a}-\sqrt{b}\right)^{9/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/2}d} + \frac{\left(\sqrt{a}+\sqrt{b}\right)^{9/2}\tan^{-1}\cdots}{2a^{3/4}\cdots}$$

Mathematica [A] time = 0.91874, size = 233, normalized size = 0.92

$$-\frac{36b(24a+35b)(c+dx)+3b(16a+95b)\sin(2(c+dx)) - \frac{96\sqrt{b}\left(\sqrt{a}+\sqrt{b}\right)^5\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}+a}} - \frac{96\sqrt{b}\left(\sqrt{a}-\sqrt{b}\right)^5\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\text{ta}\cdots}{\sqrt{\sqrt{a}\sqrt{\cdots}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}-a}}\cdots}{192b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^10/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -(36*b*(24*a + 35*b)*(c + d*x) - (96*(Sqrt[a] + Sqrt[b])^5*Sqrt[b]*ArcTan[(
(Sqrt[a] + Sqrt[b])*Tan[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt
[a + Sqrt[a]*Sqrt[b]]) - (96*(Sqrt[a] - Sqrt[b])^5*Sqrt[b]*ArcTanh[((Sqrt[a
] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[-a +
Sqrt[a]*Sqrt[b]]) + 3*b*(16*a + 95*b)*Sin[2*(c + d*x)] + 21*b^2*Sin[4*(c +
d*x)] + b^2*Sin[6*(c + d*x)])/(192*b^3*d)
```

Maple [B] time = 0.132, size = 880, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^10/(a-b*sin(d*x+c)^4),x)
```

```
[Out] 2/d/b^2/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)
+a)*(a-b))^(1/2))*a^2-1/2/d/b^2/(a*b)^(1/2)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*a
rctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*a^3-5/2/d/b/(a*b)^(1/
2)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(
a-b))^(1/2))*a^2+5/2/d*a/(a*b)^(1/2)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((
a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+2/d/b^2/(((a*b)^(1/2)-a)*(a-
b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*a^2+1/2/
d/b^2/(a*b)^(1/2)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(
((a*b)^(1/2)-a)*(a-b))^(1/2))*a^3+5/2/d/b/(a*b)^(1/2)/(((a*b)^(1/2)-a)*(a-b
))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*a^2-5/2/d
*a/(a*b)^(1/2)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a
*b)^(1/2)-a)*(a-b))^(1/2))-2/d/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*t
an(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+1/2/d*b/(a*b)^(1/2)/(((a*b)^(1/2)+
a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-2/d/
(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a
-b))^(1/2))-1/2/d*b/(a*b)^(1/2)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b
)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-1/2/d/b^2/(tan(d*x+c)^2+1)^3*ta
n(d*x+c)^5*a-41/16/d/b/(tan(d*x+c)^2+1)^3*tan(d*x+c)^5-1/d/b^2/(tan(d*x+c)^
2+1)^3*tan(d*x+c)^3*a-35/6/d/b/(tan(d*x+c)^2+1)^3*tan(d*x+c)^3-1/2/d/b^2/(t
an(d*x+c)^2+1)^3*tan(d*x+c)*a-55/16/d/b/(tan(d*x+c)^2+1)^3*tan(d*x+c)-9/2/d
/b^2*arctan(tan(d*x+c))*a-105/16/d/b*arctan(tan(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^10/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -1/192*(192*b^2*d*integrate(-4*(4*(a^2*b + 10*a*b^2 + 5*b^3)*cos(6*d*x + 6*
c)^2 + 4*(72*a^3 + 53*a^2*b - 54*a*b^2 + 9*b^3)*cos(4*d*x + 4*c)^2 + 4*(a^2
*b + 10*a*b^2 + 5*b^3)*cos(2*d*x + 2*c)^2 + 4*(a^2*b + 10*a*b^2 + 5*b^3)*si
n(6*d*x + 6*c)^2 + 4*(72*a^3 + 53*a^2*b - 54*a*b^2 + 9*b^3)*sin(4*d*x + 4*c
)^2 + 2*(8*a^3 + 113*a^2*b + 50*a*b^2 - 27*b^3)*sin(4*d*x + 4*c)*sin(2*d*x
+ 2*c) + 4*(a^2*b + 10*a*b^2 + 5*b^3)*sin(2*d*x + 2*c)^2 - ((a^2*b + 10*a*b
^2 + 5*b^3)*cos(6*d*x + 6*c) + 2*(9*a^2*b + 10*a*b^2 - 3*b^3)*cos(4*d*x + 4
```

```
*c) + (a^2*b + 10*a*b^2 + 5*b^3)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) - (a^2*
b + 10*a*b^2 + 5*b^3 - 2*(8*a^3 + 113*a^2*b + 50*a*b^2 - 27*b^3)*cos(4*d*x
+ 4*c) - 8*(a^2*b + 10*a*b^2 + 5*b^3)*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) -
2*(9*a^2*b + 10*a*b^2 - 3*b^3 - (8*a^3 + 113*a^2*b + 50*a*b^2 - 27*b^3)*cos
(2*d*x + 2*c))*cos(4*d*x + 4*c) - (a^2*b + 10*a*b^2 + 5*b^3)*cos(2*d*x + 2*
c) - ((a^2*b + 10*a*b^2 + 5*b^3)*sin(6*d*x + 6*c) + 2*(9*a^2*b + 10*a*b^2 -
 3*b^3)*sin(4*d*x + 4*c) + (a^2*b + 10*a*b^2 + 5*b^3)*sin(2*d*x + 2*c))*sin
(8*d*x + 8*c) + 2*((8*a^3 + 113*a^2*b + 50*a*b^2 - 27*b^3)*sin(4*d*x + 4*c)
 + 4*(a^2*b + 10*a*b^2 + 5*b^3)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))/(b^4*co
s(8*d*x + 8*c)^2 + 16*b^4*cos(6*d*x + 6*c)^2 + 16*b^4*cos(2*d*x + 2*c)^2 +
b^4*sin(8*d*x + 8*c)^2 + 16*b^4*sin(6*d*x + 6*c)^2 + 16*b^4*sin(2*d*x + 2*c
)^2 - 8*b^4*cos(2*d*x + 2*c) + b^4 + 4*(64*a^2*b^2 - 48*a*b^3 + 9*b^4)*cos(
4*d*x + 4*c)^2 + 4*(64*a^2*b^2 - 48*a*b^3 + 9*b^4)*sin(4*d*x + 4*c)^2 + 16*
(8*a*b^3 - 3*b^4)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) - 2*(4*b^4*cos(6*d*x +
6*c) + 4*b^4*cos(2*d*x + 2*c) - b^4 + 2*(8*a*b^3 - 3*b^4)*cos(4*d*x + 4*c))
*cos(8*d*x + 8*c) + 8*(4*b^4*cos(2*d*x + 2*c) - b^4 + 2*(8*a*b^3 - 3*b^4)*c
os(4*d*x + 4*c))*cos(6*d*x + 6*c) - 4*(8*a*b^3 - 3*b^4 - 4*(8*a*b^3 - 3*b^4
)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 4*(2*b^4*sin(6*d*x + 6*c) + 2*b^4*si
n(2*d*x + 2*c) + (8*a*b^3 - 3*b^4)*sin(4*d*x + 4*c))*sin(8*d*x + 8*c) + 16*
(2*b^4*sin(2*d*x + 2*c) + (8*a*b^3 - 3*b^4)*sin(4*d*x + 4*c))*sin(6*d*x + 6
*c)), x) + 36*(24*a + 35*b)*d*x + b*sin(6*d*x + 6*c) + 21*b*sin(4*d*x + 4*c
) + 3*(16*a + 95*b)*sin(2*d*x + 2*c))/(b^2*d)
```

Fricas [B] time = 13.9979, size = 6926, normalized size = 27.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^10/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/48*(6*b^2*d*sqrt((a*b^5*d^2*sqrt((81*a^8 + 1512*a^7*b + 9324*a^6*b^2 + 21
816*a^5*b^3 + 21942*a^4*b^4 + 9240*a^3*b^5 + 1548*a^2*b^6 + 72*a*b^7 + b^8)
/(a^3*b^9*d^4)) - a^4 - 36*a^3*b - 126*a^2*b^2 - 84*a*b^3 - 9*b^4)/(a*b^5*d
^2))*log(9/4*a^8 + 12*a^7*b - 39*a^6*b^2 + 143/2*a^4*b^4 - 52*a^3*b^5 - 3*a
^2*b^6 + 8*a*b^7 + 1/4*b^8 - 1/4*(9*a^8 + 48*a^7*b - 156*a^6*b^2 + 286*a^4*
b^4 - 208*a^3*b^5 - 12*a^2*b^6 + 32*a*b^7 + b^8)*cos(d*x + c)^2 + 1/2*(4*(a
^4*b^7 + a^3*b^8)*d^3*sqrt((81*a^8 + 1512*a^7*b + 9324*a^6*b^2 + 21816*a^5*
b^3 + 21942*a^4*b^4 + 9240*a^3*b^5 + 1548*a^2*b^6 + 72*a*b^7 + b^8)/(a^3*b^
9*d^4))*cos(d*x + c)*sin(d*x + c) + (9*a^7*b^2 + 138*a^6*b^3 + 639*a^5*b^4
+ 876*a^4*b^5 + 343*a^3*b^6 + 42*a^2*b^7 + a*b^8)*d*cos(d*x + c)*sin(d*x +
c))*sqrt((a*b^5*d^2*sqrt((81*a^8 + 1512*a^7*b + 9324*a^6*b^2 + 21816*a^5*b^
3 + 21942*a^4*b^4 + 9240*a^3*b^5 + 1548*a^2*b^6 + 72*a*b^7 + b^8)/(a^3*b^9*
d^4)) - a^4 - 36*a^3*b - 126*a^2*b^2 - 84*a*b^3 - 9*b^4)/(a*b^5*d^2)) + 1/4
*(2*(a^6*b^4 - 4*a^5*b^5 + 6*a^4*b^6 - 4*a^3*b^7 + a^2*b^8)*d^2*cos(d*x + c
)^2 - (a^6*b^4 - 4*a^5*b^5 + 6*a^4*b^6 - 4*a^3*b^7 + a^2*b^8)*d^2)*sqrt((81
*a^8 + 1512*a^7*b + 9324*a^6*b^2 + 21816*a^5*b^3 + 21942*a^4*b^4 + 9240*a^3
*b^5 + 1548*a^2*b^6 + 72*a*b^7 + b^8)/(a^3*b^9*d^4))) - 6*b^2*d*sqrt((a*b^5
*d^2*sqrt((81*a^8 + 1512*a^7*b + 9324*a^6*b^2 + 21816*a^5*b^3 + 21942*a^4*b
^4 + 9240*a^3*b^5 + 1548*a^2*b^6 + 72*a*b^7 + b^8)/(a^3*b^9*d^4)) - a^4 - 3
6*a^3*b - 126*a^2*b^2 - 84*a*b^3 - 9*b^4)/(a*b^5*d^2))*log(9/4*a^8 + 12*a^7
*b - 39*a^6*b^2 + 143/2*a^4*b^4 - 52*a^3*b^5 - 3*a^2*b^6 + 8*a*b^7 + 1/4*b^
8 - 1/4*(9*a^8 + 48*a^7*b - 156*a^6*b^2 + 286*a^4*b^4 - 208*a^3*b^5 - 12*a^
2*b^6 + 32*a*b^7 + b^8)*cos(d*x + c)^2 - 1/2*(4*(a^4*b^7 + a^3*b^8)*d^3*sqr
t((81*a^8 + 1512*a^7*b + 9324*a^6*b^2 + 21816*a^5*b^3 + 21942*a^4*b^4 + 924
0*a^3*b^5 + 1548*a^2*b^6 + 72*a*b^7 + b^8)/(a^3*b^9*d^4))*cos(d*x + c)*sin(
d*x + c) + (9*a^7*b^2 + 138*a^6*b^3 + 639*a^5*b^4 + 876*a^4*b^5 + 343*a^3*b
```

```
^6 + 42*a^2*b^7 + a*b^8)*d*cos(d*x + c)*sin(d*x + c))*sqrt((a*b^5*d^2*sqrt(
(81*a^8 + 1512*a^7*b + 9324*a^6*b^2 + 21816*a^5*b^3 + 21942*a^4*b^4 + 9240*
a^3*b^5 + 1548*a^2*b^6 + 72*a*b^7 + b^8)/(a^3*b^9*d^4)) - a^4 - 36*a^3*b -
126*a^2*b^2 - 84*a*b^3 - 9*b^4)/(a*b^5*d^2)) + 1/4*(2*(a^6*b^4 - 4*a^5*b^5
+ 6*a^4*b^6 - 4*a^3*b^7 + a^2*b^8)*d^2*cos(d*x + c)^2 - (a^6*b^4 - 4*a^5*b^
5 + 6*a^4*b^6 - 4*a^3*b^7 + a^2*b^8)*d^2)*sqrt((81*a^8 + 1512*a^7*b + 9324*
a^6*b^2 + 21816*a^5*b^3 + 21942*a^4*b^4 + 9240*a^3*b^5 + 1548*a^2*b^6 + 72*
a*b^7 + b^8)/(a^3*b^9*d^4))) + 6*b^2*d*sqrt(-(a*b^5*d^2*sqrt((81*a^8 + 1512
*a^7*b + 9324*a^6*b^2 + 21816*a^5*b^3 + 21942*a^4*b^4 + 9240*a^3*b^5 + 1548
*a^2*b^6 + 72*a*b^7 + b^8)/(a^3*b^9*d^4)) + a^4 + 36*a^3*b + 126*a^2*b^2 +
84*a*b^3 + 9*b^4)/(a*b^5*d^2))*log(-9/4*a^8 - 12*a^7*b + 39*a^6*b^2 - 143/2
*a^4*b^4 + 52*a^3*b^5 + 3*a^2*b^6 - 8*a*b^7 - 1/4*b^8 + 1/4*(9*a^8 + 48*a^7
*b - 156*a^6*b^2 + 286*a^4*b^4 - 208*a^3*b^5 - 12*a^2*b^6 + 32*a*b^7 + b^8)
*cos(d*x + c)^2 + 1/2*(4*(a^4*b^7 + a^3*b^8)*d^3*sqrt((81*a^8 + 1512*a^7*b
+ 9324*a^6*b^2 + 21816*a^5*b^3 + 21942*a^4*b^4 + 9240*a^3*b^5 + 1548*a^2*b^
6 + 72*a*b^7 + b^8)/(a^3*b^9*d^4))*cos(d*x + c)*sin(d*x + c) - (9*a^7*b^2 +
 138*a^6*b^3 + 639*a^5*b^4 + 876*a^4*b^5 + 343*a^3*b^6 + 42*a^2*b^7 + a*b^8
)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-(a*b^5*d^2*sqrt((81*a^8 + 1512*a^7*b +
 9324*a^6*b^2 + 21816*a^5*b^3 + 21942*a^4*b^4 + 9240*a^3*b^5 + 1548*a^2*b^6
 + 72*a*b^7 + b^8)/(a^3*b^9*d^4)) + a^4 + 36*a^3*b + 126*a^2*b^2 + 84*a*b^3
 + 9*b^4)/(a*b^5*d^2)) + 1/4*(2*(a^6*b^4 - 4*a^5*b^5 + 6*a^4*b^6 - 4*a^3*b^
7 + a^2*b^8)*d^2*cos(d*x + c)^2 - (a^6*b^4 - 4*a^5*b^5 + 6*a^4*b^6 - 4*a^3*
b^7 + a^2*b^8)*d^2)*sqrt((81*a^8 + 1512*a^7*b + 9324*a^6*b^2 + 21816*a^5*b^
3 + 21942*a^4*b^4 + 9240*a^3*b^5 + 1548*a^2*b^6 + 72*a*b^7 + b^8)/(a^3*b^9*
d^4))) - 6*b^2*d*sqrt(-(a*b^5*d^2*sqrt((81*a^8 + 1512*a^7*b + 9324*a^6*b^2
+ 21816*a^5*b^3 + 21942*a^4*b^4 + 9240*a^3*b^5 + 1548*a^2*b^6 + 72*a*b^7 +
b^8)/(a^3*b^9*d^4)) + a^4 + 36*a^3*b + 126*a^2*b^2 + 84*a*b^3 + 9*b^4)/(a*b
^5*d^2))*log(-9/4*a^8 - 12*a^7*b + 39*a^6*b^2 - 143/2*a^4*b^4 + 52*a^3*b^5
+ 3*a^2*b^6 - 8*a*b^7 - 1/4*b^8 + 1/4*(9*a^8 + 48*a^7*b - 156*a^6*b^2 + 286
*a^4*b^4 - 208*a^3*b^5 - 12*a^2*b^6 + 32*a*b^7 + b^8)*cos(d*x + c)^2 - 1/2*
(4*(a^4*b^7 + a^3*b^8)*d^3*sqrt((81*a^8 + 1512*a^7*b + 9324*a^6*b^2 + 21816
*a^5*b^3 + 21942*a^4*b^4 + 9240*a^3*b^5 + 1548*a^2*b^6 + 72*a*b^7 + b^8)/(a
^3*b^9*d^4))*cos(d*x + c)*sin(d*x + c) - (9*a^7*b^2 + 138*a^6*b^3 + 639*a^5
*b^4 + 876*a^4*b^5 + 343*a^3*b^6 + 42*a^2*b^7 + a*b^8)*d*cos(d*x + c)*sin(d
*x + c))*sqrt(-(a*b^5*d^2*sqrt((81*a^8 + 1512*a^7*b + 9324*a^6*b^2 + 21816*
a^5*b^3 + 21942*a^4*b^4 + 9240*a^3*b^5 + 1548*a^2*b^6 + 72*a*b^7 + b^8)/(a^
3*b^9*d^4)) + a^4 + 36*a^3*b + 126*a^2*b^2 + 84*a*b^3 + 9*b^4)/(a*b^5*d^2))
 + 1/4*(2*(a^6*b^4 - 4*a^5*b^5 + 6*a^4*b^6 - 4*a^3*b^7 + a^2*b^8)*d^2*cos(d
*x + c)^2 - (a^6*b^4 - 4*a^5*b^5 + 6*a^4*b^6 - 4*a^3*b^7 + a^2*b^8)*d^2)*sq
rt((81*a^8 + 1512*a^7*b + 9324*a^6*b^2 + 21816*a^5*b^3 + 21942*a^4*b^4 + 92
40*a^3*b^5 + 1548*a^2*b^6 + 72*a*b^7 + b^8)/(a^3*b^9*d^4))) - 9*(24*a + 35*
b)*d*x - (8*b*cos(d*x + c)^5 + 34*b*cos(d*x + c)^3 + 3*(8*a + 41*b)*cos(d*x
 + c))*sin(d*x + c))/(b^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**10/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^10/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.412 $\int \frac{\cos^8(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=186

$$\frac{\left(\sqrt{a}-\sqrt{b}\right)^{7/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^2d}+\frac{\left(\sqrt{a}+\sqrt{b}\right)^{7/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^2d}-\frac{x(a+3b)}{b^2}-\frac{\sin(c+dx)\cos^3(c+d}{4bd}$$

```
[Out] (-11*x)/(8*b) - ((a + 3*b)*x)/b^2 + ((Sqrt[a] - Sqrt[b])^(7/2)*ArcTan[(Sqrt
[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(3/4)*b^2*d) + ((Sqrt[a] +
 Sqrt[b])^(7/2)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*
a^(3/4)*b^2*d) - (11*Cos[c + d*x]*Sin[c + d*x])/(8*b*d) - (Cos[c + d*x]^3*S
in[c + d*x])/(4*b*d)
```

Rubi [A] time = 0.326102, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3224, 1170, 199, 203, 1166, 205}

$$\frac{\left(\sqrt{a}-\sqrt{b}\right)^{7/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^2d}+\frac{\left(\sqrt{a}+\sqrt{b}\right)^{7/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^2d}-\frac{x(a+3b)}{b^2}-\frac{\sin(c+dx)\cos^3(c+d}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^8/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (-11*x)/(8*b) - ((a + 3*b)*x)/b^2 + ((Sqrt[a] - Sqrt[b])^(7/2)*ArcTan[(Sqrt
[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(3/4)*b^2*d) + ((Sqrt[a] +
 Sqrt[b])^(7/2)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*
a^(3/4)*b^2*d) - (11*Cos[c + d*x]*Sin[c + d*x])/(8*b*d) - (Cos[c + d*x]^3*S
in[c + d*x])/(4*b*d)
```

Rule 3224

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(m/2 + 2*p +
1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2]
 && IntegerQ[p]
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^8(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^3\left(a+2ax^2+(a-b)x^4\right)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{b(1+x^2)^3}-\frac{2}{b(1+x^2)^2}+\frac{-a-3b}{b^2(1+x^2)}+\frac{a^2+6ab+b^2+(a-b)(a+3b)x^2}{b^2\left(a+2ax^2+(a-b)x^4\right)}\right)dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{a^2+6ab+b^2+(a-b)(a+3b)x^2}{a+2ax^2+(a-b)x^4}\,dx, x, \tan(c+dx)\right)}{b^2 d} - \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^3}\,dx, x, \tan(c+dx)\right)}{bd} - \frac{2}{}$$

$$= -\frac{(a+3b)x}{b^2} - \frac{\cos(c+dx)\sin(c+dx)}{bd} - \frac{\cos^3(c+dx)\sin(c+dx)}{4bd} + \frac{\left(\left(\sqrt{a}-\sqrt{b}\right)^4\left(\sqrt{a}+\sqrt{b}\right)\right)}{}$$

$$= -\frac{x}{b} - \frac{(a+3b)x}{b^2} + \frac{\left(\sqrt{a}-\sqrt{b}\right)^{7/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^2 d} + \frac{\left(\sqrt{a}+\sqrt{b}\right)^{7/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+}{\sqrt[4]{a}}\right)}{2a^{3/4}b^2 d}$$

$$= -\frac{11x}{8b} - \frac{(a+3b)x}{b^2} + \frac{\left(\sqrt{a}-\sqrt{b}\right)^{7/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^2 d} + \frac{\left(\sqrt{a}+\sqrt{b}\right)^{7/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan}{\sqrt[4]{a}}\right)}{2a^{3/4}b^2 d}$$

Mathematica [A] time = 0.681536, size = 200, normalized size = 1.08

$$-\frac{4(8a+35b)(c+dx) - \frac{16\left(\sqrt{a}+\sqrt{b}\right)^4\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}+a}} + \frac{16\left(\sqrt{a}-\sqrt{b}\right)^4\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}-a}} + 24b\sin(2(c+dx)) + b\sin(4(c}{32b^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^8/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -(4*(8*a + 35*b)*(c + d*x) - (16*(Sqrt[a] + Sqrt[b])^4*ArcTan[((Sqrt[a] + S
qrt[b])*Tan[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[a + Sqrt[a]
*Sqrt[b]]) + (16*(Sqrt[a] - Sqrt[b])^4*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c +
```

```
 d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[-a + Sqrt[a]*Sqrt[b]]) +
24*b*Sin[2*(c + d*x)] + b*Sin[4*(c + d*x)])/(32*b^2*d)
```

Maple [B] time = 0.127, size = 750, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8/(a-b*sin(d*x+c)^4),x)
```

```
[Out] 1/2/d/b^2/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/
2)+a)*(a-b))^(1/2))*a^2+1/d/b*a/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*
tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-3/2/d/b/(a*b)^(1/2)/(((a*b)^(1/2)
+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*a^2
+1/d*a/(a*b)^(1/2)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/((
(a*b)^(1/2)+a)*(a-b))^(1/2))+1/2/d/b^2/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctan
h((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*a^2+1/d/b*a/(((a*b)^(1/2
)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))+
3/2/d/b/(a*b)^(1/2)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)
/(((a*b)^(1/2)-a)*(a-b))^(1/2))*a^2-1/d*a/(a*b)^(1/2)/(((a*b)^(1/2)-a)*(a-b
))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-3/2/d/(((
a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^
(1/2))+1/2/d*b/(a*b)^(1/2)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d
*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-3/2/d/(((a*b)^(1/2)-a)*(a-b))^(1/2)*ar
ctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-1/2/d*b/(a*b)^(1/2)/
(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a
-b))^(1/2))-11/8/d/b/(tan(d*x+c)^2+1)^2*tan(d*x+c)^3-13/8/d/b/(tan(d*x+c)^2
+1)^2*tan(d*x+c)-35/8/d/b*arctan(tan(d*x+c))-1/d/b^2*arctan(tan(d*x+c))*a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -1/32*(32*b^2*d*integrate(-16*(4*(a*b^2 + b^3)*cos(6*d*x + 6*c)^2 + 2*(8*a^
3 + 29*a^2*b - 20*a*b^2 + 3*b^3)*cos(4*d*x + 4*c)^2 + 4*(a*b^2 + b^3)*cos(2
*d*x + 2*c)^2 + 4*(a*b^2 + b^3)*sin(6*d*x + 6*c)^2 + 2*(8*a^3 + 29*a^2*b -
20*a*b^2 + 3*b^3)*sin(4*d*x + 4*c)^2 + 2*(10*a^2*b + 13*a*b^2 - 5*b^3)*sin(
4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*(a*b^2 + b^3)*sin(2*d*x + 2*c)^2 - ((a*b^
2 + b^3)*cos(6*d*x + 6*c) + (a^2*b + 4*a*b^2 - b^3)*cos(4*d*x + 4*c) + (a*b
^2 + b^3)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) - (a*b^2 + b^3 - 2*(10*a^2*b +
 13*a*b^2 - 5*b^3)*cos(4*d*x + 4*c) - 8*(a*b^2 + b^3)*cos(2*d*x + 2*c))*cos
(6*d*x + 6*c) - (a^2*b + 4*a*b^2 - b^3 - 2*(10*a^2*b + 13*a*b^2 - 5*b^3)*co
s(2*d*x + 2*c))*cos(4*d*x + 4*c) - (a*b^2 + b^3)*cos(2*d*x + 2*c) - ((a*b^2
 + b^3)*sin(6*d*x + 6*c) + (a^2*b + 4*a*b^2 - b^3)*sin(4*d*x + 4*c) + (a*b^
2 + b^3)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 2*((10*a^2*b + 13*a*b^2 - 5*b
^3)*sin(4*d*x + 4*c) + 4*(a*b^2 + b^3)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))/
(b^4*cos(8*d*x + 8*c)^2 + 16*b^4*cos(6*d*x + 6*c)^2 + 16*b^4*cos(2*d*x + 2*
c)^2 + b^4*sin(8*d*x + 8*c)^2 + 16*b^4*sin(6*d*x + 6*c)^2 + 16*b^4*sin(2*d*
x + 2*c)^2 - 8*b^4*cos(2*d*x + 2*c) + b^4 + 4*(64*a^2*b^2 - 48*a*b^3 + 9*b^
```

```
4)*cos(4*d*x + 4*c)^2 + 4*(64*a^2*b^2 - 48*a*b^3 + 9*b^4)*sin(4*d*x + 4*c)^
2 + 16*(8*a*b^3 - 3*b^4)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) - 2*(4*b^4*cos(6
*d*x + 6*c) + 4*b^4*cos(2*d*x + 2*c) - b^4 + 2*(8*a*b^3 - 3*b^4)*cos(4*d*x
+ 4*c))*cos(8*d*x + 8*c) + 8*(4*b^4*cos(2*d*x + 2*c) - b^4 + 2*(8*a*b^3 - 3
*b^4)*cos(4*d*x + 4*c))*cos(6*d*x + 6*c) - 4*(8*a*b^3 - 3*b^4 - 4*(8*a*b^3
- 3*b^4)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 4*(2*b^4*sin(6*d*x + 6*c) + 2
*b^4*sin(2*d*x + 2*c) + (8*a*b^3 - 3*b^4)*sin(4*d*x + 4*c))*sin(8*d*x + 8*c
) + 16*(2*b^4*sin(2*d*x + 2*c) + (8*a*b^3 - 3*b^4)*sin(4*d*x + 4*c))*sin(6*
d*x + 6*c)), x) + 4*(8*a + 35*b)*d*x + b*sin(4*d*x + 4*c) + 24*b*sin(2*d*x
+ 2*c))/(b^2*d)
```

Fricas [B] time = 8.86691, size = 5565, normalized size = 29.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/8*(b^2*d*sqrt(-(a*b^4*d^2*sqrt((49*a^6 + 490*a^5*b + 1519*a^4*b^2 + 1484*
a^3*b^3 + 511*a^2*b^4 + 42*a*b^5 + b^6)/(a^3*b^7*d^4)) + a^3 + 21*a^2*b + 3
5*a*b^2 + 7*b^3)/(a*b^4*d^2))*log(7/4*a^6 + 7/2*a^5*b - 63/4*a^4*b^2 + 9*a^
3*b^3 + 25/4*a^2*b^4 - 9/2*a*b^5 - 1/4*b^6 - 1/4*(7*a^6 + 14*a^5*b - 63*a^4
*b^2 + 36*a^3*b^3 + 25*a^2*b^4 - 18*a*b^5 - b^6)*cos(d*x + c)^2 + 1/2*((a^4
*b^5 + 3*a^3*b^6)*d^3*sqrt((49*a^6 + 490*a^5*b + 1519*a^4*b^2 + 1484*a^3*b^
3 + 511*a^2*b^4 + 42*a*b^5 + b^6)/(a^3*b^7*d^4))*cos(d*x + c)*sin(d*x + c)
- (21*a^5*b^2 + 112*a^4*b^3 + 98*a^3*b^4 + 24*a^2*b^5 + a*b^6)*d*cos(d*x +
c)*sin(d*x + c))*sqrt(-(a*b^4*d^2*sqrt((49*a^6 + 490*a^5*b + 1519*a^4*b^2 +
 1484*a^3*b^3 + 511*a^2*b^4 + 42*a*b^5 + b^6)/(a^3*b^7*d^4)) + a^3 + 21*a^2
*b + 35*a*b^2 + 7*b^3)/(a*b^4*d^2)) - 1/4*(2*(a^5*b^3 - 3*a^4*b^4 + 3*a^3*b
^5 - a^2*b^6)*d^2*cos(d*x + c)^2 - (a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b
^6)*d^2)*sqrt((49*a^6 + 490*a^5*b + 1519*a^4*b^2 + 1484*a^3*b^3 + 511*a^2*b
^4 + 42*a*b^5 + b^6)/(a^3*b^7*d^4))) - b^2*d*sqrt(-(a*b^4*d^2*sqrt((49*a^6
+ 490*a^5*b + 1519*a^4*b^2 + 1484*a^3*b^3 + 511*a^2*b^4 + 42*a*b^5 + b^6)/(
a^3*b^7*d^4)) + a^3 + 21*a^2*b + 35*a*b^2 + 7*b^3)/(a*b^4*d^2))*log(7/4*a^6
 + 7/2*a^5*b - 63/4*a^4*b^2 + 9*a^3*b^3 + 25/4*a^2*b^4 - 9/2*a*b^5 - 1/4*b^
6 - 1/4*(7*a^6 + 14*a^5*b - 63*a^4*b^2 + 36*a^3*b^3 + 25*a^2*b^4 - 18*a*b^5
 - b^6)*cos(d*x + c)^2 - 1/2*((a^4*b^5 + 3*a^3*b^6)*d^3*sqrt((49*a^6 + 490*
a^5*b + 1519*a^4*b^2 + 1484*a^3*b^3 + 511*a^2*b^4 + 42*a*b^5 + b^6)/(a^3*b^
7*d^4))*cos(d*x + c)*sin(d*x + c) - (21*a^5*b^2 + 112*a^4*b^3 + 98*a^3*b^4
+ 24*a^2*b^5 + a*b^6)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-(a*b^4*d^2*sqrt((4
9*a^6 + 490*a^5*b + 1519*a^4*b^2 + 1484*a^3*b^3 + 511*a^2*b^4 + 42*a*b^5 +
b^6)/(a^3*b^7*d^4)) + a^3 + 21*a^2*b + 35*a*b^2 + 7*b^3)/(a*b^4*d^2)) - 1/4
*(2*(a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*d^2*cos(d*x + c)^2 - (a^5*b
^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*d^2)*sqrt((49*a^6 + 490*a^5*b + 1519*
a^4*b^2 + 1484*a^3*b^3 + 511*a^2*b^4 + 42*a*b^5 + b^6)/(a^3*b^7*d^4))) + b^
2*d*sqrt((a*b^4*d^2*sqrt((49*a^6 + 490*a^5*b + 1519*a^4*b^2 + 1484*a^3*b^3
+ 511*a^2*b^4 + 42*a*b^5 + b^6)/(a^3*b^7*d^4)) - a^3 - 21*a^2*b - 35*a*b^2
- 7*b^3)/(a*b^4*d^2))*log(-7/4*a^6 - 7/2*a^5*b + 63/4*a^4*b^2 - 9*a^3*b^3 -
 25/4*a^2*b^4 + 9/2*a*b^5 + 1/4*b^6 + 1/4*(7*a^6 + 14*a^5*b - 63*a^4*b^2 +
36*a^3*b^3 + 25*a^2*b^4 - 18*a*b^5 - b^6)*cos(d*x + c)^2 + 1/2*((a^4*b^5 +
3*a^3*b^6)*d^3*sqrt((49*a^6 + 490*a^5*b + 1519*a^4*b^2 + 1484*a^3*b^3 + 511
*a^2*b^4 + 42*a*b^5 + b^6)/(a^3*b^7*d^4))*cos(d*x + c)*sin(d*x + c) + (21*a
^5*b^2 + 112*a^4*b^3 + 98*a^3*b^4 + 24*a^2*b^5 + a*b^6)*d*cos(d*x + c)*sin(
d*x + c))*sqrt((a*b^4*d^2*sqrt((49*a^6 + 490*a^5*b + 1519*a^4*b^2 + 1484*a^
3*b^3 + 511*a^2*b^4 + 42*a*b^5 + b^6)/(a^3*b^7*d^4)) - a^3 - 21*a^2*b - 35*
a*b^2 - 7*b^3)/(a*b^4*d^2)) - 1/4*(2*(a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2
```

```
*b^6)*d^2*cos(d*x + c)^2 - (a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*d^2)
*sqrt((49*a^6 + 490*a^5*b + 1519*a^4*b^2 + 1484*a^3*b^3 + 511*a^2*b^4 + 42*
a*b^5 + b^6)/(a^3*b^7*d^4))) - b^2*d*sqrt((a*b^4*d^2*sqrt((49*a^6 + 490*a^5
*b + 1519*a^4*b^2 + 1484*a^3*b^3 + 511*a^2*b^4 + 42*a*b^5 + b^6)/(a^3*b^7*d
^4)) - a^3 - 21*a^2*b - 35*a*b^2 - 7*b^3)/(a*b^4*d^2))*log(-7/4*a^6 - 7/2*a
^5*b + 63/4*a^4*b^2 - 9*a^3*b^3 - 25/4*a^2*b^4 + 9/2*a*b^5 + 1/4*b^6 + 1/4*
(7*a^6 + 14*a^5*b - 63*a^4*b^2 + 36*a^3*b^3 + 25*a^2*b^4 - 18*a*b^5 - b^6)*
cos(d*x + c)^2 - 1/2*((a^4*b^5 + 3*a^3*b^6)*d^3*sqrt((49*a^6 + 490*a^5*b +
1519*a^4*b^2 + 1484*a^3*b^3 + 511*a^2*b^4 + 42*a*b^5 + b^6)/(a^3*b^7*d^4))*
cos(d*x + c)*sin(d*x + c) + (21*a^5*b^2 + 112*a^4*b^3 + 98*a^3*b^4 + 24*a^2
*b^5 + a*b^6)*d*cos(d*x + c)*sin(d*x + c))*sqrt((a*b^4*d^2*sqrt((49*a^6 + 4
90*a^5*b + 1519*a^4*b^2 + 1484*a^3*b^3 + 511*a^2*b^4 + 42*a*b^5 + b^6)/(a^3
*b^7*d^4)) - a^3 - 21*a^2*b - 35*a*b^2 - 7*b^3)/(a*b^4*d^2)) - 1/4*(2*(a^5*
b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*d^2*cos(d*x + c)^2 - (a^5*b^3 - 3*a^
4*b^4 + 3*a^3*b^5 - a^2*b^6)*d^2)*sqrt((49*a^6 + 490*a^5*b + 1519*a^4*b^2 +
 1484*a^3*b^3 + 511*a^2*b^4 + 42*a*b^5 + b^6)/(a^3*b^7*d^4))) - (8*a + 35*b
)*d*x - (2*b*cos(d*x + c)^3 + 11*b*cos(d*x + c))*sin(d*x + c))/(b^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.413 $\int \frac{\cos^6(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=155

$$-\frac{\left(\sqrt{a}-\sqrt{b}\right)^{5/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/2}d}+\frac{\left(\sqrt{a}+\sqrt{b}\right)^{5/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/2}d}-\frac{\sin(c+dx)\cos(c+dx)}{2bd}-\frac{5x}{2b}$$

```
[Out] (-5*x)/(2*b) - ((Sqrt[a] - Sqrt[b])^(5/2)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*T
an[c + d*x])/a^(1/4)])/(2*a^(3/4)*b^(3/2)*d) + ((Sqrt[a] + Sqrt[b])^(5/2)*A
rcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(3/4)*b^(3/2)*d
) - (Cos[c + d*x]*Sin[c + d*x])/(2*b*d)
```

Rubi [A] time = 0.285637, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3224, 1170, 199, 203, 1166, 205}

$$-\frac{\left(\sqrt{a}-\sqrt{b}\right)^{5/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/2}d}+\frac{\left(\sqrt{a}+\sqrt{b}\right)^{5/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/2}d}-\frac{\sin(c+dx)\cos(c+dx)}{2bd}-\frac{5x}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (-5*x)/(2*b) - ((Sqrt[a] - Sqrt[b])^(5/2)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*T
an[c + d*x])/a^(1/4)])/(2*a^(3/4)*b^(3/2)*d) + ((Sqrt[a] + Sqrt[b])^(5/2)*A
rcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(3/4)*b^(3/2)*d
) - (Cos[c + d*x]*Sin[c + d*x])/(2*b*d)
```

Rule 3224

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(m/2 + 2*p +
1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2]
 && IntegerQ[p]
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^2\left(a+2ax^2+(a-b)x^4\right)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{b\left(1+x^2\right)^2}-\frac{2}{b\left(1+x^2\right)}+\frac{3a+b+2(a-b)x^2}{b\left(a+2ax^2+(a-b)x^4\right)}\right)dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^2}\,dx, x, \tan(c+dx)\right)}{bd}+\frac{\text{Subst}\left(\int \frac{3a+b+2(a-b)x^2}{a+2ax^2+(a-b)x^4}\,dx, x, \tan(c+dx)\right)}{bd}-\frac{2\,\text{Su}\cdots}{}$$

$$= -\frac{2x}{b}-\frac{\cos(c+dx)\sin(c+dx)}{2bd}-\frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(c+dx)\right)}{2bd}+\frac{\left(2a-2b+\frac{(a-b)(a+b)}{\sqrt{a}\sqrt{b}}\cdots\right.}{}$$

$$= -\frac{5x}{2b}-\frac{\left(\sqrt{a}-\sqrt{b}\right)^{5/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/2}d}+\frac{\left(\sqrt{a}+\sqrt{b}\right)^{5/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/2}d}-\cdots$$

Mathematica [A] time = 0.489132, size = 194, normalized size = 1.25

$$\frac{\frac{2\sqrt{b}\left(\sqrt{a}+\sqrt{b}\right)^3\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}+a}}+\frac{2\sqrt{b}\left(\sqrt{a}-\sqrt{b}\right)^3\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}-a}}-10b(c+dx)-b\sin(2(c+dx))}{4b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (-10*b*(c + d*x) + (2*(Sqrt[a] + Sqrt[b])^3*Sqrt[b]*ArcTan[((Sqrt[a] + Sqrt
[b])*Tan[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[a + Sqrt[a]*Sq
rt[b]]) + (2*(Sqrt[a] - Sqrt[b])^3*Sqrt[b]*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan
[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[-a + Sqrt[a]*Sqrt[b]]
) - b*Sin[2*(c + d*x)])/(4*b^2*d)
```

Maple [B] time = 0.125, size = 483, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(a-b*sin(d*x+c)^4),x)
```

```
[Out] -1/2/d/b/(a*b)^(1/2)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/
(((a*b)^(1/2)+a)*(a-b))^(1/2))*a^2+1/d/b*a/(((a*b)^(1/2)+a)*(a-b))^(1/2)*ar
ctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+1/2/d/b/(a*b)^(1/2)/((
(a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b
))^(1/2))*a^2+1/d/b*a/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+
c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-1/d/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan(
(a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+1/2/d*b/(a*b)^(1/2)/(((a*b)
^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2
))-1/d/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2
)-a)*(a-b))^(1/2))-1/2/d*b/(a*b)^(1/2)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctan
h((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-1/2/d/b*tan(d*x+c)/(tan(
d*x+c)^2+1)-5/2/d/b*arctan(tan(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -1/4*(4*b*d*integrate(-4*(4*(a*b + 3*b^2)*cos(6*d*x + 6*c)^2 + 4*(40*a^2 -
23*a*b + 3*b^2)*cos(4*d*x + 4*c)^2 + 4*(a*b + 3*b^2)*cos(2*d*x + 2*c)^2 + 4
*(a*b + 3*b^2)*sin(6*d*x + 6*c)^2 + 4*(40*a^2 - 23*a*b + 3*b^2)*sin(4*d*x +
 4*c)^2 + 2*(8*a^2 + 41*a*b - 13*b^2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4
*(a*b + 3*b^2)*sin(2*d*x + 2*c)^2 - ((a*b + 3*b^2)*cos(6*d*x + 6*c) + 2*(5*
a*b - b^2)*cos(4*d*x + 4*c) + (a*b + 3*b^2)*cos(2*d*x + 2*c))*cos(8*d*x + 8
*c) - (a*b + 3*b^2 - 2*(8*a^2 + 41*a*b - 13*b^2)*cos(4*d*x + 4*c) - 8*(a*b
+ 3*b^2)*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) - 2*(5*a*b - b^2 - (8*a^2 + 41*
a*b - 13*b^2)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - (a*b + 3*b^2)*cos(2*d*x
+ 2*c) - ((a*b + 3*b^2)*sin(6*d*x + 6*c) + 2*(5*a*b - b^2)*sin(4*d*x + 4*c)
 + (a*b + 3*b^2)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 2*((8*a^2 + 41*a*b -
13*b^2)*sin(4*d*x + 4*c) + 4*(a*b + 3*b^2)*sin(2*d*x + 2*c))*sin(6*d*x + 6*
c))/(b^3*cos(8*d*x + 8*c)^2 + 16*b^3*cos(6*d*x + 6*c)^2 + 16*b^3*cos(2*d*x
+ 2*c)^2 + b^3*sin(8*d*x + 8*c)^2 + 16*b^3*sin(6*d*x + 6*c)^2 + 16*b^3*sin(
2*d*x + 2*c)^2 - 8*b^3*cos(2*d*x + 2*c) + b^3 + 4*(64*a^2*b - 48*a*b^2 + 9*
b^3)*cos(4*d*x + 4*c)^2 + 4*(64*a^2*b - 48*a*b^2 + 9*b^3)*sin(4*d*x + 4*c)^
2 + 16*(8*a*b^2 - 3*b^3)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) - 2*(4*b^3*cos(6
*d*x + 6*c) + 4*b^3*cos(2*d*x + 2*c) - b^3 + 2*(8*a*b^2 - 3*b^3)*cos(4*d*x
+ 4*c))*cos(8*d*x + 8*c) + 8*(4*b^3*cos(2*d*x + 2*c) - b^3 + 2*(8*a*b^2 - 3
*b^3)*cos(4*d*x + 4*c))*cos(6*d*x + 6*c) - 4*(8*a*b^2 - 3*b^3 - 4*(8*a*b^2
- 3*b^3)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 4*(2*b^3*sin(6*d*x + 6*c) + 2
*b^3*sin(2*d*x + 2*c) + (8*a*b^2 - 3*b^3)*sin(4*d*x + 4*c))*sin(8*d*x + 8*c
) + 16*(2*b^3*sin(2*d*x + 2*c) + (8*a*b^2 - 3*b^3)*sin(4*d*x + 4*c))*sin(6*
d*x + 6*c)), x) + 10*d*x + sin(2*d*x + 2*c))/(b*d)
```

Fricas [B] time = 5.90708, size = 3996, normalized size = 25.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/8*(b*d*sqrt((a*b^3*d^2*sqrt((25*a^4 + 100*a^3*b + 110*a^2*b^2 + 20*a*b^3
+ b^4)/(a^3*b^5*d^4)) - a^2 - 10*a*b - 5*b^2)/(a*b^3*d^2))*log(5/4*a^4 - 7/
2*a^2*b^2 + 2*a*b^3 + 1/4*b^4 - 1/4*(5*a^4 - 14*a^2*b^2 + 8*a*b^3 + b^4)*co
s(d*x + c)^2 + 1/2*(2*a^3*b^4*d^3*sqrt((25*a^4 + 100*a^3*b + 110*a^2*b^2 +
20*a*b^3 + b^4)/(a^3*b^5*d^4))*cos(d*x + c)*sin(d*x + c) + (5*a^4*b + 15*a^
3*b^2 + 11*a^2*b^3 + a*b^4)*d*cos(d*x + c)*sin(d*x + c))*sqrt((a*b^3*d^2*sq
rt((25*a^4 + 100*a^3*b + 110*a^2*b^2 + 20*a*b^3 + b^4)/(a^3*b^5*d^4)) - a^2
 - 10*a*b - 5*b^2)/(a*b^3*d^2)) + 1/4*(2*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d^
2*cos(d*x + c)^2 - (a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d^2)*sqrt((25*a^4 + 100*
a^3*b + 110*a^2*b^2 + 20*a*b^3 + b^4)/(a^3*b^5*d^4))) - b*d*sqrt((a*b^3*d^2
*sqrt((25*a^4 + 100*a^3*b + 110*a^2*b^2 + 20*a*b^3 + b^4)/(a^3*b^5*d^4)) -
a^2 - 10*a*b - 5*b^2)/(a*b^3*d^2))*log(5/4*a^4 - 7/2*a^2*b^2 + 2*a*b^3 + 1/
4*b^4 - 1/4*(5*a^4 - 14*a^2*b^2 + 8*a*b^3 + b^4)*cos(d*x + c)^2 - 1/2*(2*a^
3*b^4*d^3*sqrt((25*a^4 + 100*a^3*b + 110*a^2*b^2 + 20*a*b^3 + b^4)/(a^3*b^5
*d^4))*cos(d*x + c)*sin(d*x + c) + (5*a^4*b + 15*a^3*b^2 + 11*a^2*b^3 + a*b
^4)*d*cos(d*x + c)*sin(d*x + c))*sqrt((a*b^3*d^2*sqrt((25*a^4 + 100*a^3*b +
 110*a^2*b^2 + 20*a*b^3 + b^4)/(a^3*b^5*d^4)) - a^2 - 10*a*b - 5*b^2)/(a*b^
3*d^2)) + 1/4*(2*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d^2*cos(d*x + c)^2 - (a^4*
b^2 - 2*a^3*b^3 + a^2*b^4)*d^2)*sqrt((25*a^4 + 100*a^3*b + 110*a^2*b^2 + 20
*a*b^3 + b^4)/(a^3*b^5*d^4))) + b*d*sqrt(-(a*b^3*d^2*sqrt((25*a^4 + 100*a^3
*b + 110*a^2*b^2 + 20*a*b^3 + b^4)/(a^3*b^5*d^4)) + a^2 + 10*a*b + 5*b^2)/(
a*b^3*d^2))*log(-5/4*a^4 + 7/2*a^2*b^2 - 2*a*b^3 - 1/4*b^4 + 1/4*(5*a^4 - 1
4*a^2*b^2 + 8*a*b^3 + b^4)*cos(d*x + c)^2 + 1/2*(2*a^3*b^4*d^3*sqrt((25*a^4
 + 100*a^3*b + 110*a^2*b^2 + 20*a*b^3 + b^4)/(a^3*b^5*d^4))*cos(d*x + c)*si
n(d*x + c) - (5*a^4*b + 15*a^3*b^2 + 11*a^2*b^3 + a*b^4)*d*cos(d*x + c)*sin
(d*x + c))*sqrt(-(a*b^3*d^2*sqrt((25*a^4 + 100*a^3*b + 110*a^2*b^2 + 20*a*b
^3 + b^4)/(a^3*b^5*d^4)) + a^2 + 10*a*b + 5*b^2)/(a*b^3*d^2)) + 1/4*(2*(a^4
*b^2 - 2*a^3*b^3 + a^2*b^4)*d^2*cos(d*x + c)^2 - (a^4*b^2 - 2*a^3*b^3 + a^2
*b^4)*d^2)*sqrt((25*a^4 + 100*a^3*b + 110*a^2*b^2 + 20*a*b^3 + b^4)/(a^3*b^
5*d^4))) - b*d*sqrt(-(a*b^3*d^2*sqrt((25*a^4 + 100*a^3*b + 110*a^2*b^2 + 20
*a*b^3 + b^4)/(a^3*b^5*d^4)) + a^2 + 10*a*b + 5*b^2)/(a*b^3*d^2))*log(-5/4*
a^4 + 7/2*a^2*b^2 - 2*a*b^3 - 1/4*b^4 + 1/4*(5*a^4 - 14*a^2*b^2 + 8*a*b^3 +
 b^4)*cos(d*x + c)^2 - 1/2*(2*a^3*b^4*d^3*sqrt((25*a^4 + 100*a^3*b + 110*a^
2*b^2 + 20*a*b^3 + b^4)/(a^3*b^5*d^4))*cos(d*x + c)*sin(d*x + c) - (5*a^4*b
 + 15*a^3*b^2 + 11*a^2*b^3 + a*b^4)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-(a*b
^3*d^2*sqrt((25*a^4 + 100*a^3*b + 110*a^2*b^2 + 20*a*b^3 + b^4)/(a^3*b^5*d^
4)) + a^2 + 10*a*b + 5*b^2)/(a*b^3*d^2)) + 1/4*(2*(a^4*b^2 - 2*a^3*b^3 + a^
2*b^4)*d^2*cos(d*x + c)^2 - (a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d^2)*sqrt((25*a
^4 + 100*a^3*b + 110*a^2*b^2 + 20*a*b^3 + b^4)/(a^3*b^5*d^4))) - 20*d*x - 4
*cos(d*x + c)*sin(d*x + c))/(b*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.414 $\int \frac{\cos^4(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=127

$$\frac{\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}bd}+\frac{\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}bd}-\frac{x}{b}$$

```
[Out] -(x/b) + ((Sqrt[a] - Sqrt[b])^(3/2)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c +
 d*x])/a^(1/4)])/(2*a^(3/4)*b*d) + ((Sqrt[a] + Sqrt[b])^(3/2)*ArcTan[(Sqrt[
Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(3/4)*b*d)
```

Rubi [A] time = 0.23546, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3224, 1170, 203, 1166, 205}

$$\frac{\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}bd}+\frac{\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}bd}-\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -(x/b) + ((Sqrt[a] - Sqrt[b])^(3/2)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c +
 d*x])/a^(1/4)])/(2*a^(3/4)*b*d) + ((Sqrt[a] + Sqrt[b])^(3/2)*ArcTan[(Sqrt[
Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(3/4)*b*d)
```

Rule 3224

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(m/2 + 2*p +
1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2]
 && IntegerQ[p]
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+2ax^2+(a-b)x^4)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{b(1+x^2)}+\frac{a+b+(a-b)x^2}{b(a+2ax^2+(a-b)x^4)}\right) dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(c+dx)\right)}{bd}+\frac{\text{Subst}\left(\int \frac{a+b+(a-b)x^2}{a+2ax^2+(a-b)x^4}\,dx, x, \tan(c+dx)\right)}{bd}$$

$$= -\frac{x}{b}+\frac{\left(\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)(a-b)\right)\text{Subst}\left(\int \frac{1}{a+\sqrt{a}\sqrt{b}+(a-b)x^2}\,dx, x, \tan(c+dx)\right)}{2bd}+\frac{\left(\left(1+\frac{\sqrt{b}}{\sqrt{a}}\right)(a-b)\right)\text{Sub}}{}$$

$$= -\frac{x}{b}+\frac{\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}bd}+\frac{\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}bd}$$

Mathematica [A] time = 0.251548, size = 171, normalized size = 1.35

$$\frac{\frac{\left(\sqrt{a}+\sqrt{b}\right)^2\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}+a}}-\frac{\left(\sqrt{a}-\sqrt{b}\right)^2\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}-a}}-2(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (-2*(c + d*x) + ((Sqrt[a] + Sqrt[b])^2*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c +
d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[a + Sqrt[a]*Sqrt[b]]) - ((S
qrt[a] - Sqrt[b])^2*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sq
rt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[-a + Sqrt[a]*Sqrt[b]]))/(2*b*d)
```

Maple [B] time = 0.132, size = 449, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a-b*sin(d*x+c)^4),x)
```

```
[Out] 1/2/d/b*a/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/
2)+a)*(a-b))^(1/2))-1/2/d*a/(a*b)^(1/2)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arcta
n((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+1/2/d/b*a/(((a*b)^(1/2)-a
)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))+1/2
```

```
/d*a/(a*b)^(1/2)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/((
(a*b)^(1/2)-a)*(a-b))^(1/2))-1/2/d/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-
b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+1/2/d*b/(a*b)^(1/2)/(((a*b)^(1
/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-
1/2/d/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)
-a)*(a-b))^(1/2))-1/2/d*b/(a*b)^(1/2)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh
((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-1/d/b*arctan(tan(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -(b*integrate(-8*(4*b^2*cos(6*d*x + 6*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 + 4*b
^2*sin(6*d*x + 6*c)^2 + 4*b^2*sin(2*d*x + 2*c)^2 + 4*(8*a^2 - 3*a*b)*cos(4*
d*x + 4*c)^2 - b^2*cos(2*d*x + 2*c) + 4*(8*a^2 - 3*a*b)*sin(4*d*x + 4*c)^2
+ 6*(4*a*b - b^2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) - (b^2*cos(6*d*x + 6*c)
 + 2*a*b*cos(4*d*x + 4*c) + b^2*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) + (8*b^2
*cos(2*d*x + 2*c) - b^2 + 6*(4*a*b - b^2)*cos(4*d*x + 4*c))*cos(6*d*x + 6*c
) - 2*(a*b - 3*(4*a*b - b^2)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - (b^2*sin(
6*d*x + 6*c) + 2*a*b*sin(4*d*x + 4*c) + b^2*sin(2*d*x + 2*c))*sin(8*d*x + 8
*c) + 2*(4*b^2*sin(2*d*x + 2*c) + 3*(4*a*b - b^2)*sin(4*d*x + 4*c))*sin(6*d
*x + 6*c))/(b^3*cos(8*d*x + 8*c)^2 + 16*b^3*cos(6*d*x + 6*c)^2 + 16*b^3*cos
(2*d*x + 2*c)^2 + b^3*sin(8*d*x + 8*c)^2 + 16*b^3*sin(6*d*x + 6*c)^2 + 16*b
^3*sin(2*d*x + 2*c)^2 - 8*b^3*cos(2*d*x + 2*c) + b^3 + 4*(64*a^2*b - 48*a*b
^2 + 9*b^3)*cos(4*d*x + 4*c)^2 + 4*(64*a^2*b - 48*a*b^2 + 9*b^3)*sin(4*d*x
+ 4*c)^2 + 16*(8*a*b^2 - 3*b^3)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) - 2*(4*b^
3*cos(6*d*x + 6*c) + 4*b^3*cos(2*d*x + 2*c) - b^3 + 2*(8*a*b^2 - 3*b^3)*cos
(4*d*x + 4*c))*cos(8*d*x + 8*c) + 8*(4*b^3*cos(2*d*x + 2*c) - b^3 + 2*(8*a*
b^2 - 3*b^3)*cos(4*d*x + 4*c))*cos(6*d*x + 6*c) - 4*(8*a*b^2 - 3*b^3 - 4*(8
*a*b^2 - 3*b^3)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 4*(2*b^3*sin(6*d*x + 6
*c) + 2*b^3*sin(2*d*x + 2*c) + (8*a*b^2 - 3*b^3)*sin(4*d*x + 4*c))*sin(8*d*
x + 8*c) + 16*(2*b^3*sin(2*d*x + 2*c) + (8*a*b^2 - 3*b^3)*sin(4*d*x + 4*c))
*sin(6*d*x + 6*c)), x) + x)/b
```

Fricas [B] time = 3.91836, size = 2688, normalized size = 21.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/8*(b*sqrt((a*b^2*d^2*sqrt((9*a^2 + 6*a*b + b^2)/(a^3*b^3*d^4)) - a - 3*b)
/(a*b^2*d^2))*log(1/4*(3*a^2 - 2*a*b - b^2)*cos(d*x + c)^2 - 3/4*a^2 + 1/2*
a*b + 1/4*b^2 + 1/2*(a^3*b^2*d^3*sqrt((9*a^2 + 6*a*b + b^2)/(a^3*b^3*d^4))*
cos(d*x + c)*sin(d*x + c) + (3*a^2*b + a*b^2)*d*cos(d*x + c)*sin(d*x + c))*
sqrt((a*b^2*d^2*sqrt((9*a^2 + 6*a*b + b^2)/(a^3*b^3*d^4)) - a - 3*b)/(a*b^2
*d^2)) - 1/4*(2*(a^3*b - a^2*b^2)*d^2*cos(d*x + c)^2 - (a^3*b - a^2*b^2)*d^
2)*sqrt((9*a^2 + 6*a*b + b^2)/(a^3*b^3*d^4))) - b*sqrt((a*b^2*d^2*sqrt((9*a
^2 + 6*a*b + b^2)/(a^3*b^3*d^4)) - a - 3*b)/(a*b^2*d^2))*log(1/4*(3*a^2 - 2
*a*b - b^2)*cos(d*x + c)^2 - 3/4*a^2 + 1/2*a*b + 1/4*b^2 - 1/2*(a^3*b^2*d^3
```

```
*sqrt((9*a^2 + 6*a*b + b^2)/(a^3*b^3*d^4))*cos(d*x + c)*sin(d*x + c) + (3*a
^2*b + a*b^2)*d*cos(d*x + c)*sin(d*x + c))*sqrt((a*b^2*d^2*sqrt((9*a^2 + 6*
a*b + b^2)/(a^3*b^3*d^4)) - a - 3*b)/(a*b^2*d^2)) - 1/4*(2*(a^3*b - a^2*b^2
)*d^2*cos(d*x + c)^2 - (a^3*b - a^2*b^2)*d^2)*sqrt((9*a^2 + 6*a*b + b^2)/(a
^3*b^3*d^4))) + b*sqrt(-(a*b^2*d^2*sqrt((9*a^2 + 6*a*b + b^2)/(a^3*b^3*d^4)
) + a + 3*b)/(a*b^2*d^2))*log(-1/4*(3*a^2 - 2*a*b - b^2)*cos(d*x + c)^2 + 3
/4*a^2 - 1/2*a*b - 1/4*b^2 + 1/2*(a^3*b^2*d^3*sqrt((9*a^2 + 6*a*b + b^2)/(a
^3*b^3*d^4))*cos(d*x + c)*sin(d*x + c) - (3*a^2*b + a*b^2)*d*cos(d*x + c)*s
in(d*x + c))*sqrt(-(a*b^2*d^2*sqrt((9*a^2 + 6*a*b + b^2)/(a^3*b^3*d^4)) + a
 + 3*b)/(a*b^2*d^2)) - 1/4*(2*(a^3*b - a^2*b^2)*d^2*cos(d*x + c)^2 - (a^3*b
 - a^2*b^2)*d^2)*sqrt((9*a^2 + 6*a*b + b^2)/(a^3*b^3*d^4))) - b*sqrt(-(a*b^
2*d^2*sqrt((9*a^2 + 6*a*b + b^2)/(a^3*b^3*d^4)) + a + 3*b)/(a*b^2*d^2))*log
(-1/4*(3*a^2 - 2*a*b - b^2)*cos(d*x + c)^2 + 3/4*a^2 - 1/2*a*b - 1/4*b^2 -
1/2*(a^3*b^2*d^3*sqrt((9*a^2 + 6*a*b + b^2)/(a^3*b^3*d^4))*cos(d*x + c)*sin
(d*x + c) - (3*a^2*b + a*b^2)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-(a*b^2*d^2
*sqrt((9*a^2 + 6*a*b + b^2)/(a^3*b^3*d^4)) + a + 3*b)/(a*b^2*d^2)) - 1/4*(2
*(a^3*b - a^2*b^2)*d^2*cos(d*x + c)^2 - (a^3*b - a^2*b^2)*d^2)*sqrt((9*a^2
+ 6*a*b + b^2)/(a^3*b^3*d^4))) - 8*x)/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.415 $\int \frac{\cos^2(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=125

$$\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{b}d}-\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{b}d}$$

```
[Out] -(Sqrt[Sqrt[a] - Sqrt[b]]*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(
1/4)])/(2*a^(3/4)*Sqrt[b]*d) + (Sqrt[Sqrt[a] + Sqrt[b]]*ArcTan[(Sqrt[Sqrt[a
] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(3/4)*Sqrt[b]*d)
```

Rubi [A] time = 0.109314, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3224, 1093, 205}

$$\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{b}d}-\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{b}d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -(Sqrt[Sqrt[a] - Sqrt[b]]*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(
1/4)])/(2*a^(3/4)*Sqrt[b]*d) + (Sqrt[Sqrt[a] + Sqrt[b]]*ArcTan[(Sqrt[Sqrt[a
] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(3/4)*Sqrt[b]*d)
```

Rule 3224

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(m/2 + 2*p +
1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2]
 && IntegerQ[p]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{a+2ax^2+(a-b)x^4}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{a-\sqrt{a}\sqrt{b}+(a-b)x^2}\,dx, x, \tan(c+dx)\right)}{2\sqrt{a}\sqrt{b}d} - \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{a+\sqrt{a}\sqrt{b}+(a-b)x^2}\,dx, x, \tan\right.}{2\sqrt{a}\sqrt{b}d}$$
$$= -\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{b}d} + \frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{b}d}$$

Mathematica [A] time = 0.281721, size = 158, normalized size = 1.26

$$\frac{\frac{\left(\sqrt{a}\sqrt{b}+b\right)\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}+a}} + \frac{\left(\sqrt{a}\sqrt{b}-b\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}-a}}}{2\sqrt{a}bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (((Sqrt[a]*Sqrt[b] + b)*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/Sqrt[a +
Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]] + ((Sqrt[a]*Sqrt[b] - b)*ArcTa
nh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a
+ Sqrt[a]*Sqrt[b]])/(2*Sqrt[a]*b*d)
```

Maple [B] time = 0.112, size = 226, normalized size = 1.8

$$-\frac{a}{2d}\arctan\left((a-b)\tan(dx+c)\frac{1}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)\frac{1}{\sqrt{ab}}\frac{1}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}} + \frac{a}{2d}\text{Artanh}\left((-a+b)\tan(dx+c)\frac{}{\sqrt{(}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a-b*sin(d*x+c)^4),x)
```

```
[Out] -1/2/d*a/(a*b)^(1/2)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/
(((a*b)^(1/2)+a)*(a-b))^(1/2))+1/2/d*a/(a*b)^(1/2)/(((a*b)^(1/2)-a)*(a-b))^
(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))+1/2/d*b/(a*b
)^(1/2)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)
+a)*(a-b))^(1/2))-1/2/d*b/(a*b)^(1/2)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh
((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\cos(dx+c)^2}{b\sin(dx+c)^4 - a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -integrate(cos(d*x + c)^2/(b*sin(d*x + c)^4 - a), x)
```

Fricas [B] time = 2.76512, size = 1277, normalized size = 10.22

$$-\frac{1}{8}\sqrt{-\frac{abd^2\sqrt{\frac{1}{a^3bd^4}}+1}{abd^2}}\log\left(\frac{1}{2}ad\sqrt{-\frac{abd^2\sqrt{\frac{1}{a^3bd^4}}+1}{abd^2}}\cos\left(dx+c\right)\sin\left(dx+c\right)+\frac{1}{4}\cos\left(dx+c\right)^2+\frac{1}{4}\left(2a^2d^2\cos\left(dx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(-(a*b*d^2*sqrt(1/(a^3*b*d^4)) + 1)/(a*b*d^2))*log(1/2*a*d*sqrt(-(
a*b*d^2*sqrt(1/(a^3*b*d^4)) + 1)/(a*b*d^2))*cos(d*x + c)*sin(d*x + c) + 1/4
*cos(d*x + c)^2 + 1/4*(2*a^2*d^2*cos(d*x + c)^2 - a^2*d^2)*sqrt(1/(a^3*b*d^
4)) - 1/4) + 1/8*sqrt(-(a*b*d^2*sqrt(1/(a^3*b*d^4)) + 1)/(a*b*d^2))*log(-1/
2*a*d*sqrt(-(a*b*d^2*sqrt(1/(a^3*b*d^4)) + 1)/(a*b*d^2))*cos(d*x + c)*sin(d
*x + c) + 1/4*cos(d*x + c)^2 + 1/4*(2*a^2*d^2*cos(d*x + c)^2 - a^2*d^2)*sqr
t(1/(a^3*b*d^4)) - 1/4) + 1/8*sqrt((a*b*d^2*sqrt(1/(a^3*b*d^4)) - 1)/(a*b*d
^2))*log(1/2*a*d*sqrt((a*b*d^2*sqrt(1/(a^3*b*d^4)) - 1)/(a*b*d^2))*cos(d*x
+ c)*sin(d*x + c) - 1/4*cos(d*x + c)^2 + 1/4*(2*a^2*d^2*cos(d*x + c)^2 - a^
2*d^2)*sqrt(1/(a^3*b*d^4)) + 1/4) - 1/8*sqrt((a*b*d^2*sqrt(1/(a^3*b*d^4)) -
 1)/(a*b*d^2))*log(-1/2*a*d*sqrt((a*b*d^2*sqrt(1/(a^3*b*d^4)) - 1)/(a*b*d^2
))*cos(d*x + c)*sin(d*x + c) - 1/4*cos(d*x + c)^2 + 1/4*(2*a^2*d^2*cos(d*x
+ c)^2 - a^2*d^2)*sqrt(1/(a^3*b*d^4)) + 1/4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.416 $\int \frac{\sec^2(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=142

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}+\frac{\tan(c+dx)}{d(a-b)}$$

```
[Out] -(Sqrt[b]*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(3/4
)*(Sqrt[a] - Sqrt[b])^(3/2)*d) + (Sqrt[b]*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*T
an[c + d*x])/a^(1/4)])/(2*a^(3/4)*(Sqrt[a] + Sqrt[b])^(3/2)*d) + Tan[c + d*
x]/((a - b)*d)
```

Rubi [A] time = 0.23203, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3224, 1170, 1166, 205}

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}+\frac{\tan(c+dx)}{d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -(Sqrt[b]*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(3/4
)*(Sqrt[a] - Sqrt[b])^(3/2)*d) + (Sqrt[b]*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*T
an[c + d*x])/a^(1/4)])/(2*a^(3/4)*(Sqrt[a] + Sqrt[b])^(3/2)*d) + Tan[c + d*
x]/((a - b)*d)
```

Rule 3224

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(m/2 + 2*p +
1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2]
 && IntegerQ[p]
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{a+2ax^2+(a-b)x^4}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a-b}-\frac{b\left(1+2x^2\right)}{(a-b)\left(a+2ax^2+(a-b)x^4\right)}\right)dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\tan(c+dx)}{(a-b)d}-\frac{b\,\text{Subst}\left(\int \frac{1+2x^2}{a+2ax^2+(a-b)x^4}\,dx, x, \tan(c+dx)\right)}{(a-b)d}$$

$$= \frac{\tan(c+dx)}{(a-b)d}-\frac{\left(\left(\sqrt{a}+\sqrt{b}\right)^2\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{a+\sqrt{a}\sqrt{b}+(a-b)x^2}\,dx, x, \tan(c+dx)\right)}{2\sqrt{a}(a-b)d}-\frac{\left(b\left(2-\frac{a+b}{\sqrt{a}\sqrt{\vphantom{b}}}\right.\right.}{}$$

$$= -\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}d}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}d}+\frac{\tan(c+dx)}{(a-b)d}$$

Mathematica [A] time = 0.497502, size = 175, normalized size = 1.23

$$\frac{\frac{\left(\sqrt{a}\sqrt{b}-b\right)\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}+a}}+\frac{\left(\sqrt{a}\sqrt{b}+b\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}-a}}+2\tan(c+dx)}{2d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (((Sqrt[a]*Sqrt[b] - b)*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/Sqrt[a +
Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[a + Sqrt[a]*Sqrt[b]]) + ((Sqrt[a]*Sqrt[b]
+ b)*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]]
)/(Sqrt[a]*Sqrt[-a + Sqrt[a]*Sqrt[b]]) + 2*Tan[c + d*x])/(2*(a - b)*d)
```

Maple [B] time = 0.128, size = 393, normalized size = 2.8

$$\frac{\tan(dx+c)}{(a-b)d}-\frac{ab}{2(a-b)d}\arctan\left((a-b)\tan(dx+c)\frac{1}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)\frac{1}{\sqrt{ab}}\frac{1}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}-\frac{b^2}{2(a-b)d}\text{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a-b*sin(d*x+c)^4),x)
```

```
[Out] tan(d*x+c)/(a-b)/d-1/2/d*a/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*
arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*b-1/2/d/(a*b)^(1/2)/
(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a
```

```
)*(a-b))^(1/2))*b^2-1/d*b/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*
tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+1/2/d*a/(a*b)^(1/2)/(a-b)/(((a*b)
^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1
/2))*b+1/2/d/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)
*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*b^2-1/d*b/(a-b)/(((a*b)^(1/2)-a)
*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] (((a - b)*d*cos(2*d*x + 2*c)^2 + (a - b)*d*sin(2*d*x + 2*c)^2 + 2*(a - b)*d
*cos(2*d*x + 2*c) + (a - b)*d)*integrate(4*(4*b^2*cos(6*d*x + 6*c)^2 + 4*b^
2*cos(2*d*x + 2*c)^2 + 4*b^2*sin(6*d*x + 6*c)^2 + 4*b^2*sin(2*d*x + 2*c)^2
- 12*(8*a*b - 3*b^2)*cos(4*d*x + 4*c)^2 - b^2*cos(2*d*x + 2*c) - 12*(8*a*b
- 3*b^2)*sin(4*d*x + 4*c)^2 + 2*(8*a*b - 15*b^2)*sin(4*d*x + 4*c)*sin(2*d*x
 + 2*c) - (b^2*cos(6*d*x + 6*c) - 6*b^2*cos(4*d*x + 4*c) + b^2*cos(2*d*x +
2*c))*cos(8*d*x + 8*c) + (8*b^2*cos(2*d*x + 2*c) - b^2 + 2*(8*a*b - 15*b^2)
*cos(4*d*x + 4*c))*cos(6*d*x + 6*c) + 2*(3*b^2 + (8*a*b - 15*b^2)*cos(2*d*x
 + 2*c))*cos(4*d*x + 4*c) - (b^2*sin(6*d*x + 6*c) - 6*b^2*sin(4*d*x + 4*c)
+ b^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 2*(4*b^2*sin(2*d*x + 2*c) + (8*a
*b - 15*b^2)*sin(4*d*x + 4*c))*sin(6*d*x + 6*c))/(a*b^2 - b^3 + (a*b^2 - b^
3)*cos(8*d*x + 8*c)^2 + 16*(a*b^2 - b^3)*cos(6*d*x + 6*c)^2 + 4*(64*a^3 - 1
12*a^2*b + 57*a*b^2 - 9*b^3)*cos(4*d*x + 4*c)^2 + 16*(a*b^2 - b^3)*cos(2*d*
x + 2*c)^2 + (a*b^2 - b^3)*sin(8*d*x + 8*c)^2 + 16*(a*b^2 - b^3)*sin(6*d*x
+ 6*c)^2 + 4*(64*a^3 - 112*a^2*b + 57*a*b^2 - 9*b^3)*sin(4*d*x + 4*c)^2 + 1
6*(8*a^2*b - 11*a*b^2 + 3*b^3)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a*b^
2 - b^3)*sin(2*d*x + 2*c)^2 + 2*(a*b^2 - b^3 - 4*(a*b^2 - b^3)*cos(6*d*x +
6*c) - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*cos(4*d*x + 4*c) - 4*(a*b^2 - b^3)*co
s(2*d*x + 2*c))*cos(8*d*x + 8*c) - 8*(a*b^2 - b^3 - 2*(8*a^2*b - 11*a*b^2 +
 3*b^3)*cos(4*d*x + 4*c) - 4*(a*b^2 - b^3)*cos(2*d*x + 2*c))*cos(6*d*x + 6*
c) - 4*(8*a^2*b - 11*a*b^2 + 3*b^3 - 4*(8*a^2*b - 11*a*b^2 + 3*b^3)*cos(2*d
*x + 2*c))*cos(4*d*x + 4*c) - 8*(a*b^2 - b^3)*cos(2*d*x + 2*c) - 4*(2*(a*b^
2 - b^3)*sin(6*d*x + 6*c) + (8*a^2*b - 11*a*b^2 + 3*b^3)*sin(4*d*x + 4*c) +
 2*(a*b^2 - b^3)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^2*b - 11*a*b
^2 + 3*b^3)*sin(4*d*x + 4*c) + 2*(a*b^2 - b^3)*sin(2*d*x + 2*c))*sin(6*d*x
+ 6*c)), x) + 2*sin(2*d*x + 2*c))/((a - b)*d*cos(2*d*x + 2*c)^2 + (a - b)*d
*sin(2*d*x + 2*c)^2 + 2*(a - b)*d*cos(2*d*x + 2*c) + (a - b)*d)
```

Fricas [B] time = 5.37704, size = 5536, normalized size = 38.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/8*((a - b)*d*sqrt(((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d^2*sqrt((9*a^2*b^
3 + 6*a*b^4 + b^5)/((a^9 - 6*a^8*b + 15*a^7*b^2 - 20*a^6*b^3 + 15*a^5*b^4 -
 6*a^4*b^5 + a^3*b^6)*d^4)) - a*b - 3*b^2)/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*
b^3)*d^2))*cos(d*x + c)*log(3/4*a*b^2 + 1/4*b^3 - 1/4*(3*a*b^2 + b^3)*cos(d
```

```
*x + c)^2 + 1/2*(2*(a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^3*sqrt((9*a^2*b^
3 + 6*a*b^4 + b^5)/((a^9 - 6*a^8*b + 15*a^7*b^2 - 20*a^6*b^3 + 15*a^5*b^4 -
 6*a^4*b^5 + a^3*b^6)*d^4))*cos(d*x + c)*sin(d*x + c) + (3*a^3*b + 4*a^2*b^
2 + a*b^3)*d*cos(d*x + c)*sin(d*x + c))*sqrt(((a^4 - 3*a^3*b + 3*a^2*b^2 -
a*b^3)*d^2*sqrt((9*a^2*b^3 + 6*a*b^4 + b^5)/((a^9 - 6*a^8*b + 15*a^7*b^2 -
20*a^6*b^3 + 15*a^5*b^4 - 6*a^4*b^5 + a^3*b^6)*d^4)) - a*b - 3*b^2)/((a^4 -
 3*a^3*b + 3*a^2*b^2 - a*b^3)*d^2)) - 1/4*(2*(a^5 - 3*a^4*b + 3*a^3*b^2 - a
^2*b^3)*d^2*cos(d*x + c)^2 - (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d^2)*sqr
t((9*a^2*b^3 + 6*a*b^4 + b^5)/((a^9 - 6*a^8*b + 15*a^7*b^2 - 20*a^6*b^3 + 1
5*a^5*b^4 - 6*a^4*b^5 + a^3*b^6)*d^4))) - (a - b)*d*sqrt(((a^4 - 3*a^3*b +
3*a^2*b^2 - a*b^3)*d^2*sqrt((9*a^2*b^3 + 6*a*b^4 + b^5)/((a^9 - 6*a^8*b + 1
5*a^7*b^2 - 20*a^6*b^3 + 15*a^5*b^4 - 6*a^4*b^5 + a^3*b^6)*d^4)) - a*b - 3*
b^2)/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d^2))*cos(d*x + c)*log(3/4*a*b^2
+ 1/4*b^3 - 1/4*(3*a*b^2 + b^3)*cos(d*x + c)^2 - 1/2*(2*(a^6 - 3*a^5*b + 3*
a^4*b^2 - a^3*b^3)*d^3*sqrt((9*a^2*b^3 + 6*a*b^4 + b^5)/((a^9 - 6*a^8*b + 1
5*a^7*b^2 - 20*a^6*b^3 + 15*a^5*b^4 - 6*a^4*b^5 + a^3*b^6)*d^4))*cos(d*x +
c)*sin(d*x + c) + (3*a^3*b + 4*a^2*b^2 + a*b^3)*d*cos(d*x + c)*sin(d*x + c)
)*sqrt(((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d^2*sqrt((9*a^2*b^3 + 6*a*b^4 +
 b^5)/((a^9 - 6*a^8*b + 15*a^7*b^2 - 20*a^6*b^3 + 15*a^5*b^4 - 6*a^4*b^5 +
a^3*b^6)*d^4)) - a*b - 3*b^2)/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d^2)) -
1/4*(2*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d^2*cos(d*x + c)^2 - (a^5 - 3*
a^4*b + 3*a^3*b^2 - a^2*b^3)*d^2)*sqrt((9*a^2*b^3 + 6*a*b^4 + b^5)/((a^9 -
6*a^8*b + 15*a^7*b^2 - 20*a^6*b^3 + 15*a^5*b^4 - 6*a^4*b^5 + a^3*b^6)*d^4))
) + (a - b)*d*sqrt(-((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d^2*sqrt((9*a^2*b^
3 + 6*a*b^4 + b^5)/((a^9 - 6*a^8*b + 15*a^7*b^2 - 20*a^6*b^3 + 15*a^5*b^4 -
 6*a^4*b^5 + a^3*b^6)*d^4)) + a*b + 3*b^2)/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*
b^3)*d^2))*cos(d*x + c)*log(-3/4*a*b^2 - 1/4*b^3 + 1/4*(3*a*b^2 + b^3)*cos(
d*x + c)^2 + 1/2*(2*(a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^3*sqrt((9*a^2*b
^3 + 6*a*b^4 + b^5)/((a^9 - 6*a^8*b + 15*a^7*b^2 - 20*a^6*b^3 + 15*a^5*b^4
- 6*a^4*b^5 + a^3*b^6)*d^4))*cos(d*x + c)*sin(d*x + c) - (3*a^3*b + 4*a^2*b
^2 + a*b^3)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^4 - 3*a^3*b + 3*a^2*b^2
- a*b^3)*d^2*sqrt((9*a^2*b^3 + 6*a*b^4 + b^5)/((a^9 - 6*a^8*b + 15*a^7*b^2
- 20*a^6*b^3 + 15*a^5*b^4 - 6*a^4*b^5 + a^3*b^6)*d^4)) + a*b + 3*b^2)/((a^4
 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d^2)) - 1/4*(2*(a^5 - 3*a^4*b + 3*a^3*b^2 -
 a^2*b^3)*d^2*cos(d*x + c)^2 - (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d^2)*s
qrt((9*a^2*b^3 + 6*a*b^4 + b^5)/((a^9 - 6*a^8*b + 15*a^7*b^2 - 20*a^6*b^3 +
 15*a^5*b^4 - 6*a^4*b^5 + a^3*b^6)*d^4))) - (a - b)*d*sqrt(-((a^4 - 3*a^3*b
 + 3*a^2*b^2 - a*b^3)*d^2*sqrt((9*a^2*b^3 + 6*a*b^4 + b^5)/((a^9 - 6*a^8*b
+ 15*a^7*b^2 - 20*a^6*b^3 + 15*a^5*b^4 - 6*a^4*b^5 + a^3*b^6)*d^4)) + a*b +
 3*b^2)/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d^2))*cos(d*x + c)*log(-3/4*a*
b^2 - 1/4*b^3 + 1/4*(3*a*b^2 + b^3)*cos(d*x + c)^2 - 1/2*(2*(a^6 - 3*a^5*b
+ 3*a^4*b^2 - a^3*b^3)*d^3*sqrt((9*a^2*b^3 + 6*a*b^4 + b^5)/((a^9 - 6*a^8*b
 + 15*a^7*b^2 - 20*a^6*b^3 + 15*a^5*b^4 - 6*a^4*b^5 + a^3*b^6)*d^4))*cos(d*
x + c)*sin(d*x + c) - (3*a^3*b + 4*a^2*b^2 + a*b^3)*d*cos(d*x + c)*sin(d*x
+ c))*sqrt(-((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d^2*sqrt((9*a^2*b^3 + 6*a*
b^4 + b^5)/((a^9 - 6*a^8*b + 15*a^7*b^2 - 20*a^6*b^3 + 15*a^5*b^4 - 6*a^4*b
^5 + a^3*b^6)*d^4)) + a*b + 3*b^2)/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d^2
)) - 1/4*(2*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d^2*cos(d*x + c)^2 - (a^5
 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d^2)*sqrt((9*a^2*b^3 + 6*a*b^4 + b^5)/((a
^9 - 6*a^8*b + 15*a^7*b^2 - 20*a^6*b^3 + 15*a^5*b^4 - 6*a^4*b^5 + a^3*b^6)*
d^4))) + 8*sin(d*x + c))/((a - b)*d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.417 $\int \frac{\sec^4(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=161

$$\frac{b\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{b\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}+\frac{\tan^3(c+dx)}{3d(a-b)}+\frac{(a-3b)\tan(c+dx)}{d(a-b)^2}$$

```
[Out] (b*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(3/4)*(Sqrt
[a] - Sqrt[b])^(5/2)*d) + (b*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/
a^(1/4)])/(2*a^(3/4)*(Sqrt[a] + Sqrt[b])^(5/2)*d) + ((a - 3*b)*Tan[c + d*x]
)/((a - b)^2*d) + Tan[c + d*x]^3/(3*(a - b)*d)
```

Rubi [A] time = 0.347604, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3224, 1170, 1166, 205}

$$\frac{b\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{b\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}+\frac{\tan^3(c+dx)}{3d(a-b)}+\frac{(a-3b)\tan(c+dx)}{d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (b*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(3/4)*(Sqrt
[a] - Sqrt[b])^(5/2)*d) + (b*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/
a^(1/4)])/(2*a^(3/4)*(Sqrt[a] + Sqrt[b])^(5/2)*d) + ((a - 3*b)*Tan[c + d*x]
)/((a - b)^2*d) + Tan[c + d*x]^3/(3*(a - b)*d)
```

Rule 3224

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(m/2 + 2*p +
1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2]
 && IntegerQ[p]
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{a+2ax^2+(a-b)x^4}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a-3b}{(a-b)^2}+\frac{x^2}{a-b}+\frac{b(a+b)+b(a+3b)x^2}{(a-b)^2\left(a+2ax^2+(a-b)x^4\right)}\right)dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{(a-3b)\tan(c+dx)}{(a-b)^2d}+\frac{\tan^3(c+dx)}{3(a-b)d}+\frac{\text{Subst}\left(\int \frac{b(a+b)+b(a+3b)x^2}{a+2ax^2+(a-b)x^4}\,dx, x, \tan(c+dx)\right)}{(a-b)^2d}$$
$$= \frac{(a-3b)\tan(c+dx)}{(a-b)^2d}+\frac{\tan^3(c+dx)}{3(a-b)d}+\frac{\left(\left(\sqrt{a}-\sqrt{b}\right)^3b\right)\text{Subst}\left(\int \frac{1}{a-\sqrt{a}\sqrt{b}+(a-b)x^2}\,dx, x, \tan(c+\right.}{2\sqrt{a}(a-b)^2d}$$
$$= \frac{b\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}d}+\frac{b\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}d}+\frac{(a-3b)\tan(c+dx)}{(a-b)^2d}+\frac{\tan^3(c+dx}{3(a-b)d}$$

Mathematica [A] time = 1.01141, size = 205, normalized size = 1.27

$$\frac{\frac{3b\left(-2\sqrt{a}\sqrt{b}+a+b\right)\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}+a}}+4(a-4b)\tan(c+dx)-\frac{3b\left(\sqrt{a}+\sqrt{b}\right)^2\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}-a}}+2(a-b)\tan(c+dx)\sec^2(}{6d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] ((3*b*(a - 2*Sqrt[a]*Sqrt[b] + b)*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])
/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[a + Sqrt[a]*Sqrt[b]]) - (3*(Sqrt
[a] + Sqrt[b])^2*b*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqr
t[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[-a + Sqrt[a]*Sqrt[b]]) + 4*(a - 4*b)*Tan[c +
d*x] + 2*(a - b)*Sec[c + d*x]^2*Tan[c + d*x])/(6*(a - b)^2*d)
```

Maple [B] time = 0.157, size = 581, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a-b*sin(d*x+c)^4),x)
```

```
[Out] 1/3/d/(a-b)^2*tan(d*x+c)^3*a-1/3/d/(a-b)^2*tan(d*x+c)^3*b+1/d/(a-b)^2*tan(d
*x+c)*a-3/d/(a-b)^2*tan(d*x+c)*b+1/2/d*b/(a-b)^2/(((a*b)^(1/2)+a)*(a-b))^(1
/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*a+3/2/d*b^2/(a-b
)^2/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*
```

```
(a-b))^(1/2))+3/2/d*b^2/(a-b)^2/(a*b)^(1/2)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*a
rctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*a+1/2/d*b^3/(a-b)^2/(
a*b)^(1/2)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1
/2)+a)*(a-b))^(1/2))+1/2/d*b/(a-b)^2/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh(
(-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*a+3/2/d*b^2/(a-b)^2/(((a*b
)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(
1/2))-3/2/d*b^2/(a-b)^2/(a*b)^(1/2)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((
-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*a-1/2/d*b^3/(a-b)^2/(a*b)^(
1/2)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-
a)*(a-b))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -1/3*(36*(a - 2*b)*cos(4*d*x + 4*c)*sin(2*d*x + 2*c) - 12*(b*sin(4*d*x + 4*
c) - (a - 3*b)*sin(2*d*x + 2*c))*cos(6*d*x + 6*c) - 3*((a^2 - 2*a*b + b^2)*
d*cos(6*d*x + 6*c)^2 + 9*(a^2 - 2*a*b + b^2)*d*cos(4*d*x + 4*c)^2 + 9*(a^2
- 2*a*b + b^2)*d*cos(2*d*x + 2*c)^2 + (a^2 - 2*a*b + b^2)*d*sin(6*d*x + 6*c
)^2 + 9*(a^2 - 2*a*b + b^2)*d*sin(4*d*x + 4*c)^2 + 18*(a^2 - 2*a*b + b^2)*d
*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*(a^2 - 2*a*b + b^2)*d*sin(2*d*x + 2*
c)^2 + 6*(a^2 - 2*a*b + b^2)*d*cos(2*d*x + 2*c) + (a^2 - 2*a*b + b^2)*d + 2
*(3*(a^2 - 2*a*b + b^2)*d*cos(4*d*x + 4*c) + 3*(a^2 - 2*a*b + b^2)*d*cos(2*
d*x + 2*c) + (a^2 - 2*a*b + b^2)*d)*cos(6*d*x + 6*c) + 6*(3*(a^2 - 2*a*b +
b^2)*d*cos(2*d*x + 2*c) + (a^2 - 2*a*b + b^2)*d)*cos(4*d*x + 4*c) + 6*((a^2
 - 2*a*b + b^2)*d*sin(4*d*x + 4*c) + (a^2 - 2*a*b + b^2)*d*sin(2*d*x + 2*c)
)*sin(6*d*x + 6*c))*integrate(-8*(4*b^3*cos(6*d*x + 6*c)^2 + 4*b^3*cos(2*d*
x + 2*c)^2 + 4*b^3*sin(6*d*x + 6*c)^2 + 4*b^3*sin(2*d*x + 2*c)^2 - b^3*cos(
2*d*x + 2*c) - 4*(8*a^2*b + 13*a*b^2 - 6*b^3)*cos(4*d*x + 4*c)^2 - 4*(8*a^2
*b + 13*a*b^2 - 6*b^3)*sin(4*d*x + 4*c)^2 + 2*(4*a*b^2 - 11*b^3)*sin(4*d*x
+ 4*c)*sin(2*d*x + 2*c) - (b^3*cos(6*d*x + 6*c) + b^3*cos(2*d*x + 2*c) - 2*
(a*b^2 + 2*b^3)*cos(4*d*x + 4*c))*cos(8*d*x + 8*c) + (8*b^3*cos(2*d*x + 2*c
) - b^3 + 2*(4*a*b^2 - 11*b^3)*cos(4*d*x + 4*c))*cos(6*d*x + 6*c) + 2*(a*b^
2 + 2*b^3 + (4*a*b^2 - 11*b^3)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - (b^3*si
n(6*d*x + 6*c) + b^3*sin(2*d*x + 2*c) - 2*(a*b^2 + 2*b^3)*sin(4*d*x + 4*c))
*sin(8*d*x + 8*c) + 2*(4*b^3*sin(2*d*x + 2*c) + (4*a*b^2 - 11*b^3)*sin(4*d*
x + 4*c))*sin(6*d*x + 6*c))/(a^2*b^2 - 2*a*b^3 + b^4 + (a^2*b^2 - 2*a*b^3 +
 b^4)*cos(8*d*x + 8*c)^2 + 16*(a^2*b^2 - 2*a*b^3 + b^4)*cos(6*d*x + 6*c)^2
+ 4*(64*a^4 - 176*a^3*b + 169*a^2*b^2 - 66*a*b^3 + 9*b^4)*cos(4*d*x + 4*c)^
2 + 16*(a^2*b^2 - 2*a*b^3 + b^4)*cos(2*d*x + 2*c)^2 + (a^2*b^2 - 2*a*b^3 +
b^4)*sin(8*d*x + 8*c)^2 + 16*(a^2*b^2 - 2*a*b^3 + b^4)*sin(6*d*x + 6*c)^2 +
 4*(64*a^4 - 176*a^3*b + 169*a^2*b^2 - 66*a*b^3 + 9*b^4)*sin(4*d*x + 4*c)^2
 + 16*(8*a^3*b - 19*a^2*b^2 + 14*a*b^3 - 3*b^4)*sin(4*d*x + 4*c)*sin(2*d*x
+ 2*c) + 16*(a^2*b^2 - 2*a*b^3 + b^4)*sin(2*d*x + 2*c)^2 + 2*(a^2*b^2 - 2*a
*b^3 + b^4 - 4*(a^2*b^2 - 2*a*b^3 + b^4)*cos(6*d*x + 6*c) - 2*(8*a^3*b - 19
*a^2*b^2 + 14*a*b^3 - 3*b^4)*cos(4*d*x + 4*c) - 4*(a^2*b^2 - 2*a*b^3 + b^4)
*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) - 8*(a^2*b^2 - 2*a*b^3 + b^4 - 2*(8*a^3
*b - 19*a^2*b^2 + 14*a*b^3 - 3*b^4)*cos(4*d*x + 4*c) - 4*(a^2*b^2 - 2*a*b^3
 + b^4)*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) - 4*(8*a^3*b - 19*a^2*b^2 + 14*a
*b^3 - 3*b^4 - 4*(8*a^3*b - 19*a^2*b^2 + 14*a*b^3 - 3*b^4)*cos(2*d*x + 2*c)
)*cos(4*d*x + 4*c) - 8*(a^2*b^2 - 2*a*b^3 + b^4)*cos(2*d*x + 2*c) - 4*(2*(a
^2*b^2 - 2*a*b^3 + b^4)*sin(6*d*x + 6*c) + (8*a^3*b - 19*a^2*b^2 + 14*a*b^3
 - 3*b^4)*sin(4*d*x + 4*c) + 2*(a^2*b^2 - 2*a*b^3 + b^4)*sin(2*d*x + 2*c))*
```

```
sin(8*d*x + 8*c) + 16*((8*a^3*b - 19*a^2*b^2 + 14*a*b^3 - 3*b^4)*sin(4*d*x
+ 4*c) + 2*(a^2*b^2 - 2*a*b^3 + b^4)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)), x
) + 4*(3*b*cos(4*d*x + 4*c) - 3*(a - 3*b)*cos(2*d*x + 2*c) - a + 4*b)*sin(6
*d*x + 6*c) - 12*(3*(a - 2*b)*cos(2*d*x + 2*c) + a - 3*b)*sin(4*d*x + 4*c)
+ 12*b*sin(2*d*x + 2*c))/((a^2 - 2*a*b + b^2)*d*cos(6*d*x + 6*c)^2 + 9*(a^2
 - 2*a*b + b^2)*d*cos(4*d*x + 4*c)^2 + 9*(a^2 - 2*a*b + b^2)*d*cos(2*d*x +
2*c)^2 + (a^2 - 2*a*b + b^2)*d*sin(6*d*x + 6*c)^2 + 9*(a^2 - 2*a*b + b^2)*d
*sin(4*d*x + 4*c)^2 + 18*(a^2 - 2*a*b + b^2)*d*sin(4*d*x + 4*c)*sin(2*d*x +
 2*c) + 9*(a^2 - 2*a*b + b^2)*d*sin(2*d*x + 2*c)^2 + 6*(a^2 - 2*a*b + b^2)*
d*cos(2*d*x + 2*c) + (a^2 - 2*a*b + b^2)*d + 2*(3*(a^2 - 2*a*b + b^2)*d*cos
(4*d*x + 4*c) + 3*(a^2 - 2*a*b + b^2)*d*cos(2*d*x + 2*c) + (a^2 - 2*a*b + b
^2)*d)*cos(6*d*x + 6*c) + 6*(3*(a^2 - 2*a*b + b^2)*d*cos(2*d*x + 2*c) + (a^
2 - 2*a*b + b^2)*d)*cos(4*d*x + 4*c) + 6*((a^2 - 2*a*b + b^2)*d*sin(4*d*x +
 4*c) + (a^2 - 2*a*b + b^2)*d*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))
```

Fricas [B] time = 7.96937, size = 9225, normalized size = 57.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/24*(3*(a^2 - 2*a*b + b^2)*d*sqrt(-(a^2*b^2 + 10*a*b^3 + 5*b^4 - (a^6 - 5*
a^5*b + 10*a^4*b^2 - 10*a^3*b^3 + 5*a^2*b^4 - a*b^5)*d^2*sqrt((25*a^4*b^5 +
 100*a^3*b^6 + 110*a^2*b^7 + 20*a*b^8 + b^9)/((a^13 - 10*a^12*b + 45*a^11*b
^2 - 120*a^10*b^3 + 210*a^9*b^4 - 252*a^8*b^5 + 210*a^7*b^6 - 120*a^6*b^7 +
 45*a^5*b^8 - 10*a^4*b^9 + a^3*b^10)*d^4)))/((a^6 - 5*a^5*b + 10*a^4*b^2 -
10*a^3*b^3 + 5*a^2*b^4 - a*b^5)*d^2))*cos(d*x + c)^3*log(5/4*a^2*b^4 + 5/2*
a*b^5 + 1/4*b^6 - 1/4*(5*a^2*b^4 + 10*a*b^5 + b^6)*cos(d*x + c)^2 + 1/2*((a
^9 - 2*a^8*b - 5*a^7*b^2 + 20*a^6*b^3 - 25*a^5*b^4 + 14*a^4*b^5 - 3*a^3*b^6
)*d^3*sqrt((25*a^4*b^5 + 100*a^3*b^6 + 110*a^2*b^7 + 20*a*b^8 + b^9)/((a^13
 - 10*a^12*b + 45*a^11*b^2 - 120*a^10*b^3 + 210*a^9*b^4 - 252*a^8*b^5 + 210
*a^7*b^6 - 120*a^6*b^7 + 45*a^5*b^8 - 10*a^4*b^9 + a^3*b^10)*d^4))*cos(d*x
+ c)*sin(d*x + c) + (15*a^4*b^3 + 35*a^3*b^4 + 13*a^2*b^5 + a*b^6)*d*cos(d*
x + c)*sin(d*x + c))*sqrt(-(a^2*b^2 + 10*a*b^3 + 5*b^4 - (a^6 - 5*a^5*b + 1
0*a^4*b^2 - 10*a^3*b^3 + 5*a^2*b^4 - a*b^5)*d^2*sqrt((25*a^4*b^5 + 100*a^3*
b^6 + 110*a^2*b^7 + 20*a*b^8 + b^9)/((a^13 - 10*a^12*b + 45*a^11*b^2 - 120*
a^10*b^3 + 210*a^9*b^4 - 252*a^8*b^5 + 210*a^7*b^6 - 120*a^6*b^7 + 45*a^5*b
^8 - 10*a^4*b^9 + a^3*b^10)*d^4)))/((a^6 - 5*a^5*b + 10*a^4*b^2 - 10*a^3*b^
3 + 5*a^2*b^4 - a*b^5)*d^2)) - 1/4*(2*(a^7*b - 5*a^6*b^2 + 10*a^5*b^3 - 10*
a^4*b^4 + 5*a^3*b^5 - a^2*b^6)*d^2*cos(d*x + c)^2 - (a^7*b - 5*a^6*b^2 + 10
*a^5*b^3 - 10*a^4*b^4 + 5*a^3*b^5 - a^2*b^6)*d^2)*sqrt((25*a^4*b^5 + 100*a^
3*b^6 + 110*a^2*b^7 + 20*a*b^8 + b^9)/((a^13 - 10*a^12*b + 45*a^11*b^2 - 12
0*a^10*b^3 + 210*a^9*b^4 - 252*a^8*b^5 + 210*a^7*b^6 - 120*a^6*b^7 + 45*a^5
*b^8 - 10*a^4*b^9 + a^3*b^10)*d^4))) - 3*(a^2 - 2*a*b + b^2)*d*sqrt(-(a^2*b
^2 + 10*a*b^3 + 5*b^4 - (a^6 - 5*a^5*b + 10*a^4*b^2 - 10*a^3*b^3 + 5*a^2*b^
4 - a*b^5)*d^2*sqrt((25*a^4*b^5 + 100*a^3*b^6 + 110*a^2*b^7 + 20*a*b^8 + b^
9)/((a^13 - 10*a^12*b + 45*a^11*b^2 - 120*a^10*b^3 + 210*a^9*b^4 - 252*a^8*
b^5 + 210*a^7*b^6 - 120*a^6*b^7 + 45*a^5*b^8 - 10*a^4*b^9 + a^3*b^10)*d^4))
)/((a^6 - 5*a^5*b + 10*a^4*b^2 - 10*a^3*b^3 + 5*a^2*b^4 - a*b^5)*d^2))*cos(
d*x + c)^3*log(5/4*a^2*b^4 + 5/2*a*b^5 + 1/4*b^6 - 1/4*(5*a^2*b^4 + 10*a*b^
5 + b^6)*cos(d*x + c)^2 - 1/2*((a^9 - 2*a^8*b - 5*a^7*b^2 + 20*a^6*b^3 - 25
*a^5*b^4 + 14*a^4*b^5 - 3*a^3*b^6)*d^3*sqrt((25*a^4*b^5 + 100*a^3*b^6 + 110
*a^2*b^7 + 20*a*b^8 + b^9)/((a^13 - 10*a^12*b + 45*a^11*b^2 - 120*a^10*b^3
+ 210*a^9*b^4 - 252*a^8*b^5 + 210*a^7*b^6 - 120*a^6*b^7 + 45*a^5*b^8 - 10*a
^4*b^9 + a^3*b^10)*d^4))*cos(d*x + c)*sin(d*x + c) + (15*a^4*b^3 + 35*a^3*b
```

^4 + 13*a^2*b^5 + a*b^6)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-(a^2*b^2 + 10*a
*b^3 + 5*b^4 - (a^6 - 5*a^5*b + 10*a^4*b^2 - 10*a^3*b^3 + 5*a^2*b^4 - a*b^5
)*d^2*sqrt((25*a^4*b^5 + 100*a^3*b^6 + 110*a^2*b^7 + 20*a*b^8 + b^9)/((a^13
 - 10*a^12*b + 45*a^11*b^2 - 120*a^10*b^3 + 210*a^9*b^4 - 252*a^8*b^5 + 210
*a^7*b^6 - 120*a^6*b^7 + 45*a^5*b^8 - 10*a^4*b^9 + a^3*b^10)*d^4)))/((a^6 -
 5*a^5*b + 10*a^4*b^2 - 10*a^3*b^3 + 5*a^2*b^4 - a*b^5)*d^2)) - 1/4*(2*(a^7
*b - 5*a^6*b^2 + 10*a^5*b^3 - 10*a^4*b^4 + 5*a^3*b^5 - a^2*b^6)*d^2*cos(d*x
 + c)^2 - (a^7*b - 5*a^6*b^2 + 10*a^5*b^3 - 10*a^4*b^4 + 5*a^3*b^5 - a^2*b^
6)*d^2)*sqrt((25*a^4*b^5 + 100*a^3*b^6 + 110*a^2*b^7 + 20*a*b^8 + b^9)/((a^
13 - 10*a^12*b + 45*a^11*b^2 - 120*a^10*b^3 + 210*a^9*b^4 - 252*a^8*b^5 + 2
10*a^7*b^6 - 120*a^6*b^7 + 45*a^5*b^8 - 10*a^4*b^9 + a^3*b^10)*d^4))) + 3*(
a^2 - 2*a*b + b^2)*d*sqrt(-(a^2*b^2 + 10*a*b^3 + 5*b^4 + (a^6 - 5*a^5*b + 1
0*a^4*b^2 - 10*a^3*b^3 + 5*a^2*b^4 - a*b^5)*d^2*sqrt((25*a^4*b^5 + 100*a^3*
b^6 + 110*a^2*b^7 + 20*a*b^8 + b^9)/((a^13 - 10*a^12*b + 45*a^11*b^2 - 120*
a^10*b^3 + 210*a^9*b^4 - 252*a^8*b^5 + 210*a^7*b^6 - 120*a^6*b^7 + 45*a^5*b
^8 - 10*a^4*b^9 + a^3*b^10)*d^4)))/((a^6 - 5*a^5*b + 10*a^4*b^2 - 10*a^3*b^
3 + 5*a^2*b^4 - a*b^5)*d^2))*cos(d*x + c)^3*log(-5/4*a^2*b^4 - 5/2*a*b^5 -
1/4*b^6 + 1/4*(5*a^2*b^4 + 10*a*b^5 + b^6)*cos(d*x + c)^2 + 1/2*((a^9 - 2*a
^8*b - 5*a^7*b^2 + 20*a^6*b^3 - 25*a^5*b^4 + 14*a^4*b^5 - 3*a^3*b^6)*d^3*sq
rt((25*a^4*b^5 + 100*a^3*b^6 + 110*a^2*b^7 + 20*a*b^8 + b^9)/((a^13 - 10*a^
12*b + 45*a^11*b^2 - 120*a^10*b^3 + 210*a^9*b^4 - 252*a^8*b^5 + 210*a^7*b^6
 - 120*a^6*b^7 + 45*a^5*b^8 - 10*a^4*b^9 + a^3*b^10)*d^4))*cos(d*x + c)*sin
(d*x + c) - (15*a^4*b^3 + 35*a^3*b^4 + 13*a^2*b^5 + a*b^6)*d*cos(d*x + c)*s
in(d*x + c))*sqrt(-(a^2*b^2 + 10*a*b^3 + 5*b^4 + (a^6 - 5*a^5*b + 10*a^4*b^
2 - 10*a^3*b^3 + 5*a^2*b^4 - a*b^5)*d^2*sqrt((25*a^4*b^5 + 100*a^3*b^6 + 11
0*a^2*b^7 + 20*a*b^8 + b^9)/((a^13 - 10*a^12*b + 45*a^11*b^2 - 120*a^10*b^3
 + 210*a^9*b^4 - 252*a^8*b^5 + 210*a^7*b^6 - 120*a^6*b^7 + 45*a^5*b^8 - 10*
a^4*b^9 + a^3*b^10)*d^4)))/((a^6 - 5*a^5*b + 10*a^4*b^2 - 10*a^3*b^3 + 5*a^
2*b^4 - a*b^5)*d^2)) - 1/4*(2*(a^7*b - 5*a^6*b^2 + 10*a^5*b^3 - 10*a^4*b^4
+ 5*a^3*b^5 - a^2*b^6)*d^2*cos(d*x + c)^2 - (a^7*b - 5*a^6*b^2 + 10*a^5*b^3
 - 10*a^4*b^4 + 5*a^3*b^5 - a^2*b^6)*d^2)*sqrt((25*a^4*b^5 + 100*a^3*b^6 +
110*a^2*b^7 + 20*a*b^8 + b^9)/((a^13 - 10*a^12*b + 45*a^11*b^2 - 120*a^10*b
^3 + 210*a^9*b^4 - 252*a^8*b^5 + 210*a^7*b^6 - 120*a^6*b^7 + 45*a^5*b^8 - 1
0*a^4*b^9 + a^3*b^10)*d^4))) - 3*(a^2 - 2*a*b + b^2)*d*sqrt(-(a^2*b^2 + 10*
a*b^3 + 5*b^4 + (a^6 - 5*a^5*b + 10*a^4*b^2 - 10*a^3*b^3 + 5*a^2*b^4 - a*b^
5)*d^2*sqrt((25*a^4*b^5 + 100*a^3*b^6 + 110*a^2*b^7 + 20*a*b^8 + b^9)/((a^1
3 - 10*a^12*b + 45*a^11*b^2 - 120*a^10*b^3 + 210*a^9*b^4 - 252*a^8*b^5 + 21
0*a^7*b^6 - 120*a^6*b^7 + 45*a^5*b^8 - 10*a^4*b^9 + a^3*b^10)*d^4)))/((a^6
- 5*a^5*b + 10*a^4*b^2 - 10*a^3*b^3 + 5*a^2*b^4 - a*b^5)*d^2))*cos(d*x + c)
^3*log(-5/4*a^2*b^4 - 5/2*a*b^5 - 1/4*b^6 + 1/4*(5*a^2*b^4 + 10*a*b^5 + b^6
)*cos(d*x + c)^2 - 1/2*((a^9 - 2*a^8*b - 5*a^7*b^2 + 20*a^6*b^3 - 25*a^5*b^
4 + 14*a^4*b^5 - 3*a^3*b^6)*d^3*sqrt((25*a^4*b^5 + 100*a^3*b^6 + 110*a^2*b^
7 + 20*a*b^8 + b^9)/((a^13 - 10*a^12*b + 45*a^11*b^2 - 120*a^10*b^3 + 210*a
^9*b^4 - 252*a^8*b^5 + 210*a^7*b^6 - 120*a^6*b^7 + 45*a^5*b^8 - 10*a^4*b^9
+ a^3*b^10)*d^4))*cos(d*x + c)*sin(d*x + c) - (15*a^4*b^3 + 35*a^3*b^4 + 13
*a^2*b^5 + a*b^6)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-(a^2*b^2 + 10*a*b^3 +
5*b^4 + (a^6 - 5*a^5*b + 10*a^4*b^2 - 10*a^3*b^3 + 5*a^2*b^4 - a*b^5)*d^2*s
qrt((25*a^4*b^5 + 100*a^3*b^6 + 110*a^2*b^7 + 20*a*b^8 + b^9)/((a^13 - 10*a
^12*b + 45*a^11*b^2 - 120*a^10*b^3 + 210*a^9*b^4 - 252*a^8*b^5 + 210*a^7*b^
6 - 120*a^6*b^7 + 45*a^5*b^8 - 10*a^4*b^9 + a^3*b^10)*d^4)))/((a^6 - 5*a^5*
b + 10*a^4*b^2 - 10*a^3*b^3 + 5*a^2*b^4 - a*b^5)*d^2)) - 1/4*(2*(a^7*b - 5*
a^6*b^2 + 10*a^5*b^3 - 10*a^4*b^4 + 5*a^3*b^5 - a^2*b^6)*d^2*cos(d*x + c)^2
 - (a^7*b - 5*a^6*b^2 + 10*a^5*b^3 - 10*a^4*b^4 + 5*a^3*b^5 - a^2*b^6)*d^2)
*sqrt((25*a^4*b^5 + 100*a^3*b^6 + 110*a^2*b^7 + 20*a*b^8 + b^9)/((a^13 - 10
*a^12*b + 45*a^11*b^2 - 120*a^10*b^3 + 210*a^9*b^4 - 252*a^8*b^5 + 210*a^7*
b^6 - 120*a^6*b^7 + 45*a^5*b^8 - 10*a^4*b^9 + a^3*b^10)*d^4))) + 8*(2*(a -
4*b)*cos(d*x + c)^2 + a - b)*sin(d*x + c))/((a^2 - 2*a*b + b^2)*d*cos(d*x +
 c)^3)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.418 $\int \frac{\sec^6(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=204

$$-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}-\sqrt{b}\right)^{7/2}}+\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}+\sqrt{b}\right)^{7/2}}+\frac{\left(a^2-3ab+6b^2\right)\tan(c+dx)}{d(a-b)^3}+\frac{\tan^5(c+dx)}{5d(a-b)}+\frac{2(a-}{}$$

```
[Out] -(b^(3/2)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(3/4
)*(Sqrt[a] - Sqrt[b])^(7/2)*d) + (b^(3/2)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*T
an[c + d*x])/a^(1/4)])/(2*a^(3/4)*(Sqrt[a] + Sqrt[b])^(7/2)*d) + ((a^2 - 3*
a*b + 6*b^2)*Tan[c + d*x])/((a - b)^3*d) + (2*(a - 2*b)*Tan[c + d*x]^3)/(3*
(a - b)^2*d) + Tan[c + d*x]^5/(5*(a - b)*d)
```

Rubi [A] time = 0.377966, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3224, 1170, 1166, 205}

$$-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}-\sqrt{b}\right)^{7/2}}+\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}+\sqrt{b}\right)^{7/2}}+\frac{\left(a^2-3ab+6b^2\right)\tan(c+dx)}{d(a-b)^3}+\frac{\tan^5(c+dx)}{5d(a-b)}+\frac{2(a-}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -(b^(3/2)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(3/4
)*(Sqrt[a] - Sqrt[b])^(7/2)*d) + (b^(3/2)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*T
an[c + d*x])/a^(1/4)])/(2*a^(3/4)*(Sqrt[a] + Sqrt[b])^(7/2)*d) + ((a^2 - 3*
a*b + 6*b^2)*Tan[c + d*x])/((a - b)^3*d) + (2*(a - 2*b)*Tan[c + d*x]^3)/(3*
(a - b)^2*d) + Tan[c + d*x]^5/(5*(a - b)*d)
```

Rule 3224

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(m/2 + 2*p +
1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2]
 && IntegerQ[p]
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec^6(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^4}{a+2ax^2+(a-b)x^4}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a^2-3ab+6b^2}{(a-b)^3}+\frac{2(a-2b)x^2}{(a-b)^2}+\frac{x^4}{a-b}-\frac{b^2(3a+b)+4b^2(a+b)x^2}{(a-b)^3\left(a+2ax^2+(a-b)x^4\right)}\right)dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\left(a^2-3ab+6b^2\right)\tan(c+dx)}{(a-b)^3d}+\frac{2(a-2b)\tan^3(c+dx)}{3(a-b)^2d}+\frac{\tan^5(c+dx)}{5(a-b)d}-\frac{\text{Subst}\left(\int \frac{b^2(3a+b)+4b^{\ldots}}{a+2ax^2+(\ldots}\right.}{(\ldots}$$

$$= \frac{\left(a^2-3ab+6b^2\right)\tan(c+dx)}{(a-b)^3d}+\frac{2(a-2b)\tan^3(c+dx)}{3(a-b)^2d}+\frac{\tan^5(c+dx)}{5(a-b)d}+\frac{\left(\left(\sqrt{a}-\sqrt{b}\right)^4b^{3/2}\right)\text{S}\ldots}{\ldots}$$

$$= -\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^{7/2}d}+\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^{7/2}d}+\frac{\left(a^2-3ab+6b^2\right)\tan(c+dx)}{(a-b)^3d}\ldots$$

Mathematica [A] time = 1.34661, size = 253, normalized size = 1.24

$$\frac{2\left(8a^2-21ab+73b^2\right)\tan(c+dx)+\frac{15b^{3/2}\left(\sqrt{a}-\sqrt{b}\right)^3\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}+a}}+\frac{15b^{3/2}\left(\sqrt{a}+\sqrt{b}\right)^3\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}-a}}+6(a-b)^2\,\text{ta}\ldots}{30d(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] ((15*(Sqrt[a] - Sqrt[b])^3*b^(3/2)*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x]
)/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[a + Sqrt[a]*Sqrt[b]]) + (15*(Sq
rt[a] + Sqrt[b])^3*b^(3/2)*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[
-a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[-a + Sqrt[a]*Sqrt[b]]) + 2*(8*a^2 - 2
1*a*b + 73*b^2)*Tan[c + d*x] + 4*(2*a - 7*b)*(a - b)*Sec[c + d*x]^2*Tan[c +
 d*x] + 6*(a - b)^2*Sec[c + d*x]^4*Tan[c + d*x])/(30*(a - b)^3*d)
```

Maple [B] time = 0.148, size = 839, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a-b*sin(d*x+c)^4),x)
```

```
[Out] 1/5/d/(a-b)^3*tan(d*x+c)^5*a^2-2/5/d/(a-b)^3*tan(d*x+c)^5*a*b+1/5/d/(a-b)^3
*tan(d*x+c)^5*b^2+2/3/d/(a-b)^3*tan(d*x+c)^3*a^2-2/d/(a-b)^3*tan(d*x+c)^3*a
*b+4/3/d/(a-b)^3*tan(d*x+c)^3*b^2+1/d/(a-b)^3*a^2*tan(d*x+c)-3/d/(a-b)^3*a*
b*tan(d*x+c)+6/d/(a-b)^3*b^2*tan(d*x+c)-2/d*b^2/(a-b)^3/(((a*b)^(1/2)+a)*(a
-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*a-2/d*b^3
/(a-b)^3/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2
)+a)*(a-b))^(1/2))-1/2/d*b^2/(a-b)^3/(a*b)^(1/2)/(((a*b)^(1/2)+a)*(a-b))^(1
/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*a^2-3/d*b^3/(a-b
)^3/(a*b)^(1/2)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*
b)^(1/2)+a)*(a-b))^(1/2))*a-1/2/d*b^4/(a-b)^3/(a*b)^(1/2)/(((a*b)^(1/2)+a)*
(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-2/d*b^2
/(a-b)^3/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1
/2)-a)*(a-b))^(1/2))*a-2/d*b^3/(a-b)^3/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctan
h((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))+1/2/d*b^2/(a-b)^3/(a*b)^
(1/2)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)
-a)*(a-b))^(1/2))*a^2+3/d*b^3/(a-b)^3/(a*b)^(1/2)/(((a*b)^(1/2)-a)*(a-b))^(
1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*a+1/2/d*b^4/(
a-b)^3/(a*b)^(1/2)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/
(((a*b)^(1/2)-a)*(a-b))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] 1/15*(300*(a*b - 5*b^2)*cos(4*d*x + 4*c)*sin(2*d*x + 2*c) - 10*(48*b^2*sin(
6*d*x + 6*c) + 3*(a*b + 3*b^2)*sin(8*d*x + 8*c) + 2*(8*a^2 - 21*a*b + 49*b^
2)*sin(4*d*x + 4*c) + 8*(a^2 - 3*a*b + 8*b^2)*sin(2*d*x + 2*c))*cos(10*d*x
+ 10*c) + 50*(6*(a*b - 5*b^2)*sin(6*d*x + 6*c) - 16*(a^2 - 3*a*b + 5*b^2)*s
in(4*d*x + 4*c) - (8*a^2 - 27*a*b + 55*b^2)*sin(2*d*x + 2*c))*cos(8*d*x + 8
*c) - 200*((8*a^2 - 21*a*b + 25*b^2)*sin(4*d*x + 4*c) + 4*(a^2 - 3*a*b + 5*
b^2)*sin(2*d*x + 2*c))*cos(6*d*x + 6*c) + 15*((a^3 - 3*a^2*b + 3*a*b^2 - b^
3)*d*cos(10*d*x + 10*c)^2 + 25*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(8*d*x
+ 8*c)^2 + 100*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(6*d*x + 6*c)^2 + 100*(
a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(4*d*x + 4*c)^2 + 25*(a^3 - 3*a^2*b + 3
*a*b^2 - b^3)*d*cos(2*d*x + 2*c)^2 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sin(
10*d*x + 10*c)^2 + 25*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sin(8*d*x + 8*c)^2
+ 100*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sin(6*d*x + 6*c)^2 + 100*(a^3 - 3*a
^2*b + 3*a*b^2 - b^3)*d*sin(4*d*x + 4*c)^2 + 100*(a^3 - 3*a^2*b + 3*a*b^2 -
 b^3)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 25*(a^3 - 3*a^2*b + 3*a*b^2 - b
^3)*d*sin(2*d*x + 2*c)^2 + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(2*d*x +
 2*c) + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d + 2*(5*(a^3 - 3*a^2*b + 3*a*b^2 -
 b^3)*d*cos(8*d*x + 8*c) + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(6*d*x +
 6*c) + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(4*d*x + 4*c) + 5*(a^3 - 3*
a^2*b + 3*a*b^2 - b^3)*d*cos(2*d*x + 2*c) + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)
*d)*cos(10*d*x + 10*c) + 10*(10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(6*d*x
 + 6*c) + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(4*d*x + 4*c) + 5*(a^3 -
3*a^2*b + 3*a*b^2 - b^3)*d*cos(2*d*x + 2*c) + (a^3 - 3*a^2*b + 3*a*b^2 - b^
3)*d)*cos(8*d*x + 8*c) + 20*(10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(4*d*x
 + 4*c) + 5*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(2*d*x + 2*c) + (a^3 - 3*a
^2*b + 3*a*b^2 - b^3)*d)*cos(6*d*x + 6*c) + 20*(5*(a^3 - 3*a^2*b + 3*a*b^2
- b^3)*d*cos(2*d*x + 2*c) + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d)*cos(4*d*x +
4*c) + 10*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sin(8*d*x + 8*c) + 2*(a^3 - 3*
a^2*b + 3*a*b^2 - b^3)*d*sin(6*d*x + 6*c) + 2*(a^3 - 3*a^2*b + 3*a*b^2 - b^
```

```
3)*d*sin(4*d*x + 4*c) + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sin(2*d*x + 2*c))
*sin(10*d*x + 10*c) + 50*(2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sin(6*d*x + 6
*c) + 2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sin(4*d*x + 4*c) + (a^3 - 3*a^2*b
 + 3*a*b^2 - b^3)*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 100*(2*(a^3 - 3*a^
2*b + 3*a*b^2 - b^3)*d*sin(4*d*x + 4*c) + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d
*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*integrate(4*(4*(a*b^3 + 3*b^4)*cos(6*d
*x + 6*c)^2 - 4*(56*a^2*b^2 + 19*a*b^3 - 15*b^4)*cos(4*d*x + 4*c)^2 + 4*(a*
b^3 + 3*b^4)*cos(2*d*x + 2*c)^2 + 4*(a*b^3 + 3*b^4)*sin(6*d*x + 6*c)^2 - 4*
(56*a^2*b^2 + 19*a*b^3 - 15*b^4)*sin(4*d*x + 4*c)^2 + 2*(8*a^2*b^2 - 7*a*b^
3 - 29*b^4)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*(a*b^3 + 3*b^4)*sin(2*d*x
 + 2*c)^2 - ((a*b^3 + 3*b^4)*cos(6*d*x + 6*c) - 2*(7*a*b^3 + 5*b^4)*cos(4*d
*x + 4*c) + (a*b^3 + 3*b^4)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) - (a*b^3 + 3
*b^4 - 2*(8*a^2*b^2 - 7*a*b^3 - 29*b^4)*cos(4*d*x + 4*c) - 8*(a*b^3 + 3*b^4
)*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 2*(7*a*b^3 + 5*b^4 + (8*a^2*b^2 - 7*
a*b^3 - 29*b^4)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - (a*b^3 + 3*b^4)*cos(2*
d*x + 2*c) - ((a*b^3 + 3*b^4)*sin(6*d*x + 6*c) - 2*(7*a*b^3 + 5*b^4)*sin(4*
d*x + 4*c) + (a*b^3 + 3*b^4)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 2*((8*a^2
*b^2 - 7*a*b^3 - 29*b^4)*sin(4*d*x + 4*c) + 4*(a*b^3 + 3*b^4)*sin(2*d*x + 2
*c))*sin(6*d*x + 6*c))/(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5 + (a^3*b^2 - 3*
a^2*b^3 + 3*a*b^4 - b^5)*cos(8*d*x + 8*c)^2 + 16*(a^3*b^2 - 3*a^2*b^3 + 3*a
*b^4 - b^5)*cos(6*d*x + 6*c)^2 + 4*(64*a^5 - 240*a^4*b + 345*a^3*b^2 - 235*
a^2*b^3 + 75*a*b^4 - 9*b^5)*cos(4*d*x + 4*c)^2 + 16*(a^3*b^2 - 3*a^2*b^3 +
3*a*b^4 - b^5)*cos(2*d*x + 2*c)^2 + (a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*s
in(8*d*x + 8*c)^2 + 16*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*sin(6*d*x + 6*
c)^2 + 4*(64*a^5 - 240*a^4*b + 345*a^3*b^2 - 235*a^2*b^3 + 75*a*b^4 - 9*b^5
)*sin(4*d*x + 4*c)^2 + 16*(8*a^4*b - 27*a^3*b^2 + 33*a^2*b^3 - 17*a*b^4 + 3
*b^5)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4
 - b^5)*sin(2*d*x + 2*c)^2 + 2*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5 - 4*(a^
3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*cos(6*d*x + 6*c) - 2*(8*a^4*b - 27*a^3*b
^2 + 33*a^2*b^3 - 17*a*b^4 + 3*b^5)*cos(4*d*x + 4*c) - 4*(a^3*b^2 - 3*a^2*b
^3 + 3*a*b^4 - b^5)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) - 8*(a^3*b^2 - 3*a^2
*b^3 + 3*a*b^4 - b^5 - 2*(8*a^4*b - 27*a^3*b^2 + 33*a^2*b^3 - 17*a*b^4 + 3*
b^5)*cos(4*d*x + 4*c) - 4*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*cos(2*d*x +
 2*c))*cos(6*d*x + 6*c) - 4*(8*a^4*b - 27*a^3*b^2 + 33*a^2*b^3 - 17*a*b^4 +
 3*b^5 - 4*(8*a^4*b - 27*a^3*b^2 + 33*a^2*b^3 - 17*a*b^4 + 3*b^5)*cos(2*d*x
 + 2*c))*cos(4*d*x + 4*c) - 8*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*cos(2*d
*x + 2*c) - 4*(2*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*sin(6*d*x + 6*c) + (
8*a^4*b - 27*a^3*b^2 + 33*a^2*b^3 - 17*a*b^4 + 3*b^5)*sin(4*d*x + 4*c) + 2*
(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) +
16*((8*a^4*b - 27*a^3*b^2 + 33*a^2*b^3 - 17*a*b^4 + 3*b^5)*sin(4*d*x + 4*c)
 + 2*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*sin(2*d*x + 2*c))*sin(6*d*x + 6*
c)), x) + 2*(240*b^2*cos(6*d*x + 6*c) + 8*a^2 - 21*a*b + 73*b^2 + 15*(a*b +
 3*b^2)*cos(8*d*x + 8*c) + 10*(8*a^2 - 21*a*b + 49*b^2)*cos(4*d*x + 4*c) +
40*(a^2 - 3*a*b + 8*b^2)*cos(2*d*x + 2*c))*sin(10*d*x + 10*c) + 10*(8*a^2 -
 24*a*b + 64*b^2 - 30*(a*b - 5*b^2)*cos(6*d*x + 6*c) + 80*(a^2 - 3*a*b + 5*
b^2)*cos(4*d*x + 4*c) + 5*(8*a^2 - 27*a*b + 55*b^2)*cos(2*d*x + 2*c))*sin(8
*d*x + 8*c) + 20*(8*a^2 - 21*a*b + 49*b^2 + 10*(8*a^2 - 21*a*b + 25*b^2)*co
s(4*d*x + 4*c) + 40*(a^2 - 3*a*b + 5*b^2)*cos(2*d*x + 2*c))*sin(6*d*x + 6*c
) + 60*(8*b^2 - 5*(a*b - 5*b^2)*cos(2*d*x + 2*c))*sin(4*d*x + 4*c) + 30*(a*
b + 3*b^2)*sin(2*d*x + 2*c))/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(10*d*x
+ 10*c)^2 + 25*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(8*d*x + 8*c)^2 + 100*(
a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(6*d*x + 6*c)^2 + 100*(a^3 - 3*a^2*b +
3*a*b^2 - b^3)*d*cos(4*d*x + 4*c)^2 + 25*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*
cos(2*d*x + 2*c)^2 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sin(10*d*x + 10*c)^2
 + 25*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sin(8*d*x + 8*c)^2 + 100*(a^3 - 3*a
^2*b + 3*a*b^2 - b^3)*d*sin(6*d*x + 6*c)^2 + 100*(a^3 - 3*a^2*b + 3*a*b^2 -
 b^3)*d*sin(4*d*x + 4*c)^2 + 100*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sin(4*d*
x + 4*c)*sin(2*d*x + 2*c) + 25*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sin(2*d*x
+ 2*c)^2 + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(2*d*x + 2*c) + (a^3 - 3
```

```
*a^2*b + 3*a*b^2 - b^3)*d + 2*(5*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(8*d*
x + 8*c) + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(6*d*x + 6*c) + 10*(a^3
- 3*a^2*b + 3*a*b^2 - b^3)*d*cos(4*d*x + 4*c) + 5*(a^3 - 3*a^2*b + 3*a*b^2
- b^3)*d*cos(2*d*x + 2*c) + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d)*cos(10*d*x +
 10*c) + 10*(10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(6*d*x + 6*c) + 10*(a^
3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(4*d*x + 4*c) + 5*(a^3 - 3*a^2*b + 3*a*b^
2 - b^3)*d*cos(2*d*x + 2*c) + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d)*cos(8*d*x
+ 8*c) + 20*(10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(4*d*x + 4*c) + 5*(a^3
 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(2*d*x + 2*c) + (a^3 - 3*a^2*b + 3*a*b^2 -
 b^3)*d)*cos(6*d*x + 6*c) + 20*(5*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(2*d
*x + 2*c) + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d)*cos(4*d*x + 4*c) + 10*((a^3
- 3*a^2*b + 3*a*b^2 - b^3)*d*sin(8*d*x + 8*c) + 2*(a^3 - 3*a^2*b + 3*a*b^2
- b^3)*d*sin(6*d*x + 6*c) + 2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sin(4*d*x +
 4*c) + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sin(2*d*x + 2*c))*sin(10*d*x + 10
*c) + 50*(2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sin(6*d*x + 6*c) + 2*(a^3 - 3
*a^2*b + 3*a*b^2 - b^3)*d*sin(4*d*x + 4*c) + (a^3 - 3*a^2*b + 3*a*b^2 - b^3
)*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 100*(2*(a^3 - 3*a^2*b + 3*a*b^2 -
b^3)*d*sin(4*d*x + 4*c) + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sin(2*d*x + 2*c
))*sin(6*d*x + 6*c))
```

Fricas [B] time = 14.1733, size = 13045, normalized size = 63.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/120*(15*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sqrt(-(a^3*b^3 + 21*a^2*b^4 + 3
5*a*b^5 + 7*b^6 - (a^8 - 7*a^7*b + 21*a^6*b^2 - 35*a^5*b^3 + 35*a^4*b^4 - 2
1*a^3*b^5 + 7*a^2*b^6 - a*b^7)*d^2*sqrt((49*a^6*b^7 + 490*a^5*b^8 + 1519*a^
4*b^9 + 1484*a^3*b^10 + 511*a^2*b^11 + 42*a*b^12 + b^13)/((a^17 - 14*a^16*b
 + 91*a^15*b^2 - 364*a^14*b^3 + 1001*a^13*b^4 - 2002*a^12*b^5 + 3003*a^11*b
^6 - 3432*a^10*b^7 + 3003*a^9*b^8 - 2002*a^8*b^9 + 1001*a^7*b^10 - 364*a^6*
b^11 + 91*a^5*b^12 - 14*a^4*b^13 + a^3*b^14)*d^4)))/((a^8 - 7*a^7*b + 21*a^
6*b^2 - 35*a^5*b^3 + 35*a^4*b^4 - 21*a^3*b^5 + 7*a^2*b^6 - a*b^7)*d^2))*cos
(d*x + c)^5*log(7/4*a^3*b^5 + 35/4*a^2*b^6 + 21/4*a*b^7 + 1/4*b^8 - 1/4*(7*
a^3*b^5 + 35*a^2*b^6 + 21*a*b^7 + b^8)*cos(d*x + c)^2 + 1/2*(4*(a^11 - 6*a^
10*b + 14*a^9*b^2 - 14*a^8*b^3 + 14*a^6*b^5 - 14*a^5*b^6 + 6*a^4*b^7 - a^3*
b^8)*d^3*sqrt((49*a^6*b^7 + 490*a^5*b^8 + 1519*a^4*b^9 + 1484*a^3*b^10 + 51
1*a^2*b^11 + 42*a*b^12 + b^13)/((a^17 - 14*a^16*b + 91*a^15*b^2 - 364*a^14*
b^3 + 1001*a^13*b^4 - 2002*a^12*b^5 + 3003*a^11*b^6 - 3432*a^10*b^7 + 3003*
a^9*b^8 - 2002*a^8*b^9 + 1001*a^7*b^10 - 364*a^6*b^11 + 91*a^5*b^12 - 14*a^
4*b^13 + a^3*b^14)*d^4))*cos(d*x + c)*sin(d*x + c) + (7*a^6*b^3 + 77*a^5*b^
4 + 238*a^4*b^5 + 162*a^3*b^6 + 27*a^2*b^7 + a*b^8)*d*cos(d*x + c)*sin(d*x
+ c))*sqrt(-(a^3*b^3 + 21*a^2*b^4 + 35*a*b^5 + 7*b^6 - (a^8 - 7*a^7*b + 21*
a^6*b^2 - 35*a^5*b^3 + 35*a^4*b^4 - 21*a^3*b^5 + 7*a^2*b^6 - a*b^7)*d^2*sqr
t((49*a^6*b^7 + 490*a^5*b^8 + 1519*a^4*b^9 + 1484*a^3*b^10 + 511*a^2*b^11 +
 42*a*b^12 + b^13)/((a^17 - 14*a^16*b + 91*a^15*b^2 - 364*a^14*b^3 + 1001*a
^13*b^4 - 2002*a^12*b^5 + 3003*a^11*b^6 - 3432*a^10*b^7 + 3003*a^9*b^8 - 20
02*a^8*b^9 + 1001*a^7*b^10 - 364*a^6*b^11 + 91*a^5*b^12 - 14*a^4*b^13 + a^3
*b^14)*d^4)))/((a^8 - 7*a^7*b + 21*a^6*b^2 - 35*a^5*b^3 + 35*a^4*b^4 - 21*a
^3*b^5 + 7*a^2*b^6 - a*b^7)*d^2)) - 1/4*(2*(a^9*b - 7*a^8*b^2 + 21*a^7*b^3
- 35*a^6*b^4 + 35*a^5*b^5 - 21*a^4*b^6 + 7*a^3*b^7 - a^2*b^8)*d^2*cos(d*x +
 c)^2 - (a^9*b - 7*a^8*b^2 + 21*a^7*b^3 - 35*a^6*b^4 + 35*a^5*b^5 - 21*a^4*
b^6 + 7*a^3*b^7 - a^2*b^8)*d^2)*sqrt((49*a^6*b^7 + 490*a^5*b^8 + 1519*a^4*b
^9 + 1484*a^3*b^10 + 511*a^2*b^11 + 42*a*b^12 + b^13)/((a^17 - 14*a^16*b +
```

91*a^15*b^2 - 364*a^14*b^3 + 1001*a^13*b^4 - 2002*a^12*b^5 + 3003*a^11*b^6
- 3432*a^10*b^7 + 3003*a^9*b^8 - 2002*a^8*b^9 + 1001*a^7*b^10 - 364*a^6*b^1
1 + 91*a^5*b^12 - 14*a^4*b^13 + a^3*b^14)*d^4))) - 15*(a^3 - 3*a^2*b + 3*a*
b^2 - b^3)*d*sqrt(-(a^3*b^3 + 21*a^2*b^4 + 35*a*b^5 + 7*b^6 - (a^8 - 7*a^7*
b + 21*a^6*b^2 - 35*a^5*b^3 + 35*a^4*b^4 - 21*a^3*b^5 + 7*a^2*b^6 - a*b^7)*
d^2*sqrt((49*a^6*b^7 + 490*a^5*b^8 + 1519*a^4*b^9 + 1484*a^3*b^10 + 511*a^2
*b^11 + 42*a*b^12 + b^13)/((a^17 - 14*a^16*b + 91*a^15*b^2 - 364*a^14*b^3 +
1001*a^13*b^4 - 2002*a^12*b^5 + 3003*a^11*b^6 - 3432*a^10*b^7 + 3003*a^9*b
^8 - 2002*a^8*b^9 + 1001*a^7*b^10 - 364*a^6*b^11 + 91*a^5*b^12 - 14*a^4*b^1
3 + a^3*b^14)*d^4)))/((a^8 - 7*a^7*b + 21*a^6*b^2 - 35*a^5*b^3 + 35*a^4*b^4
- 21*a^3*b^5 + 7*a^2*b^6 - a*b^7)*d^2))*cos(d*x + c)^5*log(7/4*a^3*b^5 + 3
5/4*a^2*b^6 + 21/4*a*b^7 + 1/4*b^8 - 1/4*(7*a^3*b^5 + 35*a^2*b^6 + 21*a*b^7
+ b^8)*cos(d*x + c)^2 - 1/2*(4*(a^11 - 6*a^10*b + 14*a^9*b^2 - 14*a^8*b^3
+ 14*a^6*b^5 - 14*a^5*b^6 + 6*a^4*b^7 - a^3*b^8)*d^3*sqrt((49*a^6*b^7 + 490
*a^5*b^8 + 1519*a^4*b^9 + 1484*a^3*b^10 + 511*a^2*b^11 + 42*a*b^12 + b^13)/
((a^17 - 14*a^16*b + 91*a^15*b^2 - 364*a^14*b^3 + 1001*a^13*b^4 - 2002*a^12
*b^5 + 3003*a^11*b^6 - 3432*a^10*b^7 + 3003*a^9*b^8 - 2002*a^8*b^9 + 1001*a
^7*b^10 - 364*a^6*b^11 + 91*a^5*b^12 - 14*a^4*b^13 + a^3*b^14)*d^4))*cos(d*
x + c)*sin(d*x + c) + (7*a^6*b^3 + 77*a^5*b^4 + 238*a^4*b^5 + 162*a^3*b^6 +
27*a^2*b^7 + a*b^8)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-(a^3*b^3 + 21*a^2*b
^4 + 35*a*b^5 + 7*b^6 - (a^8 - 7*a^7*b + 21*a^6*b^2 - 35*a^5*b^3 + 35*a^4*b
^4 - 21*a^3*b^5 + 7*a^2*b^6 - a*b^7)*d^2*sqrt((49*a^6*b^7 + 490*a^5*b^8 + 1
519*a^4*b^9 + 1484*a^3*b^10 + 511*a^2*b^11 + 42*a*b^12 + b^13)/((a^17 - 14*
a^16*b + 91*a^15*b^2 - 364*a^14*b^3 + 1001*a^13*b^4 - 2002*a^12*b^5 + 3003*
a^11*b^6 - 3432*a^10*b^7 + 3003*a^9*b^8 - 2002*a^8*b^9 + 1001*a^7*b^10 - 36
4*a^6*b^11 + 91*a^5*b^12 - 14*a^4*b^13 + a^3*b^14)*d^4)))/((a^8 - 7*a^7*b +
21*a^6*b^2 - 35*a^5*b^3 + 35*a^4*b^4 - 21*a^3*b^5 + 7*a^2*b^6 - a*b^7)*d^2
)) - 1/4*(2*(a^9*b - 7*a^8*b^2 + 21*a^7*b^3 - 35*a^6*b^4 + 35*a^5*b^5 - 21*
a^4*b^6 + 7*a^3*b^7 - a^2*b^8)*d^2*cos(d*x + c)^2 - (a^9*b - 7*a^8*b^2 + 21
*a^7*b^3 - 35*a^6*b^4 + 35*a^5*b^5 - 21*a^4*b^6 + 7*a^3*b^7 - a^2*b^8)*d^2)
*sqrt((49*a^6*b^7 + 490*a^5*b^8 + 1519*a^4*b^9 + 1484*a^3*b^10 + 511*a^2*b^
11 + 42*a*b^12 + b^13)/((a^17 - 14*a^16*b + 91*a^15*b^2 - 364*a^14*b^3 + 10
01*a^13*b^4 - 2002*a^12*b^5 + 3003*a^11*b^6 - 3432*a^10*b^7 + 3003*a^9*b^8
- 2002*a^8*b^9 + 1001*a^7*b^10 - 364*a^6*b^11 + 91*a^5*b^12 - 14*a^4*b^13 +
a^3*b^14)*d^4))) + 15*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sqrt(-(a^3*b^3 + 2
1*a^2*b^4 + 35*a*b^5 + 7*b^6 + (a^8 - 7*a^7*b + 21*a^6*b^2 - 35*a^5*b^3 + 3
5*a^4*b^4 - 21*a^3*b^5 + 7*a^2*b^6 - a*b^7)*d^2*sqrt((49*a^6*b^7 + 490*a^5*
b^8 + 1519*a^4*b^9 + 1484*a^3*b^10 + 511*a^2*b^11 + 42*a*b^12 + b^13)/((a^1
7 - 14*a^16*b + 91*a^15*b^2 - 364*a^14*b^3 + 1001*a^13*b^4 - 2002*a^12*b^5
+ 3003*a^11*b^6 - 3432*a^10*b^7 + 3003*a^9*b^8 - 2002*a^8*b^9 + 1001*a^7*b^
10 - 364*a^6*b^11 + 91*a^5*b^12 - 14*a^4*b^13 + a^3*b^14)*d^4)))/((a^8 - 7*
a^7*b + 21*a^6*b^2 - 35*a^5*b^3 + 35*a^4*b^4 - 21*a^3*b^5 + 7*a^2*b^6 - a*b
^7)*d^2))*cos(d*x + c)^5*log(-7/4*a^3*b^5 - 35/4*a^2*b^6 - 21/4*a*b^7 - 1/4
b^8 + 1/4(7*a^3*b^5 + 35*a^2*b^6 + 21*a*b^7 + b^8)*cos(d*x + c)^2 + 1/2*(
4*(a^11 - 6*a^10*b + 14*a^9*b^2 - 14*a^8*b^3 + 14*a^6*b^5 - 14*a^5*b^6 + 6*
a^4*b^7 - a^3*b^8)*d^3*sqrt((49*a^6*b^7 + 490*a^5*b^8 + 1519*a^4*b^9 + 1484
*a^3*b^10 + 511*a^2*b^11 + 42*a*b^12 + b^13)/((a^17 - 14*a^16*b + 91*a^15*b
^2 - 364*a^14*b^3 + 1001*a^13*b^4 - 2002*a^12*b^5 + 3003*a^11*b^6 - 3432*a^
10*b^7 + 3003*a^9*b^8 - 2002*a^8*b^9 + 1001*a^7*b^10 - 364*a^6*b^11 + 91*a^
5*b^12 - 14*a^4*b^13 + a^3*b^14)*d^4))*cos(d*x + c)*sin(d*x + c) - (7*a^6*b
^3 + 77*a^5*b^4 + 238*a^4*b^5 + 162*a^3*b^6 + 27*a^2*b^7 + a*b^8)*d*cos(d*x
+ c)*sin(d*x + c))*sqrt(-(a^3*b^3 + 21*a^2*b^4 + 35*a*b^5 + 7*b^6 + (a^8 -
7*a^7*b + 21*a^6*b^2 - 35*a^5*b^3 + 35*a^4*b^4 - 21*a^3*b^5 + 7*a^2*b^6 -
a*b^7)*d^2*sqrt((49*a^6*b^7 + 490*a^5*b^8 + 1519*a^4*b^9 + 1484*a^3*b^10 +
511*a^2*b^11 + 42*a*b^12 + b^13)/((a^17 - 14*a^16*b + 91*a^15*b^2 - 364*a^1
4*b^3 + 1001*a^13*b^4 - 2002*a^12*b^5 + 3003*a^11*b^6 - 3432*a^10*b^7 + 300
3*a^9*b^8 - 2002*a^8*b^9 + 1001*a^7*b^10 - 364*a^6*b^11 + 91*a^5*b^12 - 14*
a^4*b^13 + a^3*b^14)*d^4)))/((a^8 - 7*a^7*b + 21*a^6*b^2 - 35*a^5*b^3 + 35*
a^4*b^4 - 21*a^3*b^5 + 7*a^2*b^6 - a*b^7)*d^2)) - 1/4*(2*(a^9*b - 7*a^8*b^2

```
 + 21*a^7*b^3 - 35*a^6*b^4 + 35*a^5*b^5 - 21*a^4*b^6 + 7*a^3*b^7 - a^2*b^8)
*d^2*cos(d*x + c)^2 - (a^9*b - 7*a^8*b^2 + 21*a^7*b^3 - 35*a^6*b^4 + 35*a^5
*b^5 - 21*a^4*b^6 + 7*a^3*b^7 - a^2*b^8)*d^2)*sqrt((49*a^6*b^7 + 490*a^5*b^
8 + 1519*a^4*b^9 + 1484*a^3*b^10 + 511*a^2*b^11 + 42*a*b^12 + b^13)/((a^17
- 14*a^16*b + 91*a^15*b^2 - 364*a^14*b^3 + 1001*a^13*b^4 - 2002*a^12*b^5 +
3003*a^11*b^6 - 3432*a^10*b^7 + 3003*a^9*b^8 - 2002*a^8*b^9 + 1001*a^7*b^10
 - 364*a^6*b^11 + 91*a^5*b^12 - 14*a^4*b^13 + a^3*b^14)*d^4))) - 15*(a^3 -
3*a^2*b + 3*a*b^2 - b^3)*d*sqrt(-(a^3*b^3 + 21*a^2*b^4 + 35*a*b^5 + 7*b^6 +
 (a^8 - 7*a^7*b + 21*a^6*b^2 - 35*a^5*b^3 + 35*a^4*b^4 - 21*a^3*b^5 + 7*a^2
*b^6 - a*b^7)*d^2*sqrt((49*a^6*b^7 + 490*a^5*b^8 + 1519*a^4*b^9 + 1484*a^3*
b^10 + 511*a^2*b^11 + 42*a*b^12 + b^13)/((a^17 - 14*a^16*b + 91*a^15*b^2 -
364*a^14*b^3 + 1001*a^13*b^4 - 2002*a^12*b^5 + 3003*a^11*b^6 - 3432*a^10*b^
7 + 3003*a^9*b^8 - 2002*a^8*b^9 + 1001*a^7*b^10 - 364*a^6*b^11 + 91*a^5*b^1
2 - 14*a^4*b^13 + a^3*b^14)*d^4)))/((a^8 - 7*a^7*b + 21*a^6*b^2 - 35*a^5*b^
3 + 35*a^4*b^4 - 21*a^3*b^5 + 7*a^2*b^6 - a*b^7)*d^2))*cos(d*x + c)^5*log(-
7/4*a^3*b^5 - 35/4*a^2*b^6 - 21/4*a*b^7 - 1/4*b^8 + 1/4*(7*a^3*b^5 + 35*a^2
*b^6 + 21*a*b^7 + b^8)*cos(d*x + c)^2 - 1/2*(4*(a^11 - 6*a^10*b + 14*a^9*b^
2 - 14*a^8*b^3 + 14*a^6*b^5 - 14*a^5*b^6 + 6*a^4*b^7 - a^3*b^8)*d^3*sqrt((4
9*a^6*b^7 + 490*a^5*b^8 + 1519*a^4*b^9 + 1484*a^3*b^10 + 511*a^2*b^11 + 42*
a*b^12 + b^13)/((a^17 - 14*a^16*b + 91*a^15*b^2 - 364*a^14*b^3 + 1001*a^13*
b^4 - 2002*a^12*b^5 + 3003*a^11*b^6 - 3432*a^10*b^7 + 3003*a^9*b^8 - 2002*a
^8*b^9 + 1001*a^7*b^10 - 364*a^6*b^11 + 91*a^5*b^12 - 14*a^4*b^13 + a^3*b^1
4)*d^4))*cos(d*x + c)*sin(d*x + c) - (7*a^6*b^3 + 77*a^5*b^4 + 238*a^4*b^5
+ 162*a^3*b^6 + 27*a^2*b^7 + a*b^8)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-(a^3
*b^3 + 21*a^2*b^4 + 35*a*b^5 + 7*b^6 + (a^8 - 7*a^7*b + 21*a^6*b^2 - 35*a^5
*b^3 + 35*a^4*b^4 - 21*a^3*b^5 + 7*a^2*b^6 - a*b^7)*d^2*sqrt((49*a^6*b^7 +
490*a^5*b^8 + 1519*a^4*b^9 + 1484*a^3*b^10 + 511*a^2*b^11 + 42*a*b^12 + b^1
3)/((a^17 - 14*a^16*b + 91*a^15*b^2 - 364*a^14*b^3 + 1001*a^13*b^4 - 2002*a
^12*b^5 + 3003*a^11*b^6 - 3432*a^10*b^7 + 3003*a^9*b^8 - 2002*a^8*b^9 + 100
1*a^7*b^10 - 364*a^6*b^11 + 91*a^5*b^12 - 14*a^4*b^13 + a^3*b^14)*d^4)))/((
a^8 - 7*a^7*b + 21*a^6*b^2 - 35*a^5*b^3 + 35*a^4*b^4 - 21*a^3*b^5 + 7*a^2*b
^6 - a*b^7)*d^2)) - 1/4*(2*(a^9*b - 7*a^8*b^2 + 21*a^7*b^3 - 35*a^6*b^4 + 3
5*a^5*b^5 - 21*a^4*b^6 + 7*a^3*b^7 - a^2*b^8)*d^2*cos(d*x + c)^2 - (a^9*b -
 7*a^8*b^2 + 21*a^7*b^3 - 35*a^6*b^4 + 35*a^5*b^5 - 21*a^4*b^6 + 7*a^3*b^7
- a^2*b^8)*d^2)*sqrt((49*a^6*b^7 + 490*a^5*b^8 + 1519*a^4*b^9 + 1484*a^3*b^
10 + 511*a^2*b^11 + 42*a*b^12 + b^13)/((a^17 - 14*a^16*b + 91*a^15*b^2 - 36
4*a^14*b^3 + 1001*a^13*b^4 - 2002*a^12*b^5 + 3003*a^11*b^6 - 3432*a^10*b^7
+ 3003*a^9*b^8 - 2002*a^8*b^9 + 1001*a^7*b^10 - 364*a^6*b^11 + 91*a^5*b^12
- 14*a^4*b^13 + a^3*b^14)*d^4))) + 8*((8*a^2 - 21*a*b + 73*b^2)*cos(d*x + c
)^4 + 2*(2*a^2 - 9*a*b + 7*b^2)*cos(d*x + c)^2 + 3*a^2 - 6*a*b + 3*b^2)*sin
(d*x + c))/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.419 $\int \cos^m(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\cos^m(e+fx)\left(a+b\sin^4(e+fx)\right)^p,x\right)$$

```
[Out] Unintegrable[Cos[e + f*x]^m*(a + b*Sin[e + f*x]^4)^p, x]
```

Rubi [A] time = 0.0439154, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \cos^m(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[e + f*x]^m*(a + b*Sin[e + f*x]^4)^p,x]
```

```
[Out] Defer[Int][Cos[e + f*x]^m*(a + b*Sin[e + f*x]^4)^p, x]
```

Rubi steps

$$\int \cos^m(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx=\int \cos^m(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

Mathematica [A] time = 8.21124, size = 0, normalized size = 0.

$$\int \cos^m(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^m*(a + b*Sin[e + f*x]^4)^p,x]
```

```
[Out] Integrate[Cos[e + f*x]^m*(a + b*Sin[e + f*x]^4)^p, x]
```

Maple [A] time = 1.721, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^m\left(a+b\left(\sin\left(fx+e\right)\right)^4\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^m*(a+b*sin(f*x+e)^4)^p,x)
```

```
[Out] int(cos(f*x+e)^m*(a+b*sin(f*x+e)^4)^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sin \left(fx+e\right)^4 + a\right)^p \cos \left(fx+e\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^m*(a+b*sin(f*x+e)^4)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^4 + a)^p*cos(f*x + e)^m, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \cos \left(fx+e\right)^4 - 2\, b \cos \left(fx+e\right)^2 + a + b\right)^p \cos \left(fx+e\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^m*(a+b*sin(f*x+e)^4)^p,x, algorithm="fricas")
```

```
[Out] integral((b*cos(f*x + e)^4 - 2*b*cos(f*x + e)^2 + a + b)^p*cos(f*x + e)^m,
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**m*(a+b*sin(f*x+e)**4)**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sin \left(fx+e\right)^4 + a\right)^p \cos \left(fx+e\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^m*(a+b*sin(f*x+e)^4)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^4 + a)^p*cos(f*x + e)^m, x)
```

3.420 $\int \cos^5(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$

Optimal. Leaf size=197

$$-\frac{2\sin^3(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(\frac{b\sin^4(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{b\sin^4(e+fx)}{a}\right)}{3f}-\frac{(a-b(4p+5))\sin(e+fx)\left(a+\right.}{}$$

```
[Out] (Sin[e + f*x]*(a + b*Sin[e + f*x]^4)^(1 + p))/(b*f*(5 + 4*p)) - ((a - b*(5
+ 4*p))*Hypergeometric2F1[1/4, -p, 5/4, -((b*Sin[e + f*x]^4)/a)]*Sin[e + f*
x]*(a + b*Sin[e + f*x]^4)^p)/(b*f*(5 + 4*p)*(1 + (b*Sin[e + f*x]^4)/a)^p) -
 (2*Hypergeometric2F1[3/4, -p, 7/4, -((b*Sin[e + f*x]^4)/a)]*Sin[e + f*x]^3
*(a + b*Sin[e + f*x]^4)^p)/(3*f*(1 + (b*Sin[e + f*x]^4)/a)^p)
```

Rubi [A] time = 0.221972, antiderivative size = 191, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3223, 1207, 1204, 246, 245, 365, 364}

$$-\frac{2\sin^3(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(\frac{b\sin^4(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{b\sin^4(e+fx)}{a}\right)}{3f}+\frac{\left(1-\frac{a}{4bp+5b}\right)\sin(e+fx)\left(a+b\,\mathrm{s}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^5*(a + b*Sin[e + f*x]^4)^p,x]
```

```
[Out] (Sin[e + f*x]*(a + b*Sin[e + f*x]^4)^(1 + p))/(b*f*(5 + 4*p)) + ((1 - a/(5*
b + 4*b*p))*Hypergeometric2F1[1/4, -p, 5/4, -((b*Sin[e + f*x]^4)/a)]*Sin[e
+ f*x]*(a + b*Sin[e + f*x]^4)^p)/(f*(1 + (b*Sin[e + f*x]^4)/a)^p) - (2*Hype
rgeometric2F1[3/4, -p, 7/4, -((b*Sin[e + f*x]^4)/a)]*Sin[e + f*x]^3*(a + b*
Sin[e + f*x]^4)^p)/(3*f*(1 + (b*Sin[e + f*x]^4)/a)^p)
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rule 1207

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[(e^q*x^(2*q - 3)*(a + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c
*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x
^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x]
, x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Rule 1204

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \cos^5(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx=\frac{\text{Subst}\left(\int\left(1-x^2\right)^2\left(a+bx^4\right)^p\,dx,x,\sin(e+fx)\right)}{f}$$

$$=\frac{\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^{1+p}}{bf(5+4p)}+\frac{\text{Subst}\left(\int\left(-a+b(5+4p)-2b(5+4p)\right.\right.}{bf(5+4}$$

$$=\frac{\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^{1+p}}{bf(5+4p)}+\frac{\text{Subst}\left(\int\left(-a\left(1-\frac{b(5+4p)}{a}\right)\left(a+bx^4\right)^p-\right.\right.}{bf}$$

$$=\frac{\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^{1+p}}{bf(5+4p)}-\frac{2\,\text{Subst}\left(\int x^2\left(a+bx^4\right)^p\,dx,x,\sin(e+\right.}{f}$$

$$=\frac{\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^{1+p}}{bf(5+4p)}-\frac{\left(2\left(a+b\sin^4(e+fx)\right)^p\left(1+\frac{b\sin^4(e+fx)}{a}\right.\right.}{}$$

$$=\frac{\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^{1+p}}{bf(5+4p)}-\frac{(a-b(5+4p))\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{b\sin^4(e+fx}{a}\right.}{}$$

Mathematica [A] time = 0.147129, size = 141, normalized size = 0.72

$$\frac{\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(\frac{b\sin^4(e+fx)}{a}+1\right)^{-p}\left(3\sin^4(e+fx)\,{}_2F_1\left(\frac{5}{4},-p;\frac{9}{4};-\frac{b\sin^4(e+fx)}{a}\right)-10\sin^2(e+fx)\,{}_2F_1\left(\frac{3}{4},-\right.\right.}{15f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^5*(a + b*Sin[e + f*x]^4)^p,x]
```

```
[Out] (Sin[e + f*x]*(a + b*Sin[e + f*x]^4)^p*(15*Hypergeometric2F1[1/4, -p, 5/4,
-((b*Sin[e + f*x]^4)/a)] - 10*Hypergeometric2F1[3/4, -p, 7/4, -((b*Sin[e +
f*x]^4)/a)]*Sin[e + f*x]^2 + 3*Hypergeometric2F1[5/4, -p, 9/4, -((b*Sin[e +
 f*x]^4)/a)]*Sin[e + f*x]^4))/(15*f*(1 + (b*Sin[e + f*x]^4)/a)^p)
```

Maple [F] time = 2.021, size = 0, normalized size = 0.

$$\int \left(\cos \left(fx+e\right)\right)^{5} \left(a+b \left(\sin \left(fx+e\right)\right)^{4}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^5*(a+b*sin(f*x+e)^4)^p,x)
```

```
[Out] int(cos(f*x+e)^5*(a+b*sin(f*x+e)^4)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sin \left(fx+e\right)^{4}+a\right)^{p} \cos \left(fx+e\right)^{5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sin(f*x+e)^4)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^4 + a)^p*cos(f*x + e)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b \cos \left(fx+e\right)^{4}-2\, b \cos \left(fx+e\right)^{2}+a+b\right)^{p} \cos \left(fx+e\right)^{5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sin(f*x+e)^4)^p,x, algorithm="fricas")
```

```
[Out] integral((b*cos(f*x + e)^4 - 2*b*cos(f*x + e)^2 + a + b)^p*cos(f*x + e)^5,
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**5*(a+b*sin(f*x+e)**4)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^4+a\right)^p\cos\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sin(f*x+e)^4)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^4 + a)^p*cos(f*x + e)^5, x)
```

3.421 $\int \cos^3(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$

Optimal. Leaf size=140

$$\frac{\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(\frac{b\sin^4(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{b\sin^4(e+fx)}{a}\right)}{f}-\frac{\sin^3(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(\cdots\right.}{\cdots}$$

```
[Out] (Hypergeometric2F1[1/4, -p, 5/4, -((b*Sin[e + f*x]^4)/a)]*Sin[e + f*x]*(a +
 b*Sin[e + f*x]^4)^p)/(f*(1 + (b*Sin[e + f*x]^4)/a)^p) - (Hypergeometric2F1
[3/4, -p, 7/4, -((b*Sin[e + f*x]^4)/a)]*Sin[e + f*x]^3*(a + b*Sin[e + f*x]^
4)^p)/(3*f*(1 + (b*Sin[e + f*x]^4)/a)^p)
```

Rubi [A] time = 0.0980214, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3223, 1204, 246, 245, 365, 364}

$$\frac{\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(\frac{b\sin^4(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{b\sin^4(e+fx)}{a}\right)}{f}-\frac{\sin^3(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3*(a + b*Sin[e + f*x]^4)^p,x]
```


```
[Out] (Hypergeometric2F1[1/4, -p, 5/4, -((b*Sin[e + f*x]^4)/a)]*Sin[e + f*x]*(a +
 b*Sin[e + f*x]^4)^p)/(f*(1 + (b*Sin[e + f*x]^4)/a)^p) - (Hypergeometric2F1
[3/4, -p, 7/4, -((b*Sin[e + f*x]^4)/a)]*Sin[e + f*x]^3*(a + b*Sin[e + f*x]^
4)^p)/(3*f*(1 + (b*Sin[e + f*x]^4)/a)^p)
```


Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rule 1204

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
```

```
GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \cos^3(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \left(1-x^2\right)\left(a+bx^4\right)^p\,dx,x,\sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(\left(a+bx^4\right)^p - x^2\left(a+bx^4\right)^p\right)dx,x,\sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(a+bx^4\right)^p\,dx,x,\sin(e+fx)\right)}{f} - \frac{\text{Subst}\left(\int x^2\left(a+bx^4\right)^p\,dx,x,\sin(e\right.}{f}$$

$$= \frac{\left(\left(a+b\sin^4(e+fx)\right)^p\left(1+\frac{b\sin^4(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int\left(1+\frac{bx^4}{a}\right)^p\,dx,x,\sin(e+\right.}{f}$$

$$= \frac{{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{b\sin^4(e+fx)}{a}\right)\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(1+\frac{b\sin^4(e+fx)}{a}\right)^{-}}{f}$$

Mathematica [A] time = 0.0486709, size = 106, normalized size = 0.76

$$-\frac{\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(\frac{b\sin^4(e+fx)}{a}+1\right)^{-p}\left(\sin^2(e+fx)\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{b\sin^4(e+fx)}{a}\right)-3\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{b\sin^4(e+f}{a}\right.\right.}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3*(a + b*Sin[e + f*x]^4)^p,x]
```

```
[Out] -(Sin[e + f*x]*(-3*Hypergeometric2F1[1/4, -p, 5/4, -((b*Sin[e + f*x]^4)/a)]
 + Hypergeometric2F1[3/4, -p, 7/4, -((b*Sin[e + f*x]^4)/a)]*Sin[e + f*x]^2)
*(a + b*Sin[e + f*x]^4)^p)/(3*f*(1 + (b*Sin[e + f*x]^4)/a)^p)
```

Maple [F] time = 3.415, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^3\left(a+b\left(\sin\left(fx+e\right)\right)^4\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3*(a+b*sin(f*x+e)^4)^p,x)
```

```
[Out] int(cos(f*x+e)^3*(a+b*sin(f*x+e)^4)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^4+a\right)^p\cos\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^4)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^4 + a)^p*cos(f*x + e)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\cos\left(fx+e\right)^4-2\,b\cos\left(fx+e\right)^2+a+b\right)^p\cos\left(fx+e\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^4)^p,x, algorithm="fricas")
```

```
[Out] integral((b*cos(f*x + e)^4 - 2*b*cos(f*x + e)^2 + a + b)^p*cos(f*x + e)^3,
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3*(a+b*sin(f*x+e)**4)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^4+a\right)^p\cos\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^4)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^4 + a)^p*cos(f*x + e)^3, x)
```

3.422 $\int \cos(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$

Optimal. Leaf size=67

$$\frac{\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(\frac{b\sin^4(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{b\sin^4(e+fx)}{a}\right)}{f}$$

```
[Out] (Hypergeometric2F1[1/4, -p, 5/4, -((b*Sin[e + f*x]^4)/a)]*Sin[e + f*x]*(a +
 b*Sin[e + f*x]^4)^p)/(f*(1 + (b*Sin[e + f*x]^4)/a)^p)
```

Rubi [A] time = 0.0420269, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3223, 246, 245}

$$\frac{\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(\frac{b\sin^4(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{b\sin^4(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + b*Sin[e + f*x]^4)^p,x]
```

```
[Out] (Hypergeometric2F1[1/4, -p, 5/4, -((b*Sin[e + f*x]^4)/a)]*Sin[e + f*x]*(a +
 b*Sin[e + f*x]^4)^p)/(f*(1 + (b*Sin[e + f*x]^4)/a)^p)
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \cos(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int\left(a+bx^4\right)^p\,dx,x,\sin(e+fx)\right)}{f}$$
$$= \frac{\left(\left(a+b\sin^4(e+fx)\right)^p\left(1+\frac{b\sin^4(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int\left(1+\frac{bx^4}{a}\right)^p\,dx,x,\sin(e-\right.}{f}$$
$$= \frac{{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{b\sin^4(e+fx)}{a}\right)\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(1+\frac{b\sin^4(e+fx)}{a}\right.}{f}$$

Mathematica [A] time = 0.0163648, size = 67, normalized size = 1.

$$\frac{\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(\frac{b\sin^4(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{b\sin^4(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*(a + b*Sin[e + f*x]^4)^p,x]
```

```
[Out] (Hypergeometric2F1[1/4, -p, 5/4, -((b*Sin[e + f*x]^4)/a)]*Sin[e + f*x]*(a +
 b*Sin[e + f*x]^4)^p)/(f*(1 + (b*Sin[e + f*x]^4)/a)^p)
```

Maple [F] time = 1.985, size = 0, normalized size = 0.

$$\int \cos\left(fx+e\right)\left(a+b\left(\sin\left(fx+e\right)\right)^4\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+b*sin(f*x+e)^4)^p,x)
```

```
[Out] int(cos(f*x+e)*(a+b*sin(f*x+e)^4)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sin\left(fx+e\right)^4+a\right)^p\cos\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)^4)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^4 + a)^p*cos(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\cos\left(fx+e\right)^4-2\,b\cos\left(fx+e\right)^2+a+b\right)^p\cos\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)^4)^p,x, algorithm="fricas")
```

```
[Out] integral((b*cos(f*x + e)^4 - 2*b*cos(f*x + e)^2 + a + b)^p*cos(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)**4)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^4+a\right)^p \cos\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)^4)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^4 + a)^p*cos(f*x + e), x)
```

3.423 $\int \sec(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$

Optimal. Leaf size=158

$$\frac{\sin^3(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(\frac{b\sin^4(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{3}{4};1,-p;\frac{7}{4};\sin^4(e+fx),-\frac{b\sin^4(e+fx)}{a}\right)}{3f}+\frac{\sin(e+fx)\left(a+b\sin\right.}{}$$

```
[Out] (AppellF1[1/4, 1, -p, 5/4, Sin[e + f*x]^4, -((b*Sin[e + f*x]^4)/a)]*Sin[e +
 f*x]*(a + b*Sin[e + f*x]^4)^p)/(f*(1 + (b*Sin[e + f*x]^4)/a)^p) + (AppellF
1[3/4, 1, -p, 7/4, Sin[e + f*x]^4, -((b*Sin[e + f*x]^4)/a)]*Sin[e + f*x]^3*
(a + b*Sin[e + f*x]^4)^p)/(3*f*(1 + (b*Sin[e + f*x]^4)/a)^p)
```

Rubi [A] time = 0.150238, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3223, 1240, 430, 429, 511, 510}

$$\frac{\sin^3(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(\frac{b\sin^4(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{3}{4};1,-p;\frac{7}{4};\sin^4(e+fx),-\frac{b\sin^4(e+fx)}{a}\right)}{3f}+\frac{\sin(e+fx)\left(a+b\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + b*Sin[e + f*x]^4)^p,x]
```

```
[Out] (AppellF1[1/4, 1, -p, 5/4, Sin[e + f*x]^4, -((b*Sin[e + f*x]^4)/a)]*Sin[e +
 f*x]*(a + b*Sin[e + f*x]^4)^p)/(f*(1 + (b*Sin[e + f*x]^4)/a)^p) + (AppellF
1[3/4, 1, -p, 7/4, Sin[e + f*x]^4, -((b*Sin[e + f*x]^4)/a)]*Sin[e + f*x]^3*
(a + b*Sin[e + f*x]^4)^p)/(3*f*(1 + (b*Sin[e + f*x]^4)/a)^p)
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rule 1240

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(a + c*x^4)^p, (d/(d^2 - e^2*x^4) - (e*x^2)/(d^2 - e^2*x^4
))^(-q), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[q, 0]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
```

```
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \sec(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx &= \frac{\operatorname{Subst}\left(\int \frac{\left(a+bx^4\right)^p}{1-x^2}\,dx, x, \sin(e+fx)\right)}{f}\\
&= \frac{\operatorname{Subst}\left(\int \left(\frac{\left(a+bx^4\right)^p}{1-x^4}-\frac{x^2\left(a+bx^4\right)^p}{-1+x^4}\right)dx, x, \sin(e+fx)\right)}{f}\\
&= \frac{\operatorname{Subst}\left(\int \frac{\left(a+bx^4\right)^p}{1-x^4}\,dx, x, \sin(e+fx)\right)}{f}-\frac{\operatorname{Subst}\left(\int \frac{x^2\left(a+bx^4\right)^p}{-1+x^4}\,dx, x, \sin(e+fx)\right)}{f}\\
&= \frac{\left(\left(a+b\sin^4(e+fx)\right)^p\left(1+\frac{b\sin^4(e+fx)}{a}\right)^{-p}\right)\operatorname{Subst}\left(\int \frac{\left(1+\frac{bx^4}{a}\right)^p}{1-x^4}\,dx, x, \sin(e+fx)\right)}{f}\\
&= \frac{F_1\left(\frac{1}{4};1,-p;\frac{5}{4};\sin^4(e+fx),-\frac{b\sin^4(e+fx)}{a}\right)\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(1+\right.}{f}
\end{aligned}$$

Mathematica [F] time = 5.95045, size = 0, normalized size = 0.

$$\int \sec(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]*(a + b*Sin[e + f*x]^4)^p,x]
```

```
[Out] Integrate[Sec[e + f*x]*(a + b*Sin[e + f*x]^4)^p, x]
```

Maple [F] time = 1.202, size = 0, normalized size = 0.

$$\int \sec\left(fx+e\right)\left(a+b\left(\sin\left(fx+e\right)\right)^4\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sin(f*x+e)^4)^p,x)
```

```
[Out] int(sec(f*x+e)*(a+b*sin(f*x+e)^4)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^4+a\right)^p \sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)^4)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^4 + a)^p*sec(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\cos\left(fx+e\right)^4-2\,b\cos\left(fx+e\right)^2+a+b\right)^p \sec\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)^4)^p,x, algorithm="fricas")
```

```
[Out] integral((b*cos(f*x + e)^4 - 2*b*cos(f*x + e)^2 + a + b)^p*sec(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)**4)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^4+a\right)^p \sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)^4)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^4 + a)^p*sec(f*x + e), x)
```

3.424 $\int \sec^3(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$

Optimal. Leaf size=239

$$\frac{\sin^5(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(\frac{b\sin^4(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{5}{4};2,-p;\frac{9}{4};\sin^4(e+fx),-\frac{b\sin^4(e+fx)}{a}\right)}{5f}+\frac{2\sin^3(e+fx)\left(a+b\sin\right.}{}$$

```
[Out] (AppellF1[1/4, 2, -p, 5/4, Sin[e + f*x]^4, -((b*Sin[e + f*x]^4)/a)]*Sin[e +
 f*x]*(a + b*Sin[e + f*x]^4)^p)/(f*(1 + (b*Sin[e + f*x]^4)/a)^p) + (2*Appel
lF1[3/4, 2, -p, 7/4, Sin[e + f*x]^4, -((b*Sin[e + f*x]^4)/a)]*Sin[e + f*x]^
3*(a + b*Sin[e + f*x]^4)^p)/(3*f*(1 + (b*Sin[e + f*x]^4)/a)^p) + (AppellF1[
5/4, 2, -p, 9/4, Sin[e + f*x]^4, -((b*Sin[e + f*x]^4)/a)]*Sin[e + f*x]^5*(a
 + b*Sin[e + f*x]^4)^p)/(5*f*(1 + (b*Sin[e + f*x]^4)/a)^p)
```

Rubi [A] time = 0.216692, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3223, 1240, 430, 429, 511, 510}

$$\frac{\sin^5(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(\frac{b\sin^4(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{5}{4};2,-p;\frac{9}{4};\sin^4(e+fx),-\frac{b\sin^4(e+fx)}{a}\right)}{5f}+\frac{2\sin^3(e+fx)\left(a+b\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3*(a + b*Sin[e + f*x]^4)^p,x]
```


```
[Out] (AppellF1[1/4, 2, -p, 5/4, Sin[e + f*x]^4, -((b*Sin[e + f*x]^4)/a)]*Sin[e +
 f*x]*(a + b*Sin[e + f*x]^4)^p)/(f*(1 + (b*Sin[e + f*x]^4)/a)^p) + (2*Appel
lF1[3/4, 2, -p, 7/4, Sin[e + f*x]^4, -((b*Sin[e + f*x]^4)/a)]*Sin[e + f*x]^
3*(a + b*Sin[e + f*x]^4)^p)/(3*f*(1 + (b*Sin[e + f*x]^4)/a)^p) + (AppellF1[
5/4, 2, -p, 9/4, Sin[e + f*x]^4, -((b*Sin[e + f*x]^4)/a)]*Sin[e + f*x]^5*(a
 + b*Sin[e + f*x]^4)^p)/(5*f*(1 + (b*Sin[e + f*x]^4)/a)^p)
```


Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rule 1240

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(a + c*x^4)^p, (d/(d^2 - e^2*x^4) - (e*x^2)/(d^2 - e^2*x^4
))^(-q), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[q, 0]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \sec^3(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+bx^4\right)^p}{\left(1-x^2\right)^2}\,dx,x,\sin(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(\frac{\left(a+bx^4\right)^p}{\left(-1+x^4\right)^2}+\frac{2x^2\left(a+bx^4\right)^p}{\left(-1+x^4\right)^2}+\frac{x^4\left(a+bx^4\right)^p}{\left(-1+x^4\right)^2}\right)dx,x,\sin(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{\left(a+bx^4\right)^p}{\left(-1+x^4\right)^2}\,dx,x,\sin(e+fx)\right)}{f}+\frac{\text{Subst}\left(\int \frac{x^4\left(a+bx^4\right)^p}{\left(-1+x^4\right)^2}\,dx,x,\sin(e+f\right.}{f}\\
&= \frac{\left(\left(a+b\sin^4(e+fx)\right)^p\left(1+\frac{b\sin^4(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^4}{a}\right)^p}{\left(-1+x^4\right)^2}\,dx,x,\sin(e+\right.}{f}\\
&= \frac{F_1\left(\frac{1}{4};2,-p;\frac{5}{4};\sin^4(e+fx),-\frac{b\sin^4(e+fx)}{a}\right)\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^p}{f}
\end{aligned}$$

Mathematica [F] time = 8.72661, size = 0, normalized size = 0.

$$\int \sec^3(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^3*(a + b*Sin[e + f*x]^4)^p,x]
```

```
[Out] Integrate[Sec[e + f*x]^3*(a + b*Sin[e + f*x]^4)^p, x]
```

Maple [F] time = 1.212, size = 0, normalized size = 0.

$$\int \left(\sec \left(fx+e\right)\right)^{3} \left(a+b \left(\sin \left(fx+e\right)\right)^{4}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3*(a+b*sin(f*x+e)^4)^p,x)
```

```
[Out] int(sec(f*x+e)^3*(a+b*sin(f*x+e)^4)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sin \left(fx+e\right)^{4}+a\right)^{p} \sec \left(fx+e\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^4)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^4 + a)^p*sec(f*x + e)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b \cos \left(fx+e\right)^{4}-2\, b \cos \left(fx+e\right)^{2}+a+b\right)^{p} \sec \left(fx+e\right)^{3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^4)^p,x, algorithm="fricas")
```

```
[Out] integral((b*cos(f*x + e)^4 - 2*b*cos(f*x + e)^2 + a + b)^p*sec(f*x + e)^3,
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3*(a+b*sin(f*x+e)**4)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sin \left(fx+e\right)^{4}+a\right)^{p} \sec \left(fx+e\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^4)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^4 + a)^p*sec(f*x + e)^3, x)
```

3.425 $\int \cos^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\cos^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p,x\right)$$

```
[Out] Unintegrable[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^4)^p, x]
```

Rubi [A] time = 0.0399659, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \cos^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^4)^p,x]
```

```
[Out] Defer[Int][Cos[e + f*x]^4*(a + b*Sin[e + f*x]^4)^p, x]
```

Rubi steps

$$\int \cos^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx=\int \cos^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

Mathematica [A] time = 5.38007, size = 0, normalized size = 0.

$$\int \cos^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^4)^p,x]
```

```
[Out] Integrate[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^4)^p, x]
```

Maple [A] time = 1.881, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^4\left(a+b\left(\sin\left(fx+e\right)\right)^4\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(a+b*sin(f*x+e)^4)^p,x)
```

```
[Out] int(cos(f*x+e)^4*(a+b*sin(f*x+e)^4)^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^4+a\right)^p\cos\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^4)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^4 + a)^p*cos(f*x + e)^4, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\cos\left(fx+e\right)^4-2\,b\cos\left(fx+e\right)^2+a+b\right)^p\cos\left(fx+e\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^4)^p,x, algorithm="fricas")
```

```
[Out] integral((b*cos(f*x + e)^4 - 2*b*cos(f*x + e)^2 + a + b)^p*cos(f*x + e)^4,
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(a+b*sin(f*x+e)**4)**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^4+a\right)^p\cos\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^4)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^4 + a)^p*cos(f*x + e)^4, x)
```

3.426 $\int \cos^2(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\cos^2(e+fx)\left(a+b\sin^4(e+fx)\right)^p,x\right)$$

```
[Out] Unintegrable[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^4)^p, x]
```

Rubi [A] time = 0.0406979, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \cos^2(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^4)^p,x]
```

```
[Out] Defer[Int][Cos[e + f*x]^2*(a + b*Sin[e + f*x]^4)^p, x]
```

Rubi steps

$$\int \cos^2(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx=\int \cos^2(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

Mathematica [A] time = 17.3401, size = 0, normalized size = 0.

$$\int \cos^2(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^4)^p,x]
```

```
[Out] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^4)^p, x]
```

Maple [A] time = 2.797, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^2\left(a+b\left(\sin\left(fx+e\right)\right)^4\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sin(f*x+e)^4)^p,x)
```

```
[Out] int(cos(f*x+e)^2*(a+b*sin(f*x+e)^4)^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^4+a\right)^p\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^4)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^4 + a)^p*cos(f*x + e)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(b\cos\left(fx+e\right)^4-2\,b\cos\left(fx+e\right)^2+a+b\right)^p\cos\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^4)^p,x, algorithm="fricas")
```

```
[Out] integral((b*cos(f*x + e)^4 - 2*b*cos(f*x + e)^2 + a + b)^p*cos(f*x + e)^2,
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sin(f*x+e)**4)**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^4+a\right)^p\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^4)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^4 + a)^p*cos(f*x + e)^2, x)
```

3.427 $\int \left(a+b\sin^4(e+fx)\right)^p\,dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\left(a+b\sin^4(e+fx)\right)^p,x\right)$$

```
[Out] Unintegrable[(a + b*Sin[e + f*x]^4)^p, x]
```

Rubi [A] time = 0.010978, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(a+b\sin^4(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[e + f*x]^4)^p,x]
```

```
[Out] Defer[Int][(a + b*Sin[e + f*x]^4)^p, x]
```

Rubi steps

$$\int \left(a+b\sin^4(e+fx)\right)^p\,dx=\int \left(a+b\sin^4(e+fx)\right)^p\,dx$$

Mathematica [A] time = 1.1015, size = 0, normalized size = 0.

$$\int \left(a+b\sin^4(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[e + f*x]^4)^p,x]
```

```
[Out] Integrate[(a + b*Sin[e + f*x]^4)^p, x]
```

Maple [A] time = 0.727, size = 0, normalized size = 0.

$$\int \left(a+b\left(\sin\left(fx+e\right)\right)^4\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^4)^p,x)
```

```
[Out] int((a+b*sin(f*x+e)^4)^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^4+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^4)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^4 + a)^p, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\cos\left(fx+e\right)^4-2\,b\cos\left(fx+e\right)^2+a+b\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^4)^p,x, algorithm="fricas")
```

```
[Out] integral((b*cos(f*x + e)^4 - 2*b*cos(f*x + e)^2 + a + b)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**4)**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^4+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^4)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^4 + a)^p, x)
```

3.428 $\int \sec^2(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\sec^2(e+fx)\left(a+b\sin^4(e+fx)\right)^p,x\right)$$

```
[Out] Unintegrable[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^4)^p, x]
```

Rubi [A] time = 0.043307, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sec^2(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^4)^p,x]
```

```
[Out] Defer[Int][Sec[e + f*x]^2*(a + b*Sin[e + f*x]^4)^p, x]
```

Rubi steps

$$\int \sec^2(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx=\int \sec^2(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

Mathematica [A] time = 5.7369, size = 0, normalized size = 0.

$$\int \sec^2(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^4)^p,x]
```

```
[Out] Integrate[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^4)^p, x]
```

Maple [A] time = 1.075, size = 0, normalized size = 0.

$$\int \left(\sec\left(fx+e\right)\right)^2\left(a+b\left(\sin\left(fx+e\right)\right)^4\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+b*sin(f*x+e)^4)^p,x)
```

```
[Out] int(sec(f*x+e)^2*(a+b*sin(f*x+e)^4)^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sin\left(fx+e\right)^4+a\right)^p \sec\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^4)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^4 + a)^p*sec(f*x + e)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\cos\left(fx+e\right)^4-2\,b\cos\left(fx+e\right)^2+a+b\right)^p\sec\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^4)^p,x, algorithm="fricas")
```

```
[Out] integral((b*cos(f*x + e)^4 - 2*b*cos(f*x + e)^2 + a + b)^p*sec(f*x + e)^2,
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+b*sin(f*x+e)**4)**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sin\left(fx+e\right)^4+a\right)^p \sec\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^4)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^4 + a)^p*sec(f*x + e)^2, x)
```

3.429 $\int \sec^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\sec^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p,x\right)$$

```
[Out] Unintegrable[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^4)^p, x]
```

Rubi [A] time = 0.0414706, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sec^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^4)^p,x]
```

```
[Out] Defer[Int][Sec[e + f*x]^4*(a + b*Sin[e + f*x]^4)^p, x]
```

Rubi steps

$$\int \sec^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx=\int \sec^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

Mathematica [A] time = 7.61621, size = 0, normalized size = 0.

$$\int \sec^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^4)^p,x]
```

```
[Out] Integrate[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^4)^p, x]
```

Maple [A] time = 0.832, size = 0, normalized size = 0.

$$\int \left(\sec\left(fx+e\right)\right)^4\left(a+b\left(\sin\left(fx+e\right)\right)^4\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(a+b*sin(f*x+e)^4)^p,x)
```

```
[Out] int(sec(f*x+e)^4*(a+b*sin(f*x+e)^4)^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^4+a\right)^p \sec\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^4)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^4 + a)^p*sec(f*x + e)^4, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\cos\left(fx+e\right)^4-2\,b\cos\left(fx+e\right)^2+a+b\right)^p \sec\left(fx+e\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^4)^p,x, algorithm="fricas")
```

```
[Out] integral((b*cos(f*x + e)^4 - 2*b*cos(f*x + e)^2 + a + b)^p*sec(f*x + e)^4,
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(a+b*sin(f*x+e)**4)**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^4+a\right)^p \sec\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^4)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^4 + a)^p*sec(f*x + e)^4, x)
```

3.430 $\int \cos^m(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\cos^m(e+fx)\left(a+b\sin^n(e+fx)\right)^p,x\right)$$

```
[Out] Unintegrable[Cos[e + f*x]^m*(a + b*Sin[e + f*x]^n)^p, x]
```

Rubi [A] time = 0.0490627, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \cos^m(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[e + f*x]^m*(a + b*Sin[e + f*x]^n)^p,x]
```

```
[Out] Defer[Int][Cos[e + f*x]^m*(a + b*Sin[e + f*x]^n)^p, x]
```

Rubi steps

$$\int \cos^m(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\int \cos^m(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

Mathematica [A] time = 4.36855, size = 0, normalized size = 0.

$$\int \cos^m(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^m*(a + b*Sin[e + f*x]^n)^p,x]
```

```
[Out] Integrate[Cos[e + f*x]^m*(a + b*Sin[e + f*x]^n)^p, x]
```

Maple [A] time = 1.533, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^m\left(a+b\left(\sin\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^m*(a+b*sin(f*x+e)^n)^p,x)
```

```
[Out] int(cos(f*x+e)^m*(a+b*sin(f*x+e)^n)^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^{n}+a\right)^{p}\cos\left(fx+e\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^m*(a+b*sin(f*x+e)^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^m, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sin\left(fx+e\right)^{n}+a\right)^{p}\cos\left(fx+e\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^m*(a+b*sin(f*x+e)^n)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**m*(a+b*sin(f*x+e)**n)**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^{n}+a\right)^{p}\cos\left(fx+e\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^m*(a+b*sin(f*x+e)^n)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^m, x)
```

3.431 $\int \cos^5(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$

Optimal. Leaf size=226

$$\frac{\sin^5(e+fx)\left(a+b\sin^n(e+fx)\right)^p\left(\frac{b\sin^n(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{5}{n},-p;\frac{n+5}{n};-\frac{b\sin^n(e+fx)}{a}\right)}{5f}-\frac{2\sin^3(e+fx)\left(a+b\sin^n(e+fx)\right)^p}{}$$

```
[Out] (Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*Sin[e + f*x]^n)/a)]*Sin[e +
 f*x]*(a + b*Sin[e + f*x]^n)^p)/(f*(1 + (b*Sin[e + f*x]^n)/a)^p) - (2*Hyper
geometric2F1[3/n, -p, (3 + n)/n, -((b*Sin[e + f*x]^n)/a)]*Sin[e + f*x]^3*(a
 + b*Sin[e + f*x]^n)^p)/(3*f*(1 + (b*Sin[e + f*x]^n)/a)^p) + (Hypergeometri
c2F1[5/n, -p, (5 + n)/n, -((b*Sin[e + f*x]^n)/a)]*Sin[e + f*x]^5*(a + b*Sin
[e + f*x]^n)^p)/(5*f*(1 + (b*Sin[e + f*x]^n)/a)^p)
```

Rubi [A] time = 0.174341, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3223, 1893, 246, 245, 365, 364}

$$\frac{\sin^5(e+fx)\left(a+b\sin^n(e+fx)\right)^p\left(\frac{b\sin^n(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{5}{n},-p;\frac{n+5}{n};-\frac{b\sin^n(e+fx)}{a}\right)}{5f}-\frac{2\sin^3(e+fx)\left(a+b\sin^n(e+fx)\right)^p}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^5*(a + b*Sin[e + f*x]^n)^p,x]
```

```
[Out] (Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*Sin[e + f*x]^n)/a)]*Sin[e +
 f*x]*(a + b*Sin[e + f*x]^n)^p)/(f*(1 + (b*Sin[e + f*x]^n)/a)^p) - (2*Hyper
geometric2F1[3/n, -p, (3 + n)/n, -((b*Sin[e + f*x]^n)/a)]*Sin[e + f*x]^3*(a
 + b*Sin[e + f*x]^n)^p)/(3*f*(1 + (b*Sin[e + f*x]^n)/a)^p) + (Hypergeometri
c2F1[5/n, -p, (5 + n)/n, -((b*Sin[e + f*x]^n)/a)]*Sin[e + f*x]^5*(a + b*Sin
[e + f*x]^n)^p)/(5*f*(1 + (b*Sin[e + f*x]^n)/a)^p)
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rule 1893

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \cos^5(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\frac{\text{Subst}\left(\int\left(1-x^2\right)^2\left(a+bx^n\right)^p\,dx,x,\sin(e+fx)\right)}{f}$$
$$=\frac{\text{Subst}\left(\int\left(\left(a+bx^n\right)^p-2x^2\left(a+bx^n\right)^p+x^4\left(a+bx^n\right)^p\right)dx,x,\sin(e+fx)\right)}{f}$$
$$=\frac{\text{Subst}\left(\int\left(a+bx^n\right)^p\,dx,x,\sin(e+fx)\right)}{f}+\frac{\text{Subst}\left(\int x^4\left(a+bx^n\right)^p\,dx,x,\sin\right.}{f}$$
$$=\frac{\left(\left(a+b\sin^n(e+fx)\right)^p\left(1+\frac{b\sin^n(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int\left(1+\frac{bx^n}{a}\right)^p\,dx,x,\sin(e\right.}{f}$$
$$=\frac{{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b\sin^n(e+fx)}{a}\right)\sin(e+fx)\left(a+b\sin^n(e+fx)\right)^p\left(1+\frac{b\sin^n}{\ }\right.}{f}$$

Mathematica [A] time = 0.217832, size = 155, normalized size = 0.69

$$\frac{\sin(e+fx)\left(a+b\sin^n(e+fx)\right)^p\left(\frac{b\sin^n(e+fx)}{a}+1\right)^{-p}\left(3\sin^4(e+fx)\,{}_2F_1\left(\frac{5}{n},-p;\frac{n+5}{n};-\frac{b\sin^n(e+fx)}{a}\right)-10\sin^2(e+fx)\,{}_2F_1\right.}{15f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^5*(a + b*Sin[e + f*x]^n)^p,x]
```

```
[Out] (Sin[e + f*x]*(15*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*Sin[e + f*
x]^n)/a)] - 10*Hypergeometric2F1[3/n, -p, (3 + n)/n, -((b*Sin[e + f*x]^n)/a
)]*Sin[e + f*x]^2 + 3*Hypergeometric2F1[5/n, -p, (5 + n)/n, -((b*Sin[e + f*
x]^n)/a)]*Sin[e + f*x]^4)*(a + b*Sin[e + f*x]^n)^p)/(15*f*(1 + (b*Sin[e + f
*x]^n)/a)^p)
```

Maple [F] time = 0.886, size = 0, normalized size = 0.

$$\int\left(\cos\left(fx+e\right)\right)^5\left(a+b\left(\sin\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^5*(a+b*sin(f*x+e)^n)^p,x)
```

```
[Out] int(cos(f*x+e)^5*(a+b*sin(f*x+e)^n)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^{n}+a\right)^{p}\cos\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sin(f*x+e)^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sin\left(fx+e\right)^{n}+a\right)^{p}\cos\left(fx+e\right)^{5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sin(f*x+e)^n)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**5*(a+b*sin(f*x+e)**n)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^{n}+a\right)^{p}\cos\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sin(f*x+e)^n)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^5, x)
```

3.432 $\int \cos^3(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$

Optimal. Leaf size=148

$$\frac{\sin(e+fx)\left(a+b\sin^n(e+fx)\right)^p\left(\frac{b\sin^n(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b\sin^n(e+fx)}{a}\right)}{f}-\frac{\sin^3(e+fx)\left(a+b\sin^n(e+fx)\right.}{}$$

```
[Out] (Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*Sin[e + f*x]^n)/a)]*Sin[e +
 f*x]*(a + b*Sin[e + f*x]^n)^p)/(f*(1 + (b*Sin[e + f*x]^n)/a)^p) - (Hyperge
ometric2F1[3/n, -p, (3 + n)/n, -((b*Sin[e + f*x]^n)/a)]*Sin[e + f*x]^3*(a +
 b*Sin[e + f*x]^n)^p)/(3*f*(1 + (b*Sin[e + f*x]^n)/a)^p)
```

Rubi [A] time = 0.11801, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3223, 1893, 246, 245, 365, 364}

$$\frac{\sin(e+fx)\left(a+b\sin^n(e+fx)\right)^p\left(\frac{b\sin^n(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b\sin^n(e+fx)}{a}\right)}{f}-\frac{\sin^3(e+fx)\left(a+b\sin^n(e+fx)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3*(a + b*Sin[e + f*x]^n)^p,x]
```

```
[Out] (Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*Sin[e + f*x]^n)/a)]*Sin[e +
 f*x]*(a + b*Sin[e + f*x]^n)^p)/(f*(1 + (b*Sin[e + f*x]^n)/a)^p) - (Hyperge
ometric2F1[3/n, -p, (3 + n)/n, -((b*Sin[e + f*x]^n)/a)]*Sin[e + f*x]^3*(a +
 b*Sin[e + f*x]^n)^p)/(3*f*(1 + (b*Sin[e + f*x]^n)/a)^p)
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rule 1893

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
```

```
GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \cos^3(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\frac{\text{Subst}\left(\int\left(1-x^2\right)(a+bx^n)^p\,dx,x,\sin(e+fx)\right)}{f}$$
$$=\frac{\text{Subst}\left(\int\left((a+bx^n)^p-x^2(a+bx^n)^p\right)dx,x,\sin(e+fx)\right)}{f}$$
$$=\frac{\text{Subst}\left(\int(a+bx^n)^p\,dx,x,\sin(e+fx)\right)}{f}-\frac{\text{Subst}\left(\int x^2(a+bx^n)^p\,dx,x,\sin(e\right.}{f}$$
$$=\frac{\left(\left(a+b\sin^n(e+fx)\right)^p\left(1+\frac{b\sin^n(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int\left(1+\frac{bx^n}{a}\right)^p dx,x,\sin(e+\right.}{f}$$
$$=\frac{{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b\sin^n(e+fx)}{a}\right)\sin(e+fx)\left(a+b\sin^n(e+fx)\right)^p\left(1+\frac{b\sin^n(e+f}{a}\right.}{f}$$

Mathematica [A] time = 0.0856171, size = 114, normalized size = 0.77

$$-\frac{\sin(e+fx)\left(a+b\sin^n(e+fx)\right)^p\left(\frac{b\sin^n(e+fx)}{a}+1\right)^{-p}\left(\sin^2(e+fx)\,{}_2F_1\left(\frac{3}{n},-p;\frac{n+3}{n};-\frac{b\sin^n(e+fx)}{a}\right)-3\,{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b}{}\right.\right.}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3*(a + b*Sin[e + f*x]^n)^p,x]
```

```
[Out] -(Sin[e + f*x]*(-3*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*Sin[e + f
*x]^n)/a)] + Hypergeometric2F1[3/n, -p, (3 + n)/n, -((b*Sin[e + f*x]^n)/a)]
*Sin[e + f*x]^2)*(a + b*Sin[e + f*x]^n)^p)/(3*f*(1 + (b*Sin[e + f*x]^n)/a)^
p)
```

Maple [F] time = 0.757, size = 0, normalized size = 0.

$$\int\left(\cos\left(fx+e\right)\right)^3\left(a+b\left(\sin\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3*(a+b*sin(f*x+e)^n)^p,x)
```

```
[Out] int(cos(f*x+e)^3*(a+b*sin(f*x+e)^n)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^{n}+a\right)^{p}\cos\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sin\left(fx+e\right)^{n}+a\right)^{p}\cos\left(fx+e\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^n)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3*(a+b*sin(f*x+e)**n)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^{n}+a\right)^{p}\cos\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^n)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^3, x)
```

3.433 $\int \cos(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$

Optimal. Leaf size=69

$$\frac{\sin(e+fx)\left(a+b\sin^n(e+fx)\right)^p\left(\frac{b\sin^n(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b\sin^n(e+fx)}{a}\right)}{f}$$

```
[Out] (Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*Sin[e + f*x]^n)/a)]*Sin[e +
 f*x]*(a + b*Sin[e + f*x]^n)^p)/(f*(1 + (b*Sin[e + f*x]^n)/a)^p)
```

Rubi [A] time = 0.0493514, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3223, 246, 245}

$$\frac{\sin(e+fx)\left(a+b\sin^n(e+fx)\right)^p\left(\frac{b\sin^n(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b\sin^n(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + b*Sin[e + f*x]^n)^p,x]
```

```
[Out] (Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*Sin[e + f*x]^n)/a)]*Sin[e +
 f*x]*(a + b*Sin[e + f*x]^n)^p)/(f*(1 + (b*Sin[e + f*x]^n)/a)^p)
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \cos(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int (a+bx^n)^p\,dx,x,\sin(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b\sin^n(e+fx)\right)^p\left(1+\frac{b\sin^n(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int\left(1+\frac{bx^n}{a}\right)^p\,dx,x,\sin(e\right.}{f}$$

$$= \frac{{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b\sin^n(e+fx)}{a}\right)\sin(e+fx)\left(a+b\sin^n(e+fx)\right)^p\left(1+\frac{b\sin^n(e\right.}{a}}{f}$$

Mathematica [A] time = 0.0218818, size = 69, normalized size = 1.

$$\frac{\sin(e+fx)\left(a+b\sin^n(e+fx)\right)^p\left(\frac{b\sin^n(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b\sin^n(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*(a + b*Sin[e + f*x]^n)^p,x]
```

```
[Out] (Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*Sin[e + f*x]^n)/a)]*Sin[e +
 f*x]*(a + b*Sin[e + f*x]^n)^p)/(f*(1 + (b*Sin[e + f*x]^n)/a)^p)
```

Maple [F] time = 0.7, size = 0, normalized size = 0.

$$\int \cos\left(fx+e\right)\left(a+b\left(\sin\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+b*sin(f*x+e)^n)^p,x)
```

```
[Out] int(cos(f*x+e)*(a+b*sin(f*x+e)^n)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^n+a\right)^p\cos\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^n + a)^p*cos(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sin\left(fx+e\right)^n+a\right)^p\cos\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)^n)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e)^n + a)^p*cos(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)**n)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sin \left(fx+e\right)^{n}+a\right)^{p} \cos \left(fx+e\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)^n)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^n + a)^p*cos(f*x + e), x)
```

3.434 $\int \sec(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\sec(e+fx)\left(a+b\sin^n(e+fx)\right)^p,x\right)$$

```
[Out] Unintegrable[Sec[e + f*x]*(a + b*Sin[e + f*x]^n)^p, x]
```

Rubi [A] time = 0.039806, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sec(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[e + f*x]*(a + b*Sin[e + f*x]^n)^p,x]
```

```
[Out] Defer[Int][Sec[e + f*x]*(a + b*Sin[e + f*x]^n)^p, x]
```

Rubi steps

$$\int \sec(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\int \sec(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

Mathematica [A] time = 2.84377, size = 0, normalized size = 0.

$$\int \sec(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]*(a + b*Sin[e + f*x]^n)^p,x]
```

```
[Out] Integrate[Sec[e + f*x]*(a + b*Sin[e + f*x]^n)^p, x]
```

Maple [A] time = 0.52, size = 0, normalized size = 0.

$$\int \sec\left(fx+e\right)\left(a+b\left(\sin\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sin(f*x+e)^n)^p,x)
```

```
[Out] int(sec(f*x+e)*(a+b*sin(f*x+e)^n)^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^{n}+a\right)^{p}\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^n + a)^p*sec(f*x + e), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sin\left(fx+e\right)^{n}+a\right)^{p}\sec\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)^n)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e)^n + a)^p*sec(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)**n)**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^{n}+a\right)^{p}\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)^n)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^n + a)^p*sec(f*x + e), x)
```

3.435 $\int \sec^3(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\sec^3(e+fx)\left(a+b\sin^n(e+fx)\right)^p,x\right)$$

```
[Out] Unintegrable[Sec[e + f*x]^3*(a + b*Sin[e + f*x]^n)^p, x]
```

Rubi [A] time = 0.0511064, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sec^3(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[e + f*x]^3*(a + b*Sin[e + f*x]^n)^p,x]
```

```
[Out] Defer[Int][Sec[e + f*x]^3*(a + b*Sin[e + f*x]^n)^p, x]
```

Rubi steps

$$\int \sec^3(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\int \sec^3(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

Mathematica [A] time = 5.28814, size = 0, normalized size = 0.

$$\int \sec^3(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^3*(a + b*Sin[e + f*x]^n)^p,x]
```

```
[Out] Integrate[Sec[e + f*x]^3*(a + b*Sin[e + f*x]^n)^p, x]
```

Maple [A] time = 0.54, size = 0, normalized size = 0.

$$\int \left(\sec\left(fx+e\right)\right)^3\left(a+b\left(\sin\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3*(a+b*sin(f*x+e)^n)^p,x)
```

```
[Out] int(sec(f*x+e)^3*(a+b*sin(f*x+e)^n)^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sin \left(fx+e\right)^{n}+a\right)^{p} \sec \left(fx+e\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^n + a)^p*sec(f*x + e)^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \sin \left(fx+e\right)^{n}+a\right)^{p} \sec \left(fx+e\right)^{3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^n)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e)^n + a)^p*sec(f*x + e)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3*(a+b*sin(f*x+e)**n)**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sin \left(fx+e\right)^{n}+a\right)^{p} \sec \left(fx+e\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^n)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^n + a)^p*sec(f*x + e)^3, x)
```

3.436 $\int \cos^4(e+fx) \left(a+b\sin^n(e+fx)\right)^p \, dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\cos^4(e+fx) \left(a+b\sin^n(e+fx)\right)^p,x\right)$$

```
[Out] Unintegrable[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^n)^p, x]
```

Rubi [A] time = 0.0497227, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \cos^4(e+fx) \left(a+b\sin^n(e+fx)\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^n)^p,x]
```

```
[Out] Defer[Int][Cos[e + f*x]^4*(a + b*Sin[e + f*x]^n)^p, x]
```

Rubi steps

$$\int \cos^4(e+fx) \left(a+b\sin^n(e+fx)\right)^p \, dx = \int \cos^4(e+fx) \left(a+b\sin^n(e+fx)\right)^p \, dx$$

Mathematica [A] time = 19.3064, size = 0, normalized size = 0.

$$\int \cos^4(e+fx) \left(a+b\sin^n(e+fx)\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^n)^p,x]
```

```
[Out] Integrate[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^n)^p, x]
```

Maple [A] time = 0.807, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^4 \left(a+b\left(\sin\left(fx+e\right)\right)^n\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(a+b*sin(f*x+e)^n)^p,x)
```

```
[Out] int(cos(f*x+e)^4*(a+b*sin(f*x+e)^n)^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sin\left(fx+e\right)^{n}+a\right)^{p} \cos\left(fx+e\right)^{4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^4, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b \sin\left(fx+e\right)^{n}+a\right)^{p} \cos\left(fx+e\right)^{4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^n)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(a+b*sin(f*x+e)**n)**p,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^n)^p,x, algorithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.437 $\int \cos^2(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\cos^2(e+fx)\left(a+b\sin^n(e+fx)\right)^p,x\right)$$

```
[Out] Unintegrable[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^n)^p, x]
```

Rubi [A] time = 0.048352, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \cos^2(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^n)^p,x]
```

```
[Out] Defer[Int][Cos[e + f*x]^2*(a + b*Sin[e + f*x]^n)^p, x]
```

Rubi steps

$$\int \cos^2(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\int \cos^2(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

Mathematica [A] time = 11.8331, size = 0, normalized size = 0.

$$\int \cos^2(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^n)^p,x]
```

```
[Out] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^n)^p, x]
```

Maple [A] time = 0.701, size = 0, normalized size = 0.

$$\int \left(\cos\left(fx+e\right)\right)^2\left(a+b\left(\sin\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sin(f*x+e)^n)^p,x)
```

```
[Out] int(cos(f*x+e)^2*(a+b*sin(f*x+e)^n)^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^n+a\right)^p\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sin\left(fx+e\right)^n+a\right)^p\cos\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^n)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sin(f*x+e)**n)**p,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^n)^p,x, algorithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.438 $\int (a+b\sin^n(e+fx))^p\,dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left((a+b\sin^n(e+fx))^p,x\right)$$

```
[Out] Unintegrable[(a + b*Sin[e + f*x]^n)^p, x]
```

Rubi [A] time = 0.0129351, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (a+b\sin^n(e+fx))^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[e + f*x]^n)^p,x]
```

```
[Out] Defer[Int][(a + b*Sin[e + f*x]^n)^p, x]
```

Rubi steps

$$\int (a+b\sin^n(e+fx))^p\,dx=\int (a+b\sin^n(e+fx))^p\,dx$$

Mathematica [A] time = 1.58589, size = 0, normalized size = 0.

$$\int (a+b\sin^n(e+fx))^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[e + f*x]^n)^p,x]
```

```
[Out] Integrate[(a + b*Sin[e + f*x]^n)^p, x]
```

Maple [A] time = 0.463, size = 0, normalized size = 0.

$$\int \left(a+b\left(\sin\left(fx+e\right)\right)^{n}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^n)^p,x)
```

```
[Out] int((a+b*sin(f*x+e)^n)^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sin \left(fx+e\right)^{n}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^n + a)^p, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b \sin \left(fx+e\right)^{n}+a\right)^{p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^n)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e)^n + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**n)**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sin \left(fx+e\right)^{n}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^n)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^n + a)^p, x)
```

3.439 $\int \sec^2(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\sec^2(e+fx)\left(a+b\sin^n(e+fx)\right)^p,x\right)$$

```
[Out] Unintegrable[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^n)^p, x]
```

Rubi [A] time = 0.0484019, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sec^2(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^n)^p,x]
```

```
[Out] Defer[Int][Sec[e + f*x]^2*(a + b*Sin[e + f*x]^n)^p, x]
```

Rubi steps

$$\int \sec^2(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\int \sec^2(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

Mathematica [A] time = 2.63119, size = 0, normalized size = 0.

$$\int \sec^2(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^n)^p,x]
```

```
[Out] Integrate[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^n)^p, x]
```

Maple [A] time = 0.539, size = 0, normalized size = 0.

$$\int \left(\sec\left(fx+e\right)\right)^2\left(a+b\left(\sin\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+b*sin(f*x+e)^n)^p,x)
```

```
[Out] int(sec(f*x+e)^2*(a+b*sin(f*x+e)^n)^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^{n}+a\right)^{p}\sec\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^n + a)^p*sec(f*x + e)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sin\left(fx+e\right)^{n}+a\right)^{p}\sec\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^n)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e)^n + a)^p*sec(f*x + e)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+b*sin(f*x+e)**n)**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^{n}+a\right)^{p}\sec\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^n)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^n + a)^p*sec(f*x + e)^2, x)
```

3.440 $\int \sec^4(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\sec^4(e+fx)\left(a+b\sin^n(e+fx)\right)^p,x\right)$$

```
[Out] Unintegrable[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^n)^p, x]
```

Rubi [A] time = 0.0488473, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sec^4(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^n)^p,x]
```

```
[Out] Defer[Int][Sec[e + f*x]^4*(a + b*Sin[e + f*x]^n)^p, x]
```

Rubi steps

$$\int \sec^4(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\int \sec^4(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

Mathematica [A] time = 3.72793, size = 0, normalized size = 0.

$$\int \sec^4(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^n)^p,x]
```

```
[Out] Integrate[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^n)^p, x]
```

Maple [A] time = 0.557, size = 0, normalized size = 0.

$$\int \left(\sec\left(fx+e\right)\right)^4\left(a+b\left(\sin\left(fx+e\right)\right)^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(a+b*sin(f*x+e)^n)^p,x)
```

```
[Out] int(sec(f*x+e)^4*(a+b*sin(f*x+e)^n)^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sin\left(fx+e\right)^{n}+a\right)^{p} \sec\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^n + a)^p*sec(f*x + e)^4, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b \sin\left(fx+e\right)^{n}+a\right)^{p} \sec\left(fx+e\right)^{4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^n)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e)^n + a)^p*sec(f*x + e)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(a+b*sin(f*x+e)**n)**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sin\left(fx+e\right)^{n}+a\right)^{p} \sec\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^n)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^n + a)^p*sec(f*x + e)^4, x)
```

3.441 $\int \frac{\tan^7(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=128

$$\frac{(3a^2+3ab+b^2)\sec^2(c+dx)}{2d(a+b)^3}-\frac{a^3\log\left(a+b\sin^2(c+dx)\right)}{2d(a+b)^4}+\frac{a^3\log(\cos(c+dx))}{d(a+b)^4}+\frac{\sec^6(c+dx)}{6d(a+b)}-\frac{(3a+2b)\sec^4(c+}{4d(a+b)^2}$$

```
[Out] (a^3*Log[Cos[c + d*x]])/((a + b)^4*d) - (a^3*Log[a + b*Sin[c + d*x]^2])/(2*
(a + b)^4*d) + ((3*a^2 + 3*a*b + b^2)*Sec[c + d*x]^2)/(2*(a + b)^3*d) - ((3
*a + 2*b)*Sec[c + d*x]^4)/(4*(a + b)^2*d) + Sec[c + d*x]^6/(6*(a + b)*d)
```

Rubi [A] time = 0.13169, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3194, 88}

$$\frac{(3a^2+3ab+b^2)\sec^2(c+dx)}{2d(a+b)^3}-\frac{a^3\log\left(a+b\sin^2(c+dx)\right)}{2d(a+b)^4}+\frac{a^3\log(\cos(c+dx))}{d(a+b)^4}+\frac{\sec^6(c+dx)}{6d(a+b)}-\frac{(3a+2b)\sec^4(c+}{4d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^7/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (a^3*Log[Cos[c + d*x]])/((a + b)^4*d) - (a^3*Log[a + b*Sin[c + d*x]^2])/(2*
(a + b)^4*d) + ((3*a^2 + 3*a*b + b^2)*Sec[c + d*x]^2)/(2*(a + b)^3*d) - ((3
*a + 2*b)*Sec[c + d*x]^4)/(4*(a + b)^2*d) + Sec[c + d*x]^6/(6*(a + b)*d)
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^7(c+dx)}{a+b\sin^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x^3}{(1-x)^4(a+bx)}\,dx,x,\sin^2(c+dx)\right)}{2d}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{(a+b)(-1+x)^4}+\frac{3a+2b}{(a+b)^2(-1+x)^3}+\frac{3a^2+3ab+b^2}{(a+b)^3(-1+x)^2}+\frac{a^3}{(a+b)^4(-1+x)}-\frac{a^3b}{(a+b)^4(a+bx)}\right)dx,x,\sin^2(\right.}{2d}\\ &= \frac{a^3\log(\cos(c+dx))}{(a+b)^4d}-\frac{a^3\log\left(a+b\sin^2(c+dx)\right)}{2(a+b)^4d}+\frac{(3a^2+3ab+b^2)\sec^2(c+dx)}{2(a+b)^3d}-\frac{(3a+}{}\end{aligned}$$

Mathematica [A] time = 0.286028, size = 113, normalized size = 0.88

$$\frac{\frac{6(3a^2+3ab+b^2)\sec^2(c+dx)}{(a+b)^3} - \frac{6a^3\log\left(a+b\sin^2(c+dx)\right)}{(a+b)^4} + \frac{12a^3\log(\cos(c+dx))}{(a+b)^4} + \frac{2\sec^6(c+dx)}{a+b} - \frac{3(3a+2b)\sec^4(c+dx)}{(a+b)^2}}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^7/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] ((12*a^3*Log[Cos[c + d*x]])/(a + b)^4 - (6*a^3*Log[a + b*Sin[c + d*x]^2])/(
a + b)^4 + (6*(3*a^2 + 3*a*b + b^2)*Sec[c + d*x]^2)/(a + b)^3 - (3*(3*a + 2
*b)*Sec[c + d*x]^4)/(a + b)^2 + (2*Sec[c + d*x]^6)/(a + b))/(12*d)
```

Maple [A] time = 0.086, size = 170, normalized size = 1.3

$$\frac{a^3\ln(\cos(dx+c))}{(a+b)^4 d} - \frac{3a}{4d(a+b)^2(\cos(dx+c))^4} - \frac{b}{2d(a+b)^2(\cos(dx+c))^4} + \frac{3a^2}{2d(a+b)^3(\cos(dx+c))^2} + \frac{\quad}{2d(a+b)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^7/(a+sin(d*x+c)^2*b),x)
```

```
[Out] a^3*ln(cos(d*x+c))/(a+b)^4/d-3/4/d/(a+b)^2/cos(d*x+c)^4*a-1/2/d/(a+b)^2/cos
(d*x+c)^4*b+3/2/d/(a+b)^3/cos(d*x+c)^2*a^2+3/2/d/(a+b)^3/cos(d*x+c)^2*a*b+1
/2/d/(a+b)^3/cos(d*x+c)^2*b^2+1/6/d/(a+b)/cos(d*x+c)^6-1/2/d*a^3/(a+b)^4*ln
(b*cos(d*x+c)^2-a-b)
```

Maxima [B] time = 1.0131, size = 369, normalized size = 2.88

$$-\frac{\frac{6a^3\log\left(b\sin(dx+c)^2+a\right)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4} - \frac{6a^3\log\left(\sin(dx+c)^2-1\right)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4} + \frac{6\left(3a^2+3ab+b^2\right)\sin(dx+c)^4-3\left(9a^2+7ab+2b^2\right)\sin(dx+c)^2+11a^2+7ab}{\left(a^3+3a^2b+3ab^2+b^3\right)\sin(dx+c)^6-3\left(a^3+3a^2b+3ab^2+b^3\right)\sin(dx+c)^4-a^3-3a^2b-3ab^2-b^3+3\left(a^3\right.}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/12*(6*a^3*log(b*sin(d*x + c)^2 + a)/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3
 + b^4) - 6*a^3*log(sin(d*x + c)^2 - 1)/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^
3 + b^4) + (6*(3*a^2 + 3*a*b + b^2)*sin(d*x + c)^4 - 3*(9*a^2 + 7*a*b + 2*b
^2)*sin(d*x + c)^2 + 11*a^2 + 7*a*b + 2*b^2)/((a^3 + 3*a^2*b + 3*a*b^2 + b^
3)*sin(d*x + c)^6 - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sin(d*x + c)^4 - a^3
- 3*a^2*b - 3*a*b^2 - b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sin(d*x + c)^
2))/d
```

Fricas [A] time = 4.12375, size = 420, normalized size = 3.28

$$-\frac{6a^3\cos(dx+c)^6\log\left(-b\cos(dx+c)^2+a+b\right) - 12a^3\cos(dx+c)^6\log(-\cos(dx+c)) - 6\left(3a^3+6a^2b+4ab^2+b^3\right)}{12\left(a^4+4a^3b+6a^2b^2+4ab^3+b^4\right)d\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/12*(6*a^3*cos(d*x + c)^6*log(-b*cos(d*x + c)^2 + a + b) - 12*a^3*cos(d*x
 + c)^6*log(-cos(d*x + c)) - 6*(3*a^3 + 6*a^2*b + 4*a*b^2 + b^3)*cos(d*x +
c)^4 - 2*a^3 - 6*a^2*b - 6*a*b^2 - 2*b^3 + 3*(3*a^3 + 8*a^2*b + 7*a*b^2 + 2
*b^3)*cos(d*x + c)^2)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cos(d*
x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**7/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 6.9696, size = 814, normalized size = 6.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/60*(30*a^3*log(a - 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 4*b*(cos(
d*x + c) - 1)/(cos(d*x + c) + 1) + a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1
)^2)/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 60*a^3*log(abs(-(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) - 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 +
b^4) + (147*a^3 + 1002*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 120*a^2*
b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2925*a^3*(cos(d*x + c) - 1)^2/(co
s(d*x + c) + 1)^2 + 960*a^2*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 2
40*a*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 4780*a^3*(cos(d*x + c)
 - 1)^3/(cos(d*x + c) + 1)^3 + 3600*a^2*b*(cos(d*x + c) - 1)^3/(cos(d*x + c
) + 1)^3 + 2400*a*b^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 640*b^3*(
cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 2925*a^3*(cos(d*x + c) - 1)^4/(c
os(d*x + c) + 1)^4 + 960*a^2*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 +
240*a*b^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 1002*a^3*(cos(d*x + c
) - 1)^5/(cos(d*x + c) + 1)^5 + 120*a^2*b*(cos(d*x + c) - 1)^5/(cos(d*x + c
) + 1)^5 + 147*a^3*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6)/((a^4 + 4*a^3
*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)
^6))/d
```

3.442 $\int \frac{\tan^5(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=94

$$\frac{a^2\log\left(a+b\sin^2(c+dx)\right)}{2d(a+b)^3}-\frac{a^2\log(\cos(c+dx))}{d(a+b)^3}+\frac{\sec^4(c+dx)}{4d(a+b)}-\frac{(2a+b)\sec^2(c+dx)}{2d(a+b)^2}$$

```
[Out] -((a^2*Log[Cos[c + d*x]])/((a + b)^3*d)) + (a^2*Log[a + b*Sin[c + d*x]^2])/
(2*(a + b)^3*d) - ((2*a + b)*Sec[c + d*x]^2)/(2*(a + b)^2*d) + Sec[c + d*x]
^4/(4*(a + b)*d)
```

Rubi [A] time = 0.102406, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3194, 88}

$$\frac{a^2\log\left(a+b\sin^2(c+dx)\right)}{2d(a+b)^3}-\frac{a^2\log(\cos(c+dx))}{d(a+b)^3}+\frac{\sec^4(c+dx)}{4d(a+b)}-\frac{(2a+b)\sec^2(c+dx)}{2d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -((a^2*Log[Cos[c + d*x]])/((a + b)^3*d)) + (a^2*Log[a + b*Sin[c + d*x]^2])/
(2*(a + b)^3*d) - ((2*a + b)*Sec[c + d*x]^2)/(2*(a + b)^2*d) + Sec[c + d*x]
^4/(4*(a + b)*d)
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^5(c+dx)}{a+b\sin^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x^2}{(1-x)^3(a+bx)}\,dx, x, \sin^2(c+dx)\right)}{2d}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)(-1+x)^3}+\frac{-2a-b}{(a+b)^2(-1+x)^2}-\frac{a^2}{(a+b)^3(-1+x)}+\frac{a^2b}{(a+b)^3(a+bx)}\right)dx, x, \sin^2(c+dx)\right)}{2d}\\
&= -\frac{a^2\log(\cos(c+dx))}{(a+b)^3d}+\frac{a^2\log\left(a+b\sin^2(c+dx)\right)}{2(a+b)^3d}-\frac{(2a+b)\sec^2(c+dx)}{2(a+b)^2d}+\frac{\sec^4(c+dx)}{4(a+b)d}
\end{aligned}$$

Mathematica [A] time = 0.275684, size = 78, normalized size = 0.83

$$\frac{-2\left(2a^2+3ab+b^2\right)\sec^2(c+dx)+2a^2\left(\log\left(a+b\sin^2(c+dx)\right)-2\log(\cos(c+dx))\right)+(a+b)^2\sec^4(c+dx)}{4d(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (2*a^2*(-2*Log[Cos[c + d*x]] + Log[a + b*Sin[c + d*x]^2]) - 2*(2*a^2 + 3*a*
b + b^2)*Sec[c + d*x]^2 + (a + b)^2*Sec[c + d*x]^4)/(4*(a + b)^3*d)
```

Maple [A] time = 0.085, size = 109, normalized size = 1.2

$$-\frac{a}{d\left(a+b\right)^2\left(\cos\left(dx+c\right)\right)^2}-\frac{b}{2d\left(a+b\right)^2\left(\cos\left(dx+c\right)\right)^2}+\frac{1}{4d\left(a+b\right)\left(\cos\left(dx+c\right)\right)^4}-\frac{a^2\ln\left(\cos\left(dx+c\right)\right)}{\left(a+b\right)^3d}+\frac{a^2\ln\left(b\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+sin(d*x+c)^2*b),x)
```

```
[Out] -1/d/(a+b)^2/cos(d*x+c)^2*a-1/2/d/(a+b)^2/cos(d*x+c)^2*b+1/4/d/(a+b)/cos(d*
x+c)^4-a^2*ln(cos(d*x+c))/(a+b)^3/d+1/2/d*a^2/(a+b)^3*ln(b*cos(d*x+c)^2-a-b
)
```

Maxima [A] time = 1.03609, size = 215, normalized size = 2.29

$$\frac{\frac{2\,a^2\log\left(b\sin\left(dx+c\right)^2+a\right)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{2\,a^2\log\left(\sin\left(dx+c\right)^2-1\right)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{2\,(2\,a+b)\sin\left(dx+c\right)^2-3\,a-b}{\left(a^2+2\,ab+b^2\right)\sin\left(dx+c\right)^4-2\left(a^2+2\,ab+b^2\right)\sin\left(dx+c\right)^2+a^2+2\,ab+b^2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*a^2*log(b*sin(d*x + c)^2 + a)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 2*a^
2*log(sin(d*x + c)^2 - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + (2*(2*a + b)*si
n(d*x + c)^2 - 3*a - b)/((a^2 + 2*a*b + b^2)*sin(d*x + c)^4 - 2*(a^2 + 2*a*
b + b^2)*sin(d*x + c)^2 + a^2 + 2*a*b + b^2))/d
```

Fricas [A] time = 2.96964, size = 288, normalized size = 3.06

$$\frac{2\,a^2\cos\left(dx+c\right)^4\log\left(-b\cos\left(dx+c\right)^2+a+b\right)-4\,a^2\cos\left(dx+c\right)^4\log\left(-\cos\left(dx+c\right)\right)-2\left(2\,a^2+3\,ab+b^2\right)\cos\left(dx\right.}{4\left(a^3+3\,a^2b+3\,ab^2+b^3\right)d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^2*cos(d*x + c)^4*log(-b*cos(d*x + c)^2 + a + b) - 4*a^2*cos(d*x +
c)^4*log(-cos(d*x + c)) - 2*(2*a^2 + 3*a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*
```

```
a*b + b^2)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 3.14534, size = 531, normalized size = 5.65

$$\frac{\frac{6a^2\log\left(a-\frac{2a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{4b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)}{a^3+3a^2b+3ab^2+b^3}-\frac{12a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a^3+3a^2b+3ab^2+b^3}+\frac{25a^2+\frac{124a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{24ab(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{246a^2(\cos(dx+c)-1}{(\cos(dx+c)+1)^2}}{}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/12*(6*a^2*log(a - 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 4*b*(cos(d*
x + c) - 1)/(cos(d*x + c) + 1) + a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^
2)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 12*a^2*log(abs(-(cos(d*x + c) - 1)/(co
s(d*x + c) + 1) - 1))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + (25*a^2 + 124*a^2*(
cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 24*a*b*(cos(d*x + c) - 1)/(cos(d*x +
 c) + 1) + 246*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 144*a*b*(cos
(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 48*b^2*(cos(d*x + c) - 1)^2/(cos(d*
x + c) + 1)^2 + 124*a^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 24*a*b*
(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 25*a^2*(cos(d*x + c) - 1)^4/(co
s(d*x + c) + 1)^4)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*((cos(d*x + c) - 1)/(co
s(d*x + c) + 1) + 1)^4))/d
```

3.443 $\int \frac{\tan^3(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=64

$$\frac{\sec^2(c+dx)}{2d(a+b)} - \frac{a\log\left(a+b\sin^2(c+dx)\right)}{2d(a+b)^2} + \frac{a\log(\cos(c+dx))}{d(a+b)^2}$$

```
[Out] (a*Log[Cos[c + d*x]])/((a + b)^2*d) - (a*Log[a + b*Sin[c + d*x]^2])/(2*(a +
 b)^2*d) + Sec[c + d*x]^2/(2*(a + b)*d)
```

Rubi [A] time = 0.0757577, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3194, 77}

$$\frac{\sec^2(c+dx)}{2d(a+b)} - \frac{a\log\left(a+b\sin^2(c+dx)\right)}{2d(a+b)^2} + \frac{a\log(\cos(c+dx))}{d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (a*Log[Cos[c + d*x]])/((a + b)^2*d) - (a*Log[a + b*Sin[c + d*x]^2])/(2*(a +
 b)^2*d) + Sec[c + d*x]^2/(2*(a + b)*d)
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^3(c+dx)}{a+b\sin^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x}{(1-x)^2(a+bx)}\,dx, x, \sin^2(c+dx)\right)}{2d}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{(a+b)(-1+x)^2} + \frac{a}{(a+b)^2(-1+x)} - \frac{ab}{(a+b)^2(a+bx)}\right)dx, x, \sin^2(c+dx)\right)}{2d}\\
&= \frac{a\log(\cos(c+dx))}{(a+b)^2d} - \frac{a\log\left(a+b\sin^2(c+dx)\right)}{2(a+b)^2d} + \frac{\sec^2(c+dx)}{2(a+b)d}
\end{aligned}$$

Mathematica [A] time = 0.093298, size = 52, normalized size = 0.81

$$\frac{(a+b)\sec^2(c+dx)+a\left(2\log(\cos(c+dx))-\log\left(a+b\sin^2(c+dx)\right)\right)}{2d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (a*(2*Log[Cos[c + d*x]] - Log[a + b*Sin[c + d*x]^2]) + (a + b)*Sec[c + d*x]
^2)/(2*(a + b)^2*d)
```

Maple [A] time = 0.078, size = 66, normalized size = 1.

$$\frac{a\ln(\cos(dx+c))}{(a+b)^2 d}+\frac{1}{2d(a+b)(\cos(dx+c))^2}-\frac{a\ln\left(b(\cos(dx+c))^2-a-b\right)}{2(a+b)^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+sin(d*x+c)^2*b),x)
```

```
[Out] a*ln(cos(d*x+c))/(a+b)^2/d+1/2/d/(a+b)/cos(d*x+c)^2-1/2/d*a/(a+b)^2*ln(b*co
s(d*x+c)^2-a-b)
```

Maxima [A] time = 1.00768, size = 111, normalized size = 1.73

$$-\frac{\frac{a\log\left(b\sin(dx+c)^2+a\right)}{a^2+2ab+b^2}-\frac{a\log\left(\sin(dx+c)^2-1\right)}{a^2+2ab+b^2}+\frac{1}{(a+b)\sin(dx+c)^2-a-b}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(a*log(b*sin(d*x + c)^2 + a)/(a^2 + 2*a*b + b^2) - a*log(sin(d*x + c)^
2 - 1)/(a^2 + 2*a*b + b^2) + 1/((a + b)*sin(d*x + c)^2 - a - b))/d
```

Fricas [A] time = 2.25925, size = 193, normalized size = 3.02

$$-\frac{a\cos(dx+c)^2\log\left(-b\cos(dx+c)^2+a+b\right)-2a\cos(dx+c)^2\log(-\cos(dx+c))-a-b}{2\left(a^2+2ab+b^2\right)d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(a*cos(d*x + c)^2*log(-b*cos(d*x + c)^2 + a + b) - 2*a*cos(d*x + c)^2*
log(-cos(d*x + c)) - a - b)/((a^2 + 2*a*b + b^2)*d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^3(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral(tan(c + d*x)**3/(a + b*sin(c + d*x)**2), x)
```

Giac [B] time = 1.60109, size = 316, normalized size = 4.94

$$-\frac{\frac{a\log\left(a-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{4\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)}{a^2+2\,ab+b^2}-\frac{2\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a^2+2\,ab+b^2}+\frac{3\,a+\frac{10\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{4\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{3\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{(a^2+2\,ab+b^2)\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/2*(a*log(a - 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 4*b*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) + a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/(
a^2 + 2*a*b + b^2) - 2*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1
))/(a^2 + 2*a*b + b^2) + (3*a + 10*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
+ 4*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 3*a*(cos(d*x + c) - 1)^2/(cos
(d*x + c) + 1)^2)/((a^2 + 2*a*b + b^2)*((cos(d*x + c) - 1)/(cos(d*x + c) +
1) + 1)^2))/d
```

3.444 $\int \frac{\tan(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=43

$$\frac{\log\left(a+b\sin^2(c+dx)\right)}{2d(a+b)}-\frac{\log(\cos(c+dx))}{d(a+b)}$$

```
[Out] -(Log[Cos[c + d*x]]/((a + b)*d)) + Log[a + b*Sin[c + d*x]^2]/(2*(a + b)*d)
```

Rubi [A] time = 0.0385448, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3194, 36, 31}

$$\frac{\log\left(a+b\sin^2(c+dx)\right)}{2d(a+b)}-\frac{\log(\cos(c+dx))}{d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -(Log[Cos[c + d*x]]/((a + b)*d)) + Log[a + b*Sin[c + d*x]^2]/(2*(a + b)*d)
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan(c+dx)}{a+b\sin^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(1-x)(a+bx)}\,dx,x,\sin^2(c+dx)\right)}{2d}\\ &= \frac{\text{Subst}\left(\int \frac{1}{1-x}\,dx,x,\sin^2(c+dx)\right)}{2(a+b)d}+\frac{b\,\text{Subst}\left(\int \frac{1}{a+bx}\,dx,x,\sin^2(c+dx)\right)}{2(a+b)d}\\ &= -\frac{\log(\cos(c+dx))}{(a+b)d}+\frac{\log\left(a+b\sin^2(c+dx)\right)}{2(a+b)d}\end{aligned}$$

Mathematica [A] time = 0.0323504, size = 37, normalized size = 0.86

$$\frac{\log\left(a-b\cos^2(c+dx)+b\right)-2\log(\cos(c+dx))}{2ad+2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (-2*Log[Cos[c + d*x]] + Log[a + b - b*Cos[c + d*x]^2])/(2*a*d + 2*b*d)
```

Maple [A] time = 0.065, size = 47, normalized size = 1.1

$$-\frac{\ln\left(\cos\left(dx+c\right)\right)}{\left(a+b\right)d}+\frac{\ln\left(b\left(\cos\left(dx+c\right)\right)^{2}-a-b\right)}{2\left(a+b\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+sin(d*x+c)^2*b),x)
```

```
[Out] -ln(cos(d*x+c))/(a+b)/d+1/2/d/(a+b)*ln(b*cos(d*x+c)^2-a-b)
```

Maxima [A] time = 0.983951, size = 58, normalized size = 1.35

$$\frac{\frac{\log\left(b\sin(dx+c)^{2}+a\right)}{a+b}-\frac{\log\left(\sin(dx+c)^{2}-1\right)}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(log(b*sin(d*x + c)^2 + a)/(a + b) - log(sin(d*x + c)^2 - 1)/(a + b))/d
```

Fricas [A] time = 1.87446, size = 99, normalized size = 2.3

$$\frac{\log\left(-b\cos\left(dx+c\right)^{2}+a+b\right)-2\,\log\left(-\cos\left(dx+c\right)\right)}{2\,(a+b)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(log(-b*cos(d*x + c)^2 + a + b) - 2*log(-cos(d*x + c)))/((a + b)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\tan\left(c+dx\right)}{a+b\sin^{2}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral(tan(c + d*x)/(a + b*sin(c + d*x)**2), x)
```

Giac [B] time = 1.1399, size = 149, normalized size = 3.47

$$\frac{\frac{\log\left(a-\frac{2a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{4b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)}{a+b}-\frac{2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a+b}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*(log(a - 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 4*b*(cos(d*x + c)
- 1)/(cos(d*x + c) + 1) + a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/(a +
 b) - 2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/(a + b))/d
```

3.445 $\int \frac{\cot(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=38

$$\frac{\log(\sin(c+dx))}{ad}-\frac{\log\left(a+b\sin^2(c+dx)\right)}{2ad}$$

```
[Out] Log[Sin[c + d*x]]/(a*d) - Log[a + b*Sin[c + d*x]^2]/(2*a*d)
```

Rubi [A] time = 0.0431496, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3194, 36, 29, 31}

$$\frac{\log(\sin(c+dx))}{ad}-\frac{\log\left(a+b\sin^2(c+dx)\right)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] Log[Sin[c + d*x]]/(a*d) - Log[a + b*Sin[c + d*x]^2]/(2*a*d)
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot(c+dx)}{a+b\sin^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)}\,dx, x, \sin^2(c+dx)\right)}{2d}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, \sin^2(c+dx)\right)}{2ad}-\frac{b\,\text{Subst}\left(\int \frac{1}{a+bx}\,dx, x, \sin^2(c+dx)\right)}{2ad}\\
&= \frac{\log(\sin(c+dx))}{ad}-\frac{\log\left(a+b\sin^2(c+dx)\right)}{2ad}
\end{aligned}$$

Mathematica [A] time = 0.0215795, size = 38, normalized size = 1.

$$\frac{\log(\sin(c+dx))}{ad}-\frac{\log\left(a+b\sin^2(c+dx)\right)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] Log[Sin[c + d*x]]/(a*d) - Log[a + b*Sin[c + d*x]^2]/(2*a*d)
```

Maple [A] time = 0.044, size = 37, normalized size = 1.

$$\frac{\ln(\sin(dx+c))}{da}-\frac{\ln\left(a+(\sin(dx+c))^2 b\right)}{2\,da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+sin(d*x+c)^2*b),x)
```

```
[Out] ln(sin(d*x+c))/a/d-1/2*ln(a+sin(d*x+c)^2*b)/d/a
```

Maxima [A] time = 1.01851, size = 50, normalized size = 1.32

$$-\frac{\frac{\log\left(b\sin(dx+c)^2+a\right)}{a}-\frac{\log\left(\sin(dx+c)^2\right)}{a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(log(b*sin(d*x + c)^2 + a)/a - log(sin(d*x + c)^2)/a)/d
```

Fricas [A] time = 1.82779, size = 96, normalized size = 2.53

$$-\frac{\log\left(-b\cos(dx+c)^2+a+b\right)-2\,\log\left(\frac{1}{2}\,\sin(dx+c)\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(log(-b*cos(d*x + c)^2 + a + b) - 2*log(1/2*sin(d*x + c)))/(a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cot(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral(cot(c + d*x)/(a + b*sin(c + d*x)**2), x)
```

Giac [A] time = 1.1071, size = 51, normalized size = 1.34

$$\frac{\frac{\log\left(\sin(dx+c)^2\right)}{a} - \frac{\log\left(\left|b\sin(dx+c)^2+a\right|\right)}{a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*(log(sin(d*x + c)^2)/a - log(abs(b*sin(d*x + c)^2 + a))/a)/d
```

3.446 $\int \frac{\cot^3(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=63

$$\frac{(a+b)\log\left(a+b\sin^2(c+dx)\right)}{2a^2d}-\frac{(a+b)\log(\sin(c+dx))}{a^2d}-\frac{\csc^2(c+dx)}{2ad}$$

```
[Out] -Csc[c + d*x]^2/(2*a*d) - ((a + b)*Log[Sin[c + d*x]])/(a^2*d) + ((a + b)*Lo
g[a + b*Sin[c + d*x]^2])/(2*a^2*d)
```

Rubi [A] time = 0.0749722, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3194, 77}

$$\frac{(a+b)\log\left(a+b\sin^2(c+dx)\right)}{2a^2d}-\frac{(a+b)\log(\sin(c+dx))}{a^2d}-\frac{\csc^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -Csc[c + d*x]^2/(2*a*d) - ((a + b)*Log[Sin[c + d*x]])/(a^2*d) + ((a + b)*Lo
g[a + b*Sin[c + d*x]^2])/(2*a^2*d)
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^3(c+dx)}{a+b\sin^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1-x}{x^2(a+bx)}\,dx,x,\sin^2(c+dx)\right)}{2d}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^2}+\frac{-a-b}{a^2x}+\frac{b(a+b)}{a^2(a+bx)}\right)dx,x,\sin^2(c+dx)\right)}{2d}\\
&= -\frac{\csc^2(c+dx)}{2ad}-\frac{(a+b)\log(\sin(c+dx))}{a^2d}+\frac{(a+b)\log\left(a+b\sin^2(c+dx)\right)}{2a^2d}
\end{aligned}$$

Mathematica [A] time = 0.141051, size = 50, normalized size = 0.79

$$-\frac{(a+b)\left(2\log(\sin(c+dx))-\log\left(a+b\sin^2(c+dx)\right)\right)+a\csc^2(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -(a*Csc[c + d*x]^2 + (a + b)*(2*Log[Sin[c + d*x]] - Log[a + b*Sin[c + d*x]^
2]))/(2*a^2*d)
```

Maple [B] time = 0.094, size = 161, normalized size = 2.6

$$\frac{1}{4\,da\,(-1+\cos(dx+c))}-\frac{\ln(-1+\cos(dx+c))}{2\,da}-\frac{\ln(-1+\cos(dx+c))\,b}{2\,a^2d}-\frac{1}{4\,da\,(1+\cos(dx+c))}-\frac{\ln(1+\cos(dx}{2\,da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+sin(d*x+c)^2*b),x)
```

```
[Out] 1/4/d/a/(-1+cos(d*x+c))-1/2/d/a*ln(-1+cos(d*x+c))-1/2/d/a^2*ln(-1+cos(d*x+c
))*b-1/4/a/d/(1+cos(d*x+c))-1/2/d/a*ln(1+cos(d*x+c))-1/2/d/a^2*ln(1+cos(d*x
+c))*b+1/2/d/a*ln(b*cos(d*x+c)^2-a-b)+1/2/d/a^2*ln(b*cos(d*x+c)^2-a-b)*b
```

Maxima [A] time = 0.986388, size = 76, normalized size = 1.21

$$\frac{\frac{(a+b)\log\left(b\sin(dx+c)^2+a\right)}{a^2}-\frac{(a+b)\log\left(\sin(dx+c)^2\right)}{a^2}-\frac{1}{a\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*((a + b)*log(b*sin(d*x + c)^2 + a)/a^2 - (a + b)*log(sin(d*x + c)^2)/a^
2 - 1/(a*sin(d*x + c)^2))/d
```

Fricas [A] time = 2.0216, size = 223, normalized size = 3.54

$$\frac{\left((a+b)\cos(dx+c)^2-a-b\right)\log\left(-b\cos(dx+c)^2+a+b\right)-2\left((a+b)\cos(dx+c)^2-a-b\right)\log\left(\frac{1}{2}\sin(dx+c)\right)+a}{2\left(a^2d\cos(dx+c)^2-a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(((a + b)*cos(d*x + c)^2 - a - b)*log(-b*cos(d*x + c)^2 + a + b) - 2*((
a + b)*cos(d*x + c)^2 - a - b)*log(1/2*sin(d*x + c)) + a)/(a^2*d*cos(d*x +
c)^2 - a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^3(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral(cot(c + d*x)**3/(a + b*sin(c + d*x)**2), x)
```

Giac [A] time = 1.18514, size = 146, normalized size = 2.32

$$\frac{\frac{\frac{\cos(dx+c)+1}{\cos(dx+c)-1}+\frac{\cos(dx+c)-1}{\cos(dx+c)+1}}{a}+\frac{4\,(a+b)\log\left(\left|-a\left(\frac{\cos(dx+c)+1}{\cos(dx+c)-1}+\frac{\cos(dx+c)-1}{\cos(dx+c)+1}\right)+2\,a+4\,b\right|\right)}{a^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/8*(((cos(d*x + c) + 1)/(cos(d*x + c) - 1) + (cos(d*x + c) - 1)/(cos(d*x +
 c) + 1))/a + 4*(a + b)*log(abs(-a*((cos(d*x + c) + 1)/(cos(d*x + c) - 1) +
 (cos(d*x + c) - 1)/(cos(d*x + c) + 1)) + 2*a + 4*b))/a^2)/d
```

3.447 $\int \frac{\cot^5(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=89

$$\frac{(2a+b)\csc^2(c+dx)}{2a^2d}-\frac{(a+b)^2\log\left(a+b\sin^2(c+dx)\right)}{2a^3d}+\frac{(a+b)^2\log(\sin(c+dx))}{a^3d}-\frac{\csc^4(c+dx)}{4ad}$$

```
[Out] ((2*a + b)*Csc[c + d*x]^2)/(2*a^2*d) - Csc[c + d*x]^4/(4*a*d) + ((a + b)^2*
Log[Sin[c + d*x]])/(a^3*d) - ((a + b)^2*Log[a + b*Sin[c + d*x]^2])/(2*a^3*d
)
```

Rubi [A] time = 0.0889625, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3194, 88}

$$\frac{(2a+b)\csc^2(c+dx)}{2a^2d}-\frac{(a+b)^2\log\left(a+b\sin^2(c+dx)\right)}{2a^3d}+\frac{(a+b)^2\log(\sin(c+dx))}{a^3d}-\frac{\csc^4(c+dx)}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] ((2*a + b)*Csc[c + d*x]^2)/(2*a^2*d) - Csc[c + d*x]^4/(4*a*d) + ((a + b)^2*
Log[Sin[c + d*x]])/(a^3*d) - ((a + b)^2*Log[a + b*Sin[c + d*x]^2])/(2*a^3*d
)
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^5(c+dx)}{a+b\sin^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{(1-x)^2}{x^3(a+bx)}\,dx, x, \sin^2(c+dx)\right)}{2d}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^3}+\frac{-2a-b}{a^2x^2}+\frac{(a+b)^2}{a^3x}-\frac{b(a+b)^2}{a^3(a+bx)}\right)dx, x, \sin^2(c+dx)\right)}{2d}\\
&= \frac{(2a+b)\csc^2(c+dx)}{2a^2d}-\frac{\csc^4(c+dx)}{4ad}+\frac{(a+b)^2\log(\sin(c+dx))}{a^3d}-\frac{(a+b)^2\log\left(a+b\sin^2(c+dx)\right)}{2a^3d}
\end{aligned}$$

Mathematica [A] time = 0.563815, size = 72, normalized size = 0.81

$$\frac{-a^2\csc^4(c+dx)+2a(2a+b)\csc^2(c+dx)+2(a+b)^2\left(2\log(\sin(c+dx))-\log\left(a+b\sin^2(c+dx)\right)\right)}{4a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (2*a*(2*a + b)*Csc[c + d*x]^2 - a^2*Csc[c + d*x]^4 + 2*(a + b)^2*(2*Log[Sin
[c + d*x]] - Log[a + b*Sin[c + d*x]^2]))/(4*a^3*d)
```

Maple [B] time = 0.099, size = 302, normalized size = 3.4

$$-\frac{1}{16da(-1+\cos(dx+c))^2}-\frac{7}{16da(-1+\cos(dx+c))}-\frac{b}{4a^2d(-1+\cos(dx+c))}+\frac{\ln(-1+\cos(dx+c))}{2da}+\frac{\ln(-1+c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5/(a+sin(d*x+c)^2*b),x)
```

```
[Out] -1/16/d/a/(-1+cos(d*x+c))^2-7/16/d/a/(-1+cos(d*x+c))-1/4/d/a^2/(-1+cos(d*x+
c))*b+1/2/d/a*ln(-1+cos(d*x+c))+1/d/a^2*ln(-1+cos(d*x+c))*b+1/2/d/a^3*ln(-1
+cos(d*x+c))*b^2-1/16/a/d/(1+cos(d*x+c))^2+7/16/a/d/(1+cos(d*x+c))+1/4/d/a^
2/(1+cos(d*x+c))*b+1/2/d/a*ln(1+cos(d*x+c))+1/d/a^2*ln(1+cos(d*x+c))*b+1/2/
d/a^3*ln(1+cos(d*x+c))*b^2-1/2/d/a*ln(b*cos(d*x+c)^2-a-b)-1/d/a^2*ln(b*cos(
d*x+c)^2-a-b)*b-1/2/d/a^3*ln(b*cos(d*x+c)^2-a-b)*b^2
```

Maxima [A] time = 0.998426, size = 124, normalized size = 1.39

$$-\frac{\frac{2\left(a^2+2ab+b^2\right)\log\left(b\sin(dx+c)^2+a\right)}{a^3}-\frac{2\left(a^2+2ab+b^2\right)\log\left(\sin(dx+c)^2\right)}{a^3}-\frac{2(2a+b)\sin(dx+c)^2-a}{a^2\sin(dx+c)^4}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*(a^2 + 2*a*b + b^2)*log(b*sin(d*x + c)^2 + a)/a^3 - 2*(a^2 + 2*a*b
+ b^2)*log(sin(d*x + c)^2)/a^3 - (2*(2*a + b)*sin(d*x + c)^2 - a)/(a^2*sin(
d*x + c)^4))/d
```

Fricas [B] time = 2.28817, size = 491, normalized size = 5.52

$$-\frac{2\left(2a^2+ab\right)\cos(dx+c)^2-3a^2-2ab+2\left(\left(a^2+2ab+b^2\right)\cos(dx+c)^4-2\left(a^2+2ab+b^2\right)\cos(dx+c)^2+a^2+2ab\right.}{4\left(a^3d\cos(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/4*(2*(2*a^2 + a*b)*cos(d*x + c)^2 - 3*a^2 - 2*a*b + 2*((a^2 + 2*a*b + b^
2)*cos(d*x + c)^4 - 2*(a^2 + 2*a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^
2)*log(-b*cos(d*x + c)^2 + a + b) - 4*((a^2 + 2*a*b + b^2)*cos(d*x + c)^4 -
 2*(a^2 + 2*a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2)*log(1/2*sin(d*x
+ c)))/(a^3*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^2 + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.22261, size = 277, normalized size = 3.11

$$-\frac{\frac{a\left(\frac{\cos(dx+c)+1}{\cos(dx+c)-1}+\frac{\cos(dx+c)-1}{\cos(dx+c)+1}\right)^2+12\,a\left(\frac{\cos(dx+c)+1}{\cos(dx+c)-1}+\frac{\cos(dx+c)-1}{\cos(dx+c)+1}\right)+8\,b\left(\frac{\cos(dx+c)+1}{\cos(dx+c)-1}+\frac{\cos(dx+c)-1}{\cos(dx+c)+1}\right)}{a^2}+\frac{32\left(a^2+2\,ab+b^2\right)\log\left(\left|-a\left(\frac{\cos(dx+c)+1}{\cos(dx+c)-1}+\frac{\cos(dx+c)-1}{\cos(dx+c)+1}\right)+2\right.\right.}{a^3}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/64*((a*((cos(d*x + c) + 1)/(cos(d*x + c) - 1) + (cos(d*x + c) - 1)/(cos(
d*x + c) + 1))^2 + 12*a*((cos(d*x + c) + 1)/(cos(d*x + c) - 1) + (cos(d*x +
 c) - 1)/(cos(d*x + c) + 1)) + 8*b*((cos(d*x + c) + 1)/(cos(d*x + c) - 1) +
 (cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/a^2 + 32*(a^2 + 2*a*b + b^2)*log(a
bs(-a*((cos(d*x + c) + 1)/(cos(d*x + c) - 1) + (cos(d*x + c) - 1)/(cos(d*x
+ c) + 1)) + 2*a + 4*b))/a^3)/d
```

3.448 $\int \frac{\cot^7(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=121

$$-\frac{\left(3a^2+3ab+b^2\right)\csc^2(c+dx)}{2a^3d}+\frac{(3a+b)\csc^4(c+dx)}{4a^2d}+\frac{(a+b)^3\log\left(a+b\sin^2(c+dx)\right)}{2a^4d}-\frac{(a+b)^3\log(\sin(c+dx))}{a^4d}-$$

```
[Out] -((3*a^2 + 3*a*b + b^2)*Csc[c + d*x]^2)/(2*a^3*d) + ((3*a + b)*Csc[c + d*x]
^4)/(4*a^2*d) - Csc[c + d*x]^6/(6*a*d) - ((a + b)^3*Log[Sin[c + d*x]])/(a^4
*d) + ((a + b)^3*Log[a + b*Sin[c + d*x]^2])/(2*a^4*d)
```

Rubi [A] time = 0.111973, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3194, 88}

$$-\frac{\left(3a^2+3ab+b^2\right)\csc^2(c+dx)}{2a^3d}+\frac{(3a+b)\csc^4(c+dx)}{4a^2d}+\frac{(a+b)^3\log\left(a+b\sin^2(c+dx)\right)}{2a^4d}-\frac{(a+b)^3\log(\sin(c+dx))}{a^4d}-$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -((3*a^2 + 3*a*b + b^2)*Csc[c + d*x]^2)/(2*a^3*d) + ((3*a + b)*Csc[c + d*x]
^4)/(4*a^2*d) - Csc[c + d*x]^6/(6*a*d) - ((a + b)^3*Log[Sin[c + d*x]])/(a^4
*d) + ((a + b)^3*Log[a + b*Sin[c + d*x]^2])/(2*a^4*d)
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^7(c+dx)}{a+b\sin^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{(1-x)^3}{x^4(a+bx)}\,dx,x,\sin^2(c+dx)\right)}{2d}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^4}+\frac{-3a-b}{a^2x^3}+\frac{3a^2+3ab+b^2}{a^3x^2}-\frac{(a+b)^3}{a^4x}+\frac{b(a+b)^3}{a^4(a+bx)}\right)dx,x,\sin^2(c+dx)\right)}{2d}\\ &= -\frac{\left(3a^2+3ab+b^2\right)\csc^2(c+dx)}{2a^3d}+\frac{(3a+b)\csc^4(c+dx)}{4a^2d}-\frac{\csc^6(c+dx)}{6ad}-\frac{(a+b)^3\log(\sin(c+\right.}{a^4d}\end{aligned}$$

Mathematica [A] time = 0.268362, size = 100, normalized size = 0.83

$$-\frac{6a\left(3a^2+3ab+b^2\right)\csc^2(c+dx)-3a^2(3a+b)\csc^4(c+dx)+2a^3\csc^6(c+dx)-6(a+b)^3\log\left(a+b\sin^2(c+dx)\right)+}{12a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -(6*a*(3*a^2 + 3*a*b + b^2)*Csc[c + d*x]^2 - 3*a^2*(3*a + b)*Csc[c + d*x]^4
 + 2*a^3*Csc[c + d*x]^6 + 12*(a + b)^3*Log[Sin[c + d*x]] - 6*(a + b)^3*Log[
a + b*Sin[c + d*x]^2])/(12*a^4*d)
```

Maple [B] time = 0.108, size = 489, normalized size = 4.

$$\frac{1}{48\,da\left(-1+\cos\left(dx+c\right)\right)^3}+\frac{5}{32\,da\left(-1+\cos\left(dx+c\right)\right)^2}+\frac{b}{16\,a^2d\left(-1+\cos\left(dx+c\right)\right)^2}+\frac{19}{32\,da\left(-1+\cos\left(dx+c\right)\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^7/(a+sin(d*x+c)^2*b),x)
```

```
[Out] 1/48/d/a/(-1+cos(d*x+c))^3+5/32/d/a/(-1+cos(d*x+c))^2+1/16/d/a^2/(-1+cos(d*
x+c))^2*b+19/32/d/a/(-1+cos(d*x+c))+11/16/d/a^2/(-1+cos(d*x+c))*b+1/4/d/a^3
/(-1+cos(d*x+c))*b^2-1/2/d/a*ln(-1+cos(d*x+c))-3/2/d/a^2*ln(-1+cos(d*x+c))*
b-3/2/d/a^3*ln(-1+cos(d*x+c))*b^2-1/2/d/a^4*ln(-1+cos(d*x+c))*b^3-1/48/d/a/
(1+cos(d*x+c))^3+5/32/a/d/(1+cos(d*x+c))^2+1/16/d/a^2/(1+cos(d*x+c))^2*b-19
/32/a/d/(1+cos(d*x+c))-11/16/d/a^2/(1+cos(d*x+c))*b-1/4/d/a^3/(1+cos(d*x+c)
)*b^2-1/2/d/a*ln(1+cos(d*x+c))-3/2/d/a^2*ln(1+cos(d*x+c))*b-3/2/d/a^3*ln(1+
cos(d*x+c))*b^2-1/2/d/a^4*ln(1+cos(d*x+c))*b^3+1/2/d/a*ln(b*cos(d*x+c)^2-a-
b)+3/2/d/a^2*ln(b*cos(d*x+c)^2-a-b)*b+3/2/d/a^3*ln(b*cos(d*x+c)^2-a-b)*b^2+
1/2/d/a^4*ln(b*cos(d*x+c)^2-a-b)*b^3
```

Maxima [A] time = 1.01059, size = 185, normalized size = 1.53

$$\frac{\frac{6\left(a^3+3a^2b+3ab^2+b^3\right)\log\left(b\sin(dx+c)^2+a\right)}{a^4}-\frac{6\left(a^3+3a^2b+3ab^2+b^3\right)\log\left(\sin(dx+c)^2\right)}{a^4}-\frac{6\left(3a^2+3ab+b^2\right)\sin(dx+c)^4-3\left(3a^2+ab\right)\sin(dx+c)^2+2a^2}{a^3\sin(dx+c)^6}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/12*(6*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*log(b*sin(d*x + c)^2 + a)/a^4 - 6*(
a^3 + 3*a^2*b + 3*a*b^2 + b^3)*log(sin(d*x + c)^2)/a^4 - (6*(3*a^2 + 3*a*b
+ b^2)*sin(d*x + c)^4 - 3*(3*a^2 + a*b)*sin(d*x + c)^2 + 2*a^2)/(a^3*sin(d*
x + c)^6))/d
```

Fricas [B] time = 2.83991, size = 861, normalized size = 7.12

$$6\left(3a^3+3a^2b+ab^2\right)\cos\left(dx+c\right)^4+11a^3+15a^2b+6ab^2-3\left(9a^3+11a^2b+4ab^2\right)\cos\left(dx+c\right)^2+6\left(\left(a^3+3a^2b+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/12*(6*(3*a^3 + 3*a^2*b + a*b^2)*cos(d*x + c)^4 + 11*a^3 + 15*a^2*b + 6*a*
b^2 - 3*(9*a^3 + 11*a^2*b + 4*a*b^2)*cos(d*x + c)^2 + 6*((a^3 + 3*a^2*b + 3
*a*b^2 + b^3)*cos(d*x + c)^6 - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(d*x +
c)^4 - a^3 - 3*a^2*b - 3*a*b^2 - b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*co
s(d*x + c)^2)*log(-b*cos(d*x + c)^2 + a + b) - 12*((a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*cos(d*x + c)^6 - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(d*x + c)^4 -
 a^3 - 3*a^2*b - 3*a*b^2 - b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(d*x
+ c)^2)*log(1/2*sin(d*x + c)))/(a^4*d*cos(d*x + c)^6 - 3*a^4*d*cos(d*x + c)
^4 + 3*a^4*d*cos(d*x + c)^2 - a^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**7/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.25438, size = 477, normalized size = 3.94

$$\frac{\frac{a^2\left(\frac{\cos(dx+c)+1}{\cos(dx+c)-1}+\frac{\cos(dx+c)-1}{\cos(dx+c)+1}\right)^3+12\,a^2\left(\frac{\cos(dx+c)+1}{\cos(dx+c)-1}+\frac{\cos(dx+c)-1}{\cos(dx+c)+1}\right)^2+6\,ab\left(\frac{\cos(dx+c)+1}{\cos(dx+c)-1}+\frac{\cos(dx+c)-1}{\cos(dx+c)+1}\right)^2+84\,a^2\left(\frac{\cos(dx+c)+1}{\cos(dx+c)-1}+\frac{\cos(dx+c)-1}{\cos(dx+c)+1}\right)+120\,ab\left(\frac{\cos(dx+c)+1}{\cos(dx+c)-1}+\frac{\cos(}{\cos(}\right.}{a^3}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/384*((a^2*((cos(d*x + c) + 1)/(cos(d*x + c) - 1) + (cos(d*x + c) - 1)/(co
s(d*x + c) + 1))^3 + 12*a^2*((cos(d*x + c) + 1)/(cos(d*x + c) - 1) + (cos(d
*x + c) - 1)/(cos(d*x + c) + 1))^2 + 6*a*b*((cos(d*x + c) + 1)/(cos(d*x + c
) - 1) + (cos(d*x + c) - 1)/(cos(d*x + c) + 1))^2 + 84*a^2*((cos(d*x + c) +
 1)/(cos(d*x + c) - 1) + (cos(d*x + c) - 1)/(cos(d*x + c) + 1)) + 120*a*b*(
(cos(d*x + c) + 1)/(cos(d*x + c) - 1) + (cos(d*x + c) - 1)/(cos(d*x + c) +
1)) + 48*b^2*((cos(d*x + c) + 1)/(cos(d*x + c) - 1) + (cos(d*x + c) - 1)/(c
os(d*x + c) + 1)))/a^3 + 192*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*log(abs(-a*((c
os(d*x + c) + 1)/(cos(d*x + c) - 1) + (cos(d*x + c) - 1)/(cos(d*x + c) + 1)
) + 2*a + 4*b))/a^4)/d
```

3.449 $\int \frac{\tan^8(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=120

$$\frac{a^2\tan^3(c+dx)}{3d(a+b)^3}+\frac{a^{7/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{d(a+b)^{9/2}}-\frac{a^3\tan(c+dx)}{d(a+b)^4}+\frac{\tan^7(c+dx)}{7d(a+b)}-\frac{a\tan^5(c+dx)}{5d(a+b)^2}$$

```
[Out] (a^(7/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/((a + b)^(9/2)*d) - (a
^3*Tan[c + d*x])/((a + b)^4*d) + (a^2*Tan[c + d*x]^3)/(3*(a + b)^3*d) - (a*
Tan[c + d*x]^5)/(5*(a + b)^2*d) + Tan[c + d*x]^7/(7*(a + b)*d)
```

Rubi [A] time = 0.12543, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3195, 302, 205}

$$\frac{a^2\tan^3(c+dx)}{3d(a+b)^3}+\frac{a^{7/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{d(a+b)^{9/2}}-\frac{a^3\tan(c+dx)}{d(a+b)^4}+\frac{\tan^7(c+dx)}{7d(a+b)}-\frac{a\tan^5(c+dx)}{5d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^8/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (a^(7/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/((a + b)^(9/2)*d) - (a
^3*Tan[c + d*x])/((a + b)^4*d) + (a^2*Tan[c + d*x]^3)/(3*(a + b)^3*d) - (a*
Tan[c + d*x]^5)/(5*(a + b)^2*d) + Tan[c + d*x]^7/(7*(a + b)*d)
```

Rule 3195

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*((d_.)*tan[(e_.) + (f_.)
*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[f
f/f, Subst[Int[((d*ff*x)^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(p + 1)
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m}, x] && IntegerQ[
p]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\tan^8(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^8}{a+(a+b)x^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{a^3}{(a+b)^4} + \frac{a^2x^2}{(a+b)^3} - \frac{ax^4}{(a+b)^2} + \frac{x^6}{a+b} + \frac{a^4}{(a+b)^4(a+(a+b)x^2)}\right)dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{a^3\tan(c+dx)}{(a+b)^4d} + \frac{a^2\tan^3(c+dx)}{3(a+b)^3d} - \frac{a\tan^5(c+dx)}{5(a+b)^2d} + \frac{\tan^7(c+dx)}{7(a+b)d} + \frac{a^4\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\right.}{(a+b)}$$

$$= \frac{a^{7/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{9/2}d} - \frac{a^3\tan(c+dx)}{(a+b)^4d} + \frac{a^2\tan^3(c+dx)}{3(a+b)^3d} - \frac{a\tan^5(c+dx)}{5(a+b)^2d} + \frac{\tan^7(c+d}{7(a+b)d}$$

Mathematica [A] time = 2.40066, size = 147, normalized size = 1.22

$$\frac{\tan(c+dx)\left(\left(254a^2b+122a^3+177ab^2+45b^3\right)\sec^2(c+dx)-122a^2b-176a^3-66ab^2+15(a+b)^3\sec^6(c+dx)-3(a+\right.}{105d(a+b)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^8/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (a^(7/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/((a + b)^(9/2)*d) + ((
-176*a^3 - 122*a^2*b - 66*a*b^2 - 15*b^3 + (122*a^3 + 254*a^2*b + 177*a*b^2
 + 45*b^3)*Sec[c + d*x]^2 - 3*(a + b)^2*(22*a + 15*b)*Sec[c + d*x]^4 + 15*(
a + b)^3*Sec[c + d*x]^6)*Tan[c + d*x])/(105*(a + b)^4*d)
```

Maple [B] time = 0.118, size = 252, normalized size = 2.1

$$\frac{(\tan(dx+c))^7a^3}{7d(a+b)^4} + \frac{3(\tan(dx+c))^7a^2b}{7d(a+b)^4} + \frac{3ab^2(\tan(dx+c))^7}{7d(a+b)^4} + \frac{b^3(\tan(dx+c))^7}{7d(a+b)^4} - \frac{(\tan(dx+c))^5a^3}{5d(a+b)^4} - \frac{2(\tan(dx}{5d(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^8/(a+sin(d*x+c)^2*b),x)
```

```
[Out] 1/7/d/(a+b)^4*tan(d*x+c)^7*a^3+3/7/d/(a+b)^4*tan(d*x+c)^7*a^2*b+3/7/d/(a+b)
^4*a*b^2*tan(d*x+c)^7+1/7/d/(a+b)^4*b^3*tan(d*x+c)^7-1/5/d/(a+b)^4*tan(d*x+
c)^5*a^3-2/5/d/(a+b)^4*tan(d*x+c)^5*a^2*b-1/5/d/(a+b)^4*tan(d*x+c)^5*a*b^2+
1/3/d/(a+b)^4*tan(d*x+c)^3*a^3+1/3/d/(a+b)^4*tan(d*x+c)^3*a^2*b-a^3*tan(d*x
+c)/(a+b)^4/d+1/d*a^4/(a+b)^4/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a
+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^8/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.28648, size = 1453, normalized size = 12.11

$$\left[\frac{105\,a^3\sqrt{-\frac{a}{a+b}}\cos\left(dx+c\right)^7\log\left(\frac{\left(8\,a^2+8\,ab+b^2\right)\cos(dx+c)^4-2\left(4\,a^2+5\,ab+b^2\right)\cos(dx+c)^2-4\left(\left(2\,a^2+3\,ab+b^2\right)\cos(dx+c)^3-\left(a^2+2\,ab+b^2\right)\cos(dx+c\right.}{b^2\cos(dx+c)^4-2\left(ab+b^2\right)\cos(dx+c)^2+a^2+2\,ab+b^2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^8/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/420*(105*a^3*sqrt(-a/(a + b))*cos(d*x + c)^7*log(((8*a^2 + 8*a*b + b^2)*
cos(d*x + c)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 - 4*((2*a^2 + 3*a*b
 + b^2)*cos(d*x + c)^3 - (a^2 + 2*a*b + b^2)*cos(d*x + c))*sqrt(-a/(a + b))
*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(
d*x + c)^2 + a^2 + 2*a*b + b^2)) - 4*((176*a^3 + 122*a^2*b + 66*a*b^2 + 15*
b^3)*cos(d*x + c)^6 - (122*a^3 + 254*a^2*b + 177*a*b^2 + 45*b^3)*cos(d*x +
c)^4 - 15*a^3 - 45*a^2*b - 45*a*b^2 - 15*b^3 + 3*(22*a^3 + 59*a^2*b + 52*a*
b^2 + 15*b^3)*cos(d*x + c)^2)*sin(d*x + c))/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4
*a*b^3 + b^4)*d*cos(d*x + c)^7), -1/210*(105*a^3*sqrt(a/(a + b))*arctan(1/2
*((2*a + b)*cos(d*x + c)^2 - a - b)*sqrt(a/(a + b))/(a*cos(d*x + c)*sin(d*x
 + c)))*cos(d*x + c)^7 + 2*((176*a^3 + 122*a^2*b + 66*a*b^2 + 15*b^3)*cos(d
*x + c)^6 - (122*a^3 + 254*a^2*b + 177*a*b^2 + 45*b^3)*cos(d*x + c)^4 - 15*
a^3 - 45*a^2*b - 45*a*b^2 - 15*b^3 + 3*(22*a^3 + 59*a^2*b + 52*a*b^2 + 15*b
^3)*cos(d*x + c)^2)*sin(d*x + c))/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b
^4)*d*cos(d*x + c)^7)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**8/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 9.94162, size = 637, normalized size = 5.31

$$\frac{105\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)a^4}{\left(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4\right)\sqrt{a^2+ab}}+\frac{15\,a^6\tan(dx+c)^7+90\,a^5b\tan(dx+c)^7+225\,a^4b^2\tan(dx+c)^7+300\,a^3b^3\tan(dx+c)^7+22}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^8/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/105*(105*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x
 + c) + b*tan(d*x + c))/sqrt(a^2 + a*b)))*a^4/((a^4 + 4*a^3*b + 6*a^2*b^2 +
```

```
 4*a*b^3 + b^4)*sqrt(a^2 + a*b)) + (15*a^6*tan(d*x + c)^7 + 90*a^5*b*tan(d*
x + c)^7 + 225*a^4*b^2*tan(d*x + c)^7 + 300*a^3*b^3*tan(d*x + c)^7 + 225*a^
2*b^4*tan(d*x + c)^7 + 90*a*b^5*tan(d*x + c)^7 + 15*b^6*tan(d*x + c)^7 - 21
*a^6*tan(d*x + c)^5 - 105*a^5*b*tan(d*x + c)^5 - 210*a^4*b^2*tan(d*x + c)^5
 - 210*a^3*b^3*tan(d*x + c)^5 - 105*a^2*b^4*tan(d*x + c)^5 - 21*a*b^5*tan(d
*x + c)^5 + 35*a^6*tan(d*x + c)^3 + 140*a^5*b*tan(d*x + c)^3 + 210*a^4*b^2*
tan(d*x + c)^3 + 140*a^3*b^3*tan(d*x + c)^3 + 35*a^2*b^4*tan(d*x + c)^3 - 1
05*a^6*tan(d*x + c) - 315*a^5*b*tan(d*x + c) - 315*a^4*b^2*tan(d*x + c) - 1
05*a^3*b^3*tan(d*x + c))/(a^7 + 7*a^6*b + 21*a^5*b^2 + 35*a^4*b^3 + 35*a^3*
b^4 + 21*a^2*b^5 + 7*a*b^6 + b^7))/d
```

3.450 $\int \frac{\tan^6(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=97

$$-\frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{d(a+b)^{7/2}}+\frac{a^2\tan(c+dx)}{d(a+b)^3}+\frac{\tan^5(c+dx)}{5d(a+b)}-\frac{a\tan^3(c+dx)}{3d(a+b)^2}$$

```
[Out] -((a^(5/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/((a + b)^(7/2)*d)) +
 (a^2*Tan[c + d*x])/((a + b)^3*d) - (a*Tan[c + d*x]^3)/(3*(a + b)^2*d) + Ta
n[c + d*x]^5/(5*(a + b)*d)
```

Rubi [A] time = 0.106833, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3195, 302, 205}

$$-\frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{d(a+b)^{7/2}}+\frac{a^2\tan(c+dx)}{d(a+b)^3}+\frac{\tan^5(c+dx)}{5d(a+b)}-\frac{a\tan^3(c+dx)}{3d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^6/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -((a^(5/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/((a + b)^(7/2)*d)) +
 (a^2*Tan[c + d*x])/((a + b)^3*d) - (a*Tan[c + d*x]^3)/(3*(a + b)^2*d) + Ta
n[c + d*x]^5/(5*(a + b)*d)
```

Rule 3195

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*((d_.)*tan[(e_.) + (f_.)
*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[f
f/f, Subst[Int[((d*ff*x)^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(p + 1)
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m}, x] && IntegerQ[
p]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\tan^6(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{a+(a+b)x^2}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^2}{(a+b)^3} - \frac{ax^2}{(a+b)^2} + \frac{x^4}{a+b} - \frac{a^3}{(a+b)^3(a+(a+b)x^2)}\right) dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{a^2\tan(c+dx)}{(a+b)^3 d} - \frac{a\tan^3(c+dx)}{3(a+b)^2 d} + \frac{\tan^5(c+dx)}{5(a+b)d} - \frac{a^3\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \tan(c+dx)\right)}{(a+b)^3 d}$$
$$= -\frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{7/2}d} + \frac{a^2\tan(c+dx)}{(a+b)^3 d} - \frac{a\tan^3(c+dx)}{3(a+b)^2 d} + \frac{\tan^5(c+dx)}{5(a+b)d}$$

Mathematica [A] time = 0.784555, size = 111, normalized size = 1.14

$$\frac{\sqrt{a+b}\tan(c+dx)\left(-\left(11a^2+17ab+6b^2\right)\sec^2(c+dx)+23a^2+3(a+b)^2\sec^4(c+dx)+11ab+3b^2\right)-15a^{5/2}\tan^{-1}\left(\sqrt{a\right.}}{15d(a+b)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^6/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (-15*a^(5/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]] + Sqrt[a + b]*(23*a
^2 + 11*a*b + 3*b^2 - (11*a^2 + 17*a*b + 6*b^2)*Sec[c + d*x]^2 + 3*(a + b)^
2*Sec[c + d*x]^4)*Tan[c + d*x])/(15*(a + b)^(7/2)*d)
```

Maple [A] time = 0.121, size = 161, normalized size = 1.7

$$\frac{(\tan(dx+c))^5 a^2}{5d(a+b)^3} + \frac{2(\tan(dx+c))^5 ab}{5d(a+b)^3} + \frac{(\tan(dx+c))^5 b^2}{5d(a+b)^3} - \frac{a^2(\tan(dx+c))^3}{3d(a+b)^3} - \frac{(\tan(dx+c))^3 ab}{3d(a+b)^3} + \frac{a^2\tan(dx+c)}{d(a+b)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^6/(a+sin(d*x+c)^2*b),x)
```

```
[Out] 1/5/d/(a+b)^3*tan(d*x+c)^5*a^2+2/5/d/(a+b)^3*tan(d*x+c)^5*a*b+1/5/d/(a+b)^3
*tan(d*x+c)^5*b^2-1/3*a^2*tan(d*x+c)^3/(a+b)^3/d-1/3/d/(a+b)^3*tan(d*x+c)^3
*a*b+a^2*tan(d*x+c)/(a+b)^3/d-1/d*a^3/(a+b)^3/(a*(a+b))^(1/2)*arctan((a+b)*
tan(d*x+c)/(a*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.07295, size = 1122, normalized size = 11.57

$$\left[\frac{15\,a^2\sqrt{-\frac{a}{a+b}}\cos\left(dx+c\right)^5\log\left(\frac{\left(8\,a^2+8\,ab+b^2\right)\cos(dx+c)^4-2\left(4\,a^2+5\,ab+b^2\right)\cos(dx+c)^2+4\left(\left(2\,a^2+3\,ab+b^2\right)\cos(dx+c)^3-\left(a^2+2\,ab+b^2\right)\cos(dx+c)\right.}{b^2\cos(dx+c)^4-2\left(ab+b^2\right)\cos(dx+c)^2+a^2+2\,ab+b^2}\right.}{60\left(a^3+3\,a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/60*(15*a^2*sqrt(-a/(a + b))*cos(d*x + c)^5*log(((8*a^2 + 8*a*b + b^2)*co
s(d*x + c)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a^2 + 3*a*b +
 b^2)*cos(d*x + c)^3 - (a^2 + 2*a*b + b^2)*cos(d*x + c))*sqrt(-a/(a + b))*s
in(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*
x + c)^2 + a^2 + 2*a*b + b^2)) + 4*((23*a^2 + 11*a*b + 3*b^2)*cos(d*x + c)^
4 - (11*a^2 + 17*a*b + 6*b^2)*cos(d*x + c)^2 + 3*a^2 + 6*a*b + 3*b^2)*sin(d
*x + c))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cos(d*x + c)^5), 1/30*(15*a^2*s
qrt(a/(a + b))*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)*sqrt(a/(a + b)
)/(a*cos(d*x + c)*sin(d*x + c)))*cos(d*x + c)^5 + 2*((23*a^2 + 11*a*b + 3*b
^2)*cos(d*x + c)^4 - (11*a^2 + 17*a*b + 6*b^2)*cos(d*x + c)^2 + 3*a^2 + 6*a
*b + 3*b^2)*sin(d*x + c))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cos(d*x + c)^5
)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**6/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 4.58227, size = 400, normalized size = 4.12

$$-\frac{\frac{15\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)a^3}{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\sqrt{a^2+ab}}-\frac{3\,a^4\tan(dx+c)^5+12\,a^3b\tan(dx+c)^5+18\,a^2b^2\tan(dx+c)^5+12\,ab^3\tan(dx+c)^5+3\,b^4\tan\ldots}{\ldots}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/15*(15*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x
+ c) + b*tan(d*x + c))/sqrt(a^2 + a*b)))*a^3/((a^3 + 3*a^2*b + 3*a*b^2 + b^
3)*sqrt(a^2 + a*b)) - (3*a^4*tan(d*x + c)^5 + 12*a^3*b*tan(d*x + c)^5 + 18*
a^2*b^2*tan(d*x + c)^5 + 12*a*b^3*tan(d*x + c)^5 + 3*b^4*tan(d*x + c)^5 - 5
*a^4*tan(d*x + c)^3 - 15*a^3*b*tan(d*x + c)^3 - 15*a^2*b^2*tan(d*x + c)^3 -
 5*a*b^3*tan(d*x + c)^3 + 15*a^4*tan(d*x + c) + 30*a^3*b*tan(d*x + c) + 15*
a^2*b^2*tan(d*x + c))/(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 +
b^5))/d
```

3.451 $\int \frac{\tan^4(c+dx)}{a+b\sin^2(c+dx)} dx$

Optimal. Leaf size=74

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{d(a+b)^{5/2}}+\frac{\tan^3(c+dx)}{3d(a+b)}-\frac{a\tan(c+dx)}{d(a+b)^2}$$

```
[Out] (a^(3/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/((a + b)^(5/2)*d) - (a
*Tan[c + d*x])/((a + b)^2*d) + Tan[c + d*x]^3/(3*(a + b)*d)
```

Rubi [A] time = 0.0976003, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3195, 302, 205}

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{d(a+b)^{5/2}}+\frac{\tan^3(c+dx)}{3d(a+b)}-\frac{a\tan(c+dx)}{d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (a^(3/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/((a + b)^(5/2)*d) - (a
*Tan[c + d*x])/((a + b)^2*d) + Tan[c + d*x]^3/(3*(a + b)*d)
```

Rule 3195

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*((d_.)*tan[(e_.) + (f_.)
*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[f
f/f, Subst[Int[((d*ff*x)^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(p + 1)
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m}, x] && IntegerQ[
p]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{a+(a+b)x^2}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{a}{(a+b)^2}+\frac{x^2}{a+b}+\frac{a^2}{(a+b)^2\left(a+(a+b)x^2\right)}\right)dx, x, \tan(c+dx)\right)}{d}$$
$$= -\frac{a\tan(c+dx)}{(a+b)^2d}+\frac{\tan^3(c+dx)}{3(a+b)d}+\frac{a^2\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \tan(c+dx)\right)}{(a+b)^2d}$$
$$= \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{5/2}d}-\frac{a\tan(c+dx)}{(a+b)^2d}+\frac{\tan^3(c+dx)}{3(a+b)d}$$

Mathematica [A] time = 0.309912, size = 75, normalized size = 1.01

$$\frac{3a^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)+\sqrt{a+b}\tan(c+dx)\left((a+b)\sec^2(c+dx)-4a-b\right)}{3d(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (3*a^(3/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]] + Sqrt[a + b]*(-4*a -
 b + (a + b)*Sec[c + d*x]^2)*Tan[c + d*x])/(3*(a + b)^(5/2)*d)
```

Maple [A] time = 0.109, size = 94, normalized size = 1.3

$$\frac{a(\tan(dx+c))^3}{3(a+b)^2d}+\frac{(\tan(dx+c))^3b}{3(a+b)^2d}-\frac{a\tan(dx+c)}{(a+b)^2d}+\frac{a^2}{(a+b)^2d}\arctan\left((a+b)\tan(dx+c)\frac{1}{\sqrt{a(a+b)}}\right)\frac{1}{\sqrt{a(a+b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+sin(d*x+c)^2*b),x)
```

```
[Out] 1/3*a*tan(d*x+c)^3/(a+b)^2/d+1/3/d/(a+b)^2*tan(d*x+c)^3*b-a*tan(d*x+c)/(a+b
)^2/d+1/d*a^2/(a+b)^2/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/
2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.91128, size = 879, normalized size = 11.88

$$\left[\frac{3\,a\sqrt{-\frac{a}{a+b}}\cos\left(dx+c\right)^{3}\log\left(\frac{\left(8\,a^{2}+8\,ab+b^{2}\right)\cos(dx+c)^{4}-2\left(4\,a^{2}+5\,ab+b^{2}\right)\cos(dx+c)^{2}-4\left(\left(2\,a^{2}+3\,ab+b^{2}\right)\cos(dx+c)^{3}-\left(a^{2}+2\,ab+b^{2}\right)\cos(dx+c)\right)\sqrt{-\frac{a}{a+b}}}{b^{2}\cos(dx+c)^{4}-2\left(ab+b^{2}\right)\cos(dx+c)^{2}+a^{2}+2\,ab+b^{2}}\right.}{12\left(a^{2}+2\,ab+b^{2}\right)d\cos\left(dx+c\right)^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*a*sqrt(-a/(a + b))*cos(d*x + c)^3*log(((8*a^2 + 8*a*b + b^2)*cos(d
*x + c)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 - 4*((2*a^2 + 3*a*b + b^
2)*cos(d*x + c)^3 - (a^2 + 2*a*b + b^2)*cos(d*x + c))*sqrt(-a/(a + b))*sin(
d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x +
 c)^2 + a^2 + 2*a*b + b^2)) - 4*((4*a + b)*cos(d*x + c)^2 - a - b)*sin(d*x
+ c))/((a^2 + 2*a*b + b^2)*d*cos(d*x + c)^3), -1/6*(3*a*sqrt(a/(a + b))*arc
tan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)*sqrt(a/(a + b))/(a*cos(d*x + c)*
sin(d*x + c)))*cos(d*x + c)^3 + 2*((4*a + b)*cos(d*x + c)^2 - a - b)*sin(d*
x + c))/((a^2 + 2*a*b + b^2)*d*cos(d*x + c)^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^{4}\left(c+dx\right)}{a+b\sin^{2}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral(tan(c + d*x)**4/(a + b*sin(c + d*x)**2), x)
```

Giac [B] time = 2.18947, size = 221, normalized size = 2.99

$$\frac{\frac{3\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^{2}+ab}}\right)\right)a^{2}}{\left(a^{2}+2\,ab+b^{2}\right)\sqrt{a^{2}+ab}}+\frac{a^{2}\tan(dx+c)^{3}+2\,ab\tan(dx+c)^{3}+b^{2}\tan(dx+c)^{3}-3\,a^{2}\tan(dx+c)-3\,ab\tan(dx+c)}{a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c
) + b*tan(d*x + c))/sqrt(a^2 + a*b)))*a^2/((a^2 + 2*a*b + b^2)*sqrt(a^2 + a
*b)) + (a^2*tan(d*x + c)^3 + 2*a*b*tan(d*x + c)^3 + b^2*tan(d*x + c)^3 - 3*
a^2*tan(d*x + c) - 3*a*b*tan(d*x + c))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3))/d
```

3.452 $\int \frac{\tan^2(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=53

$$\frac{\tan(c+dx)}{d(a+b)}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{d(a+b)^{3/2}}$$

```
[Out] -((Sqrt[a]*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/((a + b)^(3/2)*d)) +
 Tan[c + d*x]/((a + b)*d)
```

Rubi [A] time = 0.0758715, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3195, 321, 205}

$$\frac{\tan(c+dx)}{d(a+b)}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{d(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -((Sqrt[a]*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/((a + b)^(3/2)*d)) +
 Tan[c + d*x]/((a + b)*d)
```

Rule 3195

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*((d_.)*tan[(e_.) + (f_.)
*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[f
f/f, Subst[Int[((d*ff*x)^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(p + 1)
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m}, x] && IntegerQ[
p]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\tan^2(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{a+(a+b)x^2}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\tan(c+dx)}{(a+b)d} - \frac{a\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \tan(c+dx)\right)}{(a+b)d}$$
$$= -\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{3/2}d} + \frac{\tan(c+dx)}{(a+b)d}$$

Mathematica [A] time = 0.129393, size = 53, normalized size = 1.

$$\frac{\tan(c+dx)}{d(a+b)} - \frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{d(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -((Sqrt[a]*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/((a + b)^(3/2)*d)) +
 Tan[c + d*x]/((a + b)*d)
```

Maple [A] time = 0.104, size = 53, normalized size = 1.

$$\frac{\tan(dx+c)}{(a+b)\,d} - \frac{a}{(a+b)\,d}\arctan\left((a+b)\tan(dx+c)\frac{1}{\sqrt{a(a+b)}}\right)\frac{1}{\sqrt{a(a+b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+sin(d*x+c)^2*b),x)
```

```
[Out] tan(d*x+c)/(a+b)/d-1/d/(a+b)*a/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(
a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.87778, size = 728, normalized size = 13.74

$$\left[\frac{\sqrt{-\frac{a}{a+b}}\cos(dx+c)\log\left(\frac{\left(8a^2+8ab+b^2\right)\cos(dx+c)^4-2\left(4a^2+5ab+b^2\right)\cos(dx+c)^2+4\left(\left(2a^2+3ab+b^2\right)\cos(dx+c)^3-\left(a^2+2ab+b^2\right)\cos(dx+c)\right)\sqrt{-\frac{a}{a+b}}\sin(}{b^2\cos(dx+c)^4-2\left(ab+b^2\right)\cos(dx+c)^2+a^2+2ab+b^2}\right.}{4(a+b)d\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(-a/(a + b))*cos(d*x + c)*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)
^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a^2 + 3*a*b + b^2)*cos(
d*x + c)^3 - (a^2 + 2*a*b + b^2)*cos(d*x + c))*sqrt(-a/(a + b))*sin(d*x + c
) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 +
 a^2 + 2*a*b + b^2)) + 4*sin(d*x + c))/((a + b)*d*cos(d*x + c)), 1/2*(sqrt(
a/(a + b))*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)*sqrt(a/(a + b))/(a
*cos(d*x + c)*sin(d*x + c)))*cos(d*x + c) + 2*sin(d*x + c))/((a + b)*d*cos(
d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^2(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral(tan(c + d*x)**2/(a + b*sin(c + d*x)**2), x)
```

Giac [A] time = 1.35371, size = 116, normalized size = 2.19

$$-\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)a}{\sqrt{a^2+ab}(a+b)}-\frac{\tan(dx+c)}{a+b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -((pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c) + b
*tan(d*x + c))/sqrt(a^2 + a*b)))*a/(sqrt(a^2 + a*b)*(a + b)) - tan(d*x + c)
/(a + b))/d
```

3.453 $\int \frac{\cot^2(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=52

$$-\frac{\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{\cot(c+dx)}{ad}$$

```
[Out] -((Sqrt[a + b]*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a^(3/2)*d)) - C
ot[c + d*x]/(a*d)
```

Rubi [A] time = 0.0675979, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3195, 325, 205}

$$-\frac{\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{\cot(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -((Sqrt[a + b]*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a^(3/2)*d)) - C
ot[c + d*x]/(a*d)
```

Rule 3195

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*((d_.)*tan[(e_.) + (f_.)
*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[f
f/f, Subst[Int[((d*ff*x)^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(p + 1)
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m}, x] && IntegerQ[
p]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(a+(a+b)x^2)}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= -\frac{\cot(c+dx)}{ad} - \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \tan(c+dx)\right)}{ad}$$
$$= -\frac{\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{\cot(c+dx)}{ad}$$

Mathematica [A] time = 0.182813, size = 52, normalized size = 1.

$$\frac{-\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right) - \sqrt{a}\cot(c+dx)}{a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (-(Sqrt[a + b]*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]]) - Sqrt[a]*Cot[c
+ d*x])/(a^(3/2)*d)
```

Maple [A] time = 0.11, size = 82, normalized size = 1.6

$$-\frac{1}{da\tan(dx+c)} - \frac{1}{d}\arctan\left((a+b)\tan(dx+c)\frac{1}{\sqrt{a(a+b)}}\right)\frac{1}{\sqrt{a(a+b)}} - \frac{b}{da}\arctan\left((a+b)\tan(dx+c)\frac{1}{\sqrt{a(a+b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+sin(d*x+c)^2*b),x)
```

```
[Out] -1/d/a/tan(d*x+c)-1/d/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/
2))-1/d/a/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.77005, size = 698, normalized size = 13.42

$$\left[\frac{\sqrt{-\frac{a+b}{a}}\log\left(\frac{\left(8a^2+8ab+b^2\right)\cos(dx+c)^4-2\left(4a^2+5ab+b^2\right)\cos(dx+c)^2+4\left(\left(2a^2+ab\right)\cos(dx+c)^3-\left(a^2+ab\right)\cos(dx+c)\right)\sqrt{-\frac{a+b}{a}}\sin(dx+c)+a^2+2ab+b^2}{b^2\cos(dx+c)^4-2\left(ab+b^2\right)\cos(dx+c)^2+a^2+2ab+b^2}\right)\text{s}}{4ad\sin(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(-(a + b)/a)*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4*a^2
 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a^2 + a*b)*cos(d*x + c)^3 - (a^2 + a
*b)*cos(d*x + c))*sqrt(-(a + b)/a)*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*c
os(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2))*sin(d*x
+ c) - 4*cos(d*x + c))/(a*d*sin(d*x + c)), 1/2*(sqrt((a + b)/a)*arctan(1/2*
((2*a + b)*cos(d*x + c)^2 - a - b)*sqrt((a + b)/a)/((a + b)*cos(d*x + c)*si
n(d*x + c)))*sin(d*x + c) - 2*cos(d*x + c))/(a*d*sin(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^2(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral(cot(c + d*x)**2/(a + b*sin(c + d*x)**2), x)
```

Giac [A] time = 1.13285, size = 115, normalized size = 2.21

$$-\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)(a+b)}{\sqrt{a^2+ab}a}+\frac{1}{a\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -((pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c) + b
*tan(d*x + c))/sqrt(a^2 + a*b)))*(a + b)/(sqrt(a^2 + a*b)*a) + 1/(a*tan(d*x
 + c)))/d
```

3.454 $\int \frac{\cot^4(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=71

$$\frac{(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{5/2}d}+\frac{(a+b)\cot(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3ad}$$

```
[Out] ((a + b)^(3/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a^(5/2)*d) + ((
a + b)*Cot[c + d*x])/(a^2*d) - Cot[c + d*x]^3/(3*a*d)
```

Rubi [A] time = 0.0793176, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3195, 325, 205}

$$\frac{(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{5/2}d}+\frac{(a+b)\cot(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + b*Sin[c + d*x]^2),x]
```


```
[Out] ((a + b)^(3/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a^(5/2)*d) + ((
a + b)*Cot[c + d*x])/(a^2*d) - Cot[c + d*x]^3/(3*a*d)
```


Rule 3195

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*((d_.)*tan[(e_.) + (f_.)
*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[f
f/f, Subst[Int[((d*ff*x)^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(p + 1)
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m}, x] && IntegerQ[
p]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4\left(a+(a+b)x^2\right)}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= -\frac{\cot^3(c+dx)}{3ad} - \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{x^2\left(a+(a+b)x^2\right)}\,dx, x, \tan(c+dx)\right)}{ad}$$
$$= \frac{(a+b)\cot(c+dx)}{a^2d} - \frac{\cot^3(c+dx)}{3ad} + \frac{(a+b)^2\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \tan(c+dx)\right)}{a^2d}$$
$$= \frac{(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{5/2}d} + \frac{(a+b)\cot(c+dx)}{a^2d} - \frac{\cot^3(c+dx)}{3ad}$$

Mathematica [A] time = 0.292023, size = 72, normalized size = 1.01

$$\frac{3(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right) + \sqrt{a}\cot(c+dx)\left(-a\csc^2(c+dx)+4a+3b\right)}{3a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (3*(a + b)^(3/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]] + Sqrt[a]*Cot[c
 + d*x]*(4*a + 3*b - a*Csc[c + d*x]^2))/(3*a^(5/2)*d)
```

Maple [B] time = 0.121, size = 147, normalized size = 2.1

$$\frac{1}{d}\arctan\left((a+b)\tan(dx+c)\frac{1}{\sqrt{a(a+b)}}\right)\frac{1}{\sqrt{a(a+b)}} + 2\frac{b}{da\sqrt{a(a+b)}}\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right) + \frac{b^2}{a^2d}\arctan\left((a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+sin(d*x+c)^2*b),x)
```

```
[Out] 1/d/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))+2/d/a/(a*(a+b)
)^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))*b+1/d/a^2*b^2/(a*(a+b))^(1
/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))-1/3/d/a/tan(d*x+c)^3+1/d/a/tan
(d*x+c)+1/d/a^2/tan(d*x+c)*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.82973, size = 973, normalized size = 13.7

$$\left[\frac{4\,(4\,a+3\,b)\cos{(dx+c)}^3+3\left((a+b)\cos{(dx+c)}^2-a-b\right)\sqrt{-\frac{a+b}{a}}\log\left(\frac{\left(8\,a^2+8\,ab+b^2\right)\cos(dx+c)^4-2\left(4\,a^2+5\,ab+b^2\right)\cos(dx+c)^2-4}{b^2\cos(dx+c)^4-2}\right.}{12\left(a^2d\cos{(dx+c)}^2-a^2d\right)\sin}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/12*(4*(4*a + 3*b)*cos(d*x + c)^3 + 3*((a + b)*cos(d*x + c)^2 - a - b)*sq
rt(-(a + b)/a)*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4*a^2 + 5*a*b
 + b^2)*cos(d*x + c)^2 - 4*((2*a^2 + a*b)*cos(d*x + c)^3 - (a^2 + a*b)*cos(
d*x + c))*sqrt(-(a + b)/a)*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x +
 c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2))*sin(d*x + c) - 1
2*(a + b)*cos(d*x + c))/((a^2*d*cos(d*x + c)^2 - a^2*d)*sin(d*x + c)), 1/6*
(2*(4*a + 3*b)*cos(d*x + c)^3 - 3*((a + b)*cos(d*x + c)^2 - a - b)*sqrt((a
+ b)/a)*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)*sqrt((a + b)/a)/((a +
 b)*cos(d*x + c)*sin(d*x + c)))*sin(d*x + c) - 6*(a + b)*cos(d*x + c))/((a^
2*d*cos(d*x + c)^2 - a^2*d)*sin(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cot^4{(c+dx)}}{a+b\sin^2{(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral(cot(c + d*x)**4/(a + b*sin(c + d*x)**2), x)
```

Giac [A] time = 1.17035, size = 162, normalized size = 2.28

$$\frac{\frac{3\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)\left(a^2+2\,ab+b^2\right)}{\sqrt{a^2+ab}\,a^2}+\frac{3\,a\tan(dx+c)^2+3\,b\tan(dx+c)^2-a}{a^2\tan(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c
) + b*tan(d*x + c))/sqrt(a^2 + a*b)))*(a^2 + 2*a*b + b^2)/(sqrt(a^2 + a*b)*
a^2) + (3*a*tan(d*x + c)^2 + 3*b*tan(d*x + c)^2 - a)/(a^2*tan(d*x + c)^3))/
d
```

3.455 $\int \frac{\cot^6(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=96

$$-\frac{(a+b)^{5/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{7/2}d}+\frac{(a+b)\cot^3(c+dx)}{3a^2d}-\frac{(a+b)^2\cot(c+dx)}{a^3d}-\frac{\cot^5(c+dx)}{5ad}$$

```
[Out] -(((a + b)^(5/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a^(7/2)*d)) -
 ((a + b)^2*Cot[c + d*x])/(a^3*d) + ((a + b)*Cot[c + d*x]^3)/(3*a^2*d) - Co
t[c + d*x]^5/(5*a*d)
```

Rubi [A] time = 0.0961273, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3195, 325, 205}

$$-\frac{(a+b)^{5/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{7/2}d}+\frac{(a+b)\cot^3(c+dx)}{3a^2d}-\frac{(a+b)^2\cot(c+dx)}{a^3d}-\frac{\cot^5(c+dx)}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -(((a + b)^(5/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a^(7/2)*d)) -
 ((a + b)^2*Cot[c + d*x])/(a^3*d) + ((a + b)*Cot[c + d*x]^3)/(3*a^2*d) - Co
t[c + d*x]^5/(5*a*d)
```

Rule 3195

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*((d_.)*tan[(e_.) + (f_.)
*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[f
f/f, Subst[Int[((d*ff*x)^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(p + 1)
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m}, x] && IntegerQ[
p]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cot^6(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^6\left(a+(a+b)x^2\right)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{\cot^5(c+dx)}{5ad} - \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{x^4\left(a+(a+b)x^2\right)}\,dx, x, \tan(c+dx)\right)}{ad}$$

$$= \frac{(a+b)\cot^3(c+dx)}{3a^2d} - \frac{\cot^5(c+dx)}{5ad} + \frac{(a+b)^2\,\text{Subst}\left(\int \frac{1}{x^2\left(a+(a+b)x^2\right)}\,dx, x, \tan(c+dx)\right)}{a^2d}$$

$$= -\frac{(a+b)^2\cot(c+dx)}{a^3d} + \frac{(a+b)\cot^3(c+dx)}{3a^2d} - \frac{\cot^5(c+dx)}{5ad} - \frac{(a+b)^3\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\right.}{a^3d}$$

$$= -\frac{(a+b)^{5/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{7/2}d} - \frac{(a+b)^2\cot(c+dx)}{a^3d} + \frac{(a+b)\cot^3(c+dx)}{3a^2d} - \frac{\cot^5(c+}{5ad}$$

Mathematica [A] time = 0.897472, size = 101, normalized size = 1.05

$$\frac{-\sqrt{a}\cot(c+dx)\left(3a^2\csc^4(c+dx)+23a^2-a(11a+5b)\csc^2(c+dx)+35ab+15b^2\right)-15(a+b)^{5/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+d}{\sqrt{a}}\right.}{15a^{7/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (-15*(a + b)^(5/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]] - Sqrt[a]*Cot
[c + d*x]*(23*a^2 + 35*a*b + 15*b^2 - a*(11*a + 5*b)*Csc[c + d*x]^2 + 3*a^2
*Csc[c + d*x]^4))/(15*a^(7/2)*d)
```

Maple [B] time = 0.126, size = 239, normalized size = 2.5

$$-\frac{1}{d}\arctan\left((a+b)\tan(dx+c)\frac{1}{\sqrt{a(a+b)}}\right)\frac{1}{\sqrt{a(a+b)}} - 3\frac{b}{da\sqrt{a(a+b)}}\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right) - 3\frac{b^2}{a^2d\sqrt{a(a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6/(a+sin(d*x+c)^2*b),x)
```

```
[Out] -1/d/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))-3/d/a/(a*(a+b
))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))*b-3/d/a^2*b^2/(a*(a+b))^(
1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))-1/d/a^3/(a*(a+b))^(1/2)*arcta
n((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))*b^3-1/d/a/tan(d*x+c)-2/d/a^2/tan(d*x+c)
*b-1/d/a^3/tan(d*x+c)*b^2-1/5/d/a/tan(d*x+c)^5+1/3/d/a/tan(d*x+c)^3+1/3/d/a
^2/tan(d*x+c)^3*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.90358, size = 1413, normalized size = 14.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/60*(4*(23*a^2 + 35*a*b + 15*b^2)*cos(d*x + c)^5 - 20*(7*a^2 + 13*a*b +
6*b^2)*cos(d*x + c)^3 - 15*((a^2 + 2*a*b + b^2)*cos(d*x + c)^4 - 2*(a^2 + 2
*a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2)*sqrt(-(a + b)/a)*log(((8*a^
2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 +
4*((2*a^2 + a*b)*cos(d*x + c)^3 - (a^2 + a*b)*cos(d*x + c))*sqrt(-(a + b)/a
)*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos
(d*x + c)^2 + a^2 + 2*a*b + b^2))*sin(d*x + c) + 60*(a^2 + 2*a*b + b^2)*cos
(d*x + c))/((a^3*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^2 + a^3*d)*sin(d*x
 + c)), -1/30*(2*(23*a^2 + 35*a*b + 15*b^2)*cos(d*x + c)^5 - 10*(7*a^2 + 13
*a*b + 6*b^2)*cos(d*x + c)^3 - 15*((a^2 + 2*a*b + b^2)*cos(d*x + c)^4 - 2*(
a^2 + 2*a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2)*sqrt((a + b)/a)*arct
an(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)*sqrt((a + b)/a)/((a + b)*cos(d*x
+ c)*sin(d*x + c)))*sin(d*x + c) + 30*(a^2 + 2*a*b + b^2)*cos(d*x + c))/((a
^3*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^2 + a^3*d)*sin(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.1902, size = 231, normalized size = 2.41

$$-\frac{\frac{15\left(a^3+3a^2b+3ab^2+b^3\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)}{\sqrt{a^2+ab}a^3}+\frac{15a^2\tan(dx+c)^4+30ab\tan(dx+c)^4+15b^2\tan(dx+c)^4-5a^2\tan(dx+c}{a^3\tan(dx+c)^5}}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/15*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(pi*floor((d*x + c)/pi + 1/2)*sgn
(2*a + 2*b) + arctan((a*tan(d*x + c) + b*tan(d*x + c))/sqrt(a^2 + a*b)))/(s
qrt(a^2 + a*b)*a^3) + (15*a^2*tan(d*x + c)^4 + 30*a*b*tan(d*x + c)^4 + 15*b
^2*tan(d*x + c)^4 - 5*a^2*tan(d*x + c)^2 - 5*a*b*tan(d*x + c)^2 + 3*a^2)/(a
^3*tan(d*x + c)^5))/d
```

3.456 $\int \frac{\cot^8(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=117

$$\frac{(a+b)^{7/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{9/2}d}+\frac{(a+b)\cot^5(c+dx)}{5a^2d}-\frac{(a+b)^2\cot^3(c+dx)}{3a^3d}+\frac{(a+b)^3\cot(c+dx)}{a^4d}-\frac{\cot^7(c+dx)}{7ad}$$

```
[Out] ((a + b)^(7/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a^(9/2)*d) + ((
a + b)^3*Cot[c + d*x])/(a^4*d) - ((a + b)^2*Cot[c + d*x]^3)/(3*a^3*d) + ((a
 + b)*Cot[c + d*x]^5)/(5*a^2*d) - Cot[c + d*x]^7/(7*a*d)
```

Rubi [A] time = 0.112005, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3195, 325, 205}

$$\frac{(a+b)^{7/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{9/2}d}+\frac{(a+b)\cot^5(c+dx)}{5a^2d}-\frac{(a+b)^2\cot^3(c+dx)}{3a^3d}+\frac{(a+b)^3\cot(c+dx)}{a^4d}-\frac{\cot^7(c+dx)}{7ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^8/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] ((a + b)^(7/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a^(9/2)*d) + ((
a + b)^3*Cot[c + d*x])/(a^4*d) - ((a + b)^2*Cot[c + d*x]^3)/(3*a^3*d) + ((a
 + b)*Cot[c + d*x]^5)/(5*a^2*d) - Cot[c + d*x]^7/(7*a*d)
```

Rule 3195

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*((d_.)*tan[(e_.) + (f_.)
*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[f
f/f, Subst[Int[((d*ff*x)^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(p + 1)
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m}, x] && IntegerQ[
p]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cot^8(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^8\left(a+(a+b)x^2\right)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{\cot^7(c+dx)}{7ad} - \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{x^6\left(a+(a+b)x^2\right)}\,dx, x, \tan(c+dx)\right)}{ad}$$

$$= \frac{(a+b)\cot^5(c+dx)}{5a^2d} - \frac{\cot^7(c+dx)}{7ad} + \frac{(a+b)^2\,\text{Subst}\left(\int \frac{1}{x^4\left(a+(a+b)x^2\right)}\,dx, x, \tan(c+dx)\right)}{a^2d}$$

$$= -\frac{(a+b)^2\cot^3(c+dx)}{3a^3d} + \frac{(a+b)\cot^5(c+dx)}{5a^2d} - \frac{\cot^7(c+dx)}{7ad} - \frac{(a+b)^3\,\text{Subst}\left(\int \frac{1}{x^2\left(a+(a+b)x^2\right)}\right.}{a^3d}$$

$$= \frac{(a+b)^3\cot(c+dx)}{a^4d} - \frac{(a+b)^2\cot^3(c+dx)}{3a^3d} + \frac{(a+b)\cot^5(c+dx)}{5a^2d} - \frac{\cot^7(c+dx)}{7ad} + \frac{(a+b)^4}{}$$

$$= \frac{(a+b)^{7/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{9/2}d} + \frac{(a+b)^3\cot(c+dx)}{a^4d} - \frac{(a+b)^2\cot^3(c+dx)}{3a^3d} + \frac{(a+b)\cot^5(}{5a^2d}$$

Mathematica [A] time = 1.08601, size = 135, normalized size = 1.15

$$\frac{\cot(c+dx)\left(-a\left(122a^2+112ab+35b^2\right)\csc^2(c+dx)+3a^2(22a+7b)\csc^4(c+dx)+406a^2b-15a^3\csc^6(c+dx)+176a^3\right.}{105a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^8/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] ((a + b)^(7/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a^(9/2)*d) + (C
ot[c + d*x]*(176*a^3 + 406*a^2*b + 350*a*b^2 + 105*b^3 - a*(122*a^2 + 112*a
*b + 35*b^2)*Csc[c + d*x]^2 + 3*a^2*(22*a + 7*b)*Csc[c + d*x]^4 - 15*a^3*Cs
c[c + d*x]^6))/(105*a^4*d)
```

Maple [B] time = 0.132, size = 342, normalized size = 2.9

$$\frac{1}{d}\arctan\left((a+b)\tan(dx+c)\frac{1}{\sqrt{a(a+b)}}\right)\frac{1}{\sqrt{a(a+b)}} + 4\frac{b}{da\sqrt{a(a+b)}}\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right) + 6\frac{b^2}{a^2d\sqrt{a(a+b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^8/(a+sin(d*x+c)^2*b),x)
```

```
[Out] 1/d/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))+4/d/a/(a*(a+b)
)^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))*b+6/d/a^2*b^2/(a*(a+b))^(1
/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))+4/d/a^3/(a*(a+b))^(1/2)*arctan
((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))*b^3+1/d/a^4/(a*(a+b))^(1/2)*arctan((a+b)
*tan(d*x+c)/(a*(a+b))^(1/2))*b^4-1/7/d/a/tan(d*x+c)^7+1/d/a/tan(d*x+c)+3/d/
a^2/tan(d*x+c)*b+3/d/a^3/tan(d*x+c)*b^2+1/d/a^4/tan(d*x+c)*b^3+1/5/d/a/tan(
d*x+c)^5+1/5/d/a^2/tan(d*x+c)^5*b-1/3/d/a/tan(d*x+c)^3-2/3/d/a^2/tan(d*x+c)
^3*b-1/3/d/a^3/tan(d*x+c)^3*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.04278, size = 2006, normalized size = 17.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/420*(4*(176*a^3 + 406*a^2*b + 350*a*b^2 + 105*b^3)*cos(d*x + c)^7 - 28*(
58*a^3 + 158*a^2*b + 145*a*b^2 + 45*b^3)*cos(d*x + c)^5 + 140*(10*a^3 + 29*
a^2*b + 28*a*b^2 + 9*b^3)*cos(d*x + c)^3 + 105*((a^3 + 3*a^2*b + 3*a*b^2 +
b^3)*cos(d*x + c)^6 - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(d*x + c)^4 - a^
3 - 3*a^2*b - 3*a*b^2 - b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(d*x + c
)^2)*sqrt(-(a + b)/a)*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4*a^2
+ 5*a*b + b^2)*cos(d*x + c)^2 - 4*((2*a^2 + a*b)*cos(d*x + c)^3 - (a^2 + a*
b)*cos(d*x + c))*sqrt(-(a + b)/a)*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*co
s(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2))*sin(d*x +
 c) - 420*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(d*x + c))/((a^4*d*cos(d*x + c
)^6 - 3*a^4*d*cos(d*x + c)^4 + 3*a^4*d*cos(d*x + c)^2 - a^4*d)*sin(d*x + c)
), 1/210*(2*(176*a^3 + 406*a^2*b + 350*a*b^2 + 105*b^3)*cos(d*x + c)^7 - 14
*(58*a^3 + 158*a^2*b + 145*a*b^2 + 45*b^3)*cos(d*x + c)^5 + 70*(10*a^3 + 29
*a^2*b + 28*a*b^2 + 9*b^3)*cos(d*x + c)^3 - 105*((a^3 + 3*a^2*b + 3*a*b^2 +
 b^3)*cos(d*x + c)^6 - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(d*x + c)^4 - a
^3 - 3*a^2*b - 3*a*b^2 - b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(d*x +
c)^2)*sqrt((a + b)/a)*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)*sqrt((a
 + b)/a)/((a + b)*cos(d*x + c)*sin(d*x + c)))*sin(d*x + c) - 210*(a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*cos(d*x + c))/((a^4*d*cos(d*x + c)^6 - 3*a^4*d*cos(d*
x + c)^4 + 3*a^4*d*cos(d*x + c)^2 - a^4*d)*sin(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**8/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.20032, size = 321, normalized size = 2.74

$$\frac{\frac{105\left(a^4+4a^3b+6a^2b^2+4ab^3+b^4\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)}{\sqrt{a^2+ab}a^4}+\frac{105a^3\tan(dx+c)^6+315a^2b\tan(dx+c)^6+315ab^2\tan(dx+c}{}}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/105*(105*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(pi*floor((d*x + c)/
pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c) + b*tan(d*x + c))/sqrt(a^
2 + a*b)))/(sqrt(a^2 + a*b)*a^4) + (105*a^3*tan(d*x + c)^6 + 315*a^2*b*tan(
d*x + c)^6 + 315*a*b^2*tan(d*x + c)^6 + 105*b^3*tan(d*x + c)^6 - 35*a^3*tan
(d*x + c)^4 - 70*a^2*b*tan(d*x + c)^4 - 35*a*b^2*tan(d*x + c)^4 + 21*a^3*ta
n(d*x + c)^2 + 21*a^2*b*tan(d*x + c)^2 - 15*a^3)/(a^4*tan(d*x + c)^7))/d
```

3.457 $\int \sqrt{a-a\sin^2(e+fx)}\tan^5(e+fx)\,dx$

Optimal. Leaf size=64

$$\frac{a^2}{3f\left(a\cos^2(e+fx)\right)^{3/2}}-\frac{2a}{f\sqrt{a\cos^2(e+fx)}}-\frac{\sqrt{a\cos^2(e+fx)}}{f}$$

```
[Out] a^2/(3*f*(a*Cos[e + f*x]^2)^(3/2)) - (2*a)/(f*Sqrt[a*Cos[e + f*x]^2]) - Sqr
t[a*Cos[e + f*x]^2]/f
```

Rubi [A] time = 0.109609, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3176, 3205, 16, 43}

$$\frac{a^2}{3f\left(a\cos^2(e+fx)\right)^{3/2}}-\frac{2a}{f\sqrt{a\cos^2(e+fx)}}-\frac{\sqrt{a\cos^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Sin[e + f*x]^2]*Tan[e + f*x]^5,x]
```

```
[Out] a^2/(3*f*(a*Cos[e + f*x]^2)^(3/2)) - (2*a)/(f*Sqrt[a*Cos[e + f*x]^2]) - Sqr
t[a*Cos[e + f*x]^2]/f
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \sqrt{a-a\sin^2(e+fx)}\tan^5(e+fx)\,dx = \int \sqrt{a\cos^2(e+fx)}\tan^5(e+fx)\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{(1-x)^2\sqrt{ax}}{x^3}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{a^3\,\text{Subst}\left(\int \frac{(1-x)^2}{(ax)^{5/2}}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{a^3\,\text{Subst}\left(\int \left(\frac{1}{(ax)^{5/2}}-\frac{2}{a(ax)^{3/2}}+\frac{1}{a^2\sqrt{ax}}\right)dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= \frac{a^2}{3f\left(a\cos^2(e+fx)\right)^{3/2}}-\frac{2a}{f\sqrt{a\cos^2(e+fx)}}-\frac{\sqrt{a\cos^2(e+fx)}}{f}$$

Mathematica [A] time = 0.0937716, size = 51, normalized size = 0.8

$$-\frac{\left(3\cos^4(e+fx)+6\cos^2(e+fx)-1\right)\sec^4(e+fx)\sqrt{a\cos^2(e+fx)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Sin[e + f*x]^2]*Tan[e + f*x]^5,x]
```

```
[Out] -(Sqrt[a*Cos[e + f*x]^2]*(-1 + 6*Cos[e + f*x]^2 + 3*Cos[e + f*x]^4)*Sec[e +
 f*x]^4)/(3*f)
```

Maple [A] time = 2.165, size = 48, normalized size = 0.8

$$-\frac{3\left(\cos\left(fx+e\right)\right)^4+6\left(\cos\left(fx+e\right)\right)^2-1}{3\left(\cos\left(fx+e\right)\right)^4 f}\sqrt{a\left(\cos\left(fx+e\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x)
```

```
[Out] -1/3/cos(f*x+e)^4*(a*cos(f*x+e)^2)^(1/2)*(3*cos(f*x+e)^4+6*cos(f*x+e)^2-1)/
f
```

Maxima [A] time = 1.01499, size = 93, normalized size = 1.45

$$-\frac{3\sqrt{-a\sin\left(fx+e\right)^2+a}\,a^3-\frac{6\left(a\sin\left(fx+e\right)^2-a\right)a^4+a^5}{\left(-a\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}}{3\,a^3 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] -1/3*(3*sqrt(-a*sin(f*x + e)^2 + a)*a^3 - (6*(a*sin(f*x + e)^2 - a)*a^4 + a
^5)/(-a*sin(f*x + e)^2 + a)^(3/2))/(a^3*f)
```

Fricas [A] time = 1.65234, size = 122, normalized size = 1.91

$$-\frac{\left(3\,\cos\left(fx+e\right)^{4}+6\,\cos\left(fx+e\right)^{2}-1\right)\sqrt{a\cos\left(fx+e\right)^{2}}}{3\,f\cos\left(fx+e\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] -1/3*(3*cos(f*x + e)^4 + 6*cos(f*x + e)^2 - 1)*sqrt(a*cos(f*x + e)^2)/(f*co
s(f*x + e)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)**2)**(1/2)*tan(f*x+e)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.74912, size = 184, normalized size = 2.88

$$\frac{2\sqrt{a}\left(\frac{3\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-1\right)}{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+1}-\frac{3\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-1\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-12\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-1\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+5\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-1\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-1\right)^{3}}\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="giac")
```

```
[Out] 2/3*sqrt(a)*(3*sgn(tan(1/2*f*x + 1/2*e)^4 - 1)/(tan(1/2*f*x + 1/2*e)^2 + 1)
 - (3*sgn(tan(1/2*f*x + 1/2*e)^4 - 1)*tan(1/2*f*x + 1/2*e)^4 - 12*sgn(tan(1
/2*f*x + 1/2*e)^4 - 1)*tan(1/2*f*x + 1/2*e)^2 + 5*sgn(tan(1/2*f*x + 1/2*e)^
4 - 1))/(tan(1/2*f*x + 1/2*e)^2 - 1)^3)/f
```

3.458 $\int \sqrt{a-a\sin^2(e+fx)}\tan^3(e+fx)\,dx$

Optimal. Leaf size=38

$$\frac{a}{f\sqrt{a\cos^2(e+fx)}}+\frac{\sqrt{a\cos^2(e+fx)}}{f}$$

```
[Out] a/(f*Sqrt[a*Cos[e + f*x]^2]) + Sqrt[a*Cos[e + f*x]^2]/f
```

Rubi [A] time = 0.104688, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3176, 3205, 16, 43}

$$\frac{a}{f\sqrt{a\cos^2(e+fx)}}+\frac{\sqrt{a\cos^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Sin[e + f*x]^2]*Tan[e + f*x]^3,x]
```

```
[Out] a/(f*Sqrt[a*Cos[e + f*x]^2]) + Sqrt[a*Cos[e + f*x]^2]/f
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \sqrt{a-a\sin^2(e+fx)}\tan^3(e+fx)\,dx = \int \sqrt{a\cos^2(e+fx)}\tan^3(e+fx)\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{(1-x)\sqrt{ax}}{x^2}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{a^2\,\text{Subst}\left(\int \frac{1-x}{(ax)^{3/2}}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{a^2\,\text{Subst}\left(\int \left(\frac{1}{(ax)^{3/2}}-\frac{1}{a\sqrt{ax}}\right)dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= \frac{a}{f\sqrt{a\cos^2(e+fx)}}+\frac{\sqrt{a\cos^2(e+fx)}}{f}$$

Mathematica [A] time = 0.081039, size = 29, normalized size = 0.76

$$\frac{a\left(\cos^2(e+fx)+1\right)}{f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Sin[e + f*x]^2]*Tan[e + f*x]^3,x]
```

```
[Out] (a*(1 + Cos[e + f*x]^2))/(f*Sqrt[a*Cos[e + f*x]^2])
```

Maple [A] time = 2.115, size = 35, normalized size = 0.9

$$\frac{\left(\cos\left(fx+e\right)\right)^2+1}{\left(\cos\left(fx+e\right)\right)^2 f}\sqrt{a\left(\cos\left(fx+e\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x)
```

```
[Out] 1/cos(f*x+e)^2*(a*cos(f*x+e)^2)^(1/2)*(cos(f*x+e)^2+1)/f
```

Maxima [A] time = 1.00257, size = 62, normalized size = 1.63

$$\frac{\sqrt{-a\sin\left(fx+e\right)^2+a}\,a^2+\frac{a^3}{\sqrt{-a\sin\left(fx+e\right)^2+a}}}{a^2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] (sqrt(-a*sin(f*x + e)^2 + a)*a^2 + a^3/sqrt(-a*sin(f*x + e)^2 + a))/(a^2*f)
```

Fricas [A] time = 1.59576, size = 86, normalized size = 2.26

$$\frac{\sqrt{a\cos\left(fx+e\right)^2}\left(\cos\left(fx+e\right)^2+1\right)}{f\cos\left(fx+e\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] sqrt(a*cos(f*x + e)^2)*(cos(f*x + e)^2 + 1)/(f*cos(f*x + e)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}\tan^3\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)**2)**(1/2)*tan(f*x+e)**3,x)
```

```
[Out] Integral(sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))*tan(e + f*x)**3, x)
```

Giac [A] time = 1.50308, size = 53, normalized size = 1.39

$$\frac{4\sqrt{a}\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-1\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-1\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="giac")
```

```
[Out] 4*sqrt(a)*sgn(tan(1/2*f*x + 1/2*e)^4 - 1)/((tan(1/2*f*x + 1/2*e)^4 - 1)*f)
```

3.459 $\int \sqrt{a-a\sin^2(e+fx)}\tan(e+fx)\,dx$

Optimal. Leaf size=19

$$-\frac{\sqrt{a\cos^2(e+fx)}}{f}$$

```
[Out] -(Sqrt[a*Cos[e + f*x]^2]/f)
```

Rubi [A] time = 0.0629348, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3176, 3205, 16, 32}

$$-\frac{\sqrt{a\cos^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Sin[e + f*x]^2]*Tan[e + f*x],x]
```

```
[Out] -(Sqrt[a*Cos[e + f*x]^2]/f)
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt{a-a\sin^2(e+fx)}\tan(e+fx)\,dx=\int \sqrt{a\cos^2(e+fx)}\tan(e+fx)\,dx$$
$$=-\frac{\operatorname{Subst}\left(\int \frac{\sqrt{ax}}{x}\,dx,x,\cos^2(e+fx)\right)}{2f}$$
$$=-\frac{a\operatorname{Subst}\left(\int \frac{1}{\sqrt{ax}}\,dx,x,\cos^2(e+fx)\right)}{2f}$$
$$=-\frac{\sqrt{a\cos^2(e+fx)}}{f}$$

Mathematica [A] time = 0.0397946, size = 19, normalized size = 1.

$$-\frac{\sqrt{a\cos^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Sin[e + f*x]^2]*Tan[e + f*x],x]
```

```
[Out] -(Sqrt[a*Cos[e + f*x]^2]/f)
```

Maple [A] time = 0.131, size = 21, normalized size = 1.1

$$-\frac{1}{f}\sqrt{a-a\left(\sin\left(fx+e\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e),x)
```

```
[Out] -1/f*(a-a*sin(f*x+e)^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.6049, size = 36, normalized size = 1.89

$$-\frac{\sqrt{a\cos\left(fx+e\right)^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="fricas")
```

```
[Out] -sqrt(a*cos(f*x + e)^2)/f
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}\tan\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)**2)**(1/2)*tan(f*x+e),x)
```

```
[Out] Integral(sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))*tan(e + f*x), x)
```

Giac [B] time = 1.1338, size = 53, normalized size = 2.79

$$\frac{2\sqrt{a}\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-1\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+1\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="giac")
```

```
[Out] 2*sqrt(a)*sgn(tan(1/2*f*x + 1/2*e)^4 - 1)/((tan(1/2*f*x + 1/2*e)^2 + 1)*f)
```

3.460 $\int \cot(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$

Optimal. Leaf size=50

$$\frac{\sqrt{a\cos^2(e+fx)}}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{f}$$

```
[Out] -((Sqrt[a]*ArcTanh[Sqrt[a*Cos[e + f*x]^2]/Sqrt[a]])/f) + Sqrt[a*Cos[e + f*x
]^2]/f
```

Rubi [A] time = 0.0790169, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3176, 3205, 50, 63, 206}

$$\frac{\sqrt{a\cos^2(e+fx)}}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a]*ArcTanh[Sqrt[a*Cos[e + f*x]^2]/Sqrt[a]])/f) + Sqrt[a*Cos[e + f*x
]^2]/f
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \cot(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx &= \int \sqrt{a\cos^2(e+fx)}\cot(e+fx)\,dx \\
&= -\frac{\text{Subst}\left(\int \frac{\sqrt{ax}}{1-x}\,dx, x, \cos^2(e+fx)\right)}{2f} \\
&= \frac{\sqrt{a\cos^2(e+fx)}}{f} - \frac{a\,\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{ax}}\,dx, x, \cos^2(e+fx)\right)}{2f} \\
&= \frac{\sqrt{a\cos^2(e+fx)}}{f} - \frac{\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a\cos^2(e+fx)}\right)}{f} \\
&= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{\sqrt{a\cos^2(e+fx)}}{f}
\end{aligned}
$$

Mathematica [A] time = 0.0692788, size = 55, normalized size = 1.1

$$
\frac{\sec(e+fx)\sqrt{a\cos^2(e+fx)}\left(\cos(e+fx)+\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)\right)}{f}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] (Sqrt[a*Cos[e + f*x]^2]*(Cos[e + f*x] - Log[Cos[(e + f*x)/2]] + Log[Sin[(e
+ f*x)/2]])*Sec[e + f*x])/f
```

Maple [A] time = 1.521, size = 55, normalized size = 1.1

$$
-\frac{1}{f}\sqrt{a}\ln\left(2\,\frac{\sqrt{a}\sqrt{a\left(\cos\left(fx+e\right)\right)^2}+a}{\sin\left(fx+e\right)}\right)+\frac{1}{f}\sqrt{a\left(\cos\left(fx+e\right)\right)^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(a-a*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/f*a^(1/2)*ln(2/sin(f*x+e)*(a^(1/2)*(a*cos(f*x+e)^2)^(1/2)+a))+(a*cos(f*x
+e)^2)^(1/2)/f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.65234, size = 146, normalized size = 2.92

$$\frac{\sqrt{a\cos\left(fx+e\right)^{2}}\left(2\cos\left(fx+e\right)-\log\left(-\frac{\cos\left(fx+e\right)+1}{\cos\left(fx+e\right)-1}\right)\right)}{2f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(a*cos(f*x + e)^2)*(2*cos(f*x + e) - log(-(cos(f*x + e) + 1)/(cos(f
*x + e) - 1)))/(f*cos(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}\cot\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a-a*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))*cot(e + f*x), x)
```

Giac [A] time = 1.09789, size = 69, normalized size = 1.38

$$\frac{\frac{a\arctan\left(\frac{\sqrt{-a\sin\left(fx+e\right)^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\sqrt{-a\sin\left(fx+e\right)^{2}+a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] (a*arctan(sqrt(-a*sin(f*x + e)^2 + a)/sqrt(-a))/sqrt(-a) + sqrt(-a*sin(f*x
+ e)^2 + a))/f
```

3.461 $\int \cot^3(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$

Optimal. Leaf size=87

$$-\frac{3\sqrt{a\cos^2(e+fx)}}{2f}-\frac{\csc^2(e+fx)\left(a\cos^2(e+fx)\right)^{3/2}}{2af}+\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{2f}$$

```
[Out] (3*Sqrt[a]*ArcTanh[Sqrt[a*Cos[e + f*x]^2]/Sqrt[a]])/(2*f) - (3*Sqrt[a*Cos[e
 + f*x]^2])/(2*f) - ((a*Cos[e + f*x]^2)^(3/2)*Csc[e + f*x]^2)/(2*a*f)
```

Rubi [A] time = 0.118243, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {3176, 3205, 16, 47, 50, 63, 206}

$$-\frac{3\sqrt{a\cos^2(e+fx)}}{2f}-\frac{\csc^2(e+fx)\left(a\cos^2(e+fx)\right)^{3/2}}{2af}+\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] (3*Sqrt[a]*ArcTanh[Sqrt[a*Cos[e + f*x]^2]/Sqrt[a]])/(2*f) - (3*Sqrt[a*Cos[e
 + f*x]^2])/(2*f) - ((a*Cos[e + f*x]^2)^(3/2)*Csc[e + f*x]^2)/(2*a*f)
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cot^3(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx = \int \sqrt{a\cos^2(e+fx)}\cot^3(e+fx)\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{x\sqrt{ax}}{(1-x)^2}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{\text{Subst}\left(\int \frac{(ax)^{3/2}}{(1-x)^2}\,dx, x, \cos^2(e+fx)\right)}{2af}$$
$$= -\frac{\left(a\cos^2(e+fx)\right)^{3/2}\csc^2(e+fx)}{2af} + \frac{3\,\text{Subst}\left(\int \frac{\sqrt{ax}}{1-x}\,dx, x, \cos^2(e+fx)\right)}{4f}$$
$$= -\frac{3\sqrt{a\cos^2(e+fx)}}{2f} - \frac{\left(a\cos^2(e+fx)\right)^{3/2}\csc^2(e+fx)}{2af} + \frac{(3a)\,\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{ax}}$$
$$= -\frac{3\sqrt{a\cos^2(e+fx)}}{2f} - \frac{\left(a\cos^2(e+fx)\right)^{3/2}\csc^2(e+fx)}{2af} + \frac{3\,\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x,$$
$$= \frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{2f} - \frac{3\sqrt{a\cos^2(e+fx)}}{2f} - \frac{\left(a\cos^2(e+fx)\right)^{3/2}\csc^2(e+f$$

Mathematica [A] time = 0.432015, size = 88, normalized size = 1.01

$$-\frac{\sec(e+fx)\sqrt{a\cos^2(e+fx)}\left(8\cos(e+fx)+\csc^2\left(\frac{1}{2}(e+fx)\right)-\sec^2\left(\frac{1}{2}(e+fx)\right)+12\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)-12\log\left(\mathrm{co}\right.\right.}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3*Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] -(Sqrt[a*Cos[e + f*x]^2]*(8*Cos[e + f*x] + Csc[(e + f*x)/2]^2 - 12*Log[Cos[
(e + f*x)/2]] + 12*Log[Sin[(e + f*x)/2]] - Sec[(e + f*x)/2]^2)*Sec[e + f*x]
```

```
)/(8*f)
```

Maple [A] time = 1.508, size = 83, normalized size = 1.

$$-\frac{1}{f}\sqrt{a\left(\cos\left(fx+e\right)\right)^{2}}+\frac{3}{2f}\sqrt{a}\ln\left(\frac{1}{\sin\left(fx+e\right)}\left(2a+2\sqrt{a}\sqrt{a\left(\cos\left(fx+e\right)\right)^{2}}\right)\right)-\frac{1}{2f\left(\sin\left(fx+e\right)\right)^{2}}\sqrt{a\left(\cos\left(fx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(a-a*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] -(a*cos(f*x+e)^2)^(1/2)/f+3/2/f*a^(1/2)*ln((2*a+2*a^(1/2)*(a*cos(f*x+e)^2)^
(1/2))/sin(f*x+e))-1/2/f/sin(f*x+e)^2*(a*cos(f*x+e)^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.6959, size = 230, normalized size = 2.64

$$-\frac{\sqrt{a\cos\left(fx+e\right)^{2}}\left(4\cos\left(fx+e\right)^{3}+3\left(\cos\left(fx+e\right)^{2}-1\right)\log\left(-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)-6\cos\left(fx+e\right)\right)}{4\left(f\cos\left(fx+e\right)^{3}-f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(a*cos(f*x + e)^2)*(4*cos(f*x + e)^3 + 3*(cos(f*x + e)^2 - 1)*log(
-(cos(f*x + e) - 1)/(cos(f*x + e) + 1)) - 6*cos(f*x + e))/(f*cos(f*x + e)^3
 - f*cos(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}\cot^{3}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(a-a*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))*cot(e + f*x)**3, x)
```

Giac [B] time = 1.24256, size = 231, normalized size = 2.66

$$-\frac{\left(\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-1\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-6\,\log\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-1\right)+\frac{3\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,fx+\right.\right.}{}\right.}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*(sgn(tan(1/2*f*x + 1/2*e)^4 - 1)*tan(1/2*f*x + 1/2*e)^2 - 6*log(tan(1/
2*f*x + 1/2*e)^2)*sgn(tan(1/2*f*x + 1/2*e)^4 - 1) + (3*sgn(tan(1/2*f*x + 1/
2*e)^4 - 1)*tan(1/2*f*x + 1/2*e)^4 - 14*sgn(tan(1/2*f*x + 1/2*e)^4 - 1)*tan
(1/2*f*x + 1/2*e)^2 - sgn(tan(1/2*f*x + 1/2*e)^4 - 1))/(tan(1/2*f*x + 1/2*e
)^4 + tan(1/2*f*x + 1/2*e)^2))*sqrt(a)/f
```

3.462 $\int \sqrt{a-a\sin^2(e+fx)}\tan^6(e+fx)\,dx$

Optimal. Leaf size=120

$$\frac{\tan^5(e+fx)\sqrt{a\cos^2(e+fx)}}{4f}-\frac{5\tan^3(e+fx)\sqrt{a\cos^2(e+fx)}}{8f}-\frac{15\tan(e+fx)\sqrt{a\cos^2(e+fx)}}{8f}+\frac{15\sec(e+fx)\sqrt{a}}{}$$

```
[Out] (15*ArcTanh[Sin[e + f*x]]*Sqrt[a*Cos[e + f*x]^2]*Sec[e + f*x])/(8*f) - (15*
Sqrt[a*Cos[e + f*x]^2]*Tan[e + f*x])/(8*f) - (5*Sqrt[a*Cos[e + f*x]^2]*Tan[
e + f*x]^3)/(8*f) + (Sqrt[a*Cos[e + f*x]^2]*Tan[e + f*x]^5)/(4*f)
```

Rubi [A] time = 0.128648, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3176, 3207, 2592, 288, 321, 206}

$$\frac{\tan^5(e+fx)\sqrt{a\cos^2(e+fx)}}{4f}-\frac{5\tan^3(e+fx)\sqrt{a\cos^2(e+fx)}}{8f}-\frac{15\tan(e+fx)\sqrt{a\cos^2(e+fx)}}{8f}+\frac{15\sec(e+fx)\sqrt{a}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Sin[e + f*x]^2]*Tan[e + f*x]^6,x]
```

```
[Out] (15*ArcTanh[Sin[e + f*x]]*Sqrt[a*Cos[e + f*x]^2]*Sec[e + f*x])/(8*f) - (15*
Sqrt[a*Cos[e + f*x]^2]*Tan[e + f*x])/(8*f) - (5*Sqrt[a*Cos[e + f*x]^2]*Tan[
e + f*x]^3)/(8*f) + (Sqrt[a*Cos[e + f*x]^2]*Tan[e + f*x]^5)/(4*f)
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a-a\sin^2(e+fx)}\tan^6(e+fx)\,dx = \int \sqrt{a\cos^2(e+fx)}\tan^6(e+fx)\,dx$$
$$= \left(\sqrt{a\cos^2(e+fx)}\sec(e+fx)\right)\int \sin(e+fx)\tan^5(e+fx)\,dx$$
$$= \frac{\left(\sqrt{a\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^6}{\left(1-x^2\right)^3}\,dx, x, \sin(e+fx)\right)}{f}$$
$$= \frac{\sqrt{a\cos^2(e+fx)}\tan^5(e+fx)}{4f} - \frac{\left(5\sqrt{a\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^4}{\left(1-x^2\right.}\right.}{4f}$$
$$= -\frac{5\sqrt{a\cos^2(e+fx)}\tan^3(e+fx)}{8f} + \frac{\sqrt{a\cos^2(e+fx)}\tan^5(e+fx)}{4f} + \frac{\left(15\sqrt{a\cos\right.}{}$$
$$= -\frac{15\sqrt{a\cos^2(e+fx)}\tan(e+fx)}{8f} - \frac{5\sqrt{a\cos^2(e+fx)}\tan^3(e+fx)}{8f} + \frac{\sqrt{a\cos^2(}{}$$
$$= \frac{15\tanh^{-1}(\sin(e+fx))\sqrt{a\cos^2(e+fx)}\sec(e+fx)}{8f} - \frac{15\sqrt{a\cos^2(e+fx)}\tan(e}{8f}$$

Mathematica [A] time = 0.332584, size = 75, normalized size = 0.62

$$\frac{\sec^5(e+fx)\sqrt{a\cos^2(e+fx)}\left(-5\sin(e+fx)-15\sin(3(e+fx))-2\sin(5(e+fx))+60\cos^4(e+fx)\tanh^{-1}(\sin(e+fx)\right.}{32f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Sin[e + f*x]^2]*Tan[e + f*x]^6,x]
```

```
[Out] (Sqrt[a*Cos[e + f*x]^2]*Sec[e + f*x]^5*(60*ArcTanh[Sin[e + f*x]]*Cos[e + f*
x]^4 - 5*Sin[e + f*x] - 15*Sin[3*(e + f*x)] - 2*Sin[5*(e + f*x)]))/(32*f)
```

Maple [A] time = 1.229, size = 120, normalized size = 1.

$$\frac{a\left(16\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^4+18\left(\cos\left(fx+e\right)\right)^2\sin\left(fx+e\right)-4\sin\left(fx+e\right)+\left(15\ln\left(-1+\sin\left(fx+e\right)\right)-15\ln\right.\right.}{\left(16+16\sin\left(fx+e\right)\right)\left(-1+\sin\left(fx+e\right)\right)\cos\left(fx+e\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x)
```

```
[Out] 1/16*a*(16*sin(f*x+e)*cos(f*x+e)^4+18*cos(f*x+e)^2*sin(f*x+e)-4*sin(f*x+e)+
(15*ln(-1+sin(f*x+e))-15*ln(1+sin(f*x+e)))*cos(f*x+e)^4)/(1+sin(f*x+e))/(-1
+sin(f*x+e))/cos(f*x+e)/(a*cos(f*x+e)^2)^(1/2)/f
```

Maxima [B] time = 6.59078, size = 2639, normalized size = 21.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x, algorithm="maxima")
```

```
[Out] 1/16*(8*(sin(9*f*x + 9*e) + 4*sin(7*f*x + 7*e) + 6*sin(5*f*x + 5*e) + 4*sin
(3*f*x + 3*e) + sin(f*x + e))*cos(10*f*x + 10*e) - 20*(3*sin(8*f*x + 8*e) +
 sin(6*f*x + 6*e) - sin(4*f*x + 4*e) - 3*sin(2*f*x + 2*e))*cos(9*f*x + 9*e)
 + 60*(4*sin(7*f*x + 7*e) + 6*sin(5*f*x + 5*e) + 4*sin(3*f*x + 3*e) + sin(f
*x + e))*cos(8*f*x + 8*e) - 80*(sin(6*f*x + 6*e) - sin(4*f*x + 4*e) - 3*sin
(2*f*x + 2*e))*cos(7*f*x + 7*e) + 20*(6*sin(5*f*x + 5*e) + 4*sin(3*f*x + 3*
e) + sin(f*x + e))*cos(6*f*x + 6*e) + 120*(sin(4*f*x + 4*e) + 3*sin(2*f*x +
 2*e))*cos(5*f*x + 5*e) - 20*(4*sin(3*f*x + 3*e) + sin(f*x + e))*cos(4*f*x
+ 4*e) + 15*(2*(4*cos(7*f*x + 7*e) + 6*cos(5*f*x + 5*e) + 4*cos(3*f*x + 3*e
) + cos(f*x + e))*cos(9*f*x + 9*e) + cos(9*f*x + 9*e)^2 + 8*(6*cos(5*f*x +
5*e) + 4*cos(3*f*x + 3*e) + cos(f*x + e))*cos(7*f*x + 7*e) + 16*cos(7*f*x +
 7*e)^2 + 12*(4*cos(3*f*x + 3*e) + cos(f*x + e))*cos(5*f*x + 5*e) + 36*cos(
5*f*x + 5*e)^2 + 16*cos(3*f*x + 3*e)^2 + 8*cos(3*f*x + 3*e)*cos(f*x + e) +
cos(f*x + e)^2 + 2*(4*sin(7*f*x + 7*e) + 6*sin(5*f*x + 5*e) + 4*sin(3*f*x +
 3*e) + sin(f*x + e))*sin(9*f*x + 9*e) + sin(9*f*x + 9*e)^2 + 8*(6*sin(5*f*
x + 5*e) + 4*sin(3*f*x + 3*e) + sin(f*x + e))*sin(7*f*x + 7*e) + 16*sin(7*f
*x + 7*e)^2 + 12*(4*sin(3*f*x + 3*e) + sin(f*x + e))*sin(5*f*x + 5*e) + 36*
sin(5*f*x + 5*e)^2 + 16*sin(3*f*x + 3*e)^2 + 8*sin(3*f*x + 3*e)*sin(f*x + e
) + sin(f*x + e)^2)*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin(f*x + e) +
1) - 15*(2*(4*cos(7*f*x + 7*e) + 6*cos(5*f*x + 5*e) + 4*cos(3*f*x + 3*e) +
cos(f*x + e))*cos(9*f*x + 9*e) + cos(9*f*x + 9*e)^2 + 8*(6*cos(5*f*x + 5*e)
 + 4*cos(3*f*x + 3*e) + cos(f*x + e))*cos(7*f*x + 7*e) + 16*cos(7*f*x + 7*e
)^2 + 12*(4*cos(3*f*x + 3*e) + cos(f*x + e))*cos(5*f*x + 5*e) + 36*cos(5*f*
x + 5*e)^2 + 16*cos(3*f*x + 3*e)^2 + 8*cos(3*f*x + 3*e)*cos(f*x + e) + cos(
f*x + e)^2 + 2*(4*sin(7*f*x + 7*e) + 6*sin(5*f*x + 5*e) + 4*sin(3*f*x + 3*e
) + sin(f*x + e))*sin(9*f*x + 9*e) + sin(9*f*x + 9*e)^2 + 8*(6*sin(5*f*x +
5*e) + 4*sin(3*f*x + 3*e) + sin(f*x + e))*sin(7*f*x + 7*e) + 16*sin(7*f*x +
 7*e)^2 + 12*(4*sin(3*f*x + 3*e) + sin(f*x + e))*sin(5*f*x + 5*e) + 36*sin(
5*f*x + 5*e)^2 + 16*sin(3*f*x + 3*e)^2 + 8*sin(3*f*x + 3*e)*sin(f*x + e) +
sin(f*x + e)^2)*log(cos(f*x + e)^2 + sin(f*x + e)^2 - 2*sin(f*x + e) + 1) -
 8*(cos(9*f*x + 9*e) + 4*cos(7*f*x + 7*e) + 6*cos(5*f*x + 5*e) + 4*cos(3*f*
x + 3*e) + cos(f*x + e))*sin(10*f*x + 10*e) + 4*(15*cos(8*f*x + 8*e) + 5*co
s(6*f*x + 6*e) - 5*cos(4*f*x + 4*e) - 15*cos(2*f*x + 2*e) - 2)*sin(9*f*x +
9*e) - 60*(4*cos(7*f*x + 7*e) + 6*cos(5*f*x + 5*e) + 4*cos(3*f*x + 3*e) + c
os(f*x + e))*sin(8*f*x + 8*e) + 16*(5*cos(6*f*x + 6*e) - 5*cos(4*f*x + 4*e)
 - 15*cos(2*f*x + 2*e) - 2)*sin(7*f*x + 7*e) - 20*(6*cos(5*f*x + 5*e) + 4*c
os(3*f*x + 3*e) + cos(f*x + e))*sin(6*f*x + 6*e) - 24*(5*cos(4*f*x + 4*e) +
 15*cos(2*f*x + 2*e) + 2)*sin(5*f*x + 5*e) + 20*(4*cos(3*f*x + 3*e) + cos(f
*x + e))*sin(4*f*x + 4*e) - 16*(15*cos(2*f*x + 2*e) + 2)*sin(3*f*x + 3*e) +
 240*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) + 60*cos(f*x + e)*sin(2*f*x + 2*e) -
 60*cos(2*f*x + 2*e)*sin(f*x + e) - 8*sin(f*x + e))*sqrt(a)/((2*(4*cos(7*f*
```

```
x + 7*e) + 6*cos(5*f*x + 5*e) + 4*cos(3*f*x + 3*e) + cos(f*x + e))*cos(9*f*
x + 9*e) + cos(9*f*x + 9*e)^2 + 8*(6*cos(5*f*x + 5*e) + 4*cos(3*f*x + 3*e)
+ cos(f*x + e))*cos(7*f*x + 7*e) + 16*cos(7*f*x + 7*e)^2 + 12*(4*cos(3*f*x
+ 3*e) + cos(f*x + e))*cos(5*f*x + 5*e) + 36*cos(5*f*x + 5*e)^2 + 16*cos(3*
f*x + 3*e)^2 + 8*cos(3*f*x + 3*e)*cos(f*x + e) + cos(f*x + e)^2 + 2*(4*sin(
7*f*x + 7*e) + 6*sin(5*f*x + 5*e) + 4*sin(3*f*x + 3*e) + sin(f*x + e))*sin(
9*f*x + 9*e) + sin(9*f*x + 9*e)^2 + 8*(6*sin(5*f*x + 5*e) + 4*sin(3*f*x + 3
*e) + sin(f*x + e))*sin(7*f*x + 7*e) + 16*sin(7*f*x + 7*e)^2 + 12*(4*sin(3*
f*x + 3*e) + sin(f*x + e))*sin(5*f*x + 5*e) + 36*sin(5*f*x + 5*e)^2 + 16*si
n(3*f*x + 3*e)^2 + 8*sin(3*f*x + 3*e)*sin(f*x + e) + sin(f*x + e)^2)*f)
```

Fricas [A] time = 1.73221, size = 232, normalized size = 1.93

$$-\frac{\left(15\,\cos\left(fx+e\right)^{4}\log\left(-\frac{\sin\left(fx+e\right)-1}{\sin\left(fx+e\right)+1}\right)+2\left(8\,\cos\left(fx+e\right)^{4}+9\,\cos\left(fx+e\right)^{2}-2\right)\sin\left(fx+e\right)\right)\sqrt{a\cos\left(fx+e\right)^{2}}}{16\,f\cos\left(fx+e\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x, algorithm="fricas")
```

```
[Out] -1/16*(15*cos(f*x + e)^4*log(-(sin(f*x + e) - 1)/(sin(f*x + e) + 1)) + 2*(8
*cos(f*x + e)^4 + 9*cos(f*x + e)^2 - 2)*sin(f*x + e))*sqrt(a*cos(f*x + e)^2
)/(f*cos(f*x + e)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)**2)**(1/2)*tan(f*x+e)**6,x)
```

```
[Out] Timed out
```

Giac [B] time = 3.91622, size = 339, normalized size = 2.82

$$-\frac{\left(15\,\log\left(\left|\frac{1}{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+2\right|\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-1\right)-15\,\log\left(\left|\frac{1}{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x, algorithm="giac")
```

```
[Out] -1/16*(15*log(abs(1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*e) + 2))*sgn(t
an(1/2*f*x + 1/2*e)^4 - 1) - 15*log(abs(1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*
x + 1/2*e) - 2))*sgn(tan(1/2*f*x + 1/2*e)^4 - 1) - 32*sgn(tan(1/2*f*x + 1/2
```

```
*e)^4 - 1)/(1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*e)) - 4*(7*(1/tan(1/
2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*e))^3*sgn(tan(1/2*f*x + 1/2*e)^4 - 1) -
36*(1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*e))*sgn(tan(1/2*f*x + 1/2*e)
^4 - 1))/((1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*e))^2 - 4)^2)*sqrt(a)
/f
```

3.463 $\int \sqrt{a-a\sin^2(e+fx)}\tan^4(e+fx)\,dx$

Optimal. Leaf size=91

$$\frac{\tan^3(e+fx)\sqrt{a\cos^2(e+fx)}}{2f}+\frac{3\tan(e+fx)\sqrt{a\cos^2(e+fx)}}{2f}-\frac{3\sec(e+fx)\sqrt{a\cos^2(e+fx)}\tanh^{-1}(\sin(e+fx))}{2f}$$

```
[Out] (-3*ArcTanh[Sin[e + f*x]]*Sqrt[a*Cos[e + f*x]^2]*Sec[e + f*x])/(2*f) + (3*S
qrt[a*Cos[e + f*x]^2]*Tan[e + f*x])/(2*f) + (Sqrt[a*Cos[e + f*x]^2]*Tan[e +
 f*x]^3)/(2*f)
```

Rubi [A] time = 0.123761, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3176, 3207, 2592, 288, 321, 206}

$$\frac{\tan^3(e+fx)\sqrt{a\cos^2(e+fx)}}{2f}+\frac{3\tan(e+fx)\sqrt{a\cos^2(e+fx)}}{2f}-\frac{3\sec(e+fx)\sqrt{a\cos^2(e+fx)}\tanh^{-1}(\sin(e+fx))}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Sin[e + f*x]^2]*Tan[e + f*x]^4,x]
```

```
[Out] (-3*ArcTanh[Sin[e + f*x]]*Sqrt[a*Cos[e + f*x]^2]*Sec[e + f*x])/(2*f) + (3*S
qrt[a*Cos[e + f*x]^2]*Tan[e + f*x])/(2*f) + (Sqrt[a*Cos[e + f*x]^2]*Tan[e +
 f*x]^3)/(2*f)
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a-a\sin^2(e+fx)}\tan^4(e+fx)\,dx = \int \sqrt{a\cos^2(e+fx)}\tan^4(e+fx)\,dx$$
$$= \left(\sqrt{a\cos^2(e+fx)}\sec(e+fx)\right)\int \sin(e+fx)\tan^3(e+fx)\,dx$$
$$= \frac{\left(\sqrt{a\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^4}{\left(1-x^2\right)^2}\,dx,x,\sin(e+fx)\right)}{f}$$
$$= \frac{\sqrt{a\cos^2(e+fx)}\tan^3(e+fx)}{2f} - \frac{\left(3\sqrt{a\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{}{1}\right.}{2f}$$
$$= \frac{3\sqrt{a\cos^2(e+fx)}\tan(e+fx)}{2f} + \frac{\sqrt{a\cos^2(e+fx)}\tan^3(e+fx)}{2f} - \frac{\left(3\sqrt{a\cos^2\right.}{}$$
$$= -\frac{3\tanh^{-1}(\sin(e+fx))\sqrt{a\cos^2(e+fx)}\sec(e+fx)}{2f} + \frac{3\sqrt{a\cos^2(e+fx)}\tan}{2f}$$

Mathematica [A] time = 0.195319, size = 55, normalized size = 0.6

$$\frac{a\left((\cos(2(e+fx))+2)\tan(e+fx)-3\cos(e+fx)\tanh^{-1}(\sin(e+fx))\right)}{2f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Sin[e + f*x]^2]*Tan[e + f*x]^4,x]
```

```
[Out] (a*(-3*ArcTanh[Sin[e + f*x]]*Cos[e + f*x] + (2 + Cos[2*(e + f*x)])*Tan[e +
f*x]))/(2*f*Sqrt[a*Cos[e + f*x]^2])
```

Maple [A] time = 1.273, size = 84, normalized size = 0.9

$$\frac{a\left(4\left(\cos\left(fx+e\right)\right)^2\sin\left(fx+e\right)+2\sin\left(fx+e\right)+\left(-3\ln\left(1+\sin\left(fx+e\right)\right)+3\ln\left(-1+\sin\left(fx+e\right)\right)\right)\left(\cos\left(fx+e\right.\right.}{4f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x)
```

```
[Out] 1/4*a*(4*cos(f*x+e)^2*sin(f*x+e)+2*sin(f*x+e)+(-3*ln(1+sin(f*x+e))+3*ln(-1+
sin(f*x+e)))*cos(f*x+e)^2)/cos(f*x+e)/(a*cos(f*x+e)^2)^(1/2)/f
```

Maxima [B] time = 1.91776, size = 1116, normalized size = 12.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] -1/4*(2*(sin(5*f*x + 5*e) + 2*sin(3*f*x + 3*e) + sin(f*x + e))*cos(6*f*x +
6*e) - 6*(sin(4*f*x + 4*e) - sin(2*f*x + 2*e))*cos(5*f*x + 5*e) + 6*(2*sin(
3*f*x + 3*e) + sin(f*x + e))*cos(4*f*x + 4*e) + 3*(2*(2*cos(3*f*x + 3*e) +
cos(f*x + e))*cos(5*f*x + 5*e) + cos(5*f*x + 5*e)^2 + 4*cos(3*f*x + 3*e)^2
+ 4*cos(3*f*x + 3*e)*cos(f*x + e) + cos(f*x + e)^2 + 2*(2*sin(3*f*x + 3*e)
+ sin(f*x + e))*sin(5*f*x + 5*e) + sin(5*f*x + 5*e)^2 + 4*sin(3*f*x + 3*e)^
2 + 4*sin(3*f*x + 3*e)*sin(f*x + e) + sin(f*x + e)^2)*log(cos(f*x + e)^2 +
sin(f*x + e)^2 + 2*sin(f*x + e) + 1) - 3*(2*(2*cos(3*f*x + 3*e) + cos(f*x +
 e))*cos(5*f*x + 5*e) + cos(5*f*x + 5*e)^2 + 4*cos(3*f*x + 3*e)^2 + 4*cos(3
*f*x + 3*e)*cos(f*x + e) + cos(f*x + e)^2 + 2*(2*sin(3*f*x + 3*e) + sin(f*x
 + e))*sin(5*f*x + 5*e) + sin(5*f*x + 5*e)^2 + 4*sin(3*f*x + 3*e)^2 + 4*sin
(3*f*x + 3*e)*sin(f*x + e) + sin(f*x + e)^2)*log(cos(f*x + e)^2 + sin(f*x +
 e)^2 - 2*sin(f*x + e) + 1) - 2*(cos(5*f*x + 5*e) + 2*cos(3*f*x + 3*e) + co
s(f*x + e))*sin(6*f*x + 6*e) + 2*(3*cos(4*f*x + 4*e) - 3*cos(2*f*x + 2*e) -
 1)*sin(5*f*x + 5*e) - 6*(2*cos(3*f*x + 3*e) + cos(f*x + e))*sin(4*f*x + 4*
e) - 4*(3*cos(2*f*x + 2*e) + 1)*sin(3*f*x + 3*e) + 12*cos(3*f*x + 3*e)*sin(
2*f*x + 2*e) + 6*cos(f*x + e)*sin(2*f*x + 2*e) - 6*cos(2*f*x + 2*e)*sin(f*x
 + e) - 2*sin(f*x + e))*sqrt(a)/((2*(2*cos(3*f*x + 3*e) + cos(f*x + e))*cos
(5*f*x + 5*e) + cos(5*f*x + 5*e)^2 + 4*cos(3*f*x + 3*e)^2 + 4*cos(3*f*x + 3
*e)*cos(f*x + e) + cos(f*x + e)^2 + 2*(2*sin(3*f*x + 3*e) + sin(f*x + e))*s
in(5*f*x + 5*e) + sin(5*f*x + 5*e)^2 + 4*sin(3*f*x + 3*e)^2 + 4*sin(3*f*x +
 3*e)*sin(f*x + e) + sin(f*x + e)^2)*f)
```

Fricas [A] time = 1.6846, size = 204, normalized size = 2.24

$$-\frac{\sqrt{a\cos\left(fx+e\right)^{2}}\left(3\cos\left(fx+e\right)^{2}\log\left(-\frac{\sin\left(fx+e\right)+1}{\sin\left(fx+e\right)-1}\right)-2\left(2\cos\left(fx+e\right)^{2}+1\right)\sin\left(fx+e\right)\right)}{4f\cos\left(fx+e\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(a*cos(f*x + e)^2)*(3*cos(f*x + e)^2*log(-(sin(f*x + e) + 1)/(sin(
f*x + e) - 1)) - 2*(2*cos(f*x + e)^2 + 1)*sin(f*x + e))/(f*cos(f*x + e)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}\tan^{4}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)**2)**(1/2)*tan(f*x+e)**4,x)
```

```
[Out] Integral(sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))*tan(e + f*x)**4, x)
```

Giac [B] time = 1.98275, size = 285, normalized size = 3.13

$$\frac{\left(3 \log\left(\left|\frac{1}{\tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)}+\tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)+2\right|\right) \operatorname{sgn}\left(\tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{4}-1\right)-3 \log\left(\left|\frac{1}{\tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)}+\tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)-\right.\right.}{4 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] 1/4*(3*log(abs(1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*e) + 2))*sgn(tan(
1/2*f*x + 1/2*e)^4 - 1) - 3*log(abs(1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x +
1/2*e) - 2))*sgn(tan(1/2*f*x + 1/2*e)^4 - 1) - 4*(3*(1/tan(1/2*f*x + 1/2*e)
 + tan(1/2*f*x + 1/2*e))^2*sgn(tan(1/2*f*x + 1/2*e)^4 - 1) - 8*sgn(tan(1/2*
f*x + 1/2*e)^4 - 1))/((1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*e))^3 - 4
/tan(1/2*f*x + 1/2*e) - 4*tan(1/2*f*x + 1/2*e)))*sqrt(a)/f
```

3.464 $\int \sqrt{a-a\sin^2(e+fx)}\tan^2(e+fx)\,dx$

Optimal. Leaf size=57

$$\frac{\sec(e+fx)\sqrt{a\cos^2(e+fx)}\tanh^{-1}(\sin(e+fx))}{f}-\frac{\tan(e+fx)\sqrt{a\cos^2(e+fx)}}{f}$$

```
[Out] (ArcTanh[Sin[e + f*x]]*Sqrt[a*Cos[e + f*x]^2]*Sec[e + f*x])/f - (Sqrt[a*Cos
[e + f*x]^2]*Tan[e + f*x])/f
```

Rubi [A] time = 0.103233, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3176, 3207, 2592, 321, 206}

$$\frac{\sec(e+fx)\sqrt{a\cos^2(e+fx)}\tanh^{-1}(\sin(e+fx))}{f}-\frac{\tan(e+fx)\sqrt{a\cos^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Sin[e + f*x]^2]*Tan[e + f*x]^2,x]
```

```
[Out] (ArcTanh[Sin[e + f*x]]*Sqrt[a*Cos[e + f*x]^2]*Sec[e + f*x])/f - (Sqrt[a*Cos
[e + f*x]^2]*Tan[e + f*x])/f
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{a-a\sin^2(e+fx)}\tan^2(e+fx)\,dx &= \int \sqrt{a\cos^2(e+fx)}\tan^2(e+fx)\,dx\\ &= \left(\sqrt{a\cos^2(e+fx)}\sec(e+fx)\right)\int \sin(e+fx)\tan(e+fx)\,dx\\ &= \frac{\left(\sqrt{a\cos^2(e+fx)}\sec(e+fx)\right)\mathrm{Subst}\left(\int \frac{x^2}{1-x^2}\,dx,x,\sin(e+fx)\right)}{f}\\ &= -\frac{\sqrt{a\cos^2(e+fx)}\tan(e+fx)}{f}+\frac{\left(\sqrt{a\cos^2(e+fx)}\sec(e+fx)\right)\mathrm{Subst}\left(\int \frac{}{1-}\right.}{f}\\ &= \frac{\tanh^{-1}(\sin(e+fx))\sqrt{a\cos^2(e+fx)}\sec(e+fx)}{f}-\frac{\sqrt{a\cos^2(e+fx)}\tan(e+}{f}\end{aligned}$$

Mathematica [A] time = 0.0461756, size = 40, normalized size = 0.7

$$\frac{\sec(e+fx)\sqrt{a\cos^2(e+fx)}\left(\tanh^{-1}(\sin(e+fx))-\sin(e+fx)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Sin[e + f*x]^2]*Tan[e + f*x]^2,x]
```

```
[Out] (Sqrt[a*Cos[e + f*x]^2]*Sec[e + f*x]*(ArcTanh[Sin[e + f*x]] - Sin[e + f*x])
)/f
```

Maple [A] time = 1.321, size = 54, normalized size = 1.

$$-\frac{a\cos(fx+e)\left(2\sin(fx+e)+\ln(-1+\sin(fx+e))-\ln(1+\sin(fx+e))\right)}{2f}\frac{1}{\sqrt{a(\cos(fx+e))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x)
```

```
[Out] -1/2*a*cos(f*x+e)*(2*sin(f*x+e)+ln(-1+sin(f*x+e))-ln(1+sin(f*x+e)))/(a*cos(
f*x+e)^2)^(1/2)/f
```

Maxima [A] time = 1.69868, size = 99, normalized size = 1.74

$$\frac{\sqrt{a}\left(\log\left(\cos(fx+e)^2+\sin(fx+e)^2+2\sin(fx+e)+1\right)-\log\left(\cos(fx+e)^2+\sin(fx+e)^2-2\sin(fx+e)+1\right.\right.}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(a)*(log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin(f*x + e) + 1) - lo
g(cos(f*x + e)^2 + sin(f*x + e)^2 - 2*sin(f*x + e) + 1) - 2*sin(f*x + e))/f
```

Fricas [A] time = 1.69902, size = 147, normalized size = 2.58

$$-\frac{\sqrt{a\cos\left(fx+e\right)^{2}}\left(\log\left(-\frac{\sin\left(fx+e\right)-1}{\sin\left(fx+e\right)+1}\right)+2\,\sin\left(fx+e\right)\right)}{2\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(a*cos(f*x + e)^2)*(log(-(sin(f*x + e) - 1)/(sin(f*x + e) + 1)) +
2*sin(f*x + e))/(f*cos(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}\tan^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)**2)**(1/2)*tan(f*x+e)**2,x)
```

```
[Out] Integral(sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))*tan(e + f*x)**2, x)
```

Giac [B] time = 1.37759, size = 184, normalized size = 3.23

$$-\frac{\left(\log\left(\left|\frac{1}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+2\right|\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-1\right)-\log\left(\left|\frac{1}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-2\right|\right)\mathrm{sg}\right.}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] -1/2*(log(abs(1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*e) + 2))*sgn(tan(1
/2*f*x + 1/2*e)^4 - 1) - log(abs(1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2
*e) - 2))*sgn(tan(1/2*f*x + 1/2*e)^4 - 1) - 4*sgn(tan(1/2*f*x + 1/2*e)^4 -
1)/(1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*e)))*sqrt(a)/f
```

3.465 $\int \cot^2(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$

Optimal. Leaf size=57

$$-\frac{\tan(e+fx)\sqrt{a\cos^2(e+fx)}}{f}-\frac{\csc(e+fx)\sec(e+fx)\sqrt{a\cos^2(e+fx)}}{f}$$

```
[Out] -((Sqrt[a*Cos[e + f*x]^2]*Csc[e + f*x]*Sec[e + f*x])/f) - (Sqrt[a*Cos[e + f
*x]^2]*Tan[e + f*x])/f
```

Rubi [A] time = 0.111669, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3176, 3207, 2590, 14}

$$-\frac{\tan(e+fx)\sqrt{a\cos^2(e+fx)}}{f}-\frac{\csc(e+fx)\sec(e+fx)\sqrt{a\cos^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a*Cos[e + f*x]^2]*Csc[e + f*x]*Sec[e + f*x])/f) - (Sqrt[a*Cos[e + f
*x]^2]*Tan[e + f*x])/f
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \cot^2(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx &= \int \sqrt{a\cos^2(e+fx)}\cot^2(e+fx)\,dx \\
&= \left(\sqrt{a\cos^2(e+fx)}\sec(e+fx)\right)\int \cos(e+fx)\cot^2(e+fx)\,dx \\
&= -\frac{\left(\sqrt{a\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{1-x^2}{x^2}\,dx,x,-\sin(e+fx)\right)}{f} \\
&= -\frac{\left(\sqrt{a\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)\,dx,x,-\sin(e+fx)\right)}{f} \\
&= -\frac{\sqrt{a\cos^2(e+fx)}\csc(e+fx)\sec(e+fx)}{f}-\frac{\sqrt{a\cos^2(e+fx)}\tan(e+fx)}{f}
\end{aligned}$$

Mathematica [A] time = 0.0753841, size = 35, normalized size = 0.61

$$-\frac{\tan(e+fx)\left(\csc^2(e+fx)+1\right)\sqrt{a\cos^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a*Cos[e + f*x]^2]*(1 + Csc[e + f*x]^2)*Tan[e + f*x])/f)
```

Maple [A] time = 0.707, size = 43, normalized size = 0.8

$$-\frac{\cos\left(fx+e\right)a\left(\left(\sin\left(fx+e\right)\right)^2+1\right)}{\sin\left(fx+e\right)f}\frac{1}{\sqrt{a\left(\cos\left(fx+e\right)\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a-a*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] -cos(f*x+e)*a*(sin(f*x+e)^2+1)/sin(f*x+e)/(a*cos(f*x+e)^2)^(1/2)/f
```

Maxima [A] time = 1.48685, size = 57, normalized size = 1.

$$-\frac{2\sqrt{a}\tan\left(fx+e\right)^2+\sqrt{a}}{\sqrt{\tan\left(fx+e\right)^2+1}f\tan\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -(2*sqrt(a)*tan(f*x + e)^2 + sqrt(a))/(sqrt(tan(f*x + e)^2 + 1)*f*tan(f*x +
 e))
```

Fricas [A] time = 1.57825, size = 101, normalized size = 1.77

$$\frac{\sqrt{a\cos\left(fx+e\right)^{2}}\left(\cos\left(fx+e\right)^{2}-2\right)}{f\cos\left(fx+e\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(a*cos(f*x + e)^2)*(cos(f*x + e)^2 - 2)/(f*cos(f*x + e)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}\cot^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a-a*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))*cot(e + f*x)**2, x)
```

Giac [A] time = 1.19357, size = 122, normalized size = 2.14

$$\frac{\left(\left(\frac{1}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-1\right)+\frac{4\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-1\right)}{\frac{1}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}\right)\sqrt{a}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*((1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*e))*sgn(tan(1/2*f*x + 1/2*
e)^4 - 1) + 4*sgn(tan(1/2*f*x + 1/2*e)^4 - 1)/(1/tan(1/2*f*x + 1/2*e) + tan
(1/2*f*x + 1/2*e)))*sqrt(a)/f
```

3.466 $\int \cot^4(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$

Optimal. Leaf size=91

$$\frac{\tan(e+fx)\sqrt{a\cos^2(e+fx)}}{f}-\frac{\csc^3(e+fx)\sec(e+fx)\sqrt{a\cos^2(e+fx)}}{3f}+\frac{2\csc(e+fx)\sec(e+fx)\sqrt{a\cos^2(e+fx)}}{f}$$

```
[Out] (2*Sqrt[a*Cos[e + f*x]^2]*Csc[e + f*x]*Sec[e + f*x])/f - (Sqrt[a*Cos[e + f*
x]^2]*Csc[e + f*x]^3*Sec[e + f*x])/(3*f) + (Sqrt[a*Cos[e + f*x]^2]*Tan[e +
f*x])/f
```

Rubi [A] time = 0.115127, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3176, 3207, 2590, 270}

$$\frac{\tan(e+fx)\sqrt{a\cos^2(e+fx)}}{f}-\frac{\csc^3(e+fx)\sec(e+fx)\sqrt{a\cos^2(e+fx)}}{3f}+\frac{2\csc(e+fx)\sec(e+fx)\sqrt{a\cos^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] (2*Sqrt[a*Cos[e + f*x]^2]*Csc[e + f*x]*Sec[e + f*x])/f - (Sqrt[a*Cos[e + f*
x]^2]*Csc[e + f*x]^3*Sec[e + f*x])/(3*f) + (Sqrt[a*Cos[e + f*x]^2]*Tan[e +
f*x])/f
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \cot^4(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx = \int \sqrt{a\cos^2(e+fx)}\cot^4(e+fx)\,dx$$
$$= \left(\sqrt{a\cos^2(e+fx)}\sec(e+fx)\right)\int \cos(e+fx)\cot^4(e+fx)\,dx$$
$$= -\frac{\left(\sqrt{a\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{(1-x^2)^2}{x^4}dx, x, -\sin(e+fx)\right)}{f}$$
$$= -\frac{\left(\sqrt{a\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \left(1+\frac{1}{x^4}-\frac{2}{x^2}\right)dx, x, -\sin(e+fx)\right)}{f}$$
$$= \frac{2\sqrt{a\cos^2(e+fx)}\csc(e+fx)\sec(e+fx)}{f} - \frac{\sqrt{a\cos^2(e+fx)}\csc^3(e+fx)\text{se}}{3f}$$

Mathematica [A] time = 0.0759273, size = 47, normalized size = 0.52

$$-\frac{\tan(e+fx)\left(\csc^4(e+fx)-6\csc^2(e+fx)-3\right)\sqrt{a\cos^2(e+fx)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4*Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] -(Sqrt[a*Cos[e + f*x]^2]*(-3 - 6*Csc[e + f*x]^2 + Csc[e + f*x]^4)*Tan[e + f
*x])/(3*f)
```

Maple [A] time = 0.624, size = 55, normalized size = 0.6

$$\frac{\cos(fx+e)\,a\left(3\left(\sin(fx+e)\right)^4+6\left(\sin(fx+e)\right)^2-1\right)}{3\left(\sin(fx+e)\right)^3 f}\frac{1}{\sqrt{a\left(\cos(fx+e)\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a-a*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3*cos(f*x+e)*a*(3*sin(f*x+e)^4+6*sin(f*x+e)^2-1)/sin(f*x+e)^3/(a*cos(f*x+
e)^2)^(1/2)/f
```

Maxima [A] time = 1.56029, size = 77, normalized size = 0.85

$$\frac{8\sqrt{a}\tan(fx+e)^4+4\sqrt{a}\tan(fx+e)^2-\sqrt{a}}{3\sqrt{\tan(fx+e)^2+1}f\tan(fx+e)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(8*sqrt(a)*tan(f*x + e)^4 + 4*sqrt(a)*tan(f*x + e)^2 - sqrt(a))/(sqrt(t
an(f*x + e)^2 + 1)*f*tan(f*x + e)^3)
```

Fricas [A] time = 1.65905, size = 166, normalized size = 1.82

$$-\frac{\left(3\,\cos\left(fx+e\right)^{4}-12\,\cos\left(fx+e\right)^{2}+8\right)\sqrt{a\cos\left(fx+e\right)^{2}}}{3\left(f\cos\left(fx+e\right)^{3}-f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(3*cos(f*x + e)^4 - 12*cos(f*x + e)^2 + 8)*sqrt(a*cos(f*x + e)^2)/((f*
cos(f*x + e)^3 - f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}\cot^{4}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a-a*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))*cot(e + f*x)**4, x)
```

Giac [A] time = 1.2405, size = 178, normalized size = 1.96

$$\frac{\left(\left(\frac{1}{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)^{3}\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-1\right)-24\left(\frac{1}{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\right.\right.}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*((1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*e))^3*sgn(tan(1/2*f*x + 1
/2*e)^4 - 1) - 24*(1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*e))*sgn(tan(1
/2*f*x + 1/2*e)^4 - 1) - 48*sgn(tan(1/2*f*x + 1/2*e)^4 - 1)/(1/tan(1/2*f*x
+ 1/2*e) + tan(1/2*f*x + 1/2*e)))*sqrt(a)/f
```

3.467 $\int \cot^6(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$

Optimal. Leaf size=124

$$-\frac{\tan(e+fx)\sqrt{a\cos^2(e+fx)}}{f}-\frac{\csc^5(e+fx)\sec(e+fx)\sqrt{a\cos^2(e+fx)}}{5f}+\frac{\csc^3(e+fx)\sec(e+fx)\sqrt{a\cos^2(e+fx)}}{f}$$

```
[Out] (-3*Sqrt[a*Cos[e + f*x]^2]*Csc[e + f*x]*Sec[e + f*x])/f + (Sqrt[a*Cos[e + f
*x]^2]*Csc[e + f*x]^3*Sec[e + f*x])/f - (Sqrt[a*Cos[e + f*x]^2]*Csc[e + f*x
]^5*Sec[e + f*x])/(5*f) - (Sqrt[a*Cos[e + f*x]^2]*Tan[e + f*x])/f
```

Rubi [A] time = 0.121511, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3176, 3207, 2590, 270}

$$-\frac{\tan(e+fx)\sqrt{a\cos^2(e+fx)}}{f}-\frac{\csc^5(e+fx)\sec(e+fx)\sqrt{a\cos^2(e+fx)}}{5f}+\frac{\csc^3(e+fx)\sec(e+fx)\sqrt{a\cos^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6*Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] (-3*Sqrt[a*Cos[e + f*x]^2]*Csc[e + f*x]*Sec[e + f*x])/f + (Sqrt[a*Cos[e + f
*x]^2]*Csc[e + f*x]^3*Sec[e + f*x])/f - (Sqrt[a*Cos[e + f*x]^2]*Csc[e + f*x
]^5*Sec[e + f*x])/(5*f) - (Sqrt[a*Cos[e + f*x]^2]*Tan[e + f*x])/f
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \cot^6(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx = \int \sqrt{a\cos^2(e+fx)}\cot^6(e+fx)\,dx$$
$$= \left(\sqrt{a\cos^2(e+fx)}\sec(e+fx)\right)\int \cos(e+fx)\cot^6(e+fx)\,dx$$
$$= -\frac{\left(\sqrt{a\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{(1-x^2)^3}{x^6}\,dx,x,-\sin(e+fx)\right)}{f}$$
$$= -\frac{\left(\sqrt{a\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \left(-1+\frac{1}{x^6}-\frac{3}{x^4}+\frac{3}{x^2}\right)dx,x,-\sin(e+f$$
$$= -\frac{3\sqrt{a\cos^2(e+fx)}\csc(e+fx)\sec(e+fx)}{f} + \frac{\sqrt{a\cos^2(e+fx)}\csc^3(e+fx)\sec($$

Mathematica [A] time = 0.195662, size = 67, normalized size = 0.54

$$\frac{(235\cos(2(e+fx))-90\cos(4(e+fx))+5\cos(6(e+fx))-182)\csc^5(e+fx)\sec(e+fx)\sqrt{a\cos^2(e+fx)}}{160f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6*Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] (Sqrt[a*Cos[e + f*x]^2]*(-182 + 235*Cos[2*(e + f*x)] - 90*Cos[4*(e + f*x)]
+ 5*Cos[6*(e + f*x)])*Csc[e + f*x]^5*Sec[e + f*x])/(160*f)
```

Maple [A] time = 0.803, size = 65, normalized size = 0.5

$$-\frac{\cos(fx+e)\,a\left(5\left(\sin(fx+e)\right)^6+15\left(\sin(fx+e)\right)^4-5\left(\sin(fx+e)\right)^2+1\right)}{5\left(\sin(fx+e)\right)^5 f}\frac{1}{\sqrt{a\left(\cos(fx+e)\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6*(a-a*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/5*cos(f*x+e)*a*(5*sin(f*x+e)^6+15*sin(f*x+e)^4-5*sin(f*x+e)^2+1)/sin(f*x
+e)^5/(a*cos(f*x+e)^2)^(1/2)/f
```

Maxima [A] time = 1.50161, size = 92, normalized size = 0.74

$$-\frac{16\sqrt{a}\tan(fx+e)^6+8\sqrt{a}\tan(fx+e)^4-2\sqrt{a}\tan(fx+e)^2+\sqrt{a}}{5\sqrt{\tan(fx+e)^2+1}\,f\tan(fx+e)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/5*(16*sqrt(a)*tan(f*x + e)^6 + 8*sqrt(a)*tan(f*x + e)^4 - 2*sqrt(a)*tan(
f*x + e)^2 + sqrt(a))/(sqrt(tan(f*x + e)^2 + 1)*f*tan(f*x + e)^5)
```

Fricas [A] time = 1.64918, size = 221, normalized size = 1.78

$$\frac{\left(5\cos\left(fx+e\right)^{6}-30\cos\left(fx+e\right)^{4}+40\cos\left(fx+e\right)^{2}-16\right)\sqrt{a\cos\left(fx+e\right)^{2}}}{5\left(f\cos\left(fx+e\right)^{5}-2f\cos\left(fx+e\right)^{3}+f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/5*(5*cos(f*x + e)^6 - 30*cos(f*x + e)^4 + 40*cos(f*x + e)^2 - 16)*sqrt(a*
cos(f*x + e)^2)/((f*cos(f*x + e)^5 - 2*f*cos(f*x + e)^3 + f*cos(f*x + e))*s
in(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6*(a-a*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24039, size = 235, normalized size = 1.9

$$\frac{\left(\left(\frac{1}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)^{5}\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-1\right)-20\left(\frac{1}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)^{3}\operatorname{sgn}\left(\tan\left(\frac{1}{2}f\right.\right.\right.}{160f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/160*((1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*e))^5*sgn(tan(1/2*f*x +
1/2*e)^4 - 1) - 20*(1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*e))^3*sgn(ta
n(1/2*f*x + 1/2*e)^4 - 1) + 240*(1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2
*e))*sgn(tan(1/2*f*x + 1/2*e)^4 - 1) + 320*sgn(tan(1/2*f*x + 1/2*e)^4 - 1)/
(1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*e)))*sqrt(a)/f
```

3.468 $\int \frac{\tan^5(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=65

$$\frac{a^2}{5f\left(a\cos^2(e+fx)\right)^{5/2}}-\frac{2a}{3f\left(a\cos^2(e+fx)\right)^{3/2}}+\frac{1}{f\sqrt{a\cos^2(e+fx)}}$$

```
[Out] a^2/(5*f*(a*Cos[e + f*x]^2)^(5/2)) - (2*a)/(3*f*(a*Cos[e + f*x]^2)^(3/2)) +
 1/(f*Sqrt[a*Cos[e + f*x]^2])
```

Rubi [A] time = 0.11512, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3176, 3205, 16, 43}

$$\frac{a^2}{5f\left(a\cos^2(e+fx)\right)^{5/2}}-\frac{2a}{3f\left(a\cos^2(e+fx)\right)^{3/2}}+\frac{1}{f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] a^2/(5*f*(a*Cos[e + f*x]^2)^(5/2)) - (2*a)/(3*f*(a*Cos[e + f*x]^2)^(3/2)) +
 1/(f*Sqrt[a*Cos[e + f*x]^2])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\tan^5(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx=\int \frac{\tan^5(e+fx)}{\sqrt{a\cos^2(e+fx)}}\,dx$$
$$=-\frac{\text{Subst}\left(\int \frac{(1-x)^2}{x^3\sqrt{ax}}\,dx,x,\cos^2(e+fx)\right)}{2f}$$
$$=-\frac{a^3\,\text{Subst}\left(\int \frac{(1-x)^2}{(ax)^{7/2}}\,dx,x,\cos^2(e+fx)\right)}{2f}$$
$$=-\frac{a^3\,\text{Subst}\left(\int \left(\frac{1}{(ax)^{7/2}}-\frac{2}{a(ax)^{5/2}}+\frac{1}{a^2(ax)^{3/2}}\right)dx,x,\cos^2(e+fx)\right)}{2f}$$
$$=\frac{a^2}{5f\left(a\cos^2(e+fx)\right)^{5/2}}-\frac{2a}{3f\left(a\cos^2(e+fx)\right)^{3/2}}+\frac{1}{f\sqrt{a\cos^2(e+fx)}}$$

Mathematica [A] time = 0.0811033, size = 43, normalized size = 0.66

$$\frac{3\sec^4(e+fx)-10\sec^2(e+fx)+15}{15f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5/Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] (15 - 10*Sec[e + f*x]^2 + 3*Sec[e + f*x]^4)/(15*f*Sqrt[a*Cos[e + f*x]^2])
```

Maple [A] time = 1.846, size = 51, normalized size = 0.8

$$\frac{15\left(\cos\left(fx+e\right)\right)^4-10\left(\cos\left(fx+e\right)\right)^2+3}{15a\left(\cos\left(fx+e\right)\right)^6f}\sqrt{a\left(\cos\left(fx+e\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a-a*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/15/a/cos(f*x+e)^6*(a*cos(f*x+e)^2)^(1/2)*(15*cos(f*x+e)^4-10*cos(f*x+e)^2
+3)/f
```

Maxima [A] time = 1.03155, size = 93, normalized size = 1.43

$$\frac{15\left(a\sin\left(fx+e\right)^2-a\right)^2a^3+10\left(a\sin\left(fx+e\right)^2-a\right)a^4+3a^5}{15\left(-a\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}a^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/15*(15*(a*sin(f*x + e)^2 - a)^2*a^3 + 10*(a*sin(f*x + e)^2 - a)*a^4 + 3*a
^5)/((-a*sin(f*x + e)^2 + a)^(5/2)*a^3*f)
```

Fricas [A] time = 1.61127, size = 127, normalized size = 1.95

$$\frac{\left(15 \cos\left(fx+e\right)^{4}-10 \cos\left(fx+e\right)^{2}+3\right)\sqrt{a\cos\left(fx+e\right)^{2}}}{15\,af\cos\left(fx+e\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(15*cos(f*x + e)^4 - 10*cos(f*x + e)^2 + 3)*sqrt(a*cos(f*x + e)^2)/(a*
f*cos(f*x + e)^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^{5}\left(e+fx\right)}{\sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5/(a-a*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**5/sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1)), x)
```

Giac [A] time = 2.91545, size = 96, normalized size = 1.48

$$\frac{16\left(10\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+1\right)}{15\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-1\right)^{5}\sqrt{a}f\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 16/15*(10*tan(1/2*f*x + 1/2*e)^4 - 5*tan(1/2*f*x + 1/2*e)^2 + 1)/((tan(1/2*
f*x + 1/2*e)^2 - 1)^5*sqrt(a)*f*sgn(tan(1/2*f*x + 1/2*e)^4 - 1))
```

3.469 $\int \frac{\tan^3(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=42

$$\frac{a}{3f\left(a\cos^2(e+fx)\right)^{3/2}}-\frac{1}{f\sqrt{a\cos^2(e+fx)}}$$

```
[Out] a/(3*f*(a*Cos[e + f*x]^2)^(3/2)) - 1/(f*Sqrt[a*Cos[e + f*x]^2])
```

Rubi [A] time = 0.107025, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3176, 3205, 16, 43}

$$\frac{a}{3f\left(a\cos^2(e+fx)\right)^{3/2}}-\frac{1}{f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] a/(3*f*(a*Cos[e + f*x]^2)^(3/2)) - 1/(f*Sqrt[a*Cos[e + f*x]^2])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\tan^3(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \int \frac{\tan^3(e+fx)}{\sqrt{a\cos^2(e+fx)}}\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{1-x}{x^2\sqrt{ax}}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{a^2\,\text{Subst}\left(\int \frac{1-x}{(ax)^{5/2}}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{a^2\,\text{Subst}\left(\int \left(\frac{1}{(ax)^{5/2}} - \frac{1}{a(ax)^{3/2}}\right)dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= \frac{a}{3f\left(a\cos^2(e+fx)\right)^{3/2}} - \frac{1}{f\sqrt{a\cos^2(e+fx)}}$$

Mathematica [A] time = 0.0638454, size = 31, normalized size = 0.74

$$\frac{\sec^2(e+fx)-3}{3f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3/Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] (-3 + Sec[e + f*x]^2)/(3*f*Sqrt[a*Cos[e + f*x]^2])
```

Maple [A] time = 1.779, size = 41, normalized size = 1.

$$-\frac{3\left(\cos\left(fx+e\right)\right)^2-1}{3a\left(\cos\left(fx+e\right)\right)^4f}\sqrt{a\left(\cos\left(fx+e\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(a-a*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/3/a/cos(f*x+e)^4*(a*cos(f*x+e)^2)^(1/2)*(3*cos(f*x+e)^2-1)/f
```

Maxima [A] time = 1.0294, size = 62, normalized size = 1.48

$$\frac{3\left(a\sin\left(fx+e\right)^2-a\right)a^2+a^3}{3\left(-a\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}a^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*(a*sin(f*x + e)^2 - a)*a^2 + a^3)/((-a*sin(f*x + e)^2 + a)^(3/2)*a^2
*f)
```

Fricas [A] time = 1.61233, size = 99, normalized size = 2.36

$$-\frac{\sqrt{a\cos\left(fx+e\right)^2}\left(3\cos\left(fx+e\right)^2-1\right)}{3\,af\cos\left(fx+e\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(a*cos(f*x + e)^2)*(3*cos(f*x + e)^2 - 1)/(a*f*cos(f*x + e)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\tan^3\left(e+fx\right)}{\sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a-a*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**3/sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1)), x)
```

Giac [A] time = 1.65157, size = 77, normalized size = 1.83

$$\frac{4\left(3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)}{3\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^3\sqrt{a}f\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 4/3*(3*tan(1/2*f*x + 1/2*e)^2 - 1)/((tan(1/2*f*x + 1/2*e)^2 - 1)^3*sqrt(a)*
f*sgn(tan(1/2*f*x + 1/2*e)^4 - 1))
```

3.470 $\int \frac{\tan(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=18

$$\frac{1}{f\sqrt{a\cos^2(e+fx)}}$$

```
[Out] 1/(f*Sqrt[a*Cos[e + f*x]^2])
```

Rubi [A] time = 0.0645136, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3176, 3205, 16, 32}

$$\frac{1}{f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] 1/(f*Sqrt[a*Cos[e + f*x]^2])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tan(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \int \frac{\tan(e+fx)}{\sqrt{a\cos^2(e+fx)}}\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{ax}}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{a\,\text{Subst}\left(\int \frac{1}{(ax)^{3/2}}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= \frac{1}{f\sqrt{a\cos^2(e+fx)}}$$

Mathematica [A] time = 0.0265371, size = 18, normalized size = 1.

$$\frac{1}{f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] 1/(f*Sqrt[a*Cos[e + f*x]^2])
```

Maple [A] time = 0.148, size = 20, normalized size = 1.1

$$\frac{1}{f}\frac{1}{\sqrt{a-a\left(\sin\left(fx+e\right)\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a-a*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/f/(a-a*sin(f*x+e)^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.5705, size = 61, normalized size = 3.39

$$\frac{\sqrt{a\cos\left(fx+e\right)^2}}{af\cos\left(fx+e\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(a*cos(f*x + e)^2)/(a*f*cos(f*x + e)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(e+fx\right)}{\sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a-a*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)/sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1)), x)
```

Giac [B] time = 1.21751, size = 55, normalized size = 3.06

$$\frac{2}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-1\right)\sqrt{a}f\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/((tan(1/2*f*x + 1/2*e)^2 - 1)*sqrt(a)*f*sgn(tan(1/2*f*x + 1/2*e)^4 - 1))
```

3.471 $\int \frac{\cot(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=31

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}$$

```
[Out] -(ArcTanh[Sqrt[a*Cos[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f))
```

Rubi [A] time = 0.0758756, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3176, 3205, 63, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[Sqrt[a*Cos[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f))
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \int \frac{\cot(e+fx)}{\sqrt{a\cos^2(e+fx)}}\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{ax}}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a\cos^2(e+fx)}\right)}{af}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}$$

Mathematica [A] time = 0.0430917, size = 49, normalized size = 1.58

$$\frac{\cos(e+fx)\left(\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)\right)}{f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]/Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] (Cos[e + f*x]*(-Log[Cos[(e + f*x)/2]] + Log[Sin[(e + f*x)/2]]))/(f*Sqrt[a*C
os[e + f*x]^2])
```

Maple [A] time = 0.797, size = 40, normalized size = 1.3

$$-\frac{1}{f}\ln\left(\frac{1}{\sin\left(fx+e\right)}\left(2a+2\sqrt{a}\sqrt{a\left(\cos\left(fx+e\right)\right)^2}\right)\right)\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a-a*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/a^(1/2)*ln((2*a+2*a^(1/2)*(a*cos(f*x+e)^2)^(1/2))/sin(f*x+e))/f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.7319, size = 208, normalized size = 6.71

$$\left[-\frac{\sqrt{a\cos\left(fx+e\right)^{2}}\log\left(-\frac{\cos\left(fx+e\right)+1}{\cos\left(fx+e\right)-1}\right)}{2\,af\cos\left(fx+e\right)}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{a\cos\left(fx+e\right)^{2}}\sqrt{-a}}{a}\right)}{af}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(a*cos(f*x + e)^2)*log(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1))/(a
*f*cos(f*x + e)), sqrt(-a)*arctan(sqrt(a*cos(f*x + e)^2)*sqrt(-a)/a)/(a*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cot\left(e+fx\right)}{\sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a-a*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)/sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1)), x)
```

Giac [A] time = 1.13288, size = 43, normalized size = 1.39

$$\frac{\arctan\left(\frac{\sqrt{-a\sin\left(fx+e\right)^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(-a*sin(f*x + e)^2 + a)/sqrt(-a))/(sqrt(-a)*f)
```

3.472 $\int \frac{\cot^3(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=66

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{a}f}-\frac{\csc^2(e+fx)\sqrt{a\cos^2(e+fx)}}{2af}$$

```
[Out] ArcTanh[Sqrt[a*Cos[e + f*x]^2]/Sqrt[a]]/(2*Sqrt[a]*f) - (Sqrt[a*Cos[e + f*x
]^2]*Csc[e + f*x]^2)/(2*a*f)
```

Rubi [A] time = 0.11372, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3176, 3205, 16, 47, 63, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{a}f}-\frac{\csc^2(e+fx)\sqrt{a\cos^2(e+fx)}}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a*Cos[e + f*x]^2]/Sqrt[a]]/(2*Sqrt[a]*f) - (Sqrt[a*Cos[e + f*x
]^2]*Csc[e + f*x]^2)/(2*a*f)
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^3(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx &= \int \frac{\cot^3(e+fx)}{\sqrt{a\cos^2(e+fx)}}\,dx\\
&= -\frac{\text{Subst}\left(\int \frac{x}{(1-x)^2\sqrt{ax}}\,dx, x, \cos^2(e+fx)\right)}{2f}\\
&= -\frac{\text{Subst}\left(\int \frac{\sqrt{ax}}{(1-x)^2}\,dx, x, \cos^2(e+fx)\right)}{2af}\\
&= -\frac{\sqrt{a\cos^2(e+fx)}\csc^2(e+fx)}{2af}+\frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{ax}}\,dx, x, \cos^2(e+fx)\right)}{4f}\\
&= -\frac{\sqrt{a\cos^2(e+fx)}\csc^2(e+fx)}{2af}+\frac{\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a\cos^2(e+fx)}\right)}{2af}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{a}f}-\frac{\sqrt{a\cos^2(e+fx)}\csc^2(e+fx)}{2af}
\end{aligned}
$$

Mathematica [A] time = 0.175847, size = 80, normalized size = 1.21

$$
\frac{\cos(e+fx)\left(-\csc^2\left(\frac{1}{2}(e+fx)\right)+\sec^2\left(\frac{1}{2}(e+fx)\right)-4\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)+4\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)\right)}{8f\sqrt{a\cos^2(e+fx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3/Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] (Cos[e + f*x]*(-Csc[(e + f*x)/2]^2 + 4*Log[Cos[(e + f*x)/2]] - 4*Log[Sin[(e
 + f*x)/2]] + Sec[(e + f*x)/2]^2))/(8*f*Sqrt[a*Cos[e + f*x]^2])
```

Maple [A] time = 1.388, size = 69, normalized size = 1.1

$$
\frac{1}{2f}\ln\left(\frac{1}{\sin(fx+e)}\left(2a+2\sqrt{a}\sqrt{a(\cos(fx+e))^2}\right)\right)\frac{1}{\sqrt{a}}-\frac{1}{2af(\sin(fx+e))^2}\sqrt{a(\cos(fx+e))^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a-a*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/2/a^(1/2)*ln((2*a+2*a^(1/2)*(a*cos(f*x+e)^2)^(1/2))/sin(f*x+e))/f-1/2/f/a
/sin(f*x+e)^2*(a*cos(f*x+e)^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.66626, size = 207, normalized size = 3.14

$$-\frac{\sqrt{a\cos\left(fx+e\right)^{2}}\left(\left(\cos\left(fx+e\right)^{2}-1\right)\log\left(-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}\right)-2\cos\left(fx+e\right)\right)}{4\left(af\cos\left(fx+e\right)^{3}-af\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(a*cos(f*x + e)^2)*((cos(f*x + e)^2 - 1)*log(-(cos(f*x + e) - 1)/(
cos(f*x + e) + 1)) - 2*cos(f*x + e))/(a*f*cos(f*x + e)^3 - a*f*cos(f*x + e)
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^{3}\left(e+fx\right)}{\sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a-a*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**3/sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1)), x)
```

Giac [B] time = 1.34266, size = 158, normalized size = 2.39

$$-\frac{\frac{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-1\right)}-\frac{2\log\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}\right)}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-1\right)}+\frac{2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-1}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-1\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}}}{8\sqrt{a}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*(tan(1/2*f*x + 1/2*e)^2/sgn(tan(1/2*f*x + 1/2*e)^4 - 1) - 2*log(tan(1/
2*f*x + 1/2*e)^2)/sgn(tan(1/2*f*x + 1/2*e)^4 - 1) + (2*tan(1/2*f*x + 1/2*e)
^2 - 1)/(sgn(tan(1/2*f*x + 1/2*e)^4 - 1)*tan(1/2*f*x + 1/2*e)^2))/(sqrt(a)*
f)
```

3.473 $\int \frac{\tan^4(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=91

$$\frac{\tan^3(e+fx)}{4f\sqrt{a\cos^2(e+fx)}}-\frac{3\tan(e+fx)}{8f\sqrt{a\cos^2(e+fx)}}+\frac{3\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{8f\sqrt{a\cos^2(e+fx)}}$$

```
[Out] (3*ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(8*f*Sqrt[a*Cos[e + f*x]^2]) - (3*Ta
n[e + f*x])/(8*f*Sqrt[a*Cos[e + f*x]^2]) + Tan[e + f*x]^3/(4*f*Sqrt[a*Cos[e
 + f*x]^2])
```

Rubi [A] time = 0.138509, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3176, 3207, 2611, 3770}

$$\frac{\tan^3(e+fx)}{4f\sqrt{a\cos^2(e+fx)}}-\frac{3\tan(e+fx)}{8f\sqrt{a\cos^2(e+fx)}}+\frac{3\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{8f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] (3*ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(8*f*Sqrt[a*Cos[e + f*x]^2]) - (3*Ta
n[e + f*x])/(8*f*Sqrt[a*Cos[e + f*x]^2]) + Tan[e + f*x]^3/(4*f*Sqrt[a*Cos[e
 + f*x]^2])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \int \frac{\tan^4(e+fx)}{\sqrt{a\cos^2(e+fx)}}\,dx$$
$$= \frac{\cos(e+fx)\int \sec(e+fx)\tan^4(e+fx)\,dx}{\sqrt{a\cos^2(e+fx)}}$$
$$= \frac{\tan^3(e+fx)}{4f\sqrt{a\cos^2(e+fx)}} - \frac{(3\cos(e+fx))\int \sec(e+fx)\tan^2(e+fx)\,dx}{4\sqrt{a\cos^2(e+fx)}}$$
$$= -\frac{3\tan(e+fx)}{8f\sqrt{a\cos^2(e+fx)}} + \frac{\tan^3(e+fx)}{4f\sqrt{a\cos^2(e+fx)}} + \frac{(3\cos(e+fx))\int \sec(e+fx)\,dx}{8\sqrt{a\cos^2(e+fx)}}$$
$$= \frac{3\tanh^{-1}(\sin(e+fx))\cos(e+fx)}{8f\sqrt{a\cos^2(e+fx)}} - \frac{3\tan(e+fx)}{8f\sqrt{a\cos^2(e+fx)}} + \frac{\tan^3(e+fx)}{4f\sqrt{a\cos^2(e+fx)}}$$

Mathematica [A] time = 0.129723, size = 66, normalized size = 0.73

$$\frac{\tan(e+fx)\left(8\tan^2(e+fx)-6\sec^2(e+fx)+3\right)+3\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{8f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] (3*ArcTanh[Sin[e + f*x]]*Cos[e + f*x] + Tan[e + f*x]*(3 - 6*Sec[e + f*x]^2
+ 8*Tan[e + f*x]^2))/(8*f*Sqrt[a*Cos[e + f*x]^2])
```

Maple [A] time = 1.333, size = 103, normalized size = 1.1

$$\frac{10\left(\cos\left(fx+e\right)\right)^2\sin\left(fx+e\right)-4\sin\left(fx+e\right)+\left(-3\ln\left(1+\sin\left(fx+e\right)\right)+3\ln\left(-1+\sin\left(fx+e\right)\right)\right)\left(\cos\left(fx+e\right)\right.}{\left(16+16\sin\left(fx+e\right)\right)\left(-1+\sin\left(fx+e\right)\right)\cos\left(fx+e\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a-a*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/16*(10*cos(f*x+e)^2*sin(f*x+e)-4*sin(f*x+e)+(-3*ln(1+sin(f*x+e))+3*ln(-1+
sin(f*x+e)))*cos(f*x+e)^4)/(1+sin(f*x+e))/(-1+sin(f*x+e))/cos(f*x+e)/(a*cos
(f*x+e)^2)^(1/2)/f
```

Maxima [B] time = 2.68279, size = 2049, normalized size = 22.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*(4*(5*sin(7*f*x + 7*e) - 3*sin(5*f*x + 5*e) + 3*sin(3*f*x + 3*e) - 5*
sin(f*x + e))*cos(8*f*x + 8*e) - 40*(2*sin(6*f*x + 6*e) + 3*sin(4*f*x + 4*e
) + 2*sin(2*f*x + 2*e))*cos(7*f*x + 7*e) - 16*(3*sin(5*f*x + 5*e) - 3*sin(3
*f*x + 3*e) + 5*sin(f*x + e))*cos(6*f*x + 6*e) + 24*(3*sin(4*f*x + 4*e) + 2
*sin(2*f*x + 2*e))*cos(5*f*x + 5*e) + 24*(3*sin(3*f*x + 3*e) - 5*sin(f*x +
e))*cos(4*f*x + 4*e) - 3*(2*(4*cos(6*f*x + 6*e) + 6*cos(4*f*x + 4*e) + 4*co
s(2*f*x + 2*e) + 1)*cos(8*f*x + 8*e) + cos(8*f*x + 8*e)^2 + 8*(6*cos(4*f*x
+ 4*e) + 4*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + 16*cos(6*f*x + 6*e)^2 +
 12*(4*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 36*cos(4*f*x + 4*e)^2 + 16*
cos(2*f*x + 2*e)^2 + 4*(2*sin(6*f*x + 6*e) + 3*sin(4*f*x + 4*e) + 2*sin(2*f
*x + 2*e))*sin(8*f*x + 8*e) + sin(8*f*x + 8*e)^2 + 16*(3*sin(4*f*x + 4*e) +
 2*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 16*sin(6*f*x + 6*e)^2 + 36*sin(4*f*
x + 4*e)^2 + 48*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 16*sin(2*f*x + 2*e)^2 +
 8*cos(2*f*x + 2*e) + 1)*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin(f*x +
e) + 1) + 3*(2*(4*cos(6*f*x + 6*e) + 6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e
) + 1)*cos(8*f*x + 8*e) + cos(8*f*x + 8*e)^2 + 8*(6*cos(4*f*x + 4*e) + 4*co
s(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + 16*cos(6*f*x + 6*e)^2 + 12*(4*cos(2*
f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 36*cos(4*f*x + 4*e)^2 + 16*cos(2*f*x + 2
*e)^2 + 4*(2*sin(6*f*x + 6*e) + 3*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*si
n(8*f*x + 8*e) + sin(8*f*x + 8*e)^2 + 16*(3*sin(4*f*x + 4*e) + 2*sin(2*f*x
+ 2*e))*sin(6*f*x + 6*e) + 16*sin(6*f*x + 6*e)^2 + 36*sin(4*f*x + 4*e)^2 +
48*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 16*sin(2*f*x + 2*e)^2 + 8*cos(2*f*x
+ 2*e) + 1)*log(cos(f*x + e)^2 + sin(f*x + e)^2 - 2*sin(f*x + e) + 1) - 4*(
5*cos(7*f*x + 7*e) - 3*cos(5*f*x + 5*e) + 3*cos(3*f*x + 3*e) - 5*cos(f*x +
e))*sin(8*f*x + 8*e) + 20*(4*cos(6*f*x + 6*e) + 6*cos(4*f*x + 4*e) + 4*cos(
2*f*x + 2*e) + 1)*sin(7*f*x + 7*e) + 16*(3*cos(5*f*x + 5*e) - 3*cos(3*f*x +
 3*e) + 5*cos(f*x + e))*sin(6*f*x + 6*e) - 12*(6*cos(4*f*x + 4*e) + 4*cos(2
*f*x + 2*e) + 1)*sin(5*f*x + 5*e) - 24*(3*cos(3*f*x + 3*e) - 5*cos(f*x + e)
)*sin(4*f*x + 4*e) + 12*(4*cos(2*f*x + 2*e) + 1)*sin(3*f*x + 3*e) - 48*cos(
3*f*x + 3*e)*sin(2*f*x + 2*e) + 80*cos(f*x + e)*sin(2*f*x + 2*e) - 80*cos(2
*f*x + 2*e)*sin(f*x + e) - 20*sin(f*x + e))/((2*(4*cos(6*f*x + 6*e) + 6*cos
(4*f*x + 4*e) + 4*cos(2*f*x + 2*e) + 1)*cos(8*f*x + 8*e) + cos(8*f*x + 8*e)
^2 + 8*(6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + 16*
cos(6*f*x + 6*e)^2 + 12*(4*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 36*cos(
4*f*x + 4*e)^2 + 16*cos(2*f*x + 2*e)^2 + 4*(2*sin(6*f*x + 6*e) + 3*sin(4*f*
x + 4*e) + 2*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + sin(8*f*x + 8*e)^2 + 16*(
3*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 16*sin(6*f*x +
6*e)^2 + 36*sin(4*f*x + 4*e)^2 + 48*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 16*
sin(2*f*x + 2*e)^2 + 8*cos(2*f*x + 2*e) + 1)*sqrt(a)*f)
```

Fricas [A] time = 1.75814, size = 208, normalized size = 2.29

$$-\frac{\left(3\,\cos\left(fx+e\right)^{4}\log\left(-\frac{\sin\left(fx+e\right)-1}{\sin\left(fx+e\right)+1}\right)+2\left(5\,\cos\left(fx+e\right)^{2}-2\right)\sin\left(fx+e\right)\right)\sqrt{a\cos\left(fx+e\right)^{2}}}{16\,af\,\cos\left(fx+e\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/16*(3*cos(f*x + e)^4*log(-(sin(f*x + e) - 1)/(sin(f*x + e) + 1)) + 2*(5*
cos(f*x + e)^2 - 2)*sin(f*x + e))*sqrt(a*cos(f*x + e)^2)/(a*f*cos(f*x + e)^
5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^{4}\left(e+fx\right)}{\sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a-a*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**4/sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1)), x)
```

Giac [B] time = 2.14093, size = 271, normalized size = 2.98

$$-\frac{\frac{3\log\left(\left|\frac{1}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+2\right|\right)}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-1\right)}-\frac{3\log\left(\left|\frac{1}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-2\right|\right)}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-1\right)}-\frac{4\left(3\left(\frac{1}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)^{3}-\frac{20}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}-20\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\left(\frac{1}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)^{2}-4\right)^{2}\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-1\right)}}{16\sqrt{a}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/16*(3*log(abs(1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*e) + 2))/sgn(ta
n(1/2*f*x + 1/2*e)^4 - 1) - 3*log(abs(1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x
+ 1/2*e) - 2))/sgn(tan(1/2*f*x + 1/2*e)^4 - 1) - 4*(3*(1/tan(1/2*f*x + 1/2*
e) + tan(1/2*f*x + 1/2*e))^3 - 20/tan(1/2*f*x + 1/2*e) - 20*tan(1/2*f*x + 1
/2*e))/(((1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*e))^2 - 4)^2*sgn(tan(1
/2*f*x + 1/2*e)^4 - 1)))/(sqrt(a)*f)
```

3.474 $\int \frac{\tan^2(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=62

$$\frac{\tan(e+fx)}{2f\sqrt{a\cos^2(e+fx)}}-\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{2f\sqrt{a\cos^2(e+fx)}}$$

```
[Out] -(ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*f*Sqrt[a*Cos[e + f*x]^2]) + Tan[e
+ f*x]/(2*f*Sqrt[a*Cos[e + f*x]^2])
```

Rubi [A] time = 0.119355, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3176, 3207, 2611, 3770}

$$\frac{\tan(e+fx)}{2f\sqrt{a\cos^2(e+fx)}}-\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{2f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*f*Sqrt[a*Cos[e + f*x]^2]) + Tan[e
+ f*x]/(2*f*Sqrt[a*Cos[e + f*x]^2])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \int \frac{\tan^2(e+fx)}{\sqrt{a\cos^2(e+fx)}}\,dx$$
$$= \frac{\cos(e+fx)\int \sec(e+fx)\tan^2(e+fx)\,dx}{\sqrt{a\cos^2(e+fx)}}$$
$$= \frac{\tan(e+fx)}{2f\sqrt{a\cos^2(e+fx)}} - \frac{\cos(e+fx)\int \sec(e+fx)\,dx}{2\sqrt{a\cos^2(e+fx)}}$$
$$= -\frac{\tanh^{-1}(\sin(e+fx))\cos(e+fx)}{2f\sqrt{a\cos^2(e+fx)}} + \frac{\tan(e+fx)}{2f\sqrt{a\cos^2(e+fx)}}$$

Mathematica [A] time = 0.0451097, size = 43, normalized size = 0.69

$$\frac{\tan(e+fx)-\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{2f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] (-(ArcTanh[Sin[e + f*x]]*Cos[e + f*x]) + Tan[e + f*x])/(2*f*Sqrt[a*Cos[e +
f*x]^2])
```

Maple [A] time = 1.423, size = 65, normalized size = 1.1

$$\frac{1}{f\cos(fx+e)}\left(\frac{\sin(fx+e)}{2}+\frac{(\ln(-1+\sin(fx+e))-\ln(1+\sin(fx+e)))(\cos(fx+e))^2}{4}\right)\frac{1}{\sqrt{a(\cos(fx+e))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a-a*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] (1/2*sin(f*x+e)+1/4*(ln(-1+sin(f*x+e))-ln(1+sin(f*x+e)))*cos(f*x+e)^2)/cos(
f*x+e)/(a*cos(f*x+e)^2)^(1/2)/f
```

Maxima [B] time = 1.73306, size = 711, normalized size = 11.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(sin(3*f*x + 3*e) - sin(f*x + e))*cos(4*f*x + 4*e) - (2*(2*cos(2*f*x
 + 2*e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x + 4*e)^2 + 4*cos(2*f*x + 2*e)^2 +
 sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*sin(2*f*x + 2
*e)^2 + 4*cos(2*f*x + 2*e) + 1)*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin
(f*x + e) + 1) + (2*(2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x +
 4*e)^2 + 4*cos(2*f*x + 2*e)^2 + sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*si
```

```
n(2*f*x + 2*e) + 4*sin(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*log(cos(f*x
 + e)^2 + sin(f*x + e)^2 - 2*sin(f*x + e) + 1) - 4*(cos(3*f*x + 3*e) - cos(
f*x + e))*sin(4*f*x + 4*e) + 4*(2*cos(2*f*x + 2*e) + 1)*sin(3*f*x + 3*e) -
8*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) + 8*cos(f*x + e)*sin(2*f*x + 2*e) - 8*c
os(2*f*x + 2*e)*sin(f*x + e) - 4*sin(f*x + e))/((2*(2*cos(2*f*x + 2*e) + 1)
*cos(4*f*x + 4*e) + cos(4*f*x + 4*e)^2 + 4*cos(2*f*x + 2*e)^2 + sin(4*f*x +
 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*sin(2*f*x + 2*e)^2 + 4*co
s(2*f*x + 2*e) + 1)*sqrt(a)*f)
```

Fricas [A] time = 1.68546, size = 173, normalized size = 2.79

$$-\frac{\sqrt{a\cos\left(fx+e\right)^2}\left(\cos\left(fx+e\right)^2\log\left(-\frac{\sin\left(fx+e\right)+1}{\sin\left(fx+e\right)-1}\right)-2\sin\left(fx+e\right)\right)}{4\,af\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(a*cos(f*x + e)^2)*(cos(f*x + e)^2*log(-(sin(f*x + e) + 1)/(sin(f*
x + e) - 1)) - 2*sin(f*x + e))/(a*f*cos(f*x + e)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\tan^2\left(e+fx\right)}{\sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a-a*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**2/sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1)), x)
```

Giac [B] time = 1.45832, size = 228, normalized size = 3.68

$$\frac{\frac{\log\left(\left|\frac{1}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+2\right|\right)}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-1\right)}-\frac{\log\left(\left|\frac{1}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-2\right|\right)}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-1\right)}-\frac{4\left(\frac{1}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\left(\frac{1}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)^2-4\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-1\right)}}{4\sqrt{a}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(log(abs(1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*e) + 2))/sgn(tan(1/
2*f*x + 1/2*e)^4 - 1) - log(abs(1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*
e) - 2))/sgn(tan(1/2*f*x + 1/2*e)^4 - 1) - 4*(1/tan(1/2*f*x + 1/2*e) + tan(
1/2*f*x + 1/2*e))/(((1/tan(1/2*f*x + 1/2*e) + tan(1/2*f*x + 1/2*e))^2 - 4)*
sgn(tan(1/2*f*x + 1/2*e)^4 - 1)))/(sqrt(a)*f)
```

3.475 $\int \frac{\cot^2(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=25

$$-\frac{\cot(e+fx)}{f\sqrt{a\cos^2(e+fx)}}$$

```
[Out] -(Cot[e + f*x]/(f*Sqrt[a*Cos[e + f*x]^2]))
```

Rubi [A] time = 0.101829, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3176, 3207, 2606, 8}

$$-\frac{\cot(e+fx)}{f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] -(Cot[e + f*x]/(f*Sqrt[a*Cos[e + f*x]^2]))
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \int \frac{\cot^2(e+fx)}{\sqrt{a\cos^2(e+fx)}}\,dx$$
$$= \frac{\cos(e+fx)\int \cot(e+fx)\csc(e+fx)\,dx}{\sqrt{a\cos^2(e+fx)}}$$
$$= -\frac{\cos(e+fx)\,\mathrm{Subst}(\int 1\,dx, x, \csc(e+fx))}{f\sqrt{a\cos^2(e+fx)}}$$
$$= -\frac{\cot(e+fx)}{f\sqrt{a\cos^2(e+fx)}}$$

Mathematica [A] time = 0.0291194, size = 25, normalized size = 1.

$$-\frac{\cot(e+fx)}{f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2/Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] -(Cot[e + f*x]/(f*Sqrt[a*Cos[e + f*x]^2]))
```

Maple [A] time = 0.246, size = 32, normalized size = 1.3

$$-\frac{\cos(fx+e)}{\sin(fx+e)\,f}\frac{1}{\sqrt{a\left(\cos(fx+e)\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a-a*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] -cos(f*x+e)/sin(f*x+e)/(a*cos(f*x+e)^2)^(1/2)/f
```

Maxima [B] time = 1.72, size = 122, normalized size = 4.88

$$-\frac{2\left(\cos(fx+e)\sin(2fx+2e)-\cos(2fx+2e)\sin(fx+e)+\sin(fx+e)\right)\sqrt{a}}{\left(a\cos(2fx+2e)^2+a\sin(2fx+2e)^2-2a\cos(2fx+2e)+a\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*(cos(f*x + e)*sin(2*f*x + 2*e) - cos(2*f*x + 2*e)*sin(f*x + e) + sin(f*x
 + e))*sqrt(a)/((a*cos(2*f*x + 2*e)^2 + a*sin(2*f*x + 2*e)^2 - 2*a*cos(2*f*
x + 2*e) + a)*f)
```

Fricas [A] time = 1.54424, size = 77, normalized size = 3.08

$$-\frac{\sqrt{a\cos\left(fx+e\right)^{2}}}{af\cos\left(fx+e\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(a*cos(f*x + e)^2)/(a*f*cos(f*x + e)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^{2}\left(e+fx\right)}{\sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a-a*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**2/sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1)), x)
```

Giac [B] time = 1.2792, size = 90, normalized size = 3.6

$$\frac{\frac{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-1\right)}+\frac{1}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-1\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}}{2\sqrt{a}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(tan(1/2*f*x + 1/2*e)/sgn(tan(1/2*f*x + 1/2*e)^4 - 1) + 1/(sgn(tan(1/2*
f*x + 1/2*e)^4 - 1)*tan(1/2*f*x + 1/2*e)))/(sqrt(a)*f)
```

3.476 $\int \frac{\cot^4(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=60

$$\frac{\cot(e+fx)}{f\sqrt{a\cos^2(e+fx)}}-\frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\cos^2(e+fx)}}$$

```
[Out] Cot[e + f*x]/(f*Sqrt[a*Cos[e + f*x]^2]) - (Cot[e + f*x]*Csc[e + f*x]^2)/(3*
f*Sqrt[a*Cos[e + f*x]^2])
```

Rubi [A] time = 0.117242, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3176, 3207, 2606}

$$\frac{\cot(e+fx)}{f\sqrt{a\cos^2(e+fx)}}-\frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] Cot[e + f*x]/(f*Sqrt[a*Cos[e + f*x]^2]) - (Cot[e + f*x]*Csc[e + f*x]^2)/(3*
f*Sqrt[a*Cos[e + f*x]^2])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \int \frac{\cot^4(e+fx)}{\sqrt{a\cos^2(e+fx)}}\,dx$$
$$= \frac{\cos(e+fx)\int \cot^3(e+fx)\csc(e+fx)\,dx}{\sqrt{a\cos^2(e+fx)}}$$
$$= -\frac{\cos(e+fx)\,\text{Subst}\left(\int \left(-1+x^2\right)dx, x, \csc(e+fx)\right)}{f\sqrt{a\cos^2(e+fx)}}$$
$$= \frac{\cot(e+fx)}{f\sqrt{a\cos^2(e+fx)}} - \frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\cos^2(e+fx)}}$$

Mathematica [A] time = 0.0603917, size = 37, normalized size = 0.62

$$-\frac{\cot(e+fx)\left(\csc^2(e+fx)-3\right)}{3f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4/Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] -(Cot[e + f*x]*(-3 + Csc[e + f*x]^2))/(3*f*Sqrt[a*Cos[e + f*x]^2])
```

Maple [A] time = 0.717, size = 44, normalized size = 0.7

$$\frac{\cos\left(fx+e\right)\left(3\left(\sin\left(fx+e\right)\right)^2-1\right)}{3\left(\sin\left(fx+e\right)\right)^3 f}\frac{1}{\sqrt{a\left(\cos\left(fx+e\right)\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a-a*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3*cos(f*x+e)*(3*sin(f*x+e)^2-1)/sin(f*x+e)^3/(a*cos(f*x+e)^2)^(1/2)/f
```

Maxima [B] time = 1.70911, size = 709, normalized size = 11.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/3*((3*sin(5*f*x + 5*e) - 2*sin(3*f*x + 3*e) + 3*sin(f*x + e))*cos(6*f*x
+ 6*e) + 9*(sin(4*f*x + 4*e) - sin(2*f*x + 2*e))*cos(5*f*x + 5*e) + 3*(2*si
n(3*f*x + 3*e) - 3*sin(f*x + e))*cos(4*f*x + 4*e) - (3*cos(5*f*x + 5*e) - 2
*cos(3*f*x + 3*e) + 3*cos(f*x + e))*sin(6*f*x + 6*e) - 3*(3*cos(4*f*x + 4*e
) - 3*cos(2*f*x + 2*e) + 1)*sin(5*f*x + 5*e) - 3*(2*cos(3*f*x + 3*e) - 3*co
s(f*x + e))*sin(4*f*x + 4*e) - 2*(3*cos(2*f*x + 2*e) - 1)*sin(3*f*x + 3*e)
+ 6*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) - 9*cos(f*x + e)*sin(2*f*x + 2*e) + 9
*cos(2*f*x + 2*e)*sin(f*x + e) - 3*sin(f*x + e))*sqrt(a)/((a*cos(6*f*x + 6*
```

```
e)^2 + 9*a*cos(4*f*x + 4*e)^2 + 9*a*cos(2*f*x + 2*e)^2 + a*sin(6*f*x + 6*e)
^2 + 9*a*sin(4*f*x + 4*e)^2 - 18*a*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 9*a*
sin(2*f*x + 2*e)^2 - 2*(3*a*cos(4*f*x + 4*e) - 3*a*cos(2*f*x + 2*e) + a)*co
s(6*f*x + 6*e) - 6*(3*a*cos(2*f*x + 2*e) - a)*cos(4*f*x + 4*e) - 6*a*cos(2*
f*x + 2*e) - 6*(a*sin(4*f*x + 4*e) - a*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) +
 a)*f)
```

Fricas [A] time = 1.58113, size = 143, normalized size = 2.38

$$\frac{\sqrt{a\cos\left(fx+e\right)^{2}}\left(3\cos\left(fx+e\right)^{2}-2\right)}{3\left(af\cos\left(fx+e\right)^{3}-af\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(a*cos(f*x + e)^2)*(3*cos(f*x + e)^2 - 2)/((a*f*cos(f*x + e)^3 - a*
f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cot^{4}\left(e+fx\right)}{\sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a-a*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**4/sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1)), x)
```

Giac [A] time = 1.34488, size = 134, normalized size = 2.23

$$\frac{\frac{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}-9\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-1\right)}-\frac{9\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-1}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-1\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}}}{24\sqrt{a}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*((tan(1/2*f*x + 1/2*e)^3 - 9*tan(1/2*f*x + 1/2*e))/sgn(tan(1/2*f*x + 1
/2*e)^4 - 1) - (9*tan(1/2*f*x + 1/2*e)^2 - 1)/(sgn(tan(1/2*f*x + 1/2*e)^4 -
 1)*tan(1/2*f*x + 1/2*e)^3))/(sqrt(a)*f)
```

3.477 $\int \frac{\cot^6(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=96

$$-\frac{\cot(e+fx)}{f\sqrt{a\cos^2(e+fx)}}-\frac{\cot(e+fx)\csc^4(e+fx)}{5f\sqrt{a\cos^2(e+fx)}}+\frac{2\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\cos^2(e+fx)}}$$

```
[Out] -(Cot[e + f*x]/(f*Sqrt[a*Cos[e + f*x]^2])) + (2*Cot[e + f*x]*Csc[e + f*x]^2
)/(3*f*Sqrt[a*Cos[e + f*x]^2]) - (Cot[e + f*x]*Csc[e + f*x]^4)/(5*f*Sqrt[a*
Cos[e + f*x]^2])
```

Rubi [A] time = 0.12132, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3176, 3207, 2606, 194}

$$-\frac{\cot(e+fx)}{f\sqrt{a\cos^2(e+fx)}}-\frac{\cot(e+fx)\csc^4(e+fx)}{5f\sqrt{a\cos^2(e+fx)}}+\frac{2\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6/Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] -(Cot[e + f*x]/(f*Sqrt[a*Cos[e + f*x]^2])) + (2*Cot[e + f*x]*Csc[e + f*x]^2
)/(3*f*Sqrt[a*Cos[e + f*x]^2]) - (Cot[e + f*x]*Csc[e + f*x]^4)/(5*f*Sqrt[a*
Cos[e + f*x]^2])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cot^6(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \int \frac{\cot^6(e+fx)}{\sqrt{a\cos^2(e+fx)}}\,dx$$
$$= \frac{\cos(e+fx)\int \cot^5(e+fx)\csc(e+fx)\,dx}{\sqrt{a\cos^2(e+fx)}}$$
$$= -\frac{\cos(e+fx)\,\text{Subst}\left(\int \left(-1+x^2\right)^2\,dx, x, \csc(e+fx)\right)}{f\sqrt{a\cos^2(e+fx)}}$$
$$= -\frac{\cos(e+fx)\,\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, \csc(e+fx)\right)}{f\sqrt{a\cos^2(e+fx)}}$$
$$= -\frac{\cot(e+fx)}{f\sqrt{a\cos^2(e+fx)}} + \frac{2\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\cos^2(e+fx)}} - \frac{\cot(e+fx)\csc^4(e+fx)}{5f\sqrt{a\cos^2(e+fx)}}$$

Mathematica [A] time = 0.0745488, size = 49, normalized size = 0.51

$$-\frac{\cot(e+fx)\left(3\csc^4(e+fx)-10\csc^2(e+fx)+15\right)}{15f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6/Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] -(Cot[e + f*x]*(15 - 10*Csc[e + f*x]^2 + 3*Csc[e + f*x]^4))/(15*f*Sqrt[a*Co
s[e + f*x]^2])
```

Maple [A] time = 0.714, size = 54, normalized size = 0.6

$$-\frac{\cos\left(fx+e\right)\left(15\left(\sin\left(fx+e\right)\right)^4-10\left(\sin\left(fx+e\right)\right)^2+3\right)}{15\left(\sin\left(fx+e\right)\right)^5 f}\frac{1}{\sqrt{a\left(\cos\left(fx+e\right)\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6/(a-a*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/15*cos(f*x+e)*(15*sin(f*x+e)^4-10*sin(f*x+e)^2+3)/sin(f*x+e)^5/(a*cos(f*
x+e)^2)^(1/2)/f
```

Maxima [B] time = 1.77746, size = 1669, normalized size = 17.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*((15*sin(9*f*x + 9*e) - 20*sin(7*f*x + 7*e) + 58*sin(5*f*x + 5*e) - 20
*sin(3*f*x + 3*e) + 15*sin(f*x + e))*cos(10*f*x + 10*e) + 75*(sin(8*f*x + 8
*e) - 2*sin(6*f*x + 6*e) + 2*sin(4*f*x + 4*e) - sin(2*f*x + 2*e))*cos(9*f*x
```

```
 + 9*e) + 5*(20*sin(7*f*x + 7*e) - 58*sin(5*f*x + 5*e) + 20*sin(3*f*x + 3*e
) - 15*sin(f*x + e))*cos(8*f*x + 8*e) + 100*(2*sin(6*f*x + 6*e) - 2*sin(4*f
*x + 4*e) + sin(2*f*x + 2*e))*cos(7*f*x + 7*e) + 10*(58*sin(5*f*x + 5*e) -
20*sin(3*f*x + 3*e) + 15*sin(f*x + e))*cos(6*f*x + 6*e) + 290*(2*sin(4*f*x
+ 4*e) - sin(2*f*x + 2*e))*cos(5*f*x + 5*e) + 50*(4*sin(3*f*x + 3*e) - 3*si
n(f*x + e))*cos(4*f*x + 4*e) - (15*cos(9*f*x + 9*e) - 20*cos(7*f*x + 7*e) +
 58*cos(5*f*x + 5*e) - 20*cos(3*f*x + 3*e) + 15*cos(f*x + e))*sin(10*f*x +
10*e) - 15*(5*cos(8*f*x + 8*e) - 10*cos(6*f*x + 6*e) + 10*cos(4*f*x + 4*e)
- 5*cos(2*f*x + 2*e) + 1)*sin(9*f*x + 9*e) - 5*(20*cos(7*f*x + 7*e) - 58*co
s(5*f*x + 5*e) + 20*cos(3*f*x + 3*e) - 15*cos(f*x + e))*sin(8*f*x + 8*e) -
20*(10*cos(6*f*x + 6*e) - 10*cos(4*f*x + 4*e) + 5*cos(2*f*x + 2*e) - 1)*sin
(7*f*x + 7*e) - 10*(58*cos(5*f*x + 5*e) - 20*cos(3*f*x + 3*e) + 15*cos(f*x
+ e))*sin(6*f*x + 6*e) - 58*(10*cos(4*f*x + 4*e) - 5*cos(2*f*x + 2*e) + 1)*
sin(5*f*x + 5*e) - 50*(4*cos(3*f*x + 3*e) - 3*cos(f*x + e))*sin(4*f*x + 4*e
) - 20*(5*cos(2*f*x + 2*e) - 1)*sin(3*f*x + 3*e) + 100*cos(3*f*x + 3*e)*sin
(2*f*x + 2*e) - 75*cos(f*x + e)*sin(2*f*x + 2*e) + 75*cos(2*f*x + 2*e)*sin(
f*x + e) - 15*sin(f*x + e))*sqrt(a)/((a*cos(10*f*x + 10*e)^2 + 25*a*cos(8*f
*x + 8*e)^2 + 100*a*cos(6*f*x + 6*e)^2 + 100*a*cos(4*f*x + 4*e)^2 + 25*a*co
s(2*f*x + 2*e)^2 + a*sin(10*f*x + 10*e)^2 + 25*a*sin(8*f*x + 8*e)^2 + 100*a
*sin(6*f*x + 6*e)^2 + 100*a*sin(4*f*x + 4*e)^2 - 100*a*sin(4*f*x + 4*e)*sin
(2*f*x + 2*e) + 25*a*sin(2*f*x + 2*e)^2 - 2*(5*a*cos(8*f*x + 8*e) - 10*a*co
s(6*f*x + 6*e) + 10*a*cos(4*f*x + 4*e) - 5*a*cos(2*f*x + 2*e) + a)*cos(10*f
*x + 10*e) - 10*(10*a*cos(6*f*x + 6*e) - 10*a*cos(4*f*x + 4*e) + 5*a*cos(2*
f*x + 2*e) - a)*cos(8*f*x + 8*e) - 20*(10*a*cos(4*f*x + 4*e) - 5*a*cos(2*f*
x + 2*e) + a)*cos(6*f*x + 6*e) - 20*(5*a*cos(2*f*x + 2*e) - a)*cos(4*f*x +
4*e) - 10*a*cos(2*f*x + 2*e) - 10*(a*sin(8*f*x + 8*e) - 2*a*sin(6*f*x + 6*e
) + 2*a*sin(4*f*x + 4*e) - a*sin(2*f*x + 2*e))*sin(10*f*x + 10*e) - 50*(2*a
*sin(6*f*x + 6*e) - 2*a*sin(4*f*x + 4*e) + a*sin(2*f*x + 2*e))*sin(8*f*x +
8*e) - 100*(2*a*sin(4*f*x + 4*e) - a*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + a
)*f)
```

Fricas [A] time = 1.61589, size = 205, normalized size = 2.14

$$-\frac{\left(15 \cos\left(fx+e\right)^{4} - 20 \cos\left(fx+e\right)^{2} + 8\right)\sqrt{a\cos\left(fx+e\right)^{2}}}{15\left(af\cos\left(fx+e\right)^{5} - 2\,af\cos\left(fx+e\right)^{3} + af\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*(15*cos(f*x + e)^4 - 20*cos(f*x + e)^2 + 8)*sqrt(a*cos(f*x + e)^2)/((
a*f*cos(f*x + e)^5 - 2*a*f*cos(f*x + e)^3 + a*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6/(a-a*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.40487, size = 173, normalized size = 1.8

$$\frac{\frac{3\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}-25\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+150\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-1\right)}+\frac{150\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-25\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+3}{\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-1\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}}}{480\,\sqrt{a}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/480*((3*tan(1/2*f*x + 1/2*e)^5 - 25*tan(1/2*f*x + 1/2*e)^3 + 150*tan(1/2*
f*x + 1/2*e))/sgn(tan(1/2*f*x + 1/2*e)^4 - 1) + (150*tan(1/2*f*x + 1/2*e)^4
 - 25*tan(1/2*f*x + 1/2*e)^2 + 3)/(sgn(tan(1/2*f*x + 1/2*e)^4 - 1)*tan(1/2*
f*x + 1/2*e)^5))/(sqrt(a)*f)
```

3.478 $\int \frac{\tan^5(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=68

$$\frac{a^2}{7f\left(a\cos^2(e+fx)\right)^{7/2}}-\frac{2a}{5f\left(a\cos^2(e+fx)\right)^{5/2}}+\frac{1}{3f\left(a\cos^2(e+fx)\right)^{3/2}}$$

```
[Out] a^2/(7*f*(a*Cos[e + f*x]^2)^(7/2)) - (2*a)/(5*f*(a*Cos[e + f*x]^2)^(5/2)) +
 1/(3*f*(a*Cos[e + f*x]^2)^(3/2))
```

Rubi [A] time = 0.12691, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3176, 3205, 16, 43}

$$\frac{a^2}{7f\left(a\cos^2(e+fx)\right)^{7/2}}-\frac{2a}{5f\left(a\cos^2(e+fx)\right)^{5/2}}+\frac{1}{3f\left(a\cos^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] a^2/(7*f*(a*Cos[e + f*x]^2)^(7/2)) - (2*a)/(5*f*(a*Cos[e + f*x]^2)^(5/2)) +
 1/(3*f*(a*Cos[e + f*x]^2)^(3/2))
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\tan^5(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\tan^5(e+fx)}{\left(a\cos^2(e+fx)\right)^{3/2}}\,dx$$
$$=-\frac{\text{Subst}\left(\int \frac{(1-x)^2}{x^3(ax)^{3/2}}\,dx,x,\cos^2(e+fx)\right)}{2f}$$
$$=-\frac{a^3\,\text{Subst}\left(\int \frac{(1-x)^2}{(ax)^{9/2}}\,dx,x,\cos^2(e+fx)\right)}{2f}$$
$$=-\frac{a^3\,\text{Subst}\left(\int \left(\frac{1}{(ax)^{9/2}}-\frac{2}{a(ax)^{7/2}}+\frac{1}{a^2(ax)^{5/2}}\right)dx,x,\cos^2(e+fx)\right)}{2f}$$
$$=\frac{a^2}{7f\left(a\cos^2(e+fx)\right)^{7/2}}-\frac{2a}{5f\left(a\cos^2(e+fx)\right)^{5/2}}+\frac{1}{3f\left(a\cos^2(e+fx)\right)^{3/2}}$$

Mathematica [A] time = 0.107422, size = 51, normalized size = 0.75

$$\frac{\left(35\cos^4(e+fx)-42\cos^2(e+fx)+15\right)\sec^4(e+fx)}{105f\left(a\cos^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] ((15 - 42*Cos[e + f*x]^2 + 35*Cos[e + f*x]^4)*Sec[e + f*x]^4)/(105*f*(a*Cos
[e + f*x]^2)^(3/2))
```

Maple [A] time = 1.991, size = 51, normalized size = 0.8

$$\frac{35\left(\cos\left(fx+e\right)\right)^4-42\left(\cos\left(fx+e\right)\right)^2+15}{105\,a^2\left(\cos\left(fx+e\right)\right)^8f}\sqrt{a\left(\cos\left(fx+e\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a-a*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/105/a^2/cos(f*x+e)^8*(a*cos(f*x+e)^2)^(1/2)*(35*cos(f*x+e)^4-42*cos(f*x+e
)^2+15)/f
```

Maxima [A] time = 1.0354, size = 93, normalized size = 1.37

$$\frac{35\left(a\sin\left(fx+e\right)^2-a\right)^2a^3+42\left(a\sin\left(fx+e\right)^2-a\right)a^4+15\,a^5}{105\left(-a\sin\left(fx+e\right)^2+a\right)^{\frac{7}{2}}a^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/105*(35*(a*sin(f*x + e)^2 - a)^2*a^3 + 42*(a*sin(f*x + e)^2 - a)*a^4 + 15
*a^5)/((-a*sin(f*x + e)^2 + a)^(7/2)*a^3*f)
```

Fricas [A] time = 1.67547, size = 132, normalized size = 1.94

$$\frac{\left(35 \cos\left(fx+e\right)^{4}-42 \cos\left(fx+e\right)^{2}+15\right)\sqrt{a \cos\left(fx+e\right)^{2}}}{105 \, a^{2} f \cos\left(fx+e\right)^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/105*(35*cos(f*x + e)^4 - 42*cos(f*x + e)^2 + 15)*sqrt(a*cos(f*x + e)^2)/(
a^2*f*cos(f*x + e)^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5/(a-a*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^{5}}{\left(-a \sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^5/(-a*sin(f*x + e)^2 + a)^(3/2), x)
```

3.479 $\int \frac{\tan^3(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=44

$$\frac{a}{5f\left(a\cos^2(e+fx)\right)^{5/2}}-\frac{1}{3f\left(a\cos^2(e+fx)\right)^{3/2}}$$

```
[Out] a/(5*f*(a*Cos[e + f*x]^2)^(5/2)) - 1/(3*f*(a*Cos[e + f*x]^2)^(3/2))
```

Rubi [A] time = 0.119488, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3176, 3205, 16, 43}

$$\frac{a}{5f\left(a\cos^2(e+fx)\right)^{5/2}}-\frac{1}{3f\left(a\cos^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] a/(5*f*(a*Cos[e + f*x]^2)^(5/2)) - 1/(3*f*(a*Cos[e + f*x]^2)^(3/2))
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\tan^3(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\tan^3(e+fx)}{\left(a\cos^2(e+fx)\right)^{3/2}}\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{1-x}{x^2(ax)^{3/2}}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{a^2\,\text{Subst}\left(\int \frac{1-x}{(ax)^{7/2}}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{a^2\,\text{Subst}\left(\int \left(\frac{1}{(ax)^{7/2}}-\frac{1}{a(ax)^{5/2}}\right)dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= \frac{a}{5f\left(a\cos^2(e+fx)\right)^{5/2}} - \frac{1}{3f\left(a\cos^2(e+fx)\right)^{3/2}}$$

Mathematica [A] time = 0.110993, size = 34, normalized size = 0.77

$$\frac{a\left(3-5\cos^2(e+fx)\right)}{15f\left(a\cos^2(e+fx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3/(a - a*Sin[e + f*x]^2)^(3/2),x]

[Out] (a*(3 - 5*Cos[e + f*x]^2))/(15*f*(a*Cos[e + f*x]^2)^(5/2))
```

Maple [A] time = 1.908, size = 41, normalized size = 0.9

$$-\frac{5\left(\cos\left(fx+e\right)\right)^2-3}{15\,a^2\left(\cos\left(fx+e\right)\right)^6 f}\sqrt{a\left(\cos\left(fx+e\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(a-a*sin(f*x+e)^2)^(3/2),x)

[Out] -1/15/a^2/cos(f*x+e)^6*(a*cos(f*x+e)^2)^(1/2)*(5*cos(f*x+e)^2-3)/f
```

Maxima [A] time = 1.03413, size = 65, normalized size = 1.48

$$\frac{5\left(a\sin\left(fx+e\right)^2-a\right)a^2+3\,a^3}{15\left(-a\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}a^2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out] 1/15*(5*(a*sin(f*x + e)^2 - a)*a^2 + 3*a^3)/((-a*sin(f*x + e)^2 + a)^(5/2)*
a^2*f)
```

Fricas [A] time = 1.59149, size = 103, normalized size = 2.34

$$-\frac{\sqrt{a\cos\left(fx+e\right)^{2}}\left(5\cos\left(fx+e\right)^{2}-3\right)}{15\,a^{2}f\cos\left(fx+e\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/15*sqrt(a*cos(f*x + e)^2)*(5*cos(f*x + e)^2 - 3)/(a^2*f*cos(f*x + e)^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\tan^{3}\left(e+fx\right)}{\left(-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a-a*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**3/(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))**(3/2),
 x)
```

Giac [A] time = 1.30612, size = 78, normalized size = 1.77

$$\frac{\left(\tan\left(fx+e\right)^{2}+1\right)^{2}\left(3\,a-\frac{5\,a}{\tan\left(fx+e\right)^{2}+1}\right)}{15\,a^{2}f\sqrt{\frac{a}{\tan\left(fx+e\right)^{2}+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/15*(tan(f*x + e)^2 + 1)^2*(3*a - 5*a/(tan(f*x + e)^2 + 1))/(a^2*f*sqrt(a/
(tan(f*x + e)^2 + 1)))
```

3.480 $\int \frac{\tan(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=21

$$\frac{1}{3f\left(a\cos^2(e+fx)\right)^{3/2}}$$

```
[Out] 1/(3*f*(a*Cos[e + f*x]^2)^(3/2))
```

Rubi [A] time = 0.0744949, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3176, 3205, 16, 32}

$$\frac{1}{3f\left(a\cos^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] 1/(3*f*(a*Cos[e + f*x]^2)^(3/2))
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tan(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\tan(e+fx)}{\left(a\cos^2(e+fx)\right)^{3/2}}\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{x(ax)^{3/2}}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{a\,\text{Subst}\left(\int \frac{1}{(ax)^{5/2}}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= \frac{1}{3f\left(a\cos^2(e+fx)\right)^{3/2}}$$

Mathematica [A] time = 0.0300188, size = 21, normalized size = 1.

$$\frac{1}{3f\left(a\cos^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] 1/(3*f*(a*Cos[e + f*x]^2)^(3/2))
```

Maple [A] time = 0.119, size = 21, normalized size = 1.

$$\frac{1}{3f}\left(a-a\left(\sin\left(fx+e\right)\right)^{2}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a-a*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/3/f/(a-a*sin(f*x+e)^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.56206, size = 69, normalized size = 3.29

$$\frac{\sqrt{a\cos\left(fx+e\right)^{2}}}{3\,a^{2}f\cos\left(fx+e\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(a*cos(f*x + e)^2)/(a^2*f*cos(f*x + e)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(e+fx\right)}{\left(-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a-a*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)/(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))**(3/2), x)
```

Giac [B] time = 1.25045, size = 49, normalized size = 2.33

$$\frac{\tan\left(fx+e\right)^2+1}{3\,af\sqrt{\frac{a}{\tan\left(fx+e\right)^2+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*(tan(f*x + e)^2 + 1)/(a*f*sqrt(a/(tan(f*x + e)^2 + 1)))
```

3.481 $\int \frac{\cot(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=53

$$\frac{1}{af\sqrt{a\cos^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}$$

```
[Out] -(ArcTanh[Sqrt[a*Cos[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f)) + 1/(a*f*Sqrt[a*Cos[
e + f*x]^2])
```

Rubi [A] time = 0.0922653, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3176, 3205, 51, 63, 206}

$$\frac{1}{af\sqrt{a\cos^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a*Cos[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f)) + 1/(a*f*Sqrt[a*Cos[
e + f*x]^2])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cot(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx &= \int \frac{\cot(e+fx)}{\left(a\cos^2(e+fx)\right)^{3/2}}\,dx\\ &= -\frac{\text{Subst}\left(\int \frac{1}{(1-x)(ax)^{3/2}}\,dx, x, \cos^2(e+fx)\right)}{2f}\\ &= \frac{1}{af\sqrt{a\cos^2(e+fx)}}-\frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{ax}}\,dx, x, \cos^2(e+fx)\right)}{2af}\\ &= \frac{1}{af\sqrt{a\cos^2(e+fx)}}-\frac{\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a\cos^2(e+fx)}\right)}{a^2f}\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}+\frac{1}{af\sqrt{a\cos^2(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.0672689, size = 55, normalized size = 1.04

$$\frac{\cos(e+fx)\left(\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)\right)+1}{af\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (1 + Cos[e + f*x]*(-Log[Cos[(e + f*x)/2]] + Log[Sin[(e + f*x)/2]]))/(a*f*Sq
rt[a*Cos[e + f*x]^2])
```

Maple [A] time = 2.359, size = 75, normalized size = 1.4

$$\frac{1}{\left(\cos\left(fx+e\right)\right)^2 f}\left(-\ln\left(2\,\frac{\sqrt{a}\sqrt{a\left(\cos\left(fx+e\right)\right)^2}+a}{\sin\left(fx+e\right)}\right)a^2\left(\cos\left(fx+e\right)\right)^2+\sqrt{a\left(\cos\left(fx+e\right)\right)^2}a^{\frac{3}{2}}\right)a^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a-a*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/a^(7/2)/cos(f*x+e)^2*(-ln(2/sin(f*x+e)*(a^(1/2)*(a*cos(f*x+e)^2)^(1/2)+a)
)*a^2*cos(f*x+e)^2+(a*cos(f*x+e)^2)^(1/2)*a^(3/2))/f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.68648, size = 155, normalized size = 2.92

$$-\frac{\sqrt{a\cos\left(fx+e\right)^{2}}\left(\cos\left(fx+e\right)\log\left(-\frac{\cos(fx+e)+1}{\cos(fx+e)-1}\right)-2\right)}{2\,a^{2}f\cos\left(fx+e\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(a*cos(f*x + e)^2)*(cos(f*x + e)*log(-(cos(f*x + e) + 1)/(cos(f*x
+ e) - 1)) - 2)/(a^2*f*cos(f*x + e)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cot\left(e+fx\right)}{\left(-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a-a*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)/(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))**(3/2), x)
```

Giac [A] time = 1.12655, size = 80, normalized size = 1.51

$$\frac{\arctan\left(\frac{\sqrt{-a\sin(fx+e)^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}af}+\frac{1}{\sqrt{-a\sin\left(fx+e\right)^{2}+a}af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(-a*sin(f*x + e)^2 + a)/sqrt(-a))/(sqrt(-a)*a*f) + 1/(sqrt(-a*si
n(f*x + e)^2 + a)*a*f)
```

3.482 $\int \frac{\cot^3(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}} dx$

Optimal. Leaf size=66

$$-\frac{\csc^2(e+fx)\sqrt{a\cos^2(e+fx)}}{2a^2f}-\frac{\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f}$$

```
[Out] -ArcTanh[Sqrt[a*Cos[e + f*x]^2]/Sqrt[a]]/(2*a^(3/2)*f) - (Sqrt[a*Cos[e + f*
x]^2]*Csc[e + f*x]^2)/(2*a^2*f)
```

Rubi [A] time = 0.126603, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3176, 3205, 16, 51, 63, 206}

$$-\frac{\csc^2(e+fx)\sqrt{a\cos^2(e+fx)}}{2a^2f}-\frac{\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -ArcTanh[Sqrt[a*Cos[e + f*x]^2]/Sqrt[a]]/(2*a^(3/2)*f) - (Sqrt[a*Cos[e + f*
x]^2]*Csc[e + f*x]^2)/(2*a^2*f)
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^3(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx &= \int \frac{\cot^3(e+fx)}{\left(a\cos^2(e+fx)\right)^{3/2}}\,dx\\
&= -\frac{\text{Subst}\left(\int \frac{x}{(1-x)^2(ax)^{3/2}}\,dx, x, \cos^2(e+fx)\right)}{2f}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{(1-x)^2\sqrt{ax}}\,dx, x, \cos^2(e+fx)\right)}{2af}\\
&= -\frac{\sqrt{a\cos^2(e+fx)}\csc^2(e+fx)}{2a^2f} - \frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{ax}}\,dx, x, \cos^2(e+fx)\right)}{4af}\\
&= -\frac{\sqrt{a\cos^2(e+fx)}\csc^2(e+fx)}{2a^2f} - \frac{\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a\cos^2(e+fx)}\right)}{2a^2f}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f} - \frac{\sqrt{a\cos^2(e+fx)}\csc^2(e+fx)}{2a^2f}
\end{aligned}$$

Mathematica [A] time = 0.172236, size = 82, normalized size = 1.24

$$-\frac{\cos^3(e+fx)\left(\csc^2\left(\frac{1}{2}(e+fx)\right)-\sec^2\left(\frac{1}{2}(e+fx)\right)-4\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)+4\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)\right)}{8f\left(a\cos^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Cos[e + f*x]^3*(Csc[(e + f*x)/2]^2 + 4*Log[Cos[(e + f*x)/2]] - 4*Log[Sin[
(e + f*x)/2]] - Sec[(e + f*x)/2]^2))/(8*f*(a*Cos[e + f*x]^2)^(3/2))
```

Maple [A] time = 1.306, size = 69, normalized size = 1.1

$$-\frac{1}{2a^2f\left(\sin\left(fx+e\right)\right)^2}\sqrt{a\left(\cos\left(fx+e\right)\right)^2}-\frac{1}{2f}\ln\left(\frac{1}{\sin\left(fx+e\right)}\left(2a+2\sqrt{a}\sqrt{a\left(\cos\left(fx+e\right)\right)^2}\right)\right)a^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a-a*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/2/f/a^2/sin(f*x+e)^2*(a*cos(f*x+e)^2)^(1/2)-1/2/f/a^(3/2)*ln((2*a+2*a^(1
/2)*(a*cos(f*x+e)^2)^(1/2))/sin(f*x+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.6094, size = 212, normalized size = 3.21

$$-\frac{\sqrt{a\cos\left(fx+e\right)^{2}}\left(\left(\cos\left(fx+e\right)^{2}-1\right)\log\left(-\frac{\cos\left(fx+e\right)+1}{\cos\left(fx+e\right)-1}\right)-2\cos\left(fx+e\right)\right)}{4\left(a^{2}f\cos\left(fx+e\right)^{3}-a^{2}f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(a*cos(f*x + e)^2)*((cos(f*x + e)^2 - 1)*log(-(cos(f*x + e) + 1)/(
cos(f*x + e) - 1)) - 2*cos(f*x + e))/(a^2*f*cos(f*x + e)^3 - a^2*f*cos(f*x
+ e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cot^{3}\left(e+fx\right)}{\left(-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a-a*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**3/(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))**(3/2),
 x)
```

Giac [B] time = 1.35796, size = 174, normalized size = 2.64

$$-\frac{\frac{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}}{a^{\frac{3}{2}}\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-1\right)}+\frac{2\log\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}\right)}{a^{\frac{3}{2}}\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-1\right)}-\frac{2\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+\sqrt{a}}{a^{2}\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-1\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}}}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/8*(tan(1/2*f*x + 1/2*e)^2/(a^(3/2)*sgn(tan(1/2*f*x + 1/2*e)^4 - 1)) + 2*
log(tan(1/2*f*x + 1/2*e)^2)/(a^(3/2)*sgn(tan(1/2*f*x + 1/2*e)^4 - 1)) - (2*
sqrt(a)*tan(1/2*f*x + 1/2*e)^2 + sqrt(a))/(a^2*sgn(tan(1/2*f*x + 1/2*e)^4 -
 1)*tan(1/2*f*x + 1/2*e)^2))/f
```

3.483 $\int \frac{\tan^2(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=106

$$-\frac{\tan(e+fx)}{8af\sqrt{a\cos^2(e+fx)}}-\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{8af\sqrt{a\cos^2(e+fx)}}+\frac{\tan(e+fx)\sec^2(e+fx)}{4af\sqrt{a\cos^2(e+fx)}}$$

```
[Out] -(ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(8*a*f*Sqrt[a*Cos[e + f*x]^2]) - Tan[
e + f*x]/(8*a*f*Sqrt[a*Cos[e + f*x]^2]) + (Sec[e + f*x]^2*Tan[e + f*x])/(4*
a*f*Sqrt[a*Cos[e + f*x]^2])
```

Rubi [A] time = 0.157268, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3176, 3207, 2611, 3768, 3770}

$$-\frac{\tan(e+fx)}{8af\sqrt{a\cos^2(e+fx)}}-\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{8af\sqrt{a\cos^2(e+fx)}}+\frac{\tan(e+fx)\sec^2(e+fx)}{4af\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(8*a*f*Sqrt[a*Cos[e + f*x]^2]) - Tan[
e + f*x]/(8*a*f*Sqrt[a*Cos[e + f*x]^2]) + (Sec[e + f*x]^2*Tan[e + f*x])/(4*
a*f*Sqrt[a*Cos[e + f*x]^2])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^2(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx &= \int \frac{\tan^2(e+fx)}{\left(a\cos^2(e+fx)\right)^{3/2}}\,dx\\ &= \frac{\cos(e+fx)\int \sec^3(e+fx)\tan^2(e+fx)\,dx}{a\sqrt{a\cos^2(e+fx)}}\\ &= \frac{\sec^2(e+fx)\tan(e+fx)}{4af\sqrt{a\cos^2(e+fx)}}-\frac{\cos(e+fx)\int \sec^3(e+fx)\,dx}{4a\sqrt{a\cos^2(e+fx)}}\\ &= -\frac{\tan(e+fx)}{8af\sqrt{a\cos^2(e+fx)}}+\frac{\sec^2(e+fx)\tan(e+fx)}{4af\sqrt{a\cos^2(e+fx)}}-\frac{\cos(e+fx)\int \sec(e+fx)\,dx}{8a\sqrt{a\cos^2(e+fx)}}\\ &= -\frac{\tanh^{-1}(\sin(e+fx))\cos(e+fx)}{8af\sqrt{a\cos^2(e+fx)}}-\frac{\tan(e+fx)}{8af\sqrt{a\cos^2(e+fx)}}+\frac{\sec^2(e+fx)\tan(e+fx)}{4af\sqrt{a\cos^2(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.0951606, size = 59, normalized size = 0.56

$$\frac{\tan(e+fx)\left(2\sec^2(e+fx)-1\right)-\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{8af\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-(ArcTanh[Sin[e + f*x]]*Cos[e + f*x]) + (-1 + 2*Sec[e + f*x]^2)*Tan[e + f*
x])/(8*a*f*Sqrt[a*Cos[e + f*x]^2])
```

Maple [A] time = 1.285, size = 104, normalized size = 1.

$$-\frac{-2\left(\cos\left(fx+e\right)\right)^{2}\sin\left(fx+e\right)+4\sin\left(fx+e\right)+\left(\ln\left(-1+\sin\left(fx+e\right)\right)-\ln\left(1+\sin\left(fx+e\right)\right)\right)\left(\cos\left(fx+e\right)\right)^{4}}{16\,a\left(1+\sin\left(fx+e\right)\right)\left(-1+\sin\left(fx+e\right)\right)\cos\left(fx+e\right)f\sqrt{a\left(\cos\left(fx+e\right)\right)^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a-a*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/16/a*(-2*cos(f*x+e)^2*sin(f*x+e)+4*sin(f*x+e)+(ln(-1+sin(f*x+e))-ln(1+si
n(f*x+e)))*cos(f*x+e)^4)/(1+sin(f*x+e))/(-1+sin(f*x+e))/cos(f*x+e)/(a*cos(f
*x+e)^2)^(1/2)/f
```

Maxima [B] time = 2.65676, size = 2068, normalized size = 19.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/16*(4*(sin(7*f*x + 7*e) - 7*sin(5*f*x + 5*e) + 7*sin(3*f*x + 3*e) - sin(
f*x + e))*cos(8*f*x + 8*e) - 8*(2*sin(6*f*x + 6*e) + 3*sin(4*f*x + 4*e) + 2
*sin(2*f*x + 2*e))*cos(7*f*x + 7*e) - 16*(7*sin(5*f*x + 5*e) - 7*sin(3*f*x
+ 3*e) + sin(f*x + e))*cos(6*f*x + 6*e) + 56*(3*sin(4*f*x + 4*e) + 2*sin(2*
f*x + 2*e))*cos(5*f*x + 5*e) + 24*(7*sin(3*f*x + 3*e) - sin(f*x + e))*cos(4
*f*x + 4*e) + (2*(4*cos(6*f*x + 6*e) + 6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2
*e) + 1)*cos(8*f*x + 8*e) + cos(8*f*x + 8*e)^2 + 8*(6*cos(4*f*x + 4*e) + 4*
cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + 16*cos(6*f*x + 6*e)^2 + 12*(4*cos(
2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 36*cos(4*f*x + 4*e)^2 + 16*cos(2*f*x +
 2*e)^2 + 4*(2*sin(6*f*x + 6*e) + 3*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*
sin(8*f*x + 8*e) + sin(8*f*x + 8*e)^2 + 16*(3*sin(4*f*x + 4*e) + 2*sin(2*f*
x + 2*e))*sin(6*f*x + 6*e) + 16*sin(6*f*x + 6*e)^2 + 36*sin(4*f*x + 4*e)^2
+ 48*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 16*sin(2*f*x + 2*e)^2 + 8*cos(2*f*
x + 2*e) + 1)*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin(f*x + e) + 1) - (
2*(4*cos(6*f*x + 6*e) + 6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e) + 1)*cos(8*
f*x + 8*e) + cos(8*f*x + 8*e)^2 + 8*(6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e
) + 1)*cos(6*f*x + 6*e) + 16*cos(6*f*x + 6*e)^2 + 12*(4*cos(2*f*x + 2*e) +
1)*cos(4*f*x + 4*e) + 36*cos(4*f*x + 4*e)^2 + 16*cos(2*f*x + 2*e)^2 + 4*(2*
sin(6*f*x + 6*e) + 3*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*sin(8*f*x + 8*e
) + sin(8*f*x + 8*e)^2 + 16*(3*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*sin(6
*f*x + 6*e) + 16*sin(6*f*x + 6*e)^2 + 36*sin(4*f*x + 4*e)^2 + 48*sin(4*f*x
+ 4*e)*sin(2*f*x + 2*e) + 16*sin(2*f*x + 2*e)^2 + 8*cos(2*f*x + 2*e) + 1)*l
og(cos(f*x + e)^2 + sin(f*x + e)^2 - 2*sin(f*x + e) + 1) - 4*(cos(7*f*x + 7
*e) - 7*cos(5*f*x + 5*e) + 7*cos(3*f*x + 3*e) - cos(f*x + e))*sin(8*f*x + 8
*e) + 4*(4*cos(6*f*x + 6*e) + 6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e) + 1)*
sin(7*f*x + 7*e) + 16*(7*cos(5*f*x + 5*e) - 7*cos(3*f*x + 3*e) + cos(f*x +
e))*sin(6*f*x + 6*e) - 28*(6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e) + 1)*sin
(5*f*x + 5*e) - 24*(7*cos(3*f*x + 3*e) - cos(f*x + e))*sin(4*f*x + 4*e) + 2
8*(4*cos(2*f*x + 2*e) + 1)*sin(3*f*x + 3*e) - 112*cos(3*f*x + 3*e)*sin(2*f*
x + 2*e) + 16*cos(f*x + e)*sin(2*f*x + 2*e) - 16*cos(2*f*x + 2*e)*sin(f*x +
 e) - 4*sin(f*x + e))/((a*cos(8*f*x + 8*e)^2 + 16*a*cos(6*f*x + 6*e)^2 + 36
*a*cos(4*f*x + 4*e)^2 + 16*a*cos(2*f*x + 2*e)^2 + a*sin(8*f*x + 8*e)^2 + 16
*a*sin(6*f*x + 6*e)^2 + 36*a*sin(4*f*x + 4*e)^2 + 48*a*sin(4*f*x + 4*e)*sin
(2*f*x + 2*e) + 16*a*sin(2*f*x + 2*e)^2 + 2*(4*a*cos(6*f*x + 6*e) + 6*a*cos
(4*f*x + 4*e) + 4*a*cos(2*f*x + 2*e) + a)*cos(8*f*x + 8*e) + 8*(6*a*cos(4*f
*x + 4*e) + 4*a*cos(2*f*x + 2*e) + a)*cos(6*f*x + 6*e) + 12*(4*a*cos(2*f*x
+ 2*e) + a)*cos(4*f*x + 4*e) + 8*a*cos(2*f*x + 2*e) + 4*(2*a*sin(6*f*x + 6*
e) + 3*a*sin(4*f*x + 4*e) + 2*a*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 16*(3*
a*sin(4*f*x + 4*e) + 2*a*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + a)*sqrt(a)*f)
```

Fricas [A] time = 1.71558, size = 205, normalized size = 1.93

$$-\frac{\left(\cos\left(fx+e\right)^{4}\log\left(-\frac{\sin\left(fx+e\right)+1}{\sin\left(fx+e\right)-1}\right)+2\left(\cos\left(fx+e\right)^{2}-2\right)\sin\left(fx+e\right)\right)\sqrt{a\cos\left(fx+e\right)^{2}}}{16\,a^{2}f\cos\left(fx+e\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/16*(cos(f*x + e)^4*log(-(sin(f*x + e) + 1)/(sin(f*x + e) - 1)) + 2*(cos(
f*x + e)^2 - 2)*sin(f*x + e))*sqrt(a*cos(f*x + e)^2)/(a^2*f*cos(f*x + e)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^2(e+fx)}{\left(-a(\sin(e+fx)-1)(\sin(e+fx)+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a-a*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**2/(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))**(3/2),
 x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(fx+e)^2}{\left(-a\sin(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^2/(-a*sin(f*x + e)^2 + a)^(3/2), x)
```

3.484 $\int \frac{\cot^2(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=63

$$\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{af\sqrt{a\cos^2(e+fx)}}-\frac{\cot(e+fx)}{af\sqrt{a\cos^2(e+fx)}}$$

```
[Out] (ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(a*f*Sqrt[a*Cos[e + f*x]^2]) - Cot[e +
 f*x]/(a*f*Sqrt[a*Cos[e + f*x]^2])
```

Rubi [A] time = 0.132467, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3176, 3207, 2621, 321, 207}

$$\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{af\sqrt{a\cos^2(e+fx)}}-\frac{\cot(e+fx)}{af\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(a*f*Sqrt[a*Cos[e + f*x]^2]) - Cot[e +
 f*x]/(a*f*Sqrt[a*Cos[e + f*x]^2])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^2(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx &= \int \frac{\cot^2(e+fx)}{\left(a\cos^2(e+fx)\right)^{3/2}}\,dx \\
&= \frac{\cos(e+fx)\int \csc^2(e+fx)\sec(e+fx)\,dx}{a\sqrt{a\cos^2(e+fx)}} \\
&= -\frac{\cos(e+fx)\,\mathrm{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \csc(e+fx)\right)}{af\sqrt{a\cos^2(e+fx)}} \\
&= -\frac{\cot(e+fx)}{af\sqrt{a\cos^2(e+fx)}} - \frac{\cos(e+fx)\,\mathrm{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(e+fx)\right)}{af\sqrt{a\cos^2(e+fx)}} \\
&= \frac{\tanh^{-1}(\sin(e+fx))\cos(e+fx)}{af\sqrt{a\cos^2(e+fx)}} - \frac{\cot(e+fx)}{af\sqrt{a\cos^2(e+fx)}}
\end{aligned}
$$

Mathematica [C] time = 0.0756979, size = 44, normalized size = 0.7

$$
-\frac{\cot(e+fx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\sin^2(e+fx)\right)}{af\sqrt{a\cos^2(e+fx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -((Cot[e + f*x]*Hypergeometric2F1[-1/2, 1, 1/2, Sin[e + f*x]^2])/(a*f*Sqrt[
a*Cos[e + f*x]^2]))
```

Maple [A] time = 1.17, size = 65, normalized size = 1.

$$
-\frac{\cos\left(fx+e\right)\left(2+\sin\left(fx+e\right)\left(\ln\left(-1+\sin\left(fx+e\right)\right)-\ln\left(1+\sin\left(fx+e\right)\right)\right)\right)}{2\,a\sin\left(fx+e\right)f}\frac{1}{\sqrt{a\left(\cos\left(fx+e\right)\right)^{2}}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a-a*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/2/a*cos(f*x+e)*(2+sin(f*x+e)*(ln(-1+sin(f*x+e))-ln(1+sin(f*x+e))))/sin(f
*x+e)/(a*cos(f*x+e)^2)^(1/2)/f
```

Maxima [B] time = 1.71972, size = 297, normalized size = 4.71

$$
\left(\cos\left(2\,fx+2\,e\right)^{2}+\sin\left(2\,fx+2\,e\right)^{2}-2\,\cos\left(2\,fx+2\,e\right)+1\right)\log\left(\cos\left(fx+e\right)^{2}+\sin\left(fx+e\right)^{2}+2\,\sin\left(fx+e\right)+1\right)-
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 - 2*cos(2*f*x + 2*e) + 1)*log
(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin(f*x + e) + 1) - (cos(2*f*x + 2*e)^
2 + sin(2*f*x + 2*e)^2 - 2*cos(2*f*x + 2*e) + 1)*log(cos(f*x + e)^2 + sin(f
*x + e)^2 - 2*sin(f*x + e) + 1) - 4*cos(f*x + e)*sin(2*f*x + 2*e) + 4*cos(2
*f*x + 2*e)*sin(f*x + e) - 4*sin(f*x + e))/((a*cos(2*f*x + 2*e)^2 + a*sin(2
*f*x + 2*e)^2 - 2*a*cos(2*f*x + 2*e) + a)*sqrt(a)*f)
```

Fricas [A] time = 1.74274, size = 170, normalized size = 2.7

$$-\frac{\sqrt{a\cos\left(fx+e\right)^{2}}\left(\log\left(-\frac{\sin\left(fx+e\right)-1}{\sin\left(fx+e\right)+1}\right)\sin\left(fx+e\right)+2\right)}{2\,a^{2}f\cos\left(fx+e\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(a*cos(f*x + e)^2)*(log(-(sin(f*x + e) - 1)/(sin(f*x + e) + 1))*si
n(f*x + e) + 2)/(a^2*f*cos(f*x + e)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^{2}\left(e+fx\right)}{\left(-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a-a*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**2/(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))**(3/2),
 x)
```

Giac [A] time = 1.30893, size = 95, normalized size = 1.51

$$\frac{\frac{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{a^{\frac{3}{2}}\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-1\right)}+\frac{1}{a^{\frac{3}{2}}\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-1\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(tan(1/2*f*x + 1/2*e)/(a^(3/2)*sgn(tan(1/2*f*x + 1/2*e)^4 - 1)) + 1/(a^
(3/2)*sgn(tan(1/2*f*x + 1/2*e)^4 - 1)*tan(1/2*f*x + 1/2*e)))/f
```

3.485 $\int \frac{\cot^4(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{\cot(e+fx)\csc^2(e+fx)}{3af\sqrt{a\cos^2(e+fx)}}$$

```
[Out] -(Cot[e + f*x]*Csc[e + f*x]^2)/(3*a*f*Sqrt[a*Cos[e + f*x]^2])
```

Rubi [A] time = 0.126279, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3176, 3207, 2606, 30}

$$-\frac{\cot(e+fx)\csc^2(e+fx)}{3af\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Cot[e + f*x]*Csc[e + f*x]^2)/(3*a*f*Sqrt[a*Cos[e + f*x]^2])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\cot^4(e+fx)}{\left(a\cos^2(e+fx)\right)^{3/2}}\,dx$$
$$= \frac{\cos(e+fx)\int \cot(e+fx)\csc^3(e+fx)\,dx}{a\sqrt{a\cos^2(e+fx)}}$$
$$= -\frac{\cos(e+fx)\,\mathrm{Subst}\left(\int x^2\,dx, x, \csc(e+fx)\right)}{af\sqrt{a\cos^2(e+fx)}}$$
$$= -\frac{\cot(e+fx)\csc^2(e+fx)}{3af\sqrt{a\cos^2(e+fx)}}$$

Mathematica [A] time = 0.0343927, size = 29, normalized size = 0.76

$$-\frac{\cot^3(e+fx)}{3f\left(a\cos^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -Cot[e + f*x]^3/(3*f*(a*Cos[e + f*x]^2)^(3/2))
```

Maple [A] time = 0.528, size = 35, normalized size = 0.9

$$-\frac{\cos\left(fx+e\right)}{3\,a\left(\sin\left(fx+e\right)\right)^3 f}\frac{1}{\sqrt{a\left(\cos\left(fx+e\right)\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a-a*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/3/a*cos(f*x+e)/sin(f*x+e)^3/(a*cos(f*x+e)^2)^(1/2)/f
```

Maxima [B] time = 1.66848, size = 516, normalized size = 13.58

$$\frac{8\left(\cos\left(3\,fx+3\,e\right)\sin\left(6\,fx+6\,e\right)-\right.}{3\left(a^2\cos\left(6\,fx+6\,e\right)^2+9\,a^2\cos\left(4\,fx+4\,e\right)^2+9\,a^2\cos\left(2\,fx+2\,e\right)^2+a^2\sin\left(6\,fx+6\,e\right)^2+9\,a^2\sin\left(4\,fx+4\,e\right)^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 8/3*(cos(3*f*x + 3*e)*sin(6*f*x + 6*e) - 3*cos(3*f*x + 3*e)*sin(4*f*x + 4*e
) - (3*cos(2*f*x + 2*e) - 1)*sin(3*f*x + 3*e) - cos(6*f*x + 6*e)*sin(3*f*x
+ 3*e) + 3*cos(4*f*x + 4*e)*sin(3*f*x + 3*e) + 3*cos(3*f*x + 3*e)*sin(2*f*x
 + 2*e))*sqrt(a)/((a^2*cos(6*f*x + 6*e)^2 + 9*a^2*cos(4*f*x + 4*e)^2 + 9*a^
2*cos(2*f*x + 2*e)^2 + a^2*sin(6*f*x + 6*e)^2 + 9*a^2*sin(4*f*x + 4*e)^2 -
18*a^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 9*a^2*sin(2*f*x + 2*e)^2 - 6*a^2
```

```
*cos(2*f*x + 2*e) + a^2 - 2*(3*a^2*cos(4*f*x + 4*e) - 3*a^2*cos(2*f*x + 2*e
) + a^2)*cos(6*f*x + 6*e) - 6*(3*a^2*cos(2*f*x + 2*e) - a^2)*cos(4*f*x + 4*
e) - 6*(a^2*sin(4*f*x + 4*e) - a^2*sin(2*f*x + 2*e))*sin(6*f*x + 6*e))*f)
```

Fricas [A] time = 1.6556, size = 117, normalized size = 3.08

$$\frac{\sqrt{a\cos\left(fx+e\right)^2}}{3\left(a^2f\cos\left(fx+e\right)^3-a^2f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(a*cos(f*x + e)^2)/((a^2*f*cos(f*x + e)^3 - a^2*f*cos(f*x + e))*sin
(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^4\left(e+fx\right)}{\left(-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a-a*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**4/(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))**(3/2),
 x)
```

Giac [B] time = 1.37892, size = 153, normalized size = 4.03

$$\frac{\frac{3\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+\sqrt{a}}{a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-1\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3}+\frac{a^{\frac{9}{2}}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+3a^{\frac{9}{2}}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a^6\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-1\right)}}{24f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/24*((3*sqrt(a)*tan(1/2*f*x + 1/2*e)^2 + sqrt(a))/(a^2*sgn(tan(1/2*f*x + 1
/2*e)^4 - 1)*tan(1/2*f*x + 1/2*e)^3) + (a^(9/2)*tan(1/2*f*x + 1/2*e)^3 + 3*
a^(9/2)*tan(1/2*f*x + 1/2*e))/(a^6*sgn(tan(1/2*f*x + 1/2*e)^4 - 1)))/f
```

3.486 $\int \frac{\cot^6(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=77

$$\frac{\cot(e+fx)\csc^2(e+fx)}{3af\sqrt{a\cos^2(e+fx)}}-\frac{\cot(e+fx)\csc^4(e+fx)}{5af\sqrt{a\cos^2(e+fx)}}$$

```
[Out] (Cot[e + f*x]*Csc[e + f*x]^2)/(3*a*f*Sqrt[a*Cos[e + f*x]^2]) - (Cot[e + f*x
]*Csc[e + f*x]^4)/(5*a*f*Sqrt[a*Cos[e + f*x]^2])
```

Rubi [A] time = 0.144077, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3176, 3207, 2606, 14}

$$\frac{\cot(e+fx)\csc^2(e+fx)}{3af\sqrt{a\cos^2(e+fx)}}-\frac{\cot(e+fx)\csc^4(e+fx)}{5af\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (Cot[e + f*x]*Csc[e + f*x]^2)/(3*a*f*Sqrt[a*Cos[e + f*x]^2]) - (Cot[e + f*x
]*Csc[e + f*x]^4)/(5*a*f*Sqrt[a*Cos[e + f*x]^2])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^6(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx &= \int \frac{\cot^6(e+fx)}{\left(a\cos^2(e+fx)\right)^{3/2}}\,dx\\
&= \frac{\cos(e+fx)\int \cot^3(e+fx)\csc^3(e+fx)\,dx}{a\sqrt{a\cos^2(e+fx)}}\\
&= -\frac{\cos(e+fx)\,\text{Subst}\left(\int x^2\left(-1+x^2\right)\,dx,x,\csc(e+fx)\right)}{af\sqrt{a\cos^2(e+fx)}}\\
&= -\frac{\cos(e+fx)\,\text{Subst}\left(\int \left(-x^2+x^4\right)\,dx,x,\csc(e+fx)\right)}{af\sqrt{a\cos^2(e+fx)}}\\
&= \frac{\cot(e+fx)\csc^2(e+fx)}{3af\sqrt{a\cos^2(e+fx)}}-\frac{\cot(e+fx)\csc^4(e+fx)}{5af\sqrt{a\cos^2(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.0932803, size = 41, normalized size = 0.53

$$-\frac{\cot^3(e+fx)\left(3\csc^2(e+fx)-5\right)}{15f\left(a\cos^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Cot[e + f*x]^3*(-5 + 3*Csc[e + f*x]^2))/(15*f*(a*Cos[e + f*x]^2)^(3/2))
```

Maple [A] time = 0.855, size = 67, normalized size = 0.9

$$-\frac{\cos\left(fx+e\right)\left(5\left(\cos\left(fx+e\right)\right)^2-2\right)}{15\left(-1+\cos\left(fx+e\right)\right)^2\left(\cos\left(fx+e\right)+1\right)^2 a\sin\left(fx+e\right)f}\frac{1}{\sqrt{a\left(\cos\left(fx+e\right)\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6/(a-a*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/15*cos(f*x+e)*(5*cos(f*x+e)^2-2)/(-1+cos(f*x+e))^2/(cos(f*x+e)+1)^2/a/si
n(f*x+e)/(a*cos(f*x+e)^2)^(1/2)/f
```

Maxima [B] time = 1.74951, size = 1435, normalized size = 18.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 8/15*((5*sin(7*f*x + 7*e) + 2*sin(5*f*x + 5*e) + 5*sin(3*f*x + 3*e))*cos(10
*f*x + 10*e) - 5*(5*sin(7*f*x + 7*e) + 2*sin(5*f*x + 5*e) + 5*sin(3*f*x + 3
*e))*cos(8*f*x + 8*e) - 25*(2*sin(6*f*x + 6*e) - 2*sin(4*f*x + 4*e) + sin(2
*f*x + 2*e))*cos(7*f*x + 7*e) + 10*(2*sin(5*f*x + 5*e) + 5*sin(3*f*x + 3*e)
```

```
)*cos(6*f*x + 6*e) + 10*(2*sin(4*f*x + 4*e) - sin(2*f*x + 2*e))*cos(5*f*x +
 5*e) - (5*cos(7*f*x + 7*e) + 2*cos(5*f*x + 5*e) + 5*cos(3*f*x + 3*e))*sin(
10*f*x + 10*e) + 5*(5*cos(7*f*x + 7*e) + 2*cos(5*f*x + 5*e) + 5*cos(3*f*x +
 3*e))*sin(8*f*x + 8*e) + 5*(10*cos(6*f*x + 6*e) - 10*cos(4*f*x + 4*e) + 5*
cos(2*f*x + 2*e) - 1)*sin(7*f*x + 7*e) - 10*(2*cos(5*f*x + 5*e) + 5*cos(3*f
*x + 3*e))*sin(6*f*x + 6*e) - 2*(10*cos(4*f*x + 4*e) - 5*cos(2*f*x + 2*e) +
 1)*sin(5*f*x + 5*e) + 50*cos(3*f*x + 3*e)*sin(4*f*x + 4*e) + 5*(5*cos(2*f*
x + 2*e) - 1)*sin(3*f*x + 3*e) - 50*cos(4*f*x + 4*e)*sin(3*f*x + 3*e) - 25*
cos(3*f*x + 3*e)*sin(2*f*x + 2*e))*sqrt(a)/((a^2*cos(10*f*x + 10*e)^2 + 25*
a^2*cos(8*f*x + 8*e)^2 + 100*a^2*cos(6*f*x + 6*e)^2 + 100*a^2*cos(4*f*x + 4
*e)^2 + 25*a^2*cos(2*f*x + 2*e)^2 + a^2*sin(10*f*x + 10*e)^2 + 25*a^2*sin(8
*f*x + 8*e)^2 + 100*a^2*sin(6*f*x + 6*e)^2 + 100*a^2*sin(4*f*x + 4*e)^2 - 1
00*a^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 25*a^2*sin(2*f*x + 2*e)^2 - 10*a
^2*cos(2*f*x + 2*e) + a^2 - 2*(5*a^2*cos(8*f*x + 8*e) - 10*a^2*cos(6*f*x +
6*e) + 10*a^2*cos(4*f*x + 4*e) - 5*a^2*cos(2*f*x + 2*e) + a^2)*cos(10*f*x +
 10*e) - 10*(10*a^2*cos(6*f*x + 6*e) - 10*a^2*cos(4*f*x + 4*e) + 5*a^2*cos(
2*f*x + 2*e) - a^2)*cos(8*f*x + 8*e) - 20*(10*a^2*cos(4*f*x + 4*e) - 5*a^2*
cos(2*f*x + 2*e) + a^2)*cos(6*f*x + 6*e) - 20*(5*a^2*cos(2*f*x + 2*e) - a^2
)*cos(4*f*x + 4*e) - 10*(a^2*sin(8*f*x + 8*e) - 2*a^2*sin(6*f*x + 6*e) + 2*
a^2*sin(4*f*x + 4*e) - a^2*sin(2*f*x + 2*e))*sin(10*f*x + 10*e) - 50*(2*a^2
*sin(6*f*x + 6*e) - 2*a^2*sin(4*f*x + 4*e) + a^2*sin(2*f*x + 2*e))*sin(8*f*
x + 8*e) - 100*(2*a^2*sin(4*f*x + 4*e) - a^2*sin(2*f*x + 2*e))*sin(6*f*x +
6*e))*f)
```

Fricas [A] time = 1.68613, size = 185, normalized size = 2.4

$$-\frac{\sqrt{a\cos\left(fx+e\right)^2}\left(5\cos\left(fx+e\right)^2-2\right)}{15\left(a^2f\cos\left(fx+e\right)^5-2\,a^2f\cos\left(fx+e\right)^3+a^2f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/15*sqrt(a*cos(f*x + e)^2)*(5*cos(f*x + e)^2 - 2)/((a^2*f*cos(f*x + e)^5
- 2*a^2*f*cos(f*x + e)^3 + a^2*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6/(a-a*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.42406, size = 204, normalized size = 2.65

$$-\frac{\frac{30\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+5\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-3\sqrt{a}}{a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-1\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5}-\frac{3a^{\frac{17}{2}}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-5a^{\frac{17}{2}}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-30a^{\frac{17}{2}}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a^{10}\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-1\right)}}{480f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/480*((30*sqrt(a)*tan(1/2*f*x + 1/2*e)^4 + 5*sqrt(a)*tan(1/2*f*x + 1/2*e)
^2 - 3*sqrt(a))/(a^2*sgn(tan(1/2*f*x + 1/2*e)^4 - 1)*tan(1/2*f*x + 1/2*e)^5
) - (3*a^(17/2)*tan(1/2*f*x + 1/2*e)^5 - 5*a^(17/2)*tan(1/2*f*x + 1/2*e)^3
- 30*a^(17/2)*tan(1/2*f*x + 1/2*e))/(a^10*sgn(tan(1/2*f*x + 1/2*e)^4 - 1)))
/f
```

3.487 $\int \frac{\cot^8(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=115

$$-\frac{\cot(e+fx)\csc^6(e+fx)}{7af\sqrt{a\cos^2(e+fx)}}+\frac{2\cot(e+fx)\csc^4(e+fx)}{5af\sqrt{a\cos^2(e+fx)}}-\frac{\cot(e+fx)\csc^2(e+fx)}{3af\sqrt{a\cos^2(e+fx)}}$$

```
[Out] -(Cot[e + f*x]*Csc[e + f*x]^2)/(3*a*f*Sqrt[a*Cos[e + f*x]^2]) + (2*Cot[e +
f*x]*Csc[e + f*x]^4)/(5*a*f*Sqrt[a*Cos[e + f*x]^2]) - (Cot[e + f*x]*Csc[e +
 f*x]^6)/(7*a*f*Sqrt[a*Cos[e + f*x]^2])
```

Rubi [A] time = 0.151866, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3176, 3207, 2606, 270}

$$-\frac{\cot(e+fx)\csc^6(e+fx)}{7af\sqrt{a\cos^2(e+fx)}}+\frac{2\cot(e+fx)\csc^4(e+fx)}{5af\sqrt{a\cos^2(e+fx)}}-\frac{\cot(e+fx)\csc^2(e+fx)}{3af\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^8/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Cot[e + f*x]*Csc[e + f*x]^2)/(3*a*f*Sqrt[a*Cos[e + f*x]^2]) + (2*Cot[e +
f*x]*Csc[e + f*x]^4)/(5*a*f*Sqrt[a*Cos[e + f*x]^2]) - (Cot[e + f*x]*Csc[e +
 f*x]^6)/(7*a*f*Sqrt[a*Cos[e + f*x]^2])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^8(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx &= \int \frac{\cot^8(e+fx)}{\left(a\cos^2(e+fx)\right)^{3/2}}\,dx\\ &= \frac{\cos(e+fx)\int \cot^5(e+fx)\csc^3(e+fx)\,dx}{a\sqrt{a\cos^2(e+fx)}}\\ &= -\frac{\cos(e+fx)\,\text{Subst}\left(\int x^2\left(-1+x^2\right)^2\,dx, x, \csc(e+fx)\right)}{af\sqrt{a\cos^2(e+fx)}}\\ &= -\frac{\cos(e+fx)\,\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)\,dx, x, \csc(e+fx)\right)}{af\sqrt{a\cos^2(e+fx)}}\\ &= -\frac{\cot(e+fx)\csc^2(e+fx)}{3af\sqrt{a\cos^2(e+fx)}}+\frac{2\cot(e+fx)\csc^4(e+fx)}{5af\sqrt{a\cos^2(e+fx)}}-\frac{\cot(e+fx)\csc^6(e+fx)}{7af\sqrt{a\cos^2(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.136375, size = 51, normalized size = 0.44

$$-\frac{\cot^3(e+fx)\left(15\csc^4(e+fx)-42\csc^2(e+fx)+35\right)}{105f\left(a\cos^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^8/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Cot[e + f*x]^3*(35 - 42*Csc[e + f*x]^2 + 15*Csc[e + f*x]^4))/(105*f*(a*Co
s[e + f*x]^2)^(3/2))
```

Maple [A] time = 0.731, size = 57, normalized size = 0.5

$$-\frac{\cos(fx+e)\left(35\left(\cos(fx+e)\right)^4-28\left(\cos(fx+e)\right)^2+8\right)}{105\,a\left(\sin(fx+e)\right)^7 f}\frac{1}{\sqrt{a\left(\cos(fx+e)\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^8/(a-a*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/105/a*cos(f*x+e)*(35*cos(f*x+e)^4-28*cos(f*x+e)^2+8)/sin(f*x+e)^7/(a*cos
(f*x+e)^2)^(1/2)/f
```

Maxima [B] time = 1.82258, size = 2735, normalized size = 23.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^8/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -8/105*((35*sin(11*f*x + 11*e) + 28*sin(9*f*x + 9*e) + 114*sin(7*f*x + 7*e)
 + 28*sin(5*f*x + 5*e) + 35*sin(3*f*x + 3*e))*cos(14*f*x + 14*e) - 7*(35*si
```

```
n(11*f*x + 11*e) + 28*sin(9*f*x + 9*e) + 114*sin(7*f*x + 7*e) + 28*sin(5*f*
x + 5*e) + 35*sin(3*f*x + 3*e))*cos(12*f*x + 12*e) - 245*(3*sin(10*f*x + 10
*e) - 5*sin(8*f*x + 8*e) + 5*sin(6*f*x + 6*e) - 3*sin(4*f*x + 4*e) + sin(2*
f*x + 2*e))*cos(11*f*x + 11*e) + 21*(28*sin(9*f*x + 9*e) + 114*sin(7*f*x +
7*e) + 28*sin(5*f*x + 5*e) + 35*sin(3*f*x + 3*e))*cos(10*f*x + 10*e) + 196*
(5*sin(8*f*x + 8*e) - 5*sin(6*f*x + 6*e) + 3*sin(4*f*x + 4*e) - sin(2*f*x +
 2*e))*cos(9*f*x + 9*e) - 35*(114*sin(7*f*x + 7*e) + 28*sin(5*f*x + 5*e) +
35*sin(3*f*x + 3*e))*cos(8*f*x + 8*e) - 798*(5*sin(6*f*x + 6*e) - 3*sin(4*f
*x + 4*e) + sin(2*f*x + 2*e))*cos(7*f*x + 7*e) + 245*(4*sin(5*f*x + 5*e) +
5*sin(3*f*x + 3*e))*cos(6*f*x + 6*e) + 196*(3*sin(4*f*x + 4*e) - sin(2*f*x
+ 2*e))*cos(5*f*x + 5*e) - (35*cos(11*f*x + 11*e) + 28*cos(9*f*x + 9*e) + 1
14*cos(7*f*x + 7*e) + 28*cos(5*f*x + 5*e) + 35*cos(3*f*x + 3*e))*sin(14*f*x
 + 14*e) + 7*(35*cos(11*f*x + 11*e) + 28*cos(9*f*x + 9*e) + 114*cos(7*f*x +
 7*e) + 28*cos(5*f*x + 5*e) + 35*cos(3*f*x + 3*e))*sin(12*f*x + 12*e) + 35*
(21*cos(10*f*x + 10*e) - 35*cos(8*f*x + 8*e) + 35*cos(6*f*x + 6*e) - 21*cos
(4*f*x + 4*e) + 7*cos(2*f*x + 2*e) - 1)*sin(11*f*x + 11*e) - 21*(28*cos(9*f
*x + 9*e) + 114*cos(7*f*x + 7*e) + 28*cos(5*f*x + 5*e) + 35*cos(3*f*x + 3*e
))*sin(10*f*x + 10*e) - 28*(35*cos(8*f*x + 8*e) - 35*cos(6*f*x + 6*e) + 21*
cos(4*f*x + 4*e) - 7*cos(2*f*x + 2*e) + 1)*sin(9*f*x + 9*e) + 35*(114*cos(7
*f*x + 7*e) + 28*cos(5*f*x + 5*e) + 35*cos(3*f*x + 3*e))*sin(8*f*x + 8*e) +
 114*(35*cos(6*f*x + 6*e) - 21*cos(4*f*x + 4*e) + 7*cos(2*f*x + 2*e) - 1)*s
in(7*f*x + 7*e) - 245*(4*cos(5*f*x + 5*e) + 5*cos(3*f*x + 3*e))*sin(6*f*x +
 6*e) - 28*(21*cos(4*f*x + 4*e) - 7*cos(2*f*x + 2*e) + 1)*sin(5*f*x + 5*e)
+ 735*cos(3*f*x + 3*e)*sin(4*f*x + 4*e) + 35*(7*cos(2*f*x + 2*e) - 1)*sin(3
*f*x + 3*e) - 735*cos(4*f*x + 4*e)*sin(3*f*x + 3*e) - 245*cos(3*f*x + 3*e)*
sin(2*f*x + 2*e))*sqrt(a)/((a^2*cos(14*f*x + 14*e)^2 + 49*a^2*cos(12*f*x +
12*e)^2 + 441*a^2*cos(10*f*x + 10*e)^2 + 1225*a^2*cos(8*f*x + 8*e)^2 + 1225
*a^2*cos(6*f*x + 6*e)^2 + 441*a^2*cos(4*f*x + 4*e)^2 + 49*a^2*cos(2*f*x + 2
*e)^2 + a^2*sin(14*f*x + 14*e)^2 + 49*a^2*sin(12*f*x + 12*e)^2 + 441*a^2*si
n(10*f*x + 10*e)^2 + 1225*a^2*sin(8*f*x + 8*e)^2 + 1225*a^2*sin(6*f*x + 6*e
)^2 + 441*a^2*sin(4*f*x + 4*e)^2 - 294*a^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e
) + 49*a^2*sin(2*f*x + 2*e)^2 - 14*a^2*cos(2*f*x + 2*e) + a^2 - 2*(7*a^2*co
s(12*f*x + 12*e) - 21*a^2*cos(10*f*x + 10*e) + 35*a^2*cos(8*f*x + 8*e) - 35
*a^2*cos(6*f*x + 6*e) + 21*a^2*cos(4*f*x + 4*e) - 7*a^2*cos(2*f*x + 2*e) +
a^2)*cos(14*f*x + 14*e) - 14*(21*a^2*cos(10*f*x + 10*e) - 35*a^2*cos(8*f*x
+ 8*e) + 35*a^2*cos(6*f*x + 6*e) - 21*a^2*cos(4*f*x + 4*e) + 7*a^2*cos(2*f*
x + 2*e) - a^2)*cos(12*f*x + 12*e) - 42*(35*a^2*cos(8*f*x + 8*e) - 35*a^2*c
os(6*f*x + 6*e) + 21*a^2*cos(4*f*x + 4*e) - 7*a^2*cos(2*f*x + 2*e) + a^2)*c
os(10*f*x + 10*e) - 70*(35*a^2*cos(6*f*x + 6*e) - 21*a^2*cos(4*f*x + 4*e) +
 7*a^2*cos(2*f*x + 2*e) - a^2)*cos(8*f*x + 8*e) - 70*(21*a^2*cos(4*f*x + 4*
e) - 7*a^2*cos(2*f*x + 2*e) + a^2)*cos(6*f*x + 6*e) - 42*(7*a^2*cos(2*f*x +
 2*e) - a^2)*cos(4*f*x + 4*e) - 14*(a^2*sin(12*f*x + 12*e) - 3*a^2*sin(10*f
*x + 10*e) + 5*a^2*sin(8*f*x + 8*e) - 5*a^2*sin(6*f*x + 6*e) + 3*a^2*sin(4*
f*x + 4*e) - a^2*sin(2*f*x + 2*e))*sin(14*f*x + 14*e) - 98*(3*a^2*sin(10*f*
x + 10*e) - 5*a^2*sin(8*f*x + 8*e) + 5*a^2*sin(6*f*x + 6*e) - 3*a^2*sin(4*f
*x + 4*e) + a^2*sin(2*f*x + 2*e))*sin(12*f*x + 12*e) - 294*(5*a^2*sin(8*f*x
 + 8*e) - 5*a^2*sin(6*f*x + 6*e) + 3*a^2*sin(4*f*x + 4*e) - a^2*sin(2*f*x +
 2*e))*sin(10*f*x + 10*e) - 490*(5*a^2*sin(6*f*x + 6*e) - 3*a^2*sin(4*f*x +
 4*e) + a^2*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) - 490*(3*a^2*sin(4*f*x + 4*e
) - a^2*sin(2*f*x + 2*e))*sin(6*f*x + 6*e))*f)
```

Fricas [A] time = 1.67666, size = 247, normalized size = 2.15

$$\frac{\left(35 \cos\left(fx+e\right)^{4}-28 \cos\left(fx+e\right)^{2}+8\right)\sqrt{a\cos\left(fx+e\right)^{2}}}{105\left(a^{2}f\cos\left(fx+e\right)^{7}-3\,a^{2}f\cos\left(fx+e\right)^{5}+3\,a^{2}f\cos\left(fx+e\right)^{3}-a^{2}f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^8/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/105*(35*cos(f*x + e)^4 - 28*cos(f*x + e)^2 + 8)*sqrt(a*cos(f*x + e)^2)/((
a^2*f*cos(f*x + e)^7 - 3*a^2*f*cos(f*x + e)^5 + 3*a^2*f*cos(f*x + e)^3 - a^
2*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**8/(a-a*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.4991, size = 248, normalized size = 2.16

$$\frac{\frac{525\,\sqrt{a}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}+35\,\sqrt{a}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-63\,\sqrt{a}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+15\,\sqrt{a}}{a^{2}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-1\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{7}}+\frac{15\,a^{\frac{25}{2}}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{7}-63\,a^{\frac{25}{2}}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+35\,a^{\frac{25}{2}}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+5}{a^{14}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-1\right)}}{13440\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^8/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/13440*((525*sqrt(a)*tan(1/2*f*x + 1/2*e)^6 + 35*sqrt(a)*tan(1/2*f*x + 1/2
*e)^4 - 63*sqrt(a)*tan(1/2*f*x + 1/2*e)^2 + 15*sqrt(a))/(a^2*sgn(tan(1/2*f*
x + 1/2*e)^4 - 1)*tan(1/2*f*x + 1/2*e)^7) + (15*a^(25/2)*tan(1/2*f*x + 1/2*
e)^7 - 63*a^(25/2)*tan(1/2*f*x + 1/2*e)^5 + 35*a^(25/2)*tan(1/2*f*x + 1/2*e
)^3 + 525*a^(25/2)*tan(1/2*f*x + 1/2*e))/(a^14*sgn(tan(1/2*f*x + 1/2*e)^4 -
 1)))/f
```

3.488 $\int \sqrt{a+b\sin^2(e+fx)}\tan^5(e+fx)\,dx$

Optimal. Leaf size=177

$$-\frac{\left(8a^2+24ab+15b^2\right)\sqrt{a+b\sin^2(e+fx)}}{8f(a+b)^2}+\frac{\left(8a^2+24ab+15b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{8f(a+b)^{3/2}}+\frac{\sec^4(e+fx)\left(a+b\sin^2(\right.}{4f(a+b)}$$

```
[Out] ((8*a^2 + 24*a*b + 15*b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])
/(8*(a + b)^(3/2)*f) - ((8*a^2 + 24*a*b + 15*b^2)*Sqrt[a + b*Sin[e + f*x]^2
])/(8*(a + b)^2*f) - ((8*a + 7*b)*Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/
2))/(8*(a + b)^2*f) + (Sec[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2))/(4*(a +
 b)*f)
```

Rubi [A] time = 0.210743, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3194, 89, 78, 50, 63, 208}

$$-\frac{\left(8a^2+24ab+15b^2\right)\sqrt{a+b\sin^2(e+fx)}}{8f(a+b)^2}+\frac{\left(8a^2+24ab+15b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{8f(a+b)^{3/2}}+\frac{\sec^4(e+fx)\left(a+b\sin^2(\right.}{4f(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x]^5,x]
```

```
[Out] ((8*a^2 + 24*a*b + 15*b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])
/(8*(a + b)^(3/2)*f) - ((8*a^2 + 24*a*b + 15*b^2)*Sqrt[a + b*Sin[e + f*x]^2
])/(8*(a + b)^2*f) - ((8*a + 7*b)*Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/
2))/(8*(a + b)^2*f) + (Sec[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2))/(4*(a +
 b)*f)
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
```

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \sqrt{a+b\sin^2(e+fx)}\tan^5(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^2\sqrt{a+bx}}{(1-x)^3}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= \frac{\sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4(a+b)f} - \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}\left(\frac{1}{2}(4a+3b)+2(a+b)x\right)}{(1-x)^2}\,dx, x, \sin\right.}{4(a+b)f}$$

$$= -\frac{(8a+7b)\sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{8(a+b)^2f} + \frac{\sec^4(e+fx)\left(a+b\sin^2(e+\right.}{4(a+b)f}$$

$$= -\frac{\left(8a^2+24ab+15b^2\right)\sqrt{a+b\sin^2(e+fx)}}{8(a+b)^2f} - \frac{(8a+7b)\sec^2(e+fx)\left(a+b\sin^2\right.}{8(a+b)^2f}$$

$$= -\frac{\left(8a^2+24ab+15b^2\right)\sqrt{a+b\sin^2(e+fx)}}{8(a+b)^2f} - \frac{(8a+7b)\sec^2(e+fx)\left(a+b\sin^2\right.}{8(a+b)^2f}$$

$$= \frac{\left(8a^2+24ab+15b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{8(a+b)^{3/2}f} - \frac{\left(8a^2+24ab+15b^2\right)\sqrt{a+b\sin\right.}{8(a+b)^2f}$$

Mathematica [A] time = 0.600288, size = 143, normalized size = 0.81

$$\frac{\left(8a^2+24ab+15b^2\right)\left(\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)-\sqrt{a+b\sin^2(e+fx)}\right)+2(a+b)\sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)}{8f(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x]^5,x]
```

```
[Out] (-((8*a + 7*b)*Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2)) + 2*(a + b)*Sec
[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2) + (8*a^2 + 24*a*b + 15*b^2)*(Sqrt[
a + b]*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]] - Sqrt[a + b*Sin[e +
 f*x]^2]))/(8*(a + b)^2*f)
```

Maple [B] time = 4.911, size = 721, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x)
```

```
[Out] 1/16*((-16*(a+b)^(3/2)*(a+b-b*cos(f*x+e)^2)^(1/2)*a^2-48*(a+b)^(3/2)*(a+b-b
*cos(f*x+e)^2)^(1/2)*a*b-30*b^2*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(3/2)+8*ln
(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*
a^4+40*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*
x+e)+a))*a^3*b+71*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/
2)-b*sin(f*x+e)+a))*a^2*b^2+54*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(
f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a*b^3+15*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*
(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*b^4+8*ln(2/(-1+sin(f*x+e))*((a+
b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^4+40*ln(2/(-1+sin(f*
x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^3*b+71*ln(
2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*
a^2*b^2+54*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*s
in(f*x+e)+a))*a*b^3+15*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^
2)^(1/2)+b*sin(f*x+e)+a))*b^4)*cos(f*x+e)^4-2*(a+b)^(3/2)*(a+b-b*cos(f*x+e)
^2)^(3/2)*(8*a+7*b)*cos(f*x+e)^2+4*(a+b)^(3/2)*(a+b-b*cos(f*x+e)^2)^(3/2)*a
+4*b*(a+b-b*cos(f*x+e)^2)^(3/2)*(a+b)^(3/2))/(a+b)^(3/2)/cos(f*x+e)^4/(a^2+
2*a*b+b^2)/f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 8.29031, size = 871, normalized size = 4.92

$$\left[\frac{\left(8\,a^2 + 24\,ab + 15\,b^2\right)\sqrt{a+b}\cos\left(fx+e\right)^4\log\left(\frac{b\cos\left(fx+e\right)^2 - 2\,\sqrt{-b\cos\left(fx+e\right)^2+a+b}\sqrt{a+b} - 2\,a - 2\,b}{\cos\left(fx+e\right)^2}\right) - 2\left(8\left(a^2+2\,ab+b^2\right)\cos}{16\left(a^2+2\,ab+b^2\right)f\cos\left(fx\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] [1/16*((8*a^2 + 24*a*b + 15*b^2)*sqrt(a + b)*cos(f*x + e)^4*log((b*cos(f*x
+ e)^2 - 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b) - 2*a - 2*b)/cos(f*x
 + e)^2) - 2*(8*(a^2 + 2*a*b + b^2)*cos(f*x + e)^4 + (8*a^2 + 17*a*b + 9*b^
2)*cos(f*x + e)^2 - 2*a^2 - 4*a*b - 2*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b))
/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^4), -1/8*((8*a^2 + 24*a*b + 15*b^2)*sq
rt(-a - b)*arctan(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(a + b))*cos
(f*x + e)^4 + (8*(a^2 + 2*a*b + b^2)*cos(f*x + e)^4 + (8*a^2 + 17*a*b + 9*b
^2)*cos(f*x + e)^2 - 2*a^2 - 4*a*b - 2*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)
)/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**(1/2)*tan(f*x+e)**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin\left(fx+e\right)^2+a}\tan\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*tan(f*x + e)^5, x)
```

3.489 $\int \sqrt{a+b\sin^2(e+fx)}\tan^3(e+fx)\,dx$

Optimal. Leaf size=118

$$\frac{(2a+3b)\sqrt{a+b\sin^2(e+fx)}}{2f(a+b)}-\frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{2f\sqrt{a+b}}+\frac{\sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{2f(a+b)}$$

```
[Out] -((2*a + 3*b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/(2*Sqrt[a +
b]*f) + ((2*a + 3*b)*Sqrt[a + b*Sin[e + f*x]^2])/(2*(a + b)*f) + (Sec[e + f
*x]^2*(a + b*Sin[e + f*x]^2)^(3/2))/(2*(a + b)*f)
```

Rubi [A] time = 0.111078, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3194, 78, 50, 63, 208}

$$\frac{(2a+3b)\sqrt{a+b\sin^2(e+fx)}}{2f(a+b)}-\frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{2f\sqrt{a+b}}+\frac{\sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{2f(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x]^3,x]
```

```
[Out] -((2*a + 3*b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/(2*Sqrt[a +
b]*f) + ((2*a + 3*b)*Sqrt[a + b*Sin[e + f*x]^2])/(2*(a + b)*f) + (Sec[e + f
*x]^2*(a + b*Sin[e + f*x]^2)^(3/2))/(2*(a + b)*f)
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \sqrt{a+b\sin^2(e+fx)}\tan^3(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x\sqrt{a+bx}}{(1-x)^2}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= \frac{\sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{2(a+b)f} - \frac{(2a+3b)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{1-x}\,dx, x, \sin^2(e+\right.}{4(a+b)f}$$

$$= \frac{(2a+3b)\sqrt{a+b\sin^2(e+fx)}}{2(a+b)f} + \frac{\sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{2(a+b)f} - \frac{(2a+3}{}$$

$$= \frac{(2a+3b)\sqrt{a+b\sin^2(e+fx)}}{2(a+b)f} + \frac{\sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{2(a+b)f} - \frac{(2a+3}{}$$

$$= -\frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{2\sqrt{a+b}f} + \frac{(2a+3b)\sqrt{a+b\sin^2(e+fx)}}{2(a+b)f} + \frac{\sec^2(e+}{}$$

Mathematica [A] time = 0.398922, size = 84, normalized size = 0.71

$$\frac{(\cos(2(e+fx))+2)\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)} - \frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x]^3,x]
```

```
[Out] (-(((2*a + 3*b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/Sqrt[a + b
]) + (2 + Cos[2*(e + f*x)])*Sec[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2])/(2*f
)
```

Maple [B] time = 4.355, size = 403, normalized size = 3.4

$$\frac{1}{4\left(\cos\left(fx+e\right)\right)^2 f}\left(-\left(-4\sqrt{a+b}\sqrt{a+b-b\left(\cos\left(fx+e\right)\right)^2}a - 6b\sqrt{a+b-b\left(\cos\left(fx+e\right)\right)^2}\sqrt{a+b} + 2\ln\left(2\frac{\sqrt{a+b}\sqrt{}}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x)
```

```
[Out] 1/4*(-(-4*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)*a-6*b*(a+b-b*cos(f*x+e)^2)
^(1/2)*(a+b)^(1/2)+2*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^
(1/2)-b*sin(f*x+e)+a))*a^2+5*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*
x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a*b+3*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-
b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*b^2+2*ln(2/(-1+sin(f*x+e))*((a+b)^(1
/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^2+5*ln(2/(-1+sin(f*x+e))*
((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a*b+3*ln(2/(-1+sin
(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*b^2)*cos(
f*x+e)^2+2*(a+b-b*cos(f*x+e)^2)^(3/2)*(a+b)^(1/2))/(a+b)^(3/2)/cos(f*x+e)^2
/f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 4.748, size = 609, normalized size = 5.16

$$\left[\frac{(2a+3b)\sqrt{a+b}\cos\left(fx+e\right)^2\log\left(\frac{b\cos\left(fx+e\right)^2+2\sqrt{-b\cos\left(fx+e\right)^2+a+b}\sqrt{a+b}-2a-2b}{\cos\left(fx+e\right)^2}\right)+2\left(2(a+b)\cos\left(fx+e\right)^2+a+b\right)\sqrt{}}{4(a+b)f\cos\left(fx+e\right)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] [1/4*((2*a + 3*b)*sqrt(a + b)*cos(f*x + e)^2*log((b*cos(f*x + e)^2 + 2*sqrt
(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b) - 2*a - 2*b)/cos(f*x + e)^2) + 2*(2
*(a + b)*cos(f*x + e)^2 + a + b)*sqrt(-b*cos(f*x + e)^2 + a + b))/((a + b)*
f*cos(f*x + e)^2), 1/2*((2*a + 3*b)*sqrt(-a - b)*arctan(sqrt(-b*cos(f*x + e
)^2 + a + b)*sqrt(-a - b)/(a + b))*cos(f*x + e)^2 + (2*(a + b)*cos(f*x + e)
^2 + a + b)*sqrt(-b*cos(f*x + e)^2 + a + b))/((a + b)*f*cos(f*x + e)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\sin^2\left(e+fx\right)}\tan^3\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**(1/2)*tan(f*x+e)**3,x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*tan(e + f*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin\left(fx+e\right)^2+a}\tan\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*tan(f*x + e)^3, x)
```

3.490 $\int \sqrt{a+b\sin^2(e+fx)}\tan(e+fx)\,dx$

Optimal. Leaf size=58

$$\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{f}-\frac{\sqrt{a+b\sin^2(e+fx)}}{f}$$

```
[Out] (Sqrt[a + b]*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/f - Sqrt[a +
b*Sin[e + f*x]^2]/f
```

Rubi [A] time = 0.0559427, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3194, 50, 63, 208}

$$\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{f}-\frac{\sqrt{a+b\sin^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x],x]
```

```
[Out] (Sqrt[a + b]*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/f - Sqrt[a +
b*Sin[e + f*x]^2]/f
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \sqrt{a+b\sin^2(e+fx)}\tan(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{1-x}\,dx,x,\sin^2(e+fx)\right)}{2f}$$
$$= -\frac{\sqrt{a+b\sin^2(e+fx)}}{f} + \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,dx,x,\sin^2(e+fx)\right)}{2f}$$
$$= -\frac{\sqrt{a+b\sin^2(e+fx)}}{f} + \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\sqrt{a+b\sin^2(e+fx)}\right)}{bf}$$
$$= \frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{f} - \frac{\sqrt{a+b\sin^2(e+fx)}}{f}$$

Mathematica [A] time = 0.0579054, size = 60, normalized size = 1.03

$$\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a-b\cos^2(e+fx)+b}}{\sqrt{a+b}}\right)-\sqrt{a-b\cos^2(e+fx)+b}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x],x]
```

```
[Out] (Sqrt[a + b]*ArcTanh[Sqrt[a + b - b*Cos[e + f*x]^2]/Sqrt[a + b]] - Sqrt[a +
 b - b*Cos[e + f*x]^2])/f
```

Maple [B] time = 3.81, size = 134, normalized size = 2.3

$$\frac{1}{2f}\sqrt{a+b}\ln\left(2\frac{\sqrt{a+b}\sqrt{a+b-b\left(\cos\left(fx+e\right)\right)^2}+b\sin\left(fx+e\right)+a}{-1+\sin\left(fx+e\right)}\right)+\frac{1}{2f}\sqrt{a+b}\ln\left(2\frac{\sqrt{a+b}\sqrt{a+b-b\left(\cos\left(fx+\right.\right.}}{1+\sin\left(f\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e),x)
```

```
[Out] 1/2/f*(a+b)^(1/2)*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1
/2)+b*sin(f*x+e)+a))+1/2/f*(a+b)^(1/2)*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+
b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))-1/f*(a+b-b*cos(f*x+e)^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="maxima")
```

[Out] Exception raised: ValueError

Fricas [A] time = 3.36449, size = 370, normalized size = 6.38

$$\left[\frac{\sqrt{a+b}\log\left(\frac{b\cos\left(fx+e\right)^{2}-2\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\sqrt{a+b}-2a-2b}{\cos\left(fx+e\right)^{2}}\right)-2\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}}{2f}, -\frac{\sqrt{-a-b}\arctan\left(\frac{\sqrt{-b\cos(fx+e}}{a}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a + b)*log((b*cos(f*x + e)^2 - 2*sqrt(-b*cos(f*x + e)^2 + a + b)
*sqrt(a + b) - 2*a - 2*b)/cos(f*x + e)^2) - 2*sqrt(-b*cos(f*x + e)^2 + a +
b))/f, -(sqrt(-a - b)*arctan(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(
a + b)) + sqrt(-b*cos(f*x + e)^2 + a + b))/f]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a+b\sin^{2}\left(e+fx\right)}\tan\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**(1/2)*tan(f*x+e),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*tan(e + f*x), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.491 $\int \cot(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=54

$$\frac{\sqrt{a+b\sin^2(e+fx)}}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{f}$$

```
[Out] -((Sqrt[a]*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/f) + Sqrt[a + b*Sin
[e + f*x]^2]/f
```

Rubi [A] time = 0.0649755, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3194, 50, 63, 208}

$$\frac{\sqrt{a+b\sin^2(e+fx)}}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a]*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/f) + Sqrt[a + b*Sin
[e + f*x]^2]/f
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \cot(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx,x,\sin^2(e+fx)\right)}{2f}\\
&= \frac{\sqrt{a+b\sin^2(e+fx)}}{f}+\frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,\sin^2(e+fx)\right)}{2f}\\
&= \frac{\sqrt{a+b\sin^2(e+fx)}}{f}+\frac{a\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b\sin^2(e+fx)}\right)}{bf}\\
&= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{\sqrt{a+b\sin^2(e+fx)}}{f}
\end{aligned}
$$

Mathematica [A] time = 0.0465664, size = 53, normalized size = 0.98

$$
-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)-\sqrt{a+b\sin^2(e+fx)}}{f}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a]*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]] - Sqrt[a + b*Sin[e +
 f*x]^2])/f)
```

Maple [A] time = 1.001, size = 61, normalized size = 1.1

$$
-\frac{1}{f}\sqrt{a}\ln\left(\frac{1}{\sin(fx+e)}\left(2a+2\sqrt{a}\sqrt{a+b\left(\sin(fx+e)\right)^2}\right)\right)+\frac{1}{f}\sqrt{a+b\left(\sin(fx+e)\right)^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/f*a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e))+(a+b*s
in(f*x+e)^2)^(1/2)/f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 6.16947, size = 347, normalized size = 6.43

$$\left[\frac{\sqrt{a}\log\left(\frac{2\left(b\cos\left(fx+e\right)^2+2\sqrt{-b\cos\left(fx+e\right)^2+a+b}\sqrt{a}-2a-b\right)}{\cos\left(fx+e\right)^2-1}\right)+2\sqrt{-b\cos\left(fx+e\right)^2+a+b}}{2f}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\sqrt{-a}}{a}\right)+}{f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*log(2*(b*cos(f*x + e)^2 + 2*sqrt(-b*cos(f*x + e)^2 + a + b)*s
qrt(a) - 2*a - b)/(cos(f*x + e)^2 - 1)) + 2*sqrt(-b*cos(f*x + e)^2 + a + b)
)/f, (sqrt(-a)*arctan(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a)/a) + sqrt(-b
*cos(f*x + e)^2 + a + b))/f]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a+b\sin^2\left(e+fx\right)}\cot\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*cot(e + f*x), x)
```

Giac [A] time = 1.09873, size = 66, normalized size = 1.22

$$\frac{\frac{a\arctan\left(\frac{\sqrt{b\sin\left(fx+e\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\sqrt{b\sin\left(fx+e\right)^2+a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] (a*arctan(sqrt(b*sin(f*x + e)^2 + a)/sqrt(-a))/sqrt(-a) + sqrt(b*sin(f*x +
e)^2 + a))/f
```

3.492 $\int \cot^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=110

$$-\frac{(2a-b)\sqrt{a+b\sin^2(e+fx)}}{2af}+\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{a}f}-\frac{\csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{2af}$$

```
[Out] ((2*a - b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/(2*Sqrt[a]*f) - ((2
*a - b)*Sqrt[a + b*Sin[e + f*x]^2])/(2*a*f) - (Csc[e + f*x]^2*(a + b*Sin[e
+ f*x]^2)^(3/2))/(2*a*f)
```

Rubi [A] time = 0.102533, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3194, 78, 50, 63, 208}

$$-\frac{(2a-b)\sqrt{a+b\sin^2(e+fx)}}{2af}+\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{a}f}-\frac{\csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] ((2*a - b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/(2*Sqrt[a]*f) - ((2
*a - b)*Sqrt[a + b*Sin[e + f*x]^2])/(2*a*f) - (Csc[e + f*x]^2*(a + b*Sin[e
+ f*x]^2)^(3/2))/(2*a*f)
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \cot^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x)\sqrt{a+bx}}{x^2}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= -\frac{\csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{2af} - \frac{(2a-b)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \sin^2(e+\ldots\right.}{4af}$$

$$= -\frac{(2a-b)\sqrt{a+b\sin^2(e+fx)}}{2af} - \frac{\csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{2af} - \frac{(2a-b)\ldots}{\ldots}$$

$$= -\frac{(2a-b)\sqrt{a+b\sin^2(e+fx)}}{2af} - \frac{\csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{2af} - \frac{(2a-b)\ldots}{\ldots}$$

$$= \frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{a}f} - \frac{(2a-b)\sqrt{a+b\sin^2(e+fx)}}{2af} - \frac{\csc^2(e+fx)\ldots}{\ldots}$$

Mathematica [A] time = 0.198836, size = 77, normalized size = 0.7

$$\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right) - \sqrt{a}\left(\csc^2(e+fx)+2\right)\sqrt{a+b\sin^2(e+fx)}}{2\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] ((2*a - b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]] - Sqrt[a]*(2 + Csc[e
 + f*x]^2)*Sqrt[a + b*Sin[e + f*x]^2])/(2*Sqrt[a]*f)
```

Maple [A] time = 1.22, size = 130, normalized size = 1.2

$$-\frac{1}{f}\sqrt{a+b\left(\sin\left(fx+e\right)\right)^2} + \frac{1}{f}\sqrt{a}\ln\left(\frac{1}{\sin\left(fx+e\right)}\left(2a+2\sqrt{a}\sqrt{a+b\left(\sin\left(fx+e\right)\right)^2}\right)\right) - \frac{b}{2f}\ln\left(\frac{1}{\sin\left(fx+e\right)}\left(2a+2\sqrt{\ldots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] -(a+b*sin(f*x+e)^2)^(1/2)/f+1/f*a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2
)^(1/2))/sin(f*x+e))-1/2/f/a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/
2))/sin(f*x+e))*b-1/2/f/sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 8.42464, size = 595, normalized size = 5.41

$$\left[-\frac{\left((2\,a-b)\cos\left(fx+e\right)^2-2\,a+b\right)\sqrt{a}\log\left(\frac{2\left(b\cos\left(fx+e\right)^2+2\sqrt{-b\cos\left(fx+e\right)^2+a+b}\sqrt{a}-2\,a-b\right)}{\cos\left(fx+e\right)^2-1}\right)+2\left(2\,a\cos\left(fx+e\right)^2-3\,a\right)\sqrt{-}}{4\left(af\cos\left(fx+e\right)^2-af\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(((2*a - b)*cos(f*x + e)^2 - 2*a + b)*sqrt(a)*log(2*(b*cos(f*x + e)^2
 + 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) - 2*a - b)/(cos(f*x + e)^2 - 1
)) + 2*(2*a*cos(f*x + e)^2 - 3*a)*sqrt(-b*cos(f*x + e)^2 + a + b))/(a*f*cos
(f*x + e)^2 - a*f), -1/2*(((2*a - b)*cos(f*x + e)^2 - 2*a + b)*sqrt(-a)*arc
tan(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a)/a) + (2*a*cos(f*x + e)^2 - 3*a
)*sqrt(-b*cos(f*x + e)^2 + a + b))/(a*f*cos(f*x + e)^2 - a*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\sin^2\left(e+fx\right)}\cot^3\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*cot(e + f*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin\left(fx+e\right)^2+a}\cot\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*cot(f*x + e)^3, x)
```

3.493 $\int \cot^5(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=165

$$\frac{(8a^2-8ab-b^2)\sqrt{a+b\sin^2(e+fx)}}{8a^2f}-\frac{(8a^2-8ab-b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{8a^{3/2}f}+\frac{(8a+b)\csc^2(e+fx)\left(a+b\sin^2(e\right.}{8a^2f}$$

```
[Out] -((8*a^2 - 8*a*b - b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/(8*a^(
3/2)*f) + ((8*a^2 - 8*a*b - b^2)*Sqrt[a + b*Sin[e + f*x]^2])/(8*a^2*f) + ((
8*a + b)*Csc[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2))/(8*a^2*f) - (Csc[e +
f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2))/(4*a*f)
```

Rubi [A] time = 0.154806, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3194, 89, 78, 50, 63, 208}

$$\frac{(8a^2-8ab-b^2)\sqrt{a+b\sin^2(e+fx)}}{8a^2f}-\frac{(8a^2-8ab-b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{8a^{3/2}f}+\frac{(8a+b)\csc^2(e+fx)\left(a+b\sin^2(e\right.}{8a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((8*a^2 - 8*a*b - b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/(8*a^(
3/2)*f) + ((8*a^2 - 8*a*b - b^2)*Sqrt[a + b*Sin[e + f*x]^2])/(8*a^2*f) + ((
8*a + b)*Csc[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2))/(8*a^2*f) - (Csc[e +
f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2))/(4*a*f)
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
```

```
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \cot^5(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x)^2\sqrt{a+bx}}{x^3}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= -\frac{\csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4af} + \frac{\text{Subst}\left(\int \frac{\left(\frac{1}{2}(-8a-b)+2ax\right)\sqrt{a+bx}}{x^2}\,dx, x, \sin^2\right.}{4af}$$

$$= \frac{(8a+b)\csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{8a^2f} - \frac{\csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)}{4af}$$

$$= \frac{\left(8a^2-8ab-b^2\right)\sqrt{a+b\sin^2(e+fx)}}{8a^2f} + \frac{(8a+b)\csc^2(e+fx)\left(a+b\sin^2(e+fx\right.}{8a^2f}$$

$$= \frac{\left(8a^2-8ab-b^2\right)\sqrt{a+b\sin^2(e+fx)}}{8a^2f} + \frac{(8a+b)\csc^2(e+fx)\left(a+b\sin^2(e+fx\right.}{8a^2f}$$

$$= -\frac{\left(8a^2-8ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{8a^{3/2}f} + \frac{\left(8a^2-8ab-b^2\right)\sqrt{a+b\sin^2(e+\right.}{8a^2f}$$

Mathematica [A] time = 0.589232, size = 103, normalized size = 0.62

$$\frac{\left(-8a^2+8ab+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)+\sqrt{a}\sqrt{a+b\sin^2(e+fx)}\left((8a-b)\csc^2(e+fx)-2a\csc^4(e+fx)+8a\right)}{8a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] ((-8*a^2 + 8*a*b + b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]] + Sqrt[
a]*(8*a + (8*a - b)*Csc[e + f*x]^2 - 2*a*Csc[e + f*x]^4)*Sqrt[a + b*Sin[e +
 f*x]^2])/(8*a^(3/2)*f)
```

Maple [A] time = 1.502, size = 230, normalized size = 1.4

$$\frac{1}{f}\sqrt{a+b\left(\sin\left(fx+e\right)\right)^{2}}-\frac{1}{f}\sqrt{a}\ln\left(\frac{1}{\sin\left(fx+e\right)}\left(2a+2\sqrt{a}\sqrt{a+b\left(\sin\left(fx+e\right)\right)^{2}}\right)\right)+\frac{b}{f}\ln\left(\frac{1}{\sin\left(fx+e\right)}\left(2a+2\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5*(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] (a+b*sin(f*x+e)^2)^(1/2)/f-1/f*a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)
^(1/2))/sin(f*x+e))+1/f/a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))
/sin(f*x+e))*b-1/8/f*b/a/sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)+1/8/f*b^2/a^
(3/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e))+1/f/sin(f*x+e
)^2*(a+b*sin(f*x+e)^2)^(1/2)-1/4/f/sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 11.5458, size = 965, normalized size = 5.85

$$\left[-\frac{\left(\left(8a^{2}-8ab-b^{2}\right)\cos\left(fx+e\right)^{4}-2\left(8a^{2}-8ab-b^{2}\right)\cos\left(fx+e\right)^{2}+8a^{2}-8ab-b^{2}\right)\sqrt{a}\log\left(\frac{2\left(b\cos\left(fx+e\right)^{2}-2\sqrt{-b\cos\right.}\right.}{\cos\left(fx\right.}\right.}{16\left(a^{2}f\cos\left(fx+e\right)^{4}-2a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(((8*a^2 - 8*a*b - b^2)*cos(f*x + e)^4 - 2*(8*a^2 - 8*a*b - b^2)*cos
(f*x + e)^2 + 8*a^2 - 8*a*b - b^2)*sqrt(a)*log(2*(b*cos(f*x + e)^2 - 2*sqrt
(-b*cos(f*x + e)^2 + a + b)*sqrt(a) - 2*a - b)/(cos(f*x + e)^2 - 1)) - 2*(8
*a^2*cos(f*x + e)^4 - (24*a^2 - a*b)*cos(f*x + e)^2 + 14*a^2 - a*b)*sqrt(-b
*cos(f*x + e)^2 + a + b))/(a^2*f*cos(f*x + e)^4 - 2*a^2*f*cos(f*x + e)^2 +
```

```
a^2*f), 1/8*(((8*a^2 - 8*a*b - b^2)*cos(f*x + e)^4 - 2*(8*a^2 - 8*a*b - b^2
)*cos(f*x + e)^2 + 8*a^2 - 8*a*b - b^2)*sqrt(-a)*arctan(sqrt(-b*cos(f*x + e
)^2 + a + b)*sqrt(-a)/a) + (8*a^2*cos(f*x + e)^4 - (24*a^2 - a*b)*cos(f*x +
 e)^2 + 14*a^2 - a*b)*sqrt(-b*cos(f*x + e)^2 + a + b))/(a^2*f*cos(f*x + e)^
4 - 2*a^2*f*cos(f*x + e)^2 + a^2*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin\left(fx+e\right)^2+a}\cot\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*cot(f*x + e)^5, x)
```

3.494 $\int \sqrt{a+b\sin^2(e+fx)}\tan^4(e+fx)\,dx$

Optimal. Leaf size=234

$$\frac{\tan^3(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{(3a+4b)\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f(a+b)}-\frac{4a\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(}{a}}}{3f\sqrt{a+b\,\mathrm{si}}}$$

```
[Out] ((7*a + 8*b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*S
ec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*(a + b)*f*Sqrt[1 + (b*Sin[e + f*
x]^2)/a]) - (4*a*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a
)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*f*Sqrt[a + b*Sin[e + f*x
]^2]) - ((3*a + 4*b)*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x])/(3*(a + b)*f)
 + (Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x]^3)/(3*f)
```

Rubi [A] time = 0.276824, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3196, 467, 578, 524, 426, 424, 421, 419}

$$\frac{\tan^3(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{(3a+4b)\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f(a+b)}-\frac{4a\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(}{a}}}{3f\sqrt{a+b\,\mathrm{si}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x]^4,x]
```


```
[Out] ((7*a + 8*b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*S
ec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*(a + b)*f*Sqrt[1 + (b*Sin[e + f*
x]^2)/a]) - (4*a*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a
)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*f*Sqrt[a + b*Sin[e + f*x
]^2]) - ((3*a + 4*b)*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x])/(3*(a + b)*f)
 + (Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x]^3)/(3*f)
```


Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \sqrt{a+b\sin^2(e+fx)}\tan^4(e+fx)\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^4\sqrt{a+bx^2}}{\left(1-x^2\right)^{5/2}}dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{a+b\sin^2(e+fx)}\tan^3(e+fx)}{3f} - \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int\right.}{3f}$$

$$= -\frac{(3a+4b)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{3(a+b)f} + \frac{\sqrt{a+b\sin^2(e+fx)}\tan^3(e+}{3f}$$

$$= -\frac{(3a+4b)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{3(a+b)f} + \frac{\sqrt{a+b\sin^2(e+fx)}\tan^3(e+}{3f}$$

$$= -\frac{(3a+4b)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{3(a+b)f} + \frac{\sqrt{a+b\sin^2(e+fx)}\tan^3(e+}{3f}$$

$$= \frac{(7a+8b)\sqrt{\cos^2(e+fx)}E\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b\sin^2(}{3(a+b)f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

Mathematica [A] time = 2.04069, size = 198, normalized size = 0.85

$$-\frac{-\frac{\tan(e+fx)\sec^2(e+fx)\left(4\left(4a^2+6ab+b^2\right)\cos(2(e+fx))+8a^2-b(4a+5b)\cos(4(e+fx))+12ab+b^2\right)}{2\sqrt{2}} - 8a(a+b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}F\left(e+fx\middle|-\frac{b}{a}\right)}{6f(a+b)\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x]^4,x]
```

```
[Out] (2*a*(7*a + 8*b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x,
-(b/a)] - 8*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e +
f*x, -(b/a)] - ((8*a^2 + 12*a*b + b^2 + 4*(4*a^2 + 6*a*b + b^2)*Cos[2*(e +
f*x)] - b*(4*a + 5*b)*Cos[4*(e + f*x)])*Sec[e + f*x]^2*Tan[e + f*x])/(2*Sqr
t[2]))/(6*(a + b)*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A] time = 2.718, size = 380, normalized size = 1.6

$$-\frac{1}{(3a+3b)\left(-1+\sin\left(fx+e\right)\right)\left(1+\sin\left(fx+e\right)\right)\cos\left(fx+e\right)f}\left(\sqrt{-b\left(\cos\left(fx+e\right)\right)^4+(a+b)\left(\cos\left(fx+e\right)\right)^2}b(4a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x)
```

```
[Out] -1/3*((-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*(4*a+5*b)*sin(f*x+e)*cos
(f*x+e)^4-2*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(2*a^2+5*a*b+3*b^2)*
```

```
cos(f*x+e)^2*sin(f*x+e)+(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a^2+2*a
*b+b^2)*sin(f*x+e)-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(cos(f*x+e)^2
)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*a*(4*EllipticF(sin(f*x+e),(-1/a*b
)^(1/2))*a+4*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-7*EllipticE(sin(f*x+e),
(-1/a*b)^(1/2))*a-8*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*b)*cos(f*x+e)^2)/(
-(a+b*sin(f*x+e)^2)*(-1+sin(f*x+e))*(1+sin(f*x+e)))^(1/2)/(a+b)/(-1+sin(f*x
+e))/(1+sin(f*x+e))/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin\left(fx+e\right)^2+a}\tan\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*tan(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\sqrt{-b\cos\left(fx+e\right)^2+a+b}\tan\left(fx+e\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)*tan(f*x + e)^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\sin^2\left(e+fx\right)}\tan^4\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**(1/2)*tan(f*x+e)**4,x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*tan(e + f*x)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin\left(fx+e\right)^2+a}\tan\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*tan(f*x + e)^4, x)
```

3.495 $\int \sqrt{a+b\sin^2(e+fx)}\tan^2(e+fx)\,dx$

Optimal. Leaf size=171

$$\frac{\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f}+\frac{a\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}-2\sqrt{\cos^2(\cdots}$$

```
[Out] (-2*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*
x]*Sqrt[a + b*Sin[e + f*x]^2])/(f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (a*Sqrt
[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[
1 + (b*Sin[e + f*x]^2)/a])/(f*Sqrt[a + b*Sin[e + f*x]^2]) + (Sqrt[a + b*Sin
[e + f*x]^2]*Tan[e + f*x])/f
```

Rubi [A] time = 0.158663, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3196, 467, 524, 426, 424, 421, 419}

$$\frac{\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f}+\frac{a\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}-2\sqrt{\cos^2(\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x]^2,x]
```


```
[Out] (-2*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*
x]*Sqrt[a + b*Sin[e + f*x]^2])/(f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (a*Sqrt
[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[
1 + (b*Sin[e + f*x]^2)/a])/(f*Sqrt[a + b*Sin[e + f*x]^2]) + (Sqrt[a + b*Sin
[e + f*x]^2]*Tan[e + f*x])/f
```


Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
```

```
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \sqrt{a+b\sin^2(e+fx)}\tan^2(e+fx)\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^2\sqrt{a+bx^2}}{\left(1-x^2\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{f} - \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{a\cdots}{\sqrt{1-x\cdots}}\cdots\right.}{f}$$

$$= \frac{\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{f} - \frac{\left(2\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{a\cdots}}{\sqrt{1\cdots}}\cdots\right.}{f}$$

$$= \frac{\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{f} - \frac{\left(2\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e\cdots}\right.}{f\sqrt{1+\frac{b\cdots}{\cdots}}}$$

$$= -\frac{2\sqrt{\cos^2(e+fx)}E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} + \cdots$$

Mathematica [A] time = 0.483579, size = 140, normalized size = 0.82

$$\frac{\tan(e+fx)(2a-b\cos(2(e+fx))+b)+\sqrt{2}a\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}F\left(e+fx\middle|-\frac{b}{a}\right)-2\sqrt{2}a\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}E\left(e+fx\middle|-\right.}{\sqrt{2}f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x]^2,x]
```

```
[Out] (-2*Sqrt[2]*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b
/a)] + Sqrt[2]*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x,
-(b/a)] + (2*a + b - b*Cos[2*(e + f*x)])*Tan[e + f*x])/(Sqrt[2]*f*Sqrt[2*a
+ b - b*Cos[2*(e + f*x)]])
```

Maple [A] time = 1.994, size = 294, normalized size = 1.7

$$\frac{1}{f\cos\left(fx+e\right)}\left(-\sqrt{-b\left(\cos\left(fx+e\right)\right)^{4}+(a+b)\left(\cos\left(fx+e\right)\right)^{2}}b\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^{2}+\sqrt{-b\left(\cos\left(fx+e\right)\right)^{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x)
```

```
[Out] (-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*sin(f*x+e)*cos(f*x+e)^2+(-b*
cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a+b)*sin(f*x+e)+a*(cos(f*x+e)^2)^(1
/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^
(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))-2*a*(cos(f*x+e)^2)^(1/2)*(-b/a*c
os(f*x+e)^2+(a+b)/a)^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*Ellip
ticE(sin(f*x+e),(-1/a*b)^(1/2)))/(-(a+b*sin(f*x+e)^2)*(-1+sin(f*x+e))*(1+si
n(f*x+e)))^(1/2)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sin\left(fx+e\right)^{2}+a}\tan\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*tan(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\tan\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)*tan(f*x + e)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\sin^2\left(e + fx\right)}\tan^2\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**(1/2)*tan(f*x+e)**2,x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*tan(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin\left(fx + e\right)^2 + a}\tan\left(fx + e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*tan(f*x + e)^2, x)
```

3.496 $\int \sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=51

$$\frac{\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

```
[Out] (EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(f*Sqrt[1 + (b*Sin[
e + f*x]^2)/a])
```

Rubi [A] time = 0.0330833, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3178, 3177}

$$\frac{\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(f*Sqrt[1 + (b*Sin[
e + f*x]^2)/a])
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+b\sin^2(e+fx)}\,dx &= \frac{\sqrt{a+b\sin^2(e+fx)}\int \sqrt{1+\frac{b\sin^2(e+fx)}{a}}\,dx}{\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}\\ &= \frac{E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}\end{aligned}$$

Mathematica [A] time = 0.0853344, size = 61, normalized size = 1.2

$$\frac{a\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)])/(f*Sq
rt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A] time = 0.66, size = 71, normalized size = 1.4

$$\frac{a}{f\cos\left(fx+e\right)}\sqrt{\left(\cos\left(fx+e\right)\right)^{2}}\sqrt{\frac{a+b\left(\sin\left(fx+e\right)\right)^{2}}{a}}\mathrm{EllipticE}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)\frac{1}{\sqrt{a+b\left(\sin\left(fx+e\right)\right)^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-
1/a*b)^(1/2))/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sin\left(fx+e\right)^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{-b\cos\left(fx+e\right)^{2}+a+b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a+b\sin^{2}\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sin \left(fx + e\right)^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a), x)
```

3.497 $\int \cot^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=174

$$-\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f}+\frac{(a+b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}-\frac{2\sqrt{\mathrm{co}}}{}$$

```
[Out] -((Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/f) - (2*Sqrt[Cos[e + f*x]^2]*El
lipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^
2])/(f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + ((a + b)*Sqrt[Cos[e + f*x]^2]*Elli
pticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2
)/a])/(f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.171677, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3196, 473, 524, 426, 424, 421, 419}

$$-\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f}+\frac{(a+b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}-\frac{2\sqrt{\mathrm{co}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/f) - (2*Sqrt[Cos[e + f*x]^2]*El
lipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^
2])/(f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + ((a + b)*Sqrt[Cos[e + f*x]^2]*Elli
pticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2
)/a])/(f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^p*(c + d*x^n)^q)/(e*(m
 + 1)), x] - Dist[n/(e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^(p - 1)*(c
 + d*x^n)^(q - 1)*Simp[b*c*p + a*d*q + b*d*(p + q)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && LtQ
[m, -1] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
```

```
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \cot^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int\frac{\sqrt{1-x^2}\sqrt{a+bx^2}}{x^2}\,dx,x,\sin(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} + \frac{\left(2\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int\right.}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} - \frac{\left(2\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int\right.}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} - \frac{\left(2\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin\right.}{f\sqrt{1}}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} - \frac{2\sqrt{\cos^2(e+fx)}E\left(\sin^{-1}(\sin(e+fx))\right| -}{f\sqrt{1+\frac{b\sin}{}}}$$

Mathematica [A] time = 0.568024, size = 143, normalized size = 0.82

$$\frac{\cot(e+fx)(-(2a-b\cos(2(e+fx))+b))+\sqrt{2}(a+b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}F\left(e+fx\middle|-\frac{b}{a}\right)-2\sqrt{2}a\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}E\left(e+\right.}{\sqrt{2}f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-((2*a + b - b*Cos[2*(e + f*x)])*Cot[e + f*x]) - 2*Sqrt[2]*a*Sqrt[(2*a + b
 - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] + Sqrt[2]*(a + b)*Sqrt
[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)])/(Sqrt[2]*f*S
qrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A] time = 1.21, size = 156, normalized size = 0.9

$$\frac{1}{\sin\left(fx+e\right)\cos\left(fx+e\right)f}\left(\sin\left(fx+e\right)\sqrt{-\frac{b\left(\cos\left(fx+e\right)\right)^{2}}{a}+\frac{a+b}{a}}\sqrt{\left(\cos\left(fx+e\right)\right)^{2}}\left(\mathrm{EllipticF}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] (sin(f*x+e)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(cos(f*x+e)^2)^(1/2)*(Ellipti
cF(sin(f*x+e),(-1/a*b)^(1/2))*a+EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-2*El
lipticE(sin(f*x+e),(-1/a*b)^(1/2))*a)+b*cos(f*x+e)^4+(-a-b)*cos(f*x+e)^2)/s
in(f*x+e)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sin\left(fx+e\right)^{2}+a}\cot\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*cot(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\cot\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)*cot(f*x + e)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\sin^2\left(e + fx\right)}\cot^2\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*cot(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sin\left(fx + e\right)^2 + a}\cot\left(fx + e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*cot(f*x + e)^2, x)
```

3.498 $\int \cot^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=232

$$-\frac{\cot^3(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{(3a-b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af}-\frac{4(a+b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin}{\ }}}{3f\sqrt{a+b\sin}}$$

```
[Out] ((3*a - b)*Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a*f) - (Cot[e + f*x]
^3*Sqrt[a + b*Sin[e + f*x]^2])/(3*f) + ((7*a - b)*Sqrt[Cos[e + f*x]^2]*Elli
pticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2]
)/(3*a*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) - (4*(a + b)*Sqrt[Cos[e + f*x]^2]*
EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*
x]^2)/a])/(3*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.267956, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3196, 473, 580, 524, 426, 424, 421, 419}

$$-\frac{\cot^3(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{(3a-b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af}-\frac{4(a+b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin}{\ }}}{3f\sqrt{a+b\sin}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*Sqrt[a + b*Sin[e + f*x]^2],x]
```


```
[Out] ((3*a - b)*Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a*f) - (Cot[e + f*x]
^3*Sqrt[a + b*Sin[e + f*x]^2])/(3*f) + ((7*a - b)*Sqrt[Cos[e + f*x]^2]*Elli
pticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2]
)/(3*a*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) - (4*(a + b)*Sqrt[Cos[e + f*x]^2]*
EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*
x]^2)/a])/(3*f*Sqrt[a + b*Sin[e + f*x]^2])
```


Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^p*(c + d*x^n)^q)/(e*(m
 + 1)), x] - Dist[n/(e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^(p - 1)*(c
 + d*x^n)^(q - 1)*Simp[b*c*p + a*d*q + b*d*(p + q)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && LtQ
[m, -1] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \cot^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\operatorname{Subst}\left(\int \frac{(1-x^2)^{3/2}\sqrt{a+bx^2}}{x^4}dx, x, \sin(e+fx)\right)}{f}$$

$$= -\frac{\cot^3(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} + \frac{\left(2\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\operatorname{Subst}\left(\int -\right.}{3f}$$

$$= \frac{(3a-b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af} - \frac{\cot^3(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} +$$

$$= \frac{(3a-b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af} - \frac{\cot^3(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} +$$

$$= \frac{(3a-b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af} - \frac{\cot^3(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} +$$

$$= \frac{(3a-b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af} - \frac{\cot^3(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} +$$

Mathematica [A] time = 3.29504, size = 197, normalized size = 0.85

$$\frac{-\frac{\cot(e+fx)\csc^2(e+fx)\left(4\left(4a^2+2ab-b^2\right)\cos(2(e+fx))-8a^2+b(b-4a)\cos(4(e+fx))-4ab+3b^2\right)}{2\sqrt{2}} - 8a(a+b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}F\left(e+fx\middle|-\frac{b}{a}\right) + 2a}{6af\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-((-8*a^2 - 4*a*b + 3*b^2 + 4*(4*a^2 + 2*a*b - b^2)*Cos[2*(e + f*x)] + b*(
-4*a + b)*Cos[4*(e + f*x)])*Cot[e + f*x]*Csc[e + f*x]^2)/(2*Sqrt[2]) + 2*a*
(7*a - b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)]
 - 8*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(
b/a)])/(6*a*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A] time = 1.322, size = 351, normalized size = 1.5

$$-\frac{1}{3a\left(\sin\left(fx+e\right)\right)^3\cos\left(fx+e\right)f}\left(4\operatorname{EllipticF}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)\sqrt{\left(\cos\left(fx+e\right)\right)^2}\sqrt{\frac{a+b\left(\sin\left(fx+e\right)\right)^2}{a}}a^2\left(\sin\left(f\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/3*(4*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*(cos(f*x+e)^2)^(1/2)*((a+b*sin
(f*x+e)^2)/a)^(1/2)*a^2*sin(f*x+e)^3+4*b*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x
```

```
+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*sin(f*x+e)^3-7*Ellip
ticE(sin(f*x+e),(-1/a*b)^(1/2))*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)
^(1/2)*a^2*sin(f*x+e)^3+EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*(cos(f*x+e)^2)
^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*a*b*sin(f*x+e)^3+4*a*b*sin(f*x+e)^6-b^2
*sin(f*x+e)^6+4*a^2*sin(f*x+e)^4-6*a*b*sin(f*x+e)^4+b^2*sin(f*x+e)^4-5*a^2*
sin(f*x+e)^2+2*a*b*sin(f*x+e)^2+a^2)/a/sin(f*x+e)^3/cos(f*x+e)/(a+b*sin(f*x
+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sin\left(fx+e\right)^2 + a} \cot\left(fx+e\right)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*cot(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-b \cos\left(fx+e\right)^2 + a + b} \cot\left(fx+e\right)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)*cot(f*x + e)^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \sin^2\left(e + fx\right)} \cot^4\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*cot(e + f*x)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sin\left(fx+e\right)^2 + a} \cot\left(fx+e\right)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*cot(f*x + e)^4, x)
```

3.499 $\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan^5(e+fx)\,dx$

Optimal. Leaf size=220

$$-\frac{\left(8a^2+40ab+35b^2\right)\left(a+b\sin^2(e+fx)\right)^{3/2}}{24f(a+b)^2}-\frac{\left(8a^2+40ab+35b^2\right)\sqrt{a+b\sin^2(e+fx)}}{8f(a+b)}+\frac{\left(8a^2+40ab+35b^2\right)\tanh^{-1}}{8f\sqrt{a+b}}$$

```
[Out] ((8*a^2 + 40*a*b + 35*b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])
/(8*Sqrt[a + b]*f) - ((8*a^2 + 40*a*b + 35*b^2)*Sqrt[a + b*Sin[e + f*x]^2])
/(8*(a + b)*f) - ((8*a^2 + 40*a*b + 35*b^2)*(a + b*Sin[e + f*x]^2)^(3/2))/(
24*(a + b)^2*f) - ((8*a + 9*b)*Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(5/2))
/(8*(a + b)^2*f) + (Sec[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(5/2))/(4*(a + b)
*f)
```

Rubi [A] time = 0.257801, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3194, 89, 78, 50, 63, 208}

$$-\frac{\left(8a^2+40ab+35b^2\right)\left(a+b\sin^2(e+fx)\right)^{3/2}}{24f(a+b)^2}-\frac{\left(8a^2+40ab+35b^2\right)\sqrt{a+b\sin^2(e+fx)}}{8f(a+b)}+\frac{\left(8a^2+40ab+35b^2\right)\tanh^{-1}}{8f\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x]^2)^(3/2)*Tan[e + f*x]^5,x]
```



```
[Out] ((8*a^2 + 40*a*b + 35*b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])
/(8*Sqrt[a + b]*f) - ((8*a^2 + 40*a*b + 35*b^2)*Sqrt[a + b*Sin[e + f*x]^2])
/(8*(a + b)*f) - ((8*a^2 + 40*a*b + 35*b^2)*(a + b*Sin[e + f*x]^2)^(3/2))/(
24*(a + b)^2*f) - ((8*a + 9*b)*Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(5/2))
/(8*(a + b)^2*f) + (Sec[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(5/2))/(4*(a + b)
*f)
```


Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan^5(e+fx)\,dx=\frac{\text{Subst}\left(\int\frac{x^2(a+bx)^{3/2}}{(1-x)^3}\,dx,x,\sin^2(e+fx)\right)}{2f}$$

$$=\frac{\sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{4(a+b)f}-\frac{\text{Subst}\left(\int\frac{(a+bx)^{3/2}\left(\frac{1}{2}(4a+5b)+2(a+b)x\right)}{(1-x)^2}\,d\right.}{4(a+b)f}$$

$$=-\frac{(8a+9b)\sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{8(a+b)^2f}+\frac{\sec^4(e+fx)\left(a+b\sin^2\right.}{4(a+b)f}$$

$$=-\frac{\left(8a^2+40ab+35b^2\right)\left(a+b\sin^2(e+fx)\right)^{3/2}}{24(a+b)^2f}-\frac{(8a+9b)\sec^2(e+fx)\left(a\right.}{8(a+b)}$$

$$=-\frac{\left(8a^2+40ab+35b^2\right)\sqrt{a+b\sin^2(e+fx)}}{8(a+b)f}-\frac{\left(8a^2+40ab+35b^2\right)\left(a+b\right.}{24(a+b)^2f}$$

$$=-\frac{\left(8a^2+40ab+35b^2\right)\sqrt{a+b\sin^2(e+fx)}}{8(a+b)f}-\frac{\left(8a^2+40ab+35b^2\right)\left(a+b\right.}{24(a+b)^2f}$$

$$=\frac{\left(8a^2+40ab+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{8\sqrt{a+b}f}-\frac{\left(8a^2+40ab+35b^2\right)\sqrt{a\right.}}{8(a+b)f}$$

Mathematica [A] time = 2.18382, size = 160, normalized size = 0.73

$$-\frac{\left(8a^2+40ab+35b^2\right)\left(\sqrt{a+b\sin^2(e+fx)}\left(4a+b\sin^2(e+fx)+3b\right)-3(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)\right)-6(a+b)\,\text{s}}{24f(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)^(3/2)*Tan[e + f*x]^5,x]
```

```
[Out] -(3*(8*a + 9*b)*Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(5/2) - 6*(a + b)*Sec
[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(5/2) + (8*a^2 + 40*a*b + 35*b^2)*(-3*(a
 + b)^(3/2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]] + Sqrt[a + b*Si
n[e + f*x]^2]*(4*a + 3*b + b*Sin[e + f*x]^2)))/(24*(a + b)^2*f)
```

Maple [B] time = 3.613, size = 711, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^5,x)
```

```
[Out] 1/48*(16*b*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(5/2)*cos(f*x+e)^6+(-64*(a+b-b*
cos(f*x+e)^2)^(1/2)*(a+b)^(5/2)*a-160*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(5/2
)*b+24*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f
*x+e)+a))*a^4+168*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1
/2)+b*sin(f*x+e)+a))*a^3*b+369*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos
(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^2*b^2+330*ln(2/(-1+sin(f*x+e))*((a+b)^(
1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a*b^3+105*ln(2/(-1+sin(f*x
+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*b^4+24*ln(2/(
1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^4+
168*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e
)+a))*a^3*b+369*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)
-b*sin(f*x+e)+a))*a^2*b^2+330*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f
*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a*b^3+105*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*
(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*b^4)*cos(f*x+e)^4-6*(a+b-b*cos(
f*x+e)^2)^(1/2)*(a+b)^(5/2)*(8*a+13*b)*cos(f*x+e)^2+12*(a+b-b*cos(f*x+e)^2)
^(1/2)*(a+b)^(5/2)*a+12*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(5/2)*b)/(a+b)^(5/
2)/cos(f*x+e)^4/f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 7.07125, size = 953, normalized size = 4.33

$$\left[\frac{3\left(8a^2+40ab+35b^2\right)\sqrt{a+b}\cos\left(fx+e\right)^4\log\left(\frac{b\cos\left(fx+e\right)^2-2\sqrt{-b\cos\left(fx+e\right)^2+a+b}\sqrt{a+b}-2a-2b}{\cos\left(fx+e\right)^2}\right)+2\left(8\left(ab+b^2\right)\cos\left(fx\cdots\right.\right.}{48\left(a\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] [1/48*(3*(8*a^2 + 40*a*b + 35*b^2)*sqrt(a + b)*cos(f*x + e)^4*log((b*cos(f*
x + e)^2 - 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b) - 2*a - 2*b)/cos(f
*x + e)^2) + 2*(8*(a*b + b^2)*cos(f*x + e)^6 - 16*(2*a^2 + 7*a*b + 5*b^2)*c
os(f*x + e)^4 - 3*(8*a^2 + 21*a*b + 13*b^2)*cos(f*x + e)^2 + 6*a^2 + 12*a*b
 + 6*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b))/((a + b)*f*cos(f*x + e)^4), -1/2
4*(3*(8*a^2 + 40*a*b + 35*b^2)*sqrt(-a - b)*arctan(sqrt(-b*cos(f*x + e)^2 +
 a + b)*sqrt(-a - b)/(a + b))*cos(f*x + e)^4 - (8*(a*b + b^2)*cos(f*x + e)^
6 - 16*(2*a^2 + 7*a*b + 5*b^2)*cos(f*x + e)^4 - 3*(8*a^2 + 21*a*b + 13*b^2)
*cos(f*x + e)^2 + 6*a^2 + 12*a*b + 6*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b))/
((a + b)*f*cos(f*x + e)^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**(3/2)*tan(f*x+e)**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^5,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^5, x)
```

3.500 $\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan^3(e+fx)\,dx$

Optimal. Leaf size=148

$$\frac{(2a+5b)\left(a+b\sin^2(e+fx)\right)^{3/2}}{6f(a+b)}+\frac{(2a+5b)\sqrt{a+b\sin^2(e+fx)}}{2f}-\frac{\sqrt{a+b}(2a+5b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{2f}+\frac{\sec^2(e+\ldots}{}$$

```
[Out] -(Sqrt[a + b]*(2*a + 5*b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/
(2*f) + ((2*a + 5*b)*Sqrt[a + b*Sin[e + f*x]^2])/(2*f) + ((2*a + 5*b)*(a +
b*Sin[e + f*x]^2)^(3/2))/(6*(a + b)*f) + (Sec[e + f*x]^2*(a + b*Sin[e + f*x
]^2)^(5/2))/(2*(a + b)*f)
```

Rubi [A] time = 0.136133, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3194, 78, 50, 63, 208}

$$\frac{(2a+5b)\left(a+b\sin^2(e+fx)\right)^{3/2}}{6f(a+b)}+\frac{(2a+5b)\sqrt{a+b\sin^2(e+fx)}}{2f}-\frac{\sqrt{a+b}(2a+5b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{2f}+\frac{\sec^2(e+\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x]^2)^(3/2)*Tan[e + f*x]^3,x]
```

```
[Out] -(Sqrt[a + b]*(2*a + 5*b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/
(2*f) + ((2*a + 5*b)*Sqrt[a + b*Sin[e + f*x]^2])/(2*f) + ((2*a + 5*b)*(a +
b*Sin[e + f*x]^2)^(3/2))/(6*(a + b)*f) + (Sec[e + f*x]^2*(a + b*Sin[e + f*x
]^2)^(5/2))/(2*(a + b)*f)
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan^3(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x(a+bx)^{3/2}}{(1-x)^2}\,dx,x,\sin^2(e+fx)\right)}{2f}$$

$$= \frac{\sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{2(a+b)f} - \frac{(2a+5b)\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{1-x}\,dx,x,\text{s}\right.}{4(a+b)f}$$

$$= \frac{(2a+5b)\left(a+b\sin^2(e+fx)\right)^{3/2}}{6(a+b)f} + \frac{\sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{2(a+b)f} -$$

$$= \frac{(2a+5b)\sqrt{a+b\sin^2(e+fx)}}{2f} + \frac{(2a+5b)\left(a+b\sin^2(e+fx)\right)^{3/2}}{6(a+b)f} + \frac{\sec^2(}{}$$

$$= \frac{(2a+5b)\sqrt{a+b\sin^2(e+fx)}}{2f} + \frac{(2a+5b)\left(a+b\sin^2(e+fx)\right)^{3/2}}{6(a+b)f} + \frac{\sec^2(}{}$$

$$= -\frac{\sqrt{a+b}(2a+5b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{2f} + \frac{(2a+5b)\sqrt{a+b\sin^2(e+f}}{2f}$$

Mathematica [A] time = 0.534811, size = 116, normalized size = 0.78

$$\frac{(2a+5b)\left(\sqrt{a+b\sin^2(e+fx)}\left(4a+b\sin^2(e+fx)+3b\right)-3(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)\right)+3\sec^2(e+fx)\left(a+\right.}{6f(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)^(3/2)*Tan[e + f*x]^3,x]
```

```
[Out] (3*Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(5/2) + (2*a + 5*b)*(-3*(a + b)^(3
/2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]] + Sqrt[a + b*Sin[e + f*
x]^2]*(4*a + 3*b + b*Sin[e + f*x]^2)))/(6*(a + b)*f)
```

Maple [B] time = 3.349, size = 567, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^3,x)
```

```
[Out] 1/12*(-4*b*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(3/2)*cos(f*x+e)^4-(-16*(a+b)^(
3/2)*(a+b-b*cos(f*x+e)^2)^(1/2)*a-28*(a+b)^(3/2)*(a+b-b*cos(f*x+e)^2)^(1/2)
*b+6*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x
+e)+a))*a^3+27*a^2*b*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)
^(1/2)+b*sin(f*x+e)+a))+36*a*b^2*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*c
os(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))+15*b^3*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2
)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))+6*ln(2/(1+sin(f*x+e))*((a+b)^
(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^3+27*a^2*b*ln(2/(1+sin(
f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))+36*a*b^2*l
n(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))
+15*b^3*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f
*x+e)+a)))*cos(f*x+e)^2+6*(a+b)^(3/2)*(a+b-b*cos(f*x+e)^2)^(1/2)*a+6*(a+b)^
(3/2)*(a+b-b*cos(f*x+e)^2)^(1/2)*b)/(a+b)^(3/2)/cos(f*x+e)^2/f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 4.52874, size = 672, normalized size = 4.54

$$\left[\frac{3\,(2\,a+5\,b)\sqrt{a+b}\cos\left(fx+e\right)^{2}\log\left(\frac{b\cos\left(fx+e\right)^{2}+2\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\sqrt{a+b}-2\,a-2\,b}{\cos\left(fx+e\right)^{2}}\right)-2\left(2\,b\cos\left(fx+e\right)^{4}-2\,(4\,a+7\,b)\cos\right.}{12\,f\cos\left(fx+e\right)^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(2*a + 5*b)*sqrt(a + b)*cos(f*x + e)^2*log((b*cos(f*x + e)^2 + 2*s
qrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b) - 2*a - 2*b)/cos(f*x + e)^2) - 2
*(2*b*cos(f*x + e)^4 - 2*(4*a + 7*b)*cos(f*x + e)^2 - 3*a - 3*b)*sqrt(-b*co
s(f*x + e)^2 + a + b))/(f*cos(f*x + e)^2), 1/6*(3*(2*a + 5*b)*sqrt(-a - b)*
arctan(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(a + b))*cos(f*x + e)^2
 - (2*b*cos(f*x + e)^4 - 2*(4*a + 7*b)*cos(f*x + e)^2 - 3*a - 3*b)*sqrt(-b*
cos(f*x + e)^2 + a + b))/(f*cos(f*x + e)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**(3/2)*tan(f*x+e)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^3, x)
```

3.501 $\int \left(a+b\sin^2(e+fx)\right)^{3/2} \tan(e+fx)\,dx$

Optimal. Leaf size=84

$$-\frac{(a+b)\sqrt{a+b\sin^2(e+fx)}}{f}-\frac{\left(a+b\sin^2(e+fx)\right)^{3/2}}{3f}+\frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{f}$$

```
[Out] ((a + b)^(3/2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/f - ((a + b
)*Sqrt[a + b*Sin[e + f*x]^2])/f - (a + b*Sin[e + f*x]^2)^(3/2)/(3*f)
```

Rubi [A] time = 0.0775713, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3194, 50, 63, 208}

$$-\frac{(a+b)\sqrt{a+b\sin^2(e+fx)}}{f}-\frac{\left(a+b\sin^2(e+fx)\right)^{3/2}}{3f}+\frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x]^2)^(3/2)*Tan[e + f*x],x]
```

```
[Out] ((a + b)^(3/2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/f - ((a + b
)*Sqrt[a + b*Sin[e + f*x]^2])/f - (a + b*Sin[e + f*x]^2)^(3/2)/(3*f)
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{1-x}\,dx,x,\sin^2(e+fx)\right)}{2f}$$

$$= -\frac{\left(a+b\sin^2(e+fx)\right)^{3/2}}{3f}+\frac{(a+b)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{1-x}\,dx,x,\sin^2(e+fx)\right)}{2f}$$

$$= -\frac{(a+b)\sqrt{a+b\sin^2(e+fx)}}{f}-\frac{\left(a+b\sin^2(e+fx)\right)^{3/2}}{3f}+\frac{(a+b)^2\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{(a+b)\sqrt{a+b\sin^2(e+fx)}}{f}-\frac{\left(a+b\sin^2(e+fx)\right)^{3/2}}{3f}+\frac{(a+b)^2\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{f}-\frac{(a+b)\sqrt{a+b\sin^2(e+fx)}}{f}-\frac{\left(a+b\,\text{s}\right.}{}$$

Mathematica [A] time = 0.158502, size = 79, normalized size = 0.94

$$\frac{\sqrt{a-b\cos^2(e+fx)+b}\left(b\cos^2(e+fx)-4(a+b)\right)+3(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b\cos^2(e+fx)+b}}{\sqrt{a+b}}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)^(3/2)*Tan[e + f*x],x]
```

```
[Out] (3*(a + b)^(3/2)*ArcTanh[Sqrt[a + b - b*Cos[e + f*x]^2]/Sqrt[a + b]] + Sqrt
[a + b - b*Cos[e + f*x]^2]*(-4*(a + b) + b*Cos[e + f*x]^2))/(3*f)
```

Maple [B] time = 2.803, size = 423, normalized size = 5.

$$\frac{b\left(\cos\left(fx+e\right)\right)^2}{3f}\sqrt{a+b-b\left(\cos\left(fx+e\right)\right)^2}-\frac{4a}{3f}\sqrt{a+b-b\left(\cos\left(fx+e\right)\right)^2}-\frac{4b}{3f}\sqrt{a+b-b\left(\cos\left(fx+e\right)\right)^2}+\frac{a^2}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e),x)
```

```
[Out] 1/3/f*b*(a+b-b*cos(f*x+e)^2)^(1/2)*cos(f*x+e)^2-4/3/f*a*(a+b-b*cos(f*x+e)^2
)^(1/2)-4/3/f*b*(a+b-b*cos(f*x+e)^2)^(1/2)+1/2/(a+b)^(1/2)/f*ln(2/(1+sin(f*
x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^2+1/(a+b)^
(1/2)/f*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f
*x+e)+a))*a*b+1/2/(a+b)^(1/2)/f*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos
(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*b^2+1/2/(a+b)^(1/2)/f*ln(2/(-1+sin(f*x+e)
)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^2+1/(a+b)^(1/2
)/f*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+
e)+a))*a*b+1/2/(a+b)^(1/2)/f*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f
*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 4.04154, size = 478, normalized size = 5.69

$$\left[\frac{3\,(a+b)^{\frac{3}{2}}\log\left(\frac{b\cos\left(fx+e\right)^2-2\sqrt{-b\cos\left(fx+e\right)^2+a+b}\sqrt{a+b}-2\,a-2\,b}{\cos\left(fx+e\right)^2}\right)+2\left(b\cos\left(fx+e\right)^2-4\,a-4\,b\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}}{6\,f},\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(a + b)^(3/2)*log((b*cos(f*x + e)^2 - 2*sqrt(-b*cos(f*x + e)^2 + a
+ b)*sqrt(a + b) - 2*a - 2*b)/cos(f*x + e)^2) + 2*(b*cos(f*x + e)^2 - 4*a -
 4*b)*sqrt(-b*cos(f*x + e)^2 + a + b))/f, -1/3*(3*(a + b)*sqrt(-a - b)*arct
an(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(a + b)) - (b*cos(f*x + e)^
2 - 4*a - 4*b)*sqrt(-b*cos(f*x + e)^2 + a + b))/f]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**(3/2)*tan(f*x+e),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*tan(f*x + e), x)
```

3.502 $\int \cot(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=78

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{a\sqrt{a+b\sin^2(e+fx)}}{f}+\frac{\left(a+b\sin^2(e+fx)\right)^{3/2}}{3f}$$

```
[Out] -((a^(3/2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/f) + (a*Sqrt[a + b*
Sin[e + f*x]^2])/f + (a + b*Sin[e + f*x]^2)^(3/2)/(3*f)
```

Rubi [A] time = 0.0791743, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3194, 50, 63, 208}

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{a\sqrt{a+b\sin^2(e+fx)}}{f}+\frac{\left(a+b\sin^2(e+fx)\right)^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a^(3/2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/f) + (a*Sqrt[a + b*
Sin[e + f*x]^2])/f + (a + b*Sin[e + f*x]^2)^(3/2)/(3*f)
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \cot(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}dx, x, \sin^2(e+fx)\right)}{2f}$$
$$= \frac{\left(a+b\sin^2(e+fx)\right)^{3/2}}{3f} + \frac{a\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}dx, x, \sin^2(e+fx)\right)}{2f}$$
$$= \frac{a\sqrt{a+b\sin^2(e+fx)}}{f} + \frac{\left(a+b\sin^2(e+fx)\right)^{3/2}}{3f} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}dx, x, \text{si}\right.}{2f}$$
$$= \frac{a\sqrt{a+b\sin^2(e+fx)}}{f} + \frac{\left(a+b\sin^2(e+fx)\right)^{3/2}}{3f} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}dx, x, \sqrt{\right.}}{bf}$$
$$= -\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{a\sqrt{a+b\sin^2(e+fx)}}{f} + \frac{\left(a+b\sin^2(e+fx)\right)^{}}{3f}$$

Mathematica [A] time = 0.127664, size = 69, normalized size = 0.88

$$\frac{\sqrt{a+b\sin^2(e+fx)}\left(4a+b\sin^2(e+fx)\right)-3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-3*a^(3/2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]] + Sqrt[a + b*Sin[e
+ f*x]^2]*(4*a + b*Sin[e + f*x]^2))/(3*f)
```

Maple [A] time = 1.359, size = 91, normalized size = 1.2

$$\frac{b\left(\sin\left(fx+e\right)\right)^2}{3\,f}\sqrt{a+b\left(\sin\left(fx+e\right)\right)^2}+\frac{4\,a}{3\,f}\sqrt{a+b\left(\sin\left(fx+e\right)\right)^2}-\frac{1}{f}a^{\frac{3}{2}}\ln\left(\frac{1}{\sin\left(fx+e\right)}\left(2\,a+2\,\sqrt{a}\sqrt{a+b\left(\sin\left(f\right.\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/3/f*b*sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)+4/3*a*(a+b*sin(f*x+e)^2)^(1/2
)/f-1/f*a^(3/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 6.37492, size = 439, normalized size = 5.63

$$\left[\frac{3\,a^{\frac{3}{2}}\log\left(\frac{2\left(b\cos\left(fx+e\right)^{2}+2\,\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\sqrt{a}-2\,a-b\right)}{\cos\left(fx+e\right)^{2}-1}\right)-2\left(b\cos\left(fx+e\right)^{2}-4\,a-b\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}}{6\,f},\ \frac{3\,\sqrt{-a}a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*a^(3/2)*log(2*(b*cos(f*x + e)^2 + 2*sqrt(-b*cos(f*x + e)^2 + a + b)
*sqrt(a) - 2*a - b)/(cos(f*x + e)^2 - 1)) - 2*(b*cos(f*x + e)^2 - 4*a - b)*
sqrt(-b*cos(f*x + e)^2 + a + b))/f, 1/3*(3*sqrt(-a)*a*arctan(sqrt(-b*cos(f*
x + e)^2 + a + b)*sqrt(-a)/a) - (b*cos(f*x + e)^2 - 4*a - b)*sqrt(-b*cos(f*
x + e)^2 + a + b))/f]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11539, size = 96, normalized size = 1.23

$$\frac{\frac{3\,a^{2}\arctan\left(\frac{\sqrt{b\sin\left(fx+e\right)^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}+3\,\sqrt{b\sin\left(fx+e\right)^{2}+a}a}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*(3*a^2*arctan(sqrt(b*sin(f*x + e)^2 + a)/sqrt(-a))/sqrt(-a) + (b*sin(f*
x + e)^2 + a)^(3/2) + 3*sqrt(b*sin(f*x + e)^2 + a)*a)/f
```

3.503 $\int \cot^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=140

$$-\frac{(2a-3b)\left(a+b\sin^2(e+fx)\right)^{3/2}}{6af}-\frac{(2a-3b)\sqrt{a+b\sin^2(e+fx)}}{2f}+\frac{\sqrt{a}(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{2f}-\frac{\csc^2(e+fx)}{}$$

```
[Out] (Sqrt[a]*(2*a - 3*b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/(2*f) - (
(2*a - 3*b)*Sqrt[a + b*Sin[e + f*x]^2])/(2*f) - ((2*a - 3*b)*(a + b*Sin[e +
 f*x]^2)^(3/2))/(6*a*f) - (Csc[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(5/2))/(2*
a*f)
```

Rubi [A] time = 0.127071, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3194, 78, 50, 63, 208}

$$-\frac{(2a-3b)\left(a+b\sin^2(e+fx)\right)^{3/2}}{6af}-\frac{(2a-3b)\sqrt{a+b\sin^2(e+fx)}}{2f}+\frac{\sqrt{a}(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{2f}-\frac{\csc^2(e+fx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[a]*(2*a - 3*b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/(2*f) - (
(2*a - 3*b)*Sqrt[a + b*Sin[e + f*x]^2])/(2*f) - ((2*a - 3*b)*(a + b*Sin[e +
 f*x]^2)^(3/2))/(6*a*f) - (Csc[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(5/2))/(2*
a*f)
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \cot^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \frac{(1-x)(a+bx)^{3/2}}{x^2}dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= -\frac{\csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{2af} - \frac{(2a-3b)\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}dx, x, \right.}{4af}$$

$$= -\frac{(2a-3b)\left(a+b\sin^2(e+fx)\right)^{3/2}}{6af} - \frac{\csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{2af}$$

$$= -\frac{(2a-3b)\sqrt{a+b\sin^2(e+fx)}}{2f} - \frac{(2a-3b)\left(a+b\sin^2(e+fx)\right)^{3/2}}{6af} - \csc^2$$

$$= -\frac{(2a-3b)\sqrt{a+b\sin^2(e+fx)}}{2f} - \frac{(2a-3b)\left(a+b\sin^2(e+fx)\right)^{3/2}}{6af} - \csc^2$$

$$= \frac{\sqrt{a}(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{2f} - \frac{(2a-3b)\sqrt{a+b\sin^2(e+fx)}}{2f} - (2$$

Mathematica [A] time = 0.481449, size = 90, normalized size = 0.64

$$\frac{3\sqrt{a}(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)+\sqrt{a+b\sin^2(e+fx)}\left(-3a\csc^2(e+fx)-8a+b\cos(2(e+fx))+5b\right)}{6f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*Sqrt[a]*(2*a - 3*b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]] + (-8*a
+ 5*b + b*Cos[2*(e + f*x)] - 3*a*Csc[e + f*x]^2)*Sqrt[a + b*Sin[e + f*x]^2]
)/(6*f)
```

Maple [A] time = 1.432, size = 179, normalized size = 1.3

$$-\frac{b\left(\sin\left(fx+e\right)\right)^2}{3f}\sqrt{a+b\left(\sin\left(fx+e\right)\right)^2} - \frac{4a}{3f}\sqrt{a+b\left(\sin\left(fx+e\right)\right)^2} + \frac{b}{f}\sqrt{a+b\left(\sin\left(fx+e\right)\right)^2} + \frac{1}{f}a^{\frac{3}{2}}\ln\left(\frac{1}{\sin\left(fx\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/3/f*b*sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)-4/3*a*(a+b*sin(f*x+e)^2)^(1/
2)/f+1/f*b*(a+b*sin(f*x+e)^2)^(1/2)+1/f*a^(3/2)*ln((2*a+2*a^(1/2)*(a+b*sin(
f*x+e)^2)^(1/2))/sin(f*x+e))-3/2/f*a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e
)^2)^(1/2))/sin(f*x+e))*b-1/2/f*a/sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 9.40473, size = 699, normalized size = 4.99

$$\left[-\frac{3\left((2a-3b)\cos\left(fx+e\right)^2-2a+3b\right)\sqrt{a}\log\left(\frac{2\left(b\cos\left(fx+e\right)^2+2\sqrt{-b\cos\left(fx+e\right)^2+a+b}\sqrt{a}-2a-b\right)}{\cos\left(fx+e\right)^2-1}\right)-2\left(2b\cos\left(fx+e\right)^4-2(4a\right.}{12\left(f\cos\left(fx+e\right)^2-f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*((2*a - 3*b)*cos(f*x + e)^2 - 2*a + 3*b)*sqrt(a)*log(2*(b*cos(f*x
 + e)^2 + 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) - 2*a - b)/(cos(f*x + e
)^2 - 1)) - 2*(2*b*cos(f*x + e)^4 - 2*(4*a - b)*cos(f*x + e)^2 + 11*a - 4*b
)*sqrt(-b*cos(f*x + e)^2 + a + b))/(f*cos(f*x + e)^2 - f), -1/6*(3*((2*a -
3*b)*cos(f*x + e)^2 - 2*a + 3*b)*sqrt(-a)*arctan(sqrt(-b*cos(f*x + e)^2 + a
 + b)*sqrt(-a)/a) - (2*b*cos(f*x + e)^4 - 2*(4*a - b)*cos(f*x + e)^2 + 11*a
 - 4*b)*sqrt(-b*cos(f*x + e)^2 + a + b))/(f*cos(f*x + e)^2 - f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^3, x)
```

3.504 $\int \cot^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=208

$$\frac{\left(8a^2-24ab+3b^2\right)\left(a+b\sin^2(e+fx)\right)^{3/2}}{24a^2f}+\frac{\left(8a^2-24ab+3b^2\right)\sqrt{a+b\sin^2(e+fx)}}{8af}-\frac{\left(8a^2-24ab+3b^2\right)\tanh^{-1}\left(\sqrt{a+b}\right.}{8\sqrt{a}f}$$

```
[Out] -((8*a^2 - 24*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/(8*
Sqrt[a]*f) + ((8*a^2 - 24*a*b + 3*b^2)*Sqrt[a + b*Sin[e + f*x]^2])/(8*a*f)
+ ((8*a^2 - 24*a*b + 3*b^2)*(a + b*Sin[e + f*x]^2)^(3/2))/(24*a^2*f) + ((8*
a - b)*Csc[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(5/2))/(8*a^2*f) - (Csc[e + f*
x]^4*(a + b*Sin[e + f*x]^2)^(5/2))/(4*a*f)
```

Rubi [A] time = 0.191996, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3194, 89, 78, 50, 63, 208}

$$\frac{\left(8a^2-24ab+3b^2\right)\left(a+b\sin^2(e+fx)\right)^{3/2}}{24a^2f}+\frac{\left(8a^2-24ab+3b^2\right)\sqrt{a+b\sin^2(e+fx)}}{8af}-\frac{\left(8a^2-24ab+3b^2\right)\tanh^{-1}\left(\sqrt{a+b}\right.}{8\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -((8*a^2 - 24*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/(8*
Sqrt[a]*f) + ((8*a^2 - 24*a*b + 3*b^2)*Sqrt[a + b*Sin[e + f*x]^2])/(8*a*f)
+ ((8*a^2 - 24*a*b + 3*b^2)*(a + b*Sin[e + f*x]^2)^(3/2))/(24*a^2*f) + ((8*
a - b)*Csc[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(5/2))/(8*a^2*f) - (Csc[e + f*
x]^4*(a + b*Sin[e + f*x]^2)^(5/2))/(4*a*f)
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
```

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \cot^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\frac{\text{Subst}\left(\int \frac{(1-x)^2(a+bx)^{3/2}}{x^3}\,dx,x,\sin^2(e+fx)\right)}{2f}$$

$$=-\frac{\csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{4af}+\frac{\text{Subst}\left(\int \frac{\left(\frac{1}{2}(-8a+b)+2ax\right)(a+bx)^{3/2}}{x^2}\,dx\right.}{4af}$$

$$=\frac{(8a-b)\csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{8a^2f}-\frac{\csc^4(e+fx)\left(a+b\sin^2(e\right.}{4af}$$

$$=\frac{\left(8a^2-3(8a-b)b\right)\left(a+b\sin^2(e+fx)\right)^{3/2}}{24a^2f}+\frac{(8a-b)\csc^2(e+fx)\left(a+b\,\text{s}\right.}{8a^2f}$$

$$=\frac{\left(8a^2-3(8a-b)b\right)\sqrt{a+b\sin^2(e+fx)}}{8af}+\frac{\left(8a^2-3(8a-b)b\right)\left(a+b\sin^2(\right.}{24a^2f}$$

$$=\frac{\left(8a^2-3(8a-b)b\right)\sqrt{a+b\sin^2(e+fx)}}{8af}+\frac{\left(8a^2-3(8a-b)b\right)\left(a+b\sin^2(\right.}{24a^2f}$$

$$=-\frac{\left(8a^2-3(8a-b)b\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{8\sqrt{a}f}+\frac{\left(8a^2-3(8a-b)b\right)\sqrt{a+b}}{8af}$$

Mathematica [A] time = 0.842136, size = 123, normalized size = 0.59

$$\frac{\sqrt{a}\sqrt{a+b\sin^2(e+fx)}\left(8\left(4a+b\sin^2(e+fx)-6b\right)+3(8a-5b)\csc^2(e+fx)-6a\csc^4(e+fx)\right)-3\left(8a^2-24ab+3b^2\right)}{24\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-3*(8*a^2 - 24*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]] +
Sqrt[a]*Sqrt[a + b*Sin[e + f*x]^2]*(3*(8*a - 5*b)*Csc[e + f*x]^2 - 6*a*Csc[
e + f*x]^4 + 8*(4*a - 6*b + b*Sin[e + f*x]^2)))/(24*Sqrt[a]*f)
```

Maple [A] time = 1.688, size = 280, normalized size = 1.4

$$\frac{b\left(\sin\left(fx+e\right)\right)^2}{3f}\sqrt{a+b\left(\sin\left(fx+e\right)\right)^2}+\frac{4a}{3f}\sqrt{a+b\left(\sin\left(fx+e\right)\right)^2}-2\frac{b\sqrt{a+b\left(\sin\left(fx+e\right)\right)^2}}{f}-\frac{a}{4f\left(\sin\left(fx+e\right)\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5*(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/3/f*b*sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)+4/3*a*(a+b*sin(f*x+e)^2)^(1/2
)/f-2/f*b*(a+b*sin(f*x+e)^2)^(1/2)-1/4/f*a/sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^
(1/2)-5/8/f*b/sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)-3/8/f/a^(1/2)*b^2*ln((2
*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e))-1/f*a^(3/2)*ln((2*a+2*a^
(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e))+3/f*a^(1/2)*ln((2*a+2*a^(1/2)*(
a+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e))*b+1/f*a/sin(f*x+e)^2*(a+b*sin(f*x+e)^2
)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 12.9466, size = 1089, normalized size = 5.24

$$\left[\frac{3\left(\left(8a^2-24ab+3b^2\right)\cos\left(fx+e\right)^4-2\left(8a^2-24ab+3b^2\right)\cos\left(fx+e\right)^2+8a^2-24ab+3b^2\right)\sqrt{a}\log\left(\frac{2\left(b\cos\left(fx+e\right)^2+\cdots\right.}{\cdots}\right.}{48\left(a\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*((8*a^2 - 24*a*b + 3*b^2)*cos(f*x + e)^4 - 2*(8*a^2 - 24*a*b + 3*b
^2)*cos(f*x + e)^2 + 8*a^2 - 24*a*b + 3*b^2)*sqrt(a)*log(2*(b*cos(f*x + e)^
2 + 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) - 2*a - b)/(cos(f*x + e)^2 -
1)) - 2*(8*a*b*cos(f*x + e)^6 - 8*(4*a^2 - 3*a*b)*cos(f*x + e)^4 + (88*a^2
- 87*a*b)*cos(f*x + e)^2 - 50*a^2 + 55*a*b)*sqrt(-b*cos(f*x + e)^2 + a + b)
)/(a*f*cos(f*x + e)^4 - 2*a*f*cos(f*x + e)^2 + a*f), 1/24*(3*((8*a^2 - 24*a
*b + 3*b^2)*cos(f*x + e)^4 - 2*(8*a^2 - 24*a*b + 3*b^2)*cos(f*x + e)^2 + 8*
a^2 - 24*a*b + 3*b^2)*sqrt(-a)*arctan(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(
-a)/a) - (8*a*b*cos(f*x + e)^6 - 8*(4*a^2 - 3*a*b)*cos(f*x + e)^4 + (88*a^2
 - 87*a*b)*cos(f*x + e)^2 - 50*a^2 + 55*a*b)*sqrt(-b*cos(f*x + e)^2 + a + b
))/(a*f*cos(f*x + e)^4 - 2*a*f*cos(f*x + e)^2 + a*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^5, x)
```

3.505 $\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan^4(e+fx)\,dx$

Optimal. Leaf size=275

$$\frac{\tan^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{3f}-\frac{(a+2b)\sin^2(e+fx)\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f}-\frac{(3a+8b)\sin(e+fx)\cos(}{3}$$

```
[Out] -((3*a + 8*b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f) +
 (8*(a + 2*b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*
Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a
]) - (a*(5*a + 8*b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(
b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*f*Sqrt[a + b*Sin[e +
f*x]^2]) - ((a + 2*b)*Sin[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x
])/f + ((a + b*Sin[e + f*x]^2)^(3/2)*Tan[e + f*x]^3)/(3*f)
```

Rubi [A] time = 0.36881, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {3196, 467, 577, 582, 524, 426, 424, 421, 419}

$$\frac{\tan^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{3f}-\frac{(a+2b)\sin^2(e+fx)\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f}-\frac{(3a+8b)\sin(e+fx)\cos(}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x]^2)^(3/2)*Tan[e + f*x]^4,x]
```


```
[Out] -((3*a + 8*b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f) +
 (8*(a + 2*b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*
Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a
]) - (a*(5*a + 8*b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(
b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*f*Sqrt[a + b*Sin[e +
f*x]^2]) - ((a + 2*b)*Sin[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x
])/f + ((a + b*Sin[e + f*x]^2)^(3/2)*Tan[e + f*x]^3)/(3*f)
```


Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 577

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*b*g*n*(p + 1)), x] + Dist[1/(a*
b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b
*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m + n
*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n, 0
] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a
*f])
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan^4(e+fx)\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^4\left(a+bx^2\right)^{3/2}}{\left(1-x^2\right)^{5/2}}dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(a+b\sin^2(e+fx)\right)^{3/2}\tan^3(e+fx)}{3f} - \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int\right.}{3f}$$

$$= -\frac{(a+2b)\sin^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{f} + \frac{\left(a+b\sin^2(e+fx)\right.}{3f}$$

$$= -\frac{(3a+8b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} - \frac{(a+2b)\sin^2(e+f}{}$$

$$= -\frac{(3a+8b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} - \frac{(a+2b)\sin^2(e+f}{}$$

$$= -\frac{(3a+8b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} - \frac{(a+2b)\sin^2(e+f}{}$$

$$= -\frac{(3a+8b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} + \frac{8(a+2b)\sqrt{\cos^2(e}}{}$$

Mathematica [A] time = 2.87417, size = 211, normalized size = 0.77

$$\frac{-\frac{\tan(e+fx)\sec^2(e+fx)\left(\left(64a^2+160ab+17b^2\right)\cos(2(e+fx))+32a^2-2b(6a+17b)\cos(4(e+fx))+108ab-b^2\cos(6(e+fx))+18b^2\right)}{4\sqrt{2}} - 4a(5a+8b)\sqrt{\frac{2a-b\cos(2(e}{a}}}{12f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)^(3/2)*Tan[e + f*x]^4,x]
```

```
[Out] (32*a*(a + 2*b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -
(b/a)] - 4*a*(5*a + 8*b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e
 + f*x, -(b/a)] - ((32*a^2 + 108*a*b + 18*b^2 + (64*a^2 + 160*a*b + 17*b^2)
*Cos[2*(e + f*x)] - 2*b*(6*a + 17*b)*Cos[4*(e + f*x)] - b^2*Cos[6*(e + f*x)
])*Sec[e + f*x]^2*Tan[e + f*x])/(4*Sqrt[2]))/(12*f*Sqrt[2*a + b - b*Cos[2*(
e + f*x)]])
```

Maple [A] time = 2.241, size = 419, normalized size = 1.5

$$-\frac{1}{\left(-3+3\sin\left(fx+e\right)\right)\left(1+\sin\left(fx+e\right)\right)\cos\left(fx+e\right)f}\left(\sqrt{-b\left(\cos\left(fx+e\right)\right)^4+(a+b)\left(\cos\left(fx+e\right)\right)^2}b^2\sin\left(fx+e\right)\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^4,x)
```

```
[Out] -1/3*((-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b^2*sin(f*x+e)*cos(f*x+e)^
6+(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*(3*a+7*b)*cos(f*x+e)^4*sin(f
*x+e)-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(4*a^2+13*a*b+9*b^2)*cos(f
*x+e)^2*sin(f*x+e)+(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a^2+2*a*b+b^
2)*sin(f*x+e)-(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(cos(f*x+e)^2)^(1/2)*(-b*co
s(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*a*(5*EllipticF(sin(f*x+e),(-1/a*b)^(1/
2))*a+8*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-8*EllipticE(sin(f*x+e),(-1/a
*b)^(1/2))*a-16*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*b)*cos(f*x+e)^2)/(-(a+
b*sin(f*x+e)^2)*(-1+sin(f*x+e))*(1+sin(f*x+e)))^(1/2)/(-1+sin(f*x+e))/(1+si
n(f*x+e))/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(b\cos\left(fx+e\right)^{2}-a-b\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\tan\left(fx+e\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] integral(-(b*cos(f*x + e)^2 - a - b)*sqrt(-b*cos(f*x + e)^2 + a + b)*tan(f*
x + e)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**(3/2)*tan(f*x+e)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^4, x)
```

3.506 $\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan^2(e+fx)\,dx$

Optimal. Leaf size=222

$$\frac{\tan(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{f}+\frac{4b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{4a(a+b)\sqrt{\cos^2(e+fx)}\sec(e}{3f}$$

```
[Out] (4*b*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f) - ((7*a +
8*b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f
*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (4*a
*(a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e
 + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*f*Sqrt[a + b*Sin[e + f*x]^2]) +
((a + b*Sin[e + f*x]^2)^(3/2)*Tan[e + f*x])/f
```

Rubi [A] time = 0.230866, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3196, 467, 528, 524, 426, 424, 421, 419}

$$\frac{\tan(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{f}+\frac{4b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{4a(a+b)\sqrt{\cos^2(e+fx)}\sec(e}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x]^2)^(3/2)*Tan[e + f*x]^2,x]
```


```
[Out] (4*b*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f) - ((7*a +
8*b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f
*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (4*a
*(a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e
 + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*f*Sqrt[a + b*Sin[e + f*x]^2]) +
((a + b*Sin[e + f*x]^2)^(3/2)*Tan[e + f*x])/f
```


Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan^2(e+fx)\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^2\left(a+bx^2\right)^{3/2}}{\left(1-x^2\right)^{3/2}}dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(a+b\sin^2(e+fx)\right)^{3/2}\tan(e+fx)}{f} - \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}}{}$$

$$= \frac{4b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} + \frac{\left(a+b\sin^2(e+fx)\right)^{3/2}\text{t}}{f}$$

$$= \frac{4b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} + \frac{\left(a+b\sin^2(e+fx)\right)^{3/2}\text{t}}{f}$$

$$= \frac{4b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} + \frac{\left(a+b\sin^2(e+fx)\right)^{3/2}\text{t}}{f}$$

$$= \frac{4b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} - \frac{(7a+8b)\sqrt{\cos^2(e+fx)}}{}$$

Mathematica [A] time = 2.93857, size = 174, normalized size = 0.78

$$\frac{\sqrt{2}\tan(e+fx)\left(24a^2-4b(2a+3b)\cos(2(e+fx))+40ab-b^2\cos(4(e+fx))+13b^2\right)+32a(a+b)\sqrt{\frac{2a-b\cos(2(e+fx))+}{a}}}{24f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)^(3/2)*Tan[e + f*x]^2,x]
```

```
[Out] (-8*a*(7*a + 8*b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x,
 -(b/a)] + 32*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e
+ f*x, -(b/a)] + Sqrt[2]*(24*a^2 + 40*a*b + 13*b^2 - 4*b*(2*a + 3*b)*Cos[2*
(e + f*x)] - b^2*Cos[4*(e + f*x)])*Tan[e + f*x])/(24*f*Sqrt[2*a + b - b*Cos
[2*(e + f*x)]])
```

Maple [B] time = 2.208, size = 515, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^2,x)
```

```
[Out] 1/3*(-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b^2*sin(f*x+e)*cos(f*x+e)^
4-2*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*(a+b)*cos(f*x+e)^2*sin(f*x
+e)+3*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a^2+2*a*b+b^2)*sin(f*x+e)
+4*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x
+e),(-1/a*b)^(1/2))*a^2*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)+4*a*(cos
```

```
(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1
/a*b)^(1/2))*b*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)-7*(cos(f*x+e)^2)^
(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2)
)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*a^2-8*(cos(f*x+e)^2)^(1/2)*(-b
/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*(-b*cos
(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*a*b)/(-(a+b*sin(f*x+e)^2)*(-1+sin(f*x+e
))*(1+sin(f*x+e)))^(1/2)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(b\cos\left(fx+e\right)^{2}-a-b\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\tan\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral(-(b*cos(f*x + e)^2 - a - b)*sqrt(-b*cos(f*x + e)^2 + a + b)*tan(f*
x + e)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**(3/2)*tan(f*x+e)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^2, x)
```

3.507 $\int \left(a+b\sin^2(e+fx)\right)^{3/2} dx$

Optimal. Leaf size=154

$$-\frac{b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(e+fx\middle|-\frac{b}{a}\right)}{3f\sqrt{a+b\sin^2(e+fx)}}+\frac{2(2a+b)\sqrt{a+b\sin^2(e+}}{3f\sqrt{\frac{b\sin^2(e+f}{a}}}$$

```
[Out] -(b*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f) + (2*(2*a +
 b)*EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f*Sqrt[1 + (b
*Sin[e + f*x]^2)/a]) - (a*(a + b)*EllipticF[e + f*x, -(b/a)]*Sqrt[1 + (b*Si
n[e + f*x]^2)/a])/(3*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.167102, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3180, 3172, 3178, 3177, 3183, 3182}

$$-\frac{b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(e+fx\middle|-\frac{b}{a}\right)}{3f\sqrt{a+b\sin^2(e+fx)}}+\frac{2(2a+b)\sqrt{a+b\sin^2(e+}}{3f\sqrt{\frac{b\sin^2(e+f}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -(b*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f) + (2*(2*a +
 b)*EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f*Sqrt[1 + (b
*Sin[e + f*x]^2)/a]) - (a*(a + b)*EllipticF[e + f*x, -(b/a)]*Sqrt[1 + (b*Si
n[e + f*x]^2)/a])/(3*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 3180

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p - 1))/(2*f*p), x] + Dist[
1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a + b
)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a +
 b, 0] && GtQ[p, 1]
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rubi steps

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = -\frac{b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{1}{3}\int \frac{a(3a+b)+2b(2a+b)\sin^2(e+fx}{\sqrt{a+b\sin^2(e+fx)}}$$

$$= -\frac{b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{1}{3}(a(a+b))\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx +$$

$$= -\frac{b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{\left(2(2a+b)\sqrt{a+b\sin^2(e+fx)}\right)\int \sqrt{1+}}{3\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

$$= -\frac{b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{2(2a+b)E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e-}}{3f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

Mathematica [A] time = 0.767113, size = 156, normalized size = 1.01

$$\frac{b\sin(2(e+fx))(-2a+b\cos(2(e+fx))-b)-2\sqrt{2}a(a+b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}F\left(e+fx\middle|-\frac{b}{a}\right)+4\sqrt{2}a(2a+b)\sqrt{\frac{2a-b\cos(2(e}{a}}}{6\sqrt{2}f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (4*Sqrt[2]*a*(2*a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e +
 f*x, -(b/a)] - 2*Sqrt[2]*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*
EllipticF[e + f*x, -(b/a)] + b*(-2*a - b + b*Cos[2*(e + f*x)])*Sin[2*(e + f
*x)])/(6*Sqrt[2]*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A] time = 1.317, size = 266, normalized size = 1.7

$$\frac{1}{f\cos\left(fx+e\right)}\left(-\frac{a^2}{3}\sqrt{\left(\cos\left(fx+e\right)\right)^2}\sqrt{\frac{a+b\left(\sin\left(fx+e\right)\right)^2}{a}}\mathrm{EllipticF}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)-\frac{ab}{3}\sqrt{\left(\cos\left(fx+e\right)\right)^2}\sqrt{\frac{a}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] (-1/3*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e
),(-1/a*b)^(1/2))*a^2-1/3*a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/
2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b+4/3*(cos(f*x+e)^2)^(1/2)*((a+b*si
n(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2+2/3*(cos(f*x+
e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2
))*a*b+1/3*sin(f*x+e)^5*b^2+1/3*sin(f*x+e)^3*a*b-1/3*sin(f*x+e)^3*b^2-1/3*s
in(f*x+e)*a*b)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(-b\cos\left(fx+e\right)^{2}+a+b\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((-b*cos(f*x + e)^2 + a + b)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.508 $\int \cot^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=223

$$\frac{4b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{\cot(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{f}+\frac{4a(a+b)\sqrt{\cos^2(e+fx)}\sec(e\cdots}{3f}$$

```
[Out] (4*b*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f) - (Cot[e +
 f*x]*(a + b*Sin[e + f*x]^2)^(3/2))/f - ((7*a - b)*Sqrt[Cos[e + f*x]^2]*Ell
ipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2
])/(3*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (4*a*(a + b)*Sqrt[Cos[e + f*x]^2]
*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f
*x]^2)/a])/(3*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.256591, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3196, 473, 528, 524, 426, 424, 421, 419}

$$\frac{4b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{\cot(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{f}+\frac{4a(a+b)\sqrt{\cos^2(e+fx)}\sec(e\cdots}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2),x]
```


```
[Out] (4*b*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f) - (Cot[e +
 f*x]*(a + b*Sin[e + f*x]^2)^(3/2))/f - ((7*a - b)*Sqrt[Cos[e + f*x]^2]*Ell
ipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2
])/(3*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (4*a*(a + b)*Sqrt[Cos[e + f*x]^2]
*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f
*x]^2)/a])/(3*f*Sqrt[a + b*Sin[e + f*x]^2])
```


Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^p*(c + d*x^n)^q)/(e*(m
 + 1)), x] - Dist[n/(e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^(p - 1)*(c
 + d*x^n)^(q - 1)*Simp[b*c*p + a*d*q + b*d*(p + q)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && LtQ
[m, -1] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \cot^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{1-x^2}\left(a+bx^2\right)^{3/2}}{x^2}dx,x,\sin(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{f} + \frac{\left(2\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Sub}}{}$$

$$= \frac{4b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} - \frac{\cot(e+fx)\left(a+b\sin^2(\right.}{f}$$

$$= \frac{4b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} - \frac{\cot(e+fx)\left(a+b\sin^2(\right.}{f}$$

$$= \frac{4b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} - \frac{\cot(e+fx)\left(a+b\sin^2(\right.}{f}$$

$$= \frac{4b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} - \frac{\cot(e+fx)\left(a+b\sin^2(\right.}{f}$$

Mathematica [A] time = 2.26562, size = 173, normalized size = 0.78

$$\frac{\sqrt{2}\cot(e+fx)\left(-24a^2+4b(2a-b)\cos(2(e+fx))-8ab+b^2\cos(4(e+fx))+3b^2\right)+32a(a+b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}F}{24f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[2]*(-24*a^2 - 8*a*b + 3*b^2 + 4*(2*a - b)*b*Cos[2*(e + f*x)] + b^2*Co
s[4*(e + f*x)])*Cot[e + f*x] - 8*a*(7*a - b)*Sqrt[(2*a + b - b*Cos[2*(e + f
*x)])/a]*EllipticE[e + f*x, -(b/a)] + 32*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*
(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)])/(24*f*Sqrt[2*a + b - b*Cos[2*(e
+ f*x)]])
```

Maple [A] time = 1.258, size = 204, normalized size = 0.9

$$\frac{1}{3\sin\left(fx+e\right)\cos\left(fx+e\right)f}\left(\sin\left(fx+e\right)\sqrt{\left(\cos\left(fx+e\right)\right)^2}\sqrt{-\frac{b\left(\cos\left(fx+e\right)\right)^2}{a}+\frac{a+b}{a}}a\left(4\,\text{EllipticF}\left(\sin\left(fx+e\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/3*(sin(f*x+e)*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*a*(4
*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a+4*EllipticF(sin(f*x+e),(-1/a*b)^(1/
```

```
2))*b-7*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a+EllipticE(sin(f*x+e),(-1/a*b
)^(1/2))*b)+b^2*cos(f*x+e)^6+(2*a*b-2*b^2)*cos(f*x+e)^4+(-3*a^2-2*a*b+b^2)*
cos(f*x+e)^2)/sin(f*x+e)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(b\cos\left(fx+e\right)^2-a-b\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}\cot\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(b*cos(f*x + e)^2 - a - b)*sqrt(-b*cos(f*x + e)^2 + a + b)*cot(f*
x + e)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^2, x)
```

3.509 $\int \cot^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=276

$$\frac{(3a-5b)\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{\cot^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{3f}+\frac{(a-b)\cos^2(e+fx)\cot\cdots}{}$$

```
[Out] ((a - b)*Cos[e + f*x]^2*Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/f + ((3*a
- 5*b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f) - (Cot[e
 + f*x]^3*(a + b*Sin[e + f*x]^2)^(3/2))/(3*f) + (8*(a - b)*Sqrt[Cos[e + f*x
]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e
+ f*x]^2])/(3*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) - ((5*a - 3*b)*(a + b)*Sqrt
[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[
1 + (b*Sin[e + f*x]^2)/a])/(3*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.350888, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {3196, 473, 580, 528, 524, 426, 424, 421, 419}

$$\frac{(3a-5b)\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{\cot^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{3f}+\frac{(a-b)\cos^2(e+fx)\cot\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2),x]
```


```
[Out] ((a - b)*Cos[e + f*x]^2*Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/f + ((3*a
- 5*b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f) - (Cot[e
 + f*x]^3*(a + b*Sin[e + f*x]^2)^(3/2))/(3*f) + (8*(a - b)*Sqrt[Cos[e + f*x
]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e
+ f*x]^2])/(3*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) - ((5*a - 3*b)*(a + b)*Sqrt
[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[
1 + (b*Sin[e + f*x]^2)/a])/(3*f*Sqrt[a + b*Sin[e + f*x]^2])
```


Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^p*(c + d*x^n)^q)/(e*(m
 + 1)), x] - Dist[n/(e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^(p - 1)*(c
 + d*x^n)^(q - 1)*Simp[b*c*p + a*d*q + b*d*(p + q)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && LtQ
[m, -1] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \cot^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{(1-x^2)^{3/2}(a+bx^2)^{3/2}}{x^4}dx,x,\sin(e+fx)\right.}{f}$$

$$= -\frac{\cot^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{3f} + \frac{\left(2\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Su}}{}$$

$$= \frac{(a-b)\cos^2(e+fx)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} - \frac{\cot^3(e+fx)\left(a+\right.}{3}$$

$$= \frac{(a-b)\cos^2(e+fx)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} + \frac{(3a-5b)\cos(e+}{}$$

$$= \frac{(a-b)\cos^2(e+fx)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} + \frac{(3a-5b)\cos(e+}{}$$

$$= \frac{(a-b)\cos^2(e+fx)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} + \frac{(3a-5b)\cos(e+}{}$$

$$= \frac{(a-b)\cos^2(e+fx)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} + \frac{(3a-5b)\cos(e+}{}$$

Mathematica [A] time = 5.05421, size = 218, normalized size = 0.79

$$\frac{-4\left(5a^2+2ab-3b^2\right)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}F\left(e+fx\middle|-\frac{b}{a}\right) - \frac{\cot(e+fx)\csc^2(e+fx)\left(\left(64a^2-32ab-79b^2\right)\cos(2(e+fx))-32a^2-2b(6a-11b)\cos(}{4\sqrt{2}}}{12f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-((-32*a^2 + 44*a*b + 58*b^2 + (64*a^2 - 32*a*b - 79*b^2)*Cos[2*(e + f*x)]
 - 2*(6*a - 11*b)*b*Cos[4*(e + f*x)] - b^2*Cos[6*(e + f*x)])*Cot[e + f*x]*C
sc[e + f*x]^2)/(4*Sqrt[2]) + 32*a*(a - b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)
])/a]*EllipticE[e + f*x, -(b/a)] - 4*(5*a^2 + 2*a*b - 3*b^2)*Sqrt[(2*a + b
- b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)])/(12*f*Sqrt[2*a + b - b
*Cos[2*(e + f*x)]])
```

Maple [A] time = 1.303, size = 419, normalized size = 1.5

$$-\frac{1}{3\left(\sin\left(fx+e\right)\right)^3\cos\left(fx+e\right)f}\left(-b^2\left(\sin\left(fx+e\right)\right)^8+5\,\text{EllipticF}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)\sqrt{\left(\cos\left(fx+e\right)\right)^2}\sqrt{\frac{a+b\left(\right.}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/3*(-b^2*sin(f*x+e)^8+5*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*(cos(f*x+e)^
2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*a^2*sin(f*x+e)^3+2*b*(cos(f*x+e)^2)^(
1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*si
n(f*x+e)^3-3*b^2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*Elliptic
F(sin(f*x+e),(-1/a*b)^(1/2))*sin(f*x+e)^3-8*EllipticE(sin(f*x+e),(-1/a*b)^(
1/2))*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*a^2*sin(f*x+e)^3+8*
EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^
2)/a)^(1/2)*a*b*sin(f*x+e)^3+3*a*b*sin(f*x+e)^6-3*b^2*sin(f*x+e)^6+4*a^2*si
n(f*x+e)^4-8*a*b*sin(f*x+e)^4+4*b^2*sin(f*x+e)^4-5*a^2*sin(f*x+e)^2+5*a*b*s
in(f*x+e)^2+a^2)/sin(f*x+e)^3/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(b\cos\left(fx+e\right)^2-a-b\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}\cot\left(fx+e\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(b*cos(f*x + e)^2 - a - b)*sqrt(-b*cos(f*x + e)^2 + a + b)*cot(f*
x + e)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^4, x)
```

3.510 $\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=134

$$\frac{(8a^2+8ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{8f(a+b)^{5/2}}+\frac{\sec^4(e+fx)\sqrt{a+b\sin^2(e+fx)}}{4f(a+b)}-\frac{(8a+5b)\sec^2(e+fx)\sqrt{a+b\sin^2(e+f}}{8f(a+b)^2}$$

```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/(
8*(a + b)^(5/2)*f) - ((8*a + 5*b)*Sec[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2]
)/(8*(a + b)^2*f) + (Sec[e + f*x]^4*Sqrt[a + b*Sin[e + f*x]^2])/(4*(a + b)*
f)
```

Rubi [A] time = 0.173842, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3194, 89, 78, 63, 208}

$$\frac{(8a^2+8ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{8f(a+b)^{5/2}}+\frac{\sec^4(e+fx)\sqrt{a+b\sin^2(e+fx)}}{4f(a+b)}-\frac{(8a+5b)\sec^2(e+fx)\sqrt{a+b\sin^2(e+f}}{8f(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/(
8*(a + b)^(5/2)*f) - ((8*a + 5*b)*Sec[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2]
)/(8*(a + b)^2*f) + (Sec[e + f*x]^4*Sqrt[a + b*Sin[e + f*x]^2])/(4*(a + b)*
f)
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
```

```
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1-x)^3\sqrt{a+bx}}\,dx,x,\sin^2(e+fx)\right)}{2f}$$

$$= \frac{\sec^4(e+fx)\sqrt{a+b\sin^2(e+fx)}}{4(a+b)f} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(4a+b)+2(a+b)x}{(1-x)^2\sqrt{a+bx}}\,dx,x,\sin^2(e+fx)\right)}{4(a+b)f}$$

$$= -\frac{(8a+5b)\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8(a+b)^2f} + \frac{\sec^4(e+fx)\sqrt{a+b\sin^2(e+fx)}}{4(a+b)f} + \frac{(8a^2 \cdots}{\cdots}$$

$$= -\frac{(8a+5b)\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8(a+b)^2f} + \frac{\sec^4(e+fx)\sqrt{a+b\sin^2(e+fx)}}{4(a+b)f} + \frac{(8a^2 \cdots}{\cdots}$$

$$= \frac{\left(8a^2+8ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{8(a+b)^{5/2}f} - \frac{(8a+5b)\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8(a+b)^2f}$$

Mathematica [A] time = 0.49333, size = 108, normalized size = 0.81

$$\frac{\left(8a^2+8ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)+\sqrt{a+b}\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\left(2(a+b)\sec^2(e+fx)-8a-5b\right)}{8f(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]] +
Sqrt[a + b]*Sec[e + f*x]^2*(-8*a - 5*b + 2*(a + b)*Sec[e + f*x]^2)*Sqrt[a +
 b*Sin[e + f*x]^2])/(8*(a + b)^(5/2)*f)
```

Maple [B] time = 3.356, size = 644, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/16*((8*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(
f*x+e)+a))*a^4+24*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/
2)-b*sin(f*x+e)+a))*a^3*b+27*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*
x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^2*b^2+14*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*
(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a*b^3+3*ln(2/(1+sin(f*x+e))*((a
+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*b^4+8*ln(2/(-1+sin(f*
x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^4+24*ln(2/
(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^
3*b+27*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f
*x+e)+a))*a^2*b^2+14*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)
^(1/2)+b*sin(f*x+e)+a))*a*b^3+3*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*co
s(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*b^4)*cos(f*x+e)^4-2*(a+b)^(5/2)*(a+b-b*c
os(f*x+e)^2)^(1/2)*(8*a+5*b)*cos(f*x+e)^2+4*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b
)^(5/2)*a+4*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(5/2)*b)/(a+b)^(5/2)/cos(f*x+e
)^4/(a^2+2*a*b+b^2)/f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.03885, size = 792, normalized size = 5.91

$$\left[\frac{\left(8\,a^2+8\,ab+3\,b^2\right)\sqrt{a+b}\cos\left(fx+e\right)^4\log\left(\frac{b\cos\left(fx+e\right)^2-2\,\sqrt{-b\cos\left(fx+e\right)^2+a+b}\sqrt{a+b}-2\,a-2\,b}{\cos\left(fx+e\right)^2}\right)-2\left(\left(8\,a^2+13\,ab+5\,b^2\right)\cos\left(f\right.\right.}{16\left(a^3+3\,a^2b+3\,ab^2+b^3\right)f\cos\left(fx+e\right)^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((8*a^2 + 8*a*b + 3*b^2)*sqrt(a + b)*cos(f*x + e)^4*log((b*cos(f*x +
e)^2 - 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b) - 2*a - 2*b)/cos(f*x +
 e)^2) - 2*((8*a^2 + 13*a*b + 5*b^2)*cos(f*x + e)^2 - 2*a^2 - 4*a*b - 2*b^2
)*sqrt(-b*cos(f*x + e)^2 + a + b))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*cos(f
*x + e)^4), -1/8*((8*a^2 + 8*a*b + 3*b^2)*sqrt(-a - b)*arctan(sqrt(-b*cos(f
*x + e)^2 + a + b)*sqrt(-a - b)/(a + b))*cos(f*x + e)^4 + ((8*a^2 + 13*a*b
+ 5*b^2)*cos(f*x + e)^2 - 2*a^2 - 4*a*b - 2*b^2)*sqrt(-b*cos(f*x + e)^2 + a
 + b))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*cos(f*x + e)^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^5\left(e+fx\right)}{\sqrt{a+b\sin^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**5/sqrt(a + b*sin(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^5}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^5/sqrt(b*sin(f*x + e)^2 + a), x)
```

3.511 $\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=81

$$\frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2f(a+b)}-\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{2f(a+b)^{3/2}}$$

```
[Out] -((2*a + b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/(2*(a + b)^(3/
2)*f) + (Sec[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2])/(2*(a + b)*f)
```

Rubi [A] time = 0.0977817, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3194, 78, 63, 208}

$$\frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2f(a+b)}-\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{2f(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((2*a + b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/(2*(a + b)^(3/
2)*f) + (Sec[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2])/(2*(a + b)*f)
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(1-x)^2\sqrt{a+bx}}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= \frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2(a+b)f} - \frac{(2a+b)\,\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,dx, x, \sin^2(e+fx)\right)}{4(a+b)f}$$

$$= \frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2(a+b)f} - \frac{(2a+b)\,\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sin^2(e+fx)}\right)}{2b(a+b)f}$$

$$= -\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{2(a+b)^{3/2}f} + \frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2(a+b)f}$$

Mathematica [A] time = 0.225869, size = 77, normalized size = 0.95

$$-\frac{\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} - \frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a+b}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -(((2*a + b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/(a + b)^(3/2)
 - (Sec[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2])/(a + b))/(2*f)
```

Maple [B] time = 2.783, size = 353, normalized size = 4.4

$$\frac{1}{4\left(\cos\left(fx+e\right)\right)^2 f}\left(-\left(2\ln\left(2\frac{\sqrt{a+b}\sqrt{a+b-b\left(\cos\left(fx+e\right)\right)^2}+b\sin\left(fx+e\right)+a}{-1+\sin\left(fx+e\right)}\right)a^2+3\ln\left(2\frac{\sqrt{a+b}\sqrt{a+b-b}}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/4*(-(2*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin
(f*x+e)+a))*a^2+3*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1
/2)+b*sin(f*x+e)+a))*a*b+ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e
)^2)^(1/2)+b*sin(f*x+e)+a))*b^2+2*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*c
os(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^2+3*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*
(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a*b+ln(2/(1+sin(f*x+e))*((a+b)^
(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*b^2)*cos(f*x+e)^2+2*(a+b-
b*cos(f*x+e)^2)^(1/2)*(a+b)^(3/2))/(a+b)^(5/2)/cos(f*x+e)^2/f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.5331, size = 563, normalized size = 6.95

$$\left[\frac{(2\,a+b)\sqrt{a+b}\cos\left(fx+e\right)^2\log\left(\frac{b\cos\left(fx+e\right)^2+2\sqrt{-b\cos\left(fx+e\right)^2+a+b}\sqrt{a+b}-2\,a-2\,b}{\cos\left(fx+e\right)^2}\right)+2\sqrt{-b\cos\left(fx+e\right)^2+a+b}(a+b)}{4\left(a^2+2\,ab+b^2\right)f\cos\left(fx+e\right)^2}, \frac{(2\,a}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((2*a + b)*sqrt(a + b)*cos(f*x + e)^2*log((b*cos(f*x + e)^2 + 2*sqrt(-
b*cos(f*x + e)^2 + a + b)*sqrt(a + b) - 2*a - 2*b)/cos(f*x + e)^2) + 2*sqrt
(-b*cos(f*x + e)^2 + a + b)*(a + b))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2)
, 1/2*((2*a + b)*sqrt(-a - b)*arctan(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-
a - b)/(a + b))*cos(f*x + e)^2 + sqrt(-b*cos(f*x + e)^2 + a + b)*(a + b))/(
(a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^3\left(e+fx\right)}{\sqrt{a+b\sin^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**3/sqrt(a + b*sin(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^3}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^3/sqrt(b*sin(f*x + e)^2 + a), x)
```

3.512 $\int \frac{\tan(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=36

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{f\sqrt{a+b}}$$

[Out] ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]]/(Sqrt[a + b]*f)

Rubi [A] time = 0.0506633, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3194, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{f\sqrt{a+b}}$$

Antiderivative was successfully verified.

[In] Int[Tan[e + f*x]/Sqrt[a + b*Sin[e + f*x]^2],x]

[Out] ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]]/(Sqrt[a + b]*f)

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,dx, x, \sin^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sin^2(e+fx)}\right)}{bf}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}f}$$

Mathematica [A] time = 0.0396189, size = 38, normalized size = 1.06

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a-b\cos^2(e+fx)+b}}{\sqrt{a+b}}\right)}{f\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b - b*Cos[e + f*x]^2]/Sqrt[a + b]]/(Sqrt[a + b]*f)
```

Maple [B] time = 2.641, size = 113, normalized size = 3.1

$$\frac{1}{2f}\ln\left(2\frac{\sqrt{a+b}\sqrt{a+b-b\left(\cos\left(fx+e\right)\right)^2}+b\sin\left(fx+e\right)+a}{-1+\sin\left(fx+e\right)}\right)\frac{1}{\sqrt{a+b}}+\frac{1}{2f}\ln\left(2\frac{\sqrt{a+b}\sqrt{a+b-b\left(\cos\left(fx+e\right)\right.}}{1+\sin\left(fx+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/2/(a+b)^(1/2)/f*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1
/2)+b*sin(f*x+e)+a))+1/2/(a+b)^(1/2)/f*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+
b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.08465, size = 286, normalized size = 7.94

$$\left[\frac{\log\left(\frac{b\cos\left(fx+e\right)^2-2\sqrt{-b\cos\left(fx+e\right)^2+a+b}\sqrt{a+b}-2\,a-2\,b}{\cos\left(fx+e\right)^2}\right)}{2\sqrt{a+b}f}, -\frac{\sqrt{-a-b}\arctan\left(\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\sqrt{-a-b}}{a+b}\right)}{(a+b)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log((b*cos(f*x + e)^2 - 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b)
- 2*a - 2*b)/cos(f*x + e)^2)/(sqrt(a + b)*f), -sqrt(-a - b)*arctan(sqrt(-b*
cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(a + b))/((a + b)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(e+fx\right)}{\sqrt{a+b\sin^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)/sqrt(a + b*sin(e + f*x)**2), x)
```

Giac [B] time = 1.26318, size = 132, normalized size = 3.67

$$-\frac{2\arctan\left(-\frac{\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+2\,a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+4\,b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}-\sqrt{a}}{2\sqrt{-a-b}}\right)}{\sqrt{-a-b}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*arctan(-1/2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e
)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a) - sqrt(a
))/sqrt(-a - b))/(sqrt(-a - b)*f)
```

3.513 $\int \frac{\cot(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=33

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}$$

```
[Out] -(ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f))
```

Rubi [A] time = 0.0638331, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3194, 63, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f))
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cot(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sin^2(e+fx)}\right)}{bf}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}$$

Mathematica [A] time = 0.0362475, size = 33, normalized size = 1.

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f))
```

Maple [A] time = 0.762, size = 42, normalized size = 1.3

$$-\frac{1}{f}\ln\left(\frac{1}{\sin(fx+e)}\left(2a+2\sqrt{a}\sqrt{a+b(\sin(fx+e))^2}\right)\right)\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e))/f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.52384, size = 255, normalized size = 7.73

$$\left[\frac{\log\left(\frac{2\left(b\cos\left(fx+e\right)^{2}+2\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\sqrt{a}-2a-b\right)}{\cos\left(fx+e\right)^{2}-1}\right)}{2\sqrt{a}f}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\sqrt{-a}}{a}\right)}{af}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(2*(b*cos(f*x + e)^2 + 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) -
2*a - b)/(cos(f*x + e)^2 - 1))/(sqrt(a)*f), sqrt(-a)*arctan(sqrt(-b*cos(f*x
 + e)^2 + a + b)*sqrt(-a)/a)/(a*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(e+fx\right)}{\sqrt{a+b\sin^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)/sqrt(a + b*sin(e + f*x)**2), x)
```

Giac [A] time = 1.09399, size = 42, normalized size = 1.27

$$\frac{\arctan\left(\frac{\sqrt{b\sin\left(fx+e\right)^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*sin(f*x + e)^2 + a)/sqrt(-a))/(sqrt(-a)*f)
```

3.514 $\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=75

$$\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f}-\frac{\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2af}$$

```
[Out] ((2*a + b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/(2*a^(3/2)*f) - (Cs
c[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2])/(2*a*f)
```

Rubi [A] time = 0.0898619, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3194, 78, 63, 208}

$$\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f}-\frac{\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] ((2*a + b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/(2*a^(3/2)*f) - (Cs
c[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2])/(2*a*f)
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1-x}{x^2\sqrt{a+bx}}\,dx, x, \sin^2(e+fx)\right)}{2f} \\
&= -\frac{\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2af} - \frac{(2a+b)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sin^2(e+fx)\right)}{4af} \\
&= -\frac{\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2af} - \frac{(2a+b)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sin^2(e+fx)}\right.}{2abf} \\
&= \frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f} - \frac{\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2af}
\end{aligned}
$$

Mathematica [A] time = 0.169127, size = 71, normalized size = 0.95

$$
\frac{\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a}}{2f}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (((2*a + b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/a^(3/2) - (Csc[e +
 f*x]^2*Sqrt[a + b*Sin[e + f*x]^2])/a)/(2*f)
```

Maple [A] time = 1.213, size = 114, normalized size = 1.5

$$
\frac{1}{f}\ln\left(\frac{1}{\sin(fx+e)}\left(2a+2\sqrt{a}\sqrt{a+b(\sin(fx+e))^2}\right)\right)\frac{1}{\sqrt{a}} - \frac{1}{2af(\sin(fx+e))^2}\sqrt{a+b(\sin(fx+e))^2} + \frac{b}{2f}\ln\Bigg(
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e))/f-1/2/f/a
/sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)+1/2/f*b/a^(3/2)*ln((2*a+2*a^(1/2)*(a
+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.73345, size = 537, normalized size = 7.16

$$\left[\frac{\left((2a+b)\cos\left(fx+e\right)^2-2a-b\right)\sqrt{a}\log\left(\frac{2\left(b\cos\left(fx+e\right)^2-2\sqrt{-b\cos\left(fx+e\right)^2+a+b}\sqrt{a}-2a-b\right)}{\cos\left(fx+e\right)^2-1}\right)+2\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,a}{4\left(a^2f\cos\left(fx+e\right)^2-a^2f\right)}, \ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((2*a + b)*cos(f*x + e)^2 - 2*a - b)*sqrt(a)*log(2*(b*cos(f*x + e)^2
- 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) - 2*a - b)/(cos(f*x + e)^2 - 1)
) + 2*sqrt(-b*cos(f*x + e)^2 + a + b)*a)/(a^2*f*cos(f*x + e)^2 - a^2*f), -1
/2*(((2*a + b)*cos(f*x + e)^2 - 2*a - b)*sqrt(-a)*arctan(sqrt(-b*cos(f*x +
e)^2 + a + b)*sqrt(-a)/a) - sqrt(-b*cos(f*x + e)^2 + a + b)*a)/(a^2*f*cos(f
*x + e)^2 - a^2*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^3\left(e+fx\right)}{\sqrt{a+b\sin^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**3/sqrt(a + b*sin(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)^3}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^3/sqrt(b*sin(f*x + e)^2 + a), x)
```

3.515 $\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=126

$$-\frac{(8a^2+8ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{8a^{5/2}f}+\frac{(8a+3b)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8a^2f}-\frac{\csc^4(e+fx)\sqrt{a+b\sin^2(e+fx)}}{4af}$$

```
[Out] -((8*a^2 + 8*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/(8*a
^(5/2)*f) + ((8*a + 3*b)*Csc[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2])/(8*a^2*
f) - (Csc[e + f*x]^4*Sqrt[a + b*Sin[e + f*x]^2])/(4*a*f)
```

Rubi [A] time = 0.128517, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3194, 89, 78, 63, 208}

$$-\frac{(8a^2+8ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{8a^{5/2}f}+\frac{(8a+3b)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8a^2f}-\frac{\csc^4(e+fx)\sqrt{a+b\sin^2(e+fx)}}{4af}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((8*a^2 + 8*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/(8*a
^(5/2)*f) + ((8*a + 3*b)*Csc[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2])/(8*a^2*
f) - (Csc[e + f*x]^4*Sqrt[a + b*Sin[e + f*x]^2])/(4*a*f)
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
```

```
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x)^2}{x^3\sqrt{a+bx}}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= -\frac{\csc^4(e+fx)\sqrt{a+b\sin^2(e+fx)}}{4af} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(-8a-3b)+2ax}{x^2\sqrt{a+bx}}\,dx, x, \sin^2(e+fx)\right)}{4af}$$

$$= \frac{(8a+3b)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8a^2f} - \frac{\csc^4(e+fx)\sqrt{a+b\sin^2(e+fx)}}{4af} + \frac{(8a^2+8al}{}$$

$$= \frac{(8a+3b)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8a^2f} - \frac{\csc^4(e+fx)\sqrt{a+b\sin^2(e+fx)}}{4af} + \frac{(8a^2+8al}{}$$

$$= -\frac{(8a^2+8ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{8a^{5/2}f} + \frac{(8a+3b)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8a^2f} -$$

Mathematica [A] time = 0.36684, size = 101, normalized size = 0.8

$$\frac{\sqrt{a}\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\left(-2a\csc^2(e+fx)+8a+3b\right)-\left(8a^2+8ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{8a^{5/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-((8*a^2 + 8*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]]) + S
qrt[a]*Csc[e + f*x]^2*(8*a + 3*b - 2*a*Csc[e + f*x]^2)*Sqrt[a + b*Sin[e + f
*x]^2])/(8*a^(5/2)*f)
```

Maple [A] time = 1.662, size = 219, normalized size = 1.7

$$-\frac{1}{f}\ln\left(\frac{1}{\sin(fx+e)}\left(2a+2\sqrt{a}\sqrt{a+b(\sin(fx+e))^2}\right)\right)\frac{1}{\sqrt{a}}+\frac{1}{af(\sin(fx+e))^2}\sqrt{a+b(\sin(fx+e))^2}-\frac{b}{f}\ln\left(\frac{1}{\sin(f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e))/f+1/f/a/
sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)-1/f*b/a^(3/2)*ln((2*a+2*a^(1/2)*(a+b*
sin(f*x+e)^2)^(1/2))/sin(f*x+e))-1/4/f/a/sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1
/2)+3/8/f/a^2*b/sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)-3/8/f/a^(5/2)*b^2*ln(
(2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.08619, size = 923, normalized size = 7.33

$$\left[\frac{\left(\left(8\,a^2+8\,ab+3\,b^2\right)\cos\left(fx+e\right)^4-2\left(8\,a^2+8\,ab+3\,b^2\right)\cos\left(fx+e\right)^2+8\,a^2+8\,ab+3\,b^2\right)\sqrt{a}\log\left(\frac{2\left(b\cos\left(fx+e\right)^2+2\,\sqrt{\ldots}\right.}{\ldots}\right.}{16\left(a^3 f\cos\left(fx+e\right)^4-2\,a^3 f\cos\left(\ldots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(((8*a^2 + 8*a*b + 3*b^2)*cos(f*x + e)^4 - 2*(8*a^2 + 8*a*b + 3*b^2)*
cos(f*x + e)^2 + 8*a^2 + 8*a*b + 3*b^2)*sqrt(a)*log(2*(b*cos(f*x + e)^2 + 2
*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) - 2*a - b)/(cos(f*x + e)^2 - 1)) -
 2*((8*a^2 + 3*a*b)*cos(f*x + e)^2 - 6*a^2 - 3*a*b)*sqrt(-b*cos(f*x + e)^2
+ a + b))/(a^3*f*cos(f*x + e)^4 - 2*a^3*f*cos(f*x + e)^2 + a^3*f), 1/8*(((8
*a^2 + 8*a*b + 3*b^2)*cos(f*x + e)^4 - 2*(8*a^2 + 8*a*b + 3*b^2)*cos(f*x +
e)^2 + 8*a^2 + 8*a*b + 3*b^2)*sqrt(-a)*arctan(sqrt(-b*cos(f*x + e)^2 + a +
b)*sqrt(-a)/a) - ((8*a^2 + 3*a*b)*cos(f*x + e)^2 - 6*a^2 - 3*a*b)*sqrt(-b*c
os(f*x + e)^2 + a + b))/(a^3*f*cos(f*x + e)^4 - 2*a^3*f*cos(f*x + e)^2 + a^
3*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^5\left(e+fx\right)}{\sqrt{a+b\sin^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**5/sqrt(a + b*sin(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot \left(fx+e\right)^{5}}{\sqrt{b\sin \left(fx+e\right)^{2}+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^5/sqrt(b*sin(f*x + e)^2 + a), x)
```

3.516 $\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=246

$$-\frac{2(2a+b)\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f(a+b)^2}+\frac{\tan(e+fx)\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f(a+b)}-\frac{a\sqrt{\cos^2(e+fx)}\sec(e+\cdots}{3f\cdots}$$

```
[Out] (2*(2*a + b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*S
ec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*(a + b)^2*f*Sqrt[1 + (b*Sin[e +
f*x]^2)/a]) - (a*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a
)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*(a + b)*f*Sqrt[a + b*Sin
[e + f*x]^2]) - (2*(2*a + b)*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x])/(3*(a
 + b)^2*f) + (Sec[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x])/(3*(a
 + b)*f)
```

Rubi [A] time = 0.237528, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3196, 470, 527, 524, 426, 424, 421, 419}

$$-\frac{2(2a+b)\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f(a+b)^2}+\frac{\tan(e+fx)\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f(a+b)}-\frac{a\sqrt{\cos^2(e+fx)}\sec(e+\cdots}{3f\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/Sqrt[a + b*Sin[e + f*x]^2],x]
```


```
[Out] (2*(2*a + b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*S
ec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*(a + b)^2*f*Sqrt[1 + (b*Sin[e +
f*x]^2)/a]) - (a*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a
)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*(a + b)*f*Sqrt[a + b*Sin
[e + f*x]^2]) - (2*(2*a + b)*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x])/(3*(a
 + b)^2*f) + (Sec[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x])/(3*(a
 + b)*f)
```


Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
```

p, q, x]

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^4}{(1-x^2)^{5/2}\sqrt{a+bx^2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{3(a+b)f} - \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \ldots\right.}{3(a+b)f}$$

$$= -\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{3(a+b)^2f} + \frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\tan(e+\ldots}{3(a+b)f}$$

$$= -\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{3(a+b)^2f} + \frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\tan(e+\ldots}{3(a+b)f}$$

$$= -\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{3(a+b)^2f} + \frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\tan(e+\ldots}{3(a+b)f}$$

$$= \frac{2(2a+b)\sqrt{\cos^2(e+fx)}E\left(\sin^{-1}(\sin(e+fx))\middle| -\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3(a+b)^2f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} - \ldots$$

Mathematica [A] time = 2.18323, size = 188, normalized size = 0.76

$$\frac{-\frac{\tan(e+fx)\sec^2(e+fx)\left(2\left(4a^2+3ab+b^2\right)\cos(2(e+fx))+(2a+b)(2a-b\cos(4(e+fx))-b)\right)}{\sqrt{2}} - 2a(a+b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}F\left(e+fx\middle|-\frac{b}{a}\right) + 4a(2\ldots}{6f(a+b)^2\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (4*a*(2*a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(
b/a)] - 2*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*
x, -(b/a)] - ((2*(4*a^2 + 3*a*b + b^2)*Cos[2*(e + f*x)] + (2*a + b)*(2*a -
b - b*Cos[4*(e + f*x)]))*Sec[e + f*x]^2*Tan[e + f*x])/Sqrt[2])/(6*(a + b)^2
*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A] time = 2.59, size = 377, normalized size = 1.5

$$-\frac{1}{\left(-3+3\sin\left(fx+e\right)\right)\left(1+\sin\left(fx+e\right)\right)(a+b)^2\cos\left(fx+e\right)f}\left(2\sqrt{-b\left(\cos\left(fx+e\right)\right)^4+(a+b)\left(\cos\left(fx+e\right)\right)^2}b(2\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/3*(2*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*(2*a+b)*sin(f*x+e)*cos
(f*x+e)^4-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(4*a^2+7*a*b+3*b^2)*co
```

```
s(f*x+e)^2*sin(f*x+e)+(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a^2+2*a*b
+b^2)*sin(f*x+e)-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(cos(f*x+e)^2)^
(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*a*(EllipticF(sin(f*x+e),(-1/a*b)^(1
/2))*a+EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-4*EllipticE(sin(f*x+e),(-1/a*
b)^(1/2))*a-2*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*b)*cos(f*x+e)^2)/(-1+sin
(f*x+e))/(1+sin(f*x+e))/(a+b)^2/(-(a+b*sin(f*x+e)^2)*(-1+sin(f*x+e))*(1+sin
(f*x+e)))^(1/2)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\tan\left(fx+e\right)^{4}}{b\cos\left(fx+e\right)^{2}-a-b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)*tan(f*x + e)^4/(b*cos(f*x + e)^2
 - a - b), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^{4}\left(e+fx\right)}{\sqrt{a+b\sin^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**4/sqrt(a + b*sin(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^{4}}{\sqrt{b\sin\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^4/sqrt(b*sin(f*x + e)^2 + a), x)
```

3.517 $\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=109

$$\frac{\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f(a+b)}-\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}E\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{f(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

```
[Out] -((Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x
]*Sqrt[a + b*Sin[e + f*x]^2])/((a + b)*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a])) +
 (Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x])/((a + b)*f)
```

Rubi [A] time = 0.117775, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3196, 471, 426, 424}

$$\frac{\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f(a+b)}-\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}E\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{f(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x
]*Sqrt[a + b*Sin[e + f*x]^2])/((a + b)*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a])) +
 (Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x])/((a + b)*f)
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^2}{\left(1-x^2\right)^{3/2}\sqrt{a+bx^2}}\,dx,x,\sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{(a+b)f} - \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^2}}\,dx,x,\text{s}\right.}{(a+b)f}$$

$$= \frac{\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{(a+b)f} - \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\right)\text{Sul}}{(a+b)f\sqrt{1+\frac{b\sin^2(e+f}{a}}}$$

$$= -\frac{\sqrt{\cos^2(e+fx)}E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{(a+b)f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} + \frac{\sqrt{a+b\sin}}{}$$

Mathematica [A] time = 0.393447, size = 100, normalized size = 0.92

$$\frac{\sqrt{2}\tan(e+fx)(2a-b\cos(2(e+fx))+b)-2a\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)}{2f(a+b)\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-2*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] + S
qrt[2]*(2*a + b - b*Cos[2*(e + f*x)])*Tan[e + f*x])/(2*(a + b)*f*Sqrt[2*a +
 b - b*Cos[2*(e + f*x)]])
```

Maple [B] time = 2.065, size = 222, normalized size = 2.

$$\frac{1}{(a+b)\cos\left(fx+e\right)f}\left(-\sqrt{-b\left(\cos\left(fx+e\right)\right)^4+(a+b)\left(\cos\left(fx+e\right)\right)^2}b\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^2+\sqrt{-b\left(\cos\left(fx\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] (-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*sin(f*x+e)*cos(f*x+e)^2+(-b*
cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a+b)*sin(f*x+e)-a*(cos(f*x+e)^2)^(1
/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^
```

```
(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2)))/(a+b)/(-(a+b*sin(f*x+e)^2)*(-1+
sin(f*x+e))*(1+sin(f*x+e)))^(1/2)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^2}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^2/sqrt(b*sin(f*x + e)^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\tan\left(fx+e\right)^2}{b\cos\left(fx+e\right)^2-a-b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)*tan(f*x + e)^2/(b*cos(f*x + e)^2
- a - b), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^2\left(e+fx\right)}{\sqrt{a+b\sin^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**2/sqrt(a + b*sin(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^2}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^2/sqrt(b*sin(f*x + e)^2 + a), x)
```

3.518 $\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=51

$$\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] (EllipticF[e + f*x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(f*Sqrt[a + b*S
in[e + f*x]^2])
```

Rubi [A] time = 0.033892, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3183, 3182}

$$\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (EllipticF[e + f*x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(f*Sqrt[a + b*S
in[e + f*x]^2])
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx &= \frac{\sqrt{1+\frac{b\sin^2(e+fx)}{a}}\int \frac{1}{\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}\,dx}{\sqrt{a+b\sin^2(e+fx)}}\\
&= \frac{F\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{f\sqrt{a+b\sin^2(e+fx)}}
\end{aligned}$$

Mathematica [A] time = 0.0767491, size = 60, normalized size = 1.18

$$\frac{\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}F\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)])/(f*Sqrt
[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [C] time = 0.207, size = 60, normalized size = 1.2

$$\frac{1}{f}\sqrt{-\frac{b\left(\cos\left(fx+e\right)\right)^{2}-a-b}{a}}\mathrm{InverseJacobiAM}\left(fx+e,i\sqrt{b}\frac{1}{\sqrt{a}}\right)\frac{1}{\sqrt{a+b-b\left(\cos\left(fx+e\right)\right)^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/f/(a+b-b*cos(f*x+e)^2)^(1/2)*(-(b*cos(f*x+e)^2-a-b)/a)^(1/2)*InverseJacob
iAM(f*x+e,I/a^(1/2)*b^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sin\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sin(f*x + e)^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}}{b\cos\left(fx+e\right)^{2}-a-b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)/(b*cos(f*x + e)^2 - a - b), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+b\sin^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sin(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sin(f*x + e)^2 + a), x)
```

3.519 $\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=106

$$-\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{af}-\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}E\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{af\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

```
[Out] -((Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(a*f)) - (Sqrt[Cos[e + f*x]^2]*
EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x
]^2])/(a*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a])
```

Rubi [A] time = 0.115462, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3196, 475, 21, 426, 424}

$$-\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{af}-\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}E\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{af\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(a*f)) - (Sqrt[Cos[e + f*x]^2]*
EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x
]^2])/(a*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a])
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\mathrm{Subst}\left(\int\frac{\sqrt{1-x^2}}{x^2\sqrt{a+bx^2}}\,dx,x,\sin(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{af}+\frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\mathrm{Subst}\left(\int\frac{-a-bx^2}{\sqrt{1-x^2}\sqrt{a+bx^2}}\,d\right.}{af}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{af}-\frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\mathrm{Subst}\left(\int\frac{\sqrt{a+bx^2}}{\sqrt{1-x^2}}\,dx,x,\right.}{af}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{af}-\frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\right)\mathrm{Su}}{af\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{af}-\frac{\sqrt{\cos^2(e+fx)}E\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)\sec(e+f}{af\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

Mathematica [A] time = 0.371811, size = 101, normalized size = 0.95

$$-\frac{\cot(e+fx)\sqrt{2a-b\cos(2(e+fx))+b}}{\sqrt{2}af}-\frac{\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((Sqrt[2*a + b - b*Cos[2*(e + f*x)]]*Cot[e + f*x])/(Sqrt[2]*a*f)) - (Sqrt[
(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)])/(f*Sqrt[2*a +
 b - b*Cos[2*(e + f*x)]])
```

Maple [A] time = 1.09, size = 120, normalized size = 1.1

$$-\frac{1}{a\sin\left(fx+e\right)\cos\left(fx+e\right)f}\left(\sqrt{\left(\cos\left(fx+e\right)\right)^2}\sqrt{-\frac{b\left(\cos\left(fx+e\right)\right)^2}{a}+\frac{a+b}{a}}a\sin\left(fx+e\right)\mathrm{EllipticE}\left(\sin\left(fx+e\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] -((cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*a*sin(f*x+e)*Ellip
ticE(sin(f*x+e),(-1/a*b)^(1/2))-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)/a/sin(f*
x+e)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)^{2}}{\sqrt{b\sin\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(f*x + e)^2/sqrt(b*sin(f*x + e)^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\cot\left(fx+e\right)^{2}}{b\cos\left(fx+e\right)^{2}-a-b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)*cot(f*x + e)^2/(b*cos(f*x + e)^2
- a - b), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^{2}\left(e+fx\right)}{\sqrt{a+b\sin^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**2/sqrt(a + b*sin(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)^{2}}{\sqrt{b\sin\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^2/sqrt(b*sin(f*x + e)^2 + a), x)
```

3.520 $\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=240

$$\frac{2(2a+b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^2f}+\frac{2(2a+b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}E\left(\sin^{-1}(\sin(e+fx)\right.}{3a^2f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

```
[Out] (2*(2*a + b)*Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a^2*f) - (Cot[e +
f*x]*Csc[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2])/(3*a*f) + (2*(2*a + b)*Sqrt
[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[
a + b*Sin[e + f*x]^2])/(3*a^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) - ((a + b)*
Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*S
qrt[1 + (b*Sin[e + f*x]^2)/a])/(3*a*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.267876, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3196, 474, 583, 524, 426, 424, 421, 419}

$$\frac{2(2a+b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^2f}+\frac{2(2a+b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}E\left(\sin^{-1}(\sin(e+fx)\right.}{3a^2f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/Sqrt[a + b*Sin[e + f*x]^2],x]
```


```
[Out] (2*(2*a + b)*Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a^2*f) - (Cot[e +
f*x]*Csc[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2])/(3*a*f) + (2*(2*a + b)*Sqrt
[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[
a + b*Sin[e + f*x]^2])/(3*a^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) - ((a + b)*
Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*S
qrt[1 + (b*Sin[e + f*x]^2)/a])/(3*a*f*Sqrt[a + b*Sin[e + f*x]^2])
```


Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{\left(1-x^2\right)^{3/2}}{x^4\sqrt{a+bx^2}}\,dx,x,\sin(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af} + \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{-2(}{x^2}\right.}{3af}$$

$$= \frac{2(2a+b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^2f} - \frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af} -$$

$$= \frac{2(2a+b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^2f} - \frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af} -$$

$$= \frac{2(2a+b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^2f} - \frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af} +$$

$$= \frac{2(2a+b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^2f} - \frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af} +$$

Mathematica [A] time = 4.107, size = 186, normalized size = 0.78

$$\frac{\frac{\cot(e+fx)\csc^2(e+fx)\left((2a+b)(2a+b\cos(4(e+fx))+3b)-2\left(4a^2+5ab+2b^2\right)\cos(2(e+fx))\right)}{\sqrt{2}} - 2a(a+b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}F\left(e+fx\middle|-\frac{b}{a}\right)+4a(2a}{6a^2f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (((-2*(4*a^2 + 5*a*b + 2*b^2)*Cos[2*(e + f*x)] + (2*a + b)*(2*a + 3*b + b*C
os[4*(e + f*x)]))*Cot[e + f*x]*Csc[e + f*x]^2)/Sqrt[2] + 4*a*(2*a + b)*Sqrt
[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] - 2*a*(a + b)
*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)])/(6*a^2*
f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A] time = 1.243, size = 351, normalized size = 1.5

$$-\frac{1}{3a^2\left(\sin\left(fx+e\right)\right)^3\cos\left(fx+e\right)f}\left(\text{EllipticF}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)\sqrt{\left(\cos\left(fx+e\right)\right)^2}\sqrt{\frac{a+b\left(\sin\left(fx+e\right)\right)^2}{a}}a^2\left(\sin\left(f\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/3*(EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f
*x+e)^2)/a)^(1/2)*a^2*sin(f*x+e)^3+b*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^
2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*sin(f*x+e)^3-4*EllipticE
```

```
(sin(f*x+e),(-1/a*b)^(1/2))*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/
2)*a^2*sin(f*x+e)^3-2*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*(cos(f*x+e)^2)^(
1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*a*b*sin(f*x+e)^3+4*a*b*sin(f*x+e)^6+2*b^2
*sin(f*x+e)^6+4*a^2*sin(f*x+e)^4-3*a*b*sin(f*x+e)^4-2*b^2*sin(f*x+e)^4-5*a^
2*sin(f*x+e)^2-a*b*sin(f*x+e)^2+a^2)/a^2/sin(f*x+e)^3/cos(f*x+e)/(a+b*sin(f
*x+e)^2)^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\cot\left(fx+e\right)^4}{b\cos\left(fx+e\right)^2-a-b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)*cot(f*x + e)^4/(b*cos(f*x + e)^2
- a - b), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cot^4\left(e+fx\right)}{\sqrt{a+b\sin^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**4/sqrt(a + b*sin(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cot\left(fx+e\right)^4}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^4/sqrt(b*sin(f*x + e)^2 + a), x)
```

3.521 $\int \frac{\tan^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=177

$$-\frac{8a^2-8ab-b^2}{8f(a+b)^3\sqrt{a+b\sin^2(e+fx)}}+\frac{\left(8a^2-8ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{8f(a+b)^{7/2}}+\frac{\sec^4(e+fx)}{4f(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{(8}{8f(a}$$

```
[Out] ((8*a^2 - 8*a*b - b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/(8*
(a + b)^(7/2)*f) - (8*a^2 - 8*a*b - b^2)/(8*(a + b)^3*f*Sqrt[a + b*Sin[e +
f*x]^2]) - ((8*a + 3*b)*Sec[e + f*x]^2)/(8*(a + b)^2*f*Sqrt[a + b*Sin[e + f
*x]^2]) + Sec[e + f*x]^4/(4*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.229394, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3194, 89, 78, 51, 63, 208}

$$-\frac{8a^2-8ab-b^2}{8f(a+b)^3\sqrt{a+b\sin^2(e+fx)}}+\frac{\left(8a^2-8ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{8f(a+b)^{7/2}}+\frac{\sec^4(e+fx)}{4f(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{(8}{8f(a}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] ((8*a^2 - 8*a*b - b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/(8*
(a + b)^(7/2)*f) - (8*a^2 - 8*a*b - b^2)/(8*(a + b)^3*f*Sqrt[a + b*Sin[e +
f*x]^2]) - ((8*a + 3*b)*Sec[e + f*x]^2)/(8*(a + b)^2*f*Sqrt[a + b*Sin[e + f
*x]^2]) + Sec[e + f*x]^4/(4*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
```

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\tan^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1-x)^3(a+bx)^{3/2}}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= \frac{\sec^4(e+fx)}{4(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(4a-b)+2(a+b)x}{(1-x)^2(a+bx)^{3/2}}\,dx, x, \sin^2(e+fx)\right)}{4(a+b)f}$$

$$= -\frac{(8a+3b)\sec^2(e+fx)}{8(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\sec^4(e+fx)}{4(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\left(8a^2-8ab-b^2\right)\text{Sub}}{}$$

$$= -\frac{8a^2-8ab-b^2}{8(a+b)^3f\sqrt{a+b\sin^2(e+fx)}} - \frac{(8a+3b)\sec^2(e+fx)}{8(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\sec^4(e+fx}{4(a+b)f\sqrt{a+b\sin}}$$

$$= -\frac{8a^2-8ab-b^2}{8(a+b)^3f\sqrt{a+b\sin^2(e+fx)}} - \frac{(8a+3b)\sec^2(e+fx)}{8(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\sec^4(e+fx}{4(a+b)f\sqrt{a+b\sin}}$$

$$= \frac{\left(8a^2-8ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{8(a+b)^{7/2}f} - \frac{8a^2-8ab-b^2}{8(a+b)^3f\sqrt{a+b\sin^2(e+fx)}} - \frac{(8a+3}{8(a+b)^2f\sqrt{}}$$

Mathematica [C] time = 0.486389, size = 107, normalized size = 0.6

$$\frac{\left(-8a^2+8ab+b^2\right){}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\sin^2(e+fx)+a}{a+b}\right)-\frac{1}{2}(a+b)\sec^4(e+fx)((8a+3b)\cos(2(e+fx))+4a-b)}{8f(a+b)^3\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] ((-8*a^2 + 8*a*b + b^2)*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Sin[e + f*x]
^2)/(a + b)] - ((a + b)*(4*a - b + (8*a + 3*b)*Cos[2*(e + f*x)])*Sec[e + f*
x]^4)/2)/(8*(a + b)^3*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Maple [B] time = 12.073, size = 3763, normalized size = 21.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/16/(a^4*b^2*cos(f*x+e)^4+4*a^3*b^3*cos(f*x+e)^4+6*a^2*b^4*cos(f*x+e)^4+4
*a*b^5*cos(f*x+e)^4+b^6*cos(f*x+e)^4-2*a^5*b*cos(f*x+e)^2-10*a^4*b^2*cos(f*
x+e)^2-20*a^3*b^3*cos(f*x+e)^2-20*a^2*b^4*cos(f*x+e)^2-10*a*b^5*cos(f*x+e)^
2-2*b^6*cos(f*x+e)^2+a^6+6*a^5*b+15*a^4*b^2+20*a^3*b^3+15*a^2*b^4+6*a*b^5+b
^6)/cos(f*x+e)^4/(a+b)^(3/2)*(20*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*co
s(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^3*b^3*cos(f*x+e)^4+30*ln(2/(1+sin(f*x+
e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^2*b^4*cos(f*
x+e)^4+12*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin
(f*x+e)+a))*a*b^5*cos(f*x+e)^4-8*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*c
os(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^4*b^2*cos(f*x+e)^8-8*ln(2/(-1+sin(f*x
+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^3*b^3*cos(f
*x+e)^8+9*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*si
n(f*x+e)+a))*a^2*b^4*cos(f*x+e)^8+10*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b
-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a*b^5*cos(f*x+e)^8-8*ln(2/(1+sin(f*
x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^4*b^2*cos(
f*x+e)^8-8*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*si
n(f*x+e)+a))*a^3*b^3*cos(f*x+e)^8+9*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b
*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^2*b^4*cos(f*x+e)^8+10*ln(2/(1+sin(f
*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a*b^5*cos(f
*x+e)^8+16*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*s
in(f*x+e)+a))*a^5*b*cos(f*x+e)^6+32*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-
b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^4*b^2*cos(f*x+e)^6-2*ln(2/(-1+sin(
f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^3*b^3*co
s(f*x+e)^6-38*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+
b*sin(f*x+e)+a))*a^2*b^4*cos(f*x+e)^6-22*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*
(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a*b^5*cos(f*x+e)^6+16*ln(2/(1+s
in(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^5*b*c
os(f*x+e)^6+16*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(3/2)*a^4*cos(f*x+e)^4-2*(a
+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(3/2)*b^4*cos(f*x+e)^4-8*(a+b)^(3/2)*(-b*cos
(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*a^4*cos(f*x+e)^4+16*(a+b-b*cos(f*x+e)^2)^(
3/2)*(a+b)^(3/2)*a^3*cos(f*x+e)^2-32*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(3/2)
*a^3*b*cos(f*x+e)^6+36*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(3/2)*a*b^3*cos(f*x
+e)^6+16*(a+b)^(3/2)*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*a^3*b*cos(f*x+
e)^6-64*(a+b)^(3/2)*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*a^2*b^2*cos(f*x
```

```
+e)^6-80*(a+b)^(3/2)*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*a*b^3*cos(f*x+
e)^6-32*(a+b-b*cos(f*x+e)^2)^(3/2)*(a+b)^(3/2)*a^2*b*cos(f*x+e)^4-32*(a+b-b
*cos(f*x+e)^2)^(3/2)*(a+b)^(3/2)*a*b^2*cos(f*x+e)^4-4*(a+b-b*cos(f*x+e)^2)^
(3/2)*(a+b)^(3/2)*a^3-4*(a+b-b*cos(f*x+e)^2)^(3/2)*(a+b)^(3/2)*b^3+ln(2/(-1
+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*b^6*c
os(f*x+e)^8+ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*s
in(f*x+e)+a))*b^6*cos(f*x+e)^8-2*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*c
os(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*b^6*cos(f*x+e)^6-2*ln(2/(1+sin(f*x+e))*
((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*b^6*cos(f*x+e)^6-8
*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+
a))*a^6*cos(f*x+e)^4+ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)
^(1/2)+b*sin(f*x+e)+a))*b^6*cos(f*x+e)^4-8*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)
*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^6*cos(f*x+e)^4+ln(2/(1+sin(f
*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*b^6*cos(f*x
+e)^4+6*(a+b-b*cos(f*x+e)^2)^(3/2)*(a+b)^(3/2)*b^3*cos(f*x+e)^2-12*(a+b-b*c
os(f*x+e)^2)^(3/2)*(a+b)^(3/2)*a^2*b-12*(a+b-b*cos(f*x+e)^2)^(3/2)*(a+b)^(3
/2)*a*b^2-2*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(3/2)*b^4*cos(f*x+e)^8-2*(a+b-
b*cos(f*x+e)^2)^(3/2)*(a+b)^(3/2)*b^3*cos(f*x+e)^6+4*(a+b-b*cos(f*x+e)^2)^(
1/2)*(a+b)^(3/2)*b^4*cos(f*x+e)^6+8*(a+b)^(3/2)*(-b*cos(f*x+e)^2+(a*b^2+b^3
)/b^2)^(3/2)*a^3*cos(f*x+e)^4+32*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*co
s(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^4*b^2*cos(f*x+e)^6-2*ln(2/(1+sin(f*x+e
))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^3*b^3*cos(f*x
+e)^6-38*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(
f*x+e)+a))*a^2*b^4*cos(f*x+e)^6-22*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*
cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a*b^5*cos(f*x+e)^6-24*ln(2/(-1+sin(f*x
+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^5*b*cos(f*x
+e)^4-15*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin
(f*x+e)+a))*a^4*b^2*cos(f*x+e)^4+20*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-
b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^3*b^3*cos(f*x+e)^4+30*ln(2/(-1+sin
(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^2*b^4*c
os(f*x+e)^4+12*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)
+b*sin(f*x+e)+a))*a*b^5*cos(f*x+e)^4-24*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a
+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^5*b*cos(f*x+e)^4-15*ln(2/(1+sin
(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^4*b^2*c
os(f*x+e)^4-8*(a+b)^(3/2)*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(3/2)*a*b^2*cos
(f*x+e)^4+16*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(3/2)*a^3*b*cos(f*x+e)^4-18*(
a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(3/2)*a^2*b^2*cos(f*x+e)^4-20*(a+b-b*cos(f*
x+e)^2)^(1/2)*(a+b)^(3/2)*a*b^3*cos(f*x+e)^4+24*(a+b)^(3/2)*(-b*cos(f*x+e)^
2+(a*b^2+b^3)/b^2)^(1/2)*a^3*b*cos(f*x+e)^4+72*(a+b)^(3/2)*(-b*cos(f*x+e)^2
+(a*b^2+b^3)/b^2)^(1/2)*a^2*b^2*cos(f*x+e)^4+40*(a+b)^(3/2)*(-b*cos(f*x+e)^
2+(a*b^2+b^3)/b^2)^(1/2)*a*b^3*cos(f*x+e)^4+38*(a+b-b*cos(f*x+e)^2)^(3/2)*(
a+b)^(3/2)*a^2*b*cos(f*x+e)^2+28*(a+b-b*cos(f*x+e)^2)^(3/2)*(a+b)^(3/2)*a*b
^2*cos(f*x+e)^2+16*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(3/2)*a^2*b^2*cos(f*x+e
)^8-16*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(3/2)*a*b^3*cos(f*x+e)^8-8*(a+b)^(3
/2)*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*a^2*b^2*cos(f*x+e)^8+40*(a+b)^(
3/2)*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*a*b^3*cos(f*x+e)^8+16*(a+b-b*c
os(f*x+e)^2)^(3/2)*(a+b)^(3/2)*a*b^2*cos(f*x+e)^6+8*(a+b)^(3/2)*(-b*cos(f*x
+e)^2+(a*b^2+b^3)/b^2)^(3/2)*a^2*b*cos(f*x+e)^6+8*(a+b)^(3/2)*(-b*cos(f*x+e
)^2+(a*b^2+b^3)/b^2)^(3/2)*a*b^2*cos(f*x+e)^6)/f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 4.16715, size = 1345, normalized size = 7.6

$$\left[-\frac{\left(\left(8\,a^2b-8\,ab^2-b^3\right)\cos\left(fx+e\right)^6-\left(8\,a^3-9\,ab^2-b^3\right)\cos\left(fx+e\right)^4\right)\sqrt{a+b}\log\left(\frac{b\cos(fx+e)^2+2\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{a+}}{\cos(fx+e)^2}\right.}{16\left(\left(a^4b+4\,a^3b^2+6\,a^2b^3+4\,ab^4+b^5\right)f\,c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(((8*a^2*b - 8*a*b^2 - b^3)*cos(f*x + e)^6 - (8*a^3 - 9*a*b^2 - b^3)
*cos(f*x + e)^4)*sqrt(a + b)*log((b*cos(f*x + e)^2 + 2*sqrt(-b*cos(f*x + e)
^2 + a + b)*sqrt(a + b) - 2*a - 2*b)/cos(f*x + e)^2) - 2*((8*a^3 - 9*a*b^2
- b^3)*cos(f*x + e)^4 - 2*a^3 - 6*a^2*b - 6*a*b^2 - 2*b^3 + (8*a^3 + 19*a^2
*b + 14*a*b^2 + 3*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b))/((a
^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*f*cos(f*x + e)^6 - (a^5 + 5*a
^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*f*cos(f*x + e)^4), -1/8*(((
8*a^2*b - 8*a*b^2 - b^3)*cos(f*x + e)^6 - (8*a^3 - 9*a*b^2 - b^3)*cos(f*x +
 e)^4)*sqrt(-a - b)*arctan(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(a
+ b)) - ((8*a^3 - 9*a*b^2 - b^3)*cos(f*x + e)^4 - 2*a^3 - 6*a^2*b - 6*a*b^2
 - 2*b^3 + (8*a^3 + 19*a^2*b + 14*a*b^2 + 3*b^3)*cos(f*x + e)^2)*sqrt(-b*co
s(f*x + e)^2 + a + b))/((a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*f*c
os(f*x + e)^6 - (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*f
*cos(f*x + e)^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^5\left(e+fx\right)}{\left(a+b\sin^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**5/(a + b*sin(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^5}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^5/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.522 $\int \frac{\tan^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=118

$$\frac{2a-b}{2f(a+b)^2\sqrt{a+b\sin^2(e+fx)}}-\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{2f(a+b)^{5/2}}+\frac{\sec^2(e+fx)}{2f(a+b)\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] -((2*a - b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/(2*(a + b)^(5/
2)*f) + (2*a - b)/(2*(a + b)^2*f*Sqrt[a + b*Sin[e + f*x]^2]) + Sec[e + f*x]
^2/(2*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.122448, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3194, 78, 51, 63, 208}

$$\frac{2a-b}{2f(a+b)^2\sqrt{a+b\sin^2(e+fx)}}-\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{2f(a+b)^{5/2}}+\frac{\sec^2(e+fx)}{2f(a+b)\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -((2*a - b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/(2*(a + b)^(5/
2)*f) + (2*a - b)/(2*(a + b)^2*f*Sqrt[a + b*Sin[e + f*x]^2]) + Sec[e + f*x]
^2/(2*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
```

```
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\tan^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(1-x)^2(a+bx)^{3/2}}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= \frac{\sec^2(e+fx)}{2(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{(2a-b)\,\text{Subst}\left(\int \frac{1}{(1-x)(a+bx)^{3/2}}\,dx, x, \sin^2(e+fx)\right)}{4(a+b)f}$$

$$= \frac{2a-b}{2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\sec^2(e+fx)}{2(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{(2a-b)\,\text{Subst}\left(\int \frac{}{(1-x)}\right.}{4(a}$$

$$= \frac{2a-b}{2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\sec^2(e+fx)}{2(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{(2a-b)\,\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-}\right.}{2b}$$

$$= -\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{2(a+b)^{5/2}f} + \frac{2a-b}{2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\sec^2(e+f}{2(a+b)f\sqrt{a+b\sin}}$$

Mathematica [C] time = 0.1166, size = 75, normalized size = 0.64

$$\frac{(2a-b)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\sin^2(e+fx)+a}{a+b}\right)+(a+b)\sec^2(e+fx)}{2f(a+b)^2\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] ((2*a - b)*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Sin[e + f*x]^2)/(a + b)]
+ (a + b)*Sec[e + f*x]^2)/(2*(a + b)^2*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Maple [B] time = 9.714, size = 2199, normalized size = 18.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/4/(a^3*b^2*cos(f*x+e)^4+3*a^2*b^3*cos(f*x+e)^4+3*a*b^4*cos(f*x+e)^4+b^5*c
os(f*x+e)^4-2*a^4*b*cos(f*x+e)^2-8*a^3*b^2*cos(f*x+e)^2-12*a^2*b^3*cos(f*x+
e)^2-8*a*b^4*cos(f*x+e)^2-2*b^5*cos(f*x+e)^2+a^5+5*a^4*b+10*a^3*b^2+10*a^2*
b^3+5*a*b^4+b^5)/cos(f*x+e)^2/(a+b)^(1/2)*(2*b^3/(a+b)^(1/2)*(a+b-b*cos(f*x
+e)^2)^(3/2)+2/(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(3/2)*a^3+6*b/(a+b)^(1/2)*(
a+b-b*cos(f*x+e)^2)^(3/2)*a^2+6*b^2/(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(3/2)*
a-cos(f*x+e)^6*(2*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/
2)-b*sin(f*x+e)+a))*a^2+ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^
2)^(1/2)-b*sin(f*x+e)+a))*a*b-ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f
*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*b^2-4*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2
)*a+2*b*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(1/2)+2*(a+b)^(1/2)*(-b*cos(f*x+e)
^2+(a*b^2+b^3)/b^2)^(1/2)*a-6*(a+b)^(1/2)*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)
^(1/2)*b+2*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*s
in(f*x+e)+a))*a^2+ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1
/2)+b*sin(f*x+e)+a))*a*b-ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e
)^2)^(1/2)+b*sin(f*x+e)+a))*b^2)*b^2+2*cos(f*x+e)^4*((a+b-b*cos(f*x+e)^2)^(
3/2)*(a+b)^(1/2)*b+(a+b)^(1/2)*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(3/2)*a+(a
+b)^(1/2)*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(3/2)*b+2*ln(2/(1+sin(f*x+e))*(
(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^3+3*a^2*b*ln(2/(1
+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))-b^3*l
n(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))
-4*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(1/2)*a^2-2*(a+b-b*cos(f*x+e)^2)^(1/2)*
(a+b)^(1/2)*a*b+2*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(1/2)*b^2+2*(a+b)^(1/2)*
(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*a^2-4*(a+b)^(1/2)*(-b*cos(f*x+e)^2+
(a*b^2+b^3)/b^2)^(1/2)*a*b-6*(a+b)^(1/2)*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^
(1/2)*b^2+2*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*
sin(f*x+e)+a))*a^3+3*a^2*b*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x
+e)^2)^(1/2)+b*sin(f*x+e)+a))-b^3*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*
cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a)))*b-cos(f*x+e)^2*(4*(a+b-b*cos(f*x+e)^2
)^(3/2)*(a+b)^(1/2)*a*b+4*(a+b-b*cos(f*x+e)^2)^(3/2)*(a+b)^(1/2)*b^2-2*(a+b
)^(1/2)*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(3/2)*a^2+2*(a+b)^(1/2)*(-b*cos(f
*x+e)^2+(a*b^2+b^3)/b^2)^(3/2)*b^2+2*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-
b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^4+5*ln(2/(1+sin(f*x+e))*((a+b)^(1/
2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^3*b+3*ln(2/(1+sin(f*x+e))*
((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^2*b^2-ln(2/(1+si
n(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a*b^3-ln
(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*
b^4-4*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(1/2)*a^3-6*(a+b-b*cos(f*x+e)^2)^(1/
2)*(a+b)^(1/2)*a^2*b+2*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(1/2)*b^3+2*(a+b)^(
1/2)*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*a^3-2*(a+b)^(1/2)*(-b*cos(f*x+
e)^2+(a*b^2+b^3)/b^2)^(1/2)*a^2*b-10*(a+b)^(1/2)*(-b*cos(f*x+e)^2+(a*b^2+b^
3)/b^2)^(1/2)*a*b^2-6*(a+b)^(1/2)*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*b
^3+2*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x
+e)+a))*a^4+5*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+
b*sin(f*x+e)+a))*a^3*b+3*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e
)^2)^(1/2)+b*sin(f*x+e)+a))*a^2*b^2-ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-
b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a*b^3-ln(2/(-1+sin(f*x+e))*((a+b)^(1
/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*b^4))/f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.27331, size = 1026, normalized size = 8.69

$$\left[-\frac{\left(\left(2\,ab-b^{2}\right)\cos\left(fx+e\right)^{4}-\left(2\,a^{2}+ab-b^{2}\right)\cos\left(fx+e\right)^{2}\right)\sqrt{a+b}\log\left(\frac{b\cos\left(fx+e\right)^{2}-2\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\sqrt{a+b}-2\,a-2\,b}{\cos\left(fx+e\right)^{2}}\right)+2\left(\cdots\right.}{4\left(\left(a^{3}b+3\,a^{2}b^{2}+3\,ab^{3}+b^{4}\right)f\cos\left(fx+e\right)^{4}-\left(a^{4}+4\,a^{3}b+6\,a^{2}b^{2}+4\cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(((2*a*b - b^2)*cos(f*x + e)^4 - (2*a^2 + a*b - b^2)*cos(f*x + e)^2)*
sqrt(a + b)*log((b*cos(f*x + e)^2 - 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(
a + b) - 2*a - 2*b)/cos(f*x + e)^2) + 2*((2*a^2 + a*b - b^2)*cos(f*x + e)^2
 + a^2 + 2*a*b + b^2)*sqrt(-b*cos(f*x + e)^2 + a + b))/((a^3*b + 3*a^2*b^2
+ 3*a*b^3 + b^4)*f*cos(f*x + e)^4 - (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 +
b^4)*f*cos(f*x + e)^2), 1/2*(((2*a*b - b^2)*cos(f*x + e)^4 - (2*a^2 + a*b -
 b^2)*cos(f*x + e)^2)*sqrt(-a - b)*arctan(sqrt(-b*cos(f*x + e)^2 + a + b)*s
qrt(-a - b)/(a + b)) - ((2*a^2 + a*b - b^2)*cos(f*x + e)^2 + a^2 + 2*a*b +
b^2)*sqrt(-b*cos(f*x + e)^2 + a + b))/((a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*
f*cos(f*x + e)^4 - (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*f*cos(f*x +
e)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^{3}\left(e+fx\right)}{\left(a+b\sin^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**3/(a + b*sin(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^{3}}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^3/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.523 $\int \frac{\tan(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=63

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{f(a+b)^{3/2}}-\frac{1}{f(a+b)\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]]/((a + b)^(3/2)*f) - 1/((a +
 b)*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.0677903, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3194, 51, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{f(a+b)^{3/2}}-\frac{1}{f(a+b)\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]]/((a + b)^(3/2)*f) - 1/((a +
 b)*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\tan(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x)(a+bx)^{3/2}}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= -\frac{1}{(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,dx, x, \sin^2(e+fx)\right)}{2(a+b)f}$$

$$= -\frac{1}{(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sin^2(e+fx)}\right)}{b(a+b)f}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}f} - \frac{1}{(a+b)f\sqrt{a+b\sin^2(e+fx)}}$$

Mathematica [C] time = 0.0706664, size = 54, normalized size = 0.86

$$-\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1-\frac{b\cos^2(e+fx)}{a+b}\right)}{f(a+b)\sqrt{a-b\cos^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Hypergeometric2F1[-1/2, 1, 1/2, 1 - (b*Cos[e + f*x]^2)/(a + b)]/((a + b)*
f*Sqrt[a + b - b*Cos[e + f*x]^2]))
```

Maple [B] time = 8.316, size = 1317, normalized size = 20.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/2/a/(a^2*b^2*cos(f*x+e)^4+2*a*b^3*cos(f*x+e)^4+b^4*cos(f*x+e)^4-2*a^3*b*c
os(f*x+e)^2-6*a^2*b^2*cos(f*x+e)^2-6*a*b^3*cos(f*x+e)^2-2*b^4*cos(f*x+e)^2+
a^4+4*a^3*b+6*a^2*b^2+4*a*b^3+b^4)*(-(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2
)*b^3+(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(3/2)*b^2-2*(a+b-b*cos(f*x+e)^2)^(1
/2)*a^3-(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(3/2)*a^2+a^2*b*(-b*cos(f*x+e)^2+
(a*b^2+b^3)/b^2)^(1/2)+(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*a^3-2*(a+b-b
*cos(f*x+e)^2)^(1/2)*a*b^2-4*a^2*b*(a+b-b*cos(f*x+e)^2)^(1/2)-a*b^2*(-b*cos
(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)+ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*c
os(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*(a+b)^(1/2)*a*b^2+ln(2/(-1+sin(f*x+e))*
((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*(a+b)^(1/2)*a^3+ln
(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*
```

```
(a+b)^(1/2)*a^3+2*a^2*b*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^
2)^(1/2)-b*sin(f*x+e)+a))*(a+b)^(1/2)+2*a^2*b*ln(2/(-1+sin(f*x+e))*((a+b)^(
1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*(a+b)^(1/2)+ln(2/(1+sin(f*
x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*(a+b)^(1/2)*
a*b^2+b^2*(ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*si
n(f*x+e)+a))*(a+b)^(1/2)*a+ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x
+e)^2)^(1/2)+b*sin(f*x+e)+a))*(a+b)^(1/2)*a-2*(a+b-b*cos(f*x+e)^2)^(1/2)*a+
(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*a-(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)
^(1/2)*b)*cos(f*x+e)^4-b*(2*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x
+e)^2)^(1/2)-b*sin(f*x+e)+a))*(a+b)^(1/2)*a^2+2*ln(2/(1+sin(f*x+e))*((a+b)^
(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*(a+b)^(1/2)*a*b+2*ln(2/(-
1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*(a+b
)^(1/2)*a^2+2*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+
b*sin(f*x+e)+a))*(a+b)^(1/2)*a*b+(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(3/2)*a+
(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(3/2)*b-4*(a+b-b*cos(f*x+e)^2)^(1/2)*a^2-
4*(a+b-b*cos(f*x+e)^2)^(1/2)*a*b+2*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*
a^2-2*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*b^2)*cos(f*x+e)^2)/f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.39673, size = 672, normalized size = 10.67

$$\left[\frac{\left(b\cos\left(fx+e\right)^2-a-b\right)\sqrt{a+b}\log\left(\frac{b\cos\left(fx+e\right)^2-2\sqrt{-b\cos\left(fx+e\right)^2+a+b}\sqrt{a+b}-2a-2b}{\cos\left(fx+e\right)^2}\right)+2\sqrt{-b\cos\left(fx+e\right)^2+a+b}(a+b)}{2\left(\left(a^2b+2ab^2+b^3\right)f\cos\left(fx+e\right)^2-\left(a^3+3a^2b+3ab^2+b^3\right)f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((b*cos(f*x + e)^2 - a - b)*sqrt(a + b)*log((b*cos(f*x + e)^2 - 2*sqrt
(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b) - 2*a - 2*b)/cos(f*x + e)^2) + 2*sq
rt(-b*cos(f*x + e)^2 + a + b)*(a + b))/((a^2*b + 2*a*b^2 + b^3)*f*cos(f*x +
 e)^2 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f), -((b*cos(f*x + e)^2 - a - b)*sq
rt(-a - b)*arctan(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(a + b)) - s
qrt(-b*cos(f*x + e)^2 + a + b)*(a + b))/((a^2*b + 2*a*b^2 + b^3)*f*cos(f*x
+ e)^2 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\tan\left(e+fx\right)}{\left(a+b\sin^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)/(a + b*sin(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.524 $\int \frac{\cot(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=57

$$\frac{1}{af\sqrt{a+b\sin^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}$$

```
[Out] -(ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f)) + 1/(a*f*Sqrt[a
+ b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.0756698, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3194, 51, 63, 208}

$$\frac{1}{af\sqrt{a+b\sin^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f)) + 1/(a*f*Sqrt[a
+ b*Sin[e + f*x]^2])
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cot(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= \frac{1}{af\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sin^2(e+fx)\right)}{2af}$$

$$= \frac{1}{af\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sin^2(e+fx)}\right)}{abf}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} + \frac{1}{af\sqrt{a+b\sin^2(e+fx)}}$$

Mathematica [C] time = 0.0567884, size = 46, normalized size = 0.81

$$\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\sin^2(e+fx)}{a}+1\right)}{af\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Sin[e + f*x]^2)/a]/(a*f*Sqrt[a + b*S
in[e + f*x]^2])
```

Maple [A] time = 1.235, size = 64, normalized size = 1.1

$$\frac{1}{af}\frac{1}{\sqrt{a+b\left(\sin\left(fx+e\right)\right)^2}} - \frac{1}{f}\ln\left(\frac{1}{\sin\left(fx+e\right)}\left(2a+2\sqrt{a}\sqrt{a+b\left(\sin\left(fx+e\right)\right)^2}\right)\right)a^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/a/f/(a+b*sin(f*x+e)^2)^(1/2)-1/f/a^(3/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e
)^2)^(1/2))/sin(f*x+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 2.54335, size = 536, normalized size = 9.4

$$\left[\frac{\left(b\cos\left(fx+e\right)^2-a-b\right)\sqrt{a}\log\left(\frac{2\left(b\cos\left(fx+e\right)^2+2\sqrt{-b\cos\left(fx+e\right)^2+a+b}\sqrt{a}-2a-b\right)}{\cos\left(fx+e\right)^2-1}\right)-2\sqrt{-b\cos\left(fx+e\right)^2+a+b}a}{2\left(a^2bf\cos\left(fx+e\right)^2-\left(a^3+a^2b\right)f\right)}, \frac{\left(b\cos\left(f\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] [1/2*((b*cos(f*x + e)^2 - a - b)*sqrt(a)*log(2*(b*cos(f*x + e)^2 + 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) - 2*a - b)/(cos(f*x + e)^2 - 1)) - 2*sqrt(-b*cos(f*x + e)^2 + a + b)*a)/(a^2*b*f*cos(f*x + e)^2 - (a^3 + a^2*b)*f), ((b*cos(f*x + e)^2 - a - b)*sqrt(-a)*arctan(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a)/a) - sqrt(-b*cos(f*x + e)^2 + a + b)*a)/(a^2*b*f*cos(f*x + e)^2 - (a^3 + a^2*b)*f)]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(e+fx\right)}{\left(a+b\sin^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)/(a+b*sin(f*x+e)**2)**(3/2),x)

[Out] Integral(cot(e + f*x)/(a + b*sin(e + f*x)**2)**(3/2), x)

Giac [A] time = 1.1096, size = 77, normalized size = 1.35

$$\frac{\arctan\left(\frac{\sqrt{b\sin\left(fx+e\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}af}+\frac{1}{\sqrt{b\sin\left(fx+e\right)^2+a}af}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")

[Out] arctan(sqrt(b*sin(f*x + e)^2 + a)/sqrt(-a))/(sqrt(-a)*a*f) + 1/(sqrt(b*sin(f*x + e)^2 + a)*a*f)

3.525 $\int \frac{\cot^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=110

$$-\frac{2a+3b}{2a^2f\sqrt{a+b\sin^2(e+fx)}}+\frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{2a^{5/2}f}-\frac{\csc^2(e+fx)}{2af\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] ((2*a + 3*b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/(2*a^(5/2)*f) - (
2*a + 3*b)/(2*a^2*f*Sqrt[a + b*Sin[e + f*x]^2]) - Csc[e + f*x]^2/(2*a*f*Sqr
t[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.122642, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3194, 78, 51, 63, 208}

$$-\frac{2a+3b}{2a^2f\sqrt{a+b\sin^2(e+fx)}}+\frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{2a^{5/2}f}-\frac{\csc^2(e+fx)}{2af\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(3/2),x]
```


```
[Out] ((2*a + 3*b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/(2*a^(5/2)*f) - (
2*a + 3*b)/(2*a^2*f*Sqrt[a + b*Sin[e + f*x]^2]) - Csc[e + f*x]^2/(2*a*f*Sqr
t[a + b*Sin[e + f*x]^2])
```


Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
```

```
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cot^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1-x}{x^2(a+bx)^{3/2}}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= -\frac{\csc^2(e+fx)}{2af\sqrt{a+b\sin^2(e+fx)}} - \frac{(2a+3b)\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \sin^2(e+fx)\right)}{4af}$$

$$= -\frac{2a+3b}{2a^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{\csc^2(e+fx)}{2af\sqrt{a+b\sin^2(e+fx)}} - \frac{(2a+3b)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx\right.}{4a^2f}$$

$$= -\frac{2a+3b}{2a^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{\csc^2(e+fx)}{2af\sqrt{a+b\sin^2(e+fx)}} - \frac{(2a+3b)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,\right.}{2a^2bf}$$

$$= \frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{2a^{5/2}f} - \frac{2a+3b}{2a^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{\csc^2(e+fx)}{2af\sqrt{a+b\sin^2(e+f}}$$

Mathematica [C] time = 0.102085, size = 70, normalized size = 0.64

$$\frac{-(2a+3b)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\sin^2(e+fx)}{a}+1\right)-a\csc^2(e+fx)}{2a^2f\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-(a*Csc[e + f*x]^2) - (2*a + 3*b)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*S
in[e + f*x]^2)/a])/(2*a^2*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Maple [A] time = 1.547, size = 159, normalized size = 1.5

$$-\frac{1}{af}\frac{1}{\sqrt{a+b\left(\sin\left(fx+e\right)\right)^2}}+\frac{1}{f}\ln\left(\frac{1}{\sin\left(fx+e\right)}\left(2a+2\sqrt{a}\sqrt{a+b\left(\sin\left(fx+e\right)\right)^2}\right)\right)a^{-\frac{3}{2}}-\frac{1}{2af\left(\sin\left(fx+e\right)\right)^2}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/a/f/(a+b*sin(f*x+e)^2)^(1/2)+1/f/a^(3/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+
e)^2)^(1/2))/sin(f*x+e))-1/2/f/a/sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2)-3/2/
f/a^2*b/(a+b*sin(f*x+e)^2)^(1/2)+3/2/f/a^(5/2)*b*ln((2*a+2*a^(1/2)*(a+b*sin
(f*x+e)^2)^(1/2))/sin(f*x+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.6528, size = 956, normalized size = 8.69

$$\left[\frac{\left(\left(2\,ab+3\,b^2\right)\cos\left(fx+e\right)^4-\left(2\,a^2+7\,ab+6\,b^2\right)\cos\left(fx+e\right)^2+2\,a^2+5\,ab+3\,b^2\right)\sqrt{a}\log\left(\frac{2\left(b\cos\left(fx+e\right)^2-2\,\sqrt{-b\cos\left(fx+e\right)^2 \cdots}\right.}{\cos\left(fx+e\right)^2-1}\right.}{4\left(a^3bf\cos\left(fx+e\right)^4-\left(a^4+2\,a^3b\right)f\cos\left(fx+e\right)\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((2*a*b + 3*b^2)*cos(f*x + e)^4 - (2*a^2 + 7*a*b + 6*b^2)*cos(f*x + e
)^2 + 2*a^2 + 5*a*b + 3*b^2)*sqrt(a)*log(2*(b*cos(f*x + e)^2 - 2*sqrt(-b*co
s(f*x + e)^2 + a + b)*sqrt(a) - 2*a - b)/(cos(f*x + e)^2 - 1)) + 2*((2*a^2
+ 3*a*b)*cos(f*x + e)^2 - 3*a^2 - 3*a*b)*sqrt(-b*cos(f*x + e)^2 + a + b))/(
a^3*b*f*cos(f*x + e)^4 - (a^4 + 2*a^3*b)*f*cos(f*x + e)^2 + (a^4 + a^3*b)*f
), -1/2*(((2*a*b + 3*b^2)*cos(f*x + e)^4 - (2*a^2 + 7*a*b + 6*b^2)*cos(f*x
+ e)^2 + 2*a^2 + 5*a*b + 3*b^2)*sqrt(-a)*arctan(sqrt(-b*cos(f*x + e)^2 + a
+ b)*sqrt(-a)/a) - ((2*a^2 + 3*a*b)*cos(f*x + e)^2 - 3*a^2 - 3*a*b)*sqrt(-b
*cos(f*x + e)^2 + a + b))/(a^3*b*f*cos(f*x + e)^4 - (a^4 + 2*a^3*b)*f*cos(f
*x + e)^2 + (a^4 + a^3*b)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^3\left(e+fx\right)}{\left(a+b\sin^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**3/(a + b*sin(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)^{3}}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^3/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.526 $\int \frac{\cot^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=167

$$\frac{8a^2+24ab+15b^2}{8a^3f\sqrt{a+b\sin^2(e+fx)}}-\frac{\left(8a^2+24ab+15b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{8a^{7/2}f}+\frac{(8a+5b)\csc^2(e+fx)}{8a^2f\sqrt{a+b\sin^2(e+fx)}}-\frac{\csc^4(e+fx)}{4af\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] -((8*a^2 + 24*a*b + 15*b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/(8
*a^(7/2)*f) + (8*a^2 + 24*a*b + 15*b^2)/(8*a^3*f*Sqrt[a + b*Sin[e + f*x]^2]
) + ((8*a + 5*b)*Csc[e + f*x]^2)/(8*a^2*f*Sqrt[a + b*Sin[e + f*x]^2]) - Csc
[e + f*x]^4/(4*a*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.162834, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3194, 89, 78, 51, 63, 208}

$$\frac{8a^2+24ab+15b^2}{8a^3f\sqrt{a+b\sin^2(e+fx)}}-\frac{\left(8a^2+24ab+15b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{8a^{7/2}f}+\frac{(8a+5b)\csc^2(e+fx)}{8a^2f\sqrt{a+b\sin^2(e+fx)}}-\frac{\csc^4(e+fx)}{4af\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5/(a + b*Sin[e + f*x]^2)^(3/2),x]
```


```
[Out] -((8*a^2 + 24*a*b + 15*b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/(8
*a^(7/2)*f) + (8*a^2 + 24*a*b + 15*b^2)/(8*a^3*f*Sqrt[a + b*Sin[e + f*x]^2]
) + ((8*a + 5*b)*Csc[e + f*x]^2)/(8*a^2*f*Sqrt[a + b*Sin[e + f*x]^2]) - Csc
[e + f*x]^4/(4*a*f*Sqrt[a + b*Sin[e + f*x]^2])
```


Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
```

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cot^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x)^2}{x^3(a+bx)^{3/2}}\,dx,x,\sin^2(e+fx)\right)}{2f}$$

$$= -\frac{\csc^4(e+fx)}{4af\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(-8a-5b)+2ax}{x^2(a+bx)^{3/2}}\,dx,x,\sin^2(e+fx)\right)}{4af}$$

$$= \frac{(8a+5b)\csc^2(e+fx)}{8a^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{\csc^4(e+fx)}{4af\sqrt{a+b\sin^2(e+fx)}} + \frac{\left(8a^2+24ab+15b^2\right)\text{Subst}\left(\int \right.}{16a^2}$$

$$= \frac{8a^2+24ab+15b^2}{8a^3f\sqrt{a+b\sin^2(e+fx)}} + \frac{(8a+5b)\csc^2(e+fx)}{8a^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{\csc^4(e+fx)}{4af\sqrt{a+b\sin^2(e+fx)}} + \frac{(8}{}$$

$$= \frac{8a^2+24ab+15b^2}{8a^3f\sqrt{a+b\sin^2(e+fx)}} + \frac{(8a+5b)\csc^2(e+fx)}{8a^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{\csc^4(e+fx)}{4af\sqrt{a+b\sin^2(e+fx)}} + \frac{(8}{}$$

$$= -\frac{\left(8a^2+24ab+15b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{8a^{7/2}f} + \frac{8a^2+24ab+15b^2}{8a^3f\sqrt{a+b\sin^2(e+fx)}} + \frac{(8a+5}{8a^2f\sqrt{a}}$$

Mathematica [C] time = 0.340294, size = 94, normalized size = 0.56

$$\frac{(8a^2+24ab+15b^2)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\sin^2(e+fx)}{a}+1\right)+a\csc^2(e+fx)\left(-2a\csc^2(e+fx)+8a+5b\right)}{8a^3f\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (a*Csc[e + f*x]^2*(8*a + 5*b - 2*a*Csc[e + f*x]^2) + (8*a^2 + 24*a*b + 15*b
^2)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Sin[e + f*x]^2)/a])/(8*a^3*f*Sqr
t[a + b*Sin[e + f*x]^2])
```

Maple [A] time = 1.669, size = 288, normalized size = 1.7

$$-\frac{1}{4af\left(\sin\left(fx+e\right)\right)^{4}}\frac{1}{\sqrt{a+b\left(\sin\left(fx+e\right)\right)^{2}}}+\frac{5b}{8a^{2}f\left(\sin\left(fx+e\right)\right)^{2}}\frac{1}{\sqrt{a+b\left(\sin\left(fx+e\right)\right)^{2}}}+\frac{15b^{2}}{8fa^{3}}\frac{1}{\sqrt{a+b\left(\sin\left(fx+e\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/4/f/a/sin(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2)+5/8/f/a^2*b/sin(f*x+e)^2/(a+
b*sin(f*x+e)^2)^(1/2)+15/8/f/a^3*b^2/(a+b*sin(f*x+e)^2)^(1/2)-15/8/f/a^(7/2
)*b^2*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e))+1/a/f/(a+b*si
n(f*x+e)^2)^(1/2)-1/f/a^(3/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/s
in(f*x+e))+1/f/a/sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2)+3/f/a^2*b/(a+b*sin(f
*x+e)^2)^(1/2)-3/f/a^(5/2)*b*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/si
n(f*x+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.91502, size = 1539, normalized size = 9.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(((8*a^2*b + 24*a*b^2 + 15*b^3)*cos(f*x + e)^6 - (8*a^3 + 48*a^2*b +
87*a*b^2 + 45*b^3)*cos(f*x + e)^4 - 8*a^3 - 32*a^2*b - 39*a*b^2 - 15*b^3 +
(16*a^3 + 72*a^2*b + 102*a*b^2 + 45*b^3)*cos(f*x + e)^2)*sqrt(a)*log(2*(b*c
os(f*x + e)^2 + 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) - 2*a - b)/(cos(f
*x + e)^2 - 1)) - 2*((8*a^3 + 24*a^2*b + 15*a*b^2)*cos(f*x + e)^4 + 14*a^3
+ 29*a^2*b + 15*a*b^2 - (24*a^3 + 53*a^2*b + 30*a*b^2)*cos(f*x + e)^2)*sqrt
(-b*cos(f*x + e)^2 + a + b))/(a^4*b*f*cos(f*x + e)^6 - (a^5 + 3*a^4*b)*f*co
s(f*x + e)^4 + (2*a^5 + 3*a^4*b)*f*cos(f*x + e)^2 - (a^5 + a^4*b)*f), 1/8*(
((8*a^2*b + 24*a*b^2 + 15*b^3)*cos(f*x + e)^6 - (8*a^3 + 48*a^2*b + 87*a*b^
2 + 45*b^3)*cos(f*x + e)^4 - 8*a^3 - 32*a^2*b - 39*a*b^2 - 15*b^3 + (16*a^3
 + 72*a^2*b + 102*a*b^2 + 45*b^3)*cos(f*x + e)^2)*sqrt(-a)*arctan(sqrt(-b*c
os(f*x + e)^2 + a + b)*sqrt(-a)/a) - ((8*a^3 + 24*a^2*b + 15*a*b^2)*cos(f*x
 + e)^4 + 14*a^3 + 29*a^2*b + 15*a*b^2 - (24*a^3 + 53*a^2*b + 30*a*b^2)*cos
(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b))/(a^4*b*f*cos(f*x + e)^6 - (a^
5 + 3*a^4*b)*f*cos(f*x + e)^4 + (2*a^5 + 3*a^4*b)*f*cos(f*x + e)^2 - (a^5 +
 a^4*b)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**5/(a + b*sin(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(fx+e)^5}{\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^5/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.527 $\int \frac{\tan^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=292

$$-\frac{4a\tan(e+fx)}{3f(a+b)^2\sqrt{a+b\sin^2(e+fx)}}+\frac{b(7a-b)\sin(e+fx)\cos(e+fx)}{3f(a+b)^3\sqrt{a+b\sin^2(e+fx)}}+\frac{\tan(e+fx)\sec^2(e+fx)}{3f(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{4a\sqrt{\cos^2(e+fx)}}{}$$

```
[Out] ((7*a - b)*b*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)^3*f*Sqrt[a + b*Sin[e + f
*x]^2]) + ((7*a - b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -
(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*(a + b)^3*f*Sqrt[1 + (b*
Sin[e + f*x]^2)/a]) - (4*a*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*
x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*(a + b)^2*f*Sq
rt[a + b*Sin[e + f*x]^2]) - (4*a*Tan[e + f*x])/(3*(a + b)^2*f*Sqrt[a + b*Si
n[e + f*x]^2]) + (Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)*f*Sqrt[a + b*Sin[
e + f*x]^2])
```

Rubi [A] time = 0.314868, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3196, 470, 527, 524, 426, 424, 421, 419}

$$-\frac{4a\tan(e+fx)}{3f(a+b)^2\sqrt{a+b\sin^2(e+fx)}}+\frac{b(7a-b)\sin(e+fx)\cos(e+fx)}{3f(a+b)^3\sqrt{a+b\sin^2(e+fx)}}+\frac{\tan(e+fx)\sec^2(e+fx)}{3f(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{4a\sqrt{\cos^2(e+fx)}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(3/2),x]
```


```
[Out] ((7*a - b)*b*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)^3*f*Sqrt[a + b*Sin[e + f
*x]^2]) + ((7*a - b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -
(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*(a + b)^3*f*Sqrt[1 + (b*
Sin[e + f*x]^2)/a]) - (4*a*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*
x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*(a + b)^2*f*Sq
rt[a + b*Sin[e + f*x]^2]) - (4*a*Tan[e + f*x])/(3*(a + b)^2*f*Sqrt[a + b*Si
n[e + f*x]^2]) + (Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)*f*Sqrt[a + b*Sin[
e + f*x]^2])
```


Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
```

```
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\operatorname{Subst}\left(\int \frac{x^4}{\left(1-x^2\right)^{5/2}\left(a+bx^2\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sec^2(e+fx)\tan(e+fx)}{3(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\operatorname{Subst}\left(\int \frac{a+3ax^2}{\left(1-x^2\right)^{3/2}\left(a+bx^2\right)^{3/2}}\,d\right.}{3(a+b)f}$$

$$= -\frac{4a\tan(e+fx)}{3(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\sec^2(e+fx)\tan(e+fx)}{3(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e\right.}{}$$

$$= \frac{(7a-b)b\cos(e+fx)\sin(e+fx)}{3(a+b)^3f\sqrt{a+b\sin^2(e+fx)}} - \frac{4a\tan(e+fx)}{3(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\sec^2(e+fx)\tan(}{3(a+b)f\sqrt{a+b\sin}}$$

$$= \frac{(7a-b)b\cos(e+fx)\sin(e+fx)}{3(a+b)^3f\sqrt{a+b\sin^2(e+fx)}} - \frac{4a\tan(e+fx)}{3(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\sec^2(e+fx)\tan(}{3(a+b)f\sqrt{a+b\sin}}$$

$$= \frac{(7a-b)b\cos(e+fx)\sin(e+fx)}{3(a+b)^3f\sqrt{a+b\sin^2(e+fx)}} - \frac{4a\tan(e+fx)}{3(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\sec^2(e+fx)\tan(}{3(a+b)f\sqrt{a+b\sin}}$$

$$= \frac{(7a-b)b\cos(e+fx)\sin(e+fx)}{3(a+b)^3f\sqrt{a+b\sin^2(e+fx)}} + \frac{(7a-b)\sqrt{\cos^2(e+fx)}E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)\sec}{3(a+b)^3f\sqrt{1+\frac{b\sin^2(e+fx}{a}}}$$

Mathematica [A] time = 2.3262, size = 197, normalized size = 0.67

$$\frac{-\frac{\tan(e+fx)\sec^2(e+fx)\left(4\left(4a^2-3ab+b^2\right)\cos(2(e+fx))+8a^2+b(b-7a)\cos(4(e+fx))-21ab-5b^2\right)}{2\sqrt{2}} - 8a(a+b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}F\left(e+fx\middle|-\frac{b}{a}\right)+2}{6f(a+b)^3\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (2*a*(7*a - b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(
b/a)] - 8*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*
x, -(b/a)] - ((8*a^2 - 21*a*b - 5*b^2 + 4*(4*a^2 - 3*a*b + b^2)*Cos[2*(e +
f*x)] + b*(-7*a + b)*Cos[4*(e + f*x)])*Sec[e + f*x]^2*Tan[e + f*x])/(2*Sqrt
[2]))/(6*(a + b)^3*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A] time = 2.556, size = 368, normalized size = 1.3

$$-\frac{1}{\left(-3+3\sin\left(fx+e\right)\right)\left(1+\sin\left(fx+e\right)\right)(a+b)^3\cos\left(fx+e\right)f}\left(\sqrt{-b\left(\cos\left(fx+e\right)\right)^4+(a+b)\left(\cos\left(fx+e\right)\right)^2}b(7a-b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/3*((-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*(7*a-b)*sin(f*x+e)*cos(f
*x+e)^4-4*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*a*(a+b)*cos(f*x+e)^2*s
in(f*x+e)+(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a^2+2*a*b+b^2)*sin(f*
x+e)-(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)
^(1/2)*(cos(f*x+e)^2)^(1/2)*a*(4*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a+4*E
llipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-7*EllipticE(sin(f*x+e),(-1/a*b)^(1/2)
)*a+EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*b)*cos(f*x+e)^2)/(-1+sin(f*x+e))/(
1+sin(f*x+e))/(-(a+b*sin(f*x+e)^2)*(-1+sin(f*x+e))*(1+sin(f*x+e)))^(1/2)/(a
+b)^3/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\tan\left(fx+e\right)^4}{b^2\cos\left(fx+e\right)^4-2\left(ab+b^2\right)\cos\left(fx+e\right)^2+a^2+2\,ab+b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)*tan(f*x + e)^4/(b^2*cos(f*x + e)^4
 - 2*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\tan^4\left(e+fx\right)}{\left(a+b\sin^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**4/(a + b*sin(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^{4}}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^4/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.528 $\int \frac{\tan^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=224

$$\frac{\tan(e+fx)}{f(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{2b\sin(e+fx)\cos(e+fx)}{f(a+b)^2\sqrt{a+b\sin^2(e+fx)}}+\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(\sin^{-1}(\mathrm{si}\right.}{f(a+b)\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] (-2*b*Cos[e + f*x]*Sin[e + f*x])/((a + b)^2*f*Sqrt[a + b*Sin[e + f*x]^2]) -
 (2*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*
x]*Sqrt[a + b*Sin[e + f*x]^2])/((a + b)^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a])
 + (Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*
x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/((a + b)*f*Sqrt[a + b*Sin[e + f*x]^2]) +
 Tan[e + f*x]/((a + b)*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.225916, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3196, 471, 527, 524, 426, 424, 421, 419}

$$\frac{\tan(e+fx)}{f(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{2b\sin(e+fx)\cos(e+fx)}{f(a+b)^2\sqrt{a+b\sin^2(e+fx)}}+\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}F\left(\sin^{-1}(\mathrm{si}\right.}{f(a+b)\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(3/2),x]
```


```
[Out] (-2*b*Cos[e + f*x]*Sin[e + f*x])/((a + b)^2*f*Sqrt[a + b*Sin[e + f*x]^2]) -
 (2*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*
x]*Sqrt[a + b*Sin[e + f*x]^2])/((a + b)^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a])
 + (Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*
x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/((a + b)*f*Sqrt[a + b*Sin[e + f*x]^2]) +
 Tan[e + f*x]/((a + b)*f*Sqrt[a + b*Sin[e + f*x]^2])
```


Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^2}{(1-x^2)^{3/2}(a+bx^2)^{3/2}}dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)}{(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{a-bx^2}{\sqrt{1-x^2}(a+bx^2)^{3/2}}d\right.}{(a+b)f}$$

$$= -\frac{2b\cos(e+fx)\sin(e+fx)}{(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\tan(e+fx)}{(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\left(\sqrt{\cos^2(e+fx)}\sec(\right.}{}$$

$$= -\frac{2b\cos(e+fx)\sin(e+fx)}{(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\tan(e+fx)}{(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(2\sqrt{\cos^2(e+fx)}\sec\right.}{}$$

$$= -\frac{2b\cos(e+fx)\sin(e+fx)}{(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\tan(e+fx)}{(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(2\sqrt{\cos^2(e+fx)}\sec\right.}{}$$

$$= -\frac{2b\cos(e+fx)\sin(e+fx)}{(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{2\sqrt{\cos^2(e+fx)}E\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)\sec(e+f}{(a+b)^2f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

Mathematica [A] time = 0.885533, size = 145, normalized size = 0.65

$$\frac{2\tan(e+fx)(a-b\cos(2(e+fx)))+\sqrt{2}(a+b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}F\left(e+fx\middle|-\frac{b}{a}\right)-2\sqrt{2}a\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}E\left(e+fx\right.}{\sqrt{2}f(a+b)^2\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[2]*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b
/a)] + Sqrt[2]*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e +
 f*x, -(b/a)] + 2*(a - b*Cos[2*(e + f*x)])*Tan[e + f*x])/(Sqrt[2]*(a + b)^2
*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A] time = 2.174, size = 278, normalized size = 1.2

$$\frac{1}{(a+b)^2\cos(fx+e)f}\sqrt{-b(\cos(fx+e))^4+(a+b)(\cos(fx+e))^2}\left(a\sqrt{(\cos(fx+e))^2}\sqrt{-\frac{b(\cos(fx+e))^2}{a}+\frac{a+}{a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] (-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a*(cos(f*x+e)^2)^(1/2)*(-b/a*co
s(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))+b*(cos(f*x+e
```

```
)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^
(1/2))-2*a*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE
(sin(f*x+e),(-1/a*b)^(1/2))-2*cos(f*x+e)^2*sin(f*x+e)*b+a*sin(f*x+e)+b*sin(
f*x+e))/(a+b)^2/(-(a+b*sin(f*x+e)^2)*(-1+sin(f*x+e))*(1+sin(f*x+e)))^(1/2)/
cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\tan\left(fx+e\right)^{2}}{b^{2}\cos\left(fx+e\right)^{4}-2\left(ab+b^{2}\right)\cos\left(fx+e\right)^{2}+a^{2}+2\,ab+b^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)*tan(f*x + e)^2/(b^2*cos(f*x + e)^4
 - 2*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\tan^{2}\left(e+fx\right)}{\left(a+b\sin^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**2/(a + b*sin(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\tan\left(fx+e\right)^{2}}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.529 $\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=101

$$\frac{b\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}+\frac{\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{af(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

```
[Out] (b*Cos[e + f*x]*Sin[e + f*x])/(a*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2]) + (E
llipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(a*(a + b)*f*Sqrt[1 +
 (b*Sin[e + f*x]^2)/a])
```

Rubi [A] time = 0.0621282, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3184, 21, 3178, 3177}

$$\frac{b\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}+\frac{\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{af(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x]^2)^(-3/2),x]
```

```
[Out] (b*Cos[e + f*x]*Sin[e + f*x])/(a*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2]) + (E
llipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(a*(a + b)*f*Sqrt[1 +
 (b*Sin[e + f*x]^2)/a])
```

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a +
 b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)
*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{b\cos(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\int \frac{-a-b\sin^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx}{a(a+b)}$$
$$= \frac{b\cos(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\int \sqrt{a+b\sin^2(e+fx)}\,dx}{a(a+b)}$$
$$= \frac{b\cos(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\sqrt{a+b\sin^2(e+fx)}\int \sqrt{1+\frac{b\sin^2(e+fx)}{a}}\,dx}{a(a+b)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$
$$= \frac{b\cos(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{a(a+b)f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

Mathematica [A] time = 0.149299, size = 90, normalized size = 0.89

$$\frac{2a\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)+\sqrt{2}b\sin(2(e+fx))}{2af(a+b)\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)^(-3/2),x]
```

```
[Out] (2*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] + Sq
rt[2]*b*Sin[2*(e + f*x)])/(2*a*(a + b)*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]]
)
```

Maple [A] time = 1.346, size = 103, normalized size = 1.

$$\frac{1}{a\left(a+b\right)\cos\left(fx+e\right)f}\left(a\sqrt{\left(\cos\left(fx+e\right)\right)^2}\sqrt{-\frac{b\left(\cos\left(fx+e\right)\right)^2}{a}+\frac{a+b}{a}}\mathrm{EllipticE}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)+\left(\cos\left(fx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] (a*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x
+e),(-1/a*b)^(1/2))+cos(f*x+e)^2*sin(f*x+e)*b)/a/(a+b)/cos(f*x+e)/(a+b*sin(
f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}}{b^{2}\cos\left(fx+e\right)^{4}-2\left(ab+b^{2}\right)\cos\left(fx+e\right)^{2}+a^{2}+2\,ab+b^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)/(b^2*cos(f*x + e)^4 - 2*(a*b + b^2
)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(-3/2), x)
```

3.530 $\int \frac{\cot^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=209

$$-\frac{2\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a^2 f}-\frac{2\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}E\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{a^2 f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+$$

```
[Out] Cot[e + f*x]/(a*f*Sqrt[a + b*Sin[e + f*x]^2]) - (2*Cot[e + f*x]*Sqrt[a + b*
Sin[e + f*x]^2])/(a^2*f) - (2*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e +
 f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(a^2*f*Sqrt[1 + (b
*Sin[e + f*x]^2)/a]) + (Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]]
, -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(a*f*Sqrt[a + b*Sin[
e + f*x]^2])
```

Rubi [A] time = 0.241605, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3196, 469, 583, 524, 426, 424, 421, 419}

$$-\frac{2\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a^2 f}-\frac{2\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}E\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{a^2 f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] Cot[e + f*x]/(a*f*Sqrt[a + b*Sin[e + f*x]^2]) - (2*Cot[e + f*x]*Sqrt[a + b*
Sin[e + f*x]^2])/(a^2*f) - (2*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e +
 f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(a^2*f*Sqrt[1 + (b
*Sin[e + f*x]^2)/a]) + (Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]]
, -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(a*f*Sqrt[a + b*Sin[
e + f*x]^2])
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 469

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
q)/(a*e*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p +
 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1) +
 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e, m,
 n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{x^2\left(a+bx^2\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\cot(e+fx)}{af\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{-2+x^2}{x^2\sqrt{1-x^2}\sqrt{a+bx^2}}\,dx, x, \text{si}\right.}{af}$$

$$= \frac{\cot(e+fx)}{af\sqrt{a+b\sin^2(e+fx)}} - \frac{2\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a^2f} + \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+\right.}{}$$

$$= \frac{\cot(e+fx)}{af\sqrt{a+b\sin^2(e+fx)}} - \frac{2\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a^2f} - \frac{\left(2\sqrt{\cos^2(e+fx)}\sec(e\right.}{}$$

$$= \frac{\cot(e+fx)}{af\sqrt{a+b\sin^2(e+fx)}} - \frac{2\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a^2f} - \frac{\left(2\sqrt{\cos^2(e+fx)}\sec(e\right.}{}$$

$$= \frac{\cot(e+fx)}{af\sqrt{a+b\sin^2(e+fx)}} - \frac{2\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a^2f} - \frac{2\sqrt{\cos^2(e+fx)}E\left(\sin^{-1}\right.}{}$$

Mathematica [A] time = 0.763714, size = 142, normalized size = 0.68

$$\frac{-2\cot(e+fx)(a-b\cos(2(e+fx))+b)+\sqrt{2}a\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}F\left(e+fx\middle|-\frac{b}{a}\right)-2\sqrt{2}a\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}E\left(e+fx\right.}{\sqrt{2}a^2f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-2*(a + b - b*Cos[2*(e + f*x)])*Cot[e + f*x] - 2*Sqrt[2]*a*Sqrt[(2*a + b -
 b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] + Sqrt[2]*a*Sqrt[(2*a +
b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)])/(Sqrt[2]*a^2*f*Sqrt[
2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A] time = 1.23, size = 141, normalized size = 0.7

$$\frac{1}{\sin\left(fx+e\right)a^2\cos\left(fx+e\right)f}\left(\sin\left(fx+e\right)\sqrt{-\frac{b\left(\cos\left(fx+e\right)\right)^2}{a}+\frac{a+b}{a}}\sqrt{\left(\cos\left(fx+e\right)\right)^2}a\left(\text{EllipticF}\left(\sin\left(fx+e\right)\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] (sin(f*x+e)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(cos(f*x+e)^2)^(1/2)*a*(Ellip
ticF(sin(f*x+e),(-1/a*b)^(1/2))-2*EllipticE(sin(f*x+e),(-1/a*b)^(1/2)))+2*b
```

```
*cos(f*x+e)^4+(-a-2*b)*cos(f*x+e)^2)/sin(f*x+e)/a^2/cos(f*x+e)/(a+b*sin(f*x
+e)^2)^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\cot\left(fx+e\right)^{2}}{b^{2}\cos\left(fx+e\right)^{4}-2\left(ab+b^{2}\right)\cos\left(fx+e\right)^{2}+a^{2}+2\,ab+b^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)*cot(f*x + e)^2/(b^2*cos(f*x + e)^4
 - 2*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cot^{2}\left(e+fx\right)}{\left(a+b\sin^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**2/(a + b*sin(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cot\left(fx+e\right)^{2}}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.531 $\int \frac{\cot^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=297

$$\frac{(7a+8b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^3f}-\frac{(3a+4b)\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^2bf}-\frac{4(a+b)\sqrt{\cos^2(}}{}$$

```
[Out] ((a + b)*Cot[e + f*x]*Csc[e + f*x]^2)/(a*b*f*Sqrt[a + b*Sin[e + f*x]^2]) +
((7*a + 8*b)*Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a^3*f) - ((3*a + 4
*b)*Cot[e + f*x]*Csc[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2])/(3*a^2*b*f) + (
(7*a + 8*b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Se
c[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a^3*f*Sqrt[1 + (b*Sin[e + f*x]^2)
/a]) - (4*(a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/
a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*a^2*f*Sqrt[a + b*Sin[e
+ f*x]^2])
```

Rubi [A] time = 0.374123, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3196, 468, 583, 524, 426, 424, 421, 419}

$$\frac{(7a+8b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^3f}-\frac{(3a+4b)\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^2bf}-\frac{4(a+b)\sqrt{\cos^2(}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(3/2),x]
```



```
[Out] ((a + b)*Cot[e + f*x]*Csc[e + f*x]^2)/(a*b*f*Sqrt[a + b*Sin[e + f*x]^2]) +
((7*a + 8*b)*Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a^3*f) - ((3*a + 4
*b)*Cot[e + f*x]*Csc[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2])/(3*a^2*b*f) + (
(7*a + 8*b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Se
c[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a^3*f*Sqrt[1 + (b*Sin[e + f*x]^2)
/a]) - (4*(a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/
a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*a^2*f*Sqrt[a + b*Sin[e
+ f*x]^2])
```


Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
```

```
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{(1-x^2)^{3/2}}{x^4(a+bx^2)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{(a+b)\cot(e+fx)\csc^2(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{-3a-4b+(2a+3b}{x^4\sqrt{1-x^2}\sqrt{a+b}}\right.}{abf}$$

$$= \frac{(a+b)\cot(e+fx)\csc^2(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}} - \frac{(3a+4b)\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e+}}{3a^2bf}$$

$$= \frac{(a+b)\cot(e+fx)\csc^2(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}} + \frac{(7a+8b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^3f} - \frac{(3a+}{}$$

$$= \frac{(a+b)\cot(e+fx)\csc^2(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}} + \frac{(7a+8b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^3f} - \frac{(3a+}{}$$

$$= \frac{(a+b)\cot(e+fx)\csc^2(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}} + \frac{(7a+8b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^3f} - \frac{(3a+}{}$$

$$= \frac{(a+b)\cot(e+fx)\csc^2(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}} + \frac{(7a+8b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^3f} - \frac{(3a+}{}$$

Mathematica [A] time = 4.01837, size = 199, normalized size = 0.67

$$\frac{\frac{\cot(e+fx)\csc^2(e+fx)\left(-4\left(4a^2+11ab+8b^2\right)\cos(2(e+fx))+8a^2+b(7a+8b)\cos(4(e+fx))+37ab+24b^2\right)}{2\sqrt{2}} - 8a(a+b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}F\left(e+fx\right|-}{6a^3f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (((8*a^2 + 37*a*b + 24*b^2 - 4*(4*a^2 + 11*a*b + 8*b^2)*Cos[2*(e + f*x)] +
b*(7*a + 8*b)*Cos[4*(e + f*x)])*Cot[e + f*x]*Csc[e + f*x]^2)/(2*Sqrt[2]) +
2*a*(7*a + 8*b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -
(b/a)] - 8*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f
*x, -(b/a)])/(6*a^3*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

Maple [A] time = 1.395, size = 353, normalized size = 1.2

$$-\frac{1}{3a^3\left(\sin\left(fx+e\right)\right)^3\cos\left(fx+e\right)f}\left(4\,\text{EllipticF}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)\sqrt{\left(\cos\left(fx+e\right)\right)^2}\sqrt{\frac{a+b\left(\sin\left(fx+e\right)\right)^2}{a}}a^2\left(\sin\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/3*(4*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*(cos(f*x+e)^2)^(1/2)*((a+b*sin
(f*x+e)^2)/a)^(1/2)*a^2*sin(f*x+e)^3+4*b*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x
+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*sin(f*x+e)^3-7*Ellip
ticE(sin(f*x+e),(-1/a*b)^(1/2))*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)
^(1/2)*a^2*sin(f*x+e)^3-8*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*(cos(f*x+e)^
2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*a*b*sin(f*x+e)^3+7*a*b*sin(f*x+e)^6+8
*b^2*sin(f*x+e)^6+4*a^2*sin(f*x+e)^4-3*a*b*sin(f*x+e)^4-8*b^2*sin(f*x+e)^4-
5*a^2*sin(f*x+e)^2-4*a*b*sin(f*x+e)^2+a^2)/a^3/sin(f*x+e)^3/cos(f*x+e)/(a+b
*sin(f*x+e)^2)^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\cot\left(fx+e\right)^{4}}{b^{2}\cos\left(fx+e\right)^{4}-2\left(ab+b^{2}\right)\cos\left(fx+e\right)^{2}+a^{2}+2\,ab+b^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)*cot(f*x + e)^4/(b^2*cos(f*x + e)^4
 - 2*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cot^{4}\left(e+fx\right)}{\left(a+b\sin^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**4/(a + b*sin(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(fx+e)^4}{\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^4/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

3.532 $\int \frac{\tan^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=218

$$-\frac{8a^2-24ab+3b^2}{8f(a+b)^4\sqrt{a+b\sin^2(e+fx)}}-\frac{8a^2-24ab+3b^2}{24f(a+b)^3\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{\left(8a^2-24ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{8f(a+b)^{9/2}}+ -\frac{}{4}$$

```
[Out] ((8*a^2 - 24*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/
(8*(a + b)^(9/2)*f) - (8*a^2 - 24*a*b + 3*b^2)/(24*(a + b)^3*f*(a + b*Sin[e
 + f*x]^2)^(3/2)) - ((8*a + b)*Sec[e + f*x]^2)/(8*(a + b)^2*f*(a + b*Sin[e
+ f*x]^2)^(3/2)) + Sec[e + f*x]^4/(4*(a + b)*f*(a + b*Sin[e + f*x]^2)^(3/2)
) - (8*a^2 - 24*a*b + 3*b^2)/(8*(a + b)^4*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.27721, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3194, 89, 78, 51, 63, 208}

$$-\frac{8a^2-24ab+3b^2}{8f(a+b)^4\sqrt{a+b\sin^2(e+fx)}}-\frac{8a^2-24ab+3b^2}{24f(a+b)^3\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{\left(8a^2-24ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{8f(a+b)^{9/2}}+ -\frac{}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] ((8*a^2 - 24*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/
(8*(a + b)^(9/2)*f) - (8*a^2 - 24*a*b + 3*b^2)/(24*(a + b)^3*f*(a + b*Sin[e
 + f*x]^2)^(3/2)) - ((8*a + b)*Sec[e + f*x]^2)/(8*(a + b)^2*f*(a + b*Sin[e
+ f*x]^2)^(3/2)) + Sec[e + f*x]^4/(4*(a + b)*f*(a + b*Sin[e + f*x]^2)^(3/2)
) - (8*a^2 - 24*a*b + 3*b^2)/(8*(a + b)^4*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\tan^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1-x)^3(a+bx)^{5/2}}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= \frac{\sec^4(e+fx)}{4(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(4a-3b)+2(a+b)x}{(1-x)^2(a+bx)^{5/2}}\,dx, x, \sin^2(e+fx)\right)}{4(a+b)f}$$

$$= -\frac{(8a+b)\sec^2(e+fx)}{8(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{\sec^4(e+fx)}{4(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{\left(8a^2-24ab\right.}{}$$

$$= -\frac{8a^2-24ab+3b^2}{24(a+b)^3f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{(8a+b)\sec^2(e+fx)}{8(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{}{4(a+b)f\left(\right.}$$

$$= -\frac{8a^2-24ab+3b^2}{24(a+b)^3f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{(8a+b)\sec^2(e+fx)}{8(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{}{4(a+b)f\left(\right.}$$

$$= -\frac{8a^2-24ab+3b^2}{24(a+b)^3f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{(8a+b)\sec^2(e+fx)}{8(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{}{4(a+b)f\left(\right.}$$

$$= \frac{\left(8a^2-24ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{8(a+b)^{9/2}f} - \frac{8a^2-24ab+3b^2}{24(a+b)^3f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{}{8(}$$

Mathematica [C] time = 0.484806, size = 107, normalized size = 0.49

$$\frac{\left(-8a^2+24ab-3b^2\right)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b\sin^2(e+fx)+a}{a+b}\right)-\frac{3}{2}(a+b)\sec^4(e+fx)((8a+b)\cos(2(e+fx))+4a-3b)}{24f(a+b)^3\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] ((-8*a^2 + 24*a*b - 3*b^2)*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Sin[e +
f*x]^2)/(a + b)] - (3*(a + b)*(4*a - 3*b + (8*a + b)*Cos[2*(e + f*x)])*Sec[
e + f*x]^4)/2)/(24*(a + b)^3*f*(a + b*Sin[e + f*x]^2)^(3/2))
```

Maple [B] time = 7.612, size = 2139, normalized size = 9.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/12/f*b^2*a/(b+(-a*b)^(1/2))^3/(-b+(-a*b)^(1/2))^3/(sin(f*x+e)+(-a*b)^(1/
2)/b)^2*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)+1/12/f*b^2*(-a*b)^(1/2)/(b+(-a*
b)^(1/2))^3/(-b+(-a*b)^(1/2))^3/(sin(f*x+e)+(-a*b)^(1/2)/b)*(-b*cos(f*x+e)^
2+(a*b+b^2)/b)^(1/2)+1/16/f*b^2/(b+(-a*b)^(1/2))^2/(-b+(-a*b)^(1/2))^2/(a+b
)/(-1+sin(f*x+e))^2*(a+b-b*cos(f*x+e)^2)^(1/2)-3/16/f*b^3/(b+(-a*b)^(1/2))^
2/(-b+(-a*b)^(1/2))^2/(a+b)^2/(-1+sin(f*x+e))*(a+b-b*cos(f*x+e)^2)^(1/2)+1/
16/f*b^4/(b+(-a*b)^(1/2))^3/(-b+(-a*b)^(1/2))^3/(a+b)/(-1+sin(f*x+e))*(a+b-
b*cos(f*x+e)^2)^(1/2)-1/f*b^5*a/(b+(-a*b)^(1/2))^4/(-b+(-a*b)^(1/2))^4/(a+b
)^(1/2)*ln((2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+2*b*sin(f*x+e)+2*a)/(-
1+sin(f*x+e)))+1/2/f*b^4*a^2/(b+(-a*b)^(1/2))^4/(-b+(-a*b)^(1/2))^4/(a+b)^(
1/2)*ln((2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+2*b*sin(f*x+e)+2*a)/(-1+s
in(f*x+e)))+1/16/f*b^2/(b+(-a*b)^(1/2))^2/(-b+(-a*b)^(1/2))^2/(a+b)/(1+sin(
f*x+e))^2*(a+b-b*cos(f*x+e)^2)^(1/2)-1/12/f*b^2*a/(b+(-a*b)^(1/2))^3/(-b+(-
a*b)^(1/2))^3/(sin(f*x+e)-(-a*b)^(1/2)/b)^2*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(
1/2)-1/12/f*b^2*(-a*b)^(1/2)/(b+(-a*b)^(1/2))^3/(-b+(-a*b)^(1/2))^3/(sin(f*
x+e)-(-a*b)^(1/2)/b)*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)-1/16/f*b^4/(b+(-a*
b)^(1/2))^3/(-b+(-a*b)^(1/2))^3/(a+b)/(1+sin(f*x+e))*(a+b-b*cos(f*x+e)^2)^(
1/2)+7/16/f*b^4/(b+(-a*b)^(1/2))^3/(-b+(-a*b)^(1/2))^3*a/(a+b)^(3/2)*ln((2*
(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-2*b*sin(f*x+e)+2*a)/(1+sin(f*x+e)))+
7/16/f*b^4/(b+(-a*b)^(1/2))^3/(-b+(-a*b)^(1/2))^3*a/(a+b)^(3/2)*ln((2*(a+b)
^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+2*b*sin(f*x+e)+2*a)/(-1+sin(f*x+e)))+1/2/
f*b^4*a^2/(b+(-a*b)^(1/2))^4/(-b+(-a*b)^(1/2))^4/(a+b)^(1/2)*ln((2*(a+b)^(1
/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-2*b*sin(f*x+e)+2*a)/(1+sin(f*x+e)))-1/f*b^5*
a/(b+(-a*b)^(1/2))^4/(-b+(-a*b)^(1/2))^4/(a+b)^(1/2)*ln((2*(a+b)^(1/2)*(a+b
-b*cos(f*x+e)^2)^(1/2)-2*b*sin(f*x+e)+2*a)/(1+sin(f*x+e)))+3/16/f*b^3/(b+(-
a*b)^(1/2))^2/(-b+(-a*b)^(1/2))^2/(a+b)^2/(1+sin(f*x+e))*(a+b-b*cos(f*x+e)^
2)^(1/2)+1/2/f*b^4*a^2/(b+(-a*b)^(1/2))^4/(-b+(-a*b)^(1/2))^4/(-a*b)^(1/2)/
(sin(f*x+e)+(-a*b)^(1/2)/b)*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)-1/2/f*b^4*a
^2/(b+(-a*b)^(1/2))^4/(-b+(-a*b)^(1/2))^4/(-a*b)^(1/2)/(sin(f*x+e)-(-a*b)^(
1/2)/b)*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)+1/f*b^5*a/(b+(-a*b)^(1/2))^4/(-
b+(-a*b)^(1/2))^4/(-a*b)^(1/2)/(sin(f*x+e)-(-a*b)^(1/2)/b)*(-b*cos(f*x+e)^2
+(a*b+b^2)/b)^(1/2)-7/16/f*b^3/(b+(-a*b)^(1/2))^3/(-b+(-a*b)^(1/2))^3*a/(a+
b)/(-1+sin(f*x+e))*(a+b-b*cos(f*x+e)^2)^(1/2)-1/f*b^5*a/(b+(-a*b)^(1/2))^4/
(-b+(-a*b)^(1/2))^4/(-a*b)^(1/2)/(sin(f*x+e)+(-a*b)^(1/2)/b)*(-b*cos(f*x+e)
```

```
^2+(a*b+b^2)/b)^(1/2)+7/16/f*b^3/(b+(-a*b)^(1/2))^3/(-b+(-a*b)^(1/2))^3*a/(
a+b)/(1+sin(f*x+e))*(a+b-b*cos(f*x+e)^2)^(1/2)+3/16/f*b^4/(b+(-a*b)^(1/2))^
2/(-b+(-a*b)^(1/2))^2/(a+b)^(5/2)*ln((2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1
/2)+2*b*sin(f*x+e)+2*a)/(-1+sin(f*x+e)))-1/16/f*b^3/(b+(-a*b)^(1/2))^2/(-b+
(-a*b)^(1/2))^2/(a+b)^(3/2)*ln((2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+2*
b*sin(f*x+e)+2*a)/(-1+sin(f*x+e)))-1/16/f*b^3/(b+(-a*b)^(1/2))^2/(-b+(-a*b)
^(1/2))^2/(a+b)^(3/2)*ln((2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-2*b*sin(
f*x+e)+2*a)/(1+sin(f*x+e)))-1/16/f*b^5/(b+(-a*b)^(1/2))^3/(-b+(-a*b)^(1/2))
^3/(a+b)^(3/2)*ln((2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+2*b*sin(f*x+e)+
2*a)/(-1+sin(f*x+e)))+3/16/f*b^4/(b+(-a*b)^(1/2))^2/(-b+(-a*b)^(1/2))^2/(a+
b)^(5/2)*ln((2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-2*b*sin(f*x+e)+2*a)/(
1+sin(f*x+e)))-1/16/f*b^5/(b+(-a*b)^(1/2))^3/(-b+(-a*b)^(1/2))^3/(a+b)^(3/2
)*ln((2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-2*b*sin(f*x+e)+2*a)/(1+sin(f
*x+e)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.63943, size = 2287, normalized size = 10.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*((8*a^2*b^2 - 24*a*b^3 + 3*b^4)*cos(f*x + e)^8 - 2*(8*a^3*b - 16*a
^2*b^2 - 21*a*b^3 + 3*b^4)*cos(f*x + e)^6 + (8*a^4 - 8*a^3*b - 37*a^2*b^2 -
 18*a*b^3 + 3*b^4)*cos(f*x + e)^4)*sqrt(a + b)*log((b*cos(f*x + e)^2 - 2*sq
rt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b) - 2*a - 2*b)/cos(f*x + e)^2) + 2*
(3*(8*a^3*b - 16*a^2*b^2 - 21*a*b^3 + 3*b^4)*cos(f*x + e)^6 - 4*(8*a^4 - 8*
a^3*b - 37*a^2*b^2 - 18*a*b^3 + 3*b^4)*cos(f*x + e)^4 + 6*a^4 + 24*a^3*b +
36*a^2*b^2 + 24*a*b^3 + 6*b^4 - 3*(8*a^4 + 25*a^3*b + 27*a^2*b^2 + 11*a*b^3
 + b^4)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b))/((a^5*b^2 + 5*a^4*
b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*f*cos(f*x + e)^8 - 2*(a^6*b
+ 6*a^5*b^2 + 15*a^4*b^3 + 20*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)*f*cos(f
*x + e)^6 + (a^7 + 7*a^6*b + 21*a^5*b^2 + 35*a^4*b^3 + 35*a^3*b^4 + 21*a^2*
b^5 + 7*a*b^6 + b^7)*f*cos(f*x + e)^4), -1/24*(3*((8*a^2*b^2 - 24*a*b^3 + 3
*b^4)*cos(f*x + e)^8 - 2*(8*a^3*b - 16*a^2*b^2 - 21*a*b^3 + 3*b^4)*cos(f*x
+ e)^6 + (8*a^4 - 8*a^3*b - 37*a^2*b^2 - 18*a*b^3 + 3*b^4)*cos(f*x + e)^4)*
sqrt(-a - b)*arctan(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(a + b)) -
 (3*(8*a^3*b - 16*a^2*b^2 - 21*a*b^3 + 3*b^4)*cos(f*x + e)^6 - 4*(8*a^4 - 8
*a^3*b - 37*a^2*b^2 - 18*a*b^3 + 3*b^4)*cos(f*x + e)^4 + 6*a^4 + 24*a^3*b +
 36*a^2*b^2 + 24*a*b^3 + 6*b^4 - 3*(8*a^4 + 25*a^3*b + 27*a^2*b^2 + 11*a*b^
3 + b^4)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b))/((a^5*b^2 + 5*a^4
*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*f*cos(f*x + e)^8 - 2*(a^6*b
 + 6*a^5*b^2 + 15*a^4*b^3 + 20*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)*f*cos(
f*x + e)^6 + (a^7 + 7*a^6*b + 21*a^5*b^2 + 35*a^4*b^3 + 35*a^3*b^4 + 21*a^2
```

```
*b^5 + 7*a*b^6 + b^7)*f*cos(f*x + e)^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5/(a+b*sin(f*x+e)**2)**(5/2),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^5}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")

[Out] integrate(tan(f*x + e)^5/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

3.533 $\int \frac{\tan^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=153

$$\frac{2a-3b}{2f(a+b)^3\sqrt{a+b\sin^2(e+fx)}}+\frac{2a-3b}{6f(a+b)^2\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{2f(a+b)^{7/2}}+\frac{\mathrm{s}}{2f(a+b)\left(\right.}$$

```
[Out] -((2*a - 3*b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/(2*(a + b)^(
7/2)*f) + (2*a - 3*b)/(6*(a + b)^2*f*(a + b*Sin[e + f*x]^2)^(3/2)) + Sec[e
+ f*x]^2/(2*(a + b)*f*(a + b*Sin[e + f*x]^2)^(3/2)) + (2*a - 3*b)/(2*(a + b
)^3*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.154389, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3194, 78, 51, 63, 208}

$$\frac{2a-3b}{2f(a+b)^3\sqrt{a+b\sin^2(e+fx)}}+\frac{2a-3b}{6f(a+b)^2\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{2f(a+b)^{7/2}}+\frac{\mathrm{s}}{2f(a+b)\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(5/2),x]
```


```
[Out] -((2*a - 3*b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/(2*(a + b)^(
7/2)*f) + (2*a - 3*b)/(6*(a + b)^2*f*(a + b*Sin[e + f*x]^2)^(3/2)) + Sec[e
+ f*x]^2/(2*(a + b)*f*(a + b*Sin[e + f*x]^2)^(3/2)) + (2*a - 3*b)/(2*(a + b
)^3*f*Sqrt[a + b*Sin[e + f*x]^2])
```


Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
```

```
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\tan^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(1-x)^2(a+bx)^{5/2}}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= \frac{\sec^2(e+fx)}{2(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{(2a-3b)\,\text{Subst}\left(\int \frac{1}{(1-x)(a+bx)^{5/2}}\,dx, x, \sin^2(e+fx)\right)}{4(a+b)f}$$

$$= \frac{2a-3b}{6(a+b)^2 f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{\sec^2(e+fx)}{2(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{(2a-3b)\,\text{Subst}\left(\right.}{}$$

$$= \frac{2a-3b}{6(a+b)^2 f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{\sec^2(e+fx)}{2(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2a-}{2(a+b)^3 f\sqrt{a+b}}$$

$$= \frac{2a-3b}{6(a+b)^2 f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{\sec^2(e+fx)}{2(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2a-}{2(a+b)^3 f\sqrt{a+b}}$$

$$= -\frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{2(a+b)^{7/2}f} + \frac{2a-3b}{6(a+b)^2 f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{\sec^2(}{2(a+b)f\left(a+\right.}$$

Mathematica [C] time = 0.118391, size = 76, normalized size = 0.5

$$\frac{(2a-3b)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b\sin^2(e+fx)+a}{a+b}\right)+3(a+b)\sec^2(e+fx)}{6f(a+b)^2\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] ((2*a - 3*b)*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Sin[e + f*x]^2)/(a + b
)] + 3*(a + b)*Sec[e + f*x]^2)/(6*(a + b)^2*f*(a + b*Sin[e + f*x]^2)^(3/2))
```

Maple [B] time = 6.123, size = 1256, normalized size = 8.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/2/f*b^3/(b+(-a*b)^(1/2))^3/(-b+(-a*b)^(1/2))^3/(a+b)^(1/2)*ln((2*(a+b)^(1
/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+2*b*sin(f*x+e)+2*a)/(-1+sin(f*x+e)))*a-1/2/f
*b^4/(b+(-a*b)^(1/2))^3/(-b+(-a*b)^(1/2))^3/(a+b)^(1/2)*ln((2*(a+b)^(1/2)*(
a+b-b*cos(f*x+e)^2)^(1/2)+2*b*sin(f*x+e)+2*a)/(-1+sin(f*x+e)))+1/2/f*b^3/(b
+(-a*b)^(1/2))^3/(-b+(-a*b)^(1/2))^3/(-a*b)^(1/2)/(sin(f*x+e)+(-a*b)^(1/2)/
b)*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)*a-1/2/f*b^4/(b+(-a*b)^(1/2))^3/(-b+(
-a*b)^(1/2))^3/(-a*b)^(1/2)/(sin(f*x+e)+(-a*b)^(1/2)/b)*(-b*cos(f*x+e)^2+(a
*b+b^2)/b)^(1/2)-1/2/f*b^3/(b+(-a*b)^(1/2))^3/(-b+(-a*b)^(1/2))^3/(-a*b)^(1
/2)/(sin(f*x+e)-(-a*b)^(1/2)/b)*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)*a+1/2/f
*b^4/(b+(-a*b)^(1/2))^3/(-b+(-a*b)^(1/2))^3/(-a*b)^(1/2)/(sin(f*x+e)-(-a*b)
^(1/2)/b)*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)-1/12/f*b/(b+(-a*b)^(1/2))^2/(
-b+(-a*b)^(1/2))^2/(sin(f*x+e)-(-a*b)^(1/2)/b)^2*(-b*cos(f*x+e)^2+(a*b+b^2)
/b)^(1/2)-1/12/f*b*(-a*b)^(1/2)/(b+(-a*b)^(1/2))^2/(-b+(-a*b)^(1/2))^2/a/(s
in(f*x+e)-(-a*b)^(1/2)/b)*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)-1/12/f*b/(b+(
-a*b)^(1/2))^2/(-b+(-a*b)^(1/2))^2/(sin(f*x+e)+(-a*b)^(1/2)/b)^2*(-b*cos(f*
x+e)^2+(a*b+b^2)/b)^(1/2)+1/12/f*b*(-a*b)^(1/2)/(b+(-a*b)^(1/2))^2/(-b+(-a*
b)^(1/2))^2/a/(sin(f*x+e)+(-a*b)^(1/2)/b)*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/
2)+1/4/f*b^2/(b+(-a*b)^(1/2))^2/(-b+(-a*b)^(1/2))^2/(a+b)/(1+sin(f*x+e))*(a
+b-b*cos(f*x+e)^2)^(1/2)+1/4/f*b^3/(b+(-a*b)^(1/2))^2/(-b+(-a*b)^(1/2))^2/(
a+b)^(3/2)*ln((2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-2*b*sin(f*x+e)+2*a)
/(1+sin(f*x+e)))+1/2/f*b^3/(b+(-a*b)^(1/2))^3/(-b+(-a*b)^(1/2))^3/(a+b)^(1/
2)*ln((2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-2*b*sin(f*x+e)+2*a)/(1+sin(
f*x+e)))*a-1/2/f*b^4/(b+(-a*b)^(1/2))^3/(-b+(-a*b)^(1/2))^3/(a+b)^(1/2)*ln(
(2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-2*b*sin(f*x+e)+2*a)/(1+sin(f*x+e)
))-1/4/f*b^2/(b+(-a*b)^(1/2))^2/(-b+(-a*b)^(1/2))^2/(a+b)/(-1+sin(f*x+e))*(
a+b-b*cos(f*x+e)^2)^(1/2)+1/4/f*b^3/(b+(-a*b)^(1/2))^2/(-b+(-a*b)^(1/2))^2/
(a+b)^(3/2)*ln((2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+2*b*sin(f*x+e)+2*a
)/(-1+sin(f*x+e)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.42309, size = 1746, normalized size = 11.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*((2*a*b^2 - 3*b^3)*cos(f*x + e)^6 - 2*(2*a^2*b - a*b^2 - 3*b^3)*c
os(f*x + e)^4 + (2*a^3 + a^2*b - 4*a*b^2 - 3*b^3)*cos(f*x + e)^2)*sqrt(a +
b)*log((b*cos(f*x + e)^2 - 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b) -
2*a - 2*b)/cos(f*x + e)^2) + 2*(3*(2*a^2*b - a*b^2 - 3*b^3)*cos(f*x + e)^4
- 3*a^3 - 9*a^2*b - 9*a*b^2 - 3*b^3 - 4*(2*a^3 + a^2*b - 4*a*b^2 - 3*b^3)*c
os(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b))/((a^4*b^2 + 4*a^3*b^3 + 6*a
^2*b^4 + 4*a*b^5 + b^6)*f*cos(f*x + e)^6 - 2*(a^5*b + 5*a^4*b^2 + 10*a^3*b^
3 + 10*a^2*b^4 + 5*a*b^5 + b^6)*f*cos(f*x + e)^4 + (a^6 + 6*a^5*b + 15*a^4*
b^2 + 20*a^3*b^3 + 15*a^2*b^4 + 6*a*b^5 + b^6)*f*cos(f*x + e)^2), 1/6*(3*((
2*a*b^2 - 3*b^3)*cos(f*x + e)^6 - 2*(2*a^2*b - a*b^2 - 3*b^3)*cos(f*x + e)^
4 + (2*a^3 + a^2*b - 4*a*b^2 - 3*b^3)*cos(f*x + e)^2)*sqrt(-a - b)*arctan(s
qrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(a + b)) - (3*(2*a^2*b - a*b^2
- 3*b^3)*cos(f*x + e)^4 - 3*a^3 - 9*a^2*b - 9*a*b^2 - 3*b^3 - 4*(2*a^3 + a^
2*b - 4*a*b^2 - 3*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b))/((a
^4*b^2 + 4*a^3*b^3 + 6*a^2*b^4 + 4*a*b^5 + b^6)*f*cos(f*x + e)^6 - 2*(a^5*b
 + 5*a^4*b^2 + 10*a^3*b^3 + 10*a^2*b^4 + 5*a*b^5 + b^6)*f*cos(f*x + e)^4 +
(a^6 + 6*a^5*b + 15*a^4*b^2 + 20*a^3*b^3 + 15*a^2*b^4 + 6*a*b^5 + b^6)*f*co
s(f*x + e)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^{3}}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^3/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

3.534 $\int \frac{\tan(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=91

$$-\frac{1}{f(a+b)^2\sqrt{a+b\sin^2(e+fx)}}-\frac{1}{3f(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{f(a+b)^{5/2}}$$

```
[Out] ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]]/((a + b)^(5/2)*f) - 1/(3*(a
 + b)*f*(a + b*Sin[e + f*x]^2)^(3/2)) - 1/((a + b)^2*f*Sqrt[a + b*Sin[e + f
*x]^2])
```

Rubi [A] time = 0.0845923, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3194, 51, 63, 208}

$$-\frac{1}{f(a+b)^2\sqrt{a+b\sin^2(e+fx)}}-\frac{1}{3f(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{f(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]]/((a + b)^(5/2)*f) - 1/(3*(a
 + b)*f*(a + b*Sin[e + f*x]^2)^(3/2)) - 1/((a + b)^2*f*Sqrt[a + b*Sin[e + f
*x]^2])
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\tan(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x)(a+bx)^{5/2}}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= -\frac{1}{3(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{(1-x)(a+bx)^{3/2}}\,dx, x, \sin^2(e+fx)\right)}{2(a+b)f}$$

$$= -\frac{1}{3(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{1}{(a+b)^2 f\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\right.}{2(a+}$$

$$= -\frac{1}{3(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{1}{(a+b)^2 f\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx\right.}{b(}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}f} - \frac{1}{3(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{1}{(a+b)^2 f\sqrt{a+b\sin^2(e+f}}$$

Mathematica [C] time = 0.0821613, size = 56, normalized size = 0.62

$$-\frac{{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};1-\frac{b\cos^2(e+fx)}{a+b}\right)}{3f(a+b)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] -Hypergeometric2F1[-3/2, 1, -1/2, 1 - (b*Cos[e + f*x]^2)/(a + b)]/(3*(a + b
)*f*(a + b - b*Cos[e + f*x]^2)^(3/2))
```

Maple [B] time = 5.415, size = 895, normalized size = 9.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/6/b^3/(a+b)^(1/2)/a^2/(a^2*b^2*cos(f*x+e)^4+2*a*b^3*cos(f*x+e)^4+b^4*cos(
f*x+e)^4-2*a^3*b*cos(f*x+e)^2-6*a^2*b^2*cos(f*x+e)^2-6*a*b^3*cos(f*x+e)^2-2
*b^4*cos(f*x+e)^2+a^4+4*a^3*b+6*a^2*b^2+4*a*b^3+b^4)*(-8*a^3*b^3*(-b*cos(f*
x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*(a+b)^(1/2)-8*a^2*b^4*(-b*cos(f*x+e)^2+(a*b^2
+b^3)/b^2)^(1/2)*(a+b)^(1/2)+3*a^4*b^3*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a
+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))+3*a^4*b^3*ln(2/(1+sin(f*x+e))*((a
```

```
+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))+3*ln(2/(-1+sin(f*x+e)
)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^2*b^5+3*ln(2/(
1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^2*
b^5+6*a^3*b^4*ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+
b*sin(f*x+e)+a))+6*a^3*b^4*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+
e)^2)^(1/2)-b*sin(f*x+e)+a))+3*a^2*b^5*(ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a
+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))+ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)
*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a)))*cos(f*x+e)^4-6*cos(f*x+e)^2*a
^2*b^4*(ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f
*x+e)+a))*a+ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*s
in(f*x+e)+a))*b+ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2
)+b*sin(f*x+e)+a))*a+ln(2/(-1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)
^(1/2)+b*sin(f*x+e)+a))*b-(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*(a+b)^(1/
2)))/f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.62347, size = 1226, normalized size = 13.47

$$\left[\frac{3\left(b^2\cos\left(fx+e\right)^4-2\left(ab+b^2\right)\cos\left(fx+e\right)^2+a^2+2\,ab+b^2\right)\sqrt{a+b}\log\left(\frac{b\cos\left(fx+e\right)^2-2\sqrt{-b\cos\left(fx+e\right)^2+a+b}\sqrt{a+b}-2\,a-2\,b}{\cos\left(fx+e\right)^2}\right.}{6\left(\left(a^3b^2+3\,a^2b^3+3\,ab^4+b^5\right)f\cos\left(fx+e\right)^4-2\left(a^4b+4\,a^3b^2+6\,a^2b^3+4\,ab^4+b^5\right)f\cos\left(f\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^2*cos(f*x + e)^4 - 2*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b +
b^2)*sqrt(a + b)*log((b*cos(f*x + e)^2 - 2*sqrt(-b*cos(f*x + e)^2 + a + b)*
sqrt(a + b) - 2*a - 2*b)/cos(f*x + e)^2) + 2*(3*(a*b + b^2)*cos(f*x + e)^2
- 4*a^2 - 8*a*b - 4*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b))/((a^3*b^2 + 3*a^2
*b^3 + 3*a*b^4 + b^5)*f*cos(f*x + e)^4 - 2*(a^4*b + 4*a^3*b^2 + 6*a^2*b^3 +
 4*a*b^4 + b^5)*f*cos(f*x + e)^2 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3
 + 5*a*b^4 + b^5)*f), -1/3*(3*(b^2*cos(f*x + e)^4 - 2*(a*b + b^2)*cos(f*x +
 e)^2 + a^2 + 2*a*b + b^2)*sqrt(-a - b)*arctan(sqrt(-b*cos(f*x + e)^2 + a +
 b)*sqrt(-a - b)/(a + b)) - (3*(a*b + b^2)*cos(f*x + e)^2 - 4*a^2 - 8*a*b -
 4*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b))/((a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 +
b^5)*f*cos(f*x + e)^4 - 2*(a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*f
*cos(f*x + e)^2 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)
*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

3.535 $\int \frac{\cot(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=83

$$\frac{1}{a^2f\sqrt{a+b\sin^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}+\frac{1}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

```
[Out] -(ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]]/(a^(5/2)*f)) + 1/(3*a*f*(a +
b*Sin[e + f*x]^2)^(3/2)) + 1/(a^2*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.0876517, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3194, 51, 63, 208}

$$\frac{1}{a^2f\sqrt{a+b\sin^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}+\frac{1}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]]/(a^(5/2)*f)) + 1/(3*a*f*(a +
b*Sin[e + f*x]^2)^(3/2)) + 1/(a^2*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cot(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= \frac{1}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}dx, x, \sin^2(e+fx)\right)}{2af}$$

$$= \frac{1}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{1}{a^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}dx, x, \sin^2(e+\right.}{2a^2f}$$

$$= \frac{1}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{1}{a^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}dx, x, \sqrt{a+b\sin}\right.}{a^2bf}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} + \frac{1}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{1}{a^2f\sqrt{a+b\sin^2(e+fx)}}$$

Mathematica [C] time = 0.0643929, size = 49, normalized size = 0.59

$$\frac{{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b\sin^2(e+fx)}{a}+1\right)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*Sin[e + f*x]^2)/a]/(3*a*f*(a + b*Si
n[e + f*x]^2)^(3/2))
```

Maple [B] time = 2.888, size = 271, normalized size = 3.3

$$-\frac{7}{12\,a^2f}\sqrt{-b\left(\cos\left(fx+e\right)\right)^2+\frac{ab+b^2}{b}}\frac{1}{\sqrt{-ab}}\left(\sin\left(fx+e\right)+\frac{1}{b}\sqrt{-ab}\right)^{-1}+\frac{7}{12\,a^2f}\sqrt{-b\left(\cos\left(fx+e\right)\right)^2+\frac{ab+b^2}{b}}\frac{1}{\sqrt{-ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] -7/12/f/a^2/(-a*b)^(1/2)/(sin(f*x+e)+(-a*b)^(1/2)/b)*(-b*cos(f*x+e)^2+(a*b+
b^2)/b)^(1/2)+7/12/f/a^2/(-a*b)^(1/2)/(sin(f*x+e)-(-a*b)^(1/2)/b)*(-b*cos(f
*x+e)^2+(a*b+b^2)/b)^(1/2)-1/12/f/a^2/b/(sin(f*x+e)+(-a*b)^(1/2)/b)^2*(-b*c
os(f*x+e)^2+(a*b+b^2)/b)^(1/2)-1/12/f/a^2/b/(sin(f*x+e)-(-a*b)^(1/2)/b)^2*(
-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)-1/f/a^(5/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f
```

```
*x+e)^2)^(1/2))/sin(f*x+e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.57794, size = 911, normalized size = 10.98

$$\left[\frac{3\left(b^2\cos\left(fx+e\right)^4-2\left(ab+b^2\right)\cos\left(fx+e\right)^2+a^2+2\,ab+b^2\right)\sqrt{a}\log\left(\frac{2\left(b\cos\left(fx+e\right)^2+2\sqrt{-b\cos\left(fx+e\right)^2+a+b}\sqrt{a}-2\,a-b\right)}{\cos\left(fx+e\right)^2-1}\right)-2}{6\left(a^3b^2f\cos\left(fx+e\right)^4-2\left(a^4b+a^3b^2\right)f\cos\left(fx+e\right)^2+\left(a^5+2\,a^4b\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^2*cos(f*x + e)^4 - 2*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b +
b^2)*sqrt(a)*log(2*(b*cos(f*x + e)^2 + 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sq
rt(a) - 2*a - b)/(cos(f*x + e)^2 - 1)) - 2*(3*a*b*cos(f*x + e)^2 - 4*a^2 -
3*a*b)*sqrt(-b*cos(f*x + e)^2 + a + b))/(a^3*b^2*f*cos(f*x + e)^4 - 2*(a^4*
b + a^3*b^2)*f*cos(f*x + e)^2 + (a^5 + 2*a^4*b + a^3*b^2)*f), 1/3*(3*(b^2*c
os(f*x + e)^4 - 2*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(-a)*
arctan(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a)/a) - (3*a*b*cos(f*x + e)^2
- 4*a^2 - 3*a*b)*sqrt(-b*cos(f*x + e)^2 + a + b))/(a^3*b^2*f*cos(f*x + e)^4
 - 2*(a^4*b + a^3*b^2)*f*cos(f*x + e)^2 + (a^5 + 2*a^4*b + a^3*b^2)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10856, size = 100, normalized size = 1.2

$$\frac{\arctan\left(\frac{\sqrt{b\sin\left(fx+e\right)^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^{2}f}+\frac{3\,b\sin\left(fx+e\right)^{2}+4\,a}{3\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}a^{2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*sin(f*x + e)^2 + a)/sqrt(-a))/(sqrt(-a)*a^2*f) + 1/3*(3*b*sin
(f*x + e)^2 + 4*a)/((b*sin(f*x + e)^2 + a)^(3/2)*a^2*f)
```

3.536 $\int \frac{\cot^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=143

$$-\frac{2a+5b}{2a^3f\sqrt{a+b\sin^2(e+fx)}}-\frac{2a+5b}{6a^2f\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{(2a+5b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{2a^{7/2}f}-\frac{\csc^2(e+fx)}{2af\left(a+b\sin^2(e+f\right.}$$

```
[Out] ((2*a + 5*b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/(2*a^(7/2)*f) - (
2*a + 5*b)/(6*a^2*f*(a + b*Sin[e + f*x]^2)^(3/2)) - Csc[e + f*x]^2/(2*a*f*(
a + b*Sin[e + f*x]^2)^(3/2)) - (2*a + 5*b)/(2*a^3*f*Sqrt[a + b*Sin[e + f*x]
^2])
```

Rubi [A] time = 0.13723, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3194, 78, 51, 63, 208}

$$-\frac{2a+5b}{2a^3f\sqrt{a+b\sin^2(e+fx)}}-\frac{2a+5b}{6a^2f\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{(2a+5b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{2a^{7/2}f}-\frac{\csc^2(e+fx)}{2af\left(a+b\sin^2(e+f\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(5/2),x]
```


```
[Out] ((2*a + 5*b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/(2*a^(7/2)*f) - (
2*a + 5*b)/(6*a^2*f*(a + b*Sin[e + f*x]^2)^(3/2)) - Csc[e + f*x]^2/(2*a*f*(
a + b*Sin[e + f*x]^2)^(3/2)) - (2*a + 5*b)/(2*a^3*f*Sqrt[a + b*Sin[e + f*x]
^2])
```


Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
```

```
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1-x}{x^2(a+bx)^{5/2}}\,dx, x, \sin^2(e+fx)\right)}{2f}\\
&= -\frac{\csc^2(e+fx)}{2af\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{(2a+5b)\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}\,dx, x, \sin^2(e+fx)\right)}{4af}\\
&= -\frac{2a+5b}{6a^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\csc^2(e+fx)}{2af\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{(2a+5b)\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/}}\right.}{4a^2f}\\
&= -\frac{2a+5b}{6a^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\csc^2(e+fx)}{2af\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{2a+5b}{2a^3f\sqrt{a+b\sin^2(e+fx)}} -\\
&= -\frac{2a+5b}{6a^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\csc^2(e+fx)}{2af\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{2a+5b}{2a^3f\sqrt{a+b\sin^2(e+fx)}} -\\
&= \frac{(2a+5b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{2a^{7/2}f} - \frac{2a+5b}{6a^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\csc^2(e+fx)}{2af\left(a+b\sin^2(e+fx\right.}
\end{aligned}
$$

Mathematica [C] time = 0.261647, size = 69, normalized size = 0.48

$$
-\frac{(2a+5b)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b\sin^2(e+fx)}{a}+1\right)+3a\csc^2(e+fx)}{6a^2f\left(a+b\sin^2(e+fx)\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] -(3*a*Csc[e + f*x]^2 + (2*a + 5*b)*Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*
Sin[e + f*x]^2)/a])/(6*a^2*f*(a + b*Sin[e + f*x]^2)^(3/2))
```

Maple [B] time = 4.118, size = 1038, normalized size = 7.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/6/a^(13/2)/b^2/(b^2*cos(f*x+e)^6-2*a*b*cos(f*x+e)^4-3*b^2*cos(f*x+e)^4+a^
2*cos(f*x+e)^2+4*a*b*cos(f*x+e)^2+3*b^2*cos(f*x+e)^2-a^2-2*a*b-b^2)*(3*(a+b
-b*cos(f*x+e)^2)^(1/2)*a^(11/2)*b^2-6*ln(2/sin(f*x+e)*(a^(1/2)*(a+b-b*cos(f
*x+e)^2)^(1/2)+a))*a^6*b^2+3*(a+b-b*cos(f*x+e)^2)^(1/2)*a^(7/2)*b^4+8*a^(11
/2)*b^2*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)+20*a^(9/2)*(-b*cos(f*x+e)^2
+(a*b^2+b^3)/b^2)^(1/2)*b^3+6*(a+b-b*cos(f*x+e)^2)^(1/2)*a^(9/2)*b^3+12*a^(
7/2)*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*b^4-27*ln(2/sin(f*x+e)*(a^(1/2
)*(a+b-b*cos(f*x+e)^2)^(1/2)+a))*a^5*b^3-36*ln(2/sin(f*x+e)*(a^(1/2)*(a+b-b
*cos(f*x+e)^2)^(1/2)+a))*a^4*b^4-15*ln(2/sin(f*x+e)*(a^(1/2)*(a+b-b*cos(f*x
+e)^2)^(1/2)+a))*a^3*b^5+3*ln(2/sin(f*x+e)*(a^(1/2)*(a+b-b*cos(f*x+e)^2)^(1
/2)+a))*a^3*b^4*(2*a+5*b)*cos(f*x+e)^6-3*cos(f*x+e)^4*b^3*(-2*a^(9/2)*(-b*c
os(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)-(a+b-b*cos(f*x+e)^2)^(1/2)*a^(7/2)*b-4*a
^(7/2)*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*b+4*ln(2/sin(f*x+e)*(a^(1/2)
*(a+b-b*cos(f*x+e)^2)^(1/2)+a))*a^5+16*ln(2/sin(f*x+e)*(a^(1/2)*(a+b-b*cos(
f*x+e)^2)^(1/2)+a))*a^4*b+15*ln(2/sin(f*x+e)*(a^(1/2)*(a+b-b*cos(f*x+e)^2)^
(1/2)+a))*a^3*b^2)+cos(f*x+e)^2*b^2*(-8*a^(11/2)*(-b*cos(f*x+e)^2+(a*b^2+b^
3)/b^2)^(1/2)-6*(a+b-b*cos(f*x+e)^2)^(1/2)*a^(9/2)*b-26*a^(9/2)*(-b*cos(f*x
+e)^2+(a*b^2+b^3)/b^2)^(1/2)*b-6*(a+b-b*cos(f*x+e)^2)^(1/2)*a^(7/2)*b^2-24*
a^(7/2)*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*b^2+6*ln(2/sin(f*x+e)*(a^(1
/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+a))*a^6+39*ln(2/sin(f*x+e)*(a^(1/2)*(a+b-b*c
os(f*x+e)^2)^(1/2)+a))*a^5*b+78*ln(2/sin(f*x+e)*(a^(1/2)*(a+b-b*cos(f*x+e)^
2)^(1/2)+a))*a^4*b^2+45*ln(2/sin(f*x+e)*(a^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)
+a))*a^3*b^3))/f
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.77743, size = 1540, normalized size = 10.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((2*a*b^2 + 5*b^3)*cos(f*x + e)^6 - (4*a^2*b + 16*a*b^2 + 15*b^3)*
cos(f*x + e)^4 - 2*a^3 - 9*a^2*b - 12*a*b^2 - 5*b^3 + (2*a^3 + 13*a^2*b + 2
6*a*b^2 + 15*b^3)*cos(f*x + e)^2)*sqrt(a)*log(2*(b*cos(f*x + e)^2 - 2*sqrt(
-b*cos(f*x + e)^2 + a + b)*sqrt(a) - 2*a - b)/(cos(f*x + e)^2 - 1)) + 2*(3*
```

```
(2*a^2*b + 5*a*b^2)*cos(f*x + e)^4 + 11*a^3 + 26*a^2*b + 15*a*b^2 - 2*(4*a^
3 + 16*a^2*b + 15*a*b^2)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b))/(
a^4*b^2*f*cos(f*x + e)^6 - (2*a^5*b + 3*a^4*b^2)*f*cos(f*x + e)^4 + (a^6 +
4*a^5*b + 3*a^4*b^2)*f*cos(f*x + e)^2 - (a^6 + 2*a^5*b + a^4*b^2)*f), -1/6*
(3*((2*a*b^2 + 5*b^3)*cos(f*x + e)^6 - (4*a^2*b + 16*a*b^2 + 15*b^3)*cos(f*
x + e)^4 - 2*a^3 - 9*a^2*b - 12*a*b^2 - 5*b^3 + (2*a^3 + 13*a^2*b + 26*a*b^
2 + 15*b^3)*cos(f*x + e)^2)*sqrt(-a)*arctan(sqrt(-b*cos(f*x + e)^2 + a + b)
*sqrt(-a)/a) - (3*(2*a^2*b + 5*a*b^2)*cos(f*x + e)^4 + 11*a^3 + 26*a^2*b +
15*a*b^2 - 2*(4*a^3 + 16*a^2*b + 15*a*b^2)*cos(f*x + e)^2)*sqrt(-b*cos(f*x
+ e)^2 + a + b))/(a^4*b^2*f*cos(f*x + e)^6 - (2*a^5*b + 3*a^4*b^2)*f*cos(f*
x + e)^4 + (a^6 + 4*a^5*b + 3*a^4*b^2)*f*cos(f*x + e)^2 - (a^6 + 2*a^5*b +
a^4*b^2)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)^3}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^3/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

3.537 $\int \frac{\cot^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=208

$$\frac{8a^2+40ab+35b^2}{8a^4f\sqrt{a+b\sin^2(e+fx)}}+\frac{8a^2+40ab+35b^2}{24a^3f\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\left(8a^2+40ab+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{8a^{9/2}f}+\frac{(8a+7}{8a^2f\left(a+\right.}$$

```
[Out] -((8*a^2 + 40*a*b + 35*b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/(8
*a^(9/2)*f) + (8*a^2 + 40*a*b + 35*b^2)/(24*a^3*f*(a + b*Sin[e + f*x]^2)^(3
/2)) + ((8*a + 7*b)*Csc[e + f*x]^2)/(8*a^2*f*(a + b*Sin[e + f*x]^2)^(3/2))
- Csc[e + f*x]^4/(4*a*f*(a + b*Sin[e + f*x]^2)^(3/2)) + (8*a^2 + 40*a*b + 3
5*b^2)/(8*a^4*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.195977, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3194, 89, 78, 51, 63, 208}

$$\frac{8a^2+40ab+35b^2}{8a^4f\sqrt{a+b\sin^2(e+fx)}}+\frac{8a^2+40ab+35b^2}{24a^3f\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\left(8a^2+40ab+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{8a^{9/2}f}+\frac{(8a+7}{8a^2f\left(a+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] -((8*a^2 + 40*a*b + 35*b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/(8
*a^(9/2)*f) + (8*a^2 + 40*a*b + 35*b^2)/(24*a^3*f*(a + b*Sin[e + f*x]^2)^(3
/2)) + ((8*a + 7*b)*Csc[e + f*x]^2)/(8*a^2*f*(a + b*Sin[e + f*x]^2)^(3/2))
- Csc[e + f*x]^4/(4*a*f*(a + b*Sin[e + f*x]^2)^(3/2)) + (8*a^2 + 40*a*b + 3
5*b^2)/(8*a^4*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cot^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x)^2}{x^3(a+bx)^{5/2}}\,dx,x,\sin^2(e+fx)\right)}{2f}$$

$$= -\frac{\csc^4(e+fx)}{4af\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(-8a-7b)+2ax}{x^2(a+bx)^{5/2}}\,dx,x,\sin^2(e+fx)\right)}{4af}$$

$$= \frac{(8a+7b)\csc^2(e+fx)}{8a^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\csc^4(e+fx)}{4af\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{\left(8a^2+40ab+35b^2\right)\text{Subst}\left(\ldots\right.}{16\ldots}$$

$$= \frac{8a^2+40ab+35b^2}{24a^3f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{(8a+7b)\csc^2(e+fx)}{8a^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\csc^4(e+fx)}{4af\left(a+b\sin^2(e+fx)\right)^{3/2}}\ldots$$

$$= \frac{8a^2+40ab+35b^2}{24a^3f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{(8a+7b)\csc^2(e+fx)}{8a^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\csc^4(e+fx)}{4af\left(a+b\sin^2(e+fx)\right)^{3/2}}\ldots$$

$$= \frac{8a^2+40ab+35b^2}{24a^3f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{(8a+7b)\csc^2(e+fx)}{8a^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\csc^4(e+fx)}{4af\left(a+b\sin^2(e+fx)\right)^{3/2}}\ldots$$

$$= -\frac{\left(8a^2+40ab+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{8a^{9/2}f} + \frac{8a^2+40ab+35b^2}{24a^3f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{(8a+\ldots}{8a^2f\left(a\ldots\right.}$$

Mathematica [C] time = 0.867869, size = 117, normalized size = 0.56

$$\frac{(8a^2+40ab+35b^2)\csc^2(e+fx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b\sin^2(e+fx)}{a}+1\right)+3a\csc^4(e+fx)\left(-2a\csc^2(e+fx)+8a+7b\right)}{24a^3f\sqrt{a+b\sin^2(e+fx)}\left(a\csc^2(e+fx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (3*a*Csc[e + f*x]^4*(8*a + 7*b - 2*a*Csc[e + f*x]^2) + (8*a^2 + 40*a*b + 35
*b^2)*Csc[e + f*x]^2*Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*Sin[e + f*x]^2
)/a])/(24*a^3*f*(b + a*Csc[e + f*x]^2)*Sqrt[a + b*Sin[e + f*x]^2])
```

Maple [B] time = 4.849, size = 901, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] -7/12/f/a^2/(-a*b)^(1/2)/(sin(f*x+e)+(-a*b)^(1/2)/b)*(-b*cos(f*x+e)^2+(a*b+
b^2)/b)^(1/2)-13/6/f/a^3/(-a*b)^(1/2)*b/(sin(f*x+e)+(-a*b)^(1/2)/b)*(-b*cos
(f*x+e)^2+(a*b+b^2)/b)^(1/2)-19/12/f/a^4/(-a*b)^(1/2)/(sin(f*x+e)+(-a*b)^(1
/2)/b)*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)*b^2-1/12/f/a^2/b/(sin(f*x+e)-(-a
*b)^(1/2)/b)^2*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)-1/6/f/a^3/(sin(f*x+e)-(-
a*b)^(1/2)/b)^2*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)-1/12/f/a^4*b/(sin(f*x+e
)-(-a*b)^(1/2)/b)^2*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)+7/12/f/a^2/(-a*b)^(
1/2)/(sin(f*x+e)-(-a*b)^(1/2)/b)*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)+13/6/f
/a^3/(-a*b)^(1/2)*b/(sin(f*x+e)-(-a*b)^(1/2)/b)*(-b*cos(f*x+e)^2+(a*b+b^2)/
b)^(1/2)+19/12/f/a^4/(-a*b)^(1/2)/(sin(f*x+e)-(-a*b)^(1/2)/b)*(-b*cos(f*x+e
)^2+(a*b+b^2)/b)^(1/2)*b^2-1/12/f/a^2/b/(sin(f*x+e)+(-a*b)^(1/2)/b)^2*(-b*c
os(f*x+e)^2+(a*b+b^2)/b)^(1/2)-1/6/f/a^3/(sin(f*x+e)+(-a*b)^(1/2)/b)^2*(-b*
cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)-1/12/f/a^4*b/(sin(f*x+e)+(-a*b)^(1/2)/b)^2*
(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)-1/4/f/a^3/sin(f*x+e)^4*(a+b*sin(f*x+e)^
2)^(1/2)+11/8/f/a^4*b/sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)-35/8/f/a^(9/2)*
b^2*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e))-1/f/a^(5/2)*ln(
(2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e))-5/f/a^(7/2)*ln((2*a+2*
a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e))*b+1/f/a^3/sin(f*x+e)^2*(a+b*s
in(f*x+e)^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.16792, size = 2334, normalized size = 11.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*((8*a^2*b^2 + 40*a*b^3 + 35*b^4)*cos(f*x + e)^8 - 2*(8*a^3*b + 56*
a^2*b^2 + 115*a*b^3 + 70*b^4)*cos(f*x + e)^6 + (8*a^4 + 88*a^3*b + 323*a^2*
b^2 + 450*a*b^3 + 210*b^4)*cos(f*x + e)^4 + 8*a^4 + 56*a^3*b + 123*a^2*b^2
+ 110*a*b^3 + 35*b^4 - 2*(8*a^4 + 64*a^3*b + 171*a^2*b^2 + 185*a*b^3 + 70*b
^4)*cos(f*x + e)^2)*sqrt(a)*log(2*(b*cos(f*x + e)^2 + 2*sqrt(-b*cos(f*x + e
)^2 + a + b)*sqrt(a) - 2*a - b)/(cos(f*x + e)^2 - 1)) - 2*(3*(8*a^3*b + 40*
a^2*b^2 + 35*a*b^3)*cos(f*x + e)^6 - (32*a^4 + 232*a^3*b + 500*a^2*b^2 + 31
5*a*b^3)*cos(f*x + e)^4 - 50*a^4 - 205*a^3*b - 260*a^2*b^2 - 105*a*b^3 + (8
8*a^4 + 413*a^3*b + 640*a^2*b^2 + 315*a*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*
x + e)^2 + a + b))/(a^5*b^2*f*cos(f*x + e)^8 - 2*(a^6*b + 2*a^5*b^2)*f*cos(
f*x + e)^6 + (a^7 + 6*a^6*b + 6*a^5*b^2)*f*cos(f*x + e)^4 - 2*(a^7 + 3*a^6*
b + 2*a^5*b^2)*f*cos(f*x + e)^2 + (a^7 + 2*a^6*b + a^5*b^2)*f), 1/24*(3*((8
*a^2*b^2 + 40*a*b^3 + 35*b^4)*cos(f*x + e)^8 - 2*(8*a^3*b + 56*a^2*b^2 + 11
5*a*b^3 + 70*b^4)*cos(f*x + e)^6 + (8*a^4 + 88*a^3*b + 323*a^2*b^2 + 450*a*
b^3 + 210*b^4)*cos(f*x + e)^4 + 8*a^4 + 56*a^3*b + 123*a^2*b^2 + 110*a*b^3
+ 35*b^4 - 2*(8*a^4 + 64*a^3*b + 171*a^2*b^2 + 185*a*b^3 + 70*b^4)*cos(f*x
+ e)^2)*sqrt(-a)*arctan(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a)/a) - (3*(8
*a^3*b + 40*a^2*b^2 + 35*a*b^3)*cos(f*x + e)^6 - (32*a^4 + 232*a^3*b + 500*
a^2*b^2 + 315*a*b^3)*cos(f*x + e)^4 - 50*a^4 - 205*a^3*b - 260*a^2*b^2 - 10
5*a*b^3 + (88*a^4 + 413*a^3*b + 640*a^2*b^2 + 315*a*b^3)*cos(f*x + e)^2)*sq
rt(-b*cos(f*x + e)^2 + a + b))/(a^5*b^2*f*cos(f*x + e)^8 - 2*(a^6*b + 2*a^5
*b^2)*f*cos(f*x + e)^6 + (a^7 + 6*a^6*b + 6*a^5*b^2)*f*cos(f*x + e)^4 - 2*(
a^7 + 3*a^6*b + 2*a^5*b^2)*f*cos(f*x + e)^2 + (a^7 + 2*a^6*b + a^5*b^2)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)^5}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^5/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

3.538 $\int \frac{\tan^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=348

$$-\frac{2(2a-b)\tan(e+fx)}{3f(a+b)^2\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{8b(a-b)\sin(e+fx)\cos(e+fx)}{3f(a+b)^4\sqrt{a+b\sin^2(e+fx)}}+\frac{b(5a-3b)\sin(e+fx)\cos(e+fx)}{3f(a+b)^3\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{\text{ta}}{3f(a+}$$

```
[Out] ((5*a - 3*b)*b*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)^3*f*(a + b*Sin[e + f*x
]^2)^(3/2)) + (8*(a - b)*b*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)^4*f*Sqrt[a
 + b*Sin[e + f*x]^2]) + (8*(a - b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Si
n[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*(a + b)^4*
f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) - ((5*a - 3*b)*Sqrt[Cos[e + f*x]^2]*Ellip
ticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)
/a])/(3*(a + b)^3*f*Sqrt[a + b*Sin[e + f*x]^2]) - (2*(2*a - b)*Tan[e + f*x]
)/(3*(a + b)^2*f*(a + b*Sin[e + f*x]^2)^(3/2)) + (Sec[e + f*x]^2*Tan[e + f*
x])/(3*(a + b)*f*(a + b*Sin[e + f*x]^2)^(3/2))
```

Rubi [A] time = 0.426439, antiderivative size = 348, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3196, 470, 527, 524, 426, 424, 421, 419}

$$-\frac{2(2a-b)\tan(e+fx)}{3f(a+b)^2\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{8b(a-b)\sin(e+fx)\cos(e+fx)}{3f(a+b)^4\sqrt{a+b\sin^2(e+fx)}}+\frac{b(5a-3b)\sin(e+fx)\cos(e+fx)}{3f(a+b)^3\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{\text{ta}}{3f(a+}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(5/2),x]
```


```
[Out] ((5*a - 3*b)*b*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)^3*f*(a + b*Sin[e + f*x
]^2)^(3/2)) + (8*(a - b)*b*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)^4*f*Sqrt[a
 + b*Sin[e + f*x]^2]) + (8*(a - b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Si
n[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*(a + b)^4*
f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) - ((5*a - 3*b)*Sqrt[Cos[e + f*x]^2]*Ellip
ticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)
/a])/(3*(a + b)^3*f*Sqrt[a + b*Sin[e + f*x]^2]) - (2*(2*a - b)*Tan[e + f*x]
)/(3*(a + b)^2*f*(a + b*Sin[e + f*x]^2)^(3/2)) + (Sec[e + f*x]^2*Tan[e + f*
x])/(3*(a + b)*f*(a + b*Sin[e + f*x]^2)^(3/2))
```


Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
```

```
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^4}{(1-x^2)^{5/2}(a+bx^2)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sec^2(e+fx)\tan(e+fx)}{3(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{a+(3a-2b)x}{(1-x^2)^{3/2}(a+bx}\right.}{3(a+b)f}$$

$$= -\frac{2(2a-b)\tan(e+fx)}{3(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{\sec^2(e+fx)\tan(e+fx)}{3(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\left(\sqrt{\cos^2(e+}\right.}{}$$

$$= \frac{(5a-3b)b\cos(e+fx)\sin(e+fx)}{3(a+b)^3f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{2(2a-b)\tan(e+fx)}{3(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{\sec^2(e+}{3(a+b)f(a}$$

$$= \frac{(5a-3b)b\cos(e+fx)\sin(e+fx)}{3(a+b)^3f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{8(a-b)b\cos(e+fx)\sin(e+fx)}{3(a+b)^4f\sqrt{a+b\sin^2(e+fx)}} - \frac{2(2a-}{3(a+b)^2f(a}$$

$$= \frac{(5a-3b)b\cos(e+fx)\sin(e+fx)}{3(a+b)^3f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{8(a-b)b\cos(e+fx)\sin(e+fx)}{3(a+b)^4f\sqrt{a+b\sin^2(e+fx)}} - \frac{2(2a-}{3(a+b)^2f(a}$$

$$= \frac{(5a-3b)b\cos(e+fx)\sin(e+fx)}{3(a+b)^3f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{8(a-b)b\cos(e+fx)\sin(e+fx)}{3(a+b)^4f\sqrt{a+b\sin^2(e+fx)}} - \frac{2(2a-}{3(a+b)^2f(a}$$

$$= \frac{(5a-3b)b\cos(e+fx)\sin(e+fx)}{3(a+b)^3f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{8(a-b)b\cos(e+fx)\sin(e+fx)}{3(a+b)^4f\sqrt{a+b\sin^2(e+fx)}} + \frac{8(a-b)\sqrt{\text{co}}}{}$$

Mathematica [A] time = 3.44579, size = 235, normalized size = 0.68

$$\frac{2ab\left(\frac{2a-b\cos(2(e+fx))+b}{a}\right)^{3/2}\left(\left(-5a^2-2ab+3b^2\right)F\left(e+fx\middle|-\frac{b}{a}\right)+8a(a-b)E\left(e+fx\middle|-\frac{b}{a}\right)\right)+\sqrt{2}b\left(2ab(a+b)\sin(2(e+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (2*a*b*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*(8*a*(a - b)*EllipticE[e +
f*x, -(b/a)] + (-5*a^2 - 2*a*b + 3*b^2)*EllipticF[e + f*x, -(b/a)]) + Sqrt[
2]*b*(2*a*b*(a + b)*Sin[2*(e + f*x)] + 4*(a - b)*b*(2*a + b - b*Cos[2*(e +
f*x)])*Sin[2*(e + f*x)] - 4*(a - b)*(2*a + b - b*Cos[2*(e + f*x)])^2*Tan[e
+ f*x] + (a + b)*(2*a + b - b*Cos[2*(e + f*x)])^2*Sec[e + f*x]^2*Tan[e + f*
x]))/(6*b*(a + b)^4*f*(2*a + b - b*Cos[2*(e + f*x)])^(3/2))
```

Maple [B] time = 3.062, size = 666, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/3*(-8*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b^2*(a-b)*sin(f*x+e)*co
s(f*x+e)^6+(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*(13*a^2+2*a*b-11*b^
2)*cos(f*x+e)^4*sin(f*x+e)-2*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(2*
a^3+3*a^2*b-b^3)*cos(f*x+e)^2*sin(f*x+e)+(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^
2)^(1/2)*(a^3+3*a^2*b+3*a*b^2+b^3)*sin(f*x+e)-(-b*cos(f*x+e)^4+(a+b)*cos(f*
x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(cos(f*x+e)^2)^(1/2)*b*(8*E
llipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2-8*EllipticE(sin(f*x+e),(-1/a*b)^(1/
2))*a*b-5*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2-2*EllipticF(sin(f*x+e),(
-1/a*b)^(1/2))*a*b+3*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^2)*cos(f*x+e)^4
+(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/
2)*(cos(f*x+e)^2)^(1/2)*(8*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3-8*Ellip
ticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2-5*EllipticF(sin(f*x+e),(-1/a*b)^(1/2)
)*a^3-7*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b+EllipticF(sin(f*x+e),(-1
/a*b)^(1/2))*a*b^2+3*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^3)*cos(f*x+e)^2
)/(-1+sin(f*x+e))/(a+b*sin(f*x+e)^2)^(3/2)/(1+sin(f*x+e))/(-(a+b*sin(f*x+e)
^2)*(-1+sin(f*x+e))*(1+sin(f*x+e)))^(1/2)/(a+b)^4/cos(f*x+e)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\tan\left(fx+e\right)^{4}}{b^{3}\cos\left(fx+e\right)^{6}-3\left(ab^{2}+b^{3}\right)\cos\left(fx+e\right)^{4}-a^{3}-3a^{2}b-3ab^{2}-b^{3}+3\left(a^{2}b+2ab^{2}+b^{3}\right)\cos\left(fx+e\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)*tan(f*x + e)^4/(b^3*cos(f*x + e)^
6 - 3*(a*b^2 + b^3)*cos(f*x + e)^4 - a^3 - 3*a^2*b - 3*a*b^2 - b^3 + 3*(a^2
*b + 2*a*b^2 + b^3)*cos(f*x + e)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^{4}}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^4/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

3.539 $\int \frac{\tan^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=292

$$\frac{\tan(e+fx)}{f(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{b(7a-b)\sin(e+fx)\cos(e+fx)}{3af(a+b)^3\sqrt{a+b\sin^2(e+fx)}}-\frac{4b\sin(e+fx)\cos(e+fx)}{3f(a+b)^2\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{4\sqrt{\cos^2(e+f}}{}$$

```
[Out] (-4*b*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)^2*f*(a + b*Sin[e + f*x]^2)^(3/2
)) - ((7*a - b)*b*Cos[e + f*x]*Sin[e + f*x])/(3*a*(a + b)^3*f*Sqrt[a + b*Si
n[e + f*x]^2]) - ((7*a - b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f
*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a*(a + b)^3*f*Sqr
t[1 + (b*Sin[e + f*x]^2)/a]) + (4*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin
[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*(a + b)
^2*f*Sqrt[a + b*Sin[e + f*x]^2]) + Tan[e + f*x]/((a + b)*f*(a + b*Sin[e + f
*x]^2)^(3/2))
```

Rubi [A] time = 0.309345, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3196, 471, 527, 524, 426, 424, 421, 419}

$$\frac{\tan(e+fx)}{f(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{b(7a-b)\sin(e+fx)\cos(e+fx)}{3af(a+b)^3\sqrt{a+b\sin^2(e+fx)}}-\frac{4b\sin(e+fx)\cos(e+fx)}{3f(a+b)^2\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{4\sqrt{\cos^2(e+f}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (-4*b*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)^2*f*(a + b*Sin[e + f*x]^2)^(3/2
)) - ((7*a - b)*b*Cos[e + f*x]*Sin[e + f*x])/(3*a*(a + b)^3*f*Sqrt[a + b*Si
n[e + f*x]^2]) - ((7*a - b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f
*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a*(a + b)^3*f*Sqr
t[1 + (b*Sin[e + f*x]^2)/a]) + (4*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin
[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*(a + b)
^2*f*Sqrt[a + b*Sin[e + f*x]^2]) + Tan[e + f*x]/((a + b)*f*(a + b*Sin[e + f
*x]^2)^(3/2))
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
```

```
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^2}{(1-x^2)^{3/2}(a+bx^2)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)}{(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{a-3bx^2}{\sqrt{1-x^2}(a+bx^2)^{5/2}}\,dx\right.}{(a+b)f}$$

$$= -\frac{4b\cos(e+fx)\sin(e+fx)}{3(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{\tan(e+fx)}{(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{\left(\sqrt{\cos^2(e+fx)}\right.}{}$$

$$= -\frac{4b\cos(e+fx)\sin(e+fx)}{3(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{(7a-b)b\cos(e+fx)\sin(e+fx)}{3a(a+b)^3f\sqrt{a+b\sin^2(e+fx)}} + \frac{\tan(e}{(a+b)f(a+b}$$

$$= -\frac{4b\cos(e+fx)\sin(e+fx)}{3(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{(7a-b)b\cos(e+fx)\sin(e+fx)}{3a(a+b)^3f\sqrt{a+b\sin^2(e+fx)}} + \frac{\tan(e}{(a+b)f(a+b}$$

$$= -\frac{4b\cos(e+fx)\sin(e+fx)}{3(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{(7a-b)b\cos(e+fx)\sin(e+fx)}{3a(a+b)^3f\sqrt{a+b\sin^2(e+fx)}} + \frac{\tan(e}{(a+b)f(a+b}$$

$$= -\frac{4b\cos(e+fx)\sin(e+fx)}{3(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{(7a-b)b\cos(e+fx)\sin(e+fx)}{3a(a+b)^3f\sqrt{a+b\sin^2(e+fx)}} - \frac{(7a-b)\sqrt{\cos^2(}}{}$$

Mathematica [A] time = 2.69507, size = 199, normalized size = 0.68

$$\frac{\frac{\tan(e+fx)\left(4a^2b+24a^3+b^2(7a-b)\cos(4(e+fx))+5ab^2-4ab(11a+3b)\cos(2(e+fx))+b^3\right)}{\sqrt{2}} + 8a^2(a+b)\left(\frac{2a-b\cos(2(e+fx))+b}{a}\right)^{3/2}F\left(e+fx\middle|-\frac{b}{a}\right) - 2a^2(}{6af(a+b)^3(2a-b\cos(2(e+fx))+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (-2*a^2*(7*a - b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticE[e + f*
x, -(b/a)] + 8*a^2*(a + b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*Ellipti
cF[e + f*x, -(b/a)] + ((24*a^3 + 4*a^2*b + 5*a*b^2 + b^3 - 4*a*b*(11*a + 3*
b)*Cos[2*(e + f*x)] + (7*a - b)*b^2*Cos[4*(e + f*x)])*Tan[e + f*x])/Sqrt[2]
)/(6*a*(a + b)^3*f*(2*a + b - b*Cos[2*(e + f*x)])^(3/2))
```

Maple [B] time = 2.592, size = 851, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3*((-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b^2*(7*a-b)*sin(f*x+e)*cos(
f*x+e)^4-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*(11*a^2+10*a*b-b^2)*c
os(f*x+e)^2*sin(f*x+e)+3*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*a*(a^2+
2*a*b+b^2)*sin(f*x+e)-(cos(f*x+e)^2)^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e
)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*a*b*(4*EllipticF(sin(f*x+e),(-
1/a*b)^(1/2))*a+4*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-7*EllipticE(sin(f*
x+e),(-1/a*b)^(1/2))*a+EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*b)*cos(f*x+e)^2
+4*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(cos(f*x+e)^2)^(1/2)*(-b/a*co
s(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3+8*(-b*co
s(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^
2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b+4*(-b*cos(f*x+e
)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)
/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2-7*(-b*cos(f*x+e)^4+(a+
b)*cos(f*x+e)^2)^(1/2)*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/
2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3-6*(-b*cos(f*x+e)^4+(a+b)*cos(f*
x+e)^2)^(1/2)*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*Ellipt
icE(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b+(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(
1/2)*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f
*x+e),(-1/a*b)^(1/2))*a*b^2)/(a+b*sin(f*x+e)^2)^(3/2)/(-(a+b*sin(f*x+e)^2)*
(-1+sin(f*x+e))*(1+sin(f*x+e)))^(1/2)/(a+b)^3/a/cos(f*x+e)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\tan\left(fx+e\right)^{2}}{b^{3}\cos\left(fx+e\right)^{6}-3\left(ab^{2}+b^{3}\right)\cos\left(fx+e\right)^{4}-a^{3}-3a^{2}b-3ab^{2}-b^{3}+3\left(a^{2}b+2ab^{2}+b^{3}\right)\cos\left(fx+e\right)^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)*tan(f*x + e)^2/(b^3*cos(f*x + e)^
6 - 3*(a*b^2 + b^3)*cos(f*x + e)^4 - a^3 - 3*a^2*b - 3*a*b^2 - b^3 + 3*(a^2
*b + 2*a*b^2 + b^3)*cos(f*x + e)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan \left(fx+e\right)^{2}}{\left(b\sin \left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

3.540 $\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=223

$$\frac{2b(2a+b)\sin(e+fx)\cos(e+fx)}{3a^2f(a+b)^2\sqrt{a+b\sin^2(e+fx)}}+\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{3a^2f(a+b)^2\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{b\sin(e+fx)\cos(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

```
[Out] (b*Cos[e + f*x]*Sin[e + f*x])/(3*a*(a + b)*f*(a + b*Sin[e + f*x]^2)^(3/2))
+ (2*b*(2*a + b)*Cos[e + f*x]*Sin[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[a + b*S
in[e + f*x]^2]) + (2*(2*a + b)*EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e
+ f*x]^2])/(3*a^2*(a + b)^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) - (EllipticF[
e + f*x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*a*(a + b)*f*Sqrt[a + b*
Sin[e + f*x]^2])
```

Rubi [A] time = 0.258804, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3184, 3173, 3172, 3178, 3177, 3183, 3182}

$$\frac{2b(2a+b)\sin(e+fx)\cos(e+fx)}{3a^2f(a+b)^2\sqrt{a+b\sin^2(e+fx)}}+\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}E\left(e+fx\middle|-\frac{b}{a}\right)}{3a^2f(a+b)^2\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{b\sin(e+fx)\cos(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x]^2)^(-5/2),x]
```

```
[Out] (b*Cos[e + f*x]*Sin[e + f*x])/(3*a*(a + b)*f*(a + b*Sin[e + f*x]^2)^(3/2))
+ (2*b*(2*a + b)*Cos[e + f*x]*Sin[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[a + b*S
in[e + f*x]^2]) + (2*(2*a + b)*EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e
+ f*x]^2])/(3*a^2*(a + b)^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) - (EllipticF[
e + f*x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*a*(a + b)*f*Sqrt[a + b*
Sin[e + f*x]^2])
```

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a +
 b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)
*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rule 3173

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b - a*B)*Cos[e + f*x]*Sin[e + f*x]
*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(a + b)*(p + 1)), x] - Dist[1/(2*a*
(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p +
 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{b\cos(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\int \frac{-3a-2b+b\sin^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx}{3a(a+b)}$$

$$= \frac{b\cos(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(2a+b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{\int \frac{-a(3a+b)-2b(2a\cdots}{\sqrt{a+b\sin\cdots}}}{3a^2(a\cdots}$$

$$= \frac{b\cos(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(2a+b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{\int \frac{1}{\sqrt{a+b\sin^2(e+fx\cdots}}}{3a(a+b)}$$

$$= \frac{b\cos(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(2a+b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\left(2(2a+b)\sqrt{a\cdots}\right.}{3a\cdots}$$

$$= \frac{b\cos(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(2a+b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{2(2a+b)E\left(e+\cdots\right.}{3a^2(a+\cdots}$$

Mathematica [A] time = 1.38533, size = 172, normalized size = 0.77

$$\frac{-\sqrt{2}b\sin(2(e+fx))\left(-5a^2+b(2a+b)\cos(2(e+fx))-5ab-b^2\right)-a^2(a+b)\left(\frac{2a-b\cos(2(e+fx))+b}{a}\right)^{3/2}F\left(e+fx\middle|-\frac{b}{a}\right)+2a^2(\cdots}{3a^2f(a+b)^2(2a-b\cos(2(e+fx))+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)^(-5/2),x]
```

```
[Out] (2*a^2*(2*a + b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticE[e + f*x
, -(b/a)] - a^2*(a + b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticF[
e + f*x, -(b/a)] - Sqrt[2]*b*(-5*a^2 - 5*a*b - b^2 + b*(2*a + b)*Cos[2*(e +
 f*x)])*Sin[2*(e + f*x)])/(3*a^2*(a + b)^2*f*(2*a + b - b*Cos[2*(e + f*x)])
^(3/2))
```

Maple [B] time = 1.721, size = 547, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/3*((cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e
),(-1/a*b)^(1/2))*a^2*b*sin(f*x+e)^2+(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^
2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2*sin(f*x+e)^2-4*(cos(
f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^
(1/2))*a^2*b*sin(f*x+e)^2-2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/
2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2*sin(f*x+e)^2+4*a*b^2*sin(f*x+
e)^5+2*b^3*sin(f*x+e)^5+(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*E
llipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3+a^2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(
f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-4*(cos(f*x+e)^2)^
(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3
-2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(
-1/a*b)^(1/2))*a^2*b+5*a^2*b*sin(f*x+e)^3-a*b^2*sin(f*x+e)^3-2*sin(f*x+e)^3
*b^3-5*sin(f*x+e)*a^2*b-3*a*b^2*sin(f*x+e))/(a+b*sin(f*x+e)^2)^(3/2)/a^2/(a
+b)^2/cos(f*x+e)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b \sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(-5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}}{b^3\cos\left(fx+e\right)^6-3\left(ab^2+b^3\right)\cos\left(fx+e\right)^4-a^3-3\,a^2b-3\,ab^2-b^3+3\left(a^2b+2\,ab^2+b^3\right)\cos\left(fx+e\right)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)/(b^3*cos(f*x + e)^6 - 3*(a*b^2 +
b^3)*cos(f*x + e)^4 - a^3 - 3*a^2*b - 3*a*b^2 - b^3 + 3*(a^2*b + 2*a*b^2 +
b^3)*cos(f*x + e)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b \sin\left(fx+e\right)^2 + a\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(-5/2), x)
```

3.541 $\int \frac{\cot^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=287

$$-\frac{(7a+8b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^3f(a+b)}+\frac{(3a+4b)\cot(e+fx)}{3a^2f(a+b)\sqrt{a+b\sin^2(e+fx)}}+\frac{4\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e}{a}}}{3a^2f\sqrt{a+b\,s}}$$

```
[Out] Cot[e + f*x]/(3*a*f*(a + b*Sin[e + f*x]^2)^(3/2)) + ((3*a + 4*b)*Cot[e + f*
x])/(3*a^2*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2]) - ((7*a + 8*b)*Cot[e + f*x
]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a^3*(a + b)*f) - ((7*a + 8*b)*Sqrt[Cos[e +
 f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Si
n[e + f*x]^2])/(3*a^3*(a + b)*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (4*Sqrt[C
os[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1
+ (b*Sin[e + f*x]^2)/a])/(3*a^2*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.365945, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {3196, 469, 579, 583, 524, 426, 424, 421, 419}

$$-\frac{(7a+8b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^3f(a+b)}+\frac{(3a+4b)\cot(e+fx)}{3a^2f(a+b)\sqrt{a+b\sin^2(e+fx)}}+\frac{4\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e}{a}}}{3a^2f\sqrt{a+b\,s}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] Cot[e + f*x]/(3*a*f*(a + b*Sin[e + f*x]^2)^(3/2)) + ((3*a + 4*b)*Cot[e + f*
x])/(3*a^2*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2]) - ((7*a + 8*b)*Cot[e + f*x
]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a^3*(a + b)*f) - ((7*a + 8*b)*Sqrt[Cos[e +
 f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Si
n[e + f*x]^2])/(3*a^3*(a + b)*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (4*Sqrt[C
os[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1
+ (b*Sin[e + f*x]^2)/a])/(3*a^2*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]
```

Rule 469

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
q)/(a*e*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p +
 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1) +
 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e, m,
```

```
n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{x^2(a+bx^2)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\cot(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{-4+3x^2}{x^2\sqrt{1-x^2}(a+bx^2)^{3/2}}\,d\right.}{3af}$$

$$= \frac{\cot(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{(3a+4b)\cot(e+fx)}{3a^2(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e\right.}{}$$

$$= \frac{\cot(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{(3a+4b)\cot(e+fx)}{3a^2(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{(7a+8b)\cot(e+fx)}{3a^3(a}$$

$$= \frac{\cot(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{(3a+4b)\cot(e+fx)}{3a^2(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{(7a+8b)\cot(e+fx)}{3a^3(a}$$

$$= \frac{\cot(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{(3a+4b)\cot(e+fx)}{3a^2(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{(7a+8b)\cot(e+fx)}{3a^3(a}$$

$$= \frac{\cot(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{(3a+4b)\cot(e+fx)}{3a^2(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{(7a+8b)\cot(e+fx)}{3a^3(a}$$

Mathematica [A] time = 2.70433, size = 209, normalized size = 0.73

$$\frac{-\frac{\cot(e+fx)\left(-4b\left(11a^2+19ab+8b^2\right)\cos(2(e+fx))+68a^2b+24a^3+b^2(7a+8b)\cos(4(e+fx))+69ab^2+24b^3\right)}{\sqrt{2}} + 8a^2(a+b)\left(\frac{2a-b\cos(2(e+fx))+b}{a}\right)^{3/2}F\left(e\right.}{6a^3f(a+b)(2a-b\cos(2(e+fx))+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (-(((24*a^3 + 68*a^2*b + 69*a*b^2 + 24*b^3 - 4*b*(11*a^2 + 19*a*b + 8*b^2)*
Cos[2*(e + f*x)] + b^2*(7*a + 8*b)*Cos[4*(e + f*x)])*Cot[e + f*x])/Sqrt[2])
 - 2*a^2*(7*a + 8*b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticE[e +
 f*x, -(b/a)] + 8*a^2*(a + b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*Elli
pticF[e + f*x, -(b/a)])/(6*a^3*(a + b)*f*(2*a + b - b*Cos[2*(e + f*x)])^(3/
2))
```

Maple [A] time = 1.698, size = 411, normalized size = 1.4

$$\frac{1}{3\sin\left(fx+e\right)a^3(a+b)\cos\left(fx+e\right)f}\left(-\sqrt{-\frac{b\left(\cos\left(fx+e\right)\right)^2}{a}+\frac{a+b}{a}}\sqrt{\left(\cos\left(fx+e\right)\right)^2}ab\left(4\,\text{EllipticF}\left(\sin\left(fx+e\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3*(-(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(cos(f*x+e)^2)^(1/2)*a*b*(4*Ellipti
cF(sin(f*x+e),(-1/a*b)^(1/2))*a+4*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-7*
EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a-8*EllipticE(sin(f*x+e),(-1/a*b)^(1/2
))*b)*sin(f*x+e)*cos(f*x+e)^2+(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(cos(f*x+e)
^2)^(1/2)*a*(4*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2+8*EllipticF(sin(f*x
+e),(-1/a*b)^(1/2))*a*b+4*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^2-7*Ellipt
icE(sin(f*x+e),(-1/a*b)^(1/2))*a^2-15*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*
a*b-8*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*b^2)*sin(f*x+e)+(-7*a*b^2-8*b^3)
*cos(f*x+e)^6+(11*a^2*b+26*a*b^2+16*b^3)*cos(f*x+e)^4+(-3*a^3-14*a^2*b-19*a
*b^2-8*b^3)*cos(f*x+e)^2)/sin(f*x+e)/a^3/(a+b)/(a+b*sin(f*x+e)^2)^(3/2)/cos
(f*x+e)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\cot\left(fx+e\right)^2}{b^3\cos\left(fx+e\right)^6-3\left(ab^2+b^3\right)\cos\left(fx+e\right)^4-a^3-3a^2b-3ab^2-b^3+3\left(a^2b+2ab^2+b^3\right)\cos\left(fx+e\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)*cot(f*x + e)^2/(b^3*cos(f*x + e)^
6 - 3*(a*b^2 + b^3)*cos(f*x + e)^4 - a^3 - 3*a^2*b - 3*a*b^2 - b^3 + 3*(a^2
*b + 2*a*b^2 + b^3)*cos(f*x + e)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)^2}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

3.542 $\int \frac{\cot^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=348

$$\frac{8(a+2b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^4f} - \frac{(3a+8b)\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^3bf} + \frac{2(a+3b)\cot(e+f}{3a^2bf\sqrt{a+b\,\mathrm{s}}}$$

```
[Out] ((a + b)*Cot[e + f*x]*Csc[e + f*x]^2)/(3*a*b*f*(a + b*Sin[e + f*x]^2)^(3/2)
) + (2*(a + 3*b)*Cot[e + f*x]*Csc[e + f*x]^2)/(3*a^2*b*f*Sqrt[a + b*Sin[e +
 f*x]^2]) + (8*(a + 2*b)*Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a^4*f)
 - ((3*a + 8*b)*Cot[e + f*x]*Csc[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2])/(3*
a^3*b*f) + (8*(a + 2*b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]]
, -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a^4*f*Sqrt[1 + (b*Sin
[e + f*x]^2)/a]) - ((5*a + 8*b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e
 + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*a^3*f*Sqr
t[a + b*Sin[e + f*x]^2])
```

Rubi [A] time = 0.522608, antiderivative size = 348, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {3196, 468, 579, 583, 524, 426, 424, 421, 419}

$$\frac{8(a+2b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^4f} - \frac{(3a+8b)\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^3bf} + \frac{2(a+3b)\cot(e+f}{3a^2bf\sqrt{a+b\,\mathrm{s}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] ((a + b)*Cot[e + f*x]*Csc[e + f*x]^2)/(3*a*b*f*(a + b*Sin[e + f*x]^2)^(3/2)
) + (2*(a + 3*b)*Cot[e + f*x]*Csc[e + f*x]^2)/(3*a^2*b*f*Sqrt[a + b*Sin[e +
 f*x]^2]) + (8*(a + 2*b)*Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a^4*f)
 - ((3*a + 8*b)*Cot[e + f*x]*Csc[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2])/(3*
a^3*b*f) + (8*(a + 2*b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]]
, -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a^4*f*Sqrt[1 + (b*Sin
[e + f*x]^2)/a]) - ((5*a + 8*b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e
 + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*a^3*f*Sqr
t[a + b*Sin[e + f*x]^2])
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
```

```
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
```

```
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{\left(1-x^2\right)^{3/2}}{x^4\left(a+bx^2\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{(a+b)\cot(e+fx)\csc^2(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{-3(a+2b)+(2a+5b)x}{x^4\sqrt{1-x^2}\left(a+bx^2\right)^{3/2}}\right.}{3abf}$$

$$= \frac{(a+b)\cot(e+fx)\csc^2(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2(a+3b)\cot(e+fx)\csc^2(e+fx)}{3a^2bf\sqrt{a+b\sin^2(e+fx)}} + \frac{\left(\sqrt{\cos^2(e+fx)}\right.}{}$$

$$= \frac{(a+b)\cot(e+fx)\csc^2(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2(a+3b)\cot(e+fx)\csc^2(e+fx)}{3a^2bf\sqrt{a+b\sin^2(e+fx)}} - \frac{(3a+8b)\cot(e+}{}$$

$$= \frac{(a+b)\cot(e+fx)\csc^2(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2(a+3b)\cot(e+fx)\csc^2(e+fx)}{3a^2bf\sqrt{a+b\sin^2(e+fx)}} + \frac{8(a+2b)\cot(e+}{}$$

$$= \frac{(a+b)\cot(e+fx)\csc^2(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2(a+3b)\cot(e+fx)\csc^2(e+fx)}{3a^2bf\sqrt{a+b\sin^2(e+fx)}} + \frac{8(a+2b)\cot(e+}{}$$

$$= \frac{(a+b)\cot(e+fx)\csc^2(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2(a+3b)\cot(e+fx)\csc^2(e+fx)}{3a^2bf\sqrt{a+b\sin^2(e+fx)}} + \frac{8(a+2b)\cot(e+}{}$$

$$= \frac{(a+b)\cot(e+fx)\csc^2(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2(a+3b)\cot(e+fx)\csc^2(e+fx)}{3a^2bf\sqrt{a+b\sin^2(e+fx)}} + \frac{8(a+2b)\cot(e+}{}$$

Mathematica [A] time = 3.11643, size = 226, normalized size = 0.65

$$\frac{2a^2b\left(\frac{2a-b\cos(2(e+fx))+b}{a}\right)^{3/2}\left(8(a+2b)E\left(e+fx\middle|-\frac{b}{a}\right)-(5a+8b)F\left(e+fx\middle|-\frac{b}{a}\right)\right)+\sqrt{2}b\left(2ab(a+b)\sin(2(e+fx))+4b(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (2*a^2*b*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*(8*(a + 2*b)*EllipticE[e
+ f*x, -(b/a)] - (5*a + 8*b)*EllipticF[e + f*x, -(b/a)]) + Sqrt[2]*b*(4*(a
+ 2*b)*(2*a + b - b*Cos[2*(e + f*x)])^2*Cot[e + f*x] - a*(2*a + b - b*Cos[2
*(e + f*x)])^2*Cot[e + f*x]*Csc[e + f*x]^2 + 2*a*b*(a + b)*Sin[2*(e + f*x)]
 + 4*b*(a + 2*b)*(2*a + b - b*Cos[2*(e + f*x)])*Sin[2*(e + f*x)]))/(6*a^4*b
*f*(2*a + b - b*Cos[2*(e + f*x)])^(3/2))
```

Maple [A] time = 1.55, size = 633, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/3*(5*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x
+e),(-1/a*b)^(1/2))*a^2*b*sin(f*x+e)^5+8*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x
+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2*sin(f*x+e)^5-8*(
cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a
*b)^(1/2))*a^2*b*sin(f*x+e)^5-16*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a
)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2*sin(f*x+e)^5+8*a*b^2*sin
(f*x+e)^8+16*b^3*sin(f*x+e)^8+5*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)
^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3*sin(f*x+e)^3+8*(cos(f*x+e)^
2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*
a^2*b*sin(f*x+e)^3-8*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*Elli
pticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3*sin(f*x+e)^3-16*(cos(f*x+e)^2)^(1/2)*(
(a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b*sin(
f*x+e)^3+13*a^2*b*sin(f*x+e)^6+16*a*b^2*sin(f*x+e)^6-16*b^3*sin(f*x+e)^6+4*
a^3*sin(f*x+e)^4-7*a^2*b*sin(f*x+e)^4-24*a*b^2*sin(f*x+e)^4-5*a^3*sin(f*x+e
)^2-6*a^2*b*sin(f*x+e)^2+a^3)/sin(f*x+e)^3/a^4/(a+b*sin(f*x+e)^2)^(3/2)/cos
(f*x+e)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\cot\left(fx+e\right)^{4}}{b^{3}\cos\left(fx+e\right)^{6}-3\left(ab^{2}+b^{3}\right)\cos\left(fx+e\right)^{4}-a^{3}-3\,a^{2}b-3\,ab^{2}-b^{3}+3\left(a^{2}b+2\,ab^{2}+b^{3}\right)\cos\left(fx+e\right)^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)*cot(f*x + e)^4/(b^3*cos(f*x + e)^
6 - 3*(a*b^2 + b^3)*cos(f*x + e)^4 - a^3 - 3*a^2*b - 3*a*b^2 - b^3 + 3*(a^2
*b + 2*a*b^2 + b^3)*cos(f*x + e)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)^4}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^4/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

3.543 $\int \left(a+b\sin^2(e+fx)\right)^p (d\tan(e+fx))^m\,dx$

Optimal. Leaf size=120

$$\frac{\cos^2(e+fx)^{\frac{m+1}{2}}(d\tan(e+fx))^{m+1}\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2};\frac{m+1}{2},-p;\frac{m+3}{2};\sin^2(e+fx),-\frac{b\sin^2}{}\right.}{df(m+1)}$$

```
[Out] (AppellF1[(1 + m)/2, (1 + m)/2, -p, (3 + m)/2, Sin[e + f*x]^2, -((b*Sin[e +
 f*x]^2)/a)]*(Cos[e + f*x]^2)^((1 + m)/2)*(a + b*Sin[e + f*x]^2)^p*(d*Tan[e
 + f*x])^(1 + m))/(d*f*(1 + m)*(1 + (b*Sin[e + f*x]^2)/a)^p)
```

Rubi [A] time = 0.121811, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3197, 511, 510}

$$\frac{\cos^2(e+fx)^{\frac{m+1}{2}}(d\tan(e+fx))^{m+1}\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2};\frac{m+1}{2},-p;\frac{m+3}{2};\sin^2(e+fx),-\frac{b\sin^2}{}\right.}{df(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x]^2)^p*(d*Tan[e + f*x])^m,x]
```



```
[Out] (AppellF1[(1 + m)/2, (1 + m)/2, -p, (3 + m)/2, Sin[e + f*x]^2, -((b*Sin[e +
 f*x]^2)/a)]*(Cos[e + f*x]^2)^((1 + m)/2)*(a + b*Sin[e + f*x]^2)^p*(d*Tan[e
 + f*x])^(1 + m))/(d*f*(1 + m)*(1 + (b*Sin[e + f*x]^2)/a)^p)
```


Rule 3197

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*((d_.)*tan[(e_.) + (f_.)
*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(
ff*(d*Tan[e + f*x])^(m + 1)*(Cos[e + f*x]^2)^((m + 1)/2))/(d*f*Sin[e + f*x]
^(m + 1)), Subst[Int[((ff*x)^m*(a + b*ff^2*x^2)^p)/(1 - ff^2*x^2)^((m + 1)/
2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] &&  !In
tegerQ[m]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \left(a+b\sin^2(e+fx)\right)^p (d\tan(e+fx))^m\,dx = \frac{\left(\cos^2(e+fx)^{\frac{1+m}{2}}\sin^{-1-m}(e+fx)(d\tan(e+fx))^{1+m}\right)\text{Subst}\left(\int x^m\left(1-x^2\right.\right.}{df}$$

$$= \frac{\left(\cos^2(e+fx)^{\frac{1+m}{2}}\sin^{-1-m}(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+fx)}{a}\right)^{-p}\right.}{}$$

$$= \frac{F_1\left(\frac{1+m}{2};\frac{1+m}{2},-p;\frac{3+m}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\cos^2(e+fx)^{\frac{1+m}{2}}\left(a+b\sin\right.}{df(1+m)}$$

Mathematica [A] time = 0.501859, size = 121, normalized size = 1.01

$$\frac{\tan(e+fx)\cos^2(e+fx)^{\frac{m+1}{2}}(d\tan(e+fx))^m\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2};\frac{m+1}{2},-p;\frac{m+3}{2};\sin^2(e+fx)\right.}{f(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)^p*(d*Tan[e + f*x])^m,x]
```

```
[Out] (AppellF1[(1 + m)/2, (1 + m)/2, -p, (3 + m)/2, Sin[e + f*x]^2, -((b*Sin[e +
 f*x]^2)/a)]*(Cos[e + f*x]^2)^((1 + m)/2)*(a + b*Sin[e + f*x]^2)^p*Tan[e +
f*x]*(d*Tan[e + f*x])^m)/(f*(1 + m)*(1 + (b*Sin[e + f*x]^2)/a)^p)
```

Maple [F] time = 1.605, size = 0, normalized size = 0.

$$\int \left(a+b\left(\sin\left(fx+e\right)\right)^2\right)^p\left(d\tan\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^p*(d*tan(f*x+e))^m,x)
```

```
[Out] int((a+b*sin(f*x+e)^2)^p*(d*tan(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p\left(d\tan\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^p*(d*tan(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*(d*tan(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-b\cos\left(fx+e\right)^2+a+b\right)^p\left(d\tan\left(fx+e\right)\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^p*(d*tan(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*(d*tan(f*x + e))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**p*(d*tan(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^{2}+a\right)^{p}\left(d\tan\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^p*(d*tan(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*(d*tan(f*x + e))^m, x)
```

3.544 $\int \left(a+b\sin^2(c+dx)\right)^p \tan^3(c+dx)\,dx$

Optimal. Leaf size=102

$$\frac{\sec^2(c+dx)\left(a+b\sin^2(c+dx)\right)^{p+1}}{2d(a+b)}-\frac{(a+bp+b)\left(a+b\sin^2(c+dx)\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\sin^2(c+dx)+a}{a+b}\right)}{2d(p+1)(a+b)^2}$$

```
[Out] -((a + b + b*p)*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sin[c + d*x]^2)/(
a + b)]*(a + b*Sin[c + d*x]^2)^(1 + p))/(2*(a + b)^2*d*(1 + p)) + (Sec[c +
d*x]^2*(a + b*Sin[c + d*x]^2)^(1 + p))/(2*(a + b)*d)
```

Rubi [A] time = 0.0936865, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3194, 78, 68}

$$\frac{\sec^2(c+dx)\left(a+b\sin^2(c+dx)\right)^{p+1}}{2d(a+b)}-\frac{(a+bp+b)\left(a+b\sin^2(c+dx)\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\sin^2(c+dx)+a}{a+b}\right)}{2d(p+1)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x]^2)^p*Tan[c + d*x]^3,x]
```

```
[Out] -((a + b + b*p)*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sin[c + d*x]^2)/(
a + b)]*(a + b*Sin[c + d*x]^2)^(1 + p))/(2*(a + b)^2*d*(1 + p)) + (Sec[c +
d*x]^2*(a + b*Sin[c + d*x]^2)^(1 + p))/(2*(a + b)*d)
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \left(a+b\sin^2(c+dx)\right)^p \tan^3(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x(a+bx)^p}{(1-x)^2}dx, x, \sin^2(c+dx)\right)}{2d}$$

$$= \frac{\sec^2(c+dx)\left(a+b\sin^2(c+dx)\right)^{1+p}}{2(a+b)d} - \frac{(a+b+bp)\,\text{Subst}\left(\int \frac{(a+bx)^p}{1-x}dx, x, \text{s}\right.}{2(a+b)d}$$

$$= -\frac{(a+b+bp)\,{}_2F_1\left(1,1+p;2+p;\frac{a+b\sin^2(c+dx)}{a+b}\right)\left(a+b\sin^2(c+dx)\right)^{1+p}}{2(a+b)^2d(1+p)} + \text{se}$$

Mathematica [A] time = 0.235893, size = 83, normalized size = 0.81

$$\frac{\left(a+b\sin^2(c+dx)\right)^{p+1}\left((p+1)(a+b)\sec^2(c+dx)-(a+bp+b)\,{}_2F_1\left(1,p+1;p+2;\frac{b\sin^2(c+dx)+a}{a+b}\right)\right)}{2d(p+1)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x]^2)^p*Tan[c + d*x]^3,x]
```

```
[Out] ((-((a + b + b*p)*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sin[c + d*x]^2)
/(a + b)]) + (a + b)*(1 + p)*Sec[c + d*x]^2)*(a + b*Sin[c + d*x]^2)^(1 + p)
)/(2*(a + b)^2*d*(1 + p))
```

Maple [F] time = 0.889, size = 0, normalized size = 0.

$$\int \left(a+(\sin(dx+c))^2 b\right)^p (\tan(dx+c))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+sin(d*x+c)^2*b)^p*tan(d*x+c)^3,x)
```

```
[Out] int((a+sin(d*x+c)^2*b)^p*tan(d*x+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin(dx+c)^2+a\right)^p \tan(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^2)^p*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^2 + a)^p*tan(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-b\cos(dx+c)^2+a+b\right)^p\tan(dx+c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^2)^p*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(d*x + c)^2 + a + b)^p*tan(d*x + c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**2)**p*tan(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(dx+c\right)^{2}+a\right)^{p}\tan\left(dx+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^2)^p*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^2 + a)^p*tan(d*x + c)^3, x)
```

3.545 $\int \left(a+b\sin^2(c+dx)\right)^p \tan(c+dx)\,dx$

Optimal. Leaf size=59

$$\frac{\left(a+b\sin^2(c+dx)\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\sin^2(c+dx)+a}{a+b}\right)}{2d(p+1)(a+b)}$$

```
[Out] (Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sin[c + d*x]^2)/(a + b)]*(a + b*
Sin[c + d*x]^2)^(1 + p))/(2*(a + b)*d*(1 + p))
```

Rubi [A] time = 0.0434174, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3194, 68}

$$\frac{\left(a+b\sin^2(c+dx)\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\sin^2(c+dx)+a}{a+b}\right)}{2d(p+1)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x]^2)^p*Tan[c + d*x],x]
```

```
[Out] (Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sin[c + d*x]^2)/(a + b)]*(a + b*
Sin[c + d*x]^2)^(1 + p))/(2*(a + b)*d*(1 + p))
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sin^2(c+dx)\right)^p \tan(c+dx)\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{1-x}\,dx,x,\sin^2(c+dx)\right)}{2d}\\ &= \frac{{}_2F_1\left(1,1+p;2+p;\frac{a+b\sin^2(c+dx)}{a+b}\right)\left(a+b\sin^2(c+dx)\right)^{1+p}}{2(a+b)d(1+p)}\end{aligned}$$

Mathematica [A] time = 0.0615149, size = 61, normalized size = 1.03

$$\frac{\left(a-b\cos^2(c+dx)+b\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;1-\frac{b\cos^2(c+dx)}{a+b}\right)}{2d(p+1)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x]^2)^p*Tan[c + d*x],x]
```

```
[Out] ((a + b - b*Cos[c + d*x]^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 -
(b*Cos[c + d*x]^2)/(a + b)])/(2*(a + b)*d*(1 + p))
```

Maple [F] time = 1.234, size = 0, normalized size = 0.

$$\int \left(a+(\sin (dx+c))^2 b\right)^p \tan (dx+c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+sin(d*x+c)^2*b)^p*tan(d*x+c),x)
```

```
[Out] int((a+sin(d*x+c)^2*b)^p*tan(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin (dx+c)^2+a\right)^p \tan (dx+c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^2)^p*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^2 + a)^p*tan(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(-b\cos (dx+c)^2+a+b\right)^p \tan (dx+c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^2)^p*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((-b*cos(d*x + c)^2 + a + b)^p*tan(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**2)**p*tan(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(dx+c\right)^{2}+a\right)^{p}\tan\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^2)^p*tan(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^2 + a)^p*tan(d*x + c), x)
```

3.546 $\int \cot(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx$

Optimal. Leaf size=54

$$-\frac{\left(a+b\sin^2(c+dx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\sin^2(c+dx)}{a}+1\right)}{2ad(p+1)}$$

```
[Out] -(Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sin[c + d*x]^2)/a]*(a + b*Sin[c
 + d*x]^2)^(1 + p))/(2*a*d*(1 + p))
```

Rubi [A] time = 0.0557917, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3194, 65}

$$-\frac{\left(a+b\sin^2(c+dx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\sin^2(c+dx)}{a}+1\right)}{2ad(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + b*Sin[c + d*x]^2)^p,x]
```

```
[Out] -(Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sin[c + d*x]^2)/a]*(a + b*Sin[c
 + d*x]^2)^(1 + p))/(2*a*d*(1 + p))
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}\int \cot(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx,x,\sin^2(c+dx)\right)}{2d}\\ &= -\frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b\sin^2(c+dx)}{a}\right)\left(a+b\sin^2(c+dx)\right)^{1+p}}{2ad(1+p)}\end{aligned}$$

Mathematica [A] time = 0.0594202, size = 54, normalized size = 1.

$$-\frac{\left(a+b\sin^2(c+dx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\sin^2(c+dx)}{a}+1\right)}{2ad(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + b*Sin[c + d*x]^2)^p,x]
```

```
[Out] -(Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sin[c + d*x]^2)/a]*(a + b*Sin[c
 + d*x]^2)^(1 + p))/(2*a*d*(1 + p))
```

Maple [F] time = 1.125, size = 0, normalized size = 0.

$$\int \cot(dx+c)\left(a+(\sin(dx+c))^2 b\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+sin(d*x+c)^2*b)^p,x)
```

```
[Out] int(cot(d*x+c)*(a+sin(d*x+c)^2*b)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin(dx+c)^2+a\right)^p \cot(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^2 + a)^p*cot(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(-b\cos(dx+c)^2+a+b\right)^p \cot(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(d*x + c)^2 + a + b)^p*cot(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(dx+c\right)^2+a\right)^p\cot\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^2 + a)^p*cot(d*x + c), x)
```

3.547 $\int \cot^3(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx$

Optimal. Leaf size=95

$$\frac{(a-bp)\left(a+b\sin^2(c+dx)\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\sin^2(c+dx)}{a}+1\right)}{2a^2d(p+1)}-\frac{\csc^2(c+dx)\left(a+b\sin^2(c+dx)\right)^{p+1}}{2ad}$$

```
[Out] -(Csc[c + d*x]^2*(a + b*Sin[c + d*x]^2)^(1 + p))/(2*a*d) + ((a - b*p)*Hyper
geometric2F1[1, 1 + p, 2 + p, 1 + (b*Sin[c + d*x]^2)/a]*(a + b*Sin[c + d*x]
^2)^(1 + p))/(2*a^2*d*(1 + p))
```

Rubi [A] time = 0.075998, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3194, 78, 65}

$$\frac{(a-bp)\left(a+b\sin^2(c+dx)\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\sin^2(c+dx)}{a}+1\right)}{2a^2d(p+1)}-\frac{\csc^2(c+dx)\left(a+b\sin^2(c+dx)\right)^{p+1}}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + b*Sin[c + d*x]^2)^p,x]
```


```
[Out] -(Csc[c + d*x]^2*(a + b*Sin[c + d*x]^2)^(1 + p))/(2*a*d) + ((a - b*p)*Hyper
geometric2F1[1, 1 + p, 2 + p, 1 + (b*Sin[c + d*x]^2)/a]*(a + b*Sin[c + d*x]
^2)^(1 + p))/(2*a^2*d*(1 + p))
```


Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \cot^3(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{(1-x)(a+bx)^p}{x^2}\,dx, x, \sin^2(c+dx)\right)}{2d}$$
$$= -\frac{\csc^2(c+dx)\left(a+b\sin^2(c+dx)\right)^{1+p}}{2ad} - \frac{(a-bp)\,\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, \sin^2(c\right.}{2ad}$$
$$= -\frac{\csc^2(c+dx)\left(a+b\sin^2(c+dx)\right)^{1+p}}{2ad} + \frac{(a-bp)\,{}_2F_1\left(1, 1+p; 2+p; 1+\frac{b\sin^2(c}{a}\right.}{2a^2d(1+p}$$

Mathematica [A] time = 0.444903, size = 73, normalized size = 0.77

$$-\frac{\left(a+b\sin^2(c+dx)\right)^{p+1}\left(\frac{(bp-a)\,{}_2F_1\left(1,p+1;p+2;\frac{b\sin^2(c+dx)}{a}+1\right)}{p+1}+a\csc^2(c+dx)\right)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + b*Sin[c + d*x]^2)^p,x]
```

```
[Out] -((a*Csc[c + d*x]^2 + ((-a + b*p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b
*Sin[c + d*x]^2)/a])/(1 + p))*(a + b*Sin[c + d*x]^2)^(1 + p))/(2*a^2*d)
```

Maple [F] time = 0.809, size = 0, normalized size = 0.

$$\int (\cot(dx+c))^3\left(a+(\sin(dx+c))^2 b\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+sin(d*x+c)^2*b)^p,x)
```

```
[Out] int(cot(d*x+c)^3*(a+sin(d*x+c)^2*b)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin(dx+c)^2+a\right)^p\cot(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^2 + a)^p*cot(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-b\cos(dx+c)^2+a+b\right)^p\cot(dx+c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(d*x + c)^2 + a + b)^p*cot(d*x + c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+b*sin(d*x+c)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(dx+c\right)^{2}+a\right)^{p}\cot\left(dx+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^2 + a)^p*cot(d*x + c)^3, x)
```

3.548 $\int \left(a+b\sin^2(c+dx)\right)^p \tan^4(c+dx)\,dx$

Optimal. Leaf size=101

$$\frac{\sin^4(c+dx)\sqrt{\cos^2(c+dx)}\tan(c+dx)\left(a+b\sin^2(c+dx)\right)^p\left(\frac{b\sin^2(c+dx)}{a}+1\right)^{-p}F_1\left(\frac{5}{2};\frac{5}{2},-p;\frac{7}{2};\sin^2(c+dx),-\frac{b\sin^2(c+dx)}{a}\right)}{5d}$$

```
[Out] (AppellF1[5/2, 5/2, -p, 7/2, Sin[c + d*x]^2, -((b*Sin[c + d*x]^2)/a)]*Sqrt[
Cos[c + d*x]^2]*Sin[c + d*x]^4*(a + b*Sin[c + d*x]^2)^p*Tan[c + d*x])/(5*d*
(1 + (b*Sin[c + d*x]^2)/a)^p)
```

Rubi [A] time = 0.127491, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3196, 511, 510}

$$\frac{\sin^4(c+dx)\sqrt{\cos^2(c+dx)}\tan(c+dx)\left(a+b\sin^2(c+dx)\right)^p\left(\frac{b\sin^2(c+dx)}{a}+1\right)^{-p}F_1\left(\frac{5}{2};\frac{5}{2},-p;\frac{7}{2};\sin^2(c+dx),-\frac{b\sin^2(c+dx)}{a}\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x]^2)^p*Tan[c + d*x]^4,x]
```

```
[Out] (AppellF1[5/2, 5/2, -p, 7/2, Sin[c + d*x]^2, -((b*Sin[c + d*x]^2)/a)]*Sqrt[
Cos[c + d*x]^2]*Sin[c + d*x]^4*(a + b*Sin[c + d*x]^2)^p*Tan[c + d*x])/(5*d*
(1 + (b*Sin[c + d*x]^2)/a)^p)
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \left(a+b\sin^2(c+dx)\right)^p \tan^4(c+dx)\,dx = \frac{\left(\sqrt{\cos^2(c+dx)}\sec(c+dx)\right)\text{Subst}\left(\int \frac{x^4(a+bx^2)^p}{(1-x^2)^{5/2}}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\left(\sqrt{\cos^2(c+dx)}\sec(c+dx)\left(a+b\sin^2(c+dx)\right)^p\left(1+\frac{b\sin^2(c+dx)}{a}\right)^{-p}\right)\text{Subst}}{d}$$

$$= \frac{F_1\left(\frac{5}{2};\frac{5}{2},-p;\frac{7}{2};\sin^2(c+dx),-\frac{b\sin^2(c+dx)}{a}\right)\sqrt{\cos^2(c+dx)}\sin^4(c+dx)\left(a+b\right.}{5d}$$

Mathematica [A] time = 0.538817, size = 102, normalized size = 1.01

$$\frac{\sin^4(c+dx)\sqrt{\cos^2(c+dx)}\tan(c+dx)\left(a+b\sin^2(c+dx)\right)^p\left(\frac{a+b\sin^2(c+dx)}{a}\right)^{-p}F_1\left(\frac{5}{2};\frac{5}{2},-p;\frac{7}{2};\sin^2(c+dx),-\frac{b\sin^2(c+dx)}{a}\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x]^2)^p*Tan[c + d*x]^4,x]
```

```
[Out] (AppellF1[5/2, 5/2, -p, 7/2, Sin[c + d*x]^2, -((b*Sin[c + d*x]^2)/a)]*Sqrt[
Cos[c + d*x]^2]*Sin[c + d*x]^4*(a + b*Sin[c + d*x]^2)^p*Tan[c + d*x])/(5*d*
((a + b*Sin[c + d*x]^2)/a)^p)
```

Maple [F] time = 0.517, size = 0, normalized size = 0.

$$\int \left(a+(\sin(dx+c))^2 b\right)^p (\tan(dx+c))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+sin(d*x+c)^2*b)^p*tan(d*x+c)^4,x)
```

```
[Out] int((a+sin(d*x+c)^2*b)^p*tan(d*x+c)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin(dx+c)^2+a\right)^p \tan(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^2)^p*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^2 + a)^p*tan(d*x + c)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-b\cos(dx+c)^2+a+b\right)^p\tan(dx+c)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^2)^p*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(d*x + c)^2 + a + b)^p*tan(d*x + c)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**2)**p*tan(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sin \left(dx+c\right)^{2}+a\right)^{p} \tan \left(dx+c\right)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^2)^p*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^2 + a)^p*tan(d*x + c)^4, x)
```

3.549 $\int \left(a+b\sin^2(c+dx)\right)^p \tan^2(c+dx)\,dx$

Optimal. Leaf size=101

$$\frac{\sin^2(c+dx)\sqrt{\cos^2(c+dx)}\tan(c+dx)\left(a+b\sin^2(c+dx)\right)^p\left(\frac{b\sin^2(c+dx)}{a}+1\right)^{-p}F_1\left(\frac{3}{2};\frac{3}{2},-p;\frac{5}{2};\sin^2(c+dx),-\frac{b\sin^2(c+dx)}{a}\right)}{3d}$$

```
[Out] (AppellF1[3/2, 3/2, -p, 5/2, Sin[c + d*x]^2, -((b*Sin[c + d*x]^2)/a)]*Sqrt[
Cos[c + d*x]^2]*Sin[c + d*x]^2*(a + b*Sin[c + d*x]^2)^p*Tan[c + d*x])/(3*d*
(1 + (b*Sin[c + d*x]^2)/a)^p)
```

Rubi [A] time = 0.104974, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3196, 511, 510}

$$\frac{\sin^2(c+dx)\sqrt{\cos^2(c+dx)}\tan(c+dx)\left(a+b\sin^2(c+dx)\right)^p\left(\frac{b\sin^2(c+dx)}{a}+1\right)^{-p}F_1\left(\frac{3}{2};\frac{3}{2},-p;\frac{5}{2};\sin^2(c+dx),-\frac{b\sin^2(c+dx)}{a}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x]^2)^p*Tan[c + d*x]^2,x]
```


```
[Out] (AppellF1[3/2, 3/2, -p, 5/2, Sin[c + d*x]^2, -((b*Sin[c + d*x]^2)/a)]*Sqrt[
Cos[c + d*x]^2]*Sin[c + d*x]^2*(a + b*Sin[c + d*x]^2)^p*Tan[c + d*x])/(3*d*
(1 + (b*Sin[c + d*x]^2)/a)^p)
```


Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \left(a+b\sin^2(c+dx)\right)^p \tan^2(c+dx)\,dx = \frac{\left(\sqrt{\cos^2(c+dx)}\sec(c+dx)\right)\text{Subst}\left(\int \frac{x^2\left(a+bx^2\right)^p}{\left(1-x^2\right)^{3/2}}dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\left(\sqrt{\cos^2(c+dx)}\sec(c+dx)\left(a+b\sin^2(c+dx)\right)^p\left(1+\frac{b\sin^2(c+dx)}{a}\right)^{-p}\right)\text{Subst}\left(\int\right.}{d}$$

$$= \frac{F_1\left(\frac{3}{2};\frac{3}{2},-p;\frac{5}{2};\sin^2(c+dx),-\frac{b\sin^2(c+dx)}{a}\right)\sqrt{\cos^2(c+dx)}\sin^2(c+dx)\left(a+b\sin\right.}{3d}$$

Mathematica [A] time = 0.288817, size = 102, normalized size = 1.01

$$\frac{\sin^2(c+dx)\sqrt{\cos^2(c+dx)}\tan(c+dx)\left(a+b\sin^2(c+dx)\right)^p\left(\frac{a+b\sin^2(c+dx)}{a}\right)^{-p}F_1\left(\frac{3}{2};\frac{3}{2},-p;\frac{5}{2};\sin^2(c+dx),-\frac{b\sin^2(c+dx)}{a}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x]^2)^p*Tan[c + d*x]^2,x]
```

```
[Out] (AppellF1[3/2, 3/2, -p, 5/2, Sin[c + d*x]^2, -((b*Sin[c + d*x]^2)/a)]*Sqrt[
Cos[c + d*x]^2]*Sin[c + d*x]^2*(a + b*Sin[c + d*x]^2)^p*Tan[c + d*x])/(3*d*
((a + b*Sin[c + d*x]^2)/a)^p)
```

Maple [F] time = 0.664, size = 0, normalized size = 0.

$$\int \left(a+(\sin(dx+c))^2 b\right)^p (\tan(dx+c))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+sin(d*x+c)^2*b)^p*tan(d*x+c)^2,x)
```

```
[Out] int((a+sin(d*x+c)^2*b)^p*tan(d*x+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin(dx+c)^2+a\right)^p \tan(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^2)^p*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^2 + a)^p*tan(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-b\cos(dx+c)^2+a+b\right)^p\tan(dx+c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^2)^p*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(d*x + c)^2 + a + b)^p*tan(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**2)**p*tan(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(dx+c\right)^2+a\right)^p\tan\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^2)^p*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^2 + a)^p*tan(d*x + c)^2, x)
```

3.550 $\int \cot^2(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx$

Optimal. Leaf size=97

$$-\frac{\sqrt{\cos^2(c+dx)}\csc(c+dx)\sec(c+dx)\left(a+b\sin^2(c+dx)\right)^p\left(\frac{b\sin^2(c+dx)}{a}+1\right)^{-p}F_1\left(-\frac{1}{2};-\frac{1}{2},-p;\frac{1}{2};\sin^2(c+dx),-\frac{b\sin^2(c+d}{a}\right.}{d}$$

```
[Out] -((AppellF1[-1/2, -1/2, -p, 1/2, Sin[c + d*x]^2, -((b*Sin[c + d*x]^2)/a)]*S
qrt[Cos[c + d*x]^2]*Csc[c + d*x]*Sec[c + d*x]*(a + b*Sin[c + d*x]^2)^p)/(d*
(1 + (b*Sin[c + d*x]^2)/a)^p))
```

Rubi [A] time = 0.100471, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3196, 511, 510}

$$-\frac{\sqrt{\cos^2(c+dx)}\csc(c+dx)\sec(c+dx)\left(a+b\sin^2(c+dx)\right)^p\left(\frac{b\sin^2(c+dx)}{a}+1\right)^{-p}F_1\left(-\frac{1}{2};-\frac{1}{2},-p;\frac{1}{2};\sin^2(c+dx),-\frac{b\sin^2(c+d}{a}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + b*Sin[c + d*x]^2)^p,x]
```

```
[Out] -((AppellF1[-1/2, -1/2, -p, 1/2, Sin[c + d*x]^2, -((b*Sin[c + d*x]^2)/a)]*S
qrt[Cos[c + d*x]^2]*Csc[c + d*x]*Sec[c + d*x]*(a + b*Sin[c + d*x]^2)^p)/(d*
(1 + (b*Sin[c + d*x]^2)/a)^p))
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \cot^2(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx = \frac{\left(\sqrt{\cos^2(c+dx)}\sec(c+dx)\right)\text{Subst}\left(\int \frac{\sqrt{1-x^2}\left(a+bx^2\right)^p}{x^2}dx,x,\sin(c+dx)\right)}{d}$$

$$= \frac{\left(\sqrt{\cos^2(c+dx)}\sec(c+dx)\left(a+b\sin^2(c+dx)\right)^p\left(1+\frac{b\sin^2(c+dx)}{a}\right)^{-p}\right)\text{Subst}}{d}$$

$$= -\frac{F_1\left(-\frac{1}{2};-\frac{1}{2},-p;\frac{1}{2};\sin^2(c+dx),-\frac{b\sin^2(c+dx)}{a}\right)\sqrt{\cos^2(c+dx)}\csc(c+dx)\text{se}}{d}$$

Mathematica [A] time = 0.197504, size = 98, normalized size = 1.01

$$-\frac{\sqrt{\cos^2(c+dx)}\csc(c+dx)\sec(c+dx)\left(a+b\sin^2(c+dx)\right)^p\left(\frac{a+b\sin^2(c+dx)}{a}\right)^{-p}F_1\left(-\frac{1}{2};-\frac{1}{2},-p;\frac{1}{2};\sin^2(c+dx),-\frac{b\sin^2(c+}{a}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Sin[c + d*x]^2)^p,x]
```

```
[Out] -((AppellF1[-1/2, -1/2, -p, 1/2, Sin[c + d*x]^2, -((b*Sin[c + d*x]^2)/a)]*S
qrt[Cos[c + d*x]^2]*Csc[c + d*x]*Sec[c + d*x]*(a + b*Sin[c + d*x]^2)^p)/(d*
((a + b*Sin[c + d*x]^2)/a)^p))
```

Maple [F] time = 0.686, size = 0, normalized size = 0.

$$\int (\cot(dx+c))^2\left(a+(\sin(dx+c))^2b\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+sin(d*x+c)^2*b)^p,x)
```

```
[Out] int(cot(d*x+c)^2*(a+sin(d*x+c)^2*b)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin(dx+c)^2+a\right)^p\cot(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^2 + a)^p*cot(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-b\cos(dx+c)^2+a+b\right)^p\cot(dx+c)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(d*x + c)^2 + a + b)^p*cot(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+b*sin(d*x+c)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(dx+c\right)^{2}+a\right)^{p}\cot\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^2 + a)^p*cot(d*x + c)^2, x)
```

3.551 $\int \cot^4(c+dx) \left(a+b\sin^2(c+dx)\right)^p \, dx$

Optimal. Leaf size=101

$$-\frac{\sqrt{\cos^2(c+dx)}\csc^3(c+dx)\sec(c+dx)\left(a+b\sin^2(c+dx)\right)^p\left(\frac{b\sin^2(c+dx)}{a}+1\right)^{-p}F_1\left(-\frac{3}{2};-\frac{3}{2},-p;-\frac{1}{2};\sin^2(c+dx),-\frac{b\sin^2(c+dx)}{a}\right)}{3d}$$

```
[Out] -(AppellF1[-3/2, -3/2, -p, -1/2, Sin[c + d*x]^2, -((b*Sin[c + d*x]^2)/a)]*S
qrt[Cos[c + d*x]^2]*Csc[c + d*x]^3*Sec[c + d*x]*(a + b*Sin[c + d*x]^2)^p)/(
3*d*(1 + (b*Sin[c + d*x]^2)/a)^p)
```

Rubi [A] time = 0.102305, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3196, 511, 510}

$$-\frac{\sqrt{\cos^2(c+dx)}\csc^3(c+dx)\sec(c+dx)\left(a+b\sin^2(c+dx)\right)^p\left(\frac{b\sin^2(c+dx)}{a}+1\right)^{-p}F_1\left(-\frac{3}{2};-\frac{3}{2},-p;-\frac{1}{2};\sin^2(c+dx),-\frac{b\sin^2(c+dx)}{a}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + b*Sin[c + d*x]^2)^p,x]
```

```
[Out] -(AppellF1[-3/2, -3/2, -p, -1/2, Sin[c + d*x]^2, -((b*Sin[c + d*x]^2)/a)]*S
qrt[Cos[c + d*x]^2]*Csc[c + d*x]^3*Sec[c + d*x]*(a + b*Sin[c + d*x]^2)^p)/(
3*d*(1 + (b*Sin[c + d*x]^2)/a)^p)
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \cot^4(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx = \frac{\left(\sqrt{\cos^2(c+dx)}\sec(c+dx)\right)\text{Subst}\left(\int\frac{(1-x^2)^{3/2}(a+bx^2)^p}{x^4}dx,x,\sin(c+dx)\right)}{d}$$

$$= \frac{\left(\sqrt{\cos^2(c+dx)}\sec(c+dx)\left(a+b\sin^2(c+dx)\right)^p\left(1+\frac{b\sin^2(c+dx)}{a}\right)^{-p}\right)\text{Subst}\left(\int\right.}{d}$$

$$= -\frac{F_1\left(-\frac{3}{2};-\frac{3}{2},-p;-\frac{1}{2};\sin^2(c+dx),-\frac{b\sin^2(c+dx)}{a}\right)\sqrt{\cos^2(c+dx)}\csc^3(c+dx)\sec}{3d}$$

Mathematica [A] time = 0.237309, size = 102, normalized size = 1.01

$$-\frac{\sqrt{\cos^2(c+dx)}\csc^3(c+dx)\sec(c+dx)\left(a+b\sin^2(c+dx)\right)^p\left(\frac{a+b\sin^2(c+dx)}{a}\right)^{-p}F_1\left(-\frac{3}{2};-\frac{3}{2},-p;-\frac{1}{2};\sin^2(c+dx),-\frac{b\sin^2(c+}{a}\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Sin[c + d*x]^2)^p,x]
```

```
[Out] -(AppellF1[-3/2, -3/2, -p, -1/2, Sin[c + d*x]^2, -((b*Sin[c + d*x]^2)/a)]*S
qrt[Cos[c + d*x]^2]*Csc[c + d*x]^3*Sec[c + d*x]*(a + b*Sin[c + d*x]^2)^p)/(
3*d*((a + b*Sin[c + d*x]^2)/a)^p)
```

Maple [F] time = 0.506, size = 0, normalized size = 0.

$$\int (\cot(dx+c))^4\left(a+(\sin(dx+c))^2 b\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+sin(d*x+c)^2*b)^p,x)
```

```
[Out] int(cot(d*x+c)^4*(a+sin(d*x+c)^2*b)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin(dx+c)^2+a\right)^p\cot(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^2 + a)^p*cot(d*x + c)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-b\cos(dx+c)^2+a+b\right)^p\cot(dx+c)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(d*x + c)^2 + a + b)^p*cot(d*x + c)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+b*sin(d*x+c)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(dx+c\right)^2+a\right)^p\cot\left(dx+c\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^2 + a)^p*cot(d*x + c)^4, x)
```

3.552 $\int \frac{\cot^3(x)}{a+b\sin^3(x)}\,dx$

Optimal. Leaf size=153

$$\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(x)+b^{2/3}\sin^2(x)\right)}{6a^{5/3}}-\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(x)\right)}{3a^{5/3}}+\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(x)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}}+\frac{\log\left(a+b\sin^3(x)\right)}{3a}$$

```
[Out] (b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[x])/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a
^(5/3)) - Csc[x]^2/(2*a) - Log[Sin[x]]/a - (b^(2/3)*Log[a^(1/3) + b^(1/3)*S
in[x]])/(3*a^(5/3)) + (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[x] + b^(2/
3)*Sin[x]^2])/(6*a^(5/3)) + Log[a + b*Sin[x]^3]/(3*a)
```

Rubi [A] time = 0.190871, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3230, 1834, 1871, 200, 31, 634, 617, 204, 628, 260}

$$\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(x)+b^{2/3}\sin^2(x)\right)}{6a^{5/3}}-\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(x)\right)}{3a^{5/3}}+\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(x)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}}+\frac{\log\left(a+b\sin^3(x)\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^3/(a + b*Sin[x]^3),x]
```

```
[Out] (b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[x])/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a
^(5/3)) - Csc[x]^2/(2*a) - Log[Sin[x]]/a - (b^(2/3)*Log[a^(1/3) + b^(1/3)*S
in[x]])/(3*a^(5/3)) + (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[x] + b^(2/
3)*Sin[x]^2])/(6*a^(5/3)) + Log[a + b*Sin[x]^3]/(3*a)
```

Rule 3230

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff^(m + 1)/f, Subst[Int[(x^m*(a + b*(c*ff*x)^n)^p)/(1 - ff^2*x^2)^((m +
1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
```

```
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\cot^3(x)}{a+b\sin^3(x)}\,dx = \text{Subst}\left(\int \frac{1-x^2}{x^3\left(a+bx^3\right)}\,dx, x, \sin(x)\right)$$
$$= \text{Subst}\left(\int \left(\frac{1}{ax^3}-\frac{1}{ax}+\frac{b\left(-1+x^2\right)}{a\left(a+bx^3\right)}\right)dx, x, \sin(x)\right)$$
$$= -\frac{\csc^2(x)}{2a}-\frac{\log(\sin(x))}{a}+\frac{b\,\text{Subst}\left(\int \frac{-1+x^2}{a+bx^3}\,dx, x, \sin(x)\right)}{a}$$
$$= -\frac{\csc^2(x)}{2a}-\frac{\log(\sin(x))}{a}-\frac{b\,\text{Subst}\left(\int \frac{1}{a+bx^3}\,dx, x, \sin(x)\right)}{a}+\frac{b\,\text{Subst}\left(\int \frac{x^2}{a+bx^3}\,dx, x, \sin(x)\right)}{a}$$
$$= -\frac{\csc^2(x)}{2a}-\frac{\log(\sin(x))}{a}+\frac{\log\left(a+b\sin^3(x)\right)}{3a}-\frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx, x, \sin(x)\right)}{3a^{5/3}}-\frac{b\,\text{Subst}\left(\int \frac{\cdots}{a^{2/3}\cdots}\right.}{\cdots}$$
$$= -\frac{\csc^2(x)}{2a}-\frac{\log(\sin(x))}{a}-\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(x)\right)}{3a^{5/3}}+\frac{\log\left(a+b\sin^3(x)\right)}{3a}+\frac{b^{2/3}\,\text{Subst}\left(\int \frac{-\sqrt[3]{a}\sqrt[3]{\cdots}}{a^{2/3}-\sqrt[3]{a}\cdots}\right.}{6a\cdots}$$
$$= -\frac{\csc^2(x)}{2a}-\frac{\log(\sin(x))}{a}-\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(x)\right)}{3a^{5/3}}+\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(x)+b^{2/3}\sin^2(x)\right)}{6a^{5/3}}\cdots$$
$$= \frac{b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sin(x)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{5/3}}-\frac{\csc^2(x)}{2a}-\frac{\log(\sin(x))}{a}-\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(x)\right)}{3a^{5/3}}+\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{\cdots}\right.}{6\cdots}$$

Mathematica [A] time = 0.297401, size = 143, normalized size = 0.93

$$\frac{2\left(a^{2/3}-(-1)^{2/3}b^{2/3}\right)\log\left(-(-1)^{2/3}\sqrt[3]{a}-\sqrt[3]{b}\sin(x)\right)+2\left(a^{2/3}-b^{2/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(x)\right)+2\left(a^{2/3}+\sqrt[3]{-1}b^{2/3}\right)\log\left(\sqrt[3]{a}+\cdots\right.}{6a^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^3/(a + b*Sin[x]^3),x]
```

```
[Out] (-3*a^(2/3)*Csc[x]^2 - 6*a^(2/3)*Log[Sin[x]] + 2*(a^(2/3) - (-1)^(2/3)*b^(2
/3))*Log[-((-1)^(2/3)*a^(1/3)) - b^(1/3)*Sin[x]] + 2*(a^(2/3) - b^(2/3))*Lo
g[a^(1/3) + b^(1/3)*Sin[x]] + 2*(a^(2/3) + (-1)^(1/3)*b^(2/3))*Log[a^(1/3)
+ (-1)^(2/3)*b^(1/3)*Sin[x]])/(6*a^(5/3))
```

Maple [A] time = 0.078, size = 126, normalized size = 0.8

$$-\frac{1}{3a}\ln\left(\sin(x)+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{1}{6a}\ln\left((\sin(x))^2-\sqrt[3]{\frac{a}{b}}\sin(x)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{\sqrt{3}}{3a}\arctan\left(\frac{\sqrt{3}}{3}\left(2\sin(x)\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^3/(a+b*sin(x)^3),x)
```

```
[Out] -1/3/a/(a/b)^(2/3)*ln(sin(x)+(a/b)^(1/3))+1/6/a/(a/b)^(2/3)*ln(sin(x)^2-(a/
b)^(1/3)*sin(x)+(a/b)^(2/3))-1/3/a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(
2/(a/b)^(1/3)*sin(x)-1))+1/3*ln(a+b*sin(x)^3)/a-1/2/a/sin(x)^2-ln(sin(x))/a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3/(a+b*sin(x)^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3/(a+b*sin(x)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^{3}(x)}{a+b\sin^{3}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**3/(a+b*sin(x)**3),x)
```

```
[Out] Integral(cot(x)**3/(a + b*sin(x)**3), x)
```

Giac [A] time = 1.11203, size = 194, normalized size = 1.27

$$\frac{b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\sin(x)\right|\right)}{3\,a^{2}}+\frac{\log\left(\left|b\sin(x)^{3}+a\right|\right)}{3\,a}-\frac{\log\left(\left|\sin(x)\right|\right)}{a}-\frac{\sqrt{3}\left(-ab^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\sin(x)\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^{2}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3/(a+b*sin(x)^3),x, algorithm="giac")
```

```
[Out] 1/3*b*(-a/b)^(1/3)*log(abs(-(-a/b)^(1/3) + sin(x)))/a^2 + 1/3*log(abs(b*sin
(x)^3 + a))/a - log(abs(sin(x)))/a - 1/3*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*
sqrt(3)*((-a/b)^(1/3) + 2*sin(x))/(-a/b)^(1/3))/a^2 - 1/6*(-a*b^2)^(1/3)*lo
g(sin(x)^2 + (-a/b)^(1/3)*sin(x) + (-a/b)^(2/3))/a^2 - 1/2/(a*sin(x)^2)
```

3.553 $\int \cot(x)\sqrt{a+b\sin^3(x)}\,dx$

Optimal. Leaf size=45

$$\frac{2}{3}\sqrt{a+b\sin^3(x)}-\frac{2}{3}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^3(x)}}{\sqrt{a}}\right)$$

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sin[x]^3]/Sqrt[a]])/3 + (2*Sqrt[a + b*Sin[x]
^3])/3
```

Rubi [A] time = 0.0740431, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3230, 266, 50, 63, 208}

$$\frac{2}{3}\sqrt{a+b\sin^3(x)}-\frac{2}{3}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^3(x)}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]*Sqrt[a + b*Sin[x]^3],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sin[x]^3]/Sqrt[a]])/3 + (2*Sqrt[a + b*Sin[x]
^3])/3
```

Rule 3230

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff^(m + 1)/f, Subst[Int[(x^m*(a + b*(c*ff*x)^n)^p)/(1 - ff^2*x^2)^((m +
1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \cot(x)\sqrt{a+b\sin^3(x)}\,dx &= \text{Subst}\left(\int \frac{\sqrt{a+bx^3}}{x}\,dx, x, \sin(x)\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \sin^3(x)\right)\\
&= \frac{2}{3}\sqrt{a+b\sin^3(x)} + \frac{1}{3}a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sin^3(x)\right)\\
&= \frac{2}{3}\sqrt{a+b\sin^3(x)} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sin^3(x)}\right)}{3b}\\
&= -\frac{2}{3}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^3(x)}}{\sqrt{a}}\right) + \frac{2}{3}\sqrt{a+b\sin^3(x)}
\end{aligned}
$$

Mathematica [A] time = 0.0276282, size = 45, normalized size = 1.

$$\frac{2}{3}\sqrt{a+b\sin^3(x)} - \frac{2}{3}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^3(x)}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]*Sqrt[a + b*Sin[x]^3],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sin[x]^3]/Sqrt[a]])/3 + (2*Sqrt[a + b*Sin[x]
^3])/3
```

Maple [A] time = 0.48, size = 34, normalized size = 0.8

$$-\frac{2}{3}\text{Artanh}\left(\sqrt{a+b(\sin(x))^3}\frac{1}{\sqrt{a}}\right)\sqrt{a} + \frac{2}{3}\sqrt{a+b(\sin(x))^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(a+b*sin(x)^3)^(1/2),x)
```

```
[Out] -2/3*arctanh((a+b*sin(x)^3)^(1/2)/a^(1/2))*a^(1/2)+2/3*(a+b*sin(x)^3)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*sin(x)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*sin(x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\sin^3(x)}\cot(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*sin(x)**3)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(x)**3)*cot(x), x)
```

Giac [A] time = 1.09472, size = 51, normalized size = 1.13

$$\frac{2\,a\arctan\left(\frac{\sqrt{b\sin(x)^3+a}}{\sqrt{-a}}\right)}{3\sqrt{-a}} + \frac{2}{3}\sqrt{b\sin(x)^3 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*sin(x)^3)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*a*arctan(sqrt(b*sin(x)^3 + a)/sqrt(-a))/sqrt(-a) + 2/3*sqrt(b*sin(x)^3
+ a)
```

3.554 $\int \frac{\cot(x)}{\sqrt{a+b\sin^3(x)}}\,dx$

Optimal. Leaf size=28

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sin^3(x)}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sin[x]^3]/Sqrt[a]])/(3*Sqrt[a])
```

Rubi [A] time = 0.0715143, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3230, 266, 63, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sin^3(x)}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/Sqrt[a + b*Sin[x]^3],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sin[x]^3]/Sqrt[a]])/(3*Sqrt[a])
```

Rule 3230

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff^(m + 1)/f, Subst[Int[(x^m*(a + b*(c*ff*x)^n)^p)/(1 - ff^2*x^2)^((m +
1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cot(x)}{\sqrt{a+b\sin^3(x)}}\,dx = \text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^3}}\,dx, x, \sin(x)\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sin^3(x)\right)$$
$$= \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sin^3(x)}\right)}{3b}$$
$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sin^3(x)}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Mathematica [A] time = 0.0175116, size = 28, normalized size = 1.

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sin^3(x)}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/Sqrt[a + b*Sin[x]^3],x]

[Out] (-2*ArcTanh[Sqrt[a + b*Sin[x]^3]/Sqrt[a]])/(3*Sqrt[a])
```

Maple [A] time = 0.115, size = 21, normalized size = 0.8

$$-\frac{2}{3}\text{Artanh}\left(\sqrt{a+b\left(\sin\left(x\right)\right)^3}\frac{1}{\sqrt{a}}\right)\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a+b*sin(x)^3)^(1/2),x)

[Out] -2/3*arctanh((a+b*sin(x)^3)^(1/2)/a^(1/2))/a^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*sin(x)^3)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*sin(x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot (x)}{\sqrt{a + b\sin^3 (x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*sin(x)**3)**(1/2),x)
```

```
[Out] Integral(cot(x)/sqrt(a + b*sin(x)**3), x)
```

Giac [A] time = 1.08353, size = 32, normalized size = 1.14

$$\frac{2 \arctan\left(\frac{\sqrt{b\sin(x)^3 + a}}{\sqrt{-a}}\right)}{3\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*sin(x)^3)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*arctan(sqrt(b*sin(x)^3 + a)/sqrt(-a))/sqrt(-a)
```

3.555 $\int \cot(c+dx)\sqrt{a+b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=59

$$\frac{\sqrt{a+b\sin^4(c+dx)}}{2d}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)}{2d}$$

```
[Out] -(Sqrt[a]*ArcTanh[Sqrt[a + b*Sin[c + d*x]^4]/Sqrt[a]])/(2*d) + Sqrt[a + b*S
in[c + d*x]^4]/(2*d)
```

Rubi [A] time = 0.0889988, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3229, 266, 50, 63, 208}

$$\frac{\sqrt{a+b\sin^4(c+dx)}}{2d}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] -(Sqrt[a]*ArcTanh[Sqrt[a + b*Sin[c + d*x]^4]/Sqrt[a]])/(2*d) + Sqrt[a + b*S
in[c + d*x]^4]/(2*d)
```

Rule 3229

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^
((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff^(n/2)*x^(n/2))^p)/(1
- ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \cot(c+dx)\sqrt{a+b\sin^4(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x}\,dx, x, \sin^2(c+dx)\right)}{2d} \\
&= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \sin^4(c+dx)\right)}{4d} \\
&= \frac{\sqrt{a+b\sin^4(c+dx)}}{2d} + \frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sin^4(c+dx)\right)}{4d} \\
&= \frac{\sqrt{a+b\sin^4(c+dx)}}{2d} + \frac{a\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sin^4(c+dx)}\right)}{2bd} \\
&= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)}{2d} + \frac{\sqrt{a+b\sin^4(c+dx)}}{2d}
\end{aligned}$$

Mathematica [A] time = 0.053884, size = 55, normalized size = 0.93

$$-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)-\sqrt{a+b\sin^4(c+dx)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] -(Sqrt[a]*ArcTanh[Sqrt[a + b*Sin[c + d*x]^4]/Sqrt[a]] - Sqrt[a + b*Sin[c +
d*x]^4])/(2*d)
```

Maple [F] time = 0.754, size = 0, normalized size = 0.

$$\int \cot(dx+c)\sqrt{a+b(\sin(dx+c))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x)
```

```
[Out] int(cot(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.92246, size = 514, normalized size = 8.71

$$\left[\frac{\sqrt{a}\log\left(\frac{8\left(b\cos(dx+c)^4-2\,b\cos(dx+c)^2-2\,\sqrt{b\cos(dx+c)^4-2\,b\cos(dx+c)^2+a+b}\sqrt{a}+2\,a+b\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}\right)+2\,\sqrt{b\cos\left(dx+c\right)^4-2\,b\cos\left(dx+c\right)^2+a+b}}{4\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a)*log(8*(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d
*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)*sqrt(a) + 2*a + b)/(cos(d*x + c)^4
- 2*cos(d*x + c)^2 + 1)) + 2*sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a
 + b))/d, 1/2*(sqrt(-a)*arctan(sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 +
 a + b)*sqrt(-a)/a) + sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b))/
d]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a+b\sin^4\left(c+dx\right)}\cot\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)**4)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(c + d*x)**4)*cot(c + d*x), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.556 $\int \frac{\tan^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

Optimal. Leaf size=89

$$\frac{\sec^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{2d(a+b)}-\frac{a\tanh^{-1}\left(\frac{a+b\sin^2(c+dx)}{\sqrt{a+b}\sqrt{a+b\sin^4(c+dx)}}\right)}{2d(a+b)^{3/2}}$$

```
[Out] -(a*ArcTanh[(a + b*Sin[c + d*x]^2)/(Sqrt[a + b]*Sqrt[a + b*Sin[c + d*x]^4])
])/(2*(a + b)^(3/2)*d) + (Sec[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]^4])/(2*(a
+ b)*d)
```

Rubi [A] time = 0.116921, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3229, 807, 725, 206}

$$\frac{\sec^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{2d(a+b)}-\frac{a\tanh^{-1}\left(\frac{a+b\sin^2(c+dx)}{\sqrt{a+b}\sqrt{a+b\sin^4(c+dx)}}\right)}{2d(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] -(a*ArcTanh[(a + b*Sin[c + d*x]^2)/(Sqrt[a + b]*Sqrt[a + b*Sin[c + d*x]^4])
])/(2*(a + b)^(3/2)*d) + (Sec[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]^4])/(2*(a
+ b)*d)
```

Rule 3229

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^
((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff^(n/2)*x^(n/2))^p)/(1
- ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{x}{(1-x)^2\sqrt{a+bx^2}}\,dx, x, \sin^2(c+dx)\right)}{2d} \\
&= \frac{\sec^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{2(a+b)d} - \frac{a\,\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx^2}}\,dx, x, \sin^2(c+dx)\right)}{2(a+b)d} \\
&= \frac{\sec^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{2(a+b)d} + \frac{a\,\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \frac{-a-b\sin^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\right)}{2(a+b)d} \\
&= -\frac{a\tanh^{-1}\left(\frac{a+b\sin^2(c+dx)}{\sqrt{a+b}\sqrt{a+b\sin^4(c+dx)}}\right)}{2(a+b)^{3/2}d} + \frac{\sec^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{2(a+b)d}
\end{aligned}$$

Mathematica [A] time = 0.185625, size = 85, normalized size = 0.96

$$-\frac{\frac{a\tanh^{-1}\left(\frac{a+b\sin^2(c+dx)}{\sqrt{a+b}\sqrt{a+b\sin^4(c+dx)}}\right)}{(a+b)^{3/2}} - \frac{\sec^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{a+b}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] -((a*ArcTanh[(a + b*Sin[c + d*x]^2)/(Sqrt[a + b]*Sqrt[a + b*Sin[c + d*x]^4]
)])/(a + b)^(3/2) - (Sec[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]^4])/(a + b))/(2
*d)
```

Maple [F] time = 0.72, size = 0, normalized size = 0.

$$\int (\tan(dx+c))^3 \frac{1}{\sqrt{a+b(\sin(dx+c))^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x)
```

```
[Out] int(tan(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 3.04745, size = 895, normalized size = 10.06

$$\left[\frac{\sqrt{a+b}a\cos\left(dx+c\right)^2\log\left(\frac{\left(ab+2b^2\right)\cos(dx+c)^4-4\left(ab+b^2\right)\cos(dx+c)^2+2\sqrt{b\cos(dx+c)^4-2b\cos(dx+c)^2+a+b}\left(b\cos(dx+c)^2-a-b\right)\sqrt{a+b}+2a^2+4a}{\cos(dx+c)^4}\right.}{4\left(a^2+2ab+b^2\right)d\cos\left(dx+c\right)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a + b)*a*cos(d*x + c)^2*log(((a*b + 2*b^2)*cos(d*x + c)^4 - 4*(a
*b + b^2)*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a
 + b)*(b*cos(d*x + c)^2 - a - b)*sqrt(a + b) + 2*a^2 + 4*a*b + 2*b^2)/cos(d
*x + c)^4) + 2*sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)*(a + b))
/((a^2 + 2*a*b + b^2)*d*cos(d*x + c)^2), -1/2*(a*sqrt(-a - b)*arctan(sqrt(b
*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)*(b*cos(d*x + c)^2 - a - b)*sq
rt(-a - b)/((a*b + b^2)*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2
 + 2*a*b + b^2))*cos(d*x + c)^2 - sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^
2 + a + b)*(a + b))/((a^2 + 2*a*b + b^2)*d*cos(d*x + c)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^3\left(c+dx\right)}{\sqrt{a+b\sin^4\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+b*sin(d*x+c)**4)**(1/2),x)
```

```
[Out] Integral(tan(c + d*x)**3/sqrt(a + b*sin(c + d*x)**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(dx+c\right)^3}{\sqrt{b\sin\left(dx+c\right)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tan(d*x + c)^3/sqrt(b*sin(d*x + c)^4 + a), x)
```

3.557 $\int \frac{\tan(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

Optimal. Leaf size=51

$$\frac{\tanh^{-1}\left(\frac{a+b\sin^2(c+dx)}{\sqrt{a+b}\sqrt{a+b\sin^4(c+dx)}}\right)}{2d\sqrt{a+b}}$$

```
[Out] ArcTanh[(a + b*Sin[c + d*x]^2)/(Sqrt[a + b]*Sqrt[a + b*Sin[c + d*x]^4])]/(2
*Sqrt[a + b]*d)
```

Rubi [A] time = 0.0555082, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3229, 725, 206}

$$\frac{\tanh^{-1}\left(\frac{a+b\sin^2(c+dx)}{\sqrt{a+b}\sqrt{a+b\sin^4(c+dx)}}\right)}{2d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] ArcTanh[(a + b*Sin[c + d*x]^2)/(Sqrt[a + b]*Sqrt[a + b*Sin[c + d*x]^4])]/(2
*Sqrt[a + b]*d)
```

Rule 3229

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^
((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff^(n/2)*x^(n/2))^p)/(1
- ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx^2}}\,dx, x, \sin^2(c+dx)\right)}{2d}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \frac{-a-b\sin^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\right)}{2d}$$
$$= \frac{\tanh^{-1}\left(\frac{a+b\sin^2(c+dx)}{\sqrt{a+b}\sqrt{a+b\sin^4(c+dx)}}\right)}{2\sqrt{a+b}d}$$

Mathematica [A] time = 0.0906018, size = 65, normalized size = 1.27

$$\frac{\tanh^{-1}\left(\frac{a-b\cos^2(c+dx)+b}{\sqrt{a+b}\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}\right)}{2d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] ArcTanh[(a + b - b*Cos[c + d*x]^2)/(Sqrt[a + b]*Sqrt[a + b - 2*b*Cos[c + d*
x]^2 + b*Cos[c + d*x]^4])]/(2*Sqrt[a + b]*d)
```

Maple [A] time = 0.203, size = 72, normalized size = 1.4

$$\frac{1}{2d}\ln\left(\frac{1}{(\cos(dx+c))^2}\left(2a+2b-2b(\cos(dx+c))^2+2\sqrt{a+b}\sqrt{a+b-2b(\cos(dx+c))^2+b(\cos(dx+c))^4}\right)\right)\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x)
```

```
[Out] 1/2/d/(a+b)^(1/2)*ln((2*a+2*b-2*b*cos(d*x+c)^2+2*(a+b)^(1/2)*(a+b-2*b*cos(d
*x+c)^2+b*cos(d*x+c)^4)^(1/2))/cos(d*x+c)^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(dx+c)}{\sqrt{b\sin(dx+c)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(d*x + c)/sqrt(b*sin(d*x + c)^4 + a), x)
```

Fricas [B] time = 2.66844, size = 589, normalized size = 11.55

$$\left[\frac{\log\left(\frac{(ab+2\,b^2)\cos(dx+c)^4-4\,(ab+b^2)\cos(dx+c)^2-2\,\sqrt{b\cos(dx+c)^4-2\,b\cos(dx+c)^2+a+b}\left(b\cos(dx+c)^2-a-b\right)\sqrt{a+b}+2\,a^2+4\,ab+2\,b^2}{\cos(dx+c)^4}\right)}{4\,\sqrt{a+b}d}, \frac{\sqrt{-a-b}\arctan\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(((a*b + 2*b^2)*cos(d*x + c)^4 - 4*(a*b + b^2)*cos(d*x + c)^2 - 2*s
qrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)*(b*cos(d*x + c)^2 - a -
b)*sqrt(a + b) + 2*a^2 + 4*a*b + 2*b^2)/cos(d*x + c)^4)/(sqrt(a + b)*d), 1/
2*sqrt(-a - b)*arctan(sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)*(
b*cos(d*x + c)^2 - a - b)*sqrt(-a - b)/((a*b + b^2)*cos(d*x + c)^4 - 2*(a*b
 + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2))/((a + b)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c)**4)**(1/2),x)
```

```
[Out] Integral(tan(c + d*x)/sqrt(a + b*sin(c + d*x)**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(dx+c)}{\sqrt{b\sin(dx+c)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tan(d*x + c)/sqrt(b*sin(d*x + c)^4 + a), x)
```

3.558 $\int \frac{\cot(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

Optimal. Leaf size=35

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)}{2\sqrt{a}d}$$

```
[Out] -ArcTanh[Sqrt[a + b*Sin[c + d*x]^4]/Sqrt[a]]/(2*Sqrt[a]*d)
```

Rubi [A] time = 0.0711073, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3229, 266, 63, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)}{2\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] -ArcTanh[Sqrt[a + b*Sin[c + d*x]^4]/Sqrt[a]]/(2*Sqrt[a]*d)
```

Rule 3229

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^
((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff^(n/2)*x^(n/2))^p)/(1
- ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cot(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^2}}\,dx, x, \sin^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sin^4(c+dx)\right)}{4d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sin^4(c+dx)}\right)}{2bd}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)}{2\sqrt{a}d}$$

Mathematica [A] time = 0.0254163, size = 35, normalized size = 1.

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)}{2\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] -ArcTanh[Sqrt[a + b*Sin[c + d*x]^4]/Sqrt[a]]/(2*Sqrt[a]*d)
```

Maple [F] time = 0.717, size = 0, normalized size = 0.

$$\int \cot(dx+c)\frac{1}{\sqrt{a+b(\sin(dx+c))^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x)
```

```
[Out] int(cot(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.47433, size = 369, normalized size = 10.54

$$\left[\frac{\log\left(\frac{8\left(b\cos(dx+c)^4-2\,b\cos(dx+c)^2-2\,\sqrt{b\cos(dx+c)^4-2\,b\cos(dx+c)^2+a+b}\sqrt{a}+2\,a+b\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}\right)}{4\,\sqrt{a}d},\ \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{b\cos(dx+c)^4-2\,b\cos(dx+c)^2+a+b}\sqrt{-a}}{a}\right)}{2\,ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(8*(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c)^4
 - 2*b*cos(d*x + c)^2 + a + b)*sqrt(a) + 2*a + b)/(cos(d*x + c)^4 - 2*cos(d
*x + c)^2 + 1))/(sqrt(a)*d), 1/2*sqrt(-a)*arctan(sqrt(b*cos(d*x + c)^4 - 2*
b*cos(d*x + c)^2 + a + b)*sqrt(-a)/a)/(a*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c)**4)**(1/2),x)
```

```
[Out] Integral(cot(c + d*x)/sqrt(a + b*sin(c + d*x)**4), x)
```

Giac [A] time = 1.14464, size = 42, normalized size = 1.2

$$\frac{\arctan\left(\frac{\sqrt{b\sin(dx+c)^4+a}}{\sqrt{-a}}\right)}{2\,\sqrt{-a}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*arctan(sqrt(b*sin(d*x + c)^4 + a)/sqrt(-a))/(sqrt(-a)*d)
```

3.559 $\int \frac{\cot^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

Optimal. Leaf size=70

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)}{2\sqrt{a}d}-\frac{\csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{2ad}$$

```
[Out] ArcTanh[Sqrt[a + b*Sin[c + d*x]^4]/Sqrt[a]]/(2*Sqrt[a]*d) - (Csc[c + d*x]^2
*Sqrt[a + b*Sin[c + d*x]^4])/(2*a*d)
```

Rubi [A] time = 0.0989119, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3229, 807, 266, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)}{2\sqrt{a}d}-\frac{\csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sin[c + d*x]^4]/Sqrt[a]]/(2*Sqrt[a]*d) - (Csc[c + d*x]^2
*Sqrt[a + b*Sin[c + d*x]^4])/(2*a*d)
```

Rule 3229

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^
((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff^(n/2)*x^(n/2))^p)/(1
- ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1-x}{x^2\sqrt{a+bx^2}}\,dx, x, \sin^2(c+dx)\right)}{2d}\\
&= -\frac{\csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{2ad} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^2}}\,dx, x, \sin^2(c+dx)\right)}{2d}\\
&= -\frac{\csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{2ad} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sin^4(c+dx)\right)}{4d}\\
&= -\frac{\csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{2ad} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sin^4(c+dx)}\right)}{2bd}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)}{2\sqrt{a}d} - \frac{\csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{2ad}
\end{aligned}$$

Mathematica [A] time = 0.0815787, size = 66, normalized size = 0.94

$$\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right) - \csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] (Sqrt[a]*ArcTanh[Sqrt[a + b*Sin[c + d*x]^4]/Sqrt[a]] - Csc[c + d*x]^2*Sqrt[
a + b*Sin[c + d*x]^4])/(2*a*d)
```

Maple [F] time = 0.71, size = 0, normalized size = 0.

$$\int (\cot(dx+c))^3 \frac{1}{\sqrt{a+b(\sin(dx+c))^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x)
```

```
[Out] int(cot(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.54301, size = 640, normalized size = 9.14

$$\left[\frac{\left(\cos\left(dx+c\right)^2-1\right)\sqrt{a}\log\left(\frac{8\left(b\cos(dx+c)^4-2\,b\cos(dx+c)^2+2\,\sqrt{b\cos(dx+c)^4-2\,b\cos(dx+c)^2+a+b}\sqrt{a}+2\,a+b\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}\right)+2\,\sqrt{b\cos\left(dx+c\right)^4-2\,b\,\mathrm{co}}}{4\left(ad\cos\left(dx+c\right)^2-ad\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((cos(d*x + c)^2 - 1)*sqrt(a)*log(8*(b*cos(d*x + c)^4 - 2*b*cos(d*x +
c)^2 + 2*sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)*sqrt(a) + 2*a
+ b)/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)) + 2*sqrt(b*cos(d*x + c)^4 - 2
*b*cos(d*x + c)^2 + a + b))/(a*d*cos(d*x + c)^2 - a*d), -1/2*((cos(d*x + c)
^2 - 1)*sqrt(-a)*arctan(sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)
*sqrt(-a)/a) - sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b))/(a*d*co
s(d*x + c)^2 - a*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+b*sin(d*x+c)**4)**(1/2),x)
```

```
[Out] Integral(cot(c + d*x)**3/sqrt(a + b*sin(c + d*x)**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(dx+c\right)^3}{\sqrt{b\sin\left(dx+c\right)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(d*x + c)^3/sqrt(b*sin(d*x + c)^4 + a), x)
```

3.560 $\int \frac{\cot^5(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

Optimal. Leaf size=108

$$-\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)}{4a^{3/2}d}-\frac{\csc^4(c+dx)\sqrt{a+b\sin^4(c+dx)}}{4ad}+\frac{\csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{ad}$$

```
[Out] -((2*a - b)*ArcTanh[Sqrt[a + b*Sin[c + d*x]^4]/Sqrt[a]])/(4*a^(3/2)*d) + (C
sc[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]^4])/(a*d) - (Csc[c + d*x]^4*Sqrt[a +
b*Sin[c + d*x]^4])/(4*a*d)
```

Rubi [A] time = 0.175569, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3229, 1807, 807, 266, 63, 208}

$$-\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)}{4a^{3/2}d}-\frac{\csc^4(c+dx)\sqrt{a+b\sin^4(c+dx)}}{4ad}+\frac{\csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] -((2*a - b)*ArcTanh[Sqrt[a + b*Sin[c + d*x]^4]/Sqrt[a]])/(4*a^(3/2)*d) + (C
sc[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]^4])/(a*d) - (Csc[c + d*x]^4*Sqrt[a +
b*Sin[c + d*x]^4])/(4*a*d)
```

Rule 3229

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^
((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff^(n/2)*x^(n/2))^p)/(1
- ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cot^5(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x)^2}{x^3\sqrt{a+bx^2}}\,dx, x, \sin^2(c+dx)\right)}{2d}$$

$$= -\frac{\csc^4(c+dx)\sqrt{a+b\sin^4(c+dx)}}{4ad} - \frac{\text{Subst}\left(\int \frac{4a-(2a-b)x}{x^2\sqrt{a+bx^2}}\,dx, x, \sin^2(c+dx)\right)}{4ad}$$

$$= \frac{\csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{ad} - \frac{\csc^4(c+dx)\sqrt{a+b\sin^4(c+dx)}}{4ad} + \frac{(2a-b)\,\text{Subst}\left(\int \frac{}{x\sqrt{\cdots}}\right.}{\cdots}$$

$$= \frac{\csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{ad} - \frac{\csc^4(c+dx)\sqrt{a+b\sin^4(c+dx)}}{4ad} + \frac{(2a-b)\,\text{Subst}\left(\int \frac{}{x\sqrt{\cdots}}\right.}{\cdots}$$

$$= \frac{\csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{ad} - \frac{\csc^4(c+dx)\sqrt{a+b\sin^4(c+dx)}}{4ad} + \frac{(2a-b)\,\text{Subst}\left(\int \frac{}{-\frac{a}{b}\cdots}\right.}{\cdots}$$

$$= -\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)}{4a^{3/2}d} + \frac{\csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{ad} - \frac{\csc^4(c+dx)\sqrt{a+b\cdots}}{4ad}$$

Mathematica [A] time = 2.99334, size = 141, normalized size = 1.31

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right) - 4a\csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)} + b\sqrt{a+b\sin^4(c+dx)}\left(\frac{a\csc^4(c+dx)}{b} - \frac{\tanh^{-1}\left(\sqrt{\frac{b\sin^4(c+\cdots}{a}}\right.}{\sqrt{\frac{b\sin^4(c+dx)}{a}\cdots}}\right.}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] -(2*a^(3/2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]^4]/Sqrt[a]] - 4*a*Csc[c + d*x]^
2*Sqrt[a + b*Sin[c + d*x]^4] + b*Sqrt[a + b*Sin[c + d*x]^4]*((a*Csc[c + d*x
]^4)/b - ArcTanh[Sqrt[1 + (b*Sin[c + d*x]^4)/a]]/Sqrt[1 + (b*Sin[c + d*x]^4
)/a]))/(4*a^2*d)
```

Maple [F] time = 0.736, size = 0, normalized size = 0.

$$\int (\cot (dx + c))^5 \frac{1}{\sqrt{a + b (\sin (dx + c))^4}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5/(a+b*sin(d*x+c)^4)^(1/2),x)
```

```
[Out] int(cot(d*x+c)^5/(a+b*sin(d*x+c)^4)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.77907, size = 913, normalized size = 8.45

$$\left[-\frac{\left((2a - b) \cos (dx + c)^4 - 2 (2a - b) \cos (dx + c)^2 + 2a - b\right) \sqrt{a} \log \left(\frac{8 \left(b \cos(dx+c)^4 - 2 b \cos(dx+c)^2 + 2 \sqrt{b \cos(dx+c)^4 - 2 b \cos(dx+c}}{\cos(dx+c)^4 - 2 \cos(dx+c)^2 + 1} \right.}{8 \left(a^2 d \cos (dx + c)^4 - 2 a^2 d \cos (dx + c\right.} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(((2*a - b)*cos(d*x + c)^4 - 2*(2*a - b)*cos(d*x + c)^2 + 2*a - b)*sq
rt(a)*log(8*(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c)^
4 - 2*b*cos(d*x + c)^2 + a + b)*sqrt(a) + 2*a + b)/(cos(d*x + c)^4 - 2*cos(
d*x + c)^2 + 1)) + 2*sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)*(4
*a*cos(d*x + c)^2 - 3*a))/(a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2 +
a^2*d), 1/4*(((2*a - b)*cos(d*x + c)^4 - 2*(2*a - b)*cos(d*x + c)^2 + 2*a -
 b)*sqrt(-a)*arctan(sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)*sqr
t(-a)/a) - sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)*(4*a*cos(d*x
 + c)^2 - 3*a))/(a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2 + a^2*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^5 (c + dx)}{\sqrt{a + b \sin^4 (c + dx)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5/(a+b*sin(d*x+c)**4)**(1/2),x)
```

```
[Out] Integral(cot(c + d*x)**5/sqrt(a + b*sin(c + d*x)**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(dx+c)^5}{\sqrt{b\sin(dx+c)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(d*x + c)^5/sqrt(b*sin(d*x + c)^4 + a), x)
```

3.561 $\int \frac{\tan^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

Optimal. Leaf size=411

$$\frac{\sin(c+dx)\cos(c+dx)\left((a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}{d\sqrt{a+b}\sqrt{a+b\sin^4(c+dx)}\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)}+\frac{\sqrt[4]{a}\cos^2(c+dx)\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)\ldots}{2d(\ldots}$$

```
[Out] (Cos[c + d*x]*Sin[c + d*x]*(a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4
))/(Sqrt[a + b]*d*Sqrt[a + b*Sin[c + d*x]^4]*(Sqrt[a] + Sqrt[a + b]*Tan[c +
 d*x]^2)) - (a^(1/4)*Cos[c + d*x]^2*EllipticE[2*ArcTan[((a + b)^(1/4)*Tan[c
 + d*x])/a^(1/4)], (1 - Sqrt[a]/Sqrt[a + b])/2]*(Sqrt[a] + Sqrt[a + b]*Tan[
c + d*x]^2)*Sqrt[(a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4)/(Sqrt[a]
 + Sqrt[a + b]*Tan[c + d*x]^2)^2])/((a + b)^(3/4)*d*Sqrt[a + b*Sin[c + d*x]
^4]) + (a^(1/4)*Cos[c + d*x]^2*EllipticF[2*ArcTan[((a + b)^(1/4)*Tan[c + d*
x])/a^(1/4)], (1 - Sqrt[a]/Sqrt[a + b])/2]*(Sqrt[a] + Sqrt[a + b]*Tan[c + d
*x]^2)*Sqrt[(a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4)/(Sqrt[a] + Sq
rt[a + b]*Tan[c + d*x]^2)^2])/(2*(a + b)^(3/4)*d*Sqrt[a + b*Sin[c + d*x]^4]
)
```

Rubi [A] time = 0.379546, antiderivative size = 411, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3232, 1139, 1103, 1195}

$$\frac{\sin(c+dx)\cos(c+dx)\left((a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}{d\sqrt{a+b}\sqrt{a+b\sin^4(c+dx)}\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)}+\frac{\sqrt[4]{a}\cos^2(c+dx)\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)\ldots}{2d(\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] (Cos[c + d*x]*Sin[c + d*x]*(a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4
))/(Sqrt[a + b]*d*Sqrt[a + b*Sin[c + d*x]^4]*(Sqrt[a] + Sqrt[a + b]*Tan[c +
 d*x]^2)) - (a^(1/4)*Cos[c + d*x]^2*EllipticE[2*ArcTan[((a + b)^(1/4)*Tan[c
 + d*x])/a^(1/4)], (1 - Sqrt[a]/Sqrt[a + b])/2]*(Sqrt[a] + Sqrt[a + b]*Tan[
c + d*x]^2)*Sqrt[(a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4)/(Sqrt[a]
 + Sqrt[a + b]*Tan[c + d*x]^2)^2])/((a + b)^(3/4)*d*Sqrt[a + b*Sin[c + d*x]
^4]) + (a^(1/4)*Cos[c + d*x]^2*EllipticF[2*ArcTan[((a + b)^(1/4)*Tan[c + d*
x])/a^(1/4)], (1 - Sqrt[a]/Sqrt[a + b])/2]*(Sqrt[a] + Sqrt[a + b]*Tan[c + d
*x]^2)*Sqrt[(a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4)/(Sqrt[a] + Sq
rt[a + b]*Tan[c + d*x]^2)^2])/(2*(a + b)^(3/4)*d*Sqrt[a + b*Sin[c + d*x]^4]
)
```

Rule 3232

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_)*((d_.)*tan[(e_.) + (f_.)*
(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(f
f*(a + b*Sin[e + f*x]^4)^p*(Sec[e + f*x]^2)^(2*p))/(f*Apart[a*(1 + Tan[e +
f*x]^2)^2 + b*Tan[e + f*x]^4]^p), Subst[Int[((d*ff*x)^m*ExpandToSum[a*(1 +
ff^2*x^2)^2 + b*ff^4*x^4, x]^p)/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*
x]/ff], x]] /; FreeQ[{a, b, d, e, f, m}, x] && IntegerQ[p - 1/2]
```

Rule 1139

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[1/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[1/q, I
nt[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\tan^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \frac{\left(\cos^2(c+dx)\sqrt{a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+2ax^2+(a+b)x^4}}\,dx,x,\text{t}\right.}{d\sqrt{a+b\sin^4(c+dx)}}$$

$$= \frac{\left(\sqrt{a}\cos^2(c+dx)\sqrt{a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+2ax^2+(a+b)x^4}}\,dx,\right.}{\sqrt{a+b}\,d\sqrt{a+b\sin^4(c+dx)}}$$

$$= \frac{\cos(c+dx)\sin(c+dx)\left(a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)\right)}{\sqrt{a+b}\,d\sqrt{a+b\sin^4(c+dx)}\left(\sqrt{a}+\sqrt{a+b}\tan^2(c+dx)\right)} - \frac{\sqrt[4]{a}\cos^2(c+dx)E\left(2\right.}{}$$

Mathematica [C] time = 6.07386, size = 291, normalized size = 0.71

$$-\frac{2i\sqrt{2}\sqrt{a}\cos^2(c+dx)\sqrt{1+\left(1-\frac{i\sqrt{b}}{\sqrt{a}}\right)\tan^2(c+dx)}\sqrt{1+\left(1+\frac{i\sqrt{b}}{\sqrt{a}}\right)\tan^2(c+dx)}\left(E\left(i\sinh^{-1}\left(\sqrt{1-\frac{i\sqrt{b}}{\sqrt{a}}}\tan(c+dx)\right)|\frac{\sqrt{a+}}{\sqrt{a-}}\right.\right.}{d\left(\sqrt{a}+i\sqrt{b}\right)\sqrt{1-\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{8a-4b\cos(2(c+dx))+b\cos(4(c+dx))+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] ((-2*I)*Sqrt[2]*Sqrt[a]*Cos[c + d*x]^2*(EllipticE[I*ArcSinh[Sqrt[1 - (I*Sqr
t[b])/Sqrt[a]]*Tan[c + d*x]], (Sqrt[a] + I*Sqrt[b])/(Sqrt[a] - I*Sqrt[b])]
- EllipticF[I*ArcSinh[Sqrt[1 - (I*Sqrt[b])/Sqrt[a]]*Tan[c + d*x]], (Sqrt[a]
 + I*Sqrt[b])/(Sqrt[a] - I*Sqrt[b])])*Sqrt[1 + (1 - (I*Sqrt[b])/Sqrt[a])*Ta
n[c + d*x]^2]*Sqrt[1 + (1 + (I*Sqrt[b])/Sqrt[a])*Tan[c + d*x]^2])/((Sqrt[a]
 + I*Sqrt[b])*Sqrt[1 - (I*Sqrt[b])/Sqrt[a]]*d*Sqrt[8*a + 3*b - 4*b*Cos[2*(c
```

```
+ d*x)] + b*Cos[4*(c + d*x)]])
```

Maple [F] time = 0.71, size = 0, normalized size = 0.

$$\int (\tan (dx+c))^2 \frac{1}{\sqrt{a+b (\sin (dx+c))^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x)
```

```
[Out] int(tan(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan (dx+c)^2}{\sqrt{b \sin (dx+c)^4 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(d*x + c)^2/sqrt(b*sin(d*x + c)^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\tan (dx+c)^2}{\sqrt{b \cos (dx+c)^4 - 2\, b \cos (dx+c)^2 + a + b}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(tan(d*x + c)^2/sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^2 (c+dx)}{\sqrt{a + b \sin^4 (c+dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+b*sin(d*x+c)**4)**(1/2),x)
```

```
[Out] Integral(tan(c + d*x)**2/sqrt(a + b*sin(c + d*x)**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan (dx + c)^2}{\sqrt{b \sin (dx + c)^4 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tan(d*x + c)^2/sqrt(b*sin(d*x + c)^4 + a), x)
```

3.562 $\int \frac{1}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

Optimal. Leaf size=162

$$\frac{\cos^2(c+dx)\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)\sqrt{\frac{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}{\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a+b}\tan(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\right)}{2\sqrt[4]{a}d\sqrt[4]{a+b}\sqrt{a+b\sin^4(c+dx)}}$$

```
[Out] (Cos[c + d*x]^2*EllipticF[2*ArcTan[((a + b)^(1/4)*Tan[c + d*x])/a^(1/4)], (
1 - Sqrt[a]/Sqrt[a + b])/2]*(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2)*Sqrt[(a
+ 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4)/(Sqrt[a] + Sqrt[a + b]*Tan[c
 + d*x]^2)^2])/(2*a^(1/4)*(a + b)^(1/4)*d*Sqrt[a + b*Sin[c + d*x]^4])
```

Rubi [A] time = 0.08279, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3210, 1103}

$$\frac{\cos^2(c+dx)\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)\sqrt{\frac{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}{\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a+b}\tan(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\right)}{2\sqrt[4]{a}d\sqrt[4]{a+b}\sqrt{a+b\sin^4(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] (Cos[c + d*x]^2*EllipticF[2*ArcTan[((a + b)^(1/4)*Tan[c + d*x])/a^(1/4)], (
1 - Sqrt[a]/Sqrt[a + b])/2]*(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2)*Sqrt[(a
+ 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4)/(Sqrt[a] + Sqrt[a + b]*Tan[c
 + d*x]^2)^2])/(2*a^(1/4)*(a + b)^(1/4)*d*Sqrt[a + b*Sin[c + d*x]^4])
```

Rule 3210

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[(ff*(a + b*Sin[e + f*x]^4)^p*(Sec[e + f
*x]^2)^(2*p))/(f*(a + 2*a*Tan[e + f*x]^2 + (a + b)*Tan[e + f*x]^4)^p), Subs
t[Int[(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x],
 x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[p - 1/2]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \frac{\left(\cos^2(c+dx)\sqrt{a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+2ax^2+(a+b)x^4}}\,dx,x,\text{t}\right.}{d\sqrt{a+b\sin^4(c+dx)}}$$

$$= \frac{\cos^2(c+dx)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a+b}\tan(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\right)\left(\sqrt{a}+\sqrt{a+b}\tan^2(c+dx)\right)\sqrt{\frac{a+2a\tan\ldots}{(\sqrt{\ldots}}}}{2\sqrt[4]{a}\sqrt[4]{a+b}d\sqrt{a+b\sin^4(c+dx)}}$$

Mathematica [C] time = 2.78717, size = 304, normalized size = 1.88

$$\frac{2\sqrt{2}\left(\sqrt{b}+i\sqrt{a}\right)\sin^2(c+dx)\tan(c+dx)\left(2\sqrt{a}+i\sqrt{b}\cos(2(c+dx))-i\sqrt{b}\right)\left(2i\sqrt{a}+\sqrt{b}\cos(2(c+dx))-\sqrt{b}\right)\sqrt{\csc^2(c+\ldots}}{\sqrt{a}d(8a-4b\cos(2(c+dx))+b\cos(4\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] (2*Sqrt[2]*(I*Sqrt[a] + Sqrt[b])*(2*Sqrt[a] - I*Sqrt[b] + I*Sqrt[b]*Cos[2*(
c + d*x)])*((2*I)*Sqrt[a] - Sqrt[b] + Sqrt[b]*Cos[2*(c + d*x)])*Sqrt[(1 - (
(2*I)*Sqrt[a])/Sqrt[b] - Cos[2*(c + d*x)])*Csc[c + d*x]^2]*Sqrt[(Cot[c + d*
x]^2*(I*Sqrt[a]*Sqrt[b] - a*Csc[c + d*x]^2))/(Sqrt[a] - I*Sqrt[b])^2]*Ellip
ticF[ArcSin[Sqrt[((-I)*Sqrt[b] + Sqrt[a]*Csc[c + d*x]^2)/(Sqrt[a] - I*Sqrt[
b])]], 1/2 + ((I/2)*Sqrt[a])/Sqrt[b]]*Sin[c + d*x]^2*Tan[c + d*x])/(Sqrt[a]
*d*(8*a + 3*b - 4*b*Cos[2*(c + d*x)] + b*Cos[4*(c + d*x)])^(3/2))
```

Maple [B] time = 2.372, size = 396, normalized size = 2.4

$$-\frac{(\cos(2dx+2c)+1)^2}{\sin(2dx+2c)d}\sqrt{\left(4a+(\cos(2dx+2c))^2b+b-2b\cos(2dx+2c)\right)(\sin(2dx+2c))^2}\sqrt{-ab}\sqrt{\frac{-1+\cos(2dx+\ldots}{\cos(2dx+2c)\ldots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(d*x+c)^4)^(1/2),x)
```

```
[Out] -((4*a+cos(2*d*x+2*c)^2*b+b-2*b*cos(2*d*x+2*c))*sin(2*d*x+2*c)^2)^(1/2)*(-a
*b)^(1/2)*((-b+(-a*b)^(1/2))*(-1+cos(2*d*x+2*c))/(-a*b)^(1/2)/(cos(2*d*x+2*
c)+1))^(1/2)*(cos(2*d*x+2*c)+1)^2*((-b*cos(2*d*x+2*c)+2*(-a*b)^(1/2)+b)/(-a
*b)^(1/2)/(cos(2*d*x+2*c)+1))^(1/2)*((b*cos(2*d*x+2*c)+2*(-a*b)^(1/2)-b)/(-
a*b)^(1/2)/(cos(2*d*x+2*c)+1))^(1/2)*EllipticF(((-b+(-a*b)^(1/2))*(-1+cos(2
*d*x+2*c))/(-a*b)^(1/2)/(cos(2*d*x+2*c)+1))^(1/2),((b+(-a*b)^(1/2))/(-b+(-a
*b)^(1/2)))^(1/2))/(-b+(-a*b)^(1/2))/(1/b*(-1+cos(2*d*x+2*c))*(cos(2*d*x+2*
c)+1)*(-b*cos(2*d*x+2*c)+2*(-a*b)^(1/2)+b)*(b*cos(2*d*x+2*c)+2*(-a*b)^(1/2)
-b))^(1/2)/sin(2*d*x+2*c)/(4*a+cos(2*d*x+2*c)^2*b+b-2*b*cos(2*d*x+2*c))^(1/
2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sin(dx+c)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sin(d*x + c)^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{b\cos\left(dx+c\right)^{4}-2\,b\cos\left(dx+c\right)^{2}+a+b}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)**4)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sin\left(dx+c\right)^{4}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sin(d*x + c)^4 + a), x)
```

3.563 $\int \frac{\cot^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

Optimal. Leaf size=477

$$\frac{\sqrt[4]{a+b}\cos^2(c+dx)\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)\sqrt{\frac{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}{\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a+b}\tan(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\right)}{2a^{3/4}d\sqrt{a+b\sin^4(c+dx)}}$$

```
[Out] -((Cos[c + d*x]^2*Cot[c + d*x]*(a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*
x]^4))/(a*d*Sqrt[a + b*Sin[c + d*x]^4])) + (Sqrt[a + b]*Cos[c + d*x]*Sin[c
+ d*x]*(a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4))/(a*d*Sqrt[a + b*S
in[c + d*x]^4]*(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2)) - ((a + b)^(1/4)*Cos
[c + d*x]^2*EllipticE[2*ArcTan[((a + b)^(1/4)*Tan[c + d*x])/a^(1/4)], (1 -
Sqrt[a]/Sqrt[a + b])/2]*(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2)*Sqrt[(a + 2*
a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4)/(Sqrt[a] + Sqrt[a + b]*Tan[c + d
*x]^2)^2])/(a^(3/4)*d*Sqrt[a + b*Sin[c + d*x]^4]) + ((a + b)^(1/4)*Cos[c +
d*x]^2*EllipticF[2*ArcTan[((a + b)^(1/4)*Tan[c + d*x])/a^(1/4)], (1 - Sqrt[
a]/Sqrt[a + b])/2]*(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2)*Sqrt[(a + 2*a*Tan
[c + d*x]^2 + (a + b)*Tan[c + d*x]^4)/(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2
)^2])/(2*a^(3/4)*d*Sqrt[a + b*Sin[c + d*x]^4])
```

Rubi [A] time = 0.365576, antiderivative size = 477, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3232, 1123, 12, 1139, 1103, 1195}

$$\frac{\sqrt[4]{a+b}\cos^2(c+dx)\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)\sqrt{\frac{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}{\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a+b}\tan(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\right)}{2a^{3/4}d\sqrt{a+b\sin^4(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/Sqrt[a + b*Sin[c + d*x]^4],x]
```


```
[Out] -((Cos[c + d*x]^2*Cot[c + d*x]*(a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*
x]^4))/(a*d*Sqrt[a + b*Sin[c + d*x]^4])) + (Sqrt[a + b]*Cos[c + d*x]*Sin[c
+ d*x]*(a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4))/(a*d*Sqrt[a + b*S
in[c + d*x]^4]*(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2)) - ((a + b)^(1/4)*Cos
[c + d*x]^2*EllipticE[2*ArcTan[((a + b)^(1/4)*Tan[c + d*x])/a^(1/4)], (1 -
Sqrt[a]/Sqrt[a + b])/2]*(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2)*Sqrt[(a + 2*
a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4)/(Sqrt[a] + Sqrt[a + b]*Tan[c + d
*x]^2)^2])/(a^(3/4)*d*Sqrt[a + b*Sin[c + d*x]^4]) + ((a + b)^(1/4)*Cos[c +
d*x]^2*EllipticF[2*ArcTan[((a + b)^(1/4)*Tan[c + d*x])/a^(1/4)], (1 - Sqrt[
a]/Sqrt[a + b])/2]*(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2)*Sqrt[(a + 2*a*Tan
[c + d*x]^2 + (a + b)*Tan[c + d*x]^4)/(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2
)^2])/(2*a^(3/4)*d*Sqrt[a + b*Sin[c + d*x]^4])
```


Rule 3232

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_)*((d_.)*tan[(e_.) + (f_.)*
(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(f
f*(a + b*Sin[e + f*x]^4)^p*(Sec[e + f*x]^2)^(2*p))/(f*Apart[a*(1 + Tan[e +
f*x]^2)^2 + b*Tan[e + f*x]^4]^p), Subst[Int[((d*ff*x)^m*ExpandToSum[a*(1 +
ff^2*x^2)^2 + b*ff^4*x^4, x]^p)/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*
```

```
x]/ff], x]] /; FreeQ[{a, b, d, e, f, m}, x] && IntegerQ[p - 1/2]
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1139

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[1/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[1/q, I
nt[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \frac{\left(\cos^2(c+dx)\sqrt{a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)}\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+2ax^2+(a+b)x^4}}\,dx, x\right.}{d\sqrt{a+b\sin^4(c+dx)}}$$

$$= -\frac{\cos^2(c+dx)\cot(c+dx)\left(a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)\right)}{ad\sqrt{a+b\sin^4(c+dx)}} + \frac{\left(\cos^2(c+dx)\sqrt{a\right.}{}$$

$$= -\frac{\cos^2(c+dx)\cot(c+dx)\left(a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)\right)}{ad\sqrt{a+b\sin^4(c+dx)}} + \frac{\left((a+b)\cos^2(c+\right.}{}$$

$$= -\frac{\cos^2(c+dx)\cot(c+dx)\left(a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)\right)}{ad\sqrt{a+b\sin^4(c+dx)}} + \frac{\left(\sqrt{a+b}\cos^2(c+\right.}{}$$

$$= -\frac{\cos^2(c+dx)\cot(c+dx)\left(a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)\right)}{ad\sqrt{a+b\sin^4(c+dx)}} + \frac{\sqrt{a+b}\cos(c+dx}{ad\sqrt{a}}$$

Mathematica [C] time = 11.231, size = 378, normalized size = 0.79

$$-\frac{\cot(c+dx)\sqrt{8a-4b\cos(2(c+dx))+b\cos(4(c+dx))+3b}}{2\sqrt{2}ad} - \frac{\cos^4(c+dx)\left(\frac{\left(\sqrt{a}\sqrt{b}+ia\right)\sec^2(c+dx)\sqrt{1+\left(1-\frac{i\sqrt{b}}{\sqrt{a}}\right)\tan^2(c+dx)}\sqrt{1+\left(1\right.}}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] -(Sqrt[8*a + 3*b - 4*b*Cos[2*(c + d*x)] + b*Cos[4*(c + d*x)]]*Cot[c + d*x])
/(2*Sqrt[2]*a*d) - (Cos[c + d*x]^4*(a*Sec[c + d*x]^4*Tan[c + d*x] + b*Tan[c
 + d*x]^5 + ((I*a + Sqrt[a]*Sqrt[b])*(EllipticE[I*ArcSinh[Sqrt[1 - (I*Sqrt[
b])/Sqrt[a]]*Tan[c + d*x]], (Sqrt[a] + I*Sqrt[b])/(Sqrt[a] - I*Sqrt[b])] -
EllipticF[I*ArcSinh[Sqrt[1 - (I*Sqrt[b])/Sqrt[a]]*Tan[c + d*x]], (Sqrt[a] +
 I*Sqrt[b])/(Sqrt[a] - I*Sqrt[b])])*Sec[c + d*x]^2*Sqrt[1 + (1 - (I*Sqrt[b]
)/Sqrt[a])*Tan[c + d*x]^2]*Sqrt[1 + (1 + (I*Sqrt[b])/Sqrt[a])*Tan[c + d*x]^
2])/Sqrt[1 - (I*Sqrt[b])/Sqrt[a]]))/(a*d*Sqrt[Cos[c + d*x]^4*(a + 2*a*Tan[c
 + d*x]^2 + (a + b)*Tan[c + d*x]^4)])
```

Maple [F] time = 0.71, size = 0, normalized size = 0.

$$\int (\cot(dx+c))^2 \frac{1}{\sqrt{a+b(\sin(dx+c))^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x)
```

```
[Out] int(cot(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot (dx+c)^2}{\sqrt{b \sin (dx+c)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)^2/sqrt(b*sin(d*x + c)^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\cot (dx+c)^2}{\sqrt{b \cos (dx+c)^4-2\, b \cos (dx+c)^2+a+b}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(cot(d*x + c)^2/sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^2 (c+dx)}{\sqrt{a+b \sin^4 (c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+b*sin(d*x+c)**4)**(1/2),x)
```

```
[Out] Integral(cot(c + d*x)**2/sqrt(a + b*sin(c + d*x)**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot (dx+c)^2}{\sqrt{b \sin (dx+c)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(d*x + c)^2/sqrt(b*sin(d*x + c)^4 + a), x)
```

3.564 $\int \left(a+b\sin^4(c+dx)\right)^p \tan^m(c+dx)\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\tan^m(c+dx)\left(a+b\sin^4(c+dx)\right)^p,x\right)$$

```
[Out] Unintegrable[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^m, x]
```

Rubi [A] time = 0.0419558, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(a+b\sin^4(c+dx)\right)^p \tan^m(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^m,x]

[Out] Defer[Int][(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^m, x]
```

Rubi steps

$$\int \left(a+b\sin^4(c+dx)\right)^p \tan^m(c+dx)\,dx=\int \left(a+b\sin^4(c+dx)\right)^p \tan^m(c+dx)\,dx$$

Mathematica [A] time = 5.4124, size = 0, normalized size = 0.

$$\int \left(a+b\sin^4(c+dx)\right)^p \tan^m(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^m,x]

[Out] Integrate[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^m, x]
```

Maple [A] time = 1.996, size = 0, normalized size = 0.

$$\int \left(a+b\left(\sin\left(dx+c\right)\right)^4\right)^p \left(\tan\left(dx+c\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^m,x)

[Out] int((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^m,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(dx+c\right)^4+a\right)^p\tan\left(dx+c\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^4 + a)^p*tan(d*x + c)^m, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\cos\left(dx+c\right)^4-2\,b\cos\left(dx+c\right)^2+a+b\right)^p\tan\left(dx+c\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^m,x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)^p*tan(d*x + c)^m,
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**4)**p*tan(d*x+c)**m,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(dx+c\right)^4+a\right)^p\tan\left(dx+c\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^m,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^4 + a)^p*tan(d*x + c)^m, x)
```

3.565 $\int \left(a+b\sin^4(c+dx)\right)^p \tan^3(c+dx)\,dx$

Optimal. Leaf size=279

$$-\frac{(a+2bp+b)\sin^2(c+dx)\left(a+b\sin^4(c+dx)\right)^p\left(\frac{b\sin^4(c+dx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\sin^4(c+dx),-\frac{b\sin^4(c+dx)}{a}\right)}{2d(a+b)}+\frac{b(2p+1)}{}$$

```
[Out] -((a + b + 2*b*p)*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sin[c + d*x]^4)
/(a + b)]*(a + b*Sin[c + d*x]^4)^(1 + p))/(4*(a + b)^2*d*(1 + p)) + (Sec[c
+ d*x]^2*(a + b*Sin[c + d*x]^4)^(1 + p))/(2*(a + b)*d) - ((a + b + 2*b*p)*A
ppellF1[1/2, 1, -p, 3/2, Sin[c + d*x]^4, -((b*Sin[c + d*x]^4)/a)]*Sin[c + d
*x]^2*(a + b*Sin[c + d*x]^4)^p)/(2*(a + b)*d*(1 + (b*Sin[c + d*x]^4)/a)^p)
+ (b*(1 + 2*p)*Hypergeometric2F1[1/2, -p, 3/2, -((b*Sin[c + d*x]^4)/a)]*Sin
[c + d*x]^2*(a + b*Sin[c + d*x]^4)^p)/(2*(a + b)*d*(1 + (b*Sin[c + d*x]^4)/
a)^p)
```

Rubi [A] time = 0.293776, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {3229, 835, 844, 246, 245, 757, 430, 429, 444, 68}

$$-\frac{(a+2bp+b)\sin^2(c+dx)\left(a+b\sin^4(c+dx)\right)^p\left(\frac{b\sin^4(c+dx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\sin^4(c+dx),-\frac{b\sin^4(c+dx)}{a}\right)}{2d(a+b)}+\frac{b(2p+1)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^3,x]
```

```
[Out] -((a + b + 2*b*p)*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sin[c + d*x]^4)
/(a + b)]*(a + b*Sin[c + d*x]^4)^(1 + p))/(4*(a + b)^2*d*(1 + p)) + (Sec[c
+ d*x]^2*(a + b*Sin[c + d*x]^4)^(1 + p))/(2*(a + b)*d) - ((a + b + 2*b*p)*A
ppellF1[1/2, 1, -p, 3/2, Sin[c + d*x]^4, -((b*Sin[c + d*x]^4)/a)]*Sin[c + d
*x]^2*(a + b*Sin[c + d*x]^4)^p)/(2*(a + b)*d*(1 + (b*Sin[c + d*x]^4)/a)^p)
+ (b*(1 + 2*p)*Hypergeometric2F1[1/2, -p, 3/2, -((b*Sin[c + d*x]^4)/a)]*Sin
[c + d*x]^2*(a + b*Sin[c + d*x]^4)^p)/(2*(a + b)*d*(1 + (b*Sin[c + d*x]^4)/
a)^p)
```

Rule 3229

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^
((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff^(n/2)*x^(n/2))^p)/(1
- ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \left(a+b\sin^4(c+dx)\right)^p \tan^3(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x(a+bx^2)^p}{(1-x)^2}\,dx, x, \sin^2(c+dx)\right)}{2d}$$

$$= \frac{\sec^2(c+dx)\left(a+b\sin^4(c+dx)\right)^{1+p}}{2(a+b)d} - \frac{\text{Subst}\left(\int \frac{(a+b(1+2p)x)(a+bx^2)^p}{1-x}\,dx, x, \sin^2(\right.}{2(a+b)d}$$

$$= \frac{\sec^2(c+dx)\left(a+b\sin^4(c+dx)\right)^{1+p}}{2(a+b)d} + \frac{(b(1+2p))\,\text{Subst}\left(\int \left(a+bx^2\right)^p\,dx, x, \text{s}\right.}{2(a+b)d}$$

$$= \frac{\sec^2(c+dx)\left(a+b\sin^4(c+dx)\right)^{1+p}}{2(a+b)d} - \frac{(a+b+2bp)\,\text{Subst}\left(\int \left(\frac{(a+bx^2)^p}{1-x^2} - \frac{x(a+}{-1-}\right.\right.}{2(a+b)d}$$

$$= \frac{\sec^2(c+dx)\left(a+b\sin^4(c+dx)\right)^{1+p}}{2(a+b)d} + \frac{b(1+2p)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sin^4(c+dx)}{a}\right)\sin}{}$$

$$= \frac{\sec^2(c+dx)\left(a+b\sin^4(c+dx)\right)^{1+p}}{2(a+b)d} + \frac{b(1+2p)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sin^4(c+dx)}{a}\right)\sin}{}$$

$$= -\frac{(a+b+2bp)\,{}_2F_1\left(1,1+p;2+p;\frac{a+b\sin^4(c+dx)}{a+b}\right)\left(a+b\sin^4(c+dx)\right)^{1+p}}{4(a+b)^2d(1+p)} + \sec$$

Mathematica [B] time = 17.2688, size = 2007, normalized size = 7.19

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^3,x]
```

```
[Out] -(((1 - 2*p)*AppellF1[-2*p, -p, -p, 1 - 2*p, -(((a + b)*Sec[c + d*x]^2)/(-b
 + Sqrt[-(a*b)])), ((a + b)*Sec[c + d*x]^2)/(b + Sqrt[-(a*b)])] + 2*p*Appel
lF1[1 - 2*p, -p, -p, 2 - 2*p, -(((a + b)*Sec[c + d*x]^2)/(-b + Sqrt[-(a*b)]
)), ((a + b)*Sec[c + d*x]^2)/(b + Sqrt[-(a*b)])]*Sec[c + d*x]^2)*(a + b*Sin
[c + d*x]^4)^p*Tan[c + d*x]^3*(Cos[c + d*x]^4*(a + 2*a*Tan[c + d*x]^2 + (a
+ b)*Tan[c + d*x]^4))^p)/(4*d*p*(-1 + 2*p)*((-a + Sqrt[-(a*b)] - (a + b)*Ta
n[c + d*x]^2)/(b + Sqrt[-(a*b)]))^p*((a + Sqrt[-(a*b)] + (a + b)*Tan[c + d*
x]^2)/(-b + Sqrt[-(a*b)]))^p*(((a + b)*Sec[c + d*x]^2*((1 - 2*p)*AppellF1[-
2*p, -p, -p, 1 - 2*p, -(((a + b)*Sec[c + d*x]^2)/(-b + Sqrt[-(a*b)])), ((a
+ b)*Sec[c + d*x]^2)/(b + Sqrt[-(a*b)])] + 2*p*AppellF1[1 - 2*p, -p, -p, 2
- 2*p, -(((a + b)*Sec[c + d*x]^2)/(-b + Sqrt[-(a*b)])), ((a + b)*Sec[c + d*
x]^2)/(b + Sqrt[-(a*b)])]*Sec[c + d*x]^2)*Tan[c + d*x]*((a + Sqrt[-(a*b)] +
 (a + b)*Tan[c + d*x]^2)/(-b + Sqrt[-(a*b)]))^(-1 - p)*(Cos[c + d*x]^4*(a +
 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4))^p)/(2*(-b + Sqrt[-(a*b)])*(-
1 + 2*p)*((-a + Sqrt[-(a*b)] - (a + b)*Tan[c + d*x]^2)/(b + Sqrt[-(a*b)]))^
p) - ((a + b)*Sec[c + d*x]^2*((1 - 2*p)*AppellF1[-2*p, -p, -p, 1 - 2*p, -((
(a + b)*Sec[c + d*x]^2)/(-b + Sqrt[-(a*b)])), ((a + b)*Sec[c + d*x]^2)/(b +
 Sqrt[-(a*b)])] + 2*p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, -(((a + b)*Sec[c +
 d*x]^2)/(-b + Sqrt[-(a*b)])), ((a + b)*Sec[c + d*x]^2)/(b + Sqrt[-(a*b)])]
*Sec[c + d*x]^2)*Tan[c + d*x]*((-a + Sqrt[-(a*b)] - (a + b)*Tan[c + d*x]^2)
/(b + Sqrt[-(a*b)]))^(-1 - p)*(Cos[c + d*x]^4*(a + 2*a*Tan[c + d*x]^2 + (a
+ b)*Tan[c + d*x]^4))^p)/(2*(b + Sqrt[-(a*b)])*(-1 + 2*p)*((a + Sqrt[-(a*b)
] + (a + b)*Tan[c + d*x]^2)/(-b + Sqrt[-(a*b)]))^p) - ((Cos[c + d*x]^4*(a +
```

```
2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4))^p*(4*p*AppellF1[1 - 2*p, -p,
 -p, 2 - 2*p, -(((a + b)*Sec[c + d*x]^2)/(-b + Sqrt[-(a*b)])), ((a + b)*Sec
[c + d*x]^2)/(b + Sqrt[-(a*b)])]*Sec[c + d*x]^2*Tan[c + d*x] + (1 - 2*p)*((
-4*(a + b)*p^2*AppellF1[1 - 2*p, 1 - p, -p, 2 - 2*p, -(((a + b)*Sec[c + d*x
]^2)/(-b + Sqrt[-(a*b)])), ((a + b)*Sec[c + d*x]^2)/(b + Sqrt[-(a*b)])]*Sec
[c + d*x]^2*Tan[c + d*x])/((-b + Sqrt[-(a*b)])*(1 - 2*p)) + (4*(a + b)*p^2*
AppellF1[1 - 2*p, -p, 1 - p, 2 - 2*p, -(((a + b)*Sec[c + d*x]^2)/(-b + Sqrt
[-(a*b)])), ((a + b)*Sec[c + d*x]^2)/(b + Sqrt[-(a*b)])]*Sec[c + d*x]^2*Tan
[c + d*x])/((b + Sqrt[-(a*b)])*(1 - 2*p))) + 2*p*Sec[c + d*x]^2*((2*(a + b)
*(1 - 2*p)*p*AppellF1[2 - 2*p, 1 - p, -p, 3 - 2*p, -(((a + b)*Sec[c + d*x]^
2)/(-b + Sqrt[-(a*b)])), ((a + b)*Sec[c + d*x]^2)/(b + Sqrt[-(a*b)])]*Sec[c
 + d*x]^2*Tan[c + d*x])/((-b + Sqrt[-(a*b)])*(2 - 2*p)) - (2*(a + b)*(1 - 2
*p)*p*AppellF1[2 - 2*p, -p, 1 - p, 3 - 2*p, -(((a + b)*Sec[c + d*x]^2)/(-b
+ Sqrt[-(a*b)])), ((a + b)*Sec[c + d*x]^2)/(b + Sqrt[-(a*b)])]*Sec[c + d*x]
^2*Tan[c + d*x])/((b + Sqrt[-(a*b)])*(2 - 2*p)))))/(4*p*(-1 + 2*p)*((-a + S
qrt[-(a*b)] - (a + b)*Tan[c + d*x]^2)/(b + Sqrt[-(a*b)]))^p*((a + Sqrt[-(a*
b)] + (a + b)*Tan[c + d*x]^2)/(-b + Sqrt[-(a*b)]))^p) - (((1 - 2*p)*AppellF
1[-2*p, -p, -p, 1 - 2*p, -(((a + b)*Sec[c + d*x]^2)/(-b + Sqrt[-(a*b)])), (
(a + b)*Sec[c + d*x]^2)/(b + Sqrt[-(a*b)])] + 2*p*AppellF1[1 - 2*p, -p, -p,
 2 - 2*p, -(((a + b)*Sec[c + d*x]^2)/(-b + Sqrt[-(a*b)])), ((a + b)*Sec[c +
 d*x]^2)/(b + Sqrt[-(a*b)])]*Sec[c + d*x]^2)*(Cos[c + d*x]^4*(a + 2*a*Tan[c
 + d*x]^2 + (a + b)*Tan[c + d*x]^4))^(-1 + p)*(Cos[c + d*x]^4*(4*a*Sec[c +
d*x]^2*Tan[c + d*x] + 4*(a + b)*Sec[c + d*x]^2*Tan[c + d*x]^3) - 4*Cos[c +
d*x]^3*Sin[c + d*x]*(a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4)))/(4*
(-1 + 2*p)*((-a + Sqrt[-(a*b)] - (a + b)*Tan[c + d*x]^2)/(b + Sqrt[-(a*b)])
)^p*((a + Sqrt[-(a*b)] + (a + b)*Tan[c + d*x]^2)/(-b + Sqrt[-(a*b)]))^p)))
```

Maple [F] time = 1.094, size = 0, normalized size = 0.

$$\int \left(a + b (\sin (dx + c))^4\right)^p (\tan (dx + c))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^3,x)

[Out] int((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sin (dx + c)^4 + a\right)^p \tan (dx + c)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^3,x, algorithm="maxima")

[Out] integrate((b*sin(d*x + c)^4 + a)^p*tan(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b \cos (dx + c)^4 - 2 b \cos (dx + c)^2 + a + b\right)^p \tan (dx + c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)^p*tan(d*x + c)^3,
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**4)**p*tan(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(dx+c\right)^{4}+a\right)^{p}\tan\left(dx+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^4 + a)^p*tan(d*x + c)^3, x)
```

3.566 $\int \left(a+b\sin^4(c+dx)\right)^p \tan(c+dx)\,dx$

Optimal. Leaf size=141

$$\frac{\sin^2(c+dx)\left(a+b\sin^4(c+dx)\right)^p\left(\frac{b\sin^4(c+dx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\sin^4(c+dx),-\frac{b\sin^4(c+dx)}{a}\right)}{2d}+\frac{\left(a+b\sin^4(c+dx)\right)^{p+}}{4}$$

```
[Out] (Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sin[c + d*x]^4)/(a + b)]*(a + b*
Sin[c + d*x]^4)^(1 + p))/(4*(a + b)*d*(1 + p)) + (AppellF1[1/2, 1, -p, 3/2,
 Sin[c + d*x]^4, -((b*Sin[c + d*x]^4)/a)]*Sin[c + d*x]^2*(a + b*Sin[c + d*x
]^4)^p)/(2*d*(1 + (b*Sin[c + d*x]^4)/a)^p)
```

Rubi [A] time = 0.119269, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3229, 757, 430, 429, 444, 68}

$$\frac{\sin^2(c+dx)\left(a+b\sin^4(c+dx)\right)^p\left(\frac{b\sin^4(c+dx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\sin^4(c+dx),-\frac{b\sin^4(c+dx)}{a}\right)}{2d}+\frac{\left(a+b\sin^4(c+dx)\right)^{p+}}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x],x]
```


```
[Out] (Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sin[c + d*x]^4)/(a + b)]*(a + b*
Sin[c + d*x]^4)^(1 + p))/(4*(a + b)*d*(1 + p)) + (AppellF1[1/2, 1, -p, 3/2,
 Sin[c + d*x]^4, -((b*Sin[c + d*x]^4)/a)]*Sin[c + d*x]^2*(a + b*Sin[c + d*x
]^4)^p)/(2*d*(1 + (b*Sin[c + d*x]^4)/a)^p)
```


Rule 3229

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^
((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff^(n/2)*x^(n/2))^p)/(1
- ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
```

```
&& (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \left(a+b\sin^4(c+dx)\right)^p\tan(c+dx)\,dx=\frac{\text{Subst}\left(\int\frac{\left(a+bx^2\right)^p}{1-x}\,dx,x,\sin^2(c+dx)\right)}{2d}$$
$$=\frac{\text{Subst}\left(\int\left(\frac{\left(a+bx^2\right)^p}{1-x^2}-\frac{x\left(a+bx^2\right)^p}{-1+x^2}\right)dx,x,\sin^2(c+dx)\right)}{2d}$$
$$=\frac{\text{Subst}\left(\int\frac{\left(a+bx^2\right)^p}{1-x^2}\,dx,x,\sin^2(c+dx)\right)}{2d}-\frac{\text{Subst}\left(\int\frac{x\left(a+bx^2\right)^p}{-1+x^2}\,dx,x,\sin^2(c+dx)\right)}{2d}$$
$$=-\frac{\text{Subst}\left(\int\frac{(a+bx)^p}{-1+x}\,dx,x,\sin^4(c+dx)\right)}{4d}+\frac{\left(\left(a+b\sin^4(c+dx)\right)^p\left(1+\frac{b\sin^4(c+dx)}{a}\right)\right.}{}$$
$$=\frac{{}_2F_1\left(1,1+p;2+p;\frac{a+b\sin^4(c+dx)}{a+b}\right)\left(a+b\sin^4(c+dx)\right)^{1+p}}{4(a+b)d(1+p)}+\frac{F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\sin^4(\right.}{}$$

Mathematica [B] time = 8.08056, size = 466, normalized size = 3.3

$$-\frac{(2p-1)\left(\sqrt{-ab}-b\right)\left(\sqrt{-ab}+b\right)\sin(c+dx)\cos(c+dx)\left(-(a+b)\tan^2\right.}{2dp(a+b)^2\left((a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)\left(b(2p-1)\sin(2(c+dx))F_1\left(-2p;-p,-p;1-2p;-\frac{(a+b)\sec^2(c+dx}{\sqrt{-ab}-b}\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x],x]
```

```
[Out] -((-b + Sqrt[-(a*b)])*(b + Sqrt[-(a*b)])*(-1 + 2*p)*AppellF1[-2*p, -p, -p,
1 - 2*p, -(((a + b)*Sec[c + d*x]^2)/(-b + Sqrt[-(a*b)])), ((a + b)*Sec[c +
d*x]^2)/(b + Sqrt[-(a*b)])]*Cos[c + d*x]*Sin[c + d*x]*(a + b*Sin[c + d*x]^4
)^p*(-a + Sqrt[-(a*b)] - (a + b)*Tan[c + d*x]^2)*(a + Sqrt[-(a*b)] + (a + b
)*Tan[c + d*x]^2))/(2*(a + b)^2*d*p*(b*(-1 + 2*p)*AppellF1[-2*p, -p, -p, 1
- 2*p, -(((a + b)*Sec[c + d*x]^2)/(-b + Sqrt[-(a*b)])), ((a + b)*Sec[c + d*
x]^2)/(b + Sqrt[-(a*b)])]*Sin[2*(c + d*x)] + 2*p*((b + Sqrt[-(a*b)])*Appell
F1[1 - 2*p, 1 - p, -p, 2 - 2*p, -(((a + b)*Sec[c + d*x]^2)/(-b + Sqrt[-(a*b
)])), ((a + b)*Sec[c + d*x]^2)/(b + Sqrt[-(a*b)])] + (b - Sqrt[-(a*b)])*App
ellF1[1 - 2*p, -p, 1 - p, 2 - 2*p, -(((a + b)*Sec[c + d*x]^2)/(-b + Sqrt[-(
```

```
a*b)])), ((a + b)*Sec[c + d*x]^2)/(b + Sqrt[-(a*b)])])*Tan[c + d*x])*(a + 2
*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4))
```

Maple [F] time = 1.684, size = 0, normalized size = 0.

$$\int \left(a+b (\sin (dx+c))^{4}\right)^{p} \tan (dx+c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c)^4)^p*tan(d*x+c),x)
```

```
[Out] int((a+b*sin(d*x+c)^4)^p*tan(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sin (dx+c)^{4}+a\right)^{p} \tan (dx+c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^4 + a)^p*tan(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b \cos (dx+c)^{4}-2\, b \cos (dx+c)^{2}+a+b\right)^{p} \tan (dx+c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)^p*tan(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**4)**p*tan(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sin (dx+c)^{4}+a\right)^{p} \tan (dx+c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^4 + a)^p*tan(d*x + c), x)
```

3.567 $\int \cot(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx$

Optimal. Leaf size=54

$$-\frac{\left(a+b\sin^4(c+dx)\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\sin^4(c+dx)}{a}+1\right)}{4ad(p+1)}$$

```
[Out] -(Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sin[c + d*x]^4)/a]*(a + b*Sin[c
 + d*x]^4)^(1 + p))/(4*a*d*(1 + p))
```

Rubi [A] time = 0.0649503, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3229, 266, 65}

$$-\frac{\left(a+b\sin^4(c+dx)\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\sin^4(c+dx)}{a}+1\right)}{4ad(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + b*Sin[c + d*x]^4)^p,x]
```

```
[Out] -(Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sin[c + d*x]^4)/a]*(a + b*Sin[c
 + d*x]^4)^(1 + p))/(4*a*d*(1 + p))
```

Rule 3229

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^
((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff^(n/2)*x^(n/2))^p)/(1
- ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \cot(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^p}{x}\,dx, x, \sin^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, \sin^4(c+dx)\right)}{4d}$$
$$= -\frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b\sin^4(c+dx)}{a}\right)\left(a+b\sin^4(c+dx)\right)^{1+p}}{4ad(1+p)}$$

Mathematica [A] time = 0.0911606, size = 54, normalized size = 1.

$$-\frac{\left(a+b\sin^4(c+dx)\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\sin^4(c+dx)}{a}+1\right)}{4ad(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + b*Sin[c + d*x]^4)^p,x]
```

```
[Out] -(Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sin[c + d*x]^4)/a]*(a + b*Sin[c
 + d*x]^4)^(1 + p))/(4*a*d*(1 + p))
```

Maple [F] time = 1.563, size = 0, normalized size = 0.

$$\int \cot(dx+c)\left(a+b(\sin(dx+c))^4\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+b*sin(d*x+c)^4)^p,x)
```

```
[Out] int(cot(d*x+c)*(a+b*sin(d*x+c)^4)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin(dx+c)^4+a\right)^p\cot(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)^4)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^4 + a)^p*cot(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\cos(dx+c)^4-2\,b\cos(dx+c)^2+a+b\right)^p\cot(dx+c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)^4)^p,x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)^p*cot(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)**4)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sin \left(dx+c\right)^4+a\right)^p \cot \left(dx+c\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)^4)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^4 + a)^p*cot(d*x + c), x)
```

3.568 $\int \cot^3(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx$

Optimal. Leaf size=127

$$\frac{\left(a+b\sin^4(c+dx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\sin^4(c+dx)}{a}+1\right)}{4ad(p+1)}-\frac{\csc^2(c+dx)\left(a+b\sin^4(c+dx)\right)^p\left(\frac{b\sin^4(c+dx)}{a}+1\right)^{-p}{}_2F_1\left(-\right.}{2d}$$

```
[Out] (Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sin[c + d*x]^4)/a]*(a + b*Sin[c
+ d*x]^4)^(1 + p))/(4*a*d*(1 + p)) - (Csc[c + d*x]^2*Hypergeometric2F1[-1/2
, -p, 1/2, -((b*Sin[c + d*x]^4)/a)]*(a + b*Sin[c + d*x]^4)^p)/(2*d*(1 + (b*
Sin[c + d*x]^4)/a)^p)
```

Rubi [A] time = 0.101792, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3229, 764, 365, 364, 266, 65}

$$\frac{\left(a+b\sin^4(c+dx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\sin^4(c+dx)}{a}+1\right)}{4ad(p+1)}-\frac{\csc^2(c+dx)\left(a+b\sin^4(c+dx)\right)^p\left(\frac{b\sin^4(c+dx)}{a}+1\right)^{-p}{}_2F_1\left(-\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + b*Sin[c + d*x]^4)^p,x]
```

```
[Out] (Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sin[c + d*x]^4)/a]*(a + b*Sin[c
+ d*x]^4)^(1 + p))/(4*a*d*(1 + p)) - (Csc[c + d*x]^2*Hypergeometric2F1[-1/2
, -p, 1/2, -((b*Sin[c + d*x]^4)/a)]*(a + b*Sin[c + d*x]^4)^p)/(2*d*(1 + (b*
Sin[c + d*x]^4)/a)^p)
```

Rule 3229

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^
((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff^(n/2)*x^(n/2))^p)/(1
- ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \cot^3(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{(1-x)\left(a+bx^2\right)^p}{x^2}\,dx, x, \sin^2(c+dx)\right)}{2d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^p}{x^2}\,dx, x, \sin^2(c+dx)\right)}{2d} - \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^p}{x}\,dx, x, \sin^2(c+d\right.}{2d}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, \sin^4(c+dx)\right)}{4d} + \frac{\left(\left(a+b\sin^4(c+dx)\right)^p\left(1+\frac{b\sin^4(c\right.\right.}{a}}{}$$

$$= \frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b\sin^4(c+dx)}{a}\right)\left(a+b\sin^4(c+dx)\right)^{1+p}}{4ad(1+p)} - \frac{\csc^2(c+dx)\,{}_2}{}$$

Mathematica [A] time = 0.649591, size = 119, normalized size = 0.94

$$\frac{\left(a+b\sin^4(c+dx)\right)^p\left(\frac{\left(a+b\sin^4(c+dx)\right){}_2F_1\left(1,p+1;p+2;\frac{b\sin^4(c+dx)}{a}+1\right)}{a(p+1)} - 2\csc^2(c+dx)\left(\frac{b\sin^4(c+dx)}{a}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{b\sin^4(}{a}\right.\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + b*Sin[c + d*x]^4)^p,x]
```

```
[Out] ((a + b*Sin[c + d*x]^4)^p*((Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sin[c
 + d*x]^4)/a]*(a + b*Sin[c + d*x]^4))/(a*(1 + p)) - (2*Csc[c + d*x]^2*Hyper
geometric2F1[-1/2, -p, 1/2, -((b*Sin[c + d*x]^4)/a)])/(1 + (b*Sin[c + d*x]^
4)/a)^p))/(4*d)
```

Maple [F] time = 1.055, size = 0, normalized size = 0.

$$\int (\cot(dx+c))^3\left(a+b(\sin(dx+c))^4\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+b*sin(d*x+c)^4)^p,x)
```

```
[Out] int(cot(d*x+c)^3*(a+b*sin(d*x+c)^4)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(dx+c\right)^{4}+a\right)^{p}\cot\left(dx+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c)^4)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^4 + a)^p*cot(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\cos\left(dx+c\right)^{4}-2\,b\cos\left(dx+c\right)^{2}+a+b\right)^{p}\cot\left(dx+c\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c)^4)^p,x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)^p*cot(d*x + c)^3,
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+b*sin(d*x+c)**4)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(dx+c\right)^{4}+a\right)^{p}\cot\left(dx+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c)^4)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^4 + a)^p*cot(d*x + c)^3, x)
```

3.569 $\int \left(a+b\sin^4(c+dx)\right)^p \tan^4(c+dx)\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\tan^4(c+dx)\left(a+b\sin^4(c+dx)\right)^p,x\right)$$

```
[Out] Unintegrable[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^4, x]
```

Rubi [A] time = 0.0411529, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(a+b\sin^4(c+dx)\right)^p \tan^4(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^4,x]
```

```
[Out] Defer[Int][(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^4, x]
```

Rubi steps

$$\int \left(a+b\sin^4(c+dx)\right)^p \tan^4(c+dx)\,dx=\int \left(a+b\sin^4(c+dx)\right)^p \tan^4(c+dx)\,dx$$

Mathematica [A] time = 3.39665, size = 0, normalized size = 0.

$$\int \left(a+b\sin^4(c+dx)\right)^p \tan^4(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^4,x]
```

```
[Out] Integrate[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^4, x]
```

Maple [A] time = 0.656, size = 0, normalized size = 0.

$$\int \left(a+b\left(\sin(dx+c)\right)^4\right)^p \left(\tan(dx+c)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^4,x)
```

```
[Out] int((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^4,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sin \left(dx + c\right)^{4} + a\right)^{p} \tan \left(dx + c\right)^{4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^4 + a)^p*tan(d*x + c)^4, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \cos \left(dx + c\right)^{4} - 2 \, b \cos \left(dx + c\right)^{2} + a + b\right)^{p} \tan \left(dx + c\right)^{4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)^p*tan(d*x + c)^4,
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**4)**p*tan(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sin \left(dx + c\right)^{4} + a\right)^{p} \tan \left(dx + c\right)^{4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^4 + a)^p*tan(d*x + c)^4, x)
```

3.570 $\int \left(a+b\sin^4(c+dx)\right)^p \tan^2(c+dx)\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\tan^2(c+dx)\left(a+b\sin^4(c+dx)\right)^p,x\right)$$

```
[Out] Unintegrable[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^2, x]
```

Rubi [A] time = 0.0423265, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(a+b\sin^4(c+dx)\right)^p \tan^2(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^2,x]
```

```
[Out] Defer[Int][(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^2, x]
```

Rubi steps

$$\int \left(a+b\sin^4(c+dx)\right)^p \tan^2(c+dx)\,dx=\int \left(a+b\sin^4(c+dx)\right)^p \tan^2(c+dx)\,dx$$

Mathematica [A] time = 1.59108, size = 0, normalized size = 0.

$$\int \left(a+b\sin^4(c+dx)\right)^p \tan^2(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^2,x]
```

```
[Out] Integrate[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^2, x]
```

Maple [A] time = 0.886, size = 0, normalized size = 0.

$$\int \left(a+b\left(\sin(dx+c)\right)^4\right)^p \left(\tan(dx+c)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^2,x)
```

```
[Out] int((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(dx+c\right)^{4}+a\right)^{p}\tan\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^4 + a)^p*tan(d*x + c)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\cos\left(dx+c\right)^{4}-2\,b\cos\left(dx+c\right)^{2}+a+b\right)^{p}\tan\left(dx+c\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)^p*tan(d*x + c)^2,
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**4)**p*tan(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(dx+c\right)^{4}+a\right)^{p}\tan\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^4 + a)^p*tan(d*x + c)^2, x)
```

3.571 $\int \left(a+b\sin^4(c+dx)\right)^p \, dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\left(a+b\sin^4(c+dx)\right)^p,x\right)$$

```
[Out] Unintegrable[(a + b*Sin[c + d*x]^4)^p, x]
```

Rubi [A] time = 0.0110318, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(a+b\sin^4(c+dx)\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[c + d*x]^4)^p,x]
```

```
[Out] Defer[Int][(a + b*Sin[c + d*x]^4)^p, x]
```

Rubi steps

$$\int \left(a+b\sin^4(c+dx)\right)^p \, dx=\int \left(a+b\sin^4(c+dx)\right)^p \, dx$$

Mathematica [A] time = 2.18168, size = 0, normalized size = 0.

$$\int \left(a+b\sin^4(c+dx)\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*x]^4)^p,x]
```

```
[Out] Integrate[(a + b*Sin[c + d*x]^4)^p, x]
```

Maple [A] time = 0.784, size = 0, normalized size = 0.

$$\int \left(a+b\left(\sin\left(dx+c\right)\right)^4\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c)^4)^p,x)
```

```
[Out] int((a+b*sin(d*x+c)^4)^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(dx+c\right)^{4}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^4)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^4 + a)^p, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\cos\left(dx+c\right)^{4}-2\,b\cos\left(dx+c\right)^{2}+a+b\right)^{p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^4)^p,x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**4)**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(dx+c\right)^{4}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^4)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^4 + a)^p, x)
```

3.572 $\int \cot^2(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\cot^2(c+dx)\left(a+b\sin^4(c+dx)\right)^p,x\right)$$

```
[Out] Unintegrable[Cot[c + d*x]^2*(a + b*Sin[c + d*x]^4)^p, x]
```

Rubi [A] time = 0.0407652, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \cot^2(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cot[c + d*x]^2*(a + b*Sin[c + d*x]^4)^p,x]
```

```
[Out] Defer[Int][Cot[c + d*x]^2*(a + b*Sin[c + d*x]^4)^p, x]
```

Rubi steps

$$\int \cot^2(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx=\int \cot^2(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx$$

Mathematica [A] time = 1.41614, size = 0, normalized size = 0.

$$\int \cot^2(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Sin[c + d*x]^4)^p,x]
```

```
[Out] Integrate[Cot[c + d*x]^2*(a + b*Sin[c + d*x]^4)^p, x]
```

Maple [A] time = 0.904, size = 0, normalized size = 0.

$$\int (\cot(dx+c))^2\left(a+b(\sin(dx+c))^4\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+b*sin(d*x+c)^4)^p,x)
```

```
[Out] int(cot(d*x+c)^2*(a+b*sin(d*x+c)^4)^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sin \left(dx + c\right)^4 + a\right)^p \cot \left(dx + c\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c)^4)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^4 + a)^p*cot(d*x + c)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(b \cos \left(dx + c\right)^4 - 2\,b \cos \left(dx + c\right)^2 + a + b\right)^p \cot \left(dx + c\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c)^4)^p,x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)^p*cot(d*x + c)^2,
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+b*sin(d*x+c)**4)**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sin \left(dx + c\right)^4 + a\right)^p \cot \left(dx + c\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c)^4)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^4 + a)^p*cot(d*x + c)^2, x)
```

3.573 $\int \cot^4(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\cot^4(c+dx)\left(a+b\sin^4(c+dx)\right)^p,x\right)$$

```
[Out] Unintegrable[Cot[c + d*x]^4*(a + b*Sin[c + d*x]^4)^p, x]
```

Rubi [A] time = 0.0401924, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \cot^4(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cot[c + d*x]^4*(a + b*Sin[c + d*x]^4)^p,x]
```

```
[Out] Defer[Int][Cot[c + d*x]^4*(a + b*Sin[c + d*x]^4)^p, x]
```

Rubi steps

$$\int \cot^4(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx=\int \cot^4(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx$$

Mathematica [A] time = 2.22763, size = 0, normalized size = 0.

$$\int \cot^4(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Sin[c + d*x]^4)^p,x]
```

```
[Out] Integrate[Cot[c + d*x]^4*(a + b*Sin[c + d*x]^4)^p, x]
```

Maple [A] time = 0.632, size = 0, normalized size = 0.

$$\int (\cot(dx+c))^4\left(a+b(\sin(dx+c))^4\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+b*sin(d*x+c)^4)^p,x)
```

```
[Out] int(cot(d*x+c)^4*(a+b*sin(d*x+c)^4)^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(dx+c\right)^4+a\right)^p\cot\left(dx+c\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c)^4)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^4 + a)^p*cot(d*x + c)^4, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\cos\left(dx+c\right)^4-2\,b\cos\left(dx+c\right)^2+a+b\right)^p\cot\left(dx+c\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c)^4)^p,x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)^p*cot(d*x + c)^4,
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+b*sin(d*x+c)**4)**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(dx+c\right)^4+a\right)^p\cot\left(dx+c\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c)^4)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^4 + a)^p*cot(d*x + c)^4, x)
```

3.574 $\int \left(a+b\sin^n(c+dx)\right)^3 \tan^m(c+dx)\,dx$

Optimal. Leaf size=306

$$\frac{3a^2b\cos^2(c+dx)^{\frac{m+1}{2}}\tan^{m+1}(c+dx)\sin^n(c+dx)\,{}_2F_1\left(\frac{m+1}{2},\frac{1}{2}(m+n+1);\frac{1}{2}(m+n+3);\sin^2(c+dx)\right)}{d(m+n+1)}+\frac{a^3\tan^{m+1}(c}{}$$

```
[Out] (a^3*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d*x]^2]*Tan[c + d*
x]^(1 + m))/(d*(1 + m)) + (3*a^2*b*(Cos[c + d*x]^2)^((1 + m)/2)*Hypergeomet
ric2F1[(1 + m)/2, (1 + m + n)/2, (3 + m + n)/2, Sin[c + d*x]^2]*Sin[c + d*x
]^n*Tan[c + d*x]^(1 + m))/(d*(1 + m + n)) + (3*a*b^2*(Cos[c + d*x]^2)^((1 +
 m)/2)*Hypergeometric2F1[(1 + m)/2, (1 + m + 2*n)/2, (3 + m + 2*n)/2, Sin[c
 + d*x]^2]*Sin[c + d*x]^(2*n)*Tan[c + d*x]^(1 + m))/(d*(1 + m + 2*n)) + (b^
3*(Cos[c + d*x]^2)^((1 + m)/2)*Hypergeometric2F1[(1 + m)/2, (1 + m + 3*n)/2
, (3 + m + 3*n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(3*n)*Tan[c + d*x]^(1 + m))
/(d*(1 + m + 3*n))
```

Rubi [A] time = 0.429391, antiderivative size = 306, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3234, 3476, 364, 2602, 2577}

$$\frac{3a^2b\cos^2(c+dx)^{\frac{m+1}{2}}\tan^{m+1}(c+dx)\sin^n(c+dx)\,{}_2F_1\left(\frac{m+1}{2},\frac{1}{2}(m+n+1);\frac{1}{2}(m+n+3);\sin^2(c+dx)\right)}{d(m+n+1)}+\frac{a^3\tan^{m+1}(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x]^n)^3*Tan[c + d*x]^m,x]
```


```
[Out] (a^3*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d*x]^2]*Tan[c + d*
x]^(1 + m))/(d*(1 + m)) + (3*a^2*b*(Cos[c + d*x]^2)^((1 + m)/2)*Hypergeomet
ric2F1[(1 + m)/2, (1 + m + n)/2, (3 + m + n)/2, Sin[c + d*x]^2]*Sin[c + d*x
]^n*Tan[c + d*x]^(1 + m))/(d*(1 + m + n)) + (3*a*b^2*(Cos[c + d*x]^2)^((1 +
 m)/2)*Hypergeometric2F1[(1 + m)/2, (1 + m + 2*n)/2, (3 + m + 2*n)/2, Sin[c
 + d*x]^2]*Sin[c + d*x]^(2*n)*Tan[c + d*x]^(1 + m))/(d*(1 + m + 2*n)) + (b^
3*(Cos[c + d*x]^2)^((1 + m)/2)*Hypergeometric2F1[(1 + m)/2, (1 + m + 3*n)/2
, (3 + m + 3*n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(3*n)*Tan[c + d*x]^(1 + m))
/(d*(1 + m + 3*n))
```


Rule 3234

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*((d_.)*tan[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(d*tan[e + f*x])^m*(a
 + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
```

```
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \left(a+b\sin^n(c+dx)\right)^3\tan^m(c+dx)\,dx=\int \left(a^3\tan^m(c+dx)+3a^2b\sin^n(c+dx)\tan^m(c+dx)+3ab^2\sin^{2n}(c+dx)\,\mathrm{ta}\right.$$

$$=a^3\int \tan^m(c+dx)\,dx+\left(3a^2b\right)\int \sin^n(c+dx)\tan^m(c+dx)\,dx+\left(3ab^2\right)\int \mathrm{si}$$

$$=\frac{a^3\,\mathrm{Subst}\left(\int \frac{x^m}{1+x^2}\,dx,x,\tan(c+dx)\right)}{d}+\left(3a^2b\cos^{1+m}(c+dx)\sin^{-1-m}(c+dx)\right.$$

$$=\frac{a^3\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\tan^2(c+dx)\right)\tan^{1+m}(c+dx)}{d(1+m)}+\frac{3a^2b\cos^2(c+dx)^{\frac{1+m}{2}}\,{}_2\cdots}{}$$

Mathematica [C] time = 19.9739, size = 3544, normalized size = 11.58

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[c + d*x]^n)^3*Tan[c + d*x]^m,x]
```

```
[Out] (2*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^m*((a^3*AppellF1[(1 + m)/2, m, 1, (3 +
 m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(1 + m) + b*(Sec[(c + d*x)
/2]^2)^n*Sin[c + d*x]^n*((3*a^2*AppellF1[(1 + m + n)/2, m, 1 + n, (3 + m +
n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(1 + m + n) + b*(Sec[(c + d
*x)/2]^2)^n*Sin[c + d*x]^n*((3*a*AppellF1[1/2 + m/2 + n, m, 1 + 2*n, 3/2 +
m/2 + n, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(1 + m + 2*n) + (b*Appel
lF1[(1 + m + 3*n)/2, m, 1 + 3*n, (3 + m + 3*n)/2, Tan[(c + d*x)/2]^2, -Tan[
(c + d*x)/2]^2]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n)/(1 + m + 3*n))))*Tan
[(c + d*x)/2]*Tan[c + d*x]^m*(a^3*Tan[c + d*x]^m + 3*a^2*b*Sin[c + d*x]^n*T
an[c + d*x]^m + 3*a*b^2*Sin[c + d*x]^(2*n)*Tan[c + d*x]^m + b^3*Sin[c + d*x
]^(3*n)*Tan[c + d*x]^m))/(d*(2*m*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^m*Sec[c
+ d*x]^2*((a^3*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2])/(1 + m) + b*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*((3*a^
2*AppellF1[(1 + m + n)/2, m, 1 + n, (3 + m + n)/2, Tan[(c + d*x)/2]^2, -Tan
[(c + d*x)/2]^2])/(1 + m + n) + b*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*((3
*a*AppellF1[1/2 + m/2 + n, m, 1 + 2*n, 3/2 + m/2 + n, Tan[(c + d*x)/2]^2, -
Tan[(c + d*x)/2]^2])/(1 + m + 2*n) + (b*AppellF1[(1 + m + 3*n)/2, m, 1 + 3*
n, (3 + m + 3*n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Sec[(c + d*x)
```

/2]^2)^n*Sin[c + d*x]^n)/(1 + m + 3*n))))*Tan[(c + d*x)/2]*Tan[c + d*x]^(-1
+ m) + Sec[(c + d*x)/2]^2*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^m*((a^3*Appell
F1[(1 + m)/2, m, 1, (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(1
+ m) + b*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*((3*a^2*AppellF1[(1 + m + n
)/2, m, 1 + n, (3 + m + n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(1
+ m + n) + b*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*((3*a*AppellF1[1/2 + m/2
+ n, m, 1 + 2*n, 3/2 + m/2 + n, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/
(1 + m + 2*n) + (b*AppellF1[(1 + m + 3*n)/2, m, 1 + 3*n, (3 + m + 3*n)/2, T
an[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]
^n)/(1 + m + 3*n))))*Tan[c + d*x]^m + 2*m*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)
^(-1 + m)*((a^3*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, Tan[(c + d*x)/2]^2, -T
an[(c + d*x)/2]^2])/(1 + m) + b*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*((3*a
^2*AppellF1[(1 + m + n)/2, m, 1 + n, (3 + m + n)/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2])/(1 + m + n) + b*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*((
3*a*AppellF1[1/2 + m/2 + n, m, 1 + 2*n, 3/2 + m/2 + n, Tan[(c + d*x)/2]^2,
-Tan[(c + d*x)/2]^2])/(1 + m + 2*n) + (b*AppellF1[(1 + m + 3*n)/2, m, 1 + 3
*n, (3 + m + 3*n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Sec[(c + d*x
)/2]^2)^n*Sin[c + d*x]^n)/(1 + m + 3*n))))*Tan[(c + d*x)/2]*(-(Sec[(c + d*x
)/2]^2*Sin[c + d*x]) + Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*Ta
n[c + d*x]^m + 2*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^m*Tan[(c + d*x)/2]*(b*n*
Cos[c + d*x]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^(-1 + n)*((3*a^2*AppellF1[
(1 + m + n)/2, m, 1 + n, (3 + m + n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/
2]^2])/(1 + m + n) + b*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*((3*a*AppellF1
[1/2 + m/2 + n, m, 1 + 2*n, 3/2 + m/2 + n, Tan[(c + d*x)/2]^2, -Tan[(c + d*
x)/2]^2])/(1 + m + 2*n) + (b*AppellF1[(1 + m + 3*n)/2, m, 1 + 3*n, (3 + m +
3*n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Sec[(c + d*x)/2]^2)^n*Si
n[c + d*x]^n)/(1 + m + 3*n))) + b*n*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*(
(3*a^2*AppellF1[(1 + m + n)/2, m, 1 + n, (3 + m + n)/2, Tan[(c + d*x)/2]^2,
-Tan[(c + d*x)/2]^2])/(1 + m + n) + b*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^
n*((3*a*AppellF1[1/2 + m/2 + n, m, 1 + 2*n, 3/2 + m/2 + n, Tan[(c + d*x)/2]
^2, -Tan[(c + d*x)/2]^2])/(1 + m + 2*n) + (b*AppellF1[(1 + m + 3*n)/2, m, 1
+ 3*n, (3 + m + 3*n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Sec[(c +
d*x)/2]^2)^n*Sin[c + d*x]^n)/(1 + m + 3*n)))*Tan[(c + d*x)/2] + (a^3*(-(((
1 + m)*AppellF1[1 + (1 + m)/2, m, 2, 1 + (3 + m)/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(3 + m)) + (m*(1 + m
)*AppellF1[1 + (1 + m)/2, 1 + m, 1, 1 + (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan
[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(3 + m)))/(1 + m) + b
*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*(b*n*Cos[c + d*x]*(Sec[(c + d*x)/2]^
2)^n*Sin[c + d*x]^(-1 + n)*((3*a*AppellF1[1/2 + m/2 + n, m, 1 + 2*n, 3/2 +
m/2 + n, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(1 + m + 2*n) + (b*Appel
lF1[(1 + m + 3*n)/2, m, 1 + 3*n, (3 + m + 3*n)/2, Tan[(c + d*x)/2]^2, -Tan[
(c + d*x)/2]^2]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n)/(1 + m + 3*n)) + b*n
*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*((3*a*AppellF1[1/2 + m/2 + n, m, 1 +
2*n, 3/2 + m/2 + n, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(1 + m + 2*n
) + (b*AppellF1[(1 + m + 3*n)/2, m, 1 + 3*n, (3 + m + 3*n)/2, Tan[(c + d*x)
/2]^2, -Tan[(c + d*x)/2]^2]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n)/(1 + m +
3*n))*Tan[(c + d*x)/2] + (3*a^2*(-(((1 + n)*(1 + m + n)*AppellF1[1 + (1 +
m + n)/2, m, 2 + n, 1 + (3 + m + n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(3 + m + n)) + (m*(1 + m + n)*App
ellF1[1 + (1 + m + n)/2, 1 + m, 1 + n, 1 + (3 + m + n)/2, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(3 + m + n)))/
(1 + m + n) + b*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*((b*n*AppellF1[(1 + m
+ 3*n)/2, m, 1 + 3*n, (3 + m + 3*n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/
2]^2]*Cos[c + d*x]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^(-1 + n))/(1 + m + 3
*n) + (b*n*AppellF1[(1 + m + 3*n)/2, m, 1 + 3*n, (3 + m + 3*n)/2, Tan[(c +
d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*Tan[(
c + d*x)/2])/(1 + m + 3*n) + (3*a*(-(((1/2 + m/2 + n)*(1 + 2*n)*AppellF1[3/
2 + m/2 + n, m, 2 + 2*n, 5/2 + m/2 + n, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/
2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(3/2 + m/2 + n)) + (m*(1/2 + m/2

```
 + n)*AppellF1[3/2 + m/2 + n, 1 + m, 1 + 2*n, 5/2 + m/2 + n, Tan[(c + d*x)/
2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(3/2 + m/2
+ n)))/(1 + m + 2*n) + (b*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*(-(((1 + 3*
n)*(1 + m + 3*n)*AppellF1[1 + (1 + m + 3*n)/2, m, 2 + 3*n, 1 + (3 + m + 3*n
)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d
*x)/2])/(3 + m + 3*n)) + (m*(1 + m + 3*n)*AppellF1[1 + (1 + m + 3*n)/2, 1 +
 m, 1 + 3*n, 1 + (3 + m + 3*n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*
Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(3 + m + 3*n)))/(1 + m + 3*n))))*Tan[c
 + d*x]^m))
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \left(a+b\left(\sin\left(dx+c\right)\right)^{n}\right)^{3}\left(\tan\left(dx+c\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c)^n)^3*tan(d*x+c)^m,x)
```

```
[Out] int((a+b*sin(d*x+c)^n)^3*tan(d*x+c)^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(dx+c\right)^{n}+a\right)^{3}\tan\left(dx+c\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^3*tan(d*x+c)^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^n + a)^3*tan(d*x + c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b^{3}\sin\left(dx+c\right)^{3\,n}+3\,ab^{2}\sin\left(dx+c\right)^{2\,n}+3\,a^{2}b\sin\left(dx+c\right)^{n}+a^{3}\right)\tan\left(dx+c\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^3*tan(d*x+c)^m,x, algorithm="fricas")
```

```
[Out] integral((b^3*sin(d*x + c)^(3*n) + 3*a*b^2*sin(d*x + c)^(2*n) + 3*a^2*b*sin
(d*x + c)^n + a^3)*tan(d*x + c)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**n)**3*tan(d*x+c)**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(dx+c\right)^{n}+a\right)^{3}\tan\left(dx+c\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^3*tan(d*x+c)^m,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^n + a)^3*tan(d*x + c)^m, x)
```

3.575 $\int \left(a+b\sin^n(c+dx)\right)^2\tan^m(c+dx)\,dx$

Optimal. Leaf size=215

$$\frac{a^2\tan^{m+1}(c+dx)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\tan^2(c+dx)\right)}{d(m+1)}+\frac{2ab\cos^2(c+dx)^{\frac{m+1}{2}}\tan^{m+1}(c+dx)\sin^n(c+dx)\,{}_2F_1\left(\frac{m+1}{2},\frac{1}{2}(m+\right.}{d(m+n+1)}$$

```
[Out] (a^2*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d*x]^2]*Tan[c + d*
x]^(1 + m))/(d*(1 + m)) + (2*a*b*(Cos[c + d*x]^2)^((1 + m)/2)*Hypergeometri
c2F1[(1 + m)/2, (1 + m + n)/2, (3 + m + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^
n*Tan[c + d*x]^(1 + m))/(d*(1 + m + n)) + (b^2*(Cos[c + d*x]^2)^((1 + m)/2)
*Hypergeometric2F1[(1 + m)/2, (1 + m + 2*n)/2, (3 + m + 2*n)/2, Sin[c + d*x
]^2]*Sin[c + d*x]^(2*n)*Tan[c + d*x]^(1 + m))/(d*(1 + m + 2*n))
```

Rubi [A] time = 0.295359, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3234, 3476, 364, 2602, 2577}

$$\frac{a^2\tan^{m+1}(c+dx)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\tan^2(c+dx)\right)}{d(m+1)}+\frac{2ab\cos^2(c+dx)^{\frac{m+1}{2}}\tan^{m+1}(c+dx)\sin^n(c+dx)\,{}_2F_1\left(\frac{m+1}{2},\frac{1}{2}(m+\right.}{d(m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x]^n)^2*Tan[c + d*x]^m,x]
```

```
[Out] (a^2*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d*x]^2]*Tan[c + d*
x]^(1 + m))/(d*(1 + m)) + (2*a*b*(Cos[c + d*x]^2)^((1 + m)/2)*Hypergeometri
c2F1[(1 + m)/2, (1 + m + n)/2, (3 + m + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^
n*Tan[c + d*x]^(1 + m))/(d*(1 + m + n)) + (b^2*(Cos[c + d*x]^2)^((1 + m)/2)
*Hypergeometric2F1[(1 + m)/2, (1 + m + 2*n)/2, (3 + m + 2*n)/2, Sin[c + d*x
]^2]*Sin[c + d*x]^(2*n)*Tan[c + d*x]^(1 + m))/(d*(1 + m + 2*n))
```

Rule 3234

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*((d_.)*tan[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(d*tan[e + f*x])^m*(a
 + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \left(a+b\sin^n(c+dx)\right)^2\tan^m(c+dx)\,dx = \int \left(a^2\tan^m(c+dx)+2ab\sin^n(c+dx)\tan^m(c+dx)+b^2\sin^{2n}(c+dx)\tan$$
$$= a^2\int \tan^m(c+dx)\,dx+(2ab)\int \sin^n(c+dx)\tan^m(c+dx)\,dx+b^2\int \sin^2$$
$$= \frac{a^2\,\text{Subst}\left(\int \frac{x^m}{1+x^2}\,dx,x,\tan(c+dx)\right)}{d}+\left(2ab\cos^{1+m}(c+dx)\sin^{-1-m}(c+dx\right.$$
$$= \frac{a^2\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\tan^2(c+dx)\right)\tan^{1+m}(c+dx)}{d(1+m)}+\frac{2ab\cos^2(c+dx)^{\frac{1+m}{2}}}{}$$

Mathematica [C] time = 15.4473, size = 2368, normalized size = 11.01

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[c + d*x]^n)^2*Tan[c + d*x]^m,x]
```

```
[Out] (2*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^m*((a^2*AppellF1[(1 + m)/2, m, 1, (3 +
 m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(1 + m) + b*(Sec[(c + d*x)
/2]^2)^n*Sin[c + d*x]^n*((2*a*AppellF1[(1 + m + n)/2, m, 1 + n, (3 + m + n)
/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(1 + m + n) + (b*AppellF1[1/2
 + m/2 + n, m, 1 + 2*n, 3/2 + m/2 + n, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n)/(1 + m + 2*n)))*Tan[(c + d*x)/2
]*Tan[c + d*x]^m*(a^2*Tan[c + d*x]^m + 2*a*b*Sin[c + d*x]^n*Tan[c + d*x]^m
+ b^2*Sin[c + d*x]^(2*n)*Tan[c + d*x]^m))/(d*(2*m*(Cos[c + d*x]*Sec[(c + d*
x)/2]^2)^m*Sec[c + d*x]^2*((a^2*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, Tan[(c
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(1 + m) + b*(Sec[(c + d*x)/2]^2)^n*Sin[
c + d*x]^n*((2*a*AppellF1[(1 + m + n)/2, m, 1 + n, (3 + m + n)/2, Tan[(c +
d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(1 + m + n) + (b*AppellF1[1/2 + m/2 + n, m
, 1 + 2*n, 3/2 + m/2 + n, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Sec[(c
+ d*x)/2]^2)^n*Sin[c + d*x]^n)/(1 + m + 2*n)))*Tan[(c + d*x)/2]*Tan[c + d*x
]^(-1 + m) + Sec[(c + d*x)/2]^2*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^m*((a^2*A
ppellF1[(1 + m)/2, m, 1, (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2
])/(1 + m) + b*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*((2*a*AppellF1[(1 + m
+ n)/2, m, 1 + n, (3 + m + n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/
(1 + m + n) + (b*AppellF1[1/2 + m/2 + n, m, 1 + 2*n, 3/2 + m/2 + n, Tan[(c
+ d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n)/(1
 + m + 2*n)))*Tan[c + d*x]^m + 2*m*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(-1 +
m)*((a^2*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c +
 d*x)/2]^2])/(1 + m) + b*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*((2*a*Appell
```

```
F1[(1 + m + n)/2, m, 1 + n, (3 + m + n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*
x)/2]^2])/(1 + m + n) + (b*AppellF1[1/2 + m/2 + n, m, 1 + 2*n, 3/2 + m/2 +
n, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Sec[(c + d*x)/2]^2)^n*Sin[c +
d*x]^n)/(1 + m + 2*n)))*Tan[(c + d*x)/2]*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]
) + Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*Tan[c + d*x]^m + 2*(C
os[c + d*x]*Sec[(c + d*x)/2]^2)^m*Tan[(c + d*x)/2]*(b*n*Cos[c + d*x]*(Sec[(
c + d*x)/2]^2)^n*Sin[c + d*x]^(-1 + n)*((2*a*AppellF1[(1 + m + n)/2, m, 1 +
 n, (3 + m + n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(1 + m + n) +
(b*AppellF1[1/2 + m/2 + n, m, 1 + 2*n, 3/2 + m/2 + n, Tan[(c + d*x)/2]^2, -
Tan[(c + d*x)/2]^2]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n)/(1 + m + 2*n)) +
 b*n*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*((2*a*AppellF1[(1 + m + n)/2, m,
 1 + n, (3 + m + n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(1 + m + n
) + (b*AppellF1[1/2 + m/2 + n, m, 1 + 2*n, 3/2 + m/2 + n, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n)/(1 + m + 2*n
))*Tan[(c + d*x)/2] + (a^2*(-(((1 + m)*AppellF1[1 + (1 + m)/2, m, 2, 1 + (3
 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c
 + d*x)/2])/(3 + m)) + (m*(1 + m)*AppellF1[1 + (1 + m)/2, 1 + m, 1, 1 + (3
+ m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c
+ d*x)/2])/(3 + m)))/(1 + m) + b*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*((b*
n*AppellF1[1/2 + m/2 + n, m, 1 + 2*n, 3/2 + m/2 + n, Tan[(c + d*x)/2]^2, -T
an[(c + d*x)/2]^2]*Cos[c + d*x]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^(-1 + n
))/(1 + m + 2*n) + (b*n*AppellF1[1/2 + m/2 + n, m, 1 + 2*n, 3/2 + m/2 + n,
Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x
]^n*Tan[(c + d*x)/2])/(1 + m + 2*n) + (b*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x
]^n*(-(((1/2 + m/2 + n)*(1 + 2*n)*AppellF1[3/2 + m/2 + n, m, 2 + 2*n, 5/2 +
 m/2 + n, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(
c + d*x)/2])/(3/2 + m/2 + n)) + (m*(1/2 + m/2 + n)*AppellF1[3/2 + m/2 + n,
1 + m, 1 + 2*n, 5/2 + m/2 + n, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec
[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(3/2 + m/2 + n)))/(1 + m + 2*n) + (2*a*(-
(((1 + n)*(1 + m + n)*AppellF1[1 + (1 + m + n)/2, m, 2 + n, 1 + (3 + m + n)
/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*
x)/2])/(3 + m + n)) + (m*(1 + m + n)*AppellF1[1 + (1 + m + n)/2, 1 + m, 1 +
 n, 1 + (3 + m + n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*
x)/2]^2*Tan[(c + d*x)/2])/(3 + m + n)))/(1 + m + n)))*Tan[c + d*x]^m))
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \left(a + b\left(\sin\left(dx + c\right)\right)^{n}\right)^{2} \left(\tan\left(dx + c\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c)^n)^2*tan(d*x+c)^m,x)
```

```
[Out] int((a+b*sin(d*x+c)^n)^2*tan(d*x+c)^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(dx + c\right)^{n} + a\right)^{2} \tan\left(dx + c\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^2*tan(d*x+c)^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^n + a)^2*tan(d*x + c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2\sin\left(dx+c\right)^{2n}+2\,ab\sin\left(dx+c\right)^n+a^2\right)\tan\left(dx+c\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^2*tan(d*x+c)^m,x, algorithm="fricas")
```

```
[Out] integral((b^2*sin(d*x + c)^(2*n) + 2*a*b*sin(d*x + c)^n + a^2)*tan(d*x + c)
^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**n)**2*tan(d*x+c)**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sin\left(dx+c\right)^n+a\right)^2\tan\left(dx+c\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^2*tan(d*x+c)^m,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^n + a)^2*tan(d*x + c)^m, x)
```

3.576 $\int \left(a+b\sin^n(c+dx)\right)\tan^m(c+dx)\,dx$

Optimal. Leaf size=124

$$\frac{a\tan^{m+1}(c+dx)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\tan^2(c+dx)\right)}{d(m+1)}+\frac{b\cos^2(c+dx)^{\frac{m+1}{2}}\tan^{m+1}(c+dx)\sin^n(c+dx)\,{}_2F_1\left(\frac{m+1}{2},\frac{1}{2}(m+n+$$

```
[Out] (a*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d*x]^2]*Tan[c + d*x]
^(1 + m))/(d*(1 + m)) + (b*(Cos[c + d*x]^2)^((1 + m)/2)*Hypergeometric2F1[(
1 + m)/2, (1 + m + n)/2, (3 + m + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^n*Tan[
c + d*x]^(1 + m))/(d*(1 + m + n))
```

Rubi [A] time = 0.161395, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3234, 3476, 364, 2602, 2577}

$$\frac{a\tan^{m+1}(c+dx)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\tan^2(c+dx)\right)}{d(m+1)}+\frac{b\cos^2(c+dx)^{\frac{m+1}{2}}\tan^{m+1}(c+dx)\sin^n(c+dx)\,{}_2F_1\left(\frac{m+1}{2},\frac{1}{2}(m+n+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x]^n)*Tan[c + d*x]^m,x]
```

```
[Out] (a*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d*x]^2]*Tan[c + d*x]
^(1 + m))/(d*(1 + m)) + (b*(Cos[c + d*x]^2)^((1 + m)/2)*Hypergeometric2F1[(
1 + m)/2, (1 + m + n)/2, (3 + m + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^n*Tan[
c + d*x]^(1 + m))/(d*(1 + m + n))
```

Rule 3234

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*((d_.)*tan[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(d*tan[e + f*x])^m*(a
 + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \left(a+b\sin^n(c+dx)\right)\tan^m(c+dx)\,dx &= \int \left(a\tan^m(c+dx)+b\sin^n(c+dx)\tan^m(c+dx)\right)\,dx\\
&= a\int \tan^m(c+dx)\,dx+b\int \sin^n(c+dx)\tan^m(c+dx)\,dx\\
&= \frac{a\,\text{Subst}\left(\int \frac{x^m}{1+x^2}\,dx,x,\tan(c+dx)\right)}{d}+\left(b\cos^{1+m}(c+dx)\sin^{-1-m}(c+dx)\,\text{ta}\right.\\
&= \frac{a\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\tan^2(c+dx)\right)\tan^{1+m}(c+dx)}{d(1+m)}+\frac{b\cos^2(c+dx)^{\frac{1+m}{2}}\,{}_2F_1}{}
\end{aligned}
$$

Mathematica [C] time = 13.5209, size = 1395, normalized size = 11.25

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[c + d*x]^n)*Tan[c + d*x]^m,x]
```

```
[Out] (2*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^m*(a*(1 + m + n)*AppellF1[(1 + m)/2, m
, 1, (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + b*(1 + m)*Appell
F1[(1 + m + n)/2, m, 1 + n, (3 + m + n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*
x)/2]^2]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n)*Tan[(c + d*x)/2]*Tan[c + d*
x]^m*(a*Tan[c + d*x]^m + b*Sin[c + d*x]^n*Tan[c + d*x]^m))/(d*(1 + m)*(1 +
m + n)*((2*m*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^m*Sec[c + d*x]^2*(a*(1 + m +
 n)*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)
/2]^2] + b*(1 + m)*AppellF1[(1 + m + n)/2, m, 1 + n, (3 + m + n)/2, Tan[(c
+ d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n)*Ta
n[(c + d*x)/2]*Tan[c + d*x]^(-1 + m))/((1 + m)*(1 + m + n)) + (Sec[(c + d*x
)/2]^2*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^m*(a*(1 + m + n)*AppellF1[(1 + m)/
2, m, 1, (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + b*(1 + m)*Ap
pellF1[(1 + m + n)/2, m, 1 + n, (3 + m + n)/2, Tan[(c + d*x)/2]^2, -Tan[(c
+ d*x)/2]^2]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n)*Tan[c + d*x]^m)/((1 + m
)*(1 + m + n)) + (2*m*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(-1 + m)*(a*(1 + m
+ n)*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x
)/2]^2] + b*(1 + m)*AppellF1[(1 + m + n)/2, m, 1 + n, (3 + m + n)/2, Tan[(c
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n)*T
an[(c + d*x)/2]*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Cos[c + d*x]*Sec[(c +
 d*x)/2]^2*Tan[(c + d*x)/2])*Tan[c + d*x]^m)/((1 + m)*(1 + m + n)) + (2*(Co
s[c + d*x]*Sec[(c + d*x)/2]^2)^m*Tan[(c + d*x)/2]*(b*(1 + m)*n*AppellF1[(1
+ m + n)/2, m, 1 + n, (3 + m + n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^
2]*Cos[c + d*x]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^(-1 + n) + b*(1 + m)*n*
AppellF1[(1 + m + n)/2, m, 1 + n, (3 + m + n)/2, Tan[(c + d*x)/2]^2, -Tan[(
c + d*x)/2]^2]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*Tan[(c + d*x)/2] + a*(
1 + m + n)*(-(((1 + m)*AppellF1[1 + (1 + m)/2, m, 2, 1 + (3 + m)/2, Tan[(c
+ d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(3 +
 m)) + (m*(1 + m)*AppellF1[1 + (1 + m)/2, 1 + m, 1, 1 + (3 + m)/2, Tan[(c +
 d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(3 +
```

```
m)) + b*(1 + m)*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*(-(((1 + n)*(1 + m +
n)*AppellF1[1 + (1 + m + n)/2, m, 2 + n, 1 + (3 + m + n)/2, Tan[(c + d*x)/2
]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(3 + m + n))
 + (m*(1 + m + n)*AppellF1[1 + (1 + m + n)/2, 1 + m, 1 + n, 1 + (3 + m + n)
/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*
x)/2])/(3 + m + n)))*Tan[c + d*x]^m)/((1 + m)*(1 + m + n))))
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \left(a+b\left(\sin\left(dx+c\right)\right)^{n}\right)\left(\tan\left(dx+c\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c)^n)*tan(d*x+c)^m,x)
```

```
[Out] int((a+b*sin(d*x+c)^n)*tan(d*x+c)^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(dx+c\right)^{n}+a\right)\tan\left(dx+c\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)*tan(d*x+c)^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^n + a)*tan(d*x + c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sin\left(dx+c\right)^{n}+a\right)\tan\left(dx+c\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)*tan(d*x+c)^m,x, algorithm="fricas")
```

```
[Out] integral((b*sin(d*x + c)^n + a)*tan(d*x + c)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**n)*tan(d*x+c)**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(dx+c\right)^{n}+a\right)\tan\left(dx+c\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)*tan(d*x+c)^m,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^n + a)*tan(d*x + c)^m, x)
```

3.577 $\int \frac{\tan^m(c+dx)}{a+b\sin^n(c+dx)} \, dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{\tan^m(c+dx)}{a+b\sin^n(c+dx)},x\right)$$

```
[Out] Unintegrable[Tan[c + d*x]^m/(a + b*Sin[c + d*x]^n), x]
```

Rubi [A] time = 0.0577452, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tan^m(c+dx)}{a+b\sin^n(c+dx)} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Tan[c + d*x]^m/(a + b*Sin[c + d*x]^n),x]
```

```
[Out] Defer[Int][Tan[c + d*x]^m/(a + b*Sin[c + d*x]^n), x]
```

Rubi steps

$$\int \frac{\tan^m(c+dx)}{a+b\sin^n(c+dx)} \, dx = \int \frac{\tan^m(c+dx)}{a+b\sin^n(c+dx)} \, dx$$

Mathematica [A] time = 2.37861, size = 0, normalized size = 0.

$$\int \frac{\tan^m(c+dx)}{a+b\sin^n(c+dx)} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Tan[c + d*x]^m/(a + b*Sin[c + d*x]^n),x]
```

```
[Out] Integrate[Tan[c + d*x]^m/(a + b*Sin[c + d*x]^n), x]
```

Maple [A] time = 0.75, size = 0, normalized size = 0.

$$\int \frac{(\tan(dx+c))^m}{a+b(\sin(dx+c))^n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^m/(a+b*sin(d*x+c)^n),x)
```

```
[Out] int(tan(d*x+c)^m/(a+b*sin(d*x+c)^n),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(dx + c)^m}{b\sin(dx + c)^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m/(a+b*sin(d*x+c)^n),x, algorithm="maxima")
```

```
[Out] integrate(tan(d*x + c)^m/(b*sin(d*x + c)^n + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\tan(dx + c)^m}{b\sin(dx + c)^n + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m/(a+b*sin(d*x+c)^n),x, algorithm="fricas")
```

```
[Out] integral(tan(d*x + c)^m/(b*sin(d*x + c)^n + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^m(c + dx)}{a + b\sin^n(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**m/(a+b*sin(d*x+c)**n),x)
```

```
[Out] Integral(tan(c + d*x)**m/(a + b*sin(c + d*x)**n), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(dx + c)^m}{b\sin(dx + c)^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m/(a+b*sin(d*x+c)^n),x, algorithm="giac")
```

```
[Out] integrate(tan(d*x + c)^m/(b*sin(d*x + c)^n + a), x)
```

3.578 $\int \frac{\tan^m(c+dx)}{(a+b\sin^n(c+dx))^2}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{\tan^m(c+dx)}{(a+b\sin^n(c+dx))^2},x\right)$$

```
[Out] Unintegrable[Tan[c + d*x]^m/(a + b*Sin[c + d*x]^n)^2, x]
```

Rubi [A] time = 0.0580189, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tan^m(c+dx)}{(a+b\sin^n(c+dx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Tan[c + d*x]^m/(a + b*Sin[c + d*x]^n)^2,x]
```

```
[Out] Defer[Int][Tan[c + d*x]^m/(a + b*Sin[c + d*x]^n)^2, x]
```

Rubi steps

$$\int \frac{\tan^m(c+dx)}{(a+b\sin^n(c+dx))^2}\,dx = \int \frac{\tan^m(c+dx)}{(a+b\sin^n(c+dx))^2}\,dx$$

Mathematica [A] time = 23.4877, size = 0, normalized size = 0.

$$\int \frac{\tan^m(c+dx)}{(a+b\sin^n(c+dx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Tan[c + d*x]^m/(a + b*Sin[c + d*x]^n)^2,x]
```

```
[Out] Integrate[Tan[c + d*x]^m/(a + b*Sin[c + d*x]^n)^2, x]
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \frac{(\tan(dx+c))^m}{(a+b(\sin(dx+c))^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^m/(a+b*sin(d*x+c)^n)^2,x)
```

```
[Out] int(tan(d*x+c)^m/(a+b*sin(d*x+c)^n)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(dx+c)^m}{\left(b\sin(dx+c)^n+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m/(a+b*sin(d*x+c)^n)^2,x, algorithm="maxima")
```

```
[Out] integrate(tan(d*x + c)^m/(b*sin(d*x + c)^n + a)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\tan(dx+c)^m}{b^2\sin(dx+c)^{2n}+2ab\sin(dx+c)^n+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m/(a+b*sin(d*x+c)^n)^2,x, algorithm="fricas")
```

```
[Out] integral(tan(d*x + c)^m/(b^2*sin(d*x + c)^(2*n) + 2*a*b*sin(d*x + c)^n + a^
2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**m/(a+b*sin(d*x+c)**n)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(dx+c)^m}{\left(b\sin(dx+c)^n+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m/(a+b*sin(d*x+c)^n)^2,x, algorithm="giac")
```

```
[Out] integrate(tan(d*x + c)^m/(b*sin(d*x + c)^n + a)^2, x)
```

3.579 $\int \cot(x)\sqrt{a+b\sin^n(x)}\,dx$

Optimal. Leaf size=47

$$\frac{2\sqrt{a+b\sin^n(x)}}{n}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^n(x)}}{\sqrt{a}}\right)}{n}$$

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sin[x]^n]/Sqrt[a]])/n + (2*Sqrt[a + b*Sin[x]
^n])/n
```

Rubi [A] time = 0.0811218, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3230, 266, 50, 63, 208}

$$\frac{2\sqrt{a+b\sin^n(x)}}{n}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^n(x)}}{\sqrt{a}}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]*Sqrt[a + b*Sin[x]^n],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sin[x]^n]/Sqrt[a]])/n + (2*Sqrt[a + b*Sin[x]
^n])/n
```

Rule 3230

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff^(m + 1)/f, Subst[Int[(x^m*(a + b*(c*ff*x)^n)^p)/(1 - ff^2*x^2)^((m +
1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \cot(x)\sqrt{a+b\sin^n(x)}\,dx &= \text{Subst}\left(\int \frac{\sqrt{a+bx^n}}{x}\,dx, x, \sin(x)\right)\\ &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \sin^n(x)\right)}{n}\\ &= \frac{2\sqrt{a+b\sin^n(x)}}{n} + \frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sin^n(x)\right)}{n}\\ &= \frac{2\sqrt{a+b\sin^n(x)}}{n} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sin^n(x)}\right)}{bn}\\ &= -\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^n(x)}}{\sqrt{a}}\right)}{n} + \frac{2\sqrt{a+b\sin^n(x)}}{n}\end{aligned}$$

Mathematica [A] time = 0.0280716, size = 45, normalized size = 0.96

$$\frac{2\sqrt{a+b\sin^n(x)} - 2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^n(x)}}{\sqrt{a}}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]*Sqrt[a + b*Sin[x]^n],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sin[x]^n]/Sqrt[a]] + 2*Sqrt[a + b*Sin[x]^n])
/n
```

Maple [A] time = 0.124, size = 38, normalized size = 0.8

$$\frac{1}{n}\left(2\sqrt{a+b(\sin(x))^n} - 2\sqrt{a}\text{Artanh}\left(\frac{\sqrt{a+b(\sin(x))^n}}{\sqrt{a}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(a+b*sin(x)^n)^(1/2),x)
```

```
[Out] 1/n*(2*(a+b*sin(x)^n)^(1/2)-2*a^(1/2)*arctanh((a+b*sin(x)^n)^(1/2)/a^(1/2))
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*sin(x)^n)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.9486, size = 252, normalized size = 5.36

$$\left[\frac{\sqrt{a}\log\left(\frac{b\sin(x)^n-2\sqrt{b\sin(x)^n+a}\sqrt{a}+2a}{\sin(x)^n}\right)+2\sqrt{b\sin(x)^n+a}}{n}, \frac{2\left(\sqrt{-a}\arctan\left(\frac{\sqrt{b\sin(x)^n+a}\sqrt{-a}}{a}\right)+\sqrt{b\sin(x)^n+a}\right)}{n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*sin(x)^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(a)*log((b*sin(x)^n - 2*sqrt(b*sin(x)^n + a)*sqrt(a) + 2*a)/sin(x)^n)
 + 2*sqrt(b*sin(x)^n + a))/n, 2*(sqrt(-a)*arctan(sqrt(b*sin(x)^n + a)*sqrt(
-a)/a) + sqrt(b*sin(x)^n + a))/n]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a+b\sin^n(x)}\cot(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*sin(x)**n)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(x)**n)*cot(x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sin(x)^n+a}\cot(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*sin(x)^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(x)^n + a)*cot(x), x)
```

3.580 $\int \frac{\cot(x)}{\sqrt{a+b\sin^n(x)}}\,dx$

Optimal. Leaf size=29

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sin^n(x)}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sin[x]^n]/Sqrt[a]])/(Sqrt[a]*n)
```

Rubi [A] time = 0.0786557, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3230, 266, 63, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sin^n(x)}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/Sqrt[a + b*Sin[x]^n],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sin[x]^n]/Sqrt[a]])/(Sqrt[a]*n)
```

Rule 3230

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff^(m + 1)/f, Subst[Int[(x^m*(a + b*(c*ff*x)^n)^p)/(1 - ff^2*x^2)^((m +
1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cot(x)}{\sqrt{a+b\sin^n(x)}}\,dx = \text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^n}}\,dx, x, \sin(x)\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sin^n(x)\right)}{n}$$
$$= \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sin^n(x)}\right)}{bn}$$
$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sin^n(x)}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

Mathematica [A] time = 0.0170192, size = 29, normalized size = 1.

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sin^n(x)}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/Sqrt[a + b*Sin[x]^n],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sin[x]^n]/Sqrt[a]])/(Sqrt[a]*n)
```

Maple [A] time = 0.04, size = 24, normalized size = 0.8

$$-2\,\frac{1}{n\sqrt{a}}\text{Artanh}\left(\frac{\sqrt{a+b\left(\sin\left(x\right)\right)^{n}}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a+b*sin(x)^n)^(1/2),x)
```

```
[Out] -2*arctanh((a+b*sin(x)^n)^(1/2)/a^(1/2))/n/a^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*sin(x)^n)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.9321, size = 190, normalized size = 6.55

$$\left[\frac{\log\left(\frac{b\sin(x)^n-2\sqrt{b\sin(x)^n+a}\sqrt{a}+2a}{\sin(x)^n}\right)}{\sqrt{a}n}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{b\sin(x)^n+a}\sqrt{-a}}{a}\right)}{an}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*sin(x)^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((b*sin(x)^n - 2*sqrt(b*sin(x)^n + a)*sqrt(a) + 2*a)/sin(x)^n)/(sqrt(a)
*n), 2*sqrt(-a)*arctan(sqrt(b*sin(x)^n + a)*sqrt(-a)/a)/(a*n)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(x)}{\sqrt{a + b\sin^n(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*sin(x)**n)**(1/2),x)
```

```
[Out] Integral(cot(x)/sqrt(a + b*sin(x)**n), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(x)}{\sqrt{b\sin(x)^n + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*sin(x)^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(x)/sqrt(b*sin(x)^n + a), x)
```

3.581 $\int \left(a+b\sin^n(c+dx)\right)^p \tan^m(c+dx)\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\tan^m(c+dx)\left(a+b\sin^n(c+dx)\right)^p,x\right)$$

```
[Out] Unintegrable[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^m, x]
```

Rubi [A] time = 0.0521793, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(a+b\sin^n(c+dx)\right)^p \tan^m(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^m,x]
```

```
[Out] Defer[Int][(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^m, x]
```

Rubi steps

$$\int \left(a+b\sin^n(c+dx)\right)^p \tan^m(c+dx)\,dx=\int \left(a+b\sin^n(c+dx)\right)^p \tan^m(c+dx)\,dx$$

Mathematica [A] time = 3.8874, size = 0, normalized size = 0.

$$\int \left(a+b\sin^n(c+dx)\right)^p \tan^m(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^m,x]
```

```
[Out] Integrate[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^m, x]
```

Maple [A] time = 2.234, size = 0, normalized size = 0.

$$\int \left(a+b\left(\sin\left(dx+c\right)\right)^{n}\right)^{p}\left(\tan\left(dx+c\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^m,x)
```

```
[Out] int((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^m,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin(dx+c)^n+a\right)^p \tan(dx+c)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^n + a)^p*tan(d*x + c)^m, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sin(dx+c)^n+a\right)^p \tan(dx+c)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^m,x, algorithm="fricas")
```

```
[Out] integral((b*sin(d*x + c)^n + a)^p*tan(d*x + c)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**n)**p*tan(d*x+c)**m,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin(dx+c)^n+a\right)^p \tan(dx+c)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^m,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^n + a)^p*tan(d*x + c)^m, x)
```

3.582 $\int \left(a+b\sin^n(c+dx)\right)^p \tan^3(c+dx)\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\tan^3(c+dx)\left(a+b\sin^n(c+dx)\right)^p,x\right)$$

```
[Out] Unintegrable[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^3, x]
```

Rubi [A] time = 0.0511328, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(a+b\sin^n(c+dx)\right)^p \tan^3(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^3,x]
```

```
[Out] Defer[Int][(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^3, x]
```

Rubi steps

$$\int \left(a+b\sin^n(c+dx)\right)^p \tan^3(c+dx)\,dx=\int \left(a+b\sin^n(c+dx)\right)^p \tan^3(c+dx)\,dx$$

Mathematica [A] time = 2.76398, size = 0, normalized size = 0.

$$\int \left(a+b\sin^n(c+dx)\right)^p \tan^3(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^3,x]
```

```
[Out] Integrate[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^3, x]
```

Maple [A] time = 0.533, size = 0, normalized size = 0.

$$\int \left(a+b\left(\sin\left(dx+c\right)\right)^{n}\right)^{p}\left(\tan\left(dx+c\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^3,x)
```

```
[Out] int((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(dx+c\right)^n+a\right)^p\tan\left(dx+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^n + a)^p*tan(d*x + c)^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sin\left(dx+c\right)^n+a\right)^p\tan\left(dx+c\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral((b*sin(d*x + c)^n + a)^p*tan(d*x + c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**n)**p*tan(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(dx+c\right)^n+a\right)^p\tan\left(dx+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^n + a)^p*tan(d*x + c)^3, x)
```

3.583 $\int \left(a+b\sin^n(c+dx)\right)^p \tan(c+dx)\,dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\tan(c+dx)\left(a+b\sin^n(c+dx)\right)^p,x\right)$$

```
[Out] Unintegrable[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x], x]
```

Rubi [A] time = 0.026456, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(a+b\sin^n(c+dx)\right)^p \tan(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x],x]
```

```
[Out] Defer[Int][(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x], x]
```

Rubi steps

$$\int \left(a+b\sin^n(c+dx)\right)^p \tan(c+dx)\,dx=\int \left(a+b\sin^n(c+dx)\right)^p \tan(c+dx)\,dx$$

Mathematica [A] time = 2.48924, size = 0, normalized size = 0.

$$\int \left(a+b\sin^n(c+dx)\right)^p \tan(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x],x]
```

```
[Out] Integrate[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x], x]
```

Maple [A] time = 0.634, size = 0, normalized size = 0.

$$\int \left(a+b\left(\sin\left(dx+c\right)\right)^{n}\right)^{p}\tan\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c)^n)^p*tan(d*x+c),x)
```

```
[Out] int((a+b*sin(d*x+c)^n)^p*tan(d*x+c),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sin (dx+c)^n+a)^p \tan (dx+c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^n + a)^p*tan(d*x + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((b\sin (dx+c)^n+a)^p \tan (dx+c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((b*sin(d*x + c)^n + a)^p*tan(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**n)**p*tan(d*x+c),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sin (dx+c)^n+a)^p \tan (dx+c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^n + a)^p*tan(d*x + c), x)
```

3.584 $\int \cot(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx$

Optimal. Leaf size=55

$$-\frac{\left(a+b\sin^n(c+dx)\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\sin^n(c+dx)}{a}+1\right)}{adn(p+1)}$$

```
[Out] -((Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sin[c + d*x]^n)/a]*(a + b*Sin[
c + d*x]^n)^(1 + p))/(a*d*n*(1 + p)))
```

Rubi [A] time = 0.0765276, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3230, 266, 65}

$$-\frac{\left(a+b\sin^n(c+dx)\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\sin^n(c+dx)}{a}+1\right)}{adn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + b*Sin[c + d*x]^n)^p,x]
```

```
[Out] -((Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sin[c + d*x]^n)/a]*(a + b*Sin[
c + d*x]^n)^(1 + p))/(a*d*n*(1 + p)))
```

Rule 3230

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff^(m + 1)/f, Subst[Int[(x^m*(a + b*(c*ff*x)^n)^p)/(1 - ff^2*x^2)^((m +
1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \cot(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^n)^p}{x}\,dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, \sin^n(c+dx)\right)}{dn}$$
$$= -\frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b\sin^n(c+dx)}{a}\right)\left(a+b\sin^n(c+dx)\right)^{1+p}}{adn(1+p)}$$

Mathematica [A] time = 0.0575393, size = 55, normalized size = 1.

$$-\frac{\left(a+b\sin^n(c+dx)\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\sin^n(c+dx)}{a}+1\right)}{adn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + b*Sin[c + d*x]^n)^p,x]
```

```
[Out] -((Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sin[c + d*x]^n)/a]*(a + b*Sin[
c + d*x]^n)^(1 + p))/(a*d*n*(1 + p)))
```

Maple [F] time = 0.624, size = 0, normalized size = 0.

$$\int \cot(dx+c)\left(a+b(\sin(dx+c))^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+b*sin(d*x+c)^n)^p,x)
```

```
[Out] int(cot(d*x+c)*(a+b*sin(d*x+c)^n)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin(dx+c)^n+a\right)^p\cot(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^n + a)^p*cot(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sin(dx+c)^n+a\right)^p\cot(dx+c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)^n)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sin(d*x + c)^n + a)^p*cot(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)**n)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(dx+c\right)^{n}+a\right)^{p}\cot\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)^n)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^n + a)^p*cot(d*x + c), x)
```

3.585 $\int \cot^3(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx$

Optimal. Leaf size=136

$$\frac{\left(a+b\sin^n(c+dx)\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\sin^n(c+dx)}{a}+1\right)}{adn(p+1)}-\frac{\csc^2(c+dx)\left(a+b\sin^n(c+dx)\right)^p\left(\frac{b\sin^n(c+dx)}{a}+1\right)^{-p}\,{}_2F_1}{2d}$$

```
[Out] (Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sin[c + d*x]^n)/a]*(a + b*Sin[c
+ d*x]^n)^(1 + p))/(a*d*n*(1 + p)) - (Csc[c + d*x]^2*Hypergeometric2F1[-2/n
, -p, -((2 - n)/n), -((b*Sin[c + d*x]^n)/a)]*(a + b*Sin[c + d*x]^n)^p)/(2*d
*(1 + (b*Sin[c + d*x]^n)/a)^p)
```

Rubi [A] time = 0.151623, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3230, 1844, 365, 364, 266, 65}

$$\frac{\left(a+b\sin^n(c+dx)\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\sin^n(c+dx)}{a}+1\right)}{adn(p+1)}-\frac{\csc^2(c+dx)\left(a+b\sin^n(c+dx)\right)^p\left(\frac{b\sin^n(c+dx)}{a}+1\right)^{-p}\,{}_2F_1}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + b*Sin[c + d*x]^n)^p,x]
```

```
[Out] (Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sin[c + d*x]^n)/a]*(a + b*Sin[c
+ d*x]^n)^(1 + p))/(a*d*n*(1 + p)) - (Csc[c + d*x]^2*Hypergeometric2F1[-2/n
, -p, -((2 - n)/n), -((b*Sin[c + d*x]^n)/a)]*(a + b*Sin[c + d*x]^n)^p)/(2*d
*(1 + (b*Sin[c + d*x]^n)/a)^p)
```

Rule 3230

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff^(m + 1)/f, Subst[Int[(x^m*(a + b*(c*ff*x)^n)^p)/(1 - ff^2*x^2)^((m +
1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rule 1844

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n
, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !IGtQ[m, 0]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \cot^3(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)(a+bx^n)^p}{x^3}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{(a+bx^n)^p}{x^3} - \frac{(a+bx^n)^p}{x}\right)dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+bx^n)^p}{x^3}\,dx, x, \sin(c+dx)\right)}{d} - \frac{\text{Subst}\left(\int \frac{(a+bx^n)^p}{x}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, \sin^n(c+dx)\right)}{dn} + \frac{\left(\left(a+b\sin^n(c+dx)\right)^p\left(1+\frac{b\sin^n(c+dx)}{a}\right.\right.}{}$$

$$= \frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b\sin^n(c+dx)}{a}\right)\left(a+b\sin^n(c+dx)\right)^{1+p}}{adn(1+p)} - \frac{\csc^2(c+dx)\,{}_2F_1}{}$$

Mathematica [A] time = 1.0117, size = 129, normalized size = 0.95

$$\frac{\left(a+b\sin^n(c+dx)\right)^p\left(\frac{2\left(a+b\sin^n(c+dx)\right){}_2F_1\left(1,p+1;p+2;\frac{b\sin^n(c+dx)}{a}+1\right)}{an(p+1)} - \csc^2(c+dx)\left(\frac{b\sin^n(c+dx)}{a}+1\right)^{-p}{}_2F_1\left(-\frac{2}{n},-p;\frac{n-2}{n};-\frac{b\sin^n(c+}{a}\right.\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + b*Sin[c + d*x]^n)^p,x]
```

```
[Out] ((a + b*Sin[c + d*x]^n)^p*((2*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sin
[c + d*x]^n)/a]*(a + b*Sin[c + d*x]^n))/(a*n*(1 + p)) - (Csc[c + d*x]^2*Hyp
ergeometric2F1[-2/n, -p, (-2 + n)/n, -((b*Sin[c + d*x]^n)/a)])/(1 + (b*Sin[
c + d*x]^n)/a)^p))/(2*d)
```

Maple [F] time = 0.56, size = 0, normalized size = 0.

$$\int (\cot(dx+c))^3\left(a+b(\sin(dx+c))^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+b*sin(d*x+c)^n)^p,x)
```

```
[Out] int(cot(d*x+c)^3*(a+b*sin(d*x+c)^n)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sin \left(dx+c\right)^{n}+a\right)^{p} \cot \left(dx+c\right)^{3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c)^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^n + a)^p*cot(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b \sin \left(dx+c\right)^{n}+a\right)^{p} \cot \left(dx+c\right)^{3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c)^n)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sin(d*x + c)^n + a)^p*cot(d*x + c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+b*sin(d*x+c)**n)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sin \left(dx+c\right)^{n}+a\right)^{p} \cot \left(dx+c\right)^{3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c)^n)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^n + a)^p*cot(d*x + c)^3, x)
```

3.586 $\int (a+b\sin^n(c+dx))^p \tan^4(c+dx)\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\tan^4(c+dx)\left(a+b\sin^n(c+dx)\right)^p,x\right)$$

```
[Out] Unintegrable[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^4, x]
```

Rubi [A] time = 0.0488318, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(a+b\sin^n(c+dx)\right)^p \tan^4(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^4,x]
```

```
[Out] Defer[Int][(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^4, x]
```

Rubi steps

$$\int \left(a+b\sin^n(c+dx)\right)^p \tan^4(c+dx)\,dx = \int \left(a+b\sin^n(c+dx)\right)^p \tan^4(c+dx)\,dx$$

Mathematica [A] time = 2.77081, size = 0, normalized size = 0.

$$\int \left(a+b\sin^n(c+dx)\right)^p \tan^4(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^4,x]
```

```
[Out] Integrate[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^4, x]
```

Maple [A] time = 0.537, size = 0, normalized size = 0.

$$\int \left(a+b\left(\sin(dx+c)\right)^n\right)^p \left(\tan(dx+c)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^4,x)
```

```
[Out] int((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^4,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)^n + a)^p \tan(dx+c)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^n + a)^p*tan(d*x + c)^4, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sin(dx+c)^n + a)^p \tan(dx+c)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] integral((b*sin(d*x + c)^n + a)^p*tan(d*x + c)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**n)**p*tan(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(dx+c)^n + a)^p \tan(dx+c)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^n + a)^p*tan(d*x + c)^4, x)
```

3.587 $\int (a+b\sin^n(c+dx))^p \tan^2(c+dx)\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\tan^2(c+dx)\left(a+b\sin^n(c+dx)\right)^p,x\right)$$

```
[Out] Unintegrable[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^2, x]
```

Rubi [A] time = 0.0511503, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(a+b\sin^n(c+dx)\right)^p \tan^2(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^2,x]
```

```
[Out] Defer[Int][(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^2, x]
```

Rubi steps

$$\int \left(a+b\sin^n(c+dx)\right)^p \tan^2(c+dx)\,dx=\int \left(a+b\sin^n(c+dx)\right)^p \tan^2(c+dx)\,dx$$

Mathematica [A] time = 2.06964, size = 0, normalized size = 0.

$$\int \left(a+b\sin^n(c+dx)\right)^p \tan^2(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^2,x]
```

```
[Out] Integrate[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^2, x]
```

Maple [A] time = 0.519, size = 0, normalized size = 0.

$$\int \left(a+b\left(\sin(dx+c)\right)^n\right)^p \left(\tan(dx+c)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^2,x)
```

```
[Out] int((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(dx+c\right)^{n}+a\right)^{p}\tan\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^2,x, algorithm="maxima")

[Out] integrate((b*sin(d*x + c)^n + a)^p*tan(d*x + c)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sin\left(dx+c\right)^{n}+a\right)^{p}\tan\left(dx+c\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^2,x, algorithm="fricas")

[Out] integral((b*sin(d*x + c)^n + a)^p*tan(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**n)**p*tan(d*x+c)**2,x)

[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(dx+c\right)^{n}+a\right)^{p}\tan\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^2,x, algorithm="giac")

[Out] integrate((b*sin(d*x + c)^n + a)^p*tan(d*x + c)^2, x)
```

3.588 $\int \left(a+b\sin^n(c+dx)\right)^p \, dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\left(a+b\sin^n(c+dx)\right)^p,x\right)$$

```
[Out] Unintegrable[(a + b*Sin[c + d*x]^n)^p, x]
```

Rubi [A] time = 0.0132638, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(a+b\sin^n(c+dx)\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[c + d*x]^n)^p,x]
```

```
[Out] Defer[Int][(a + b*Sin[c + d*x]^n)^p, x]
```

Rubi steps

$$\int \left(a+b\sin^n(c+dx)\right)^p \, dx=\int \left(a+b\sin^n(c+dx)\right)^p \, dx$$

Mathematica [A] time = 1.22628, size = 0, normalized size = 0.

$$\int \left(a+b\sin^n(c+dx)\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*x]^n)^p,x]
```

```
[Out] Integrate[(a + b*Sin[c + d*x]^n)^p, x]
```

Maple [A] time = 0.461, size = 0, normalized size = 0.

$$\int \left(a+b\left(\sin\left(dx+c\right)\right)^{n}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c)^n)^p,x)
```

```
[Out] int((a+b*sin(d*x+c)^n)^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sin \left(dx+c\right)^{n}+a\right)^{p}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^n + a)^p, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \sin \left(dx+c\right)^{n}+a\right)^{p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sin(d*x + c)^n + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**n)**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sin \left(dx+c\right)^{n}+a\right)^{p}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^n + a)^p, x)
```

3.589 $\int \cot^2(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\cot^2(c+dx)\left(a+b\sin^n(c+dx)\right)^p,x\right)$$

```
[Out] Unintegrable[Cot[c + d*x]^2*(a + b*Sin[c + d*x]^n)^p, x]
```

Rubi [A] time = 0.0491673, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \cot^2(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cot[c + d*x]^2*(a + b*Sin[c + d*x]^n)^p,x]
```

```
[Out] Defer[Int][Cot[c + d*x]^2*(a + b*Sin[c + d*x]^n)^p, x]
```

Rubi steps

$$\int \cot^2(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx=\int \cot^2(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx$$

Mathematica [A] time = 2.00576, size = 0, normalized size = 0.

$$\int \cot^2(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Sin[c + d*x]^n)^p,x]
```

```
[Out] Integrate[Cot[c + d*x]^2*(a + b*Sin[c + d*x]^n)^p, x]
```

Maple [A] time = 0.53, size = 0, normalized size = 0.

$$\int (\cot(dx+c))^2\left(a+b(\sin(dx+c))^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+b*sin(d*x+c)^n)^p,x)
```

```
[Out] int(cot(d*x+c)^2*(a+b*sin(d*x+c)^n)^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sin (dx + c)^n + a\right)^p \cot (dx + c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c)^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^n + a)^p*cot(d*x + c)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \sin (dx + c)^n + a\right)^p \cot (dx + c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c)^n)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sin(d*x + c)^n + a)^p*cot(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+b*sin(d*x+c)**n)**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sin (dx + c)^n + a\right)^p \cot (dx + c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c)^n)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^n + a)^p*cot(d*x + c)^2, x)
```

3.590 $\int \cot^4(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\cot^4(c+dx)\left(a+b\sin^n(c+dx)\right)^p,x\right)$$

```
[Out] Unintegrable[Cot[c + d*x]^4*(a + b*Sin[c + d*x]^n)^p, x]
```

Rubi [A] time = 0.0501024, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \cot^4(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cot[c + d*x]^4*(a + b*Sin[c + d*x]^n)^p,x]
```

```
[Out] Defer[Int][Cot[c + d*x]^4*(a + b*Sin[c + d*x]^n)^p, x]
```

Rubi steps

$$\int \cot^4(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx=\int \cot^4(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx$$

Mathematica [A] time = 2.72788, size = 0, normalized size = 0.

$$\int \cot^4(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Sin[c + d*x]^n)^p,x]
```

```
[Out] Integrate[Cot[c + d*x]^4*(a + b*Sin[c + d*x]^n)^p, x]
```

Maple [A] time = 0.569, size = 0, normalized size = 0.

$$\int (\cot(dx+c))^4\left(a+b(\sin(dx+c))^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+b*sin(d*x+c)^n)^p,x)
```

```
[Out] int(cot(d*x+c)^4*(a+b*sin(d*x+c)^n)^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(dx+c\right)^{n}+a\right)^{p}\cot\left(dx+c\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c)^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^n + a)^p*cot(d*x + c)^4, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sin\left(dx+c\right)^{n}+a\right)^{p}\cot\left(dx+c\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c)^n)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sin(d*x + c)^n + a)^p*cot(d*x + c)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+b*sin(d*x+c)**n)**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(dx+c\right)^{n}+a\right)^{p}\cot\left(dx+c\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c)^n)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^n + a)^p*cot(d*x + c)^4, x)
```

3.591 $\int \frac{a+b\sin^2(e+fx)}{(g\cos(e+fx))^{5/2}\sqrt{d\sin(e+fx)}}\,dx$

Optimal. Leaf size=107

$$\frac{(2a-b)\sqrt{\sin(2e+2fx)}F\left(e+fx-\frac{\pi}{4}\middle|2\right)}{3fg^2\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}+\frac{2(a+b)\sqrt{d\sin(e+fx)}}{3dfg(g\cos(e+fx))^{3/2}}$$

```
[Out] (2*(a + b)*Sqrt[d*Sin[e + f*x]])/(3*d*f*g*(g*Cos[e + f*x])^(3/2)) + ((2*a -
 b)*EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(3*f*g^2*Sqrt[g*Co
s[e + f*x]]*Sqrt[d*Sin[e + f*x]])
```

Rubi [A] time = 0.181446, antiderivative size = 114, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {3202, 457, 329, 237, 335, 275, 232}

$$\frac{2(a+b)\sqrt{d\sin(e+fx)}}{3dfg(g\cos(e+fx))^{3/2}}-\frac{2(2a-b)\left(1-\csc^2(e+fx)\right)^{3/4}(d\sin(e+fx))^{3/2}F\left(\frac{1}{2}\csc^{-1}(\sin(e+fx))\middle|2\right)}{3d^2fg(g\cos(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x]^2)/((g*Cos[e + f*x])^(5/2)*Sqrt[d*Sin[e + f*x]]),x]
```

```
[Out] (2*(a + b)*Sqrt[d*Sin[e + f*x]])/(3*d*f*g*(g*Cos[e + f*x])^(3/2)) - (2*(2*a
 - b)*(1 - Csc[e + f*x]^2)^(3/4)*EllipticF[ArcCsc[Sin[e + f*x]]/2, 2]*(d*Si
n[e + f*x])^(3/2))/(3*d^2*f*g*(g*Cos[e + f*x])^(3/2))
```

Rule 3202

```
Int[(cos[(e_.) + (f_.)*(x_)]*(c_.))^(m_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Sin[e + f*x], x]}, Dist[(ff*c^(2*IntPart[(m - 1)/2] + 1)*(c*Co
s[e + f*x])^(2*FracPart[(m - 1)/2]))/(f*(Cos[e + f*x]^2)^FracPart[(m - 1)/2
]), Subst[Int[(d*ff*x)^n*(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x],
 x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] &&  !Int
egerQ[m]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{a+b\sin^2(e+fx)}{(g\cos(e+fx))^{5/2}\sqrt{d\sin(e+fx)}}\,dx = \frac{\cos^2(e+fx)^{3/4}\,\text{Subst}\left(\int \frac{a+bx^2}{\sqrt{dx}\left(1-x^2\right)^{7/4}}\,dx,x,\sin(e+fx)\right)}{fg(g\cos(e+fx))^{3/2}}$$

$$= \frac{2(a+b)\sqrt{d\sin(e+fx)}}{3dfg(g\cos(e+fx))^{3/2}} - \frac{\left((-2a+b)\cos^2(e+fx)^{3/4}\right)\text{Subst}\left(\int \frac{1}{\sqrt{dx}\left(1-x^2\right)^{3/4}}\,d\right.}{3fg(g\cos(e+fx))^{3/2}}$$

$$= \frac{2(a+b)\sqrt{d\sin(e+fx)}}{3dfg(g\cos(e+fx))^{3/2}} - \frac{\left(2(-2a+b)\cos^2(e+fx)^{3/4}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{x^4}{d^2}\right)^{3/4}}\,dx\right.}{3dfg(g\cos(e+fx))^{3/2}}$$

$$= \frac{2(a+b)\sqrt{d\sin(e+fx)}}{3dfg(g\cos(e+fx))^{3/2}} - \frac{\left(2(-2a+b)\left(1-\csc^2(e+fx)\right)^{3/4}(d\sin(e+fx))^{3/2}\right.}{3dfg(g\cos(e\cdot}$$

$$= \frac{2(a+b)\sqrt{d\sin(e+fx)}}{3dfg(g\cos(e+fx))^{3/2}} + \frac{\left(2(-2a+b)\left(1-\csc^2(e+fx)\right)^{3/4}(d\sin(e+fx))^{3/}\right.}{3dfg(g\cos(e+}$$

$$= \frac{2(a+b)\sqrt{d\sin(e+fx)}}{3dfg(g\cos(e+fx))^{3/2}} + \frac{\left((-2a+b)\left(1-\csc^2(e+fx)\right)^{3/4}(d\sin(e+fx))^{3/2}\right.}{3dfg(g\cos(e+fx}$$

$$= \frac{2(a+b)\sqrt{d\sin(e+fx)}}{3dfg(g\cos(e+fx))^{3/2}} - \frac{2(2a-b)\left(1-\csc^2(e+fx)\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}(\csc(e+\right.}{3d^2fg(g\cos(e+fx))^{3/2}}$$

Mathematica [C] time = 0.185173, size = 102, normalized size = 0.95

$$\frac{2\cos^2(e+fx)^{3/4}\left(5a\sin(e+fx)\,{}_2F_1\left(\frac{1}{4},\frac{7}{4};\frac{5}{4};\sin^2(e+fx)\right)+b\sin^3(e+fx)\,{}_2F_1\left(\frac{5}{4},\frac{7}{4};\frac{9}{4};\sin^2(e+fx)\right)\right)}{5fg\sqrt{d\sin(e+fx)}(g\cos(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)/((g*Cos[e + f*x])^(5/2)*Sqrt[d*Sin[e + f*x
]]),x]
```

```
[Out] (2*(Cos[e + f*x]^2)^(3/4)*(5*a*Hypergeometric2F1[1/4, 7/4, 5/4, Sin[e + f*x
]^2]*Sin[e + f*x] + b*Hypergeometric2F1[5/4, 7/4, 9/4, Sin[e + f*x]^2]*Sin[
e + f*x]^3))/(5*f*g*(g*Cos[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]])
```

Maple [B] time = 0.415, size = 327, normalized size = 3.1

$$\frac{\sqrt{2}\cos\left(fx+e\right)\sin\left(fx+e\right)}{3f\left(-1+\cos\left(fx+e\right)\right)}\left(-2\,\mathrm{EllipticF}\left(\sqrt{-\frac{-1+\cos\left(fx+e\right)-\sin\left(fx+e\right)}{\sin\left(fx+e\right)}},1/2\sqrt{2}\right)\cos\left(fx+e\right)\sin\left(fx+e\right)\sqrt{-\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)/(g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/3/f*2^(1/2)*(-2*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),
1/2*2^(1/2))*cos(f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))
^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f
*x+e))^(1/2)*a+EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2
*2^(1/2))*cos(f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1
/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+
e))^(1/2)*b+2^(1/2)*cos(f*x+e)*a+2^(1/2)*cos(f*x+e)*b-2^(1/2)*a-2^(1/2)*b)*
cos(f*x+e)*sin(f*x+e)/(-1+cos(f*x+e))/(g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(
1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\sin\left(fx+e\right)^{2}+a}{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)/(g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)/((g*cos(f*x + e))^(5/2)*sqrt(d*sin(f*x + e
))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(b\cos\left(fx+e\right)^{2}-a-b\right)\sqrt{g\cos\left(fx+e\right)}\sqrt{d\sin\left(fx+e\right)}}{dg^{3}\cos\left(fx+e\right)^{3}\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)/(g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] integral(-(b*cos(f*x + e)^2 - a - b)*sqrt(g*cos(f*x + e))*sqrt(d*sin(f*x +
e))/(d*g^3*cos(f*x + e)^3*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)/(g*cos(f*x+e))**(5/2)/(d*sin(f*x+e))**(1/2),x
)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\sin\left(fx+e\right)^{2}+a}{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)/(g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)/((g*cos(f*x + e))^(5/2)*sqrt(d*sin(f*x + e
))), x)
```

3.592 $\int (c\cos(e+fx))^m (d\sin(e+fx))^n \left(a+b\sin^2(e+fx)\right)^p \, dx$

Optimal. Leaf size=137

$$\frac{c\cos^2(e+fx)^{\frac{1-m}{2}}(c\cos(e+fx))^{m-1}(d\sin(e+fx))^{n+1}\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{n+1}{2};\frac{1-m}{2},-p;\frac{n+3}{2};\sin^2(\right.}{df(n+1)}$$

```
[Out] (c*AppellF1[(1 + n)/2, (1 - m)/2, -p, (3 + n)/2, Sin[e + f*x]^2, -((b*Sin[e
 + f*x]^2)/a)]*(c*Cos[e + f*x])^(-1 + m)*(Cos[e + f*x]^2)^((1 - m)/2)*(d*Si
n[e + f*x])^(1 + n)*(a + b*Sin[e + f*x]^2)^p)/(d*f*(1 + n)*(1 + (b*Sin[e +
f*x]^2)/a)^p)
```

Rubi [A] time = 0.210649, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {3202, 511, 510}

$$\frac{c\cos^2(e+fx)^{\frac{1-m}{2}}(c\cos(e+fx))^{m-1}(d\sin(e+fx))^{n+1}\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{n+1}{2};\frac{1-m}{2},-p;\frac{n+3}{2};\sin^2(\right.}{df(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cos[e + f*x])^m*(d*Sin[e + f*x])^n*(a + b*Sin[e + f*x]^2)^p,x]
```


```
[Out] (c*AppellF1[(1 + n)/2, (1 - m)/2, -p, (3 + n)/2, Sin[e + f*x]^2, -((b*Sin[e
 + f*x]^2)/a)]*(c*Cos[e + f*x])^(-1 + m)*(Cos[e + f*x]^2)^((1 - m)/2)*(d*Si
n[e + f*x])^(1 + n)*(a + b*Sin[e + f*x]^2)^p)/(d*f*(1 + n)*(1 + (b*Sin[e +
f*x]^2)/a)^p)
```


Rule 3202

```
Int[(cos[(e_.) + (f_.)*(x_)]*(c_.))^(m_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Sin[e + f*x], x]}, Dist[(ff*c^(2*IntPart[(m - 1)/2] + 1)*(c*Co
s[e + f*x])^(2*FracPart[(m - 1)/2]))/(f*(Cos[e + f*x]^2)^FracPart[(m - 1)/2
]), Subst[Int[(d*ff*x)^n*(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x],
 x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] &&  !Int
egerQ[m]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int (c\cos(e+fx))^m(d\sin(e+fx))^n\left(a+b\sin^2(e+fx)\right)^p\,dx = \frac{\left(c(c\cos(e+fx))^{2\left(-\frac{1}{2}+\frac{m}{2}\right)}\cos^2(e+fx)^{\frac{1}{2}-\frac{m}{2}}\right)\text{Subst}\left(\int (d\right.}{f}$$

$$= \frac{\left(c(c\cos(e+fx))^{2\left(-\frac{1}{2}+\frac{m}{2}\right)}\cos^2(e+fx)^{\frac{1}{2}-\frac{m}{2}}\left(a+b\sin^2(e\right.\right.}{}$$

$$= \frac{cF_1\left(\frac{1+n}{2};\frac{1-m}{2},-p;\frac{3+n}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)(c\cos}{}$$

Mathematica [A] time = 0.779783, size = 135, normalized size = 0.99

$$\frac{\tan(e+fx)\cos^2(e+fx)^{\frac{1-m}{2}}(c\cos(e+fx))^m(d\sin(e+fx))^n\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{n+1}{2};\frac{1-m}{2},-p;\right.}{f(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cos[e + f*x])^m*(d*Sin[e + f*x])^n*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[(1 + n)/2, (1 - m)/2, -p, (3 + n)/2, Sin[e + f*x]^2, -((b*Sin[e +
 f*x]^2)/a)]*(c*Cos[e + f*x])^m*(Cos[e + f*x]^2)^((1 - m)/2)*(d*Sin[e + f*x
])^n*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x])/(f*(1 + n)*(1 + (b*Sin[e + f*x]
^2)/a)^p)
```

Maple [F] time = 3.26, size = 0, normalized size = 0.

$$\int \left(c\cos\left(fx+e\right)\right)^m\left(d\sin\left(fx+e\right)\right)^n\left(a+b\left(\sin\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(f*x+e))^m*(d*sin(f*x+e))^n*(a+b*sin(f*x+e)^2)^p,x)
```

```
[Out] int((c*cos(f*x+e))^m*(d*sin(f*x+e))^n*(a+b*sin(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p\left(c\cos\left(fx+e\right)\right)^m\left(d\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(d*sin(f*x+e))^n*(a+b*sin(f*x+e)^2)^p,x, algorit
hm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*(c*cos(f*x + e))^m*(d*sin(f*x + e))^n, x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-b\cos\left(fx+e\right)^{2}+a+b\right)^{p}\left(c\cos\left(fx+e\right)\right)^{m}\left(d\sin\left(fx+e\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(d*sin(f*x+e))^n*(a+b*sin(f*x+e)^2)^p,x, algorit
hm="fricas")
```

```
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*(c*cos(f*x + e))^m*(d*sin(f*x + e))^
n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))**m*(d*sin(f*x+e))**n*(a+b*sin(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sin\left(fx+e\right)^{2}+a\right)^{p}\left(c\cos\left(fx+e\right)\right)^{m}\left(d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(d*sin(f*x+e))^n*(a+b*sin(f*x+e)^2)^p,x, algorit
hm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*(c*cos(f*x + e))^m*(d*sin(f*x + e))^n, x
)
```

3.593 $\int \sqrt{a+(c\cos(e+fx)+b\sin(e+fx))^2}\,dx$

Optimal. Leaf size=79

$$\frac{\sqrt{a+(b\sin(e+fx)+c\cos(e+fx))^2}E\left(e+fx+\tan^{-1}(b,c)\middle|-\frac{b^2+c^2}{a}\right)}{f\sqrt{\frac{(b\sin(e+fx)+c\cos(e+fx))^2}{a}+1}}$$

```
[Out] (EllipticE[e + f*x + ArcTan[b, c], -((b^2 + c^2)/a)]*Sqrt[a + (c*Cos[e + f*
x] + b*Sin[e + f*x])^2])/(f*Sqrt[1 + (c*Cos[e + f*x] + b*Sin[e + f*x])^2/a]
)
```

Rubi [F] time = 0.698582, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \sqrt{a+(c\cos(e+fx)+b\sin(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[a + (c*Cos[e + f*x] + b*Sin[e + f*x])^2],x]
```

```
[Out] ((I/2)*Defer[Subst][Defer[Int][Sqrt[a + (c + b*x)^2/(1 + x^2)]/(I - x), x],
 x, Tan[e + f*x]])/f + ((I/2)*Defer[Subst][Defer[Int][Sqrt[a + (c + b*x)^2/
(1 + x^2)]/(I + x), x], x, Tan[e + f*x]])/f
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+(c\cos(e+fx)+b\sin(e+fx))^2}\,dx &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+\frac{(c+bx)^2}{1+x^2}}}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left(\frac{i\sqrt{a+\frac{(c+bx)^2}{1+x^2}}}{2(i-x)}+\frac{i\sqrt{a+\frac{(c+bx)^2}{1+x^2}}}{2(i+x)}\right)dx,x,\tan(e+fx)\right)}{f}\\ &= \frac{i\,\text{Subst}\left(\int \frac{\sqrt{a+\frac{(c+bx)^2}{1+x^2}}}{i-x}\,dx,x,\tan(e+fx)\right)}{2f}+\frac{i\,\text{Subst}\left(\int \frac{\sqrt{a+\frac{(c+bx)^2}{1+x^2}}}{i+x}\,dx,x,\text{t}\right.}{2f}\end{aligned}$$

Mathematica [B] time = 1.63885, size = 325, normalized size = 4.11

$$-\frac{\left(\left(b^2-c^2\right)\sin(2(e+fx))+2bc\cos(2(e+fx))\right)\sqrt{2a+\left(c^2-b^2\right)\cos(2(e+fx))+b^2+2bc\sin(2(e+fx))+c^2}E\left(\sin^{-}\right.}{\sqrt{2}f\sqrt{\left(b^2+c^2\right)^2}\sqrt{\frac{\left(\left(b^2-c^2\right)\sin(2(e+fx))+2bc\cos(2(e+fx))\right)^2}{\left(b^2+c^2\right)^2}}\sqrt{\frac{2a+\left(c^2-b^2\right)\cos(2(e+fx))+}{2a+\sqrt{\left(b^2+c^2\right.}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + (c*Cos[e + f*x] + b*Sin[e + f*x])^2],x]
```

```
[Out] -((EllipticE[ArcSin[Sqrt[(Sqrt[(b^2 + c^2)^2] + (b^2 - c^2)*Cos[2*(e + f*x)
] - 2*b*c*Sin[2*(e + f*x)])/Sqrt[(b^2 + c^2)^2]]/Sqrt[2]], (2*Sqrt[(b^2 + c
^2)^2])/(2*a + b^2 + c^2 + Sqrt[(b^2 + c^2)^2])]*Sqrt[2*a + b^2 + c^2 + (-b
^2 + c^2)*Cos[2*(e + f*x)] + 2*b*c*Sin[2*(e + f*x)]]*(2*b*c*Cos[2*(e + f*x)
] + (b^2 - c^2)*Sin[2*(e + f*x)]))/(Sqrt[2]*Sqrt[(b^2 + c^2)^2]*f*Sqrt[(2*a
 + b^2 + c^2 + (-b^2 + c^2)*Cos[2*(e + f*x)] + 2*b*c*Sin[2*(e + f*x)])/(2*a
 + b^2 + c^2 + Sqrt[(b^2 + c^2)^2])]*Sqrt[(2*b*c*Cos[2*(e + f*x)] + (b^2 -
c^2)*Sin[2*(e + f*x)])^2/(b^2 + c^2)^2]))
```

Maple [B] time = 1.51, size = 4061599, normalized size = 51412.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+(c*cos(f*x+e)+b*sin(f*x+e))^2)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\left(c\cos\left(fx+e\right)+b\sin\left(fx+e\right)\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+(c*cos(f*x+e)+b*sin(f*x+e))^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((c*cos(f*x + e) + b*sin(f*x + e))^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{2\,bc\cos\left(fx+e\right)\sin\left(fx+e\right)-\left(b^2-c^2\right)\cos\left(fx+e\right)^2+b^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+(c*cos(f*x+e)+b*sin(f*x+e))^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*b*c*cos(f*x + e)*sin(f*x + e) - (b^2 - c^2)*cos(f*x + e)^2
+ b^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+\left(b\sin\left(e+fx\right)+c\cos\left(e+fx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+(c*cos(f*x+e)+b*sin(f*x+e))**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + (b*sin(e + f*x) + c*cos(e + f*x))**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\left(c\cos\left(fx+e\right)+b\sin\left(fx+e\right)\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+(c*cos(f*x+e)+b*sin(f*x+e))^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((c*cos(f*x + e) + b*sin(f*x + e))^2 + a), x)
```

3.594 $\int \frac{1}{\sqrt{a+(c\cos(e+fx)+b\sin(e+fx))^2}}\,dx$

Optimal. Leaf size=79

$$\frac{\sqrt{\frac{(b\sin(e+fx)+c\cos(e+fx))^2}{a}+1}F\left(e+fx+\tan^{-1}(b,c)|-\frac{b^2+c^2}{a}\right)}{f\sqrt{a+(b\sin(e+fx)+c\cos(e+fx))^2}}$$

```
[Out] (EllipticF[e + f*x + ArcTan[b, c], -((b^2 + c^2)/a)]*Sqrt[1 + (c*Cos[e + f*
x] + b*Sin[e + f*x])^2/a])/(f*Sqrt[a + (c*Cos[e + f*x] + b*Sin[e + f*x])^2]
)
```

Rubi [F] time = 0.697165, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\sqrt{a+(c\cos(e+fx)+b\sin(e+fx))^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/Sqrt[a + (c*Cos[e + f*x] + b*Sin[e + f*x])^2],x]
```

```
[Out] ((I/2)*Defer[Subst][Defer[Int][1/((I - x)*Sqrt[a + (c + b*x)^2/(1 + x^2)]),
 x], x, Tan[e + f*x]])/f + ((I/2)*Defer[Subst][Defer[Int][1/((I + x)*Sqrt[a
 + (c + b*x)^2/(1 + x^2)]), x], x, Tan[e + f*x]])/f
```

Rubi steps

$$\int \frac{1}{\sqrt{a+(c\cos(e+fx)+b\sin(e+fx))^2}}\,dx=\frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+\frac{(c+bx)^2}{1+x^2}}}\,dx,x,\tan(e+fx)\right)}{f}$$

$$=\frac{\text{Subst}\left(\int \left(\frac{i}{2(i-x)\sqrt{a+\frac{(c+bx)^2}{1+x^2}}}+\frac{i}{2(i+x)\sqrt{a+\frac{(c+bx)^2}{1+x^2}}}\right)dx,x,\tan(e+fx)\right)}{f}$$

$$=\frac{i\,\text{Subst}\left(\int \frac{1}{(i-x)\sqrt{a+\frac{(c+bx)^2}{1+x^2}}}\,dx,x,\tan(e+fx)\right)}{2f}+\frac{i\,\text{Subst}\left(\int \frac{1}{(i+x)\sqrt{a+\frac{(c+bx)^2}{1+x^2}}}\,d\right.}{2f}$$

Mathematica [C] time = 1.6031, size = 529, normalized size = 6.7

$$\sqrt{2}\sec\left(\tan^{-1}\left(\frac{c^2-b^2}{2bc}\right)+2(e+fx)\right)\sqrt{-\frac{bc\sqrt{\frac{(b^2+c^2)^2}{b^2c^2}}\left(\sin\left(\tan^{-1}\left(\frac{c^2-b^2}{2bc}\right)+2(e+fx)\right)-1\right)}{2a+bc\sqrt{\frac{(b^2+c^2)^2}{b^2c^2}}+b^2+c^2}}\sqrt{-\frac{bc\sqrt{\frac{(b^2+c^2)^2}{b^2c^2}}\left(\sin\left(\tan^{-1}\left(\frac{c^2-b^2}{2bc}\right)+2(e+fx)\right)+1\right)}{2a-bc\sqrt{\frac{(b^2+c^2)^2}{b^2c^2}}+b^2+c^2}}\sqrt{2a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[a + (c*Cos[e + f*x] + b*Sin[e + f*x])^2],x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2, 1/2, 1/2, 3/2, (2*a + b^2 + c^2 + b*c*Sqrt[(b^2 + c^
2)^2/(b^2*c^2)]*Sin[2*(e + f*x) + ArcTan[(-b^2 + c^2)/(2*b*c)]])/(2*a + b^2
 + c^2 - b*c*Sqrt[(b^2 + c^2)^2/(b^2*c^2)]), (2*a + b^2 + c^2 + b*c*Sqrt[(b
^2 + c^2)^2/(b^2*c^2)]*Sin[2*(e + f*x) + ArcTan[(-b^2 + c^2)/(2*b*c)]])/(2*
a + b^2 + c^2 + b*c*Sqrt[(b^2 + c^2)^2/(b^2*c^2)])]*Sec[2*(e + f*x) + ArcTa
n[(-b^2 + c^2)/(2*b*c)]]*Sqrt[-((b*c*Sqrt[(b^2 + c^2)^2/(b^2*c^2)]*(-1 + Si
n[2*(e + f*x) + ArcTan[(-b^2 + c^2)/(2*b*c)]]))/(2*a + b^2 + c^2 + b*c*Sqrt
[(b^2 + c^2)^2/(b^2*c^2)]))]*Sqrt[-((b*c*Sqrt[(b^2 + c^2)^2/(b^2*c^2)]*(1 +
 Sin[2*(e + f*x) + ArcTan[(-b^2 + c^2)/(2*b*c)]]))/(2*a + b^2 + c^2 - b*c*S
qrt[(b^2 + c^2)^2/(b^2*c^2)]))]*Sqrt[2*a + b^2 + c^2 + b*c*Sqrt[(b^2 + c^2)
^2/(b^2*c^2)]*Sin[2*(e + f*x) + ArcTan[(-b^2 + c^2)/(2*b*c)]]])/(b*c*Sqrt[(
b^2 + c^2)^2/(b^2*c^2)]*f)
```

Maple [C] time = 0.871, size = 257865, normalized size = 3264.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+(c*cos(f*x+e)+b*sin(f*x+e))^2)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\left(c\cos\left(fx+e\right)+b\sin\left(fx+e\right)\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+(c*cos(f*x+e)+b*sin(f*x+e))^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((c*cos(f*x + e) + b*sin(f*x + e))^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2\,bc\cos\left(fx+e\right)\sin\left(fx+e\right)-\left(b^2-c^2\right)\cos\left(fx+e\right)^2+b^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+(c*cos(f*x+e)+b*sin(f*x+e))^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*b*c*cos(f*x + e)*sin(f*x + e) - (b^2 - c^2)*cos(f*x + e)^
2 + b^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + \left(b \sin\left(e + fx\right) + c \cos\left(e + fx\right)\right)^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+(c*cos(f*x+e)+b*sin(f*x+e))**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + (b*sin(e + f*x) + c*cos(e + f*x))**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\left(c \cos\left(fx + e\right) + b \sin\left(fx + e\right)\right)^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+(c*cos(f*x+e)+b*sin(f*x+e))^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt((c*cos(f*x + e) + b*sin(f*x + e))^2 + a), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
                print("both result and optimal complex");
             fi;
             #both result and optimal complex
             if leaf_count_result<=2*leaf_count_optimal then
                return "A";
             else
                return "B";
             end if
          else #result contains complex but optimal is not
             if debug then
                print("result contains complex but optimal is not");
             fi;
             return "C";
          end if
       else # result do not contain complex
            # this assumes optimal do not as well
            if debug then
                print("result do not contain complex, this assumes optimal do
    not as well");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
               if debug then
                  print("leaf_count_result<=2*leaf_count_optimal");
               fi;
               return "A";
            else
               if debug then
                  print("leaf_count_result>2*leaf_count_optimal");
               fi;
               return "B";
            end if
       end if
  else #ExpnType(result) > ExpnType(optimal)
       if debug then
          print("ExpnType(result) > ExpnType(optimal)");
       fi;
       return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len
   (flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                          #since this estimate of leaf count is bit lower than
```

```
                      #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): # is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```